Capital Allocation by Percentile Layer

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Abstract

Motivation. Capital allocation can have substantial ramifications upon measuring risk adjusted profitability as well as setting risk loads for pricing. Current allocation methods that emphasize the tail allocate too much capital to extreme events; "capital consumption" methods, which incorporate relative likelihood, tend to allocate insufficient capital to highly unlikely yet extremely severe losses.

Method. In this paper I develop a new formulation of the meaning of holding capital equal to the Value at Risk. The new formulation views the total capital of the firm as the sum of many percentile layers of capital. Thus capital allocation varies continuously by layer and the capital allocated to any particular loss scenario is the sum of allocated capital across many percentile layers.

Results. Capital allocation by percentile layer produces capital allocations that differ significantly from other common methods such as VaR, TVaR, and coTVaR.

Conclusions. Capital allocation by percentile layer has important advantages over existing methods. It highlights a new formulation of Value at Risk and other capital standards, recognizes the capital usage of losses that do not extend into the tail, and captures the disproportionate capital usage of severe losses.

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Keywords. Capital Allocation; Percentile Layer of Capital; Value at Risk; Enterprise Risk Management; Risk Load; Risk Adjusted Profitability

1. REQUIRED CAPITAL, REQUIRED RATE OF RETURN, AND CAPITAL ALLOCATION

How much capital should an insurance firm hold? And what rate of return must the firm achieve on this capital? While these questions are of critical importance to the firm, external forces in the operating environment often dictate the answers. For example, regulators and rating agencies greatly influence the amount of capital the firm must hold; in addition, investors influence both the amount of capital the firm holds and the required rate of return on this capital. Therefore, the issues of the amount of capital and the required rate of return on capital are often ultimately beyond the decision making power of the company; rather, they are demands that the operating environment imposes upon the firm.

Given that a firm must hold a certain amount of capital, the firm essentially incurs a firm-wide "overhead" cost related to the required rate of return on this capital. Management often desires to allocate this cost, like other overhead costs, to subsets of the firm such as subsidiaries, business units, and product lines. How should the firm allocate the cost of required return on capital? This is the question of "capital allocation".

1.1 Why is Capital Allocation Important?

How a firm allocates capital, similar to other cost allocation decisions, can significantly affect the measured profitability of a particular line of business. Moreover, allocating capital can affect target pricing margins and the volume of business the company writes in each line of business and product type. As a result, the topic is critically important and often the subject of contentious debate among the heads of the firm's various business units.

1.2 Defining the Scope of the Problem

We will restrict our discussion to the situation of a publicly traded insurance company that writes property catastrophe business, both insurance and reinsurance, covering several perils around the world; we will exclude long tail casualty business in an attempt to simplify our discussion to a single year time horizon problem. We will assume that investors require that the firm holds capital based upon the Value at Risk (VaR) at the 99th percentile and that the required return can be expressed as an annual percentage rate of return on this amount of capital. The issue we grapple with here relates only to allocation.

1.3 Allocating Capital to Users of Capital

Mango [4] has stressed that the entire capital of the firm is available to pay the claim of any single policy. Thus, the required rate of return on capital is a cost that accrues on the total firm level, and Kreps [1] has clarified that capital allocation is really the allocation of the required rate of return on capital. Mango [3] also has highlighted the connection between allocating capital and broader issues of cost allocation. Therefore, similar to other cost allocation situations, we want to connect the firm-wide cost of capital to those subsets of the firm which require the company to incur this cost: essentially, to match the expenditure to its source. Namely, we desire to allocate the cost of capital to those business units, products, perils, reinsurance contracts, and individual insurance policies that contribute to the loss scenarios that "use" capital.

1.4 So Who "Uses" Capital? Investigating Value at Risk (VaR) and Tail Value at Risk (TVaR)

In our situation, the company must hold capital based upon Value at Risk (VaR) at the 99th percentile. The traditional view of this requirement is that the firm is holding capital in order to pay for a catastrophically bad scenario (the 99th percentile loss), but is not concerned with other loss scenarios that are either greater than or less than this VaR (99%) scenario. Thus Kreps [1] and

Venter [5] describe (and critique) the VaR approach as allocating capital only to those components that contribute to one particular loss scenario (e.g. the 99th percentile loss) but not to scenarios that are either greater than or less than the selected VaR percentile. Similarly for Tail Value at Risk (TVaR), the traditional view is that the company holds capital "for the average loss event given that it is (at least) a catastrophic scenario"; thus, according to this view, we allocate capital to a line of business only to the extent of its contribution to loss events greater than or equal to the 99th percentile loss (or other selected threshold). Again, loss scenarios that are less than the TVaR threshold percentile receive no capital allocation.

Intuitively, this characterization of the VaR (and TVaR) capital requirement seems unsatisfying; to clarify what is bothersome, we will use a thought experiment with simplified numbers.

1.5 Thought Experiment #1

Assume we are dealing with two perils:

1) Wind 20% chance of 99M loss, else zero
2) Earthquake (EQ) 5% chance of 100M loss, else zero

Assume the perils are independent. Thus, the possible scenarios for portfolio loss are:

1) 76% probability that neither peril occurs, loss = 0
2) 19% probability that only Wind occurs, loss of 99M
3) 4% probability that only EQ occurs, loss of 100M
4) 1% probability that both Wind and EQ occur, loss of 199M

Using VaR (99%) as our capital requirement, we hold 100M of capital to pay for 99% of the loss events; only the rare, 1% chance of a Wind event plus an EQ event will exceed the capital.

Many current approaches to allocation have serious drawbacks.

<u>Method #1 ("coVaR")</u>: If we say that using VaR to set the capital requirement means that we allocate capital to the events that generate the VaR scenario of 100M, then does that mean we should only allocate capital to the EQ peril (which causes the potential loss event of 100M) – yet the Wind peril that can cause a loss event of "only" 99M receives zero capital allocation?

<u>Method #2 ("alternative coVaR"):</u> Another approach might be to use all events ≥ VaR to allocate. Then we allocate 80% [=4%/(4%+1%)] to the EQ event and 20% [=1%/(4%+1%)] to the

"Wind + EQ" event; using Kreps's "co-measures" approach, we can then further allocate the capital for the "Wind + EQ" event to its components: Wind [= 49.75% = 99 / (100 + 99)] and EQ [= 50.25% = 100 / (100+ 99)]. In total EQ would receive approximately 90% [=80% + 50.25% * 20%] and Wind would receive roughly 10% [=49.75% * 20%]. But again, the substantial possibility of a standalone Wind event of 99M has no significance?

<u>Method #3 ("coTVaR"):</u> Another approach might be to use the TVaR measure for loss events ≥ 100M to allocate. Then the EQ event receives allocation proportional to 80% * 100M and the "Wind + EQ" event receives allocation proportional to 20% * 199M. Using Kreps's co-measures again, ultimately EQ receives 83.5% and Wind 16.5%; but again, we will allocate zero capital based upon the "Only Wind" event of 99M, which is much more likely to use capital and nearly as large of a loss as the "EQ only" event!

It seems intuitively clear that Wind is not receiving the appropriate capital allocation in this situation. More broadly, tail based methods in general have been criticized for ignoring loss scenarios below the tail threshold (e.g., Wang [7]).

2. REFORMULATING AND CLARIFYING VALUE AT RISK (VAR)

It therefore seems appropriate to reformulate and clarify what it means for a firm to hold capital at the 99^{th} percentile, or VaR (99%). While the prior formulation suggests that the firm holds sufficient capital "**for** the 99th percentile loss", I believe that a better formulation of the meaning of the VaR capital requirement is that the firm holds sufficient capital "**even for** the 99th percentile loss". Once we focus on VaR requiring sufficient capital "**even for** the 99th percentile loss", we can see that this capital amount is intended to also cover losses at lower percentiles as well; thus, we must allocate capital and its cost even to loss events that fall below the VaR threshold.

We can use an analogous argument to reformulate TVaR as well. Specifically, using TVaR (99%) to set capital means we are holding capital "**even for** the average loss beyond the 99th percentile", but not "only for" these events. Beyond VaR and TVaR, the same line of reasoning may be appropriate when interpreting other capital benchmarks as well.

2.1 Ramifications of New Formulation of VaR

What are some of the ramifications of our formulation that holding capital equal to VaR (99%) means holding sufficient capital "even for a 99th percentile loss"?

It would appear to follow that we need to think about capital allocation by percentile layer. In other words, why does the firm hold capital equal to the 99th percentile loss rather than the lower amount of the 98th percentile loss? The difference between the required capital amounts at these two percentile losses can be attributed solely to those loss events that outstrip the 98th percentile. Similarly, the difference between the amount of capital at the 98th percentile loss and the 97th percentile loss can be attributed solely to those losses that exceed the 97th percentile. And so on…

Therefore, allocation of capital to loss scenarios would appear to require calculations that vary by layer of capital.

3. DEFINING A "PERCENTILE LAYER OF CAPITAL"

Thus, we can define a "Percentile Layer of Capital" as follows. Define percentile α, increment j, and percentile $\alpha + j$ on the interval [0, 1]. Then

$$\text{Percentile Layer of Capital } (\alpha, \alpha + j) = \text{Required Capital at percentile } (\alpha + j) - \text{Required Capital at percentile } (\alpha) \tag{3.0}$$

We can also define a "Layer of Capital" as follows. Define amounts a and b, then

$$\text{Layer of Capital } (a, a + b) = \text{Capital equal to amount } (a + b) - \text{Capital equal to amount } (a) \tag{3.1}$$

For example, assume we have simulated 100 discrete loss events and the 78th loss (ordered from smallest to largest) is 59M and the 77th loss is 47M, then the percentile layer of capital (77%, 78%) = 59M – 47M = 12M.

3.1 Refining the Percentile Layer of Capital

Note that we can set Capital (α) = any function of (VaR (α)). For example, if we want a 99th percentile loss to consume no more than 50% of capital, then

VaR (99%) = 50% * Capital (99%) and

Capital (99%) = 2 * VaR (99%)

For ease of use, we will assume that the capital required at a loss percentile will equal that loss amount:

Capital (α) = VaR (α) = loss percentile (α)

Also, we will assume that j, which equals the "width" or "increment" of a layer's percentiles between lower and upper bounds, equals 1/n, where n = number of available discrete values. For example, if we have 100 simulation outputs, then the layer increment j = 1%, and if we have 1000 simulated values, then j = 0.1%.

3.2 Allocating a Percentile Layer of Capital to Loss Events

We can see that each layer of capital is potentially used or depleted (or "consumed" in Mango's [4] terminology) by loss events that exceed the lower bound of the layer, but not by loss scenarios that fall short of the lower bound of the layer (i.e., those losses that do not penetrate or "hit" the layer). Thus, it is desirable to allocate each layer of capital only to those events that penetrate the layer. Another critical consideration is that some of the losses that penetrate the layer are more likely to do so than others. Therefore, each event (i) that penetrates the layer of capital receives an allocation based upon its conditional exceedance probability.

Conditional Exceedance Probability for event (i) = Probability of event (i) that penetrates the layer of capital / Probability of all events that penetrate the layer of capital

Thus, for any layer of capital, we take the amount of capital (or the "width" of the layer), we allocate this amount of capital only to loss events that penetrate the layer, and we calculate the allocation percentages based upon each loss event's conditional probability of penetrating the layer. The allocation percentages, by definition, sum to 100% on any layer.

After performing the allocation of each layer of capital (from zero up to the required VaR capital amount - but not beyond it), we will have allocated 100% of the capital to loss events.

Many loss scenarios will penetrate several different percentile layers of capital and therefore receive varying allocations of capital from many layers of capital. The total capital allocated to any particular loss event is simply the total, summed over all layers of capital that the loss event penetrates, of the capital allocated on each individual layer. As an example, take the 83rd percentile loss event. On each layer of capital (from zero up to the 83rd percentile layer of capital but not beyond) it receives varying amounts of allocated capital; sum across all of these layers to calculate total capital allocated to this event. Of course, each loss "event" or "scenario" may be an accumulation of losses from several business units, policies, and/or perils. But as Kreps [1] has shown, once we have the total allocated capital for a loss scenario, we can then allocate to the subcomponents based upon their contributions to the total.

3.2.1 Applying Capital Allocation by Percentile Layer to Thought Experiment #1

In this section we will apply the procedure of capital allocation by percentile layer to the simplified numbers of Thought Experiment #1.

In Thought Experiment #1, there are 4 potential scenarios:

1) 76% neither peril occurs, loss = 0
2) 19% only Wind occurs, loss of 99M
3) 4% only EQ occurs, loss of 100M
4) 1% both Wind and EQ occur, loss of 199M

We hold capital equal to VaR (99%) = 100M. The layer of capital of 1M x 99M can only be penetrated (or "depleted" or "consumed") by event #3 or #4. Event #3, the "Only EQ" event, has a conditional exceedance probability of 80% [4% / (4%+1%)]. Event #4, the "Wind and EQ" event, has conditional exceedance probability of 20%. Therefore, we allocate the 1M in layer capital (100M – 99M) as follows:

- 80% for EQ event,
- 20% for Wind + EQ event
- 0% for Wind only event

The next layer of capital, 99M x 0, can be used by all 3 loss events.

- "Only Wind" event has conditional exceedance probability of 79% [19% / (19%+4%+1%)]
- "Only EQ" event has conditional exceedance probability of 17% [4% / (19%+4%+1%)]
- "Wind and EQ" event has conditional exceedance probability of 4% [1% / (19%+4%+1%)]

Therefore, the allocation of 99M in capital (99M – 0) is

- 79% for Wind
- 17% for EQ

- 4% for Wind + EQ

The total capital allocation to loss event across both layers (namely, 1M x 99M and 99M x 0) is then

- "Only Wind" = 79% x 99M = 78.4M
- "Only EQ" = 17% x 99M + 80% x 1M = 17.3M
- "Wind + EQ" event = 4% x 99M + 20% x 1M = 4.3M

The total allocated capital = 78.4 + 17.3 + 4.3 = 100 = VaR(99%)

The loss event of "Wind + EQ" can then be allocated further to the underlying perils that contribute to the loss event (per Kreps [1]) as follows. In a "Wind + EQ" event, which receives a 4.3M allocation, Wind contributes 99M and EQ contributes 100M. Therefore, Wind % = (99/199) = 49.75%, EQ = (100/199) = 50.25%. The total allocation to peril is therefore

- Wind = 78.4M + 49.75% x 4.3M = 80.5M
- EQ = 17.3M + 50.25% x 4.3M = 19.5M

Comparing results of different methods at the 99th percentile, we see that

- Capital allocation by percentile layer = Wind 80.5%, EQ 19.5%
- coTVaR for all events ≥ 100M = Wind 16.5%, EQ 83.5%

Thus, capital allocation by percentile layer creates a completely different allocation than coTVaR.

3.2.2 Thought Experiment #2

In Thought Experiment #1, capital allocation by percentile layer produced allocations that are essentially proportional to the perils' average loss. So does this imply that the procedure will always result in such an allocation? After all, it would seem problematic to always allocate capital in proportion to the average loss; catastrophic perils with the capability to produce severe losses should receive a greater allocation of capital, regardless of the "average" outcome. Thought Experiment #2 shows that capital allocation by percentile layer will respond appropriately in such a situation.

Again assume we are dealing with two perils:

1) Wind 20% chance of 50M loss, else zero
2) Earthquake (EQ) 5% chance of 100M loss, else zero

Note that for Wind the average loss = 10M and for EQ the average loss = 5M.

Assume the perils are independent. Thus, the possible scenarios for portfolio loss are:

1) 76% probability that neither peril occurs, loss = 0
2) 19% probability that only Wind occurs, loss of 50M
3) 4% probability that only EQ occurs, loss of 100M
4) 1% probability that both Wind and EQ occur, loss of 150M

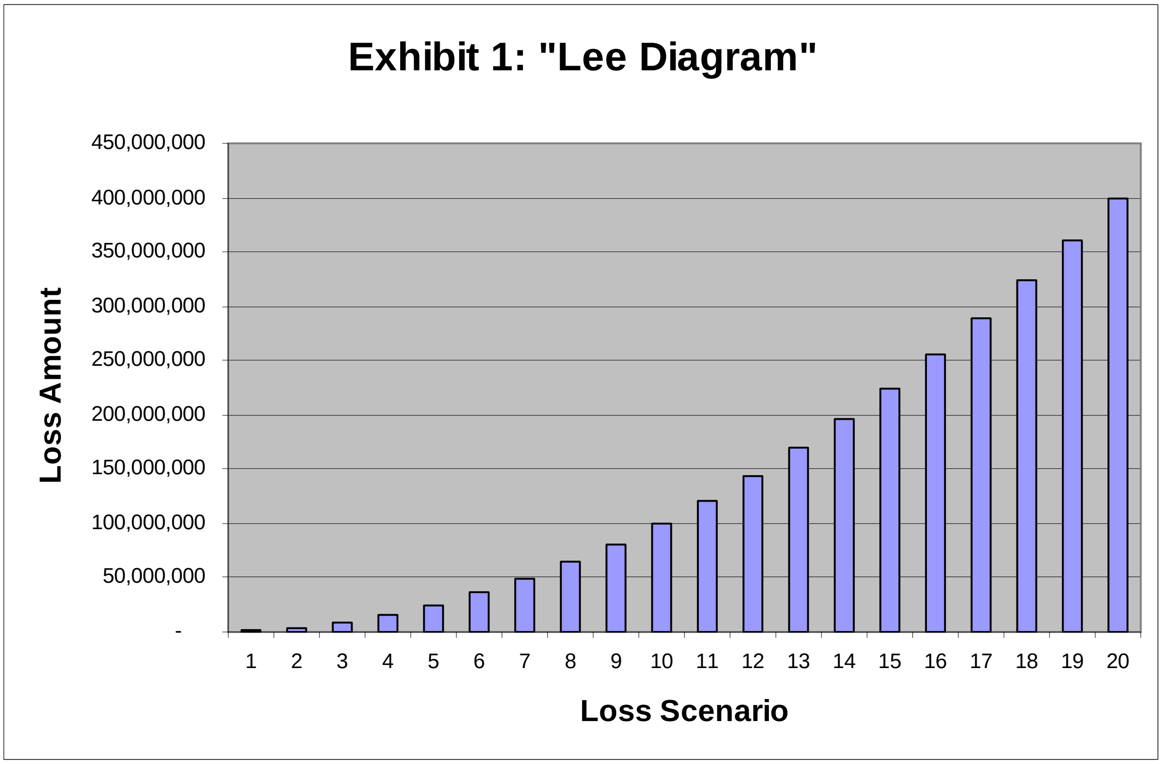
Using VaR (99%) as our capital requirement, we hold 100M of capital to pay for 99% of the loss events; only the rare, 1% chance of a Wind event plus an EQ event will exceed the capital. Applying capital allocation by percentile layer to the 50M x 50M layer of capital as well as the 50M x 0 layer of capital, we obtain the following allocation:

- Capital allocation by percentile layer = Wind 44%, EQ 56%
- Allocation in proportion to average loss = Wind 67%, EQ 33%

This example shows that capital allocation by percentile layer can produce unique allocations that are proportional neither to the average loss, nor to probability of occurrence, nor to standalone VaR.

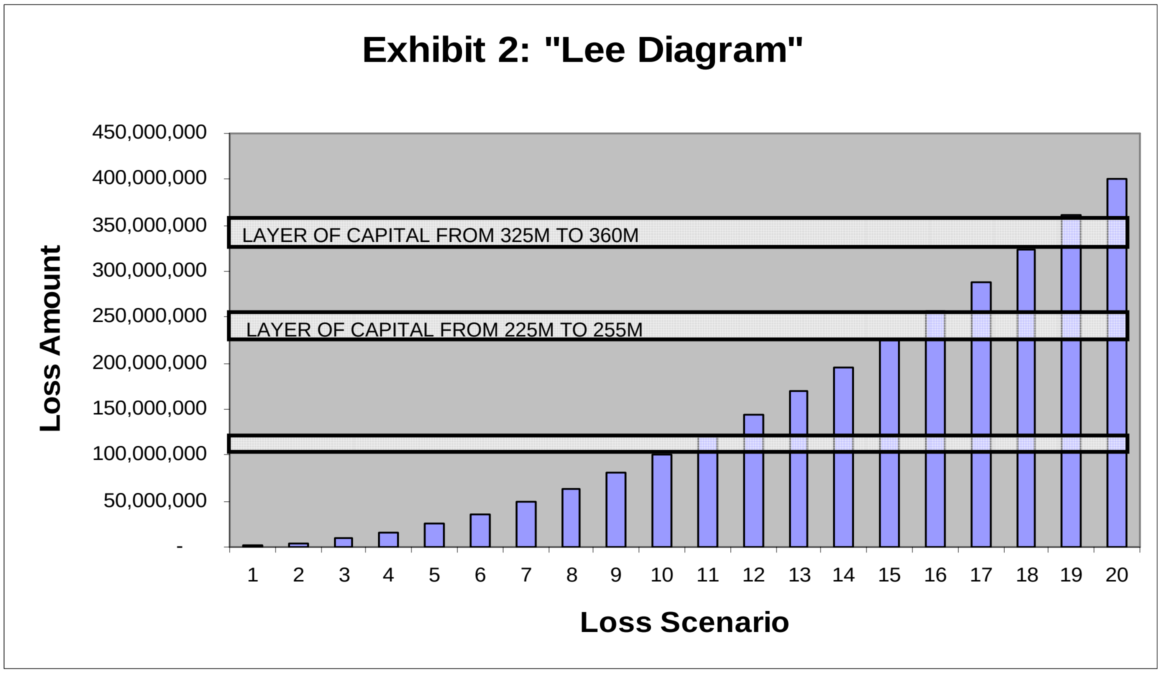
4. GRAPHICAL DESCRIPTION OF CAPITAL ALLOCATION BY PERCENTILE LAYER - DISCRETE

Let us view the "size of loss" distribution in graphical format to further clarify the approach; we will use sample numbers for simplicity. We will use "Lee Diagrams" (see Lee [2]), namely graphs where the loss scenario number (ordered in increasing size) is plotted on the X-axis and the loss amount is plotted on the Y-axis:



In this example (Exhibit 1) there are 20 loss scenarios; why is it that the firm needs to hold 360M of capital rather than just 100M of capital? It appears that loss scenarios 1 through 10, which are all less than or equal to 100M, do not require this "layer of capital". In contradistinction, loss scenarios 11 through 20, which exceed 100M, clearly do utilize this layer of capital in excess of 100M. Examining in further detail, we see that all of scenarios 11 through 20 utilize the 1M x 100M layer, but not all of them require the 1M x 200M layer, and even fewer require the 1M x 300M layer.

Thus, we must allocate each individual layer of capital to the loss events that penetrate the layer in proportion to the relative usage of the layer of capital; i.e., in proportion to the relative exceedance probability, as per Exhibit 2:



Numerical example:

- Loss scenario #19 is one of 2 events (scenarios 19 and 20) that require the 35M x 325M layer of capital.
    - Thus scenario #19 receives 1/2 allocation of this 35M of capital.
- Loss scenario #19 also is one of 5 events (scenarios 16 through 20) that require the firm to hold the 30M x 225M layer of capital.
    - Thus it receives 1/5 allocation of this 30M of capital.
- Apply the procedure to all layers; allocate to all loss events that exceed the lower bound of the layer via conditional exceedance probability.

Note that a loss event tends to receive a larger percentage allocation in the upper layers than in the lower layers for 2 reasons:

1) In the upper layers, we are allocating a full layer of capital to fewer loss events (i.e., the exceedance probability decreases as the loss amount increases); therefore, each event gets a larger share of the "overhead" of the total layer of capital.

2) In the upper layers, we are allocating a wider layer of capital because the severity of each loss event tends to outstrip the prior loss event by a greater amount (i.e., the percentile layer of capital tends to widen as the loss amount increases). This behavior will depend, however, on the particular shape of the size of loss distribution.

5. GENERALIZATION OF CAPITAL ALLOCATION BY PERCENTILE LAYER TO DISCRETE LOSS EVENTS

Let VaR(k) = total required capital = $\Sigma\ [x(\alpha+j) - x(\alpha)]$

- $x(\alpha)$ is the loss amount at percentile α
- j is selected percentile increment
- α sums from zero to (k - j)

Allocation of capital **for each percentile layer of capital, across loss events**

- A Layer of Capital = $[x(\alpha+j) - x(\alpha)]$
- Allocation of capital on layer $[x(\alpha+j) - x(\alpha)]$ to loss event x(i) =
    - $[x(\alpha+j) - x(\alpha)]$ * Probability (x = x(i)) / Probability (x > $x(\alpha)$)
- Sum across all loss events x(i) such that $i > \alpha$

For an equivalent view, we can also look at the allocation of capital **for each loss event, across all percentile layers of capital** =

- A Layer of Capital = $[x(\alpha+j) - x(\alpha)]$
- Allocation of capital on layer $[x(\alpha+j) - x(\alpha)]$ to loss event x(i) =
    - $[x(\alpha+j) - x(\alpha)]$ * Probability (x = x(i)) / Probability (x > $x(\alpha)$)
- Sum across all layers of capital such that $\alpha \geq 0$, $(\alpha+j) \leq \min(i, k)$
- Note the min(i, k) restriction. For any loss event, we sum across all layers of capital up to the amount of the given loss event, but not if the loss event exceeds the VaR threshold. In such a case, the loss beyond the VaR threshold does not generate additional allocated capital to the loss event.

6. GENERALIZATION OF CAPITAL ALLOCATION BY PERCENTILE LAYER TO CONTINUOUS LOSS FUNCTION

We can take the formulas for discrete loss events and generalize them into continuous versions.

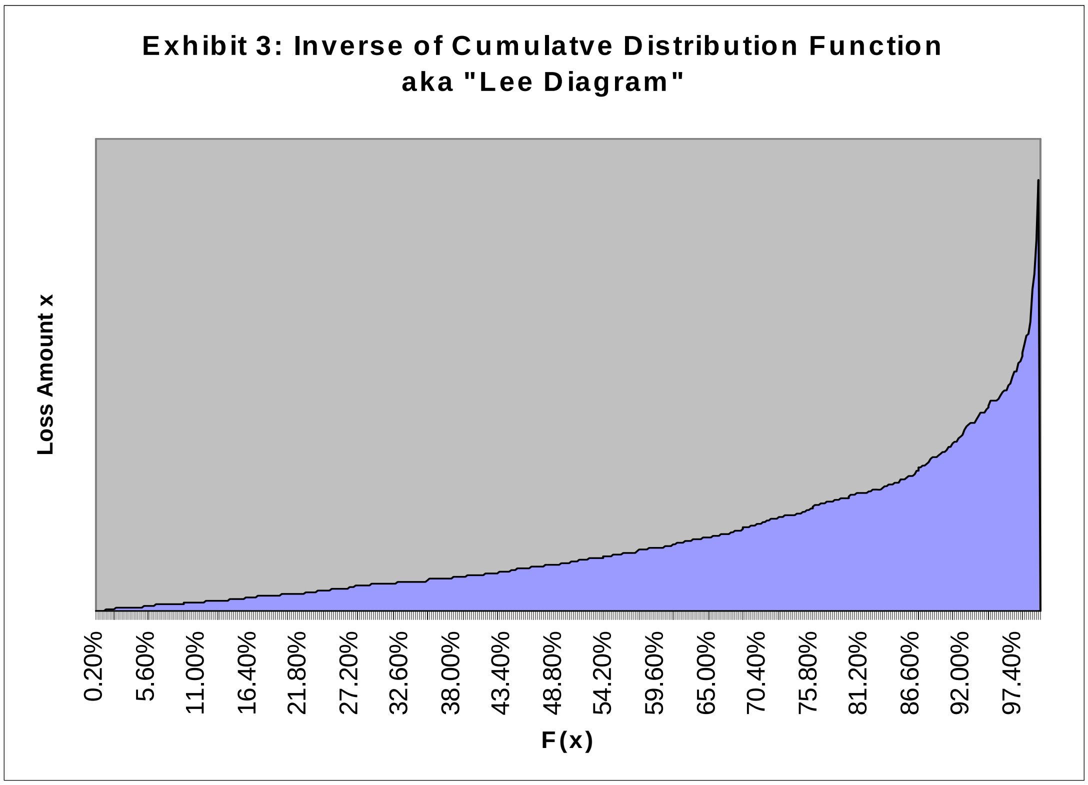
First, we will define the inverse function of F(x), a function that accepts a percentile as input and returns the loss amount as output.

Inverse function of F(x) = $F^{-1}(\alpha) = F^{-1}(F(x)) = x$

Derivative of $F^{-1}(F(x)) = dF^{-1}(F(x)) / dF(x) = dx / dF(x) = 1 / f(x)$

Incremental change in loss amount = dx

Incremental change in percentile = dF(x)



In Exhibit 4, each horizontal bar is a layer of capital.

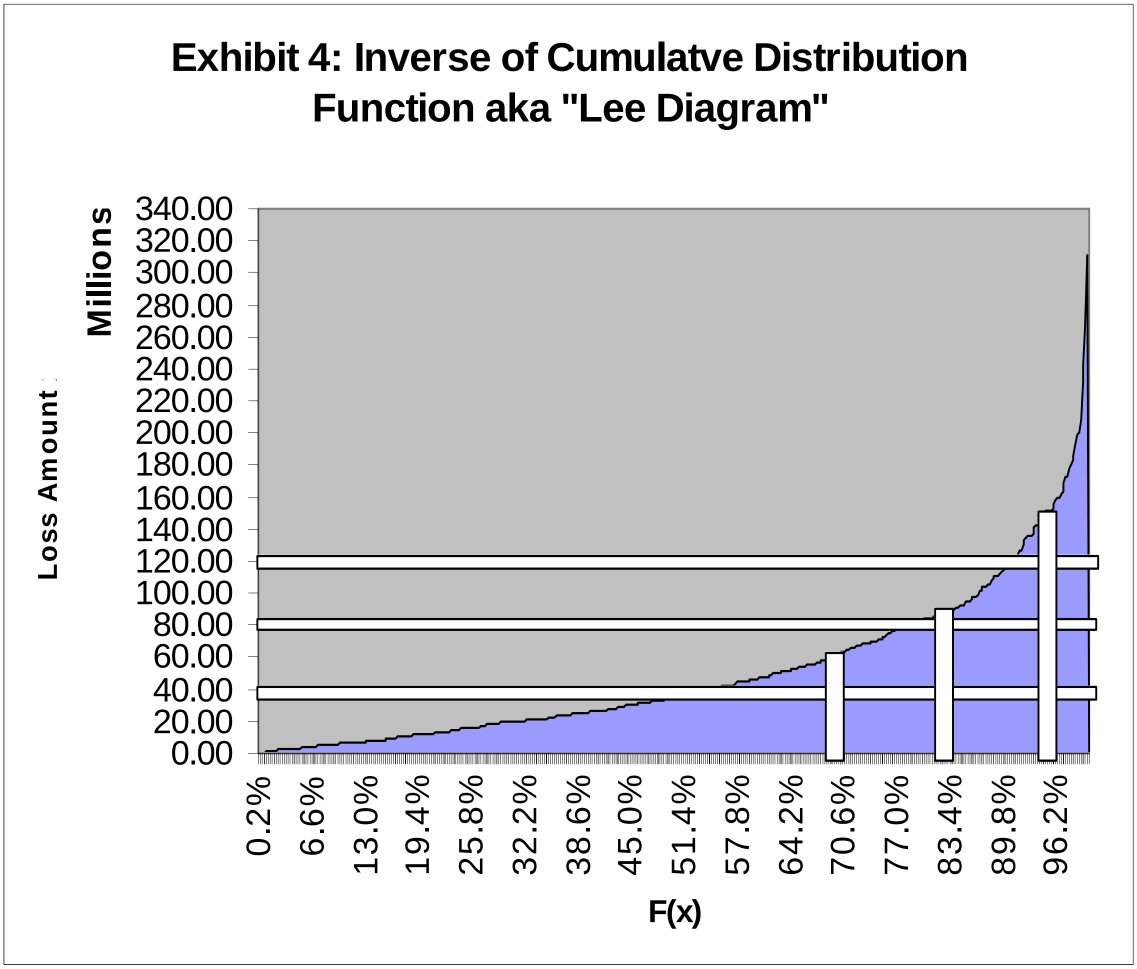
The length of the layer of capital, by definition, is 1.0.

The infinitesimally small width of each layer of capital = dx.

Each vertical bar represents a loss event.

The length = the loss amount = x.

The infinitesimally small width = dF(x) = f(x)dx.



6.1 Two Alternative Views of Capital Allocation by Percentile Layer

We can view the capital allocation as a "horizontal procedure" which takes each layer of capital and allocates to all loss events which penetrate the layer.

We can also view the allocation as a "vertical procedure" which takes each loss event and allocates capital to it for all layers that it penetrates.

6.2 Approach #1: Horizontal then Vertical

Let x represent the loss amount and let y represent the capital.

First take an infinitesimally small layer of capital (y, y+dy) and allocate it across loss events.

Integrate across all loss events x which penetrate the layer, from x = y to x = ∞

$$\int_{x=y}^{x=\infty} f(x)/(1-F(y))dx \tag{6.0}$$

The allocation weights sum to 1 on each layer.

Then perform this procedure for all layers of capital:

$$\int_{y=0}^{y=VaR(99\%)} \int_{x=y}^{x=\infty} f(x)/(1-F(y))dxdy \tag{6.1}$$

Because capital is based upon the 99th percentile, there are no "layers of capital" above the 99th percentile to allocate, so we integrate y only up to VaR(99%).

The total allocated capital equals the total amount of capital, which is VaR(99%).

6.3 Approach #2: Vertical then Horizontal

Let x represent the loss amount and let y represent the capital.

Each loss event uses capital on many layers of capital (y, y+dy).

Allocate to a loss event across each layer of capital:

$$\int_{y=0}^{y=x} f(x)/(1-F(y))dy \tag{6.2}$$

Integrate y across all layers of capital less than or equal to the loss amount x.

If the loss amount x exceeds VaR(99%), we do not allocate additional layers of capital beyond VaR(99%); in such a case when x>VaR(99%), we integrate as follows:

$$\int_{y=0}^{y=VaR(99\%)} f(x)/(1-F(y))dy \tag{6.3}$$

Then perform allocation across all loss events x:

$$\int_{x=x(0\%)}^{x=\infty} \int_{y=0}^{y=\min(x,VaR(99\%))} f(x)/(1-F(y))dydx \tag{6.4}$$

6.4 Formula for Allocating Capital to a Loss Event

The "vertical view" can provide some insight into the capital allocation to each loss event.

As we saw previously (equation (6.2)), for any loss event with amount x (assuming x is below the VaR threshold and therefore the allocated capital is not capped in any way), the Allocated Capital to loss event x = AC(x) =

$$AC(x) = \int_{y=0}^{y=x} f(x)/(1-F(y))dy \tag{6.5}$$

Because we are integrating y, we can move f(x) outside the integral and rewrite the formula: Allocated Capital to loss event x = AC(x) =

$$AC(x) = f(x) \int_{y=0}^{y=x} 1/(1-F(y))dy \qquad (6.6)$$

For completeness, also recall that if the loss event is in the tail, namely x>VaR(99%), then

$$AC(x) = f(x) \int_{y=0}^{y=VaR(99\%)} 1/(1-F(y))dy \qquad (6.7)$$

According to equation (6.6), the procedure of capital allocation by layer says that any loss event's allocated capital depends upon:

1) The probability of the event occurring (i.e., f(x)).
2) The severity of the loss event, or the extent to which the loss event penetrates layers of capital (i.e., the upper bound of integration is x, the loss amount).
3) The loss event's inability to share the burden of its required capital with other loss events (i.e., ∫ 1 / (1-F(y)) dy). We can think of this factor as the extent to which a loss event "sticks out" or is dissimilar in severity to other loss events.

6.4.1 The Derivative of the Allocated Capital to Loss Event

We can also use equation (6.6) to obtain the derivative of Allocated Capital to loss event with respect to the loss amount x:

$$d/dx\{AC(x)\} = d/dx\{f(x) \int_{y=0}^{y=x} 1/(1-F(y))dy\} \qquad (6.8)$$

$$= f(x) * d/dx\{\int_{y=0}^{y=x} 1/(1-F(y))dy\} + \int_{y=0}^{y=x} 1/(1-F(y))dy * d/dx\{f(x)\} \qquad (6.9)$$

$$d / dx\{AC(x)\} = f(x)/(1-F(x)) + f'(x)\int_{y=0}^{y=x} 1/(1-F(y))dy \qquad (6.10)$$

We can understand formula (6.10) as saying that as the loss amount x under consideration increases, 2 factors simultaneously affect the allocated capital:

1) The allocated capital increases to the extent that the loss amount receives allocation from an additional layer of capital based upon conditional probability [= f(x) / (1 – F(x))].
2) The allocated capital changes (usually decreases) to the extent that the loss amount is less likely to occur and thus receives a lower allocation on the lower layers of capital [= d/dx (f(x)) * ∫ 1/ (1-F(y)) dy].

Two observations about these 2 factors:

1) Usually, the derivative of f(x) is negative, so item #2 is usually negative, but can be positive when the derivative of f(x) is positive.
2) When dealing with simulation output of n discrete events, each discrete event has likelihood of 1/n and thus is equally likely; therefore, the allocated capital to each larger event increases only with respect to factor #1, whereas factor #2 will equal zero.

6.4.2 Utility Function

Equation (6.6) also shows how we can use capital allocation by percentile layer to describe the disutility, or "pain", given a particular loss event x.

Let r = required % rate of return on capital. Then the cost of capital associated with loss event x =

$$r * f(x)\int_{y=0}^{y=x} 1/(1-F(y))dy \qquad (6.11)$$

The cost of capital of an event, given the loss event, is then

$$r \int_{y=0}^{y=x} 1/(1-F(y))dy \tag{6.12}$$

And the total cost, given the event, equals the loss amount x plus the cost of capital =

$$x + r \int_{y=0}^{y=x} 1/(1-F(y))\,dy \tag{6.13}$$

Equation (6.13) shows the disutility as an additive loading to the loss amount x. Rearranging terms, we can also show the disutility as a multiplicative factor as well:

$$x[1 + r(1/x) \int_{y=0}^{y=x} 1/(1-F(y))\,dy\,] \tag{6.14}$$

7. INTERPRETATION, COMMENTS, AND EXTENSIONS

The procedure for capital allocation by percentile layer outlined above generates allocations that are different than many other methods, with ramifications for measuring the relative risk and profitability of various lines of business. Some methods, such as coTVaR, tend to allocate the overwhelming amount of capital only to perils that contribute to the very worst scenarios; capital allocation by percentile layer, however, recognizes that when the firm holds capital even for an extremely catastrophic scenario, some of the capital also benefits other, more likely, more moderately severe downside events. On the other hand, some other methods (e.g., Mango's "capital consumption", XTVaR, etc.) allocate capital to a broader range of loss events that consume capital; the allocation varies proportionately based upon conditional probability. Because these methods fully account for relative probability, however, they may allocate insufficient capital to severe yet unlikely events. The potentially extreme loss of such events causes firms to hold an amount of capital that far outstrips the amount required by other loss events; although the actual occurrence of one of these events is very unlikely, the cost of holding precautionary capital is quite definite. Capital allocation by percentile layer appropriately allocates more capital cost to those unlikely, severe events that require the firm to hold additional capital.

Capital allocation by percentile layer as delineated above assumes that required capital is based upon VaR, but a similar model can also apply to TVaR. In other words, we can view TVaR as saying we want to hold enough capital "even for {the 99th percentile loss + the average amount by which losses above the 99th percentile tend to exceed the 99th percentile}". In such a case, capital allocation by layer would be nearly the same, allocating capital up to the 99th percentile. The only additional step would then be to allocate one additional layer of capital (i.e., TVaR – VaR) to the losses that exceed the TVaR threshold. Consistent with TVaR's meaning as well as the layer allocation approach, this additional layer of capital should be allocated to loss events in proportion to each event's average amount of loss excess of the TVaR threshold.

7.1 Additional Areas of Application

The application highlighted here focuses on property catastrophe risk, but the reformulation of the meaning of VaR should have similar ramifications to other sources of risk as well. Specifically, risk and capital for risky assets such as equities and fixed income securities have traditionally been defined based upon VaR metrics; as a result, methods that allocate capital among various asset classes and operating units may benefit from implementing capital allocation by percentile layer.

Capital allocation by percentile layer may also be germane when the firm's total capital does not reside in one "indivisible bucket of equity capital" but rather is split into multiple tranches of capital. Because these tranches sustain capital depletion in a predetermined sequential order and, as a result, carry different cost of capital rates, it would seem appropriate to allocate capital with a procedure that explicitly accounts for the varying layers of capital and their costs. In addition, alternative forms of capital that apply on a "layered" basis (e.g., excess of loss reinsurance) and their costs (e.g., the amount of "risk load" or "margin" in the reinsurance price) would also appear to be candidates for capital allocation by percentile layer.

7.2 Implementation

In many situations in which we want to implement capital allocation by percentile layer, we will be dealing with discrete output from a simulation model. By using the previously derived discrete formulas we can program a spreadsheet and achieve numerical results. Once capital amounts are allocated to each simulated loss event, we can then (per Mango, Kreps) further allocate the capital for the total loss to those individual components that contributed to the total.

7.2.1 Contributions to Capital

The main focus of the analysis until now has been on the allocation of capital with respect to loss without considering premium. When measuring the allocated cost of capital for a business unit or peril or individual contract, one must also recognize that the associated premium (net of expenses) is essentially a contribution to capital or "offset" to allocated capital. As a result, one should subtract collected premium net of expenses from the allocated capital before multiplying by the cost of capital rate.

8. IMPLICATIONS FOR RISK LOAD

The discussion until now has related to a retrospective situation, when the price that the firm has charged for a certain transaction is a historical fact; the only question the firm asks is how to allocate capital costs in order to measure profitability. But what should the company do in a prospective situation? How does capital allocation affect what price the firm should charge? What does capital allocation by percentile layer imply about calculating risk load and determining the premium?

For the purposes of our discussion, we will ignore any provisions in the premium for expenses, parameter uncertainty, winner's curse, or other loadings. Thus we will define

$$\text{Premium net of expenses} = \text{expected loss} + \text{cost of capital} \qquad (8.0)$$

Let:

P = premium net of expenses

E[L] = expected loss

r = required % rate of return on capital

Then

$$P = E[L] + r * (\text{allocated capital} - \text{contributed capital}) \qquad (8.1)$$

Let:

Contributed capital = premium net of expenses.

Then

P = E[L] + r * (allocated capital – P)

Rearranging terms, we derive:

P (1+r) = E[L] + r * (allocated capital)

P = 1/ (1+r) * E[L] + r/(1+r) * allocated capital

Let 1/ (1+r) = (1+r-r)/(1+r) = [(1+r)/(1+r)-(r/(1+r))] = [1-r/(1+r)]. Then

P = (1 - r/(1+r)) * E[L] + r/(1+r) * allocated capital.

Then

$$P = E[L] + r/(1+r) * (\text{allocated capital} - E[L]) \qquad (8.2)$$

For any given loss event x (given it is below the VaR threshold), allocated capital is given by Equation (6.6) and E[L] = x * f(x).

Then the Premium for any loss event x =

$$P(x) = xf(x) + r/(1+r)[f(x)\int_{y=0}^{y=x} 1/(1-F(y))dy - xf(x)] \qquad (8.3)$$

Rearranging terms, we derive

$$P(x) = f(x)\{x + r/(1+r)[\int_{y=0}^{y=x} 1/(1-F(y))dy - x]\} \quad (8.4)$$

Equation (8.4) shows that the disutility function given loss event x, after taking into account its premium's contribution to capital, equals

$$x + r/(1+r)[\int_{y=0}^{y=x} 1/(1-F(y))dy - x] \quad (8.5)$$

We can also rearrange equation (8.3) to produce a multiplicative factor,

$$P(x) = xf(x)\{1 + r/(1+r)[(1/x)\int_{y=0}^{y=x} 1/(1-F(y))dy - 1]\} \quad (8.6)$$

Equation (8.6) highlights that the required premium associated with loss event x is the expected value x*f(x) multiplied by an adjustment factor. We can view the adjustment factor as either

1) an adjustment to the loss amount x
2) an adjustment to the probability f(x)

8.1 Properties of the Risk Load

Equation (8.5) shows that given a loss event, the additive risk load amount =

$$r/(1+r)[\int_{y=0}^{y=x} 1/(1-F(y))dy - x] \quad (8.7)$$

Equation (8.7) and its derivatives show that the risk load increases with respect to the loss amount x at an increasing rate. It also shows that even for very small values of the loss event x the risk load is strictly positive. This result suggests that capital allocation by percentile layer as applied above, in contradistinction to many common methods, requires that even small loss events that are less than the portfolio's mean receive an allocation of capital and a positive risk load.

Why should a loss event that is ***less*** than the average loss require an allocation of capital? In order to clarify this issue, we turn to thought experiment #3.

8.1.1 Thought Experiment #3

Again assume we are dealing with two perils:

1) Wind 20% chance of 5M loss, else zero
2) Earthquake (EQ) 5% chance of 100M loss, else zero

Assume the perils are independent. Thus, the possible scenarios for portfolio loss are:

1) 76% probability that neither peril occurs, loss = 0
2) 19% probability that only Wind occurs, loss of 5M
3) 4% probability that only EQ occurs, loss of 100M
4) 1% probability that both Wind and EQ occur, loss of 105M

Note that the average loss for Wind = E[Wind] = 1M and E[EQ] = 5M. The two perils are independent so the portfolio expected loss = 6M. For simplicity assume that the premium for each peril equals the mean.

Now what happens when there is a "Wind only" loss of 5M? The Wind loss of 5M exceeds its 1M of premium, so it clearly needs capital. Yet overall, the portfolio has 6M of premium available and so the firm can use this money to pay the "Wind only" loss of 5M. Where, however, does this 6M of premium come from? While 1M comes from Wind, the majority, 5M, comes from the premium inflow from EQ. Thus it is clear that when a "Wind only" event occurs, the Wind subline "uses" or "consumes" capital, and the EQ subline "provides" capital by contributing its premium.

Therefore, this numerical example shows that even a loss event (e.g., Wind loss of 5M) that is ***less*** than the portfolio's mean loss (e.g. 6M) can consume capital and deserves allocation of capital. As a result, many common methods, which only allocate capital to loss events that exceed the mean, may generate skewed allocations.

9. FINAL NUMERICAL EXAMPLE

Take the following situation involving 3 independent lines of business (LOB), corresponding to 3 perils

- LOB A: (e.g., Fire)
    - 25% chance of a loss;
    - If there is a loss, the amount is exponentially distributed
        - Exponential Mean = 4M
- LOB B: (e.g., Wind)
    - 5% chance of loss;
    - If there is a loss, the amount is exponentially distributed
        - Exponential Mean = 20M
- LOB C: (e.g., EQ)
    - 1% chance of loss;
    - If there is a loss, the amount is exponentially distributed
        - Exponential Mean = 100M

Each line of business has an annual average loss amount of 1M, but some lines have losses that are more infrequent and extreme than others.

We will run 10,000 simulations, set required capital equal to VaR(99%), and use capital allocation by percentile layer in order to calculate the allocated capital for each simulated loss event. Then we will take the amount of capital assigned to each loss event and allocate to the contributing perils; each peril will receive an allocation based upon the contribution of its loss to the total event loss. Finally, we will take allocated capital and subtract the amount of the mean loss (as a proxy for the contribution to capital from premium) from the allocated capital.

9.1 Final Numerical Example – Allocation Results

	Line of Business		
Method	A	B	C
Standalone TVaR @99th percentile	10%	30%	60%
coTVaR allocation @99th percentile	0%	24%	76%
coTVaR allocation @95th percentile	10%	42%	48%
coTVaR allocation @90th percentile	21%	39%	40%
coTVaR allocation @breakeven percentile	29%	35%	36%
Capital Allocation by Percentile Layer, VaR@99%	17%	53%	30%

Note that all of the tail-based methods such as VaR, TVaR, coTVaR, etc. allocate the greatest amount of capital to the severe yet extremely unlikely EQ event. Only capital allocation by percentile layer assigns the most capital to the more likely Wind event.

10. CONCLUSIONS

Capital allocation by percentile layer has several advantages, both conceptual and functional, over existing methods for allocating capital. It emerges organically from a new formulation of the meaning of holding Value at Risk capital; allocates capital to the entire range of loss events, not only the most extreme events in the tail of the distribution; tends to allocate more capital, all else equal, to those events that are more likely; tends to allocate disproportionately more capital to those loss events that are more severe; renders moot the question of which arbitrary percentile threshold to select for allocation purposes by using all relevant percentile thresholds; produces allocation weights that always add up to 100%; explicitly allocates the entire amount of the firm's capital, in contrast to other methods that allocate based upon the last dollar of "marginal" capital; and provides a framework for allocating capital by layer and by tranche.

Capital allocation by percentile layer has the potential to generate significantly different allocations than existing methods, with ramifications for calculating risk load and for measuring risk adjusted profitability.

Acknowledgment

The author would like to acknowledge Dave Clark, Vladimir Kremerman, Bill Panning, Ira Robbin, Marc Shamula, Walt Stewart, and Kyle Vrieze for commenting on earlier drafts of this paper.

Appendix A: Calculating Results for an Exponential Distribution

If the loss distribution follows an exponential distribution, F(x) = 1 – exp (-x / theta), we can solve formula (6.6) to derive a formula for allocated capital for loss event x (assuming x < VaR)

$$AC(x) = (1/\theta)\exp(-x/\theta)\int_{y=0}^{y=x}\exp(y/\theta)\,dy \qquad \text{(A.1)}$$

$$AC(x) = 1 - \exp(-x/\theta) \qquad \text{(A.2)}$$

We can also use formula (6.10) to calculate the derivative of allocated capital (x) for an exponential distribution =

$$d/dx\{AC(x)\} = (1/\theta)\exp(-x/\theta) \qquad \text{(A.3)}$$

= a positive number, confirming that allocated capital increases as the loss amount x gets larger. However, the second derivative is negative, so the rate of increase is decreasing.

We can also solve formula (6.13) to calculate the total cost (the loss amount plus the cost of allocated capital) given loss amount x =

$$x + r\theta(\exp(\ x/\theta) - 1) \qquad \text{(A.4)}$$

We can also solve formula (6.14) to express the total cost given loss amount x as the product of the loss amount x and a multiplicative loading factor =

$$x[1 + r\theta(1/x)(\exp(\ x/\theta) - 1)] \qquad \text{(A.5)}$$

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PRICING INSURANCE POLICIES:
THE INTERNAL RATE OF RETURN MODEL

Sholom Feldblum

(May 1992)

Financial models, which consider the time value of money, surplus commitments, and investment income, are increasingly being used in insurance rate making. This reading shows how an internal rate of return model can be used to price insurance policies. It discusses the framework of the IRR model, the various insurance, investment, and tax cash flows, the surplus commitments and equity flows, and two methods of estimating the opportunity cost of equity capital. It presents an application of the IRR model from a recent Workers' Compensation rate filing. Finally, it discusses the potential pitfalls in using IRR pricing models.

[This study note was written as educational material for the Part 10A CAS examination. The tables in Section V are reproduced with permission of the National Council on Compensation Insurance. The views expressed here do not necessarily represent the position of the Casualty Actuarial Society, the Liberty Mutual Insurance Company, the National Council on Compensation Insurance, or of any other organization. I am indebted to Robert Butsic, Richard Woll, and David Appel for numerous suggestions and comments on using IRR models, and to Charles Walter Stewart, Paul Kneuer, Jonathan Norton, John Kollar, Ira Robbin, John Coffin, Steve Lehmann, Paul Braithwaite, Leigh Halliwell, William Kahley, Len Gershun, Gerald Dolan, and Peter Murdza for extensive editing and corrections to earlier drafts of this reading. The remaining errors, of course, are my own.]

PRICING INSURANCE POLICIES: THE INTERNAL RATE OF RETURN MODEL

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PRICING INSURANCE POLICIES:
THE INTERNAL RATE OF RETURN MODEL

Section I: Introduction

How should an actuary determine premium rates for insurance policies? Early rating bureau pricing procedures incorporated a fixed underwriting profit margin, such as 2.5% for Workers' Compensation and 5% for other lines of business. The simplicity of this approach led to its continued use by the actuarial profession.

During the past two decades, economists, financial analysts, and casualty actuaries have proposed alternative pricing models, sparked by the lack of theoretical justification for the traditional procedure, the high interest rates in the American economy, and the increasing competitiveness of the insurance industry. More precisely, the stimuli for more accurate pricing models fall into three categories:

(1) *The time value of money:* Insurance cash flows on a given contract occur at different times. Often, premiums are collected and expenses are paid at policy inception, whereas losses are settled months or years later. Monies exchanged at different dates have different values, which we relate to economic inflation, available interest rates, or the opportunity cost of capital. Financial insurance pricing models consider both the magnitudes and the dates of cash transactions.

(2) *Competition and expected returns:* In a free market economy, the price of a product depends on the degree of competition in the industry. If a firm prices its product above the market level, it may lose sales. If it prices its product below the market level, its profits may fall. The optimal price for products whose costs are known in advance of the sale is determined by production costs and competitive constraints. Complex insurance products, however, require an *a priori* analysis of both expected costs and achievable returns.

(3) *The rate base:* The underwriting profit margin is a return on sales. Businessmen in many industries measure profits in relation to sales, though this method is not favored by financial analysts and theoretical economists. Alternative rate bases are assets, which are used in public utility rate regulation, and equity (or net worth), which is used in most financial pricing models.

There is a wide divergence between the underwriting profit margins assumed in rate filings and actual insurance experience. Over the past 15 years, underwriting profit margins have averaged about –7%, despite the +5% or +2.5% assumed in rate filings. Much of this discrepancy stems from regulatory disapproval of requested rate revisions. In addition, some insurers do not always target a positive underwriting profit margin, since the resultant rates may not be competitive.

Actuaries have responded with new, more sophisticated pricing techniques, which consider cash flows, financial constraints, and competitive pressures. Many insurers analyze their performance with realistic profitability models. But the documentation and dissemination of these models, whether in minutes of technical rating bureau committee meetings or in academic articles, has been sparse. The practicing actuary needs a clearer exposition of the various pricing models.

This paper describes the *Internal Rate of Return* (IRR) insurance pricing model. The IRR model is used extensively by the National Council on Compensation Insurance and by various private carriers for Workers' Compensation rate filings and internal profitability analyses. Moreover, the IRR model has influenced other pricing techniques, such as the *Risk Compensated Discounted Cash Flow Model* which Fireman's Fund proposed for its California rate filings.

The expansion of "open competition" rate regulatory laws, and the replacement of the Insurance Services Office and the National Council on Compensation Insurance advisory rates by loss costs in many jurisdictions, compels company actuaries to determine appropriate profit provisions. The practicing actuary must estimate the needed provisions and justify them at rate hearings. This paper emphasizes the use of IRR pricing models for statewide rate indications, with brief comments on other applications.

IRR pricing models have numerous variations. The models change continually, in conformity with changes in tax laws, insurance regulation, and financial theories. Although this paper uses a recent NCCI Workers' Compensation rate filing as an illustration, it does not attempt to document any particular model. Rather, it shows the framework of the analysis, and discusses the assumptions and results. It clarifies the working of Internal Rate of Return models, so that you can understand their use in rate filings and actuarial analyses.[1]

Point of View

One may examine insurance transactions from two points of view:

(1) *Insurer <—> Policyholder:* The policyholder pays premiums to purchase an insurance contract, which obligates the insurer to compensate the policyholder for incurred losses. These transactions occur in the *product market*, and prices are influenced by the supply of insurance coverage and the demand for insurance services.

(2) *Equity Provider <—> Insurer:* Shareholders, or equity providers, invest funds in an insurance company. The investment provides a return, whether of capital accumulation or dividends. These transactions occur in the *financial market*, and expected returns are influenced by the risks of insurance operations.

The two views are interrelated. The supply of insurance services in the product market depends on the costs that insurers pay to obtain capital, as well as the returns achievable by investors on alternative uses of that capital. Similarly, the expected returns in the financial market,

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[1] On the development of financial pricing models, see Hanson [1970], Webb [1982], and Derrig [1990]. For examples of the major models, see Fairley [1979], Hill [1979], NAIC [1984], Urrutia [1986], Myers and Cohn [1987], Mahler [1987], Woll [1990], Butsic and Lerwick [1990], Bingham [1990], and Robbin [1991]. For analyses of these models, see Hill and Modigliani [1987], Derrig [1987], And and Lai [1987], D'Arcy and Doherty [1988], Garven [1989], D'Arcy and Garven [1990], Mahler [1991], and Cummins [1990A; 1990B; 1991].

which are influenced by the risks of insurance operations, depend on consumers' demand for insurance services.

Yet the two viewpoints may require different assumptions, use different analyses, and lead to different results. The Internal Rate of Return insurance pricing model described here uses the equity-holders' viewpoint, whereas some other financial models use the insurer-policyholder perspective. For instance, the *discounted cash flow model* used in Massachusetts insurance rate regulation assumes that the capital markets are perfectly efficient (Myers and Cohn [1987]). Were there no federal income taxes on investment income, the transactions between equity providers and an insurer would have no bearing on the "fair" price of insurance policies in the Massachusetts discounted cash flow model. Although the Myers-Cohn model uses modern portfolio theory to determine the appropriate discount rate for valuing cash flows, it largely ignores the investment activities of insurance companies or of their stockholders.

Actuarial procedures traditionally approached rate making from this first perspective: the transactions between the insurer and its policyholders. Profits were related to premiums and losses; the capital structure of the insurance company was not considered. Much economic theory, as well as several sophisticated actuarial pricing models, continues along this vein.

Financial pricing models, such as the internal rate of return model, relate profits to assets or equity. Insurance cash flows in the product market, such as premiums, losses, and expenses, are of concern only insofar as they affect the transactions between the company and its stockholders. Of course, insurance cash flows are the major determinants of stock prices and therefore of stockholder profits. The focus here is *point of view*, not *cause*: how the actuary should measure profitability, not what factors influence profitability.[2]

A Non-Insurance Illustration of the IRR Model

The internal rate of return model determines premium rates by comparing (A) the internal rate of return that sets the net present value of a project's cash flows to zero, with (B) the opportunity cost of capital, or the return demanded by investors for projects of similar risk. The decision rule of the IRR model is "Accept an investment opportunity which offers a rate of return in excess of the opportunity cost of capital."[3]

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[2] Compare Cummins [1990B], page 126: ". . . actuarial models tend to focus on supply and demand in insurance markets and typically do not give much attention to the behavior of company owners beyond the assumption that they are risk averse. Financial models tend to emphasize supply and demand in the capital markets and typically neglect the product market beyond the implicit assumption that insurance buyers are willing to pay more than the actuarial values for insurance." See also Cummins [1991].

[3] See Brealey and Myers [1988], pp. 77-85, or Weston and Copeland [1986], pp. 111-120, for introductory expositions of the Internal Rate of Return model, and Sweeney and Mantripragada [1987] and Dorfman [1981] for additional treatment. Although criticized by some financial analysts, IRR models abound: ". . . the most commonly used discounted cash flow method among practitioners is the internal rate of return method" (McDaniel, McCarty , and Jessell [1988], page 369). Gitman and Forrester [1977], page 68, find that the "internal rate of return is the dominant technique" for capital budgeting analyses, with 54% of companies using it as their primary tool, compared to 10% for the net present value method. Internal rate of return models are the dominant financial pricing technique used in life insurance, though earnings, or "statutory book profits," are generally used in place of cash flows (Anderson [1959]; Sondergeld [1982]). In a recent survey of 32 insurers, internal rate of return was the most

The importance of "point of view" can be illustrated by comparing an IRR analysis of capital budgeting with an insurance pricing analysis. In capital budgeting decisions, internal rate of return analyses are often used to value investments that require an initial outlay of capital but promise increased revenues in subsequent time periods. In property/liability insurance operations, the issuance of a policy provides an immediate inflow of cash (the premium) to the insurer, but it obligates the insurer for future loss expenditures. From the viewpoint of the equityholders, though, the insurance operations are similar to other capital budgeting decisions.

Consider first a non-insurance investment decision: Using old production machinery, a firm has $250,000 of annual revenues from a particular product and $50,000 of annual expenses. For $100,000, It can buy new equipment with a two year life span and no salvage value, which would increase annual revenues to $300,000 and reduce annual expenses to $35,000. Should it purchase the new equipment? For simplicity, assume that the purchase costs are incurred at the beginning of the year, the increases in revenues and the decreases in expenses occur at the end of each year, and there are no federal income taxes.

Exhibit 1: Equipment Purchase Decision - Revenues and Expenses

Date	Purchase Cost	Old Equipment		New Equipment		Difference
		Revenues	Expenses	Revenues	Expense	
01/01/92	–$100,000					–$100,000
12/31/92		$ 250,000	$ 50,000	$ 300,000	$ 35,000	65,000
12/31/93		250,000	50,000	300,000	35,000	65,000

The table above shows the annual revenues and expenses with and without purchase of the new equipment. The right-most column summarizes the cash flow difference: the firm pays $100,000 on January 1, 1992, to purchase the equipment, and it gains $65,000 on 12/31/92 and 12/31/93 from increased revenues and lower expenses. The internal rate of return is the value of R which satisfies the equation

$$\$100{,}000 = (1+R)^{-1}(\$65{,}000) + (1+R)^{-2}(\$65{,}000),$$

or R = 19.5%.

Should the firm purchase the new equipment? The answer depends on the opportunity cost of capital: How much does it cost to raise the $100,000? If the cost is 15% per annum, then purchase the equipment. If the cost is 25% per annum, then continue with the old equipment.

Insurance IRR Models

Note the initial cash outflow in the example above: the firm invests money before it realizes

common measure of profitability, slighty exceeding "present value of profits as a percentage of premium" methods (Britton, Campbell, Davlin, and Hoch [1985], page 100; see also Exhibit 1, item III.A.I, on page 120). In life insurance terminology, return on investment (ROI) compares statutory income with statutory surplus, and return on equity (ROE) compares GAAP income with GAAP equity; see Smith [1987].

future revenues. Property/Liability insurance operations seem to show the opposite pattern: the insurer collects premiums before paying losses. But this ignores the equity commitments that support the insurance operations. From the viewpoint of the equityholders, there is indeed a "cash outflow" at the inception of the policy and "cash inflows" as the policy expires and losses are paid.[4]

Two aspects of insurance operations that reflect the equity holders' perspective are incorporated in IRR pricing models:

1. When an insurer writes a policy, part of the premium is used to pay acquisition, underwriting, and administrative expenses. The remaining premium dollars are invested in financial securities, such as stocks and bonds, to support the unearned premium reserve and the loss reserve.

2. Insurance companies "commit surplus" to support their insurance writings: that is, to assure that the company has sufficient capital to withstand unexpected losses.[5]

The cash transactions provide an inflow of funds to the insurer at policy inception, and an outflow as losses are paid. But the owners of the insurer must provide funds to allow the firm to write the policy, so there is a net cash outflow at policy inception from investors. Their return, as in the illustration of the new equipment purchase, occurs in future years, as the policy expires, losses are paid, and surplus is "freed."

Equity Flows

The Internal Rate of Return pricing model takes the viewpoint of the equityholders, who commit capital to support the insurance operations. Insurance transactions are of concern only insofar as they influence the surplus funding required. But how might one determine this influence? In other words, what equity is needed to support both the surplus account and other insurance operations?

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[4] See Cummins [1990A]: "An 'off-the-shelf' approach to insurance pricing, suggested in some rate hearings, views the problem from the company perspective, considering premiums as inflows and losses as outflows. While this is not necessarily incorrect, it can be misleading in an IRR context because the signs of the flows are opposite to those in the usual capital budgeting problem; the flows are positive initially and negative later on" (page 86), and "In the NCCI application of the IRR model, it is assumed that the insurer must make an equity commitment equal to the underwriting loss early in the policy period. Under these circumstances, the early flows are likely to be negative, paralleling the usual capital budgeting example" (footnote 13). [Actually, the equity commitment in the NCCI model is both to fund the underwriting loss and to support the risk of the insurance policy.]

Benjamin [1976] models insurer profitability by means of the commitment of surplus that supports the new business strain caused by a conservative valuation basis and then the release of surplus as losses are paid. Life insurers have a first year cash outflow (surplus strain) caused by high commissions and acquisition expenses, followed by net earnings (cash inflows net of required reserves) in subsequent years.

[5] There is no explicit obligation to commit surplus, but the practice is "enforced" by the NAIC IRIS tests and by the ratings issued by the A. M. Best Corp., Moody's, and Standard and Poor's. An insurer fails the first IRIS test if its ratio of premiums written to policyholders' surplus exceeds 300% (NAIC [1989]; Bailey [1988]). The ratio of loss reserves to surplus influences the Best's rating (Best's [1991], pages xiii-xiv). The Risk Based Capital formula being developed by the NAIC will strengthen the statutory surplus requirements (Hartman, et al. [1992]; Kaufman and Liebers [1992]).

Consider utility companies. Utilities build plants and procure equipment to provide their services. The money needed for this is termed "used and useful" capital (Hanson [1970]). But in insurance operations, there is no intrinsic relationship between policyholders' surplus and premium writings. Stockholders do not continually provide funds to support new policies, and they do not continually receive the monies back, with a return on their investment, as the losses are paid. The regulatory constraints set minimum capital levels, but they do not tell us what the appropriate surplus commitment is.

To determine appropriate surplus levels, some financial analysts examine the actual surplus held by insurer carriers, presuming an overall efficiency of capital markets (Griffin, Jones, and Smith [1983], page 383). Were the insurance industry overcapitalized, investors would withdraw their funds.[6] Conversely, they would invest additional funds if the insurance industry were undercapitalized. Capital market efficiency implies that the current industry surplus levels are necessary and sufficient for insurance operations. Note, however, that the use of IRR pricing models is not dependent on any particular assumptions about capital market efficiency, since appropriate surplus levels may be determined in other ways (Hofflander [1969]; Daykin, et al. [1987]; Pentikäinen, et al. [1989]).

Even if the amount of needed surplus is estimated from industry aggregates, the timing of the surplus commitment and of its release is an assumption in the IRR model. Both the amount of surplus and the timing of its commitment affect the equity flows and the internal rate of return.

To see the importance of equity flows, consider first the association of surplus with lines of business. (On the propriety of allocating surplus to line, see Section III below.) Often, surplus is allocated in proportion to loss reserves or premium writings, or a combination of the two. The procedure used is important, since the average lag between premium collection and loss payment is greater for the Commercial lines than for the Personal lines of business, and greater for the liability lines than for the property lines. An association of surplus with reserves attributes more surplus to the long-tailed lines of business than an association of surplus with premium does.

Both the allocation of surplus to line of business and the internal rate of return depend on the pattern of equity flows. If surplus is committed when the policy is written and is no longer needed when the policy expires, then a $1,000 Homeowners' policy requires the same surplus as a $1,000 Workers' Compensation policy does. If the surplus is committed when the unearned premium reserve is set up, and the required surplus declines as losses are paid and the unearned premium plus loss reserves decrease, then the Workers' Compensation policy needs more surplus. In most instances, the more surplus that is allocated to a policy, the lower will be that policy's internal rate of return.

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[6] Joskow [1973] argues that the efficient capital market hypothesis applies only if insurance policies are competitively priced. An overpricing of premiums may cause an overcapitalization of the industry, as investors strive for the higher returns. Danzon [1983], however, contests Joskow's analysis of rating bureau cartelization and overpriced insurance policies.

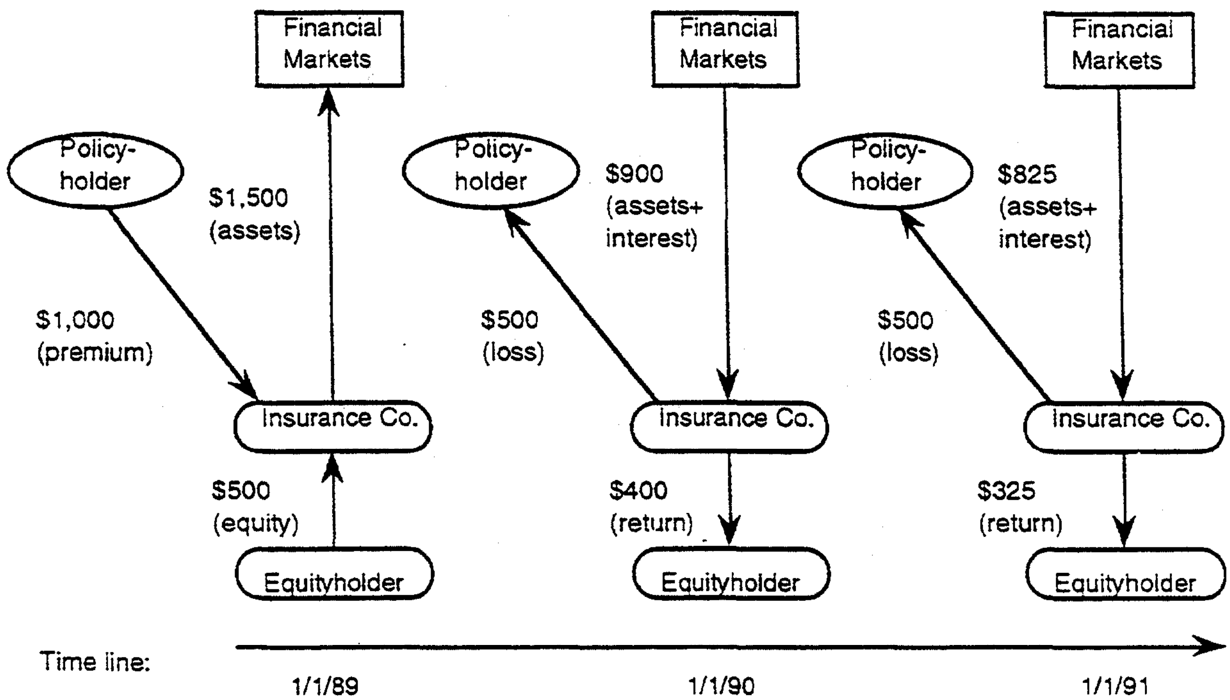
An Equity Flow Illustration

A simplified illustration of an insurance internal rate of return model should clarify the relationships between premium, loss, investment, and equity flows. There are no taxes or expenses in this heuristic example. Actual Internal Rate of Return models, of course, must realistically mirror all cash flows.

Suppose an insurer

- collects $1,000 of premium on January 1, 1989,
- pays two claims of $500 each on January 1, 1990 and January 1, 1991,
- wants a 2:1 ratio of undiscounted reserves to surplus, and
- earns 10% on its financial investments.

These cash flows are diagrammed below.



The internal rate of return analysis models the cash flows to and from investors. The cash transactions among the insurer, its policyholders, claimants, financial markets, and taxing authorities are relevant only in so far as they affect the cash flows to and from investors.

Reviewing each of these transactions should clarify the equity flows. On January 1, 1989, the insurer collects $1,000 in premium and sets up a $1,000 reserve, first as an unearned premium reserve and then as a loss reserve. Since the insurer desires a 2:1 reserves to surplus ratio, equityholders must supply $500 of surplus. The combined $1,500 is invested in the capital markets (e.g., stocks or bonds).

At 10% per annum interest, the \$1,500 in financial assets earns \$150 during 1989, for a total of \$1,650 on December 31, 1989. On January 1, 1990, the insurer pays \$500 in losses, reducing the loss reserve from \$1,000 to \$500, so the required surplus is now \$250.

The \$500 paid loss reduces the assets from \$1,650 to \$1,150. Assets of \$500 must be kept for the second anticipated loss payment, and \$250 must be held as surplus. This leaves \$400 that can be returned to the equityholders. Similar analysis leads to the \$325 cash flow to the equityholders on January 1, 1991.

Thus, the investors supplied \$500 on 1/1/89, and received \$400 on 1/1/90 and \$325 on 1/1/91. Solving the following equation for "v"

$$\$500 = (\$400)(v) + (\$325)(v^2)$$

yields "v" = 0.769, or "r" = 30%. ["V" is the discount factor and "r" is the annual interest rate, so $v = 1/(1+r)$.]

The internal rate of return to investors is 30%. If the cost of equity capital is less than 30%, the insurer has a financial incentive to write the policy. [The insurer may have other reasons for writing or not writing the policy, such a desire for market share growth, expectations about the future, or concerns about policyholder relationships; see Smith [1983] for a discussion of internal rate of return versus marketing objectives for writing insurance contracts.] Since we are analyzing these transactions from the stockholder's point of view, we compare the internal rate of return with the cost of equity capital.[7]

Actual IRR models are more complex. The following sections (i) describe the insurance cash flows, (ii) explain the surplus commitment and equity flow assumptions, (iii) show how to determine the cost of equity capital, (ii) provide an illustration from a recent Workers' Compensation filing, and (ii) discuss potential pitfalls in using IRR models.

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[7] Textbook presentations of the IRR model for other industries use the firm's weighted cost of capital, which is a combination of the cost of equity capital and the cost of debt capital. Since we are considering the equity holders' perspective, only the cost of equity capital is relevant (see Modigliani and Miller [1958] for further analysis).

Section II: Cash Flows

A. Premium Cash Flows

Insurance pricing models often assume that premium is collected at the inception of the policy. Although once true, this assumption is no longer valid. Large Commercial Lines risks may pay monthly premiums, may spread their premium payments over the first three quarters of the policy year, or may pay a deposit premium at inception and the remainder over the final three quarters of the policy year. A portion of the premium on policies subject to audit may not be collected until after the policy expires. Retrospectively rated policies, particularly for Workers' Compensation, may show return premiums at first adjustment, but additional premiums at subsequent adjustments. Insureds on "cash flow" premium payment plans may not pay the premium until shortly before the insurer expects to pay the loss. The resultant premium cash flow pattern on many retrospectively policies is from policyholder to insurers at inception, from insurer to policyholder at first adjustment (approximately 18 to 21 months after policy inception), and from policyholder to insurer at subsequent adjustments. Combining retrospectively rated policies with special payment plans, audited exposure bases, or cash flow plans produces intricate premium transactions.

An insurance pricing model must examine both premium collection and loss payment patterns. We commence with premiums, after two caveats: (1) Industry-wide *loss* payment patterns by line of business may be determined from Schedule P of the Annual Statement. Most insurers carefully analyze their own loss payment patterns, in greater detail and for more finely segmented blocks of business, to judge reserve adequacy. Industry-wide *premium* collection patterns are not available, and few insurers even track their own experience. (2) Loss payment patterns are stable from year to year, since they depend more on the external insurance environment, such as statutory compensation systems, court delays, and claim emergence, than on internal insurer operations, such as claim settlement philosophy. Premium payment patterns vary widely, since they depend on the payment plans offered by the insurer or chosen by the policyholder.

The pricing actuary can determine premium payment patterns for each block of business only after discussions with the underwriting, audit, and billing personnel at his company. The following discussion of Workers' Compensation premium collection patterns shows some of the factors that must be considered in an internal rate of return pricing model.

Premium Collection Patterns

Exhibit 2 displays sample Workers' Compensation premium cash flow patterns by quarter since policy issue.[8]

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[8] The pattern was developed by the Workers' Compensation Rating and Inspection Bureau of Massachusetts from a study of 350 risks with effective dates in 1986; see its *Filing for 1/1/91 Rates*, Section VIII, "Underwriting Profit Provision," pages 369-536. Exhibit 2 shows the "trimmed flow" from page 383 and patterns for two policy types from pages 386-389. See also MARB [1981], Section VIII, Subsection C, for an earlier study.

Exhibit 2: Workers' Compensation Premium Payment Patterns

Quarter	1	2	3	4	5	6	7	8	9	10	11	12+
All Policies	22%	21%	23%	21%	4%	3%	3%	−1%	0%	3%	1%	0%
Small GC	31	18	18	17	5	6	4	−1	1	0	0	0
Large RR	5	20	24	20	3	2	4	−9	4	22	5	1

Small GC – Guaranteed cost policies issued by stock companies; standard premium less than $5,000.
Large RR – Retrospectively rated policies issued by mutuals; standard premium greater than $500,000.

This payment pattern is for an annual policy written at the beginning of the first quarter. Since the premium is earned evenly over the year, it will be matched with an *accident year* loss payment pattern. Other applications of the IRR model, such as that used by the NCCI (see Section V), use a policy year model: that is, policies written evenly over the course of a year. This premium collection pattern must be matched with a *policy year* loss payment pattern.

Workers' Compensation premium is collected rather evenly over the policy term, since many large policyholders are billed as the premium is earned (NAIC [1990]). During the subsequent year, premium flows depend on standard premium audits (cash inflows) and first adjustments on retrospectively rated policies (cash outflows). For all insureds combined, the premium audits are generally larger than the first retrospective adjustments, and they precede them, on average, by one or two quarters. Second adjustments on retrospectively rated policies provide additional cash inflows in the tenth and eleventh quarters.

The premium cash flow pattern differs between small guaranteed cost policies, where much premium is paid up-front, and large retrospectively rated policies, where much premium is collected at the second adjustment (10th quarter). Small policyholders with guaranteed cost policies pay 50% of the premium within the first 6 months and the balance within the next 15 months. Large policyholders with retrospectively rated policies pay only 25% of the premium within the first 6 months; over 30% remains unpaid after 2 years. Note the return premiums at the first retrospective adjustment (-9% in the eighth quarter) and the additional premiums from the second adjustment (22% in the tenth quarter) for these policies. [The 9% is the net of the retrospective returns with the additional audit premiums.]

Much Commercial Lines insurance is distributed by independent agents, who sometimes hold the premiums received for a month or two before forwarding them to the insurer, at least in the first year that the policy is issued. In 1990, industry-wide agents' balances in course of collection were 7.4% of written premiums. (The percentage is greater for Commercial Casualty insurers. Premium balances, which include both agents' balances and accrued retrospective premiums, are about 10% of premium collected for Personal Lines writers but 30% of premium collected for Commercial Casualty writers; see Best's [1991A].) In other words, a portion of the premium paid by policyholders is not received by the insurer for several weeks. The magnitude of this delay, which depends on the distribution system used by the insurer, affects the premium collection pattern.

B. Loss Cash Flows

In many lines of business, losses are not paid until long after the accident has occurred. The average time lag ranges from a few months for Homeowners' and Automobile Physical Damage, to several years for Products Liability or Workers' Compensation. The investment income earned on the "float" between premium collection and loss payment is one stimulus for insurance pricing models that account for the time value of money.

Payment Lags

Several factors cause the lag between the occurrence and payment of claims:

- Loss adjustment procedures, such as claim filing and investigation, take several weeks or months, particularly when the insurer must first determine the liability of its insured.

- In Workers' Compensation and no-fault Personal Automobile, payments for lost income are paid periodically, as the income loss accrues. [Under tort liability systems, payments for anticipated future lost income are usually made as lump sum settlements.]

- An injury may not be reported by the victim until years after the accident or exposure that caused it, particularly for latent diseases, such as black lung or asbestosis, and for Professional Liability claims.

- Litigated claims often remain unpaid for long periods, since court backlogs, discovery procedures, and legal negotiations delay settlement.

Workers' Compensation claims show all these effects. Moreover, the lag between occurrence and payment is long and stable in Workers' Compensation. Annual Statement data shows an average lag of four to five years, with little fluctuation from year to year (Woll [1987]; see also Kahane [1978]; Noris [1985]).

Most insurers collect data on loss payment patterns, particularly if paid loss development methods are used for reserve indications. Alternatively, one can use industry patterns, as compiled by the A. M. Best Co. from Annual Statement Schedule P data.

The chart below shows a hypothetical 20 year countrywide loss payment pattern for Workers' Compensation.[9] Note the long tail: over 12% of losses remain unpaid after 10 years.

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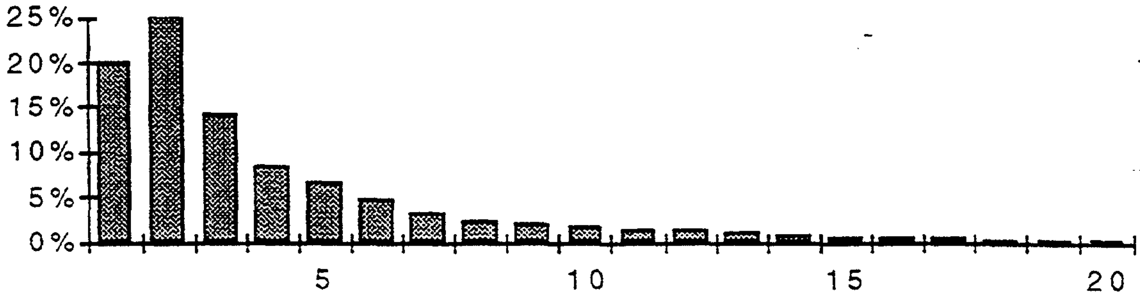
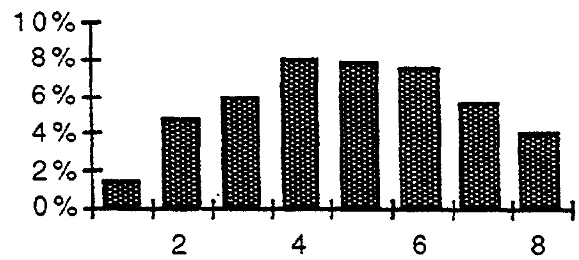
[9] The countrywide pattern is not appropriate for all states, since statutory benefit provisions and limitations on the size and duration of recoveries affect the expected payment patterns; see NCCI [1991], Section 5, pages 219-391, for paid loss patterns by state and Chamber of Commerce [1991], Part 2, pages 17-29 (or NCCI [1991], Section 4, pages 134-191) for benefit provisions by state.

WC Accident Year Loss Payment Pattern



The cash flow pattern for loss payments during the first two years is important for the IRR model, since the payments are not evenly distributed by quarter. Loss payments increase rapidly during the first four quarters since the inception of the policy, remain level from the fourth through the sixth quarters, and then slowly decline, as shown in the accompanying chart. The figures are shown as percentages of ultimate losses. 45% of losses are paid during the first two years: 1.5% in the first quarter, 5% in the second quarter, and so forth.

WC Quarterly Payment Pattern



Loss Adjustment Expenses

Loss adjustment expenses are a large component of insurance costs in several lines, particularly General Liability. Allocated loss adjustment expenses (ALAE), such as defense counsel fees, relate to specific claims. Unallocated loss adjustment expenses (ULAE), such as Claims Department salaries and overhead costs, can not be related to specific claims.

The cash flow patterns for losses, ALAE, and ULAE are different. ALAE is paid more slowly than losses, since defense counsel costs are greatest for the heavily litigated, slowly settling cases. ULAE is incurred primarily when the claim is first reported, since setting up files and investigating the cases consume a large portion of Claims Department salaries. The statutory distribution formula for paid ULAE prescribed in Schedule P uses two assumptions:[10] (1) Half of the ULAE is expended when the claim is reported, and half is expended when the claim is settled. (2) 90% of claims are reported during the calendar year during which the loss occurred, and 10% are reported in the subsequent year. The actual percentages vary by line of business. For instance, fewer than 75% of General Liability claims are reported during the calendar year in which the loss occurred, though 90% for Workers' Compensation are indeed reported by then.

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[10] See Salzmann [1988]; Feldblum [1991]. Alternatively, one may assume that ULAE is expended while a claim is open, with additional amounts paid when it is reported or settled (Kittel [1981]; Johnson [1989]).

For example, the average ULAE to loss ratio is 6% for Workers' Compensation. Many WC claims have periodic indemnity payments, with little Claims Department involvement, in contrast with GL bodily injury claims, where the insurer may negotiate with the claimant until the settlement date. Thus, most ULAE on long-duration disability cases is expended when the claim is first reported and the insurer investigates the injury. We assume that 60% of total Workers' Compensation ULAE is incurred when the claim is reported, and we distribute this figure in proportion to the claims emergence pattern: 90% in the first year and 10% in the next half year. The remaining 40% of ULAE is distributed in proportion to the WC loss payout pattern.

C. Expense Cash Flows

Traditional rate making procedures place little emphasis on expense costs. Historical loss ratios are developed and trended to project anticipated loss ratios during the future policy period. A target loss ratio is set as the complement of the expense ratio, which is determined from (i) Insurance Expense Exhibit data, adjusted for trends in expense levels, (ii) agency contracts, and (iii) company budgets. Pricing actuaries have refined the analysis of expenses, with emphasis on fixed versus variable expenses, expense flattening procedures, expenses by size of risk, and the relationship of expenses to policyholder persistency (McConnell [1952], Wade [1973]; Hunt [1978]; Childs and Currie [1980]; Feldblum [1990A]).

In theory, we should analyze expenses as rigorously as we analyze premiums and losses. In practice, several problems hinder this analysis:

1. *Data:* Few companies monitor their expense payment patterns. Loss payment patterns are shown in the Annual Statement, Schedule P, and are used to estimate bulk reserves. But most companies keep expense data only on a calendar year basis. The pricing actuary can find out the "other acquisition expense" in the month of January but he cannot always determine when that expense is paid in relation to the policy inception dates.

2. *Company:* Expense levels vary widely by company. Loss payment patterns depend more on external influences than on internal company operations. But expense levels and payment patterns depend on the company's distribution system, underwriting philosophy, and the services it provides.

3. *Risk Size:* Traditional rate making procedures assume that insurance costs vary directly with premium. For instance, premium taxes and some commissions vary with gross premium; losses and loss adjustment expenses vary with net premium. The assumption is not necessarily true for overhead expenses, such as policy issuance and billing costs, advertising, Home Office rent, and employee salaries, or for expenses that vary by policy size, such as Workers' Compensation commissions. In general, overhead expenses are a lower percentage of premium for larger risks. To set more equitable rates, actuaries use premium discounts in Workers' Compensation and expense flattening procedures in Personal Automobile.

4. *Policy Year:* Underwriting expenses, other acquisitions costs, and commissions for direct writers are higher in the first policy year than in subsequent years. Long-term expense costs depend on policyholder persistency: insureds who persist for several renewals have lower average annual expense costs that those who terminate in the first few years (Feldblum [1992B; 1993]).

In sum, the expense levels and cash flow patterns should depend on company characteristics, size of the risk, and new versus renewal underwriting. The following paragraphs discuss the types of expenses, following the Insurance Expense Exhibit categories, but they do not incorporate all the refinements mentioned above. Industry average Workers' Compensation expense levels are used as illustrations.

Types of Expenses

1. *State premium taxes* are paid quarterly as a percentage of the previous year's tax liability, with an adjustment in the first quarter of the following year to reflect the actual liability calculated on the prior year's written premium. If the volume of business is not changing, the premium tax cash flows should reflect the written premium pattern. Most Personal Lines premium is booked at policy inception, so the premium tax is paid in the first quarter. For large Commercial Lines risks, premium often is booked as it is earned, so premium taxes are incurred monthly throughout the policy year, as well as at audit dates.

Premium taxes and special assessments (such as for guarantee funds) vary by state. Workers' Compensation rates range from 2.5% to 8%, with a countrywide average of 3.5 to 4% (NCCI [1991], Section 3, pages 129-133). Certain assessments, such as those for second-injury funds and administrative costs, are sometimes based on incurred benefits, not only premium writings. Other taxes, licenses, and fees, which average 0.5% to 1% of written premium, appear with premium taxes on line 8 of the Insurance Expense Exhibit and may be included with premium taxes and assessments in the cash flow patterns.

2. The *commission* payment pattern follows the premium collection pattern. The overall commission level depends on several factors. For Personal Lines, the commission level varies by policy year: first year commissions may be 25% of premium and renewal commissions may be 5% of premium. For Workers' Compensation, commissions vary by size of risk, ranging from 2% to 15%. Contingent commissions depend on the profitability or persistency of an agent's book of business and vary by company. The industry average direct commission level for Workers' Compensation is 5.1 of adjusted direct written premium (Best's [1991A], page 119).

3. *Other acquisition expenses*, such as overhead expenses for agents' offices or for advertising, are difficult to model. First, the expenses are related to blocks of business, not to specific policies. Second, these expenses are incurred and paid several months before policies are issued. The industry average expense level for Workers' Compensation is 3.4% (Best's [1991A], page 119). A sample cash flow pattern might be 20% in the first quarter after policy inception, 50% in the quarter before policy inception, and 30% in the next preceding quarter.

4. *Underwriting and administrative expenses*, such as salaries of Home Office personnel or rent and utilities on company owned buildings, have a mixed payment pattern. Many of these expenses are incurred primarily when the policy is first written, so they vary by year since the original inception date.

General expenses incurred for Workers' Compensation average 4.9% of earned premium. Actuaries often assume that about half of these are underwriting expenses, which are incurred on or prior to the inception date. A sample cash flow pattern would be 65% in the first quarter (underwriting costs), 10% each in the next three quarters (other Home Office costs), and 5% during the second year (continuing actuarial, accounting, and systems costs for open cases).

These expense levels and sample cash flow patterns are summarized in the exhibit below. The cash flow pattern for "total expenses" is a weighted average (by expense level) of the patterns for each expense item.

Exhibit 3: Workers' Compensation Expense Levels and Cash Flow Patterns

Expense Type	Expense Level	Cash Flow Pattern by Quarter −1	0	1	2	3	4	5	6	7	8	9+
Taxes, licenses, fees	4.5%	0%	0%	50%	12%	12%	12%	4%	4%	4%	2%	0%
Commissions	5.1	0	0	25	21	20	20	4	3	2	3	2
Other acquisition	3.4	30	50	20	0	0	0	0	0	0	0	0
General expenses	4.9	10	25	25	10	10	10	6	2	1	1	0
Total expenses	17.9%	8%	16%	30%	12%	11%	11%	4%	3%	2%	2%	1%

D. Investment Cash Flows

The investment income earned by insurance companies has been the stimulus for financial pricing models. Yet the *investment* rate of return is different from the *internal* rate of return. The former relates to the insurer's transactions, whereas the latter relates to those of the equity holders in the insurance company (Smith [1987]; Griffin, Jones, and Smith [1983]).

IRR models for some other industries consider the cash flows from operations: sales (premiums), production costs (losses), and expenses. What the firm does with its extra cash, whether it invests the monies in financial securities, uses them to finance other activities, or pays larger dividends to stockholders, is not necessarily relevant to its pricing decisions. The firm's financial holdings are distinct from its business operations.

Insurance operations are different. Financial holdings corresponding to the premium, loss, and loss adjustment expense reserves are needed to pay claims covered by the policies. The remaining assets, corresponding to surplus, support the insurance operations. The financial holdings can not necessarily be used to pay stockholder dividends or to finance other activities.

Investment Yield and Internal Rate of Return

Thus, there are two "rates of return" in the IRR insurance pricing model. The internal rate of return is earned by equity holders for supplying funds to the insurance company. The investment rate of return is earned by the insurance company for supplying funds to the stock or bond markets. The investment returns are a part of the insurer's business operations from which the internal rate of return is determined.

Although the internal rate of return and investment rate of return are distinct, there are close links between the two. If the returns are expressed in nominal dollars, both will vary with economic inflation. If the returns are expressed in real terms, the connection is more subtle. The internal rate of return varies with the riskiness of the insurance operations. If the insurer invests in higher risk securities, such as common stocks and junk bonds, rather than in low risk securities, such as Treasury notes, the insurer's investment return will be high, but so will the cost of equity capital, or the internal rate of return demanded by equity holders.[11]

To illustrate the derivation of the investment rate of return and the opportunity cost of equity capital, Exhibit 4 shows the 1989 insurance industry financial portfolio and average rates of return, using data from the A. M. Best Corp., DRI, and Value Line. The investment yields are "new money rates of return," not imbedded yields or portfolio returns. The distribution of securities is taken from the actual industry portfolio, which consists predominantly of municipal bonds, Treasury securities, investment grade corporate bonds, and common stocks, with smaller proportions of mortgage loans, short term investments, and other holdings (such as real estate).[12]

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[11] When considering the inclusion of investment income in the rate making procedures, some observers have wondered: "Should policyholders receive the benefits of a successful investment strategy? And should they incur the costs of a poor strategy?" To avoid shifting investment risk to policyholders, some pricing models use "risk free" interest rates (Cooper [1974], Wohl [1987], Bingham [1990]). Other models, such as that described in this reading, use expected returns determined from the insurer's financial portfolio and base the "target" rate of return on the cost of equity capital. An insurer with an aggressive investment strategy may earn higher investment yields. The additional investment risk, though, raises the opportunity cost of equity capital, so there is an offsetting effect on premium rates.

[12] The 1989 financial holdings by type of security are taken from Best's [1990]. Bonds and stocks are from the Annual Statement Schedule D, "Summary by Country," with amortized values for bonds, market values for common stocks, and statement values for preferred stocks. The remaining invested assets are taken from the Annual Statement Balance Sheet, page 2, "Assets," lines 3 through 7.

Yields on bonds are available from such sources as DRI (= Standard & Poor's), the Wall Street Journal, or Moody's. For bonds, the most recent new money yield is a good proxy for the expected new money yield; that is, the analyst may have no better data than the current rates to forecast future rates. (On the rationale for using new money yields versus portofio returns, see Cummins and Chang [1983].) As Robert Butsic has pointed out to me, an adjustment for default risk should be subtracted from the reported returns, though it is hard to quantify the amount (Vanderhoof, et al. [1989]; Altmann [1989]). For common stocks, expected yields must be estimated, using a CAPM or DGM model (see Section IV). Average yields in 1989 for the other securities are taken from Best's [1990], page 2, except for "Other invested assets," whose high reported return (18.2% in 1989 and 17.7% in 1990) seems unusual.

Exhibit 4: Property/Liability Insurance Company Investment Portfolios and Average Yields

	1989 Holdings ($000,000)	Percent of Invested Assets	Expected Yield	Tax Rate	Expected After-Tax Yield
Treasury Bonds	81,107	18.2%	8.5%	34.0%	5.6%
Municipal Bonds	151,407	33.9	7.0	5.1	6.6
Corporate and Foreign Gov't Bonds	78,958	17.7	9.2	34.0	6.1
Preferred Stocks	10,956	2.5	8.8	13.8	7.6
Common Stocks	73,049	16.3	15.0	26.9	11.0
Mortgage Loans	6,461	1.4	9.9	34.0	6.5
Cash and Short-Term Investments	33,853	7.6	8.9	34.0	5.9
Other Invested Assets	10,748	2.4	10.0	34.0	6.6
Total:	446,539	100%	9.3%	22.5%	7.0%

Taxes on Investment Income

The Federal Income Tax rates reflect the provisions of the 1986 tax amendments. All investment returns except stock dividends and municipal bond yields are taxed at 34%. The "Dividends Received Deduction" provides a tax exemption of 70% of most common and preferred stock dividends received by corporations; the remaining 30% of dividends is taxed at 34%. The "Proration" provision subjects 15% of tax-exempt municipal bond income and the dividends received deduction to the 34% tax rate, unless the securities were owned before Aug 8, 1986.

Thus, interest on municipal bonds is taxed at (15%)(34%), or 5.1%. Stock dividends are taxed as follows:

Dividends received deduction:	(.70)	x	(0.15)	x	(.34)	= 3.6%
Remaining dividends:	(.30)	x	(1.00)	x	(.34)	= <u>10.2%</u>
Total:						13.8%

The dividends received deduction does not affect capital gains and losses, which are taxed at 34%. In 1989, insurers earned $2,509 million in common stock dividends and $12,169 million in realized plus unrealized capital gains on common stocks (Bests's [1990], page 62). This high proportion of 1989 common stock returns coming from capital gains (83%) reflects the bull market of the 1980's, not a predominance of growth stocks. Using 65% as the expected percentage of common stock returns coming from capital gains, the combined tax rate is

$$(65\%)(34\%) \quad + \quad (35\%)(13.8\%) \quad = \quad 26.9\%.$$

For the overall financial portfolio of the Property/Casualty insurance industry, the 1989 pre-tax return was 9.3%, the average tax rate was 22.5%, and the after-tax return was 7%.

Adjustments for expenses, depreciation on real estate, and other "write-in deductions" reduce the expected investment return. Investment expenses do not relate to the writing or servicing of insurance policies, so they are not included in the expense cash flows above. Rather, investment expenses are deducted from the gross investment returns to determine net investment returns. Similarly, depreciation on real estate and other "write-in deductions from investment income" must be subtracted from the gross totals above.

In 1989, the industry reported $2,176 million of such expenses, depreciation, and other deductions, or 0.5% of the $447 billion in invested assets shown above (Best's [1990],p. 62). The full 34% tax rate is applicable to these expenses and deductions, so the after-tax yield is reduced by 0.3%. The expected net investment yields are 8.8% pre-tax and 6.7% after-tax.

E. Federal Income Taxes

Federal income taxes depend both on the magnitude and incidence of the firm's earnings and on the accounting methods of the company. For all but the smallest firms, the tax rate on underwriting income is 34%. The incidence of taxes is determined by statutory accounting, federal statute, and IRS regulation.

Until 1987, taxable income was generally based upon statutory income as reported in the NAIC accounting blank, with adjustments for tax exempt income, depreciation and amortization of assets expensed on the Annual Statement, and other miscellaneous adjustments. Two provisions of the 1986 Tax Reform Act changed this relationship, by accelerating the timing of federal income taxes. First, "revenue offset" includes in current income 20% of the change in the unearned premium reserve, as a proxy for deferred acquisition expenses (DAC). Second, insurers must discount loss reserves, using a prescribed interest rate and either an industry wide or a company payment pattern.[13]

Several relationships between tax provisions and other actuarial assumptions should be noted:

1. *Revenue Offset:* The IRS assumes a 20% deferred acquisition cost expense ratio for all lines. The actual expense ratio differs by company and by line of business, ranging from 13% in Workers' Compensation to 37% in Fire Insurance (Best's [1991A]; Feldblum [1992B]).

2. *Discount Rates:* The prescribed interest rate is a 60 month moving average of yields on fixed income Treasury securities with maturities between 3 and 9 years. If interest rates are stable, this is similar to new money rates earned by insurers.

3. *Surplus Commitments:* Investment income on surplus funds is taxed. Were equity holders provided no return from insurance operations, they would prefer to invest their funds

[13] See NCCI [1987]; Gleeson and Lenrow [1987]; Almagro and Ghezzi [1988]; and Friedman and Heitz [1988]. The 1986 Act also includes a transition provision whereby 20% of the December 31, 1986 unearned premium reserve is brought into income ratably over six years commencing in 1987. In addition, insurers are subject to an Alternative Minimum Tax, if this tax is greater than the normal income tax. The AMT may be viewed as an additional cost of business, since it restricts otherwise optimal investment and underwriting strategy (see the references cited above). The complexities of the federal income tax regulations prevent their full treatment in this reading.

directly in the capital markets, rather than provide it to insurers and face double taxation.[14] To induce equity holders to invest in the company, insurers must use earnings from insurance operations to compensate equity holders for the tax on capital and surplus funds.

The incidence of this tax depends on the assumed surplus commitment pattern. If surplus is committed in proportion to premiums earned, this tax is incurred during the policy year. If surplus is committed in proportion to loss plus LAE reserves, much of this tax is incurred in subsequent years.

Federal income taxes are a complex but essential aspect of financial pricing models. The pricing actuary must consider the relationship between the tax effects of the insurance transactions and the insurer's overall tax situation. Further analysis of federal income taxes is provided in the illustration in Section V.

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[14] The "double taxation" stems from the corporate income tax paid by the insurer and the personal income tax paid by its equity holders. Suppose an investor provides $1 million to an insurer, which uses this money to buy stocks or bonds, providing a 10% return. Of the $100,000 investment income, $34,000 is paid in corporate income taxes, and $66,000 is disbursed as dividends to equity holders. Of this $66,000, an additional amount, say $22,000, is paid in personal income taxes, and the equity holders remain with $44,000. The equity holders would prefer to invest directly in stocks and bonds, and have federal income taxes deducted only once; see Myers and Cohn [1987], pages 57-58; Cummins [1990A], page 84; Garven [1987].

Section III: Surplus

Financial pricing models calculate a return on the capital used by a firm to furnish goods or services. Manufacturers use fixed assets, such as plants and equipment, to produce their output. The return on investment relates the firm's output to the capital required for its production.

The insurance contract is a promise: a commitment to compensate policyholders if certain contingent events occur. The worth of the promise depends on the integrity and financial strength of the insurer. The surplus held by an insurer backs its promises. An insurer could not fulfill its obligations if it lacked the financial assets that support them, even as a manufacturer could provide no output if it lacked the fixed assets to produce it.

There are important differences between the manufacturer's fixed assets and the insurer's surplus. The fixed assets needed for production can be objectively measured, for the firm as a whole and often for each product line which it produces. The surplus "needed" by an insurer, and its allocation to lines of business and time periods, is a theoretical construct. The pricing actuary using an IRR model may assume a relationship between surplus commitment and insurance transactions. He can examine the assumption for reasonableness, but he can not always test it empirically.

The Individual Firm and the Industry

This difference between (i) insurance pricing and (ii) the uses of internal rate of return models in other industries has several implications:

- IRR models used for capital budgeting decisions examine the funds invested by a particular firm and the returns expected by that firm. A firm may invest more funds if it anticipates larger returns. Similarly, if it invests less funds, it may generate less revenue.

  For the insurance pricing model, the assumption of capital markets efficiency implies that the industry is neither over- nor under-capitalized. But any given insurer, because of favorable or adverse operating results in the past, may have more or less surplus than it needs. Whereas manufacturers of similar products often have similar levels of operating leverage (that is, the ratio of sales to fixed assets), insurers writing identical lines of business have diverse levels of insurance leverage (that is, the ratio of premium to surplus). The actual surplus held by an insurer may reflect historical happenstance, not deliberate strategy.

- The internal rate of return model concentrates on the *capital market*: the returns earned by providers of equity to insurance firms and the opportunity cost of this capital. The prices of insurance contracts are determined in the *product market*, by the supply of and demand for insurance policies.

  At the industry level, the capital and product markets are intertwined. If marketplace prices are inadequate, and industry returns are below the opportunity cost of capital, firms will leave the industry and prices will rise. This relationship does not necessarily hold for the individual firm. Industry rates may be adequate, but the firm's internal rate of return may be depressed by operating inefficiencies.

In sum, the required surplus is a theoretical construct. Even if the industry as a whole has an appropriate surplus level, individual firms may be over- or under-capitalized. Moreover, the relationship between the product market and the capital market is strong for the industry as a whole but weak for any individual firm.

Some analysts therefore determine the surplus commitments in the IRR model from industry-wide practice (Griffin, Jones, and Smith [1983], page 383). The surplus actually held by the individual insurer is ignored, and replaced with the required surplus posited by the actuary.

Surplus Allocation

Many financial pricing models examine returns on an insurer's surplus or equity. Surplus exists for the company as a whole, not for each particular line of business. Whether surplus can be allocated to line of business is a disputed issue.[15] Even when an allocation is needed for regulatory purposes or for a financial pricing model, many allocation methods are possible.

This paper does not contend that an allocation of surplus by line of business is theoretically "correct." But an allocation of surplus is needed for the internal rate of return model, and the pricing actuary must posit surplus allocation assumptions. The following pages highlight two aspects of these assumptions:

1. How is surplus allocated? Actuaries generally allocate surplus in proportion to another base, such as premium writings, statutory reserves, or the present value of future loss

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[15] In testimony regarding California's Proposition 103, Bass [1990] says: "By its fundamental nature, surplus is not allocatable, whether to line of business, to jurisdiction, or to any other segment of an insurer's operation" (page 231). Similarly, McClenahan [1990] says: "The fact is that the entire surplus of an insurer stands behind each and every risk" (page 152). After reviewing several allocation methods, Kneuer [1987] concludes that none "addresses the philosophical questions that underlie any attempt to allocate surplus" (page 224). See also Roth [1992], who argues against surplus allocation and proposes an alternative measure of return.

These quotations relate primarily to solvency, which differs from pricing. As Peter Murdza has pointed out to me, "allocation for ratemaking purposes only does not mean that surplus is actually allocated for solvency or other purposes." Similarly, Callaghan and Derrig [1982] say: "A company's surplus is not in fact or in law allocated by line and state. A company's entire surplus is available to meet the losses on any line in any state. . . .The fact that surplus is not actually allocated by line and state does not, however, mean that it need not be allocated for purposes of determining an appropriate underwriting profit provision for each line. As noted above . . . Massachusetts law requires the determination of rates by line. Thus it is not only appropriate but required that the ratemaker . . . consider surplus by line, just as other elements of the rate-making methodology must be considered by line. . . .Such consideration requires that surplus be allocated by line and state for purposes of rate-making, even though it is not allocated by line and state by law. Indeed, such allocation is unavoidable. Any profit methodology which purports to determine profit provisions by line assumes an allocation of surplus by line and state." In a paper arguing against surplus allocation for regulatory purposes, Bass and Khury [1992], page 563, note: ". . . nothing we say here should be construed as challenging the idea of allocating surplus for the purposes of deriving estimates of branch office profitability, deriving estimates of of business profitability, etc., for purposes of *internal* management of an insurer's operation."

payments. The *timing* of the surplus allocation is equally important: when is the surplus "committed," and when is it "freed"?

2. Should the surplus allocation depend on the type of policy? For instance, does a retrospectively rated Workers' Compensation policy require less surplus than a guaranteed cost policy? Does a claims-made policy require less surplus than an occurrence policy?

Premiums and Reserves

The ratio of written premium to policyholders' surplus is a common test of surplus adequacy (NAIC [1989]; Best's [1991B]; Ludwig and McAuley [1988]). The test indicates when surplus is so low that the insurer's solidity should be examined. This surplus adequacy test may be extended to the surplus allocation issue, which implies that

- Required surplus varies directly with premium. If one line has twice the premium of a second line, the first line needs twice the surplus commitment.

- Surplus is committed when the premium is written, and it is released when the policy expires.

An alternative base is the reserves held for the block of business, or the anticipated future loss and expense payments. The corresponding implications are

1. Required surplus varies directly with reserves. If one line has twice the reserves of a second line, the first line needs twice the surplus commitment.

2. If the allocation base is loss and expense reserves, then surplus is committed when the losses occur, and it is released when the losses are paid. If the allocation base includes the loss portion of the unearned premium reserves in addition to loss reserves, then surplus is committed when the policy is written, and it is released when the losses are paid.[16]

These assumptions affect the internal rate of return for each line of business. Increasing the surplus allocated to a line moves the internal rate of return closer to the after-tax investment yield. The insurance industry's cost of equity capital, as well as its total return on capital, generally exceeds the after-tax investment yield, so increasing the surplus allocation decreases

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[16] The NCCI uses undiscounted reserves in its IRR model (see Section V). Cummins [1990A] criticizes this: ". . . the present market value, not the book value, of liabilities must be used in the NCCI approach. Thus, the surplus-to-reserves ratio should be based on the estimated market value of liabilities, not the book value." Butsic [1988] also uses the present value of reserves.

Mixtures of the premium and reserves bases are also possible. For instance, Myers and Cohn [1987] use the statutory premium to surplus ratio to allocate surplus by line of business, but they use loss payments to model the timing of surplus flows between the equity providers and the insurer. Daykin, *et al.* [1987] use technical provisions (loss reserves) to determine the required asset margin for existing business and net written premium to determine the required asset margin for future business.

the internal rate of return.[17]

Long- and Short-Tailed Lines

Slow paying lines with large loss reserves, such as Workers' Compensation and General Liability, receive a greater allocation of surplus if reserves are used as the base rather than premiums. This result may also be viewed as a timing phenomenon: when reserves are the allocation base, the surplus is committed for longer periods in the slow paying lines than in the quick paying lines.

For instance, suppose an insurer writes $100 million of annual business in each of two lines, Homeowners' and Workers' Compensation, and it holds $100 million of surplus. The expected loss ratio is 50% for Homeowners' and 75% for Workers' Compensation. The average duration between occurrence and payment of a claim is half a year for Homeowners' and four years for Workers' Compensation.

If premium is used as the allocation base, the $100 million of surplus is split equally, half to Homeowners' and half to Workers' Compensation. If reserves are used as the allocation base, the split is different. The anticipated Homeowners' losses of $50 million a year, using the expected 50% loss ratio, spend half a year as reserves. In a steady state, there are $25 million of reserves at any point in time. The anticipated Workers' Compensation losses of $75 million a year, using the expected 75% loss ratio, spend four years as reserves. In a steady state, there are $300 million of reserves at any point in time. Twelve times as much surplus would be allocated to Workers' Compensation as to Homeowners'.[18]

Insurance Risks

Surplus protects the insurer against several types of risk. Asset risk is the risk that financial assets will depreciate (e.g., bonds will default or stock prices will drop). Pricing risk is the risk that at policy expiration, incurred losses and expenses will be greater than expected. Reserving risk is the risk that loss reserves will not cover ultimate loss payments. Asset-liability mismatch risk is the risk that changes in interest rates will affect the market value of certain assets, such as bonds, differently than that of liabilities. Catastrophe risk is the risk that unforeseen losses, such as hurricanes or earthquakes, will depress the return realized by the insurer. Reinsurance risk is the risk that reinsurance recoverables will not be collected. Credit risk is the risk that agents will not remit premium balances or that insureds will not

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[17] However, if the premium rates are so inadequate that the net present value of the insurance cash flows discounted at the after-tax investment yield on surplus funds is negative, then increasing the surplus commitment increases the internal rate of return. See Section VI for further discussion of this issue.

[18] Using the present value of future loss payments instead of undiscounted statutory reserves reduces the spread between slow paying and quick paying lines of business. An alternative adjustment for the illustration in the text is to substitute a discounted loss ratio for the 75% undiscounted WC loss ratio. The higher expected loss ratio in Workers' Compensation than in Homeowners' reflects the greater investment income in the former line.

remit accrued retrospective premiums.[19]

Pricing and catastrophe risk occur during the policy period; other risks continue until all losses are paid. For instance, an insurer may write a General Liability or Workers' Compensation policy on January 1, but it may have paid only 20% of the incurred losses by December 31. Many financial pricing models therefore allocate surplus based on unpaid losses, in addition to or instead of premium writings.

If reserves are used as the surplus allocation base, what reserves should be used? Loss and loss adjustment expense reserves support claims that have already occurred, whether or not they have been reported to the insurer. Unearned premium reserves, minus the "equity" reflecting acquisition and underwriting expenses, support the claims anticipated over the policy term.

Legally, the insurer is not always "at risk" for the unexpired portion of the policy, since in many jurisdictions it may unilaterally cancel contracts, particularly in the Commercial Lines. [In Personal Lines, several states prohibit mid-term cancellations, or even non-renewals, except for non-payment of premium.] In practice, an insurer rarely cancel contracts in mid-term, so pricing risk continues through the expiration date. Moreover, the pricing risk during the policy term is generally greater than the reserving risk that remains after the policy expires (Butsic [1988]).[20]

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19 See Hofflander [1969], pages 72-74, Pentikäinen, et al. [1989], Section 3, Kaufman and Liebers [1992], and Feldblum [1992A]. Life actuaries divide insurance risk into three parts: asset risks (C-1), pricing risk (C-2), and the financial effects caused by changes in interest rates (C-3). For the SOA classification system, see Hickman, Cody, Maynard, Trowbridge, and Turner [1979] and CAS [1991]. On asset risks, see Sega [1986], Cody [1988], and Vanderhoof, Albert, Tenenbein, and Verni [1989]; on C-3 risk, see SOA [1987], Geyer [1989], and Mereu [1989]. For a recent treatment of all risks, see Steinig, et al. [1991].

20 The NCCI's IRR model does not consider pricing risk, explaining that "for workers compensation, surplus exists solely to support the capacity to pay *incurred* loss claims; there is almost no danger that unearned premium reserves will be affected by adverse exposure, as in lines subject to catastrophe" (NCCI [n.d.], page 5); but note that the NCCI illustration in Section V uses unearned premium reserves in addition to loss reserves for the surplus commitment assumption. The NCCI also uses surplus to fund the underwriting loss on the insurance contract, since it uses undiscounted loss reserves. Cummins [1990A] suggests that this is not appropriate: "The NCCI model assumes that the company fully funds the underwriting loss at policy inception. The NCCI maintains that its approach is consistent with the realities of a regulated insurance market where loss liabilities must be fully funded at nominal values. . . . In a regulated industry, there may be a justification for departing from financial theoretic principles provided that the departures realistically reflect the impact of regulation on the firm's market value. . . . The issue is whether or not the requirement that the company set up nominally valued reserves actually affects the market value of the firm; and, if so, whether its effect is captured accurately by the NCCI model. . . . it is not unknown for insurers to be significantly underreserved for sustained periods of time without incurring regulatory intervention. It seems unlikely that the statutory reserve constraint is as stringent as the NCCI model assumes" (page 95); and: "The NCCI approach is correct only if this treatment of surplus is an economic reality, i.e., if writing a block of policies requires the firm to completely forego the use of surplus equal to the underwriting loss early in the policy period rather than funding the loss more gradually out of investment income as losses are paid . . . This depends upon the stringency of regulatory reserving and premiums-to-surplus constraints" (page 101).

Arguments can be made for both the NCCI and Cummins positions, as in the following simulated dialogue. (NCCI:) The NCCI model is used for rate filings, where adherence to statutory accounting principles is required; the fact that

Policy Form

Surplus is committed to guard against the risks to the insurance company. In most insurance transactions, the insured pays a fixed premium and the insurer is liable for random loss occurrences. Surplus protects policyholders from adverse fluctuations in loss payments that might threaten the insurer's solvency.

Occurence contracts subject the insurer to liability for accidents that occur during the policy term. Claims made forms impose this liability only for claims reported to the insurer, or accidents reported to the insured, during the policy term. The elimination of much of the IBNR liability reduces the loss uncertainty and – in theory – the surplus needed to support the risk.

Service contracts, where the insurer handles claims but does not incur loss liabilities, involve no insurance risk (though the risk remains that expense charges will not cover actual expenses incurred). In fact, since the insurer receives expense fees but no premium, no surplus is required for statutory financial statements. Capital is needed to hire the requisite personnel, but no surplus is needed for insurance "risk."

Retrospective rating is a cross between an insurance policy and a service contract. There is little insurance risk in the primary layer, where losses are fully reimbursed by the policyholder, though there is the "credit risk" of the insured's failure to pay the premiums (Livingston [1982]; Greene [1988]; Brown [1992]). There is substantial risk where the loss limit or the maximum premium curtails the reimbursement. Overall, the retrospectively rated policy is more risky than a service contract but less risky than a prospectively rated insurance policy.

Although risk varies by policy type, there is no simple procedure to account for this. Some applications of the IRR pricing model make no distinction among retrospectively rated policies, excess coverage, large deductible policies, and first dollar contracts. Other applications count only premium and reserves for which true insurance protection is provided. For retrospectively rated policies, this would be the insurance charge and the reserves for claims in excess of the loss limit, or claim payments that would not be reimbursed by the policyholder because the ultimate premium exceeds the maximum premium (see Simon [1965]; Snader [1990]).

Equal treatment of all policy types overstates the risk on retrospectively rated policies and understates the risk on excess coverage. Considering only premium and reserves which cover true insurance risk understates the risk for retrospectively rated polices, excess coverage, and large deductible policies, since loss fluctuations are greater on higher layers of coverage. The ideal procedure lies somewhere in the middle, though the continual changes in policy forms and insurance services make this terrain too slippery for fixed techniques.

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many insurers are underreserved in practice is not necessarily germane. (Cummins:) If so, one should use reserves to suplus ratios derived from grossed up reserves and restated surplus. This would raise the ratios and lower the underwriting profit provision needed to obtain the desired internal rate of return. (NCCI:) The NAIC requires the low leverage ratios, in addition to full value loss reserves; the IRR model must comport with statutory requirements.

Section IV: The Cost of Equity Capital

The magnitude and timing of the insurance cash flows (premiums, losses, expenses, investment income, taxes) and the surplus commitment assumptions allow us to determine the internal rate of return on investor supplied equity. The insurer has a financial incentive to write the policy if the internal rate of return exceeds the opportunity cost of this equity capital. But what return do investors demand for the use of their money?

If the capital market is efficient, then the returns achieved by investors is an indicator of the returns needed to elicit equity capital, assuming that all other factors remain steady. If the returns that investors achieved were less than what they demanded, they would withdraw some capital (Bailey [1967]; but cf. Matison [1987]). With lower industry capacity, premium rates would rise, underwriting standards would tighten, and overall returns would increase. Conversely, if the returns that investors achieved were greater than what they demanded, they would commit additional capital. The combination of greater industry capacity, more firms, and increased competition in the insurance market would cause premium rates to fall, underwriting standards to loosen, and overall returns to decline.

If other factors are changing, the returns demanded by investors may not be equal to the returns achieved in the past. For instance, if inflation accelerates, investors demand higher returns to compensate them for the reduced value of money. Similarly, an increase in the perceived risk of holding insurance stocks, due perhaps to more stringent rate regulation, lower profits, and an increased number of insolvencies, may cause investors to demand higher returns.

This section presents two models for estimating the return on stockholder supplied equity, or the opportunity cost of equity capital. It begins with the Dividend Growth Model (DGM), which directly estimates the cost of equity capital but relies on uncertain projections about future dividend payments. It proceeds to the Capital Asset Pricing Model (CAPM), which relies on observed historical data, but whose theoretical foundations are often questioned.[21]

The Dividend Growth Model

What determines the prices of stocks? The stock certificate is a piece of paper, with no intrinsic worth. In a free market, of course, its value is determined by the forces of supply and demand: what others are willing to pay for it. But this only begs the question: What determines how much others are willing to pay for the stock certificate?

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[21] Returns on either statutory surplus or GAAP equity may be derived from financial statements. The historical returns on surplus derived from Annual Statement figures are often used by regulators for estimating "equitable" returns. Growth rates, payment lags, and accounting conventions cause the financial and accounting estimates of return on equity (or surplus) to diverge (Anderson [1972]; Feldblum [1992B]; Butsic [1990]; Bingham [1992]; see also Beaver, Kettler, and Scholes [1970]). In general, financial models are better indicators of the returns demanded by investors. On the cost of equity capital for insurers, see Haugen and Kronke [1971], Quirin and Waters [1975], Lee and Forbes [1980], and Cummins [1992]. On the application of financial methods to insurance, see D'Arcy [1989]).

A stock certificate is a financial asset, like a bond. The worth of a bond is determined by the cash payments to the owner: semiannual coupons and the par value at maturity. At any time, the worth of a bond is the present value of these future cash payments.

A stock has three differences from a bond.

- First, the stock never matures: there are periodic dividends, but no "repayment of principal at maturity."
- Second, the dividend payments are less certain. If the firm faces financial difficulties, it may eliminate or reduce a dividend payment. If it earns unusually large profits one year, it may provide a larger dividend.
- Third, bond coupons have fixed amounts. Stock dividends are not fixed in nominal terms, but generally grow with monetary inflation and with the earnings of the firm.

If we knew the amounts of all future dividend payments, we could estimate the price of the stock as the present value of the future cash flows. The actual future dividends are uncertain, but we can use historical experience to forecast them. To determine present values, we must know the appropriate discount rate, which is the opportunity cost of equity capital. So if we know the current price, and we forecast future dividends, we can solve for the discount rate.[22]

Forecasting future dividends is a difficult task. To simplify, assume that the firm's earnings, assets, dividends, and stock price are all increasing at a constant rate. This growth rate, in combination with the dividend to price ratio, determines the cost of equity capital.

For example, suppose a firm is growing 10% per annum, its stock price increases at the same rate, and it pays an annual dividend at the end of each year equal to 5% of its stock price. What is the return to the equity holders in this firm?

Imagine an investor who buys a share of common stock for $100 on January 1, receives the dividend on December 31, and then sells the stock. (The $100 price is arbitrary; any price gives the same result.) On December 31, the stock price is $110 (10% per annum capital appreciation), and the dividend is $5.50. The annual return to the investor, or the cost of equity capital, is ($10 + $5.50) / $100, or 15.5% (Butters, et al. [1981], page 140).

Derivation of the DGM

In mathematical terms, let

K be the cost of equity capital,
D be the stockholder dividend at the end of the previous year,
P be the stock price at the beginning of the year, and
G be the anticipated (uniform) growth rate of stockholder dividends.

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22 On the Dividend Growth Model, see Gordon and Shapiro [1956], Sharpe and Alexander [1990], chapter 16, Weston and Copeland [1986].

We assumed above that all financial characteristics of the firm, such as earnings, assets, stock price, and dividends, are growing at the same rate. This is the common situation, since dividends can not grow indefinitely if earnings do not keep pace. The mathematical derivation, though, only needs the growth rate of dividends (hence the name Dividend Growth Model).

On January 1, the investor pays P for the stock. If the firm grows 100G% per annum, he can sell the stock on December 31 for (P)(1 + G). In addition, he receives the stockholder dividend on December 31. The dividend the previous year was D, so this year it will be (D)(1 + G). The return to the investor, or the cost of equity capital, is

$$\{ (P)(1 + G) + (D)(1 + G) - P \} / P, \text{ or}$$

$$K = (D/P)(1 + G) + G.$$

A more rigorous derivation examines only future cash flows, the stockholder dividends. The price of the stock equals the present value of future returns. If dividends are growing at 100G% per annum, the future returns are D(1+G) in one year's time, $D(1+G)^2$ another year later, and so forth. Discounting these at the cost of equity capital ("K"), we obtain

$$P = D(1+G)/(1+K) + D(1+G)^2/(1+K)^2 + D(1+G)^3/(1+K)^3 + \ldots$$

Now $(x + x^2 + x^3 + \ldots) = x/(1-x)$ for positive $x < 1$. If dividends are positive, $K > G$, so

$$P = D \{ (1+G)/(1+K) \} / \{ 1 - [(1+G)/(1+K)] \}.$$

Simplifying this expression gives

$$P = D (1 + G) / (K - G), \text{ or}$$
$$K = (D/P)(1 + G) + G.$$

Both parameters of the dividend growth model, the ratio of stockholder dividend to stock price (or "dividend yield") and the anticipated dividend growth rate, are calculated or projected by investment firms for the major publicly traded stock companies. The dividend yield is generally stable from year to year, and averaged between 4% and 4.5% for Property/Liability insurers in 1989.

Changes in Dividend Growth Rates

The anticipated dividend growth rate is a subjective estimate, for which investment firms provide differing forecasts. Moreover, the growth rate of the firm is often inversely related to the dividend yield: "growth stocks" pay low dividends, whereas "income stocks" pay higher dividends but grow more slowly. These phenomena complicate the Dividend Growth Model.

Suppose an aggressive stock company perceives an opportunity for rapid and profitable growth. To finance its expansion, it retains most of its earnings, keeping its ratio of dividends to price at

2%. During its expansion, its stock price and its dividends both grow at 20% per annum.[23]

During this period, the return on equity to stockholders is 22.4% [= 20% + (2%)(1.2)]. An investment analyst estimating the future return on equity may reason that the insurer can not continue growing rapidly. He may reduce the growth rate to 10% per annum, and calculate the anticipated return on equity as 12.24% [= 10% + (2%)(1.12)].

This estimate is not correct, since a sustained reduction in growth may lead to an increased dividend yield. Since it no longer need finance a rapid expansion, the insurer can retain less of its earnings and pay greater stockholder dividends. Investment analysts who foresee a reduction in the growth rate often anticipate an increase in the dividend yield.

Stock versus Mutual Insurers

Cost of capital estimates derived from stock prices and dividends can be done only for publicly traded companies. Mutual companies and privately owned companies can not be included in a dividend growth model analysis. Moreover, if insurance risk differs by industry segment (life, health, Property/Casualty) and line of business, then the opportunity cost of capital may differ as well. Some insurers sell predominantly Property/Liability coverages; others have substantial life or health business as well.

For its cost of equity capital analyses, the National Council on Compensation Insurance examines 14 publicly traded stock insurers specializing in Property/Liability coverages, as well as 7 insurers who write both life/health and Property/Casualty coverages. In 1989, the NCCI

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[23] The inverse relationship of business expansion and free surplus is particularly strong for insurance companies, for several reasons:

(a) Statutory accounting disallows the capitalization of acquisition expenses, and prohibits the recognition of the "equity" in the unearned premium reserve. A growing insurer has an increasing "equity." Since all acquisition costs are expensed when they are incurred and are also held as unearned premium liabilities (pro-rata over the term of the policy), the increase in the equity is double counted as charges to the statutory income statement (Morgan [1988]). See NAIC [1984], page 131, table 8-5, and Feldblum [1992B] for the magnitude of this effect.

(b) In both statutory and GAAP financial statements, loss and loss expense reserves are shown at nominal values, not discounted values. New business in a long-tailed line generally shows an underwriting loss during the policy year and investment income gains in subsequent years (Lowe [1989]; Woll [1987]). For an insurer that is not growing, the investment income gains from previous blocks of policies often exceed the underwriting loss from the new business (if the coverage has been reasonably priced). For a rapidly growing insurer, the new business underwriting loss may outweigh the investment income gains from previous business, resulting in an apparent accounting loss.

(c) New business shows worse loss ratios and higher average loss costs than renewal business does. Reunderwriting of the existing book, the "transient" nature of many poor risks, and the attraction of "marginal" risks by rapidly growing insurers contribute to this phenomenon (Feldblum [1990B; 1993]; D'Arcy and Doherty [1989; 1990]; Conning and Co. [1988]). Rapidly growing insurers show poor accounting results but build up a potentially profitable renewal book of business.

Because of these phenomena, an insurer that successfully implements a profitable growth strategy will show lower gains or greater losses in its financial statements during its expansion as compared with "true economic" results. It will be forced to reduce its dividend yield even more than growth firms in other industries do.

estimated a dividend yield of 4.5% and a growth rate of 11%, for a cost of equity capital of 16%.

The IRR pricing model seeks the cost of *surplus* funds, which are needed by all insurers. The cost of equity capital is needed for the financial management of publicly traded stock companies, and it is a proxy for the cost of surplus funds of other insurers. The IRR model assumes that the "cost of surplus funds" for mutual and privately owned insurers is similar to the cost of equity capital of stock insurers (Launie [1971], page 265; Cummins [1992]; but see Roth [1992] who disputes this).[24]

Capital Asset Pricing Model

The Dividend Growth Model works best in an unchanging environment: inflation remains level, the firm grows steadily, and the economy expands slowly. If inflation accelerates suddenly, the economy enters a recession, or the firm's book of business changes rapidly, the Dividend Growth Model may not provide reasonable forecasts.

Consider the effects of inflation. If inflation accelerates, and investors seek the same return in inflation-adjusted dollars, then the *nominal* cost of equity capital will rise. But so will the nominal costs of other financial instruments, such as the coupon rate on bonds, or the mortgage rate on home loans.

Few pricing actuaries try forecast future inflation or economic conditions. Instead, they seek a relationship between the cost of equity capital and some steady and accessible index. The Capital Asset Pricing Model (CAPM) provides such a relationship.

Price Fluctuation

The Capital Asset Pricing Model presumes that there are two influences on common stock price fluctuations. Some price changes are peculiar to the specific firm. For instance, the stock price for an oil company may increase if the company discovers an untapped oil source. Similarly, the stock price of an auto manufacturer may drop if its employees declare a strike.

A second influence on the prices of individual stocks is the movement in the stock market as a whole. During a "bull market," the prices of most stocks increase. The prices of some stocks are highly responsive to market movements: if the market as a whole goes up 12%, the prices of these stocks may increase 15%. The prices of other stocks are less responsive, and may increase only 10% during this period.

Price fluctuations that are peculiar to individual firms are referred to as *firm-specific*, *unsystematic*, or *diversifiable* risk. Price movements that reflect overall market returns are

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[24] Modigliani and Miller [1958] argue that the cost of capital depends on the riskiness of the firm, not on its capital structure (Proposition I); see also Fama [1978]. Thus, the appropriate discount rate should not necessarily be higher for stock companies. In fact, since mutual and privately owned insurers have grown more rapidly than publicly traded stock companies during the second half of the twentieth century, the cost of capital estimates derived here may underestimate the true cost of surplus funds. The Dividend Growth Model will show low annual growth rates, providing a low estimate of the cost of capital. Rapidly expanding insurers have a higher cost of capital, but they are under-represented among publicly traded stock companies.

termed *systematic* or *undiversifiable* risk. The Capital Asset Pricing Model hypothesizes that

- The *expected* return from a common stock is related only to the stock's systematic risk;
- The difference between the *expected* return from a common stock and return on a risk-free security is proportional to the firm's systematic risk; and
- The systematic risk and the factor of proportionality are relatively constant over time.[25]

Formally, the Capital Asset Pricing Model posits the following relationship:

$$R = R_f + \beta \, (R_m - R_f),$$

where R is the expected return on a given stock,
R_f is the risk free rate, such as the rate on Treasury bills,
R_m is the overall market return, and
β quantifies the undiversifiable or systematic risk associated with this stock.

The "market risk spread," or $(R_m - R_f)$, has averaged about 8.6 percentage points over the past 60 years, if R_f is the return on short term Treasury bill.[26] The β parameters, which reflect systematic risk, are estimated from historical data, and have averaged about unity for most Property/Liability insurers.

In sum, the Capital Assets Pricing Model estimates that the cost of equity capital for Property/Liability insurers is about 8.6 percentage points higher than the return on Treasury bills. The Treasury bill returns are readily available, and they closely track monetary inflation, economic prosperity, and other external conditions that affect the cost of capital.

In 1989, the return on Treasury bills was between 7.5 and 8%, implying a cost of equity capital between 16 and 16.5%. (The returns on Treasury bills dropped sharply in 1990 and 1991; see Section V.) This is consistent with the Dividend Growth Model estimate derived above. The similarity is not unexpected: although the formulas in these two model are different, the financial data used and many of the underlying assumptions are the same. As noted above, other methods of estimating the opportunity cost of capital have been proposed, and the range of results is wider than this section implies.

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[25] See Sharpe [1970] and Lintner [1965]. Good introductions to the CAPM are Weston and Copeland [1986], chapters 16 and 17, Brealey and Myers [1988], chapter 9, or Cohen, Zinberg, and Zeikel [1982], pp. 143-241. For application of these concepts to insurance returns, see Williams [1983] and Cooper [1974]. CAPM estimates of the cost of capital have been used in public utility regulation; see the testimony of Stewart C. Myers in the 1971 AT&T rate case (Butters, et al. [1981], page 131, note 22).

[26] This figure uses the arithmetic average of the difference between stock returns and the return on Treasury bills. The averages from 1926 to 1986 are 12.12% for stock returns and 3.51% for T-Bills, for a difference of 8.61% (Sharpe and Alexander [1990], pages 5-6). Other analysts, such as Cox and Griepengrog [1988] and Quirin and Waters [1975], use geometric averages, not arithmetic averages. The geometric averages are 9.98% for stock returns and 3.45% for T-Bills, for a difference of 6.53%. See Ibbotson and Sinquefield [1982], pages 57-61, for further discussion of when to use each type of average.

Section V: A Rate Filing Illustration

The illustrations in previous sections were heuristic, using simplified cash flows to emphasize concepts, not details. This section provides selected exhibits from a 1991 NCCI Workers' Compensation rate filing, to show an actual application of the IRR model. Note several characteristics of this illustration:

- Policy year cash flows are used, since the filing sets rates for all policies issued during a future policy year. The previous sections used accident year cash flows, to show the expected internal rate of return on an individual policy.
- Quarterly cash flows are used for the first 6 years, beginning one year prior to policy inception, followed by annual cash flows for the next 19 years.
- The IRR model is used to justify the underwriting profit provision specified in the rate filing, not to independently determine the proper profit provision.
- The NCCI model incorporates some elements, such as tax credits, not discussed previously.

These are differences of technique and format. The same principles underlie the model in this illustration as those discussed throughout the paper.

Assumptions – Table I

The tables in this section are from a 1991 NCCI Workers' Compensation rate filing that uses a 0% underwriting profit and contingencies provision (Table I, row 5). To justify this factor, the NCCI shows that the implied equity cash flows generate a 10.42% internal rate of return. This rate of return must be compared with the cost of equity capital: If investors demand a return greater than 10.42%, a positive underwriting profit provision should be used; if investors would be satisfied with a lower return, a negative provision may be used.

Workers' Compensation rates may be stated on three bases. *Manual* premiums are the prices determined from bureau or company rate manuals. *Standard* premiums are the manual premium after adjustment for experience rating plan modifications. *Net* premiums are the standard premiums after adjustment for premium discounts and retrospective rating (NCCI [1984]).

Table I shows loss and expense provisions relative to *NCCI net* premiums. The experience rating plan modifications and premium discounts are incorporated in net premiums, so no adjustment for these items is shown. Member company rate deviations and schedule rating adjustments, which are not incorporated in the NCCI net premiums, average 3.2% of net premium in the state under review (row 6). Thus, if net written premium at bureau rates is $1 million, the collected premium is $968,000 (rows 10 and 11).

The differences between ratios to standard vs. net premium may be seen from rows 2 and 3. Commissions are 15% of the first $1,000 of premium, grading down at higher layers, for an average of 6.61% to *net* premium (row 2). Similarly, the average ratio of other expenses to net premiums is 9.99% (row 3).

Premium taxes are state specific. The tax and assessment provisions shown on rows 4A, 4B, and 4C are (i) state premium tax (0.63%), (ii) Uninsured Employers' Fund (0.25%), and (iii) The Workers' Compensation Account of the Insurance Guaranty Association (0.17%). These provisions have different payment patterns, so they are modeled separately.

The expense provisions amount to 17.65% (= 6.61% for commissions + 9.99% for other expenses + 1.05% for state taxes and assessments + 0% for profit and contingencies). The loss ratio of 82.35% (row 1) is the complement of the expense ratio.

Row 8B shows the after-tax yield on investments, which equals the pre-tax yield (row 8A) times the complement of the tax rate (row 8C). The pre-tax yield is determined from the distribution of assets held by Commercial Lines insurers and the current yields by asset class (see the discussion below). For Treasury securities and bonds, the NCCI used average yields in July 1991, when interest rates were low, so the yields are below those shown in Section II.D.

The reserves to surplus ratio (row 9) is the ratio of loss, loss adjustment expense, and unearned premium reserves to policyholders' surplus for Commercial Casualty predominating companies for 1985 through 1989 (see Best's [1991A]). The statewide average policyholder dividend ratio (row 7) is taken from industry Page 14 experience.

The caption at the bottom of Table 1 notes that the implied internal rate of return is 10.42%. The following exhibits show the derivation of this rate of return.

Cash Flows – Table II

Table II shows premium, loss, and expense cash flow patterns. The premium cash flow pattern (column 1) is derived from the Massachusetts study discussed in Section II.A, converted to a policy year pattern. [The Massachusetts pattern assumes one policy written at time "zero"; the NCCI pattern assumes that policies are written everly through the year.]

The first nine years of the loss payout pattern (column 2) are derived from a policy year call for experience by state. Subsequent years are modeled in a manner consistent with the payout pattern in the sixth through ninth years (cf. McClenahan [1975] and Woll [1987]).

The commission cash flow pattern is the same as the premium collection pattern, so no separate column is needed. The cash flow pattern for other expenses (column 3) is derived from the 1977 Massachusetts study converted to a policy year basis (see Section II.C above).

Tax and assessment cash flows (columns 4, 5, 6) are determined from state regulations. The premium tax and the Uninsured Employers Fund follow the premium collection pattern, but they are paid annually one quarter subsequent to the year of premium collection. Thus, the 0.19% in row 5 of columns 4 and 5 equals the sum of rows 1 through 4 in column 1, and the 50.83% in row 9 of columns 4 and 5 equals the sum of rows 5 through 8 in column 1. The guaranty fund assessment (column 6) is paid in the first quarter subsequent to the policy year.

Policyholder dividends (column 7) are paid in the second quarter after expiration of the policy. Since policies are issued evenly over the first four quarters, dividends are paid evenly in the sixth through ninth quarters.

Investment Yield – Tables III-A and III-B

Financial pricing models incorporate investment income by projecting future yields on invested assets. Some models use risk free investments and marginal tax rates, arguing that the gains or losses from more aggressive investment strategies should be allocated to stockholders, not policyholders. The IRR model illustrated here compares the internal rate of return with the cost of equity capital. Since the cost of capital reflects the investment strategy, the internal rate of return should do so as well. Tables III-A and III-B therefore show actual investment yields and the associated federal income taxes on insurers' financial portfolios.

The top half of Table III-A separates realized capital gains and losses (row 1) from net investment income earned (row 2), since different tax rates are applied to each.[27] Rows 4 and 5 show the proportion of total (statutory) investment return derived from each part. Rows 6 through 8 determine investment expenses as a percent of total assets.

Realized capital gains are taxed at 34%, and investment expenses provide a tax credit of 34%. Other investment income, such as yields on tax exempt bonds and dividends from common and preferred stocks, incur lower tax rates (see Section II.D above). The appropriate post-tax yield on these investments is shown in Table III-B.

Rows 9 and 10 shows the pre-tax and post-tax yields derived in Table III-B. Row 11 shows the pre-tax yield with an offset for post-tax investment expenses. The pre-tax yield is 7.731%, investment expenses are 0.466%, post-tax investment expenses are (1 – 34%)(0.466%) = 0.308%, so the weighted pre-tax yield is 7.424%. Row 13 shows the post-tax yield:

- *Realized capital gains and losses:* Composite pre-tax yield (7.731%) times the percentage derived from realized capital gains and losses (15.6%) times the complement of the marginal tax rate (1 – 34%) = 0.796%.

- *Net investment income earned:* Composite post-tax yield (6.160%) times the percentage derived from investment income (84.4%) = 5.199%.

- *Investment expense credit:* Investment expense ratio (0.466%) times the complement of the marginal tax rate (1 – 34%) = 0.308%.

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[27] The statutory income statement shows net investment income earned and realized capital gains and losses; unrealized capital gains and losses are a direct credit or charge to surplus. Table III-B, which shows a pre-tax yield on common stock 730 basis points above that on Treasury securities, implicitly includes unrealized capital gains and losses. Table III-B calculates the expected pre-tax and pre-expense investment yield, which is a total return; Table III-A applies the appropriate tax rates to each part of the yield. Statutory accounting ignores taxes on unrealized capital gains and losses; GAAP sets up a deferred tax liability (AICPA [1990]). This IRR model supports a state rate filing, so it follows the statutory treatment.

The estimated post-tax yield is therefore 0.796% + 5.199% – 0.308% = 5.687% (row 13). Row 12 shows the implied investment income tax rate, or the complement of the ratio of the post-tax yield to the pre-tax yield: (1 – 5.687/7.424) = 23.39%.

Table III-B shows pre- and post-tax yields on various investments. The calculations are similar to those in Section II.D; see particularly the discussion there on investment tax rates. This illustration uses July 1991 yields on fixed income investments (see the footnotes to column 2 in Table III-B), which are lower than the 1989 yields.

Cash Flows Supporting Reserves – Table IV

The illustration assumes that the carrier holds full value statutory reserves and has a 3.5 to 1 reserves to surplus ratio. Statutory reserves are greater than economically adequate reserves, since (i) the loss reserves are undiscounted, (ii) the unearned premium reserves have no offset for deferred acquisition costs, and (iii) some assets, such as overdue agents' balances, are not admitted. Statutory reserves at early durations may exceed the premium, net of expenses paid, losses paid, and non-admitted assets, received by the insurer.

Column 1 of Table IV shows the cumulative premium *collected*, derived from column 1 of Table II. For example, Table IV, column 1, row 5, shows $45,399.20 collected by the end of the first quarter. This equals the sum of rows 4 and 5 in column 1 of Table II, or 4.69%, times the total premium collected, or $968,000.

The illustration assumes that agents' balances are not overdue on individual policies unless the aggregate balances are overdue. Agents' balances are the difference between written and collected premium. For example, since policies are issued evenly through the year, first quarter writings are $968,000/4, or $242,000. Collected premium is $45,399.20 (column 1), so the difference, or $196,600.80, is being held by agents (column 2).

No agents' balances are overdue during the first two years, since much of the premium is "deferred and not yet due." [One may think of the "agents' balances" column as either funds not yet remitted by the agent to the insurer, or funds not yet remitted by the insured to the agent.] All premium is due by the end of the second year, so the difference between written and collected premium at time 2.25, or $968,000.00 – $958,900.80 = $9,099.20, is overdue (column 3). The "admitted agents' balances" (column 4) is the difference between columns 2 and 3.[28]

Losses incurred (column 5) presume that policies are written evenly through the year and losses are incurred evenly through the policy term. The full policy year incurred losses are $823,500, or 82.35% *x* $1 million. During the first quarter, 25% of the policies are written. A policy written on January 1 has been in effect for three months by the end of the quarter, so 25% of its losses are incurred by March 31. A policy written on March 31 has had no exposure during the first quarter. The average policy written during the first quarter has

[28] In practice, audit and retrospective premium are not booked until after the policy expires, so the $9,099.20 "overdue agents' balances" in column 3 may be overstated. Conversely, the zeros in the preceding four rows may be understated, since premium on some individual policies may be overdue. [This is the implication of positive figures in column for the third through fifth years, or time 2.25 through 5.00.] These adjustments are minor.

been in effect for 6 weeks by March 31, or 12.5% of the year. Combining, we have

25% x 12.5% x $823,500 = $25,734.38 (column 5).

Unearned premium (column 6) is the difference between written and earned premium. For example, $242,000 (= $968,000/4) is written during the first quarter. On average, 12.5% of the policy is earned by March 31, or 87.5% is unearned. Combining, we have

87.5% x $242,000 = $211,750.00 (column 6).

"Total premium net of reserves" is admitted premium written (columns 1 plus 4) minus premium reserves and incurred losses (column 5 plus 6). Continuing with row 5, we have

$45,399.20 + $196,600.80 – $25,734.38 – $211,750.00 = $4,515.63 (column 7)

Column 8 shows the incremental premium net of reserves, or the first differences of column 7. For example, the $13,546.88 in row 6 of column 8 equals $18,062.50 – $4,515.63 from column 7. The column 8 figures are the excess of (admitted) booked premium over statutory reserves. The figures are positive for the eight quarters during which premiums are in force, since the "zero" underwriting profit provision and the positive return on the policy implies that the premium suffices to cover the losses. During the quarter following the last policy expiration ("2.00 to 2.25"), there is no new written premium or incurred losses, but $9,099.20 of agents' balances become non-admitted, so this amount appears in column 8. Similarly, column 8 in all subsequent quarters is the change in column 3 (overdue agents' balances).[29]

Tax Credits – Table V-A

Expositions of concepts may ignore federal income taxes; practical exhibits must include them. Income tax details obscure the principles of the pricing model, but tax flows may differentiate profitable from unprofitable contracts.

Several characteristics of tax liabilities complicate the treatment:

- The statutory loss reserves in the NCCI's IRR model are undiscounted, but IRS taxable income uses discounted reserves. [In the illustration, federal income taxes exceed the insurer's statutory income during the first year, which show negative cash flows from underwriting in column 5 of Table V-B; see below for further discussion.]

- The discount rate and loss payout pattern are prescribed by the IRS. The discount rate does not necessarily equal the internal rate of return or the current investment yield in the

[29] Since statutory accounting does not recognize the equity in the unearned premium reserves (that is, no acquisition costs may be deferred and amortized over the policy term), one might expect negative entries in column 8 for the four quarters in which policies are issued and large positive amounts in the following four quarters, instead of the mirror images shown in Table IV. This does not occur because expense charges are not included until Table V-A, column 3 and Table V-B, column 3. Column 5 of Table V-B, "net cash flow from underwriting," is indeed negative during each quarter of policy issuance.

model, and the payout pattern does not necessarily equal the loss payment cash flow in column 2 of Table II.

- The NCCI model uses policy year cash flow patterns; IRS discounting procedures prescribe accident year cash flow patterns. Table V-A therefore separates the policy year's paid losses and loss reserves into "accident year 1" and "accident year 2" components.

Because federal income taxes are based on discounted reserves but statutory income is based on undiscounted reserves, taxes are paid before the net income is booked. Consider a policy issued on January 1, with income taxes determined from discounted reserves as of December 31. If there were no investment income received in subsequent years, the insurer would receive a tax refund as discounted reserves are converted into undiscounted paid losses. Since this IRR model uses an *after-tax* investment yield and statutory financial statements, part of the taxes paid during the first two years form "tax credits."[30]

Table V-A calculates the tax credit. Column 1 shows premium written, or $968,000 during the policy year. Since policies are written evenly throughout the year, half of this premium is unearned, so $484,000 appears in column 2. The "revenue offset" provision of the 1986 Federal Income Tax Amendments allows only 80% of the unearned premium reserve as a reduction of taxable income, or $387,200 in this case. [The remaining 20% represent prepaid expenses, which are included in column 3.]

Total expenses paid during the policy year are $95,622.71 (column 3). Policyholder dividends are not paid until three months after policy expiration, so column 4 contains a zero in row 2.

Since the second accident year of this policy year has not yet commenced by December 31, all losses paid and loss reserves are related to "accident year 1" (columns 5 and 7). In subsequent years, both losses paid and loss reserves must be allocated to accident year to compute the federal income tax liability. Since policies are written evenly throughout the year, the "accident year 2" figures in column 6 and 8 equal the "accident year 1" figures in column 5 and 7 in the previous row.[31]

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[30] If the financial statements used discounted reserves, or "present value accounting," or if before-tax investment yields were used, no tax credits would be available; see FASB [1990] and Woll [1987].

[31] The actual lag is two thirds of a year: The average accident date of claims occurring in the first accident year of a given policy year is September 1, or two thirds of the way through the year. The average accident date of claims occurring in the second accident year is May 1, or one third of the way through the year. The time span between these two average accident dates is two thirds of a year.

The NCCI uses an "accident year loss payout pattern" to determine the paid losses in columns 5 and 6. The computations are unclear to me. The total losses paid in the second row of columns 5 and 6 of Table V-A (the two accident year pieces of the first calendar year) should equal the total losses paid determined from the loss payout pattern in Table II, column 2, for the four quarters of the first calendar year (time 0 through time 1). Column 5 of Table V-A shows $100,611.11 for year 0. The latter calculation gives

(0.675% + 2.025% + 3.375% + 4.725%) *x* 82.35% (loss ratio) *x* $1 million (premium) = $88,938.

Columns 7 and 8 show the change in discounted loss reserves, where the discount is computed using IRS discount factors and payout patterns (not shown in the tables). Assuming a tax rate of 34% on underwriting income, income taxes are

$968,000 – (80%)($484,000) – $95,622.71 – $100,611.11 – $245,099.97 = $139,466.21

and (34%)($139,466.21) = $47,418.51

The inverse of the tax liability is the "tax credit" shown in column 9.

Cash Flow from Underwriting – Table V-B

Table V-B shows the net cash flow from underwriting. The premium flow net of reserves (column 1), taken from Table IV, column 8, equals collected premium + admitted agents' balances – losses incurred – unearned premium. The tax credits (column 2) are determined on an annual basis in Table V-A, column 9; they are spread evenly to quarter for Table V-B.

Expenses (column 3) are the sum of commission, other expenses, and premium taxes. Cash flow patterns are shown in Table II, column 1, 3, 4, 5, and 6. [The cash flow pattern for commissions is the same as the cash flow pattern for premiums (Table II, column 1).] Expense levels are shown in Table I. The expense amounts in Table V-B, column 3, are derived from the net premium, the expense levels, and the expense cash flow patterns.

For instance, the fifth row of column 3 ($17,346.94) is derived as follows:

- Commissions: Net premium of $968,000 (Table I, row 11) *x* commission rate of 6.61% (Table I, row 2) *x* the entry from premium collection pattern of 4.5% (Table II, column 1, row 5) = $2,879.

- Other expenses: Net premium of $968,000 (Table I, row 11) *x* other expense rate of 9.99% (Table I, row 3) *x* the entry from other expense payment pattern of 14.9579% (Table II, column 3, row 5) = $14,464.77.

- Tax & assess: [Note that the premium tax and the Uninsured Employer's Fund have the same payment pattern, and there is no Insurance Guaranty Fund assessment in this quarter.] Net premium of $968,000 (Table I, row 11) *x* combined premium tax and Uninsured Employer's Fund rate of 0.88% (Table I, rows 4A + 4B) *x* the entry from the premium tax payment pattern of 0.19 (Table II, column 4 , row 5) = $16.18.

Dividends (column 4) are derived from the average dividend level in Table I, row 7 (4.9%) and the dividend cash flow pattern in Table II, column 7. For example, the $11,858 is column 5, rows 10 through 13, equals

$968,000 (net premium) *x* 4.9% (dividend level) *x* 25% (cash flow pattern) = $11,858

The net cash flow from underwriting (column 5) equals the premium flow net of reserves (column 1) + the tax credit (column 2) – expenses (column 3) – dividends (column 4).

Surplus Funds and Invested Assets – Table VI

Assets support two items on the liability side of the balance sheet: (i) premium and loss reserves and (ii) policyholders' surplus. Assets may be either invested assets or admitted but not invested assets, such as agents' balances. Table VI shows how much invested assets is needed.

Loss and premium reserves are shown in columns 1 and 2, and admitted agents' balances are shown in column 3. Premium reserves are taken from Table IV, column 6. Loss reserves are incurred losses (Table IV, column 5) minus paid losses (derived from the loss payout pattern in Table II, column 2, and the expected loss ratio in Table I, row 1). Admitted agents' balances are taken from Table IV, column 4.

The "cash level" in Table VI, column 4, is the amount of invested assets needed to support the reserves, or loss and premium reserves minus admitted agents' balances. For example, row 5 gives

$20,175.75 + $211,750.00 – $196,600.80 = $35,324.95

Required policyholders' surplus equals the premium and loss reserves divided by 3.5 (Table I, row 9). For Table VI, column 5, row 5, we have

($20,175.75 + $211,750.00) / 3.5 = $66,264.50

Since admitted agents' balances have all be used to support the reserves, invested assets must support policyholders' surplus.

Equity Flows – Table VII

Table VII shows the net cash flows to and from equity holders, derived as the after tax cash flows from underwriting and investments and the required contributions or reductions to surplus.

The net cash flow from underwriting (column 1), taken from Table V-B, column 5, is already an after-tax figure.

The pre-tax income from invested cash (column 2) is derived from the cash level (Table VI, column 4) and the pre-tax investment yield (Table I, row 8A). The cash level of Table VI, column 4, is the end-of-quarter cash level. We need the average cash level during the quarter to determine investment income, so we use the mean cash levels of this and the preceding quarter. The pre-tax investment yield of 7.424% in Table III, row 11, is the annual yield; the fourth root of this figure is the quarterly yield. Thus, the $335.68 in Table VII, column 2, row 5, is derived as follows:

- Average cash level during quarter = ($1,839.20 + $35,324.95) / 2 = $18,582.08, using end-of-quarter cash levels from Table VI, column 4, rows 4 and 5.
- Quarterly pre-tax investment yield = $1.07424^{0.25} - 1 = 0.0180646$.
- $18.582.08 *x* 0.018055 = $335.68.

The tax rate on investment income is 23.391% (Table III-A, row 12). $335.68 *x* 23.391% = $78.52 (Table VII, column 3; the 2 cent discrepancy is a rounding error).

The net cash flow from the surplus account (column 4) is the first difference from Table VI, column 5, "funds in surplus account." For instance,

$126,772.29 – $66,264.50 = –$60,507.79 (Table VII, column 4, row 6).

The pre-tax income from invested surplus (column 5) and the tax on income from invested surplus (column 6) are determined in the same manner as used for columns 2 and 3.

The net cash flow to investors (column 7) equals the sum of the entries in columns 1 through 6. The internal rate of return, or 10.42%, is the interest rate that sets the present value of these cash flows to zero. Although there is no closed form equation to calculate the IRR, many microcomputer spreadsheets, such as Excel or Lotus 1-2-3, have built-in functions that perform the needed calculations.

INTERNAL RATE OF RETURN ANALYSIS
STATE OF XXXXXXXX

TABLE I: ASSUMPTIONS

	ITEMS	PERCENT
(1)	LOSS RATIO	82.35
(2)	COMMISSIONS	6.61
(3)	OTHER EXPENSES	9.99
(4)	STATE PREMIUM TAXES	
	(A) TAX1	0.63
	(B) TAX2	0.25
	(C) TAX3	0.17
(5)	PROFIT AND CONTINGENCY	0.00
(6)	DEVIATIONS AND SCHEDULE RATING	3.20
(7)	DIVIDENDS TO POLICYHOLDERS	4.90
(8)	INVESTMENT INCOME	
	(A) PRE-TAX RETURN ON ASSETS	7.42
	(B) POST-TAX RETURN ON ASSETS	5.69
	(C) INVESTMENT INCOME TAX RATE	23.4%
(9)	RESERVE TO SURPLUS RATIO	3.50
(10)	PREMIUMS WRITTEN	1,000,000
(11)	COLLECTED PREMIUM	968,000

INTERNAL RATE OF RETURN: 10.42 %

NATIONAL COUNC   COMPENSATION INSURANCE
INTERNAL k   OF RETURN ANALYSIS
STATE OF XXXXXXXX

TABLE II: CASH FLOW PATTERNS (%)

TIME INTERVAL FROM : TO	(1) PREMIUM COLLECTION	(2) LOSS PAYOUT	(3) OTHER EXPENSES	(4) TAX 1	(5) TAX 2	(6) TAX 3 *	(7) DIVIDENDS PAID
-1.00 : -0.75	0.00000	0.00000	0.1664	0	0	0	0.00
-0.75 : -0.50	0.00000	0.00000	0.6747	0	0	0	0.00
-0.50 : -0.25	0.00000	0.00000	2.7575	0	0	0	0.00
-0.25 : 0.00	0.19000	0.00000	8.3026	0	0	0	0.00
0.00 : 0.25	4.50000	0.67500	14.9579	0.19	0.19	0	0.00
0.25 : 0.50	10.60000	2.02500	18.3660	0	0	0	0.00
0.50 : 0.75	15.59000	3.37500	17.9559	0	0	0	0.00
0.75 : 1.00	20.14000	4.72500	13.9586	0	0	0	0.00
1.00 : 1.25	19.13000	9.60000	8.8096	50.83	50.83	100	0.00
1.25 : 1.50	14.13000	8.25000	5.8757	0	0	0	25.00
1.50 : 1.75	9.39000	6.90000	4.2865	0	0	0	25.00
1.75 : 2.00	4.39000	5.55000	2.7388	0	0	0	25.00
2.00 : 2.25	1.00000	4.35000	1.0661	47.04	47.04	0	25.00
2.25 : 2.50	0.06000	4.35000	0.0836	0	0	0	0.00
2.50 : 2.75	(0.12000)	4.35000	0.0000	0	0	0	0.00
2.75 : 3.00	0.10000	4.35000	0.0000	0	0	0	0.00
3.00 : 3.25	0.13000	2.60000	0.0000	1.04	1.04	0	0.00
3.25 : 3.50	(0.02000)	2.60000	0.0000	0	0	0	0.00
3.50 : 3.75	(0.03000)	2.60000	0.0000	0	0	0	0.00
3.75 : 4.00	0.11000	2.60000	0.0000	0	0	0	0.00
4.00 : 4.25	0.23000	1.65000	0.0000	0.19	0.19	0	0.00
4.25 : 4.50	0.22000	1.65000	0.0000	0	0	0	0.00
4.50 : 4.75	0.17000	1.65000	0.0000	0	0	0	0.00
4.75 : 5.00	0.08000	1.65000	0.0000	0	0	0	0.00
5.00 : 6.00	0.01000	4.40000	0.0000	0.7	0.7	0	0.00
6.00 : 7.00	0.00000	3.40000	0.0000	0.01	0.01	0	0.00
7.00 : 8.00	0.00000	2.60000	0.0000	0	0	0	0.00
8.00 : 9.00	0.00000	2.30000	0.0000	0	0	0	0.00
9.00 : 10.00	0.00000	2.06324	0.0000	0	0	0	0.00
10.00 : 11.00	0.00000	1.72765	0.0000	0	0	0	0.00
11.00 : 12.00	0.00000	1.44664	0.0000	0	0	0	0.00
12.00 : 13.00	0.00000	1.21134	0.0000	0	0	0	0.00
13.00 : 14.00	0.00000	1.01431	0.0000	0	0	0	0.00
14.00 : 15.00	0.00000	0.84932	0.0000	0	0	0	0.00
15.00 : 16.00	0.00000	0.71118	0.0000	0	0	0	0.00
16.00 : 17.00	0.00000	0.59550	0.0000	0	0	0	0.00
17.00 : 18.00	0.00000	0.49864	0.0000	0	0	0	0.00
18.00 : 19.00	0.00000	0.41753	0.0000	0	0	0	0.00
19.00 : 20.00	0.00000	0.34962	0.0000	0	0	0	0.00
20.00 : 21.00	0.00000	0.29275	0.0000	0	0	0	0.00
21.00 : 22.00	0.00000	0.24514	0.0000	0	0	0	0.00
22.00 : 23.00	0.00000	0.20526	0.0000	0	0	0	0.00
23.00 : 24.00	0.00000	0.17188	0.0000	0	0	0	0.00

* EXCLUDES POLICYHOLDER DIVIDENDS. REFUND INCLUDED IN TABLE V-B, COLUMN (3).

NATIONAL COUNCIL ON COMPENSATION INSURANCE

TOTAL ESTIMATED YIELD FOR THE COMPOSITE PORTFOLIO

STATE OF XXXXXXXX

COUNTRY-WIDE DATA:

1.	REALIZED CAPITAL GAINS, 1985-89:	23,066,973
2.	NET INVESTMENT INCOME EARNED, 1985-89:	124,412,729
3.	TOTAL REALIZED CAPITAL GAINS AND NET INVESTMENT INCOME EARNED, 1985-89:	147,479,702
4.	REALIZED CAPITAL GAINS AS A PROPORTION OF TOTAL REALIZED CAPITAL GAINS AND NET INVESTMENT INCOME EARNED:	0.156
5.	NET INVESTMENT INCOME EARNED AS A PROPORTION OF TOTAL REALIZED CAPITAL GAINS AND NET INVESTMENT INCOME EARNED:	0.844
6.	MEAN TOTAL INVESTED ASSETS, 1985-1989:	356,137,393
7.	MEAN TOTAL DEDUCTIONS FROM INVESTMENT INCOME 1985-1989:	1,660,844
8.	INVESTMENT EXPENSES AS PERCENT OF MEAN TOTAL ASSETS:	0.466

CALCULATION OF STATE SPECIFIC PRE AND POST TAX PORTFOLIO YIELDS:

9.	WEIGHTED PRE-TAX YIELD FOR THE COMPOSITE PORTFOLIO:	7.731
10.	WEIGHTED POST TAX YIELD FOR THE COMPOSITE PORTFOLIO:	6.160
11.	WEIGHTED PRE-TAX YIELD FOR THE COMPOSITE PORTFOLIO AFTER INVESTMENT EXPENSES AND EXPENSE TAX CREDITS:	7.424
12.	INVESTMENT INCOME TAX RATE:	23.391
13.	TOTAL ESTIMATED POST-TAX YIELD FOR COMPOSITE PORTFOLIO:	5.687

SOURCES- 1, 2: BEST'S AGGREGATES AND AVERAGES, EDITIONS 1986-90
6, 7: BEST'S AGGREGATES AND AVERAGES, EDITIONS 1986-90
(3) = (1) + (2)
(4) = (1) / (3)
(5) = (2) / (3)
(8) = (7) / (6)
(9), (10): TABLE III-B
(11) = (9) - {(8) x .66}
(12) = 1 - (13) / (11)
(13) = {(9) x (4) x .66} + {(10) x (5)} - {(8) x .66}

NOTES: (1) ALL DOLLAR AMOUNTS ARE IN THOUSANDS OF DOLLARS.

NATIONAL COUNCIL ON COMPENSATION INSURANCE

WEIGHTED PRE-TAX AND POST-TAX ESTIMATED YIELDS ON INVESTMENT INCOME
FOR A COMPOSITE PORTFOLIO

STATE OF XXXXXXXX

	(1) PORTFOLIO COMPOSITION	(2) PRE-TAX YIELDS	(3) TAX RATE	(4) POST-TAX YIELDS
U.S. TREASURY SECURITIES	18.7 %	7.149 %	34.0 %	4.718 %
BONDS EXEMPT FROM U.S. TAX	35.4	5.109	5.1	4.848
BONDS NOT EXEMPT FROM U.S. TAX	17.6	7.498	34.0	4.949
PREFERRED STOCKS	2.6	8.850	13.8	7.631
COMMON STOCKS	16.7	14.463	13.8	12.471
REAL ESTATE	1.3	9.803	34.0	6.470
SHORT TERM INVESTMENTS	7.8	6.375	34.0	4.208
	100.0 %	7.731 %		6.160 %

SOURCES:

COLUMN (1): BEST'S AGGREGATES AND AVERAGES, 1990 EDITION

COLUMN (2):
- FEDERAL RESERVE STATISTICAL BULLETIN: YIELDS AVAILABLE 7/7/91.
  - U.S. TREASURY BILLS AND BONDS
  - COMMERCIAL PAPER
  - MOODY'S 'AAA' SEASONED CORPORATE BONDS
  - STATE AND LOCAL BONDS
- MOODY'S BOND RECORD, 7/91 EDITION.
  - 'A' RATED PREFERRED UTILITY STOCKS
- NEW YORK TIMES, FIRST SUNDAY EDITION OF 7/91.
  - 7 DAY YIELDS ON TAX-EXEMPT MONEY MARKET FUNDS
- STOCKS, BONDS, BILLS, AND INFLATION 1989 YEARBOOK (IBBOTSON ASSOCIATES INC., CHICAGO, ILLINOIS 1989)
  - EXHIBIT 28, PAGE 86
- IBBOTSON AND SIEGEL, "REAL ESTATE RETURNS: A COMPARISON WITH
  - OTHER INVESTMENTS", AREUEA JOURNAL, VOL. 12, NO.3, 1984

COLUMN (3): THE TAX REFORM ACT OF 1986.

COLUMN (4): COLUMN(2) x (1 - COLUMN(3)).

NATIONAL COUNCIL ON COMPENSATION INSURANCE
INTERNAL RATE OF RETURN ANALYSIS
STATE OF XXXXXXXX

TABLE IV: CASH FLOWS FOR LOSS AND UNEARNED PREMIUM RESERVES

TIME INTERVAL FROM	:	TO	(1) PREMIUM COLLECTED	(2) AGENTS BALANCES	(3) OVERDUE AGENTS BALANCES	(4) ADMITTED AGENTS BALANCES	(5) LOSSES INCURRED	(6) UNEARNED PREMIUM	(7) TOTAL PREMIUM NET OF RESERVES	(8) PREMIUM NET OF RESERVES
-1.00	:	-0.75	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
-0.75	:	-0.50	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
-0.50	:	-0.25	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
-0.25	:	0.00	1,839.20	(1,839.20)	0.00	(1,839.20)	0.00	0.00	0.00	0.00
0.00	:	0.25	45,399.20	196,600.80	0.00	196,600.80	25,734.38	211,750.00	4,515.63	4,515.63
0.25	:	0.50	148,007.20	335,992.80	0.00	335,992.80	102,937.50	363,000.00	18,062.50	13,546.88
0.50	:	0.75	298,918.40	427,081.60	0.00	427,081.60	231,609.38	453,750.00	40,640.63	22,578.13
0.75	:	1.00	493,873.60	474,126.40	0.00	474,126.40	411,750.00	484,000.00	72,250.00	31,609.37
1.00	:	1.25	679,052.00	288,948.00	0.00	288,948.00	591,890.63	272,250.00	103,859.37	31,609.38
1.25	:	1.50	815,830.40	152,169.60	0.00	152,169.60	720,562.50	121,000.00	126,437.50	22,578.13
1.50	:	1.75	906,725.60	61,274.40	0.00	61,274.40	797,765.63	30,250.00	139,984.37	13,546.88
1.75	:	2.00	949,220.80	18,779.20	0.00	18,779.20	823,500.00	0.00	144,500.00	4,515.63
2.00	:	2.25	958,900.80	9,099.20	9,099.20	0.00	823,500.00	0.00	135,400.80	(9,099.20)
2.25	:	2.50	959,481.60	8,518.40	8,518.40	0.00	823,500.00	0.00	135,981.60	580.80
2.50	:	2.75	958,320.00	9,680.00	9,680.00	0.00	823,500.00	0.00	134,820.00	(1,161.60)
2.75	:	3.00	959,288.00	8,712.00	8,712.00	0.00	823,500.00	0.00	135,788.00	968.00
3.00	:	3.25	960,546.40	7,453.60	7,453.60	0.00	823,500.00	0.00	137,046.40	1,258.40
3.25	:	3.50	960,352.80	7,647.20	7,647.20	0.00	823,500.00	0.00	136,852.80	(193.60)
3.50	:	3.75	960,062.40	7,937.60	7,937.60	0.00	823,500.00	0.00	136,562.40	(290.40)
3.75	:	4.00	961,127.20	6,872.80	6,872.80	0.00	823,500.00	0.00	137,627.20	1,064.80
4.00	:	4.25	963,353.60	4,646.40	4,646.40	0.00	823,500.00	0.00	139,853.60	2,226.40
4.25	:	4.50	965,483.20	2,516.80	2,516.80	0.00	823,500.00	0.00	141,983.20	2,129.60
4.50	:	4.75	967,128.80	871.20	871.20	0.00	823,500.00	0.00	143,628.80	1,645.60
4.75	:	5.00	967,[illegible]	96.80	96.80	0.00	823,500.00	0.00	144,403.20	774.40
5.00	:	6.00	968[illegible]	0.00	0.00	0.00	823,500.00	0.00	144,500.00	96.80
6.00	:	7.00	968[illegible]	0.00	0.00	0.00	823,500.00	0.00	144,500.00	0.00
7.00	:	8.00	968,000.00	0.00	0.00	0.00	823,500.00	0.00	144,500.00	0.00
8.00	:	9.00	968,000.00	0.00	0.00	0.00	823,500.00	0.00	144,500.00	0.00
9.00	:	10.00	968,000.00	0.00	0.00	0.00	823,500.00	0.00	144,500.00	0.00
10.00	:	11.00	968,000.00	0.00	0.00	0.00	823,500.00	0.00	144,500.00	0.00
11.00	:	12.00	968,000.00	0.00	0.00	0.00	823,500.00	0.00	144,500.00	0.00
12.00	:	13.00	968,000.00	0.00	0.00	0.00	823,500.00	0.00	144,500.00	0.00
13.00	:	14.00	968,000.00	0.00	0.00	0.00	823,500.00	0.00	144,500.00	0.00
14.00	:	15.00	968,000.00	0.00	0.00	0.00	823,500.00	0.00	144,500.00	0.00
15.00	:	16.00	968,000.00	0.00	0.00	0.00	823,500.00	0.00	144,500.00	0.00
16.00	:	17.00	968,000.00	0.00	0.00	0.00	823,500.00	0.00	144,500.00	0.00
17.00	:	18.00	968,000.00	0.00	0.00	0.00	823,500.00	0.00	144,500.00	0.00
18.00	:	19.00	968,000.00	0.00	0.00	0.00	823,500.00	0.00	144,500.00	0.00
19.00	:	20.00	968,000.00	0.00	0.00	0.00	823,500.00	0.00	144,500.00	0.00
20.00	:	21.00	968,000.00	0.00	0.00	0.00	823,500.00	0.00	144,500.00	0.00
21.00	:	22.00	968,000.00	0.00	0.00	0.00	823,500.00	0.00	144,500.00	0.00
22.00	:	23.00	968,000.00	0.00	0.00	0.00	823,500.00	0.00	144,500.00	0.00
23.00	:	24.00	968,000.00	0.00	0.00	0.00	823,500.00	0.00	144,500.00	0.00

NATIONAL COUNCIL ON COMPENSATION INSURANCE
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TABLE V-A: TAX CREDITS AVAILABLE FROM UNDERWRITING OPERATIONS

YEAR	(1) PREMIUM WRITTEN (POST DEVIATION)	(2) CHANGE IN UNEARNED PREMIUM RESERVE	(3) EXPENSES	(4) DIVIDENDS	(5) LOSSES PAID: ACCIDENT YEAR 1	(6) LOSSES PAID: ACCIDENT YEAR 2	(7) CHANGE IN DISCOUNTED LOSS RESERVE: ACCIDENT YEAR 1	(8) CHANGE IN DISCOUNTED LOSS RESERVE: ACCIDENT YEAR 2	(9) TAX CREDIT
-1	0.00	0.00	11,626.64	0.00	0.00	0.00	0.00	0.00	3,953.06
0	968,000.00	484,000.00	95,622.71	0.00	100,611.11	0.00	245,099.97	0.00	(47,418.51)
1	0.00	(484,000.00)	57,016.51	35,574.00	100,858.16	100,611.11	(90,261.65)	245,099.97	20,977.35
2	0.00	0.00	5,764.10	11,858.00	58,674.38	100,858.16	(49,078.03)	(90,261.65)	12,857.08
3	0.00	0.00	210.22	0.00	35,781.08	58,674.38	(28,627.83)	(49,078.03)	5,766.33
4	0.00	0.00	464.30	0.00	23,099.18	35,781.08	(16,830.54)	(28,627.83)	4,721.30
5	0.00	0.00	66.03	0.00	16,264.13	23,099.18	(11,873.58)	(16,830.54)	3,646.57
6	0.00	0.00	0.85	0.00	12,517.20	16,264.13	(7,455.98)	(11,873.58)	3,213.89
7	0.00	0.00	0.00	0.00	10,149.64	12,517.20	(6,113.06)	(7,455.98)	3,093.25
8	0.00	0.00	0.00	0.00	9,031.57	10,149.64	(5,422.29)	(6,113.06)	2,599.59
9	0.00	0.00	0.00	0.00	7,873.58	9,031.57	(3,840.49)	(5,422.29)	2,598.41
10	0.00	0.00	0.00	0.00	6,592.91	7,873.58	(3,346.02)	(3,840.49)	2,475.19
11	0.00	0.00	0.00	0.00	5,520.54	6,592.91	(2,922.43)	(3,346.02)	1,987.30
12	0.00	0.00	0.00	0.00	4,622.60	5,520.54	(2,560.33)	(2,922.43)	1,584.53
13	0.00	0.00	0.00	0.00	3,870.72	4,622.60	(2,251.83)	(2,560.33)	1,251.59
14	0.00	0.00	0.00	0.00	3,241.13	3,870.72	(1,990.49)	(2,251.83)	975.64
15	0.00	0.00	0.00	0.00	2,713.94	3,241.13	(2,606.43)	(1,990.49)	461.77
16	0.00	0.00	0.00	0.00	2,272.51	2,713.94	(2,182.48)	(2,606.43)	67.16
17	0.00	0.00	0.00	0.00	1,902.87	2,272.51	(1,827.49)	(2,182.48)	56.24
18	0.00	0.00	0.00	0.00	1,593.36	1,902.87	(1,530.24)	(1,827.49)	47.09
19	0.00	0.00	0.00	0.00	1,334.20	1,593.36	(1,281.34)	(1,530.24)	39.43
20	0.00	0.00	0.00	0.00	1,117.18	1,334.20	(1,072.93)	(1,281.34)	33.02
21	0.00	0.00	0.00	0.00	935.47	1,117.18	(898.41)	(1,072.93)	27.65
22	0.00	0.00	0.00	0.00	783.31	935.47	(752.28)	(898.41)	23.15
23	0.00	0.00	0.00	0.00	389.24	1,172.55	(373.82)	(1,126.10)	21.04

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TABLE V-B: NET CASH FLOW FROM UNDERWRITING

TIME INTERVAL FROM	TO	(1) PREMIUM FLOWS NET OF RESERVES	(2) TAX CREDITS	(3) EXPENSES	(4) DIVIDENDS	(5) NET CASH FLOW FROM UNDERWRITING
-1.00 :	-0.75	0.00	988.26	160.85	0.00	827.41
-0.75 :	-0.50	0.00	988.26	652.27	0.00	335.99
-0.50 :	-0.25	0.00	988.26	2,665.71	0.00	(1,677.45)
-0.25 :	0.00	0.00	988.26	8,147.80	0.00	(7,159.54)
0.00 :	0.25	4,515.63	(11,854.63)	17,356.94	0.00	(24,695.94)
0.25 :	0.50	13,546.88	(11,854.63)	24,540.38	0.00	(22,848.13)
0.50 :	0.75	22,578.13	(11,854.63)	27,338.44	0.00	(16,614.94)
0.75 :	1.00	31,609.37	(11,854.63)	26,386.96	0.00	(6,632.21)
1.00 :	1.25	31,609.38	5,244.34	26,738.27	0.00	10,115.44
1.25 :	1.50	22,578.13	5,244.34	14,705.52	11,858.00	1,258.95
1.50 :	1.75	13,546.88	5,244.34	10,134.89	11,858.00	(3,201.68)
1.75 :	2.00	4,515.63	5,244.34	5,437.83	11,858.00	(7,535.87)
2.00 :	2.25	(9,099.20)	3,214.27	5,657.66	11,858.00	(23,400.59)
2.25 :	2.50	580.80	3,214.27	119.24	0.00	3,675.83
2.50 :	2.75	(1,161.60)	3,214.27	(76.82)	0.00	2,129.49
2.75 :	3.00	968.00	3,214.27	64.02	0.00	4,118.25
3.00 :	3.25	1,258.40	1,441.58	171.81	0.00	2,528.17
3.25 :	3.50	(193.60)	1,441.58	(12.80)	0.00	1,260.79
3.50 :	3.75	(290.40)	1,441.58	(19.21)	0.00	1,170.39
3.75 :	4.00	1,064.80	1,441.58	70.42	0.00	2,435.96
4.00 :	4.25	2,226.40	1,180.33	163.42	0.00	3,243.30
4.25 :	4.50	2,129.60	1,180.33	140.84	0.00	3,169.09
4.50 :	4.75	1,645.60	1,180.33	108.83	0.00	2,717.10
4.75 :	5.00	774.40	1,180.33	51.21	0.00	1,903.51
5.00 :	6.00	96.80	3,646.57	66.03	0.00	3,677.34
6.00 :	7.00	0.00	3,213.89	0.85	0.00	3,213.04
7.00 :	8.00	0.00	3,093.25	0.00	0.00	3,093.25
8.00 :	9.00	0.00	2,599.59	0.00	0.00	2,599.59
9.00 :	10.00	0.00	2,598.41	0.00	0.00	2,598.41
10.00 :	11.00	0.00	2,475.19	0.00	0.00	2,475.19
11.00 :	12.00	0.00	1,987.30	0.00	0.00	1,987.30
12.00 :	13.00	0.00	1,584.53	0.00	0.00	1,584.53
13.00 :	14.00	0.00	1,251.59	0.00	0.00	1,251.59
14.00 :	15.00	0.00	975.64	0.00	0.00	975.64
15.00 :	16.00	0.00	461.77	0.00	0.00	461.77
16.00 :	17.00	0.00	67.16	0.00	0.00	67.16
17.00 :	18.00	0.00	56.24	0.00	0.00	56.24
18.00 :	19.00	0.00	47.09	0.00	0.00	47.09
19.00 :	20.00	0.00	39.43	0.00	0.00	39.43
20.00 :	21.00	0.00	33.02	0.00	0.00	33.02
21.00 :	22.00	0.00	27.65	0.00	0.00	27.65
22.00 :	23.00	0.00	23.15	0.00	0.00	23.15
23.00 :	24.00	0.00	21.04	0.00	0.00	21.04

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TABLE VI: DERIVATION OF CASH LEVEL AND FUNDS IN SURPLUS ACCOUNT

TIME INTERVAL FROM	:	TO	(1) LOSS ADJUSTMENT AND LOSS RESERVES	(2) UNEARNED PREMIUM RESERVES	(3) ADMITTED AGENTS BALANCES	(4) CASH LEVEL	(5) FUNDS IN SURPLUS ACCOUNT
-1.00	:	-0.75	0.00	0.00	0.00	0.00	0.00
-0.75	:	-0.50	0.00	0.00	0.00	0.00	0.00
-0.50	:	-0.25	0.00	0.00	0.00	0.00	0.00
-0.25	:	0.00	0.00	0.00	(1,839.20)	1,839.20	0.00
0.00	:	0.25	20,175.75	211,750.00	196,600.80	35,324.95	66,264.50
0.25	:	0.50	80,703.00	363,000.00	335,992.80	107,710.20	126,772.29
0.50	:	0.75	181,581.75	453,750.00	427,081.60	208,250.15	181,523.36
0.75	:	1.00	322,812.00	484,000.00	474,126.40	332,685.60	230,517.71
1.00	:	1.25	423,896.63	272,250.00	288,948.00	407,198.63	198,899.04
1.25	:	1.50	484,629.75	121,000.00	152,169.60	453,460.15	173,037.07
1.50	:	1.75	505,011.38	30,250.00	61,274.40	473,986.97	152,931.82
1.75	:	2.00	485,041.50	0.00	18,779.20	466,262.30	138,583.29
2.00	:	2.25	449,219.25	0.00	0.00	449,219.25	128,348.36
2.25	:	2.50	413,397.00	0.00	0.00	413,397.00	118,113.43
2.50	:	2.75	377,574.75	0.00	0.00	377,574.75	107,878.50
2.75	:	3.00	341,752.50	0.00	0.00	341,752.50	97,643.57
3.00	:	3.25	320,341.50	0.00	0.00	320,341.50	91,526.14
3.25	:	3.50	298,930.50	0.00	0.00	298,930.50	85,408.71
3.50	:	3.75	277,519.50	0.00	0.00	277,519.50	79,291.29
3.75	:	4.00	256,108.50	0.00	0.00	256,108.50	73,173.86
4.00	:	4.25	242,520.75	0.00	0.00	242,520.75	69,291.64
4.25	:	4.50	228,933.00	0.00	0.00	228,933.00	65,409.43
4.50	:	4.75	215,345.25	0.00	0.00	215,345.25	61,527.21
4.75	:	5.00	201,757.50	0.00	0.00	201,757.50	57,645.00
5.00	:	6.00	165,523.50	0.00	0.00	165,523.50	47,292.43
6.00	:	7.00	137,524.50	0.00	0.00	137,524.50	39,292.71
7.00	:	8.00	116,113.50	0.00	0.00	116,113.50	33,175.29
8.00	:	9.00	97,173.00	0.00	0.00	97,173.00	27,763.71
9.00	:	10.00	80,182.20	0.00	0.00	80,182.20	22,909.20
10.00	:	11.00	65,955.02	0.00	0.00	65,955.02	18,844.29
11.00	:	12.00	54,041.96	0.00	0.00	54,041.96	15,440.56
12.00	:	13.00	44,066.62	0.00	0.00	44,066.62	12,590.46
13.00	:	14.00	35,713.80	0.00	0.00	35,713.80	10,203.94
14.00	:	15.00	28,719.61	0.00	0.00	28,719.61	8,205.60
15.00	:	16.00	22,863.06	0.00	0.00	22,863.06	6,532.30
16.00	:	17.00	17,959.10	0.00	0.00	17,959.10	5,131.17
17.00	:	18.00	13,852.79	0.00	0.00	13,852.79	3,957.94
18.00	:	19.00	10,414.39	0.00	0.00	10,414.39	2,975.54
19.00	:	20.00	7,535.26	0.00	0.00	7,535.26	2,152.93
20.00	:	21.00	5,124.44	0.00	0.00	5,124.44	1,464.13
21.00	:	22.00	3,105.75	0.00	0.00	3,105.75	887.36
22.00	:	23.00	1,415.40	0.00	0.00	1,415.40	404.40
23.00	:	24.00	(0.00)	0.00	0.00	(0.00)	(0.00)

NATIONAL COUNCI[illegible] COMPENSATION INSURANCE
INTERNAL R[illegible] OF RETURN ANALYSIS
STATE OF XXXXXXXX

TABLE VII: NOMINAL CASH FLOW TO INVESTORS

TIME INTERVAL FROM	TO	(1) NET CASH FLOW FROM UNDERWRITING	(2) PRE-TAX INCOME FROM INVESTED CASH	(3) TAX ON INCOME FROM INVESTED CASH	(4) NET FLOW FROM SURPLUS ACCOUNT	(5) PRE-TAX INCOME FROM INVESTED SURPLUS	(6) TAX ON INCOME FROM INVESTED SURPLUS	(7) NET CASH FLOW TO INVESTORS
-1.00 :	-0.75	827.41	0.00	0.00	0.00	0.00	0.00	827.41
-0.75 :	-0.50	335.99	0.00	0.00	0.00	0.00	0.00	335.99
-0.50 :	-0.25	(1,677.45)	0.00	0.00	0.00	0.00	0.00	(1,677.45)
-0.25 :	0.00	(7,159.54)	16.61	(3.89)	0.00	0.00	0.00	(7,146.82)
0.00 :	0.25	(24,695.94)	335.68	(78.54)	(66,264.50)	598.52	(140.04)	(90,244.82)
0.25 :	0.50	(22,848.13)	1,291.94	(302.28)	(60,507.79)	1,743.56	(407.94)	(81,030.64)
0.50 :	0.75	(16,614.94)	2,853.85	(667.72)	(54,751.07)	2,784.62	(651.52)	(67,046.78)
0.75 :	1.00	(6,632.21)	4,885.89	(1,143.16)	(48,994.36)	3,721.68	(870.76)	(49,032.92)
1.00 :	1.25	10,115.44	6,682.85	(1,563.59)	31,618.68	3,878.62	(907.48)	49,824.51
1.25 :	1.50	1,258.95	7,773.72	(1,818.82)	25,861.96	3,359.44	(786.01)	35,649.23
1.50 :	1.75	(3,201.68)	8,376.97	(1,959.97)	20,105.25	2,944.25	(688.87)	25,575.96
1.75 :	2.00	(7,535.87)	8,492.61	(1,987.02)	14,348.54	2,633.05	(616.06)	15,335.24
2.00 :	2.25	(23,400.59)	8,268.90	(1,934.68)	10,234.93	2,411.00	(564.10)	(4,984.55)
2.25 :	2.50	3,675.83	7,791.40	(1,822.96)	10,234.93	2,226.11	(520.85)	21,584.47
2.50 :	2.75	2,129.49	7,144.29	(1,671.56)	10,234.93	2,041.23	(477.59)	19,400.79
2.75 :	3.00	4,118.25	6,497.17	(1,520.15)	10,234.93	1,856.34	(434.33)	20,752.21
3.00 :	3.25	2,528.17	5,980.23	(1,399.20)	6,117.43	1,708.64	(399.77)	14,535.49
3.25 :	3.50	1,260.79	5,593.45	(1,308.70)	6,117.43	1,598.13	(373.92)	12,887.17
3.50 :	3.75	1,170.39	5,206.66	(1,218.21)	6,117.43	1,487.62	(348.06)	12,415.83
3.75 :	4.00	2,435.96	4,819.88	(1,127.71)	6,117.43	1,377.11	(322.20)	13,300.47
4.00 :	4.25	3,243.30	4,503.77	(1,053.75)	3,882.21	1,286.79	(301.07)	11,561.25
4.25 :	4.50	3,169.09	4,258.31	(996.32)	3,882.21	1,216.66	(284.66)	11,245.29
4.50 :	4.75	2,717.10	4,012.85	(938.89)	3,882.21	1,146.53	(268.25)	10,551.55
4.75 :	5.00	1,903.51	3,767.39	(881.46)	3,882.21	1,076.40	(251.85)	9,496.21
5.00 :	6.00	3,677.34	13,633.47	(3,189.84)	10,352.57	3,895.28	(911.38)	27,457.45
6.00 :	7.00	3,213.04	11,249.14	(2,631.97)	7,999.71	3,214.04	(751.99)	22,291.97
7.00 :	8.00	3,093.25	9,415.04	(2,202.85)	6,117.43	2,690.01	(629.38)	18,483.51
8.00 :	9.00	2,599.59	7,917.19	(1,852.39)	5,411.57	2,262.06	(529.26)	15,808.77
9.00 :	10.00	2,598.41	6,583.42	(1,540.33)	4,854.52	1,880.98	(440.09)	13,936.90
10.00 :	11.00	2,475.19	5,424.61	(1,269.20)	4,064.91	1,549.89	(362.63)	11,882.77
11.00 :	12.00	1,987.30	4,454.29	(1,042.17)	3,403.73	1,272.65	(297.76)	9,778.04
12.00 :	13.00	1,584.53	3,641.79	(852.07)	2,850.10	1,040.51	(243.45)	8,021.41
13.00 :	14.00	1,251.59	2,961.45	(692.89)	2,386.52	846.13	(197.97)	6,554.83
14.00 :	15.00	975.64	2,391.77	(559.60)	1,998.34	683.36	(159.89)	5,329.62
15.00 :	16.00	461.77	1,914.75	(448.00)	1,673.30	547.07	(128.00)	4,020.90
16.00 :	17.00	67.16	1,515.32	(354.54)	1,401.13	432.95	(101.30)	2,960.72
17.00 :	18.00	56.24	1,180.86	(276.29)	1,173.23	337.39	(78.94)	2,392.49
18.00 :	19.00	47.09	900.80	(210.76)	982.40	257.37	(60.22)	1,916.68
19.00 :	20.00	39.43	666.29	(155.89)	822.61	190.37	(44.54)	1,518.27
20.00 :	21.00	33.02	469.93	(109.95)	688.81	134.27	(31.41)	1,184.65
21.00 :	22.00	27.65	305.50	(71.48)	576.77	87.29	(20.42)	905.31
22.00 :	23.00	23.15	167.83	(39.27)	482.96	47.95	(11.22)	671.40
23.00 :	24.00	21.04	52.54	(12.29)	404.40	15.01	(3.51)	477.18

INTERNAL RATE OF RETURN: 10.42 %

Section VI: Potential Pitfalls in IRR Analyses

Internal rate of return capital budgeting techniques have been criticized on theoretical grounds, and IRR insurance pricing models have been criticized on practical grounds. Advocates of IRR analyses respond that the criticisms are not material, particularly compared to the benefits of the models. This section reviews the arguments on both sides.

General Criticisms

Two widely used capital budgeting techniques consider the time value of money:

- Internal rate of return analyses determine the interest rate that sets the present value of cash inflows equal to the present value of cash outflows. [When used for pricing insurance policies, as in this reading, the IRR model generally considers "equity capital" inflows and outflows, not cash inflows and outflows.[32]] If this interest rate exceeds the cost of capital, the project may be accepted; if it does not, the project should be rejected.

- Net present value analyses use the cost of capital to discount all cash flows to the same time. If the sum of the discounted values is positive, the project may be accepted; if it is negative, the project should be rejected.

The IRR model solves for the discount rate such that the net present value is zero – that is, such that the present value of outflows equals the present value of inflows. The NPV model determines the net present value of all transactions at a given discount rate.

The IRR and NPV models may be thought of as inverse functions:

- IRR: The implied discount rate is a continuous function of the net present value.
- NPV: The net present value is a continuous function of the discount rate.

In other words, the same relationship between net present value and discount rate underlies both the IRR and NPV models.

For most "accept or reject" decision with no constraints, the two methods give the same answer. When there are budget constraints, the projects are mutually exclusive, or there are unusual cash flows, the two methods may give different answers. Often, however, the different answers stem from different decision rules, not from the mathematics involved.

One goal of capital budgeting is to optimize the net worth of the corporation. Some academic theoreticians argue that net present value analyses achieve this objective, whereas internal rate of return analyses may not. Many corporate analysts find the academic arguments of little practical concern and consider the IRR analyses useful and clear.

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[32] Cf. Sondergeld [1982], page 425: "The internal rate of return is the yield rate at which the present value of the surplus transfers equals zero." (Sondergeld uses an IRR model with implied "surplus transfers" between benchmark surplus, insurance surplus, and corporate surplus.)

Cash Flow Patterns

The cash flow pattern affects the number of positive solutions to the internal rate of return equation. Most projects involve an initial cash outflow (or a series of cash outflows) followed by cash inflows. There is a single "sign reversal" between outflows and inflows in this pattern, and there is at most one positive real root to the IRR equation.[33]

The maximum number of positive real roots is equal to the number of sign reversals in the cash flow pattern (Descartes's "rule of signs"). If the project involves an outflow followed by an inflow followed by another outflow, there may be two positive real roots. Solomon [1956] provides an "oil pump" illustration of this, which has been repeated often in financial texts.

Suppose a company's present equipment enable it to extract oil from a well over two years, for revenues of $10,000 each year. By purchasing a more efficient pump for $1,600, the company can extract all the oil in one year, for revenues of $20,000.

To simplify the IRR analysis, assume that the cash outflows and inflows occur at discrete times separated by one year. Purchasing the more efficient equipment means a cash outflow of $1,600 in the first year, an extra cash inflow of $10,000 in the second year, and a cash outflow (or the loss of a cash inflow) of $10,000 in the third year. The IRR equation is

$$\$1,600 = (1+r)^{-1}(\$10,000) - (1+r)^{-2}(\$10,000)$$

R = 25% and r = 400% both satisfy the equation; there is no unique solution.

In truth, neither solution is realistic. Solomon [1956] notes that if the purchase price of the pump is $0, the internal rate of return is 0%; if the price is $827, the IRR becomes 10%; if it is $1,600, the IRR is 25%; if it is $2,500, the IRR is 100%. The more the pump costs, the more profitable the project seems.

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33 This is Descartes's "rule of signs." Roach [1987], pages 189-190, provides a formal statement:

"If f(x) represents a polynomial with real coefficients and with its terms arranged in descending powers of x, the difference v – p between the number v of variations in signs of f(x) and the number p of positive roots of f(x) = 0 is zero or an even positive integer. In symbols,

v – p = 2k, k is a positive integer or zero."

For an intuitive understanding of the statement in the text, suppose there is a cash outflow at time 0 and ten cash inflows at times 1 through 10. The IRR equation is

$$(1+r)^0 out_0 = (1+r)^1 in_1 + (1+r)^1 in_1 + (1+r)^2 in_2 + \ldots + (1+r)^{10} in_{10}.$$

Since $(1+r)^0 = 1$, the left hand side of the equation is constant, whereas the right hand side is a monotonically decreasing function of "r," so there is at most one positive real solution. If there are multiple cash outflows followed by multiple cash inflows, set the time origin to a point between the outflows and inflows. The left hand side (outflows) is an increasing function of "r," and the right hand side (inflows) is a decreasing function of "r," so again there is at most one positive real solution.

Various methods have been proposed to resolve such problems. Solomon rephrases the question as "What is it worth to the investor to receive $10,000 one year earlier than he would otherwise have received it?" If the investor can obtain x% per annum on his money, the cash inflow and outflow of $10,000 apiece can be replaced by a single cash inflow of $10,000$x$. The IRR analysis can now be used to determine whether the project is profitable.[34]

Oversimplifications

In most cases, sign reversals in the projected cash flows result from inaccuracies or oversimplifications, not true reversals in the expected flows. For example, the "net cash flow to investors" (Table VII, column 7) in the illustration of Section V shows a general pattern of net outflows followed by net inflows. There are two exceptions: net inflows in the first two quarters of the year preceding policy inception ($827.41 and $335.99), and a net outflow in the quarter following termination of the last policy ($4,984.55).

Both of the exceptions result from oversimplifications, not from true reversals. The two early "inflows" stem from prepaid expenses and tax credits. The prepaid expenses occur almost entirely in the last two quarters of the year preceding policy inception ($813.12 in the first two quarters and $10,813.51 in the latter two quarters). The tax credit is determined on an annual basis and spread evenly to all four quarters ($3,953.06 for the full year, spread as $988.26 each quarter).

The inaccuracy due to this simplification is not material to the IRR analysis as a whole, but it causes a sign reversal. A more precise analysis would spread the first year's tax credit in the same proportion as the prepaid expenses. All four quarters would show cash outflows, and there would be no sign reversal.

The net outflow of equity funds of $4,984.55 in the fifth quarter following the policy year stems from the assumption that no agents' balances are overdue until this quarter, and all remaining agents' balances become overdue at this time. This simplification is unrealistic, but correcting it requires unavailable data and extensive analysis. A precise projection of the overdue portion of agents' balances would show a decrease in this quarter, not an increase. The net outflow is results from the oversimplification.

In sum, expected cash flows in insurance pricing models generally show a single sign reversal. Additional reversals indicate oversimplifications. If these are material, the projections should be reexamined. One should not tinker with the IRR equation and leave the errors untouched.

Actual cash flows may indeed show multiple sign reversals. An unanticipated IBNR loss may cause a cash outflow from investors in the middle of the stream of inflows. Net present value

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[34] Butler and Appel [1989] expand on Solomon's work, outlining an algorithm to eliminate all but the first sign reversal from the cash flow pattern. This transformation is not always easy. The IRR model implicitly assumes that returns can be reinvested at the internal rate of return. Solomon's transformation is the text uses a fixed outside return. Paquin [1987] uses separate borrowing (fixed) and lending (IRR) discount rates for insurance transactions with multiple cash flow sign changes. Since Paquin uses an IRR model from the insurer-policyholder perspective, not the equity holders' perspective, sign changes in the cash flow pattern are not unusual.

methods can be used to estimate prospectively the expected profitability of a contract and to determine retrospectively the actual results. Internal rate of return analyses on a single policy or a small group of policies are better for projecting estimates than for determining results.

The focus differs between IRR and NPV analyses. The IRR model derives the rate of return, whereas the NPV model determines the dollar return. To determine the rate of return with an NPV analysis, one must compare the present value of profits to some base, such as the present value of surplus (Robbin [1991]; Sondergeld [1982]). If the assumed, or benchmark, surplus varies with loss, some of the problems noted here with regard to IRR models apply to NPV models as well.

Mutually Exclusive Projects and Reinvestment Rates

Some projects pose "accept or reject" decisions, where net present value and internal rate of return analyses give the same answer. Other projects are mutually exclusive, particularly if there are aggregate budget constraints or if the projects accomplish the same goal. Net present value and IRR analyses do not necessarily provide the same ranking of projects.

Consider two mutually exclusive projects, each of which requires an initial capital outlay of $12,000. Project A returns $10,000 one year hence and $6,500 two years hence; project B returns $5,000 one year hence and $12,500 two years hence. The net present values of these projects at interest rates between 10% and 30% are shown below.

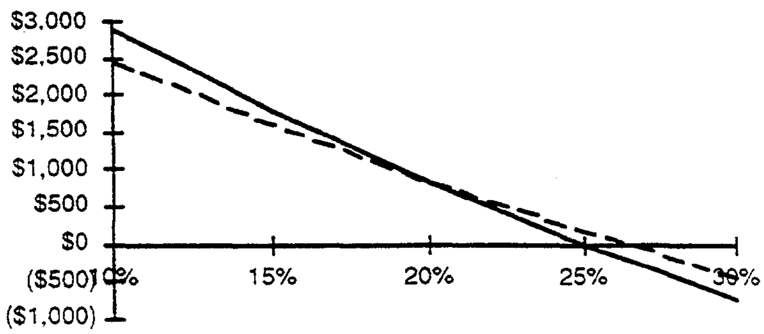
Net Present Values at Varying Interest Rates

	Cash Flows at Time			Net Present Value at Interest Rate of				
Project	0	1	2	10%	15%	20%	25%	30%
A	−12,000	10,000	6,500	2,463	1,611	847	160	−462
B	−12,000	5,000	12,500	2,876	1,800	847	0	−757

On a present value basis, if the opportunity cost of capital is less than 20%, project B is preferable; if it is more than 20%, project A is preferable. On an Internal Rate of Return basis, project A, with an IRR of 26.3%, is preferable to project B, with an IRR of 25%.

The two projects differ in the aggregate dollars of revenue and in the timing of the cash inflows. Project B has a larger nominal revenue ($17,500) than project A has ($16,500), but project A has quicker receipt of the revenue (61% in the first year) than project B has (29% in the first year). At low rates of interest, it is often wise to defer income for a larger total return; at high rates of interest, deferral of income may be expensive.

The accompanying chart graphs the net present values of the two projects at varying interest rates. The two lines cross at 20%, but they have different slopes. At interest rates below 20%, project B (the solid line) is the better investment; at interest rates above 20%, project A (the broken line) is better.



The criticism of the internal rate of return analysis runs as follows. Project A has a higher IRR (26.3%) than project B has (25%). But all this means is that project A is preferable if the revenue received in the first year ($10,000) can be invested at 26.3%. In truth, the firm's opportunity cost of capital is 15%; this is the return that the firm's owners can receive on their funds. The IRR analysis incorrectly mixes the interest rate that equates real values of inflows and outflows with the interest rate at which the firm can invest the funds it receives.[35]

This argument is valid for certain capital budgeting decisions. It is of dubious merit for many applications of the IRR insurance pricing model, for the following reasons:

1. The IRR pricing model is often used to set statewide manual rates, not to price individual policies. If the cost of capital is 15%, but the pricing model shows an IRR of 20%, the insurer can plough back the revenue it receives by writing more policies. As long as the insurer can grow at the internal rate of return and maintain the same quality of risks, the IRR assumptions are correct (Dorfman [1981]).

2. When the pricing model is used to determine the underwriting profit provision, the analyst selects a premium rate that equalizes the internal rate of return and the cost of equity capital. In such cases, there is no difference between net present value and IRR analyses.

Practical Criticisms

One IRR decision rule is: "If the internal rate of return is less than the cost of equity capital, reject the project." The internal rate of return may be positive, and may even exceed the investment yield, but if it is less than the cost of capital, then the project may be undesirable.[36]

Some regulators take another perspective. "A low rate of return may not be desired, but as long as it is positive, isn't the insurer making money? And if it exceeds the investment return, isn't

[35] This criticism is often denoted as the "reinvestment rate assumption"; see particularly Weston and Copeland [1986], Sweeney and Mantripragada [1987], and McDaniel, McCarty, and Jessell [1988].

[36] An alternative IRR decision rule is "Between two similar projects, the one with the higher internal rate of return is preferable." The decision rule used must be appropriate for the type of IRR analysis.

it more than sufficient?"

The presentation of results is crucial in rate filings. Both the net present value and the IRR analyses may show that the project is not profitable. But the former discounts the cash flows at the cost of capital and thereby shows a negative net present value – a clear indication that the project is not profitable. The latter may show a positive, though inadequate, return – a less convincing demonstration of unprofitability.

In utility regulation, this difference is less important. As costs increase, the internal rate of return drops, soon becoming negative. In the insurance pricing model, as costs increase, the internal rate of return subsides slowly, hovering at low positive values even as premiums become severely inadequate.

The difference between utility and insurance regulation stems from the equity base against which returns are measured. The "used and useful" capital in utility regulation is a fixed amount; it does not vary with the projections in the model. But the "required surplus" in the insurance pricing model is an assumption posited by the actuary. If required surplus is determined by a "reserves to surplus" ratio, then as costs increase, so do surplus and investment income. Although the total return is inadequate, the added investment income offsets some of the underwriting loss, and the internal rate of return declines more slowly with increasing loss.

Premium Inadequacies and IRR Analyses

In such cases, the implied equity flow assumptions are not reasonable. As costs rise but premium rates are depressed, equity holders will not provide more capital, as the IRR model implies. Rational investors will provide less capital, if they provide any at all.

To clarify this problem, consider a simple IRR analysis:

- Net premium (that is, premium less expenses) of $10,000 is collected at policy inception.
- One loss will be paid four years after policy inception.
- The insurer funds the loss with a four year zero-coupon bond yielding 10% per annum.
- The equity commitment assumes a 2:1 ratio of undiscounted reserves to surplus.

The assumptions are simplified, but they are realistic for Workers' Compensation. We exclude expenses and taxes, which would complicate the analysis. The loss payment four years after policy inception means a lag of 3.5 years from occurrence to settlement. Workers' Compensation claims actually have weekly benefit payments; to simplify, we use an average payment date and a single cash transaction. Semi-annual coupons are the most common, but the zero-coupon bond makes the exposition clearer. The 2:1 reserves to surplus ratio is low (a more realistic ratio would lie between 2.5:1 and 4:1) but it highlights the problem.

The expected profitability of the contract depends on the expected losses and the cost of capital. Suppose expected losses are $12,000, and the cost of capital is 15% per annum. At policy inception, the equity holders contribute $2,000 to fund the underwriting loss ($12,000 minus $10,000) plus $6,000 as supporting surplus ($12,000 / 2). The total assets of $18,000, consisting of net premium ($10,000) plus equity contribution ($8,000) are invested in a four year 10% zero coupon bond, where they grow to $26,354. The loss of $12,000 is paid, and $14,354 is returned to the equity holders.

Only two equity flows affect the equity holders: the outflow of $8,000 at policy inception and the inflow of $14,354 four years later. The internal rate of return is 15.75%, so the contract is acceptable.

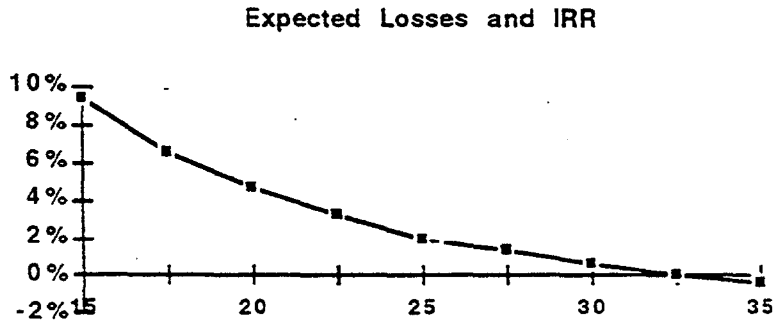
Suppose the expected loss is $15,000. The contract is clearly unprofitable, since the net premium of $10,000 accumulated at 10% interest is only $14,641 after four years. In other words, part of the loss must be funded with existing surplus, since premium plus investment income will not cover it.

The IRR analysis, when properly interpreted, says the same. The equityholders contribute $12,500 at policy inception: $5,000 to fund the underwriting loss and $7;500 as supporting surplus. The $22,500 of assets grow to $32,942 after four years, when the loss of $15,000 is paid and $17,942 is returned to the equityholders.

The internal rate of return is 9.42%. Since this is less than the cost of capital, the contract should be rejected. Since this is even less than the investment yield, the operating ratio exceeds 100%, and existing surplus must be used to fund the loss.

Let us now choose an extreme example: suppose the expected loss is $25,000. With a loss ratio above 200%, results are clearly unprofitable. But the IRR analysis says: equityholders contribute $27,500 at policy inception. Assets of $37,5000 grow to $54,904 after four years, when the $25,000 is paid and $29,904 is returned to the equityholders. The internal rate of return is 2%.

Eventually the IRR becomes negative, but expected losses rise considerably before this happens. The accompanying chart graphs the relationship between the IRR and the expected losses for this example. The horizontal axis shows expected losses in thousands of dollars, and the vertical axis shows the internal rate of return.



The problem is rate filing presentation and acceptance by regulators. When the contract is clearly unprofitable, the actuary should show the negative expected net present value and the insufficiency of net premiums plus investment income to fund the losses. For internal use, the IRR analysis is sound.

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THE RELATIONSHIP OF UNDERWRITING, INVESTMENT, LEVERAGE, AND EXPOSURE TO TOTAL RETURN ON OWNERS' EQUITY

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In recent years, insurance literature and research reflect a great deal of attention to investment return in property and liability insurance companies and a number of important considerations have been discussed. Many issues, however, have not necessarily been resolved and there remains a dearth of thoughtful material on property and liability company finance. There has been so little analysis of investment matters from an actuarial point of view that there is still a need for further development of and agreement on fundamental principles. Accordingly, this paper is written for the purpose of formulating some simple but basic relationships which depict the manner in which investment return, financial leverage, underwriting results, and the utilization of underwriting capacity (or the so-called insurance exposure) all combine to determine the return to stockholders of an insurance company.

The Choice of the Investment Base

In the Arthur D. Little study of insurance company profits and prices, the issue was raised concerning the choice between total assets (investable funds) or net worth (capital and surplus) as the appropriate investment base for computing rates of return. The study concentrated primarily on return on total investable funds to "overcome the difficulties caused by seasonal variations in assets and differences in debt/equity ratios."[2] It was argued further that from society's point of view the critical measure of return is on total assets since society is the ultimate winner or loser regardless of how the resources in a business venture are financed. While the Little study did present computations of return to net worth, it was admitted that the "study does not present a framework for making a risk/return comparison for returns on net worth."[3] These aspects of the choice of an appropriate in-

[1] The author acknowledges the assistance of Dr. Anthony J. Curley, Assistant Professor of Finance at the University of Pennsylvania, who first introduced the author to certain leverage relationships in non-insurance enterprises and by so doing unintionally stimulated this paper.

[2] *Prices and Profits in the Property and Liability Insurance Industry* (A Report to the American Insurance Association by Arthur D. Little, Inc.), 1968, p. 28.

[3] *Ibid.*, p. 40.

vestment base are subject to debate but an analysis of the objectives and methodology of the Little study is not the purpose of this article. What will be shown, however, is the exact relationship between return on assets and return on equity via the well-known concept of financial leverage.

Total Return on Equity — The Basic Equation

It can be argued sensibly that an insurance company operates with a levered capital structure. The leverage, however, does not result from the use of debt capital,[4] but, instead, is an "insurance leverage" resulting from the deferred nature of insurance liabilities. This concept of insurance leverage can be used to explain in simple terms the relationship between return on assets and return on equity.

For convenience let us establish the following notation:

T — Total after-tax return to the insurer
I — Investment gain or loss (after appropriate tax charges)
U — Underwriting profit or loss (after appropriate tax charges)
P — Premium income
A — Total assets
R — Reserves and other liabilities (excluding equity in unearned premium reserves)
S — Stockholders' equity (capital, surplus, and equity in unearned premium reserve)

Using this notation:

$$\frac{T}{S} = \text{Total return on equity}$$

$$T = I + U \text{ and } S = A - R$$

Therefore: $$\frac{T}{S} = \frac{I+U}{S}$$

or: $$\frac{T}{S} = \frac{A}{A} \cdot \frac{I+U}{S} = \frac{A\,(I+U)}{AS}$$

[4] Recently it has been recommended that property-liability insurance companies be permitted to issue debt obligations to obtain capital. See New York State Insurance Department, *Report of the Special Commitee on Insurance Holding Companies*, 1968, p. 8. It should be recognized that the introduction of true debt into the capital structure may be possible only at interest rates well above an insurer's present cost of capital.

Using simple algebra:

$$\frac{T}{S} = \frac{AI + AU + IR - IR}{AS}$$

$$= \frac{I(A-R)}{AS} + \frac{IR}{AS} + \frac{AU}{AS}$$

$$= \frac{IS}{AS} + \frac{IR}{AS} + \frac{U}{S} \qquad (1)$$

$$= \frac{I}{A} + \frac{IR}{AS} + \frac{U}{S} \cdot \frac{P}{P}$$

finally yields:

$$\frac{T}{S} = \frac{I}{A}\left(1 + \frac{R}{S}\right) + \frac{U}{P} \cdot \frac{P}{S} \qquad (2)$$

Hence, we see that the total return to stockholders is equal to the sum of investment return on assets (I/A) multiplied by an insurance leverage factor ($1 + R/S$) dependent on the size of reserves relative to surplus — plus — the underwriting profit[5] (or minus the underwriting loss) on premiums (U/P) multiplied by an insurance exposure term (P/S) relating premiums to surplus. The formula does not require a mutually exclusive choice between equity or total assets as an investment base but rather clearly points out their interdependence. In fact, the formula contains a third rate of return measure in the form of the U/P ratio, a familiar and traditional benchmark for measuring underwriting results. Thus, in one simple equation we see the relationship among return on equity (the investors' viewpoint), return on assets (society's viewpoint), and return on sales (the regulators' and actuaries' viewpoint).

Formula (2) contains the P/S ratio which is sometimes referred to as the insurance exposure and has been advocated on occasion as a rule-of-thumb indicator of insolvency risk.[6] In the basic formula, however, it can be seen that the P/S ratio and the U/P ratio contribute to the return on equity

[5] Since the primary objective of the formula is to measure return for investors and not regulators, underwriting profit or loss on an adjusted basis would be preferable to statutory results since the former would show more correctly the true incidence of expenses. Whatever adjustment is used, it should reflect the fact that it is the cash flow from underwriting that directly affects the investable assets.

[6] For example, see J. W. Middendorf, II, *Investment Policies of Fire and Casualty Insurance Companies* (New York: Wood, Struthers and Co., 1954), pp. 26–30; and Roger Kenney, *Fundamentals of Fire and Casualty Insurance Strength* (Dedham, Mass.: Kenney Insurance Studies, 1967), pp. 97–102.

in much the same manner as do sales margins multiplied by turnover rates in the analysis of return for manufacturing or merchandising concerns.

Reserves Viewed as Non-Equity Capital

Another interesting aspect of this formulation is revealed by placing it in a different form as follows:

$$\text{from (1)} \quad \frac{T}{S} = \frac{I}{A} + \frac{IR}{AS} + \frac{U}{S} \cdot \frac{R}{R}$$

$$\text{therefore} \quad \frac{T}{S} = \frac{I}{A} + \frac{R}{S} \left(\frac{I}{A} + \frac{U}{R}\right) \qquad (3)$$

An interpretation of formula (3) requires that R be viewed as "reserve capital," that is, the amount of total investable assets that has been supplied by other than the owners. In this form the leverage factor R/S is applied separately to interest income on total assets and underwriting profit or loss related to the reserve capital contributed by policyholders. In the case of underwriting losses, formula (3) is plainly analogous to the use of debt capital for financial leverage.[7] With this viewpoint, underwriting losses can be considered as the "interest" that the insurer has paid for the use of R dollars of reserve capital.[8] Naturally, reserve capital differs from the usual debt capital in that with the former the cost of "borrowing" is a variable rather than a fixed interest rate.[9] Formula (3) indicates that it is to the benefit of the owners to continue to write insurance in the event of underwriting losses as long as ratio I/A exceeds the absolute value of a negative ratio U/R. This does not mean that underwriting losses are a desirable objective, but it merely indicates the advantage of continuing to write insurance (ignoring other constraints on cutbacks) during periods of unprofitability. Only when losses make the absolute value of negative U/R larger than I/A does the leverage from the insurance portfolio become unfavorable and detract from the return to stockholders.

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[7] The development of a counterpart of this formula for analysis of leverage through debt financing appears in C. A. Westwick, "A Graphical Treatment of Gearing," *Journal of Accounting Research,* Vol. 4, No. 2, Autumn, 1966.

[8] Similarly, underwriting profits can be viewed as a negative cost of reserve capital.

[9] The bulk of the reserve liabilities obviously are not obligations that extend over durations comparable to long-term debt instruments. They do, however, resemble short- and intermediate-term debt and it can be argued that all forms of indebtedness, regardless of term, should be included in the measurement of leverage. See Ivan R. Woods "Financial 'Leverage' and 'Gearing' in Perspective," reprinted in Edward J. Mock (editor) *Financial Decision Making* (Scranton, Pennsylvania: International Textbook Co., 1967), pp. 533–534.

The Impact of Insurance Leverage

The significant impact of leverage in insurance operations can be illustrated by applying formula (3) to the four hypothetical examples of operating results shown in Table 1.[10] The percentage return on equity as calculated by formula (3) for each company and for each insurance situation is shown in Table 2. While these results can be calculated directly, formula (3) is useful for visualizing in each instance the contribution to or subtraction from the total return on equity resulting from the effect of leverage in the insurance companies. The figures in Table 2 show the increased absolute and relative variability of operating returns that result from increased leverage, and this variability would have been even more significant had the investment rate of return been allowed to vary. Hence, the leverage ratio or the reserve-surplus ratio serves as an indicator or a partial determinant of the riskiness of the owner's investment in the firm.

Actuarial Determination of the Optimum Capital Structure

The preceding view of reserves as leverage-inducing, non-equity capital, if it is accepted, has significant implications for the scope of actuarial analysis. With this view, the actuary, dealing primarily with premiums and reserves, cannot, and indeed should not, ignore one of the fundamental problems in the theory and practice of financial management — the problem of determining the optimal capital structure of the firm.

The problem of finding the optimal composition of liabilities and owners' equity at which the value of a firm will be maximized appears on the surface to be as relevant to a stock insurance company as to any other business enterprise. The two crucial variables that are generally accepted as the determinants of the value of a firm are the expected earning stream and the rate at which that stream is capitalized by the market. It is intuitively obvious and it has been shown in formula (3) that non-equity financing from reserves will add to the income stream as long as the costs of financing the reserves are less than the returns from invested assets. The central issue of the optimal capital structure is the effect of non-equity financing such as reserves on the quality (variance) of the insurer's earnings

[10] The figures in Table 1 are in no sense assumed to be realistic or representative of any one company. They are used only to point out the direction of the impact of the leverage variable and many other considerations have been ignored. For example, nothing has been said about the fact that insurance companies with such diverse leverage ratios are not likely to have identical investment or underwriting results. Also, no attempt is made to discuss the implications of the varied blends of income and gains and losses that can underlie the return on invested assets.

Table 1

Hypothetical Operating Results

Company A: An unlevered investment trust
Invested assets: $20,000,000
Owners' equity: $20,000,000
Investment return: 5%
Leverage ratio[3]: 0

Company B: Insurance company — "low" leverage
Invested assets: $20,000,000
Reserve liabilities[1]: $6,666,667
Owners' equity[2]: $13,333,333
Investment return: 5%
Leverage ratio[3]: ½

Company C: Insurance company — "medium" leverage
Invested assets: $20,000,000
Reserve liabilities[1]: $10,000,000
Owners' equity[2]: $10,000,000
Investment return: 5%
Leverage ratio[3]: 1

Company D: Insurance company — "high" leverage
Invested assets: $20,000,000
Reserve liabilities[1]: $13,333,333
Owners' equity[2]: $6,666,667
Investment return: 5%
Leverage ratio[3]: 2

Insurance operating results[4]: Situation 1 — +6% (profit)
Situation 2 — 0% (breakeven)
Situation 3 — −6% (loss)

[1] Excluding equity in unearned premium reserve.
[2] Including equity in unearned premium reserve.
[3] Reserve liabilities divided by owners' equity.
[4] Underwriting profit or loss as a percentage of reserve liabilities.

Table 2

Return on Owners' Equity Based on Data in Table 1

	Company A	Company B	Company C	Company D
Situation 1	5.0%	10.5%	16%	27%
Situation 2	5.0	7.5	10	15
Situation 3	5.0	4.5	4	3

and, hence, on the rate at which the earnings are capitalized by the market for valuation purposes. It is in the determination of the impact of insurance obligations (as reflected in reserves) on the magnitude and variance of future earnings that the talents of the actuary are required. What this suggests is that the actuarial determination of the probability of ruin or insolvency should be extended to include the determination of the probabilities of unfavorable returns to owners and the attendant lowering of market valuation of the company or at the extreme a departure of equity capital from the business.

The analysis of reserve capital (or insurance leverage) is undoubtedly more complicated than the analysis of debt capital. As was stated previously, the cost of the latter is fixed while the former has an expected cost with a variance. Additionally, an increase in the relative amount of debt capital generally entails demands by the creditors for a progressively higher interest rate to reflect the increased risk of larger fixed commitments, but the relative profitability of expanding an insurance portfolio is not as predictable. The ability to reduce the relative variance of underwriting results by sheer volume and logical diversification may offset the costs of taking additional and possibly poorer risks.

The actuarial analysis of the optimal capital structure (or optimum reserve-surplus ratio) of the insurer must also include an analysis of the quality and earning capacity of the assets. One of the major determinants of the amount of non-equity capital that may safely be undertaken by the firm is the degree of variability in the investment earning stream. The traditional position is that the greater the variability of earnings the lower the prescribed debt-equity ratio. Thus, the optimum reserve position for an insurer is not independent of the investment policy that is followed.

Of what practical application is an analysis of the optimal capital structure of a property and liability insurer? If the industry does have a capacity problem from the insuring public's viewpoint, it may be explained by a capital structure that from an investor's viewpoint is optimal at a relatively low reserve/surplus ratio. Furthermore, one can inquire whether a capacity problem is attributable only in part to rating formulas and/or regulation and is affected also by overly aggressive investment portfolios that set the optimal capital structure at a relatively low reserve/surplus ratio. Alternatively, and in the author's opinion more realistically, if the optimal capital structure is at a higher reserve/surplus ratio than is maintained currently in the typical company, then one might conclude that the industry is overcapitalized with investor capital. This situation would explain the financial motivation behind the recent emphasis on holding-company formations to absorb insurance company capital. Interestingly, the fact that investor capital might be in excess appears to have been overlooked or ignored as a possible logical explanation of the general unprofitability alleged by the Arthur D. Little study of prices and profits.

Conclusions

If present regulatory and financial trends continue, the actuary is going to be forced to narrow the analytical gap between the insurance and investment sides of the business.[11] The arguments presented here reinforce the position that investment return can no longer be ignored by the actuary, but they do not prescribe the manner in which investment should be included in the current ratemaking process. It is suggested that somehow simply plugging a rate of return into current ratemaking formulas is too narrow an approach. Once the actuary introduces investment returns into his analysis, he must logically be concerned with the rather broad financial management objectives affecting total performance of the firm. The basic formulas derived in this paper show the role that the insurance operations play in the over-all determination of total return to stockholders. According to financial theory, it is this return that management should be attempting to maximize. It appears, however, that management in general, and actuaries in particular, have been over-zealous in addressing themselves to regulators rather than the shareholders. In order to remedy this imbalance, current techniques of ratemaking and rate regulation may have to undergo traumatic procedural and philosophical changes to properly accommodate the introduction of investment considerations into the ratemaking process. Perhaps the only solution with enough flexibility is a system of open competition.

[11] The existence of this separation was described to this Society in S. Davidson Herron, Jr., "Insurance Company Investment," *Proceedings of the Casualty Actuarial Society*, 1966, pp. 238–239.

THE RELATIONSHIP OF UNDERWRITING, INVESTMENT, LEVERAGE, AND EXPOSURE TO TOTAL RETURN ON OWNERS' EQUITY

J. ROBERT FERRARI

VOLUME LV, PAGE 295

DISCUSSION BY R. J. BALCAREK

It is only very recently that the insurance industry began to acquaint itself with the concept of the return on owners' equity and its implications. Professor Ferrari's important and interesting paper presents a solid foundation for further exploration and analysis.

The reviewer found the formulas illuminating and beautiful in their simplicity. However, simplicity is not always an unqualified blessing. It may be useful to warn that the utilization of Ferrari's formulas requires a great deal of caution. As a case in point, one could easily argue on the basis of formula (3) that, provided the underwriting results do not fall below a certain standard, the premium volume should be expanded as much as possible. No doubt, such expansion would increase the total return on owners' equity but the equity would be exposed to a considerably higher risk. Therefore, it would seem that the maximization of the return should be subject to the condition that there is no appreciable increase in the degree of risk to which the owners' equity is exposed.

Secondly, the formulas lend themselves best to describe a static state. They could be used to illustrate the current or past relationships of a single insurer, a group of insurers, or the industry as a whole. Once we adopt a dynamic approach we would find that most of these relationships start interacting with each other. We cannot say: "Let us increase the premium writings in relation to surplus, assume all other relationships constant, and thus determine the effect of the increase in premium volume on the rate of return." The problem is that the other relationships will not stay constant and they will change directly as a result of the change in premium volume. Professor Ferrari anticipated this to some degree when he mentioned the possibility of the additional business being of a poorer quality, i.e., using his symbols, if P/S increases then U/P may decrease. Obviously this is a possibility, but it would appear that the majority of the companies could avoid it provided they imposed adequate controls over the process of expansion. However, there will be other, perhaps more powerful, relationships, assuming the need to keep the risk to owners' equity unchanged:

(1) When the premium to surplus ratio P/S increases, then the investment gain on assets I/A will tend to decrease because (a) the proportion of uninvested assets originating from the insurance operations, such as cash and agents' balances, will tend to rise, and (b) with a higher P/S the element of risk to owners' equity becomes greater and this would have to be compensated for by a more conservative investment policy.

(2) An insurer can safely write a larger premium volume with the same surplus if his underwriting results are more favorable. In other words, the ratio of premium volumes to surplus P/S will move in the same direction as rate of underwriting profit U/P.

(3) An examination of the relationship between the rate of underwriting profit U/P and the investment return on assets I/A leads to the conclusion that they would tend to move in the same direction. This means that if underwriting results are good the insurer could indulge in a more aggressive investment policy.

No doubt, there are more such inter-relationships and no formula or mathematical model could possibly take them all into account. However, the reviewer feels that Ferrari's formula would benefit greatly if two or three such relationships were incorporated into it. It has to be realized that a study of each of these relationships would be fairly involved, providing ample material for a separate paper. The reviewer is convinced that it is possible to determine, at least partially, the parameters involved in these relationships. Once this is done (easier said than done), then, using linear programming or a similar technique, Ferrari's formula could be used to determine an optimal solution from the stockholders' viewpoint.

The reviewer's recent paper entitled "The Capital Investment Market and the Insurance Industry"* presents a special case of the relationship

* *PCAS*, Vol. LV, p. 186 (1968).

between U/P and P/S. It describes the case when the rate of underwriting return U/P is so low that the desirable written premium to surplus ratio P/S is equal to zero.

DISCUSSION BY ROBERT A. BAILEY

Mr. Ferrari has illuminated the relationships among return on equity, return on assets, and return on sales with simple formulas. These simple relationships provide valuable insight and should be helpful to anyone who must make meaningful decisions as to the future course of an insurer, in underwriting commitments, investments, and prices.

Mr. Ferrari's formulas illustrate the effect of leverage — the relationship of premiums and liabilities to shareholders' equity — and have thereby enabled him to pose the important problem of the optimum capital structure for an insurer.

His formulas lead to two significant conclusions:

(1) Capacity depends on profits. If the net result from underwriting plus the investment gain from the investable portion of the insurance reserves is a profit, capacity will increase. If it is a loss, capacity will decrease. (Of course, profits may also be dependent on capacity — too much capacity leading to reduced profits in a competitive market.) The correct measurement of investment returns from funds attributable to the underwriting operation is therefore of critical importance to the management of an insurer.

(2) The optimum capital structure, assuming a profitable result from underwriting and the underwriting portion of investment income, is a minimum of capital and a maximum of leverage. In fact, if it is possible, the optimum capital is less than zero. Mr. Ferrari suggests that variability of earnings introduces an opposing tendency to maximize capital in order to stabilize earnings, because stable earnings are capitalized at a higher rate than variable earnings. According to this theory the optimum capital structure is attained at some mid-point between the opposing tendencies to maximize leverage and to maximize stability of earnings. However, this restraint on attaining maximum leverage applies only if the insurer is an independent entity. This restraint is largely eliminated if the insurer is owned by a holding company that holds other enterprises in addition to insurance.

A holding company can treat its insurance operation like a separate

Risk-Adjusted Performance Measurement for P&C Insurers

Richard Goldfarb, FCAS, CFA, FRM

Original Draft: December 2006 (Revised: October 2010)

Abstract

This paper was prepared as an introduction to risk-adjusted performance measurement for P&C insurance companies. A simplified numerical example is used to demonstrate how measures such as risk-adjusted return on capital (RAROC) can be used to guide certain strategic decisions. While the discussion is simplified throughout, the numerical examples are used to highlight the important challenges associated with this methodology and clarify some of its limitations.

Acknowledgments

Several individuals read earlier drafts of this paper and provided feedback that greatly improved the content and helped to streamline the presentation. I would like to particularly thank Walt Stewart, Emily Gilde, Ralph Blanchard, Don Mango, Bill Panning, Edward Chiang, Tom Conway, Jenny Lai, Parr Schoolman and Jessica Somerfeld for their contributions.

Note Regarding October 2010 Revision

The October 2010 revision merely reflects a change to the title of the study note resulting from revisions to the numbering convention used for the CAS exam for which this study note was originally produced. All of the content remains the same as the December 2006 version.

Table of Contents

1. Introduction

This paper was prepared as an introduction to risk-adjusted performance measurement for a P&C insurance company.

The paper is organized as follows. Section 2 provides an overview of risk-adjusted performance measurement, with an emphasis on one particular implementation of risk-adjusted return on capital (RAROC). The emphasis on RAROC is used solely to focus the discussion, as many of the issues presented in subsequent sections are relevant to alternative methodologies that also attempt to risk-adjust performance measures.

Section 3 discusses the techniques used to characterize the risk distributions for different risk sources and the issues associated with developing a firm's aggregate risk profile. Section 4 then presents a simplified numerical example and uses it to demonstrate various techniques used to calculate the firm's aggregate risk capital and then allocate, or attribute, this risk capital to individual business units.

In Section 5, various applications that make use of the allocated risk capital are discussed in the context of the numerical example presented in Section 4. While not intended to be exhaustive, the discussion of these applications will help to emphasize the strengths, weaknesses and limitations of the specific RAROC application presented.

Finally, Section 6 summarizes some of the refinements that might be needed for certain applications, some of which can be useful for overcoming the limitations discussed in Section 5.

2. Overview of Risk-Adjusted Performance Measurement

Risk-adjusted performance measures are intended to improve upon the metrics used to make capital planning, risk management and corporate strategy decisions by explicitly reflecting the risks inherent in different businesses.

In a simple one-period case[1] in which a business requires an investment of a specific amount of capital and earns (or is expected to earn) a given dollar amount of income (profit) during the period, the return on capital is simply calculated as:

$$\text{Return on Capital} = \frac{\text{Income}}{\text{Capital}}$$

This is, of course, a very general form of a "return" calculation and in practice there are a wide variety of approaches that can be used to determine the amounts used for both the numerator and denominator. In many instances, adjustments made to either the numerator or denominator will have the effect of transforming the resulting measure into less of a "rate of return" than is commonly acknowledged. The resulting metrics are more accurately described as profitability indices or, more generally, financial ratios. This distinction between a rate of return and a financial ratio will be explored further when the challenges associated with developing an objective benchmark for the metric is discussed. For now, the ratio of income to capital will be referred to as a return on capital measure in the usual manner.

A variety of standard return on capital measures such as return on equity (ROE), return on assets (ROA) or total shareholder return (TSR) are often reviewed to assess *ex post* or *ex ante* performance of different business units within a firm or to assess the firm's overall performance relative to peers. However, because these measures often do not explicitly distinguish between activities with varying degrees of risk or uncertainty, they can sometimes result in misleading indications of relative performance and value creation.

Insurance companies commonly attempt to overcome this weakness associated with conventional ROE measures by allocating, or more accurately attributing, their capital or surplus to different

[1] A one-period model is rarely adequate for insurance businesses, since the capital required to support these businesses is committed and the income is earned over many periods. This issue will be explored further in Section 5.5.

business units using either premium to surplus ratios or reserve to surplus ratios that vary by line of business. This can serve to "risk-adjust" the return on capital measure by attributing more capital or surplus to business segments with more perceived risk, though often the premium to surplus and reserve to surplus ratios used are selected judgmentally or without the use of quantitative models.

An alternative approach is to make the risk-adjustment more explicit. Many banks and insurance companies have adopted risk-adjusted return on capital measures in which either the "return" is risk-adjusted, the "capital" is risk-adjusted, or in some cases both are risk-adjusted. Often all three instances are generically referred to as RAROC (**R**isk **A**djusted **R**eturn **O**n **C**apital), a convention that will be used here for convenience. But for clarity, throughout this discussion the emphasis will be on a measure based on income that is not risk-adjusted and capital that is risk-adjusted[2]:

$$\text{Risk - Adjusted Return on Capital} = RAROC = \frac{\text{Income}}{\text{Risk - Adjusted Capital}}$$

2.1 Income Measures

A wide variety of income measures exist, all of which are intended to reflect the profit, in dollars, during a specific measurement period. Four relevant choices include:

- GAAP Net Income – This measures the income earned according to GAAP accounting conventions. Use of this measure is convenient when RAROC is intended to be used to guide management decision-making, since the measurement basis is already in use within the firm.
- Statutory Net Income – In countries where separate statutory (regulatory) accounting frameworks are used, the income component may also be measured using these statutory accounting conventions.
- IASB Fair Value Basis Net Income – Although not yet formally adopted, efforts are underway by the International Accounting Standards Board (IASB) to develop "fair value" accounting standards. These standards are intended to remove many existing biases in various accounting conventions used throughout the world. For insurance companies, this measure of profit differs from GAAP net income primarily due to the discounting of loss reserves to reflect their present value and the inclusion of a risk margin on loss reserves to approximate a risk charge that would typically be included in an arms-length transaction designed to transfer the risk to a third party.
- Economic Profit – A more general method for measuring profit that further eliminates many accounting biases is often referred to as *economic profit*. Unfortunately, this term is often used to refer to many different types of adjustments to the GAAP income measures. Generally it refers to the total change in the "economic value" of the assets and liabilities of the firm, where asset values reflect their market value and the liabilities are discounted to reflect their present value. Whether this discounting of the liabilities includes a risk margin, as in the IASB definition of fair value, often varies.

  Some believe that estimates of the change in the "economic value" of assets and liabilities represent a more meaningful measure of the gain or loss in a given period. But there are limitations associated with this measure:

  - To accurately reflect the change in value for a firm, changes in the value of its *future* profits must also be taken into account. This *franchise value* can be a significant source of value for firms (well in excess of the value of the assets and liabilities on its balance sheet) and changes in this value will clearly impact total shareholder returns.

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[2] To clarify, the methodology used throughout this paper is referred to as RAROC. However, because it is calculated as **R**eturn **O**ver **R**isk-**A**djusted **C**apital, it is often referred to as RORAC to indicate that it is the capital amount that is risk-adjusted. The RAROC terminology is often reserved for measures of **R**isk-**A**djusted **R**eturn **O**ver **C**apital, where the return measure is risk-adjusted. However, in both cases, Return on Capital is being measured and in both cases it is Risk-Adjusted, so in another sense *both* can legitimately be referred to as **R**isk-**A**djusted **R**eturn **O**n **C**apital, RAROC.

- The use of economic profit as an income measure also complicates reconciliation to GAAP income or other more familiar measures of profitability. This reconciliation issue is often important in practice because management may have more difficulty interpreting income measures that deviate significantly from commonly used measures.
- If the economic profit measures are not disclosed to external parties such as investors, regulators or rating agencies, management may have more difficulty communicating the basis for their decisions. These external parties may only have access to GAAP and statutory financial statements and they may be unable to reproduce internally generated economic profit estimates.

In the discussion that follows, a specific measure of "economic profit" will be used, merely for convenience. A variety of adjustments often made to the selected income measure will be ignored in Sections 2 through 5, but will be addressed briefly in Section 6.

2.2 Capital Measures

There are numerous ways to measure the capital required for a given firm or for specific business units within the firm. Some of these capital measures are risk-adjusted and some are not.

Two measures that are not risk-adjusted include:

- Actual Committed Capital – This is the actual cash capital provided to the company by its shareholders and used to generate income for the firm and its respective business units. This is typically an accounting book value equal to contributed capital plus retained earnings and can be based on GAAP, Statutory or IASB accounting conventions.
- Market Value of Equity – As discussed in Section 2.1, the committed capital measure described above could be adjusted to reflect market values of the assets and liabilities, though this will still reflect only the value of the net assets on the balance sheet. An alternative is to actually use the market value of the firm's equity, which will generally be larger than the committed capital because of the inclusion of the franchise value of the firm.

Four measures that explicitly reflect risk-adjustments, to varying degrees, include:

- Regulatory Required Capital – This is the capital required to satisfy minimum regulatory requirements. This is typically determined by explicit application of the appropriate regulatory capital requirement model.
- Rating Agency Required Capital – This is the capital required to achieve a stated credit rating from one or more credit rating agencies (S&P, A.M. Best, Moody's or Fitch). This is usually determined by explicit application of the respective credit rating agencies' capital models and by reference to the standards each rating agency has established for capital levels required to achieve specific ratings[3].
- Economic Capital – This term is commonly used but often defined differently, which leads to unnecessary confusion. In its most general sense, economic capital could be defined as *the capital required to ensure a specified probability (level of confidence) that the firm can achieve a specified objective over a given time horizon*. The objective that the risk capital is intended to achieve can vary based on the circumstances and can vary depending upon whether the focus is on the policyholder, debtholder or shareholder perspectives.

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[3] It is important to note that the capital models used by the rating agencies represent just one of many factors that are used to assign a credit rating to any particular firm. Other factors include the strength of the management team, historical experience, access to capital and other related considerations. Nonetheless, the rating agencies typically provide indications of the rating levels associated with different levels of capital adequacy that result from the application of their capital models. These indications are used by firms to determine the "required capital" for a given rating, independent of all of these other rating factors.

- Solvency Objective – The most common approach used by rating agencies and regulators could be referred to as a *solvency objective*. A solvency objective focuses on holding sufficient capital today to ensure that the firm can meet its existing obligations to policyholders (and perhaps debtholders as well). This approach clearly reflects a policyholder or debtholder perspective.
- Capital Adequacy Objective – An alternative approach is to use what could be referred to as a *capital adequacy objective.* This objective focuses on holding sufficient capital to ensure that the firm can continue to pay dividends, support premium growth in line with long-term business plans or maintain a certain degree of financial strength over an extended horizon so as to maximize the franchise value of the firm.

These two approaches can lead to substantially different indications of the capital required for the firm or any individual business. The "solvency" perspective is currently quite commonly used, so for convenience this perspective will be adopted throughout this paper[4]. When using this definition of economic capital, the focus is typically on ensuring that there are sufficient financial resources (in cash and marketable securities) to satisfy policyholder (and debtholder) obligations. However, there will necessarily be a somewhat arbitrary separation of the total financial resources into a portion that represents a "liability" and a portion that represents "capital". This separation will usually follow applicable accounting conventions, but can lead to meaningful differences in practice.

For instance, some practitioners define economic capital as the difference between the total financial resources needed *less* the undiscounted value of the (expected) liability. This is consistent with how the firm's resources would be classified under U.S. statutory accounting. Others prefer to define the economic capital as the amount that the total financial resources needed exceeds the discounted value of the (expected) liability. Still others might choose to incorporate a risk margin in the liability and treat the economic capital as the amount by which the total financial resources needed exceeds the fair value of the liability.

Any of these approaches could be used, so long as they are used consistently across different risks.

- Risk Capital – The range of different interpretations of the term *economic capital* is worrisome and can lead to a variety of inconsistent adjustments in practice. For instance, the choices described above all define economic capital as the portion in excess of the discounted expected liability, the undiscounted expected liability or the fair value of the liability under the assumption that funding for these amounts are already accounted for in the firm's financial statements. This is not the case for all risks – some could not be reflected at all on the balance sheet, in which case the economic capital has to account for all of the potential liabilities, while others could be funded by an amount well in excess of the discounted value, undiscounted value or fair value of the expected liability.

To avoid confusion in this paper, a closely related measure referred to here as *risk capital* will be used instead of any of the definitions of economic capital. Risk capital is defined as the amount of capital that must be contributed by the **shareholders** of the firm in order to absorb the risk that liabilities will exceed the funds already provided for in either the loss reserves or in the policyholder premiums. Under this definition, any conservatism in the loss reserves or any risk margins included in the premiums will reduce the amount of risk capital that must be provided by shareholders.

Notice that in the absence of a risk margin included in the premiums or the reserves, the risk capital and the economic capital may be identical. As a result, for many of the applications discussed later in this paper either amount could be used. However, for some of the main

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[4] Panning's *Managing the Invisible* contains a thorough discussion of the alternative perspective and its importance for managing a firm.

applications that involve evaluating specific business unit results or pricing for new business, the use of risk capital will more fairly account for the risk from the shareholder's perspective.

As a result, the term *risk capital* will be used here, even in instances where it is equivalent to the common definition of economic capital.

Are Risk-Based Capital Measures Superior?

Before proceeding, it is worthwhile to consider whether the risk-based measures of capital are necessarily more insightful or meaningful for various strategic decisions than the measures of actual committed capital or market value capital. These risk-based measures of "required" capital are quite often substantially lower than either the book value of the firm or the market value of the firm's equity. As a result, attempts to reflect the "cost" of the capital allocated to specific business units will potentially understate the true costs by ignoring a substantial amount of unallocated capital.

Some practitioners attempt to compensate for this weakness through the use of so-called *stranded capital* charges that further adjust the return measure to reflect a cost associated with the actual capital held in excess of the risk-based capital.

Issues associated with this adjustment will be discussed in Section 6. At this point it is sufficient to emphasize that there is an alternative approach to ensuring that all of the capital held by the firm is taken into account that preserves the risk-based nature of the allocation of capital. In this method, the firm's actual capital is used, but the *allocation method* is risk-based. In other words, measures of risk capital for each business unit serve as the basis for the allocation, but the total amount of capital allocated is simply the firm's actual book value or its market value.

3. Measuring Risk Capital

A critical component of the RAROC measure described in the previous section is the calculation of the risk capital for the firm and, more importantly, the risk capital allocated to various business units. For clarity, the allocation methods will be discussed in Section 4 in the context of a simplified numerical example. In this section, a variety of risk measures and the methods used to measure the firm's overall risk capital will be described.

3.1 Risk Measures

Four common risk measures will be described in this section and then used in Section 4 in the context of a specific numerical example.

Probability of Ruin

The Probability of Ruin is the (estimated) probability that a "ruin" scenario will occur. This is often defined specifically to refer to "default", where the assets are insufficient to fully settle all liabilities, but other definitions of ruin could easily be substituted. For instance, risk capital might be determined based upon an objective of maintaining a particular credit rating over some specified time horizon. In this context, ruin could be defined as a decline in the credit rating below some specified threshold.

Percentile Risk Measure (Value at Risk)

In practice, calculating the firm's actual probability of ruin is often of less interest than a closely related measure – the dollar amount of capital required to achieve a specific probability of ruin target.

Suppose the full distribution indicating the amount, in dollars, that could be lost over a given time horizon was known. Because each dollar of loss will destroy one dollar of "capital", each percentile of this distribution indicates the amount of starting capital required so that the losses do not exceed the capital, resulting in "ruin", with a given probability. For example, the 99th percentile of this distribution determines the amount of capital required to limit the probability of ruin to 1%. Similarly, the 95th percentile determines the amount of capital required to limit the probability of ruin to 5%. This is best demonstrated with a numerical example.

The following table represents 1,000 simulated values from an insurance liability claim distribution[5], with most of the values not shown for convenience and the values sorted in descending order. Here, the expected value of the claims equals $5,000 and the premium charged is $6,000.

Table 1: Simulated Underwriting Loss

Scenario	Liability	Premium	Loss
1,000	7,356	6,000	1,356
999	7,354	6,000	1,354
998	7,269	6,000	1,269
997	7,199	6,000	1,199
996	7,178	6,000	1,178
995	7,039	6,000	1,039
994	7,021	6,000	1,021
993	6,949	6,000	949
992	6,946	6,000	946
991	6,908	6,000	908
990	**6,908**	**6,000**	**908**
989	6,811	6,000	811
988	6,797	6,000	797
987	6,792	6,000	792
986	6,787	6,000	787
985	6,767	6,000	767
:	:	:	:
5	3,323	6,000	-2,677
4	3,261	6,000	-2,739
3	3,248	6,000	-2,752
2	3,243	6,000	-2,757
1	2,735	6,000	-3,265

[5] Simulated liability distributions are used in this section to avoid mathematical details and provide an intuitive discussion of the differences in allocation methodologies. Imprecision introduced through the use of too few simulated values should be ignored.

The last column reflects the "loss" in the profit and loss sense (e.g. as in the calculation of an underwriting loss), with losses depicted as positive amounts and profits as negative amounts. This reversal of the signs is done to facilitate the discussion of both liability claim distributions and asset distributions later in this paper. Note though that care must be taken to distinguish between losses in this profit/loss sense and claim amount distributions, which actuaries commonly refer to as "loss distributions". Note as well that the losses (again, in the profit/loss sense) are shown here net of the premiums charged and other expenses incurred.

The 99^{th} percentile risk measure is the loss amount that is exceeded only 1% of the time. In this specific example, this is equal to $908. If the firm had an additional $908 of risk capital, then it would have sufficient funds ($6,908 in total when the premiums are taken into account) to pay all claims 99% of the time and would suffer partial "default" in only 1% of the scenarios.

This percentile risk measure is essentially identical to the risk measure known as Value at Risk (VaR). There are two minor distinctions that are worth noting:

- Value vs. Nominal Loss Amount – When VaR is calculated for marketable securities such as equities, bonds or derivative instruments, the quantity of interest is the change in value of the instrument over a specific time horizon. In some applications, including the one discussed here, the quantity modeled may not necessarily be the *value* of the cash flows, which would include the effects of discounting for the time value of money and a risk margin. Instead, often the quantity being modeled is simply the total amount of the cash flows or simply the discounted value of these cash flows without consideration of a risk margin. As a result, it may be more accurate to refer to risk capital as a percentile risk measure, rather than a "value at risk".

- Relative vs. Absolute VaR – In some textbooks VaR is defined as the amount by which the percentile deviates from the mean of the profit/loss distribution rather than the amount by which it falls below zero. In the context of the previous numerical example, since the expected liability amount is $5,000 and the premium is $6,000, the expected "loss" is -$1,000 (technically, an expected profit of $1,000). The 99^{th} percentile loss amount is $908, so in a relative sense this is $1,908 worse than the expected loss.

  However, in the application discussed here the goal is to understand how much risk capital is needed. Therefore, the absolute measure of $908 is more relevant than the deviation from the mean, which can be viewed as a relative measure.

Despite these two minor distinctions, the percentile risk measure and the VaR terminology are commonly used interchangeably. This will be the case in various sections of this paper, where the VaR terminology is used to remain consistent with common practice.

Conditional Tail Expectation

The conditional tail expectation (CTE), which is also known as the Tail VaR (TVaR) or the Tail Conditional Expectation (TCE), is similar to the percentile risk measure (VaR) in some respects. The difference is that rather than reflect the value at a single percentile of the distribution, the CTE represents the average loss for those losses that exceed the chosen percentile. Once again, note that the use of the term "loss distribution" refers to the profit/loss sense of the word. When dealing with insurance liabilities, the premiums or the carried reserves should be subtracted from the "claim" amount when calculating the CTE.

Continuing with the previous example, the CTE can be calculated as the average of the 10 scenarios that exceed the 99^{th} percentile value. These scenarios have an average loss of $1,122 and are shown as the boxed values in the following table.

Table 2: Calculation of the CTE

Scenario	Liability	Premium	Loss
1,000	7,356	6,000	**1,356**
999	7,354	6,000	**1,354**
998	7,269	6,000	**1,269**
997	7,199	6,000	**1,199**
996	7,178	6,000	**1,178**
995	7,039	6,000	**1,039**
994	7,021	6,000	**1,021**
993	6,949	6,000	**949**
992	6,946	6,000	**946**
991	**6,908**	**6,000**	**908**
990	6,908	6,000	908
989	6,811	6,000	811
988	6,797	6,000	797
987	6,792	6,000	792
986	6,787	6,000	787
985	6,767	6,000	767
:	:	:	:
5	3,323	6,000	-2,677
4	3,261	6,000	-2,739
3	3,248	6,000	-2,752
2	3,243	6,000	-2,757
1	2,735	6,000	-3,265

Due to certain desirable mathematical properties[6], the CTE has become an increasingly common risk measure used in practice. Interestingly, using this risk measure results in a more ambiguous relationship between the *risk measure* and the *capital* needed to satisfy a specific objective. In the case of the percentile risk measure (VaR), it is easy to see that when the firm's capital is equal to the X^{th} percentile (the *X*% VaR) then the default probability is 1-*X*%. But when capital is equal to the *X*% CTE, the default probability is some amount less than 1-*X*%.

The precise default probability is dependent upon the particular shape of the loss distribution, though some practitioners commonly assume that it is roughly equal to (1-*X*%)/2. In the example shown here, capital equal to the 99% CTE = $1,122 would result in defaults in 5 of the scenarios, or .5% of the time. The reliability of this approximation depends heavily on the shape of the aggregate loss distribution.

Expected Policyholder Deficit Ratio

The Expected Policyholder Deficit (EPD) is closely related to the CTE risk measure. However, the CTE is conditional on the losses exceeding an arbitrarily selected percentile while the EPD is somewhat less arbitrary. The EPD is driven by the average value of the *shortfall* between the assets and liabilities. All liability scenarios are included in this calculation, in contrast to the CTE risk measure that uses only those scenarios for which the liabilities exceed a selected percentile. But in the EPD calculation, scenarios for which there is no "shortfall" are assigned a value of zero.

Again using the same example and assuming that the premiums collected represent the only assets the firm carries, the average shortfall is calculated using all of the highlighted values in the following table.

[6] See Artzner, et al.

Table 3: Calculation of the EPD

Scenario	Liability	Premium	Shortfall
1,000	7,356	6,000	**1,356**
999	7,354	6,000	**1,354**
998	7,269	6,000	**1,269**
997	7,199	6,000	**1,199**
996	7,178	6,000	**1,178**
995	7,039	6,000	**1,039**
994	7,021	6,000	**1,021**
993	6,949	6,000	**949**
992	6,946	6,000	**946**
991	6,908	6,000	**908**
:	:	:	**:**
908	6,032	6,000	**32**
907	6,024	6,000	**24**
906	6,022	6,000	**22**
905	6,019	6,000	**19**
904	6,015	6,000	**15**
903	6,012	6,000	**12**
902	6,008	6,000	**8**
901	6,006	6,000	**6**
900	6,006	6,000	**6**
899	6,003	6,000	**3**
:	:	:	**0**
5	3,323	6,000	**0**
4	3,261	6,000	**0**
3	3,248	6,000	**0**
2	3,243	6,000	**0**
1	2,735	6,000	**0**

The EPD in this case is equal to $38.72. It is closely related to the *value* of shortfall protection, though it does not take into consideration discounting for the time value of money or the inclusion of a risk margin.

To use the EPD as the basis for risk capital, a target ratio of the EPD to the expected liabilities, referred to as the EPD Ratio, is assumed. For instance, if 0.5% is used as the EPD ratio target, then the risk capital would be determined such that the EPD is equal to 0.5% of the expected liability amount, or $25. In the case of fixed assets and lognormally distributed liabilities as shown here, Butsic's formulas[7] can be used to derive risk capital equal to $253.86. In a more general case or when using simulation as the basis for the liability values, an iterative process will be needed because the EPD calculation itself depends on the total assets, which equal the policyholder provided funds as well as the risk capital.

3.2 Risk Measurement Threshold

For each of the risk measures described above, a critical input is the threshold at which the risk is measured. For instance, in the case of default probability, a specific target probability of default must be selected. In the case of the percentile risk measure or the CTE, a specific percentile must be selected. In the case of the EPD Ratio, a specific target ratio must be used.

There are a variety of methods that could be used:

- Bond Default Probabilities at Selected Credit Rating Level – Practitioners commonly rely on bond default statistics to determine a risk measurement threshold. It is often argued that once the firm's managers decide that they desire a "AA" rating they merely need to select a level of risk capital such that their probability of default is consistent with that of an "AA-rated" bond.

[7] See Butsic 1994.

An obvious weakness of this approach is that it does not address the more fundamental question of which rating to target. For the present discussion, this decision is assumed to be based upon knowledge of the firm's business strategy and target customer base.

The more important issue with this approach that is often overlooked is the need to distinguish between i) a probability of default assuming the firm is immediately (or at the end of some chosen time horizon) placed into run-off and ii) a probability of being downgraded over a specific time horizon. To manage a firm and maximize shareholder value, what should matter most to a firm that targets an "AA-rating" is their ability to **retain** that rating with a high probability. However, commonly used risk capital models do not attempt to measure this probability. Instead, they assume a run-off scenario (either immediately or after a specified time period) and assess whether the current capital base is sufficient to withstand a "tail event".

When this run-off approach is used along with a risk measurement threshold tied to default probabilities, a critical question to address is what bond default probabilities to use. One set of statistics which are often quoted are those that appeared in a paper discussing Bank of America's implementation of RAROC[8]. In that paper, the following bond default data was used:

Table 4: Estimated Default Probabilities by Rating

S&P Rating	Moody's Equivalent	1-Year Default Probability	Percentile
AAA	Aaa	0.01%	99.99%
AA	Aa3/A1	0.03%	99.97%
A	A2/A3	0.11%	99.89%
BBB	Baa2	0.30%	99.70%
BB	Ba1/Ba2	0.81%	99.19%
B	Ba3/B1	2.21%	97.79%
CCC	B2/B3	6.00%	94.00%
CC	B3/Caa	11.68%	88.32%
C	Caa/Ca	16.29%	83.71%

Based on this table, many firms have adopted the 0.03% probability of default and, by extension, the 99.97% threshold as an appropriate percentile on the distribution to measure risk. Aside from the obvious danger of placing too much reliance on risk measurements this far out in the tail, there are several subtleties that should be considered:

- Historical vs. Current Estimates – A choice between historical default rates and current market estimates of default rates must be made. The former will be somewhat more stable, but the latter will more accurately reflect current market conditions.
- Source of Historical Default Statistics – The table above contains average default rates that are not consistent with more recent estimates of long term default rates by rating. For example, the following tables show statistics based on both S&P and Moody's analysis of historical data from roughly equivalent time periods (note that some years are not shown).

[8] Source: James, "RAROC Based Capital Budgeting and Performance Evaluation: A Case Study of Bank Capital Allocation", 1996, Wharton Working Paper 96-40. Author cited Bank of America as his source.

Table 5: Alternative Estimates of Historical Default Rates by Rating

		Default % - Data 1981 through			
		1997	2000	2002	2003
S&P	AAA	0.00	0.00	0.00	0.00
	AA	0.00	0.01	0.01	0.01
	A	0.05	0.04	0.05	0.05

		Default % - Data 1970 through				
		1996	1999	2001	2002	2003
Moody's	Aaa	0.00	0.00	0.00	0.00	0.00
	Aa	0.03	0.03	0.02	0.02	0.02
	A	0.01	0.01	0.02	0.02	0.02

The default statistics from the S&P data differ noticeably from the figures quoted in the Bank of America data. The Moody's data also exhibits an unusual relationship between the AA- and A-rated categories and has significantly lower default rates for the A-rated bonds than the S&P data indicates.

- Time Horizon – All of the default rates shown above reflect *annual* default probabilities. In cases where the risk is being measured over a single annual period, these data may be applicable. In many instances though, "default" in risk capital models is often assessed over the lifetime of the liabilities, which have varying time horizons based on the nature of the risk. Some practitioners modify the threshold to account for these varying horizons, arguing that over longer horizons there is a larger probability of a bond defaulting and therefore over longer horizons it is acceptable for the insurer to have a higher probability of default.

- Management's Risk Preferences – Some practitioners argue that the risk measurement threshold that is most relevant is the one that matches the risk preferences of the firm's management. For instance, if the firm's management prefers to limit its probability of default to a particular value, then perhaps that amount should be used to measure the risk?

  Getting the firm's management to articulate and agree upon a particular threshold can be challenging. Attitudes towards risk are often inconsistent and context-specific[9]. In addition, the risk preferences of management, the risk preferences of the board of directors and the risk preferences of the firm's shareholders will often differ, which further complicates this exercise in practice.

  More importantly, effective risk preference statements go beyond articulating a "probability of default". To begin, effective risk preference statements should reflect both the *risk* and the potential *reward* for taking risk. Secondly, shareholder value for an insurer is ultimately driven by events that may cause a ratings downgrade, a weakened financial position or any other event that diminishes the firm's ability to remain a going concern and continue to write profitable insurance business in perpetuity. Risk preferences intended to capture the shareholders' perspective are unlikely to focus on the probability of default.

- Arbitrary Default Probability, Percentile or EPD Ratio – While it may not be scientific, one could choose an arbitrary threshold such that the risk measure can be reliably estimated and reflect the appropriate relative views of "risk". As will be shown in Section 5, in many applications it is the relative measures of risk associated with a firm's different activities that matter the most. In addition, even under the most ideal circumstances it may be very difficult to reliably and accurately measurer *any* loss distribution's 99.97^{th} percentile. This is especially true when dealing with insurance liability risk models for which significant model and parameter uncertainty exists.

[9] See Bazerman for a detailed discussion of the many behavioral biases that complicate this process.

For the sake of brevity, these issues will not be fully resolved here. When various applications of risk capital and RAROC measures are used in Section 5, the sensitivity of the results to the choice of risk measurement thresholds will be explored.

3.3 Risk Sources

3.3.1 Overview

While practices vary, the conventional approach to measuring a firm's aggregate risk profile segregates the risks into five main categories following the framework adopted by the NAIC and several rating agencies:

- **Market Risk –** This measures the potential loss in value, over the selected risk exposure horizon, that results from the impact that changes in equity indices, interest rates, foreign exchange rates and other similar "market" variables have on the firm's *current* investments in equities, fixed income securities or derivative securities.

  Standard practice is to estimate the distribution of portfolio profits or losses over the selected horizon and use risk measures such as VaR or CTE. A critical issue though is to identify the appropriate time horizon over which to measure the profit/loss distribution and the resulting risk measure. Calculations of VaR for these classes of investments are typically performed over a horizon on the order of 10 or fewer days, which roughly coincides with estimates of the time required to divest risky positions. Calculating the VaR or CTE over longer horizons can be quite challenging, given limitations in historical data used to calibrate the models, the need to account for potential non-stationary models, the need to reflect mean reversion and autocorrelation across periods and the need to account for changes in portfolio composition over longer horizons[10]. For risk-adjusted performance measurement within an insurance company though, the risk exposure horizon for analyzing the insurance liabilities is necessarily much longer because their underwriting and reserve risk exposures generally must be held to maturity. Aggregating market risk with the other risks is therefore inherently problematic due to differences in these time horizons.

  For the moment, and at the risk of confusing matters, this potential disparity in the time horizons will be ignored and a one-year horizon will be selected for measuring market risk. This simplified approach is consistent with current insurance industry practice and allows the discussion to focus on other aspects of this methodology. Discussion of the challenges associated with the time horizon inconsistency will be deferred until Section 6.

  The specific methods used to calculate VaR for various asset classes are covered extensively in various readings on the current CAS Syllabus and will not be discussed in detail here.

- **Credit Risk –** This measures the potential loss in value due to *credit events*, such as counterparty default, changes in counterparty credit rating or changes in credit-rating specific yield spreads[11]. These credit-related risk exposures can impact the firm in a variety of ways, but the three that are the most important include:

  - Marketable Securities, Derivative and Swap Positions – A firm's marketable securities, derivative positions and swap positions may be subject to specific exposure to the various credit events noted above. It is somewhat arbitrary to categorize these exposures within the credit risk category, as opposed to the market risk category, but for practical purposes the methods and models used for the various credit risks are likely to overlap and so it is natural to include these along with the other sources of credit risk.

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[10] See Rebonato & Pimbley for an insightful discussion of this topic.

[11] Some practitioners classify certain components of credit risk, such as changes in credit spreads unrelated to changes in the underlying counterparty's rating, along with the market risks discussed earlier. Depending on the methods used though, it may be difficult to separate these components cleanly. For presentation purposes, this discussion assumes that all credit-related risks are included as a single risk source.

- Insured's Contingent Premiums and Deductibles – These reflect policyholder obligations in the form of loss-sensitive premium adjustments, deductibles, etc. that, in some instances, cannot be readily offset against claim payments and therefore create a counterparty credit exposure.
- Reinsurance Recoveries – This category represents the most challenging source of credit risk to an insurance company. While the same methods used for the other sources of credit risk are generally applicable here, there are three unique aspects to this category:
  - Definition of Default – For reinsurers, the definition of "default" may need to be adjusted to properly account for the fact that a credit downgrade below the equivalent to an investment-grade rating could, and often does, create a "death spiral" for the firm. Their ability to write future business will be substantially impacted and many existing policyholders will rush to commute or otherwise settle outstanding and potential recoveries. This could create a severe liquidity crisis and result in settlement amounts far less than 100% of potential recoveries for the reinsureds. As a result, a broader definition of default may be necessary.

    In addition, disputes between insurers and their reinsurers are common and often result in settlements of less than 100% of potential recoveries. To the extent that the risk from such disputes can be quantified, they may be treated as the equivalent of a partial default.
  - Substantial Contingent Exposure – Potential exposure to reinsurers' credit risk can far exceed the reinsurance recoverable balances currently on the balance sheet. The balance sheet entries reflect only the receivables relating to paid claims and the expected recoveries against current estimates of *gross* loss reserves. They do not include the potential recoveries from reinsurers in the event of adverse loss development or in the event that losses on new written and earned premiums exceed their expected values. In practice, these contingent exposures need to also be reflected[12].
  - Correlation with Other Insurance Risks – It should be obvious from the previous point that reinsurance credit risk is likely to be highly correlated with the underlying insurance risks. As a result, it is harder to rely on external credit-risk only models for this category of credit risk exposure than it is for investment portfolio or other assets with credit exposure.

The specific methods used to measure credit risk are covered in other readings on the current CAS Syllabus and will not be discussed in detail here.

- **Insurance Underwriting Risk –** This category includes the three primary categories of insurance risk:
  - Loss Reserves on Prior Policy Years – Potential adverse development from existing estimates.
  - Underwriting Risk for Current Period Policy Year – Potential losses (and expenses) in excess of premiums charged for the "current" policy period. In some cases, the definition may include only unearned premiums, but in general it is assumed that one year of new business will be written and so the underwriting risk will also include the potential losses associated with those premiums as well.

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[12] Moody's P&C capital model documentation discusses this issue and presents one method for doing this.

- Property Catastrophe Risk – Due to the unique modeling needs of catastrophe risk associated with earthquakes, hurricanes and other weather-related events, these risks are often segregated.

  Each of these three categories of insurance underwriting risk will be described in more detail in the subsequent portions of Section 3.

- **Other Risk Sources –** The above list is far from exhaustive. There are a variety of additional "risks" that could impact a firm, including a wide variety of operational risks associated with the failure of people, systems or processes, as well as a wide variety of strategic risks related to competitors. While these are important risks for a firm to understand, anticipate and manage, they are generally less quantifiable and therefore do not serve a critical role in the current discussion. For convenience, they will be ignored in the discussion that follows.

Given this overview of the typical risk categories used, the rest of this section will explore the insurance risk category in more detail.

3.3.2 Loss Reserve Risk

For most P&C insurers, the magnitude of carried loss and expense reserves, as well as the uncertainty associated with the estimation of these reserves, makes the risk inherent in loss reserves the dominant risk to the firm.

To fully appreciate what is being measured with respect to loss reserve risk, it is useful to make a distinction between three components of the total risk:

- Process Risk – This is the risk that actual results will deviate from their expected value due to the random variation inherent in the underlying claim development process.
- Parameter Risk – This is the risk that the *actual*, but unknown, expected value of the liability deviates from the *estimate* of that expected value due to inaccurate parameter estimates in the models.
- Model Risk – This is the risk that the *actual*, but unknown, expected value of the liability deviates from the *estimate* of that expected value due to the use of the wrong models.

A variety of actuarial methods exist to establish loss reserves. Some of these lend themselves to a statistical analysis of two closely related concepts:

- Reserve Estimation Error – This represents the range of uncertainty associated with a given reserve estimate, rather than the uncertainty with regard to the ultimate "outcome". It is a measure of how statistically reliable a given *estimate* is relative to the true, but currently unknown, value. This uncertainty is usually depicted as a *confidence interval* for a given estimate.
- Reserve Distribution – This represents the full distribution of the unpaid loss amount and is intended to estimate the likelihood that the ultimate outcome deviates from the current estimate. It is a depiction of the full range of possible values for the unpaid loss, along with their associated probabilities, from which the realized value will be drawn. This distribution is often expressed in terms of the *percentile* values (e.g. the 98th percentile is the value that is larger than 98% of all other possible values).

The primary goal for this paper is to obtain the full distribution of unpaid losses (perhaps at some particular valuation date) and not merely a confidence interval for the estimate.

Alternative Methods for Measuring Loss Reserve Risk

With the above distinction in mind, some common methods used for determining the loss reserve distribution can be summarized as follows:

- Mack Methods (1993, 1999) – These are analytical methods for estimating the standard error of the reserves based on the traditional chain ladder model for estimating ultimate losses. Their analytical tractability makes them ideal for the current purposes, where frequent stresstesting of assumptions and methods is required, despite some inherent weaknesses of the methods[13].

- Hodes, Feldblum, Blumsohn – This is a simulation method that is also based on the Chain Ladder model for estimating ultimate losses. The approach involves simulating age-to-age loss development factors for each development period, rather than relying on various averages. This approach is intuitively appealing, and quite flexible, though the use of simulation could impact run-time and the reliability of the results.

- Bootstrapping Method – A variety of "bootstrapping" methods exists. One method discussed by England and Verrall uses the distribution of incremental paid or incurred loss amounts to simulate a new, hypothetical loss triangle, from which loss development factors can be derived and new ultimate loss amounts estimated. The result of a large number of similar simulations produces a distribution of ultimate losses and reserves.

- Zehnwirth Methods – While the methods above attempt to adapt existing actuarial methods to produce estimates of the full distribution, Zehnwirth has proposed a different modeling framework that relies on a ground-up probabilistic model of the loss development process. His model works with the (log) incremental paid losses and identifies common trends impacting accident years, calendar years and development periods simultaneously. Using these more elaborate probabilistic models, estimates of the full distribution of ultimate outcomes follow more naturally.

- Panning Econometric Approach – In a recent paper[14], Panning addressed three common weaknesses of some of the previous methods. First, they tend to be derived from chain ladder loss development estimation methods which are *ad hoc* and do not rely on objective criteria for measuring and maximizing the goodness of fit to the observed data. Second, they often rely on cumulative loss data, which introduces serial correlation. And third, they often incorrectly assume constant variance across development periods, even though the development periods should be expected to exhibit heteroskedasticity.

  Panning's method corrects for these three characteristics by relying on linear regression techniques that minimize the squared errors, uses incremental rather than cumulative data and models each development period separately to account for the non-constant variance in the error terms for each development period.

- Collective Risk Model – As will be discussed in the next section with regard to underwriting risk, it is conceptually possible to use claim frequency and severity distribution assumptions, so long as they both represent the distributions of *outstanding* frequency and *outstanding* severity. However, because the severity distributions used at inception for <u>all</u> claims will include the smaller, simpler and more quickly reported and paid claims as well as the larger, more complex and slower reported and paid claims, it is critical to use severity distributions conditional on the age of the outstanding claims. Few entities are likely to have sufficient data to accomplish this parameterization reliably, though research by the Insurance Services Office (ISO) has produced interesting results[15].

- Relationship to Underwriting Risk – In the absence of robust loss reserve data, the coefficient of variation for the ultimate loss distributions could be based on the coefficient of variation for the underwriting risk distributions for similar classes of business. To use this information, the underwriting model parameters would have to be adjusted to reflect the declining coefficient of variation relative to the ultimate losses as a given accident year ages.

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[13] See Hayne and the CAS Working Party on Quantifying Variability in Reserve Estimates

[14] See Panning, 2005.

[15] See Meyers, Klinker and Lalonde

Given the variety of methods available, this paper will not attempt to address the many technical differences that may result from each of them. Readers interested in a more thorough treatment of these various methods, and in particular their strengths and weaknesses, are encouraged to review the report issued by the CAS Working Party on Quantifying Variability in Reserve Estimates.

On a conceptual level all of these methods attempt to quantify the distribution of outstanding claims as of a given date. In the discussion that follows the Mack Method will be used. This particular method was chosen solely for convenience, though its analytical tractability is particularly appealing.

While the details of the calculations will not be shown here, the following numerical example uses industry data[16] for Commercial Auto Liability and the formulas from Mack's 1993 paper to demonstrate the method.

Table 6: Sample Paid Loss Data for Mack Method Example

Sample Insurer
Commercial Auto Liability

Accident	Valuation Month									
Year	12	24	36	48	60	72	84	96	108	120
1994	357,848	1,124,788	1,735,330	2,218,270	2,745,596	3,319,994	3,466,336	3,606,286	3,833,515	3,901,463
1995	352,118	1,236,139	2,170,033	3,353,322	3,799,067	4,120,063	4,647,867	4,914,039	5,339,085	
1996	290,507	1,292,306	2,218,525	3,235,179	3,985,995	4,132,918	4,628,910	4,909,315		
1997	310,608	1,418,858	2,195,047	3,757,447	4,029,929	4,381,982	4,588,268			
1998	443,160	1,136,350	2,128,333	2,897,821	3,402,672	3,873,311				
1999	396,132	1,333,217	2,180,715	2,985,752	3,691,712					
2000	440,832	1,288,463	2,419,861	3,483,130						
2001	359,480	1,421,128	2,864,498							
2002	376,686	1,363,294								
2003	344,014									

Accident	Age to Age Factors									
Year	12:24	24:36	36:48	48:60	60:72	72:84	84:96	96:108	108:120	120:ULT
1994	3.143	1.543	1.278	1.238	1.209	1.044	1.040	1.063	1.018	
1995	3.511	1.755	1.545	1.133	1.084	1.128	1.057	1.086		
1996	4.448	1.717	1.458	1.232	1.037	1.120	1.061			
1997	4.568	1.547	1.712	1.073	1.087	1.047				
1998	2.564	1.873	1.362	1.174	1.138					
1999	3.366	1.636	1.369	1.236						
2000	2.923	1.878	1.439							
2001	3.953	2.016								
2002	3.619									
Wtd	3.491	1.747	1.457	1.174	1.104	1.086	1.054	1.077	1.018	
Simple	3.566	1.746	1.452	1.181	1.111	1.085	1.053	1.075	1.018	
Select	3.491	1.747	1.457	1.174	1.104	1.086	1.054	1.077	1.018	1.018
To Ultimate	14.703	4.212	2.411	1.654	1.409	1.277	1.175	1.115	1.036	1.018

The following table summarizes the estimated reserve on a nominal basis and the Mack Method standard errors both by accident year and in the aggregate.

16 Source: AM Best

Table 7: Mack Method Example

Sample Insurer
Commercial Auto Liability

Accident Year	(1) Paid Loss	(2) LDF	(3) Ultimate Loss	(4) Selected Ultimate	(5) Reserve	(6) Mack Method Std Error
1994	3,901,463	1.018	3,970,615	3,970,615	69,152	0
1995	5,339,085	1.036	5,530,030	5,530,030	190,945	76,874
1996	4,909,315	1.115	5,474,165	5,474,165	564,850	123,856
1997	4,588,268	1.175	5,391,810	5,391,810	803,542	135,916
1998	3,873,311	1.277	4,944,310	4,944,310	1,070,999	266,040
1999	3,691,712	1.409	5,201,766	5,201,766	1,510,054	418,295
2000	3,483,130	1.654	5,761,106	5,761,106	2,277,976	568,213
2001	2,864,498	2.411	6,905,058	6,905,058	4,040,560	890,842
2002	1,363,294	4.212	5,742,274	5,742,274	4,378,980	988,473
2003	344,014	14.703	5,057,913	5,057,913	4,713,899	1,387,316
	34,358,090		53,979,046	53,979,046	19,620,956	2,490,469
					Coefficient of Variation	0.127

The key result for the present purposes is the estimated coefficient of variation for the aggregate unpaid liabilities. From this, and the mean of the reserve risk distribution, a lognormal distribution is assumed for the outstanding losses and the parameters estimated. The lognormal assumption was chosen arbitrarily; in practice it may be important to confirm whether this is a reasonable assumption and to consider other distributions as well.

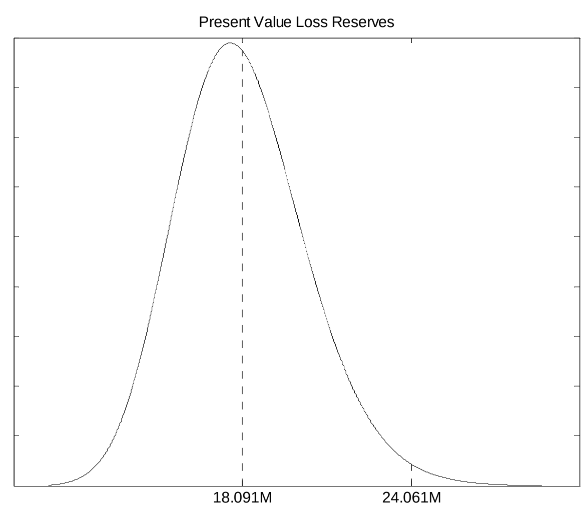
Note also that the loss reserve distribution parameters should be adjusted to reflect their *discounted* values, where the discount rate is based on a risk-free rate (4.0% in this case) and the discounting is done to the *end* of the one-year period consistent with an assumption that all payments are made at the end of the year. This is one approach to normalizing the models to account for the different time horizons over which the claim payments will be made.

The parameters of the lognormal distribution are calculated as follows using the method of moments:

Table 8: Reserve Risk – Lognormal Parameters

	Undiscounted	Discounted
Mu	16.784	16.703
Sigma	0.126	0.126

The resulting distribution of outstanding losses, on a present value basis, can be shown graphically as follows with the mean ($18.091M) and the 99th percentile ($24.061M) values highlighted:



Ultimate Liability vs. Loss Development During Horizon

Some practitioners advocate measuring the reserve risk over a finite horizon, such as one year, and reflecting only the degree to which the ultimate liability may need to be restated as of the end of this horizon. This is in contrast to the measure described above, which reflects the uncertainty in the loss reserves that comes from, for instance, unknown rates of loss severity trend over the lifetime of the liability. The one-year measurement reflects only the degree to which the *best estimate* could change over this time horizon, making it more compatible conceptually with the market VaR and credit VaR calculations discussed previously and more consistent with calendar year measures of income that are often used.

For many lines of insurance business, the differences between a lifetime of liability horizon and a one-year horizon is likely to be small and perhaps insignificant. For lines such as high-layer, excess of loss general liability where there is little new information that emerges over a short horizon, the differences can be significant[17].

For the present purposes, these potential differences will be ignored and lifetime of liability insurance risk distributions will be used. However, the distributions will be adjusted to reflect present value loss amounts as of the end of the one-year horizon, as if all claims are known and paid at the end of this period. Appendix A of this paper discusses the issues associated with this choice of *risk exposure horizon* in more detail, including the potential inconsistency with the market and credit risk measures.

Ceded Reinsurance Recoveries

The analysis of ceded reinsurance can be modeled directly using similar models as is done for the gross loss reserves, adjusted of course to take into consideration the specific nature of the reinsurance agreements, or indirectly by modeling the *net* reserve risk. Typically, modeling the net reserves will be the easiest approach. However, the need to take into consideration the credit risk on reinsurance recoveries may favor a more direct estimation of ceded reserves.

3.3.3 Underwriting Risk

The term *underwriting risk* is used to reflect the risk that total claim and expense costs on new business written and/or earned during a specified *risk assumption horizon*[18] exceed the premiums collected during the same period. This new business written reflects both renewals as well as policies for "new" customers – it is all business written during the risk assumption horizon but that is not currently reflected on the firm's balance sheet. In some rating agency and regulatory capital models, this is often referred to as *premium* risk or *new business* risk.

A variety of risk factors affect the distribution of potential claims and expenses on new business. To simplify the discussion of the quantification of this risk the following methods will be discussed:

- Loss Ratio Distribution Models
- Frequency & Severity Models
- Inference from Reserve Risk Models

Loss Ratio Distribution Models

One easy approach to implement relies on an assumed distribution of loss ratios. Combined with an estimate of written premium during the risk assumption horizon, either deterministically or

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[17] Notice that by ignoring the risk of subsequent deviations beyond the selected horizon there is an implicit assumption that the liabilities could, if necessary, be transferred to a third party subsequent to the restatement, since there will still be risk of further adverse deviations but the firm will have no capital to support this risk. For this assumption to be reasonable, the firm will *also* need to have sufficient additional resources to pay a fair market risk margin to the assuming party.

[18] The term *risk assumption horizon* is being used here to refer to the period over which new exposures to risk are being added to the firm. This is in contrast to the *risk exposure horizon*, discussed in Appendix A, which refers to the period over which the risks are assumed to affect the firm. In an insurance context, the risk assumption horizon reflects how much new business is assumed to be written.

stochastically generated, the full distribution of potential liabilities and expenses net of premium can be determined.

Among the most important considerations in applying this approach are the following:

- Source of Model Parameters – Loss ratio distribution parameters may be based on either historical loss ratio experience for the company or on industry data if the company's own data lacks sufficient credibility.

  When using the company's historical loss ratio data, it is important to adequately reflect changes in claim cost trends, premium adequacy and the relative volume over the data analysis period.

  Industry loss ratio data from external sources (e.g. ISO or NCCI in the U.S.) can either supplement or replace company data in certain circumstances.

- Choice of Distribution Models – The foundation of the risk capital framework discussed here is the explicit recognition of uncertainty. This makes the choice of distribution models, their applicability to the particular risk and their fit with the historical data critically important.

  While common models for loss ratios will often be limited to Normal, Lognormal and Gamma distributions, others are certainly feasible. Special attention however should be paid to the model fit and often-encountered inconsistencies should be avoided.

  For instance, lognormal distributions are commonly used to model loss ratios due to their desirable quality of a heavy right tail that seems to reflect reality. However, when applied to loss ratios, this model tends to exhibit a left tail that is too heavy and a right tail that is too light. In other words, a lognormal model could result in a high probability of the loss ratios being well below the mean and too little probability of loss ratios well above the mean.

  One way to correct for this is to use a mixture of two lognormal models, one with a very small coefficient of variation and one with a very large coefficient of variation. Using "small" losses to calibrate the first model and "large" losses to calibrate the second, the two models can be combined to produce a more reasonable overall loss ratio distribution[19].

As an example of this approach, consider the following hypothetical data for a commercial auto insurer. The estimated loss ratios for the past ten accident years are assumed to be representative of the prospective years' results.

[19] See Mildenhall, 1997

Table 9: Hypothetical Historical Loss Ratios

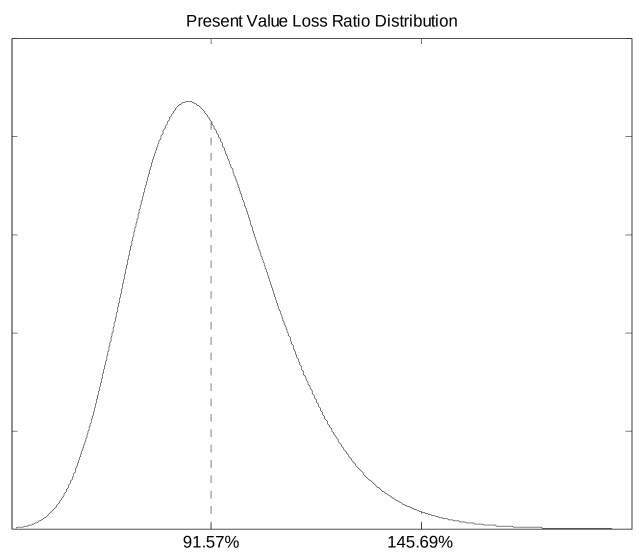
Year	Earned Premium	Ultimate Loss	Loss Ratio
1994	5,272,000	3,970,615	75.3%
1995	5,188,000	5,530,030	106.6%
1996	4,212,000	5,474,165	130.0%
1997	3,656,000	5,391,810	147.5%
1998	4,528,000	4,944,310	109.2%
1999	5,012,000	5,201,766	103.8%
2000	6,174,000	5,761,106	93.3%
2001	6,202,000	6,905,058	111.3%
2002	6,528,000	5,742,274	88.0%
2003	6,276,000	5,057,913	80.6%
		Mean	104.6%
		Std Deviation	22.1%
		CV	0.2113

Assuming a standard lognormal distribution but adjusting the parameters to reflect the discounted loss ratio as of the end of the year (i.e. assuming all claims are paid at the end of the year), the following parameters can be estimated using the method of moments:

Table 10: Discounted Distribution Parameters

Discounted Mean	91.6%
CV	0.2113
Mu	-0.1099
Sigma	0.2090

The resulting lognormal distribution is as shown in the following graph:



Frequency & Severity Models

While the loss ratio distribution model can be based on rather limited historical data, more robust models can be developed which rely on separate frequency and severity models to determine the aggregate loss distribution. When sufficiently detailed and relevant claim data is available, this approach can have a number of key advantages. Klugman, Panjer and Wilmot provide the following list of advantages that this approach has over the previous method[20]:

1. Growth in volume of business can be more easily accounted for;
2. Inflation can be more accurately reflected, particularly when there are deductibles and policy limits;
3. Changes in limit and deductible profiles can be directly reflected;
4. Impact of deductibles on claim frequency can be reflected;
5. Estimates of the split of losses between insured, insurer and reinsurer can be mutually consistent.

Using these models, separate probability distributions for claim frequency can be developed based upon Poisson, Negative Binomial or Normal distributions and separate claim severity models can be developed using any number of distributions such as the Lognormal, Gamma, Exponential or Beta distributions. The aggregate loss distribution can then be determined via a variety of methods:

- Analytical Solution – For certain choices of frequency and severity models, it may be possible to determine a closed form solution for the aggregate loss distribution (sometimes referred to as the *collective risk model*) based on the frequency and severity parameters.
- Numerical Methods – Numerical approximations based upon the Fast Fourier Transform can be used to determine the aggregate loss distribution based on the frequency and severity parameters.
- Approximations – Using the mean, variance and possibly higher moments of the collective risk model, an aggregate loss distribution can be approximated with parameters that provide a best fit to these moments.

  If N is random number of claims and S_i is the random severity for each claim, the collective risk model (without parameter uncertainty) suggests the following mean and variance of the aggregate distribution:

  Aggregate Loss Mean $= E(N)E(S)$

  Aggregate Loss Variance $= E(N)Var(S) + Var(N)E(S)^2$

  The previous formulas can be trivially adjusted to reflect parameter uncertainty by introducing a "shock" or random deviation to both of the claim counts, N, and the severity S_i. Heckman and Meyers[21] introduce *contagion* parameters c for the frequency shock and b for the severity shock and derive the following modified formulas for the mean and variance of the aggregate distribution:

  Aggregate Loss Mean $= E(N)E(S)$

  Aggregate Loss Variance $= E(N)E(S^2)(1 + b) + Var(N)E(S)^2(b + c + bc)$

  To demonstrate the flexibility of this method, consider the following example taken from the IAA Report on Insurer Solvency. Assume that all (ground-up, unlimited) commercial auto liability claims follow a lognormal distribution with a mean value of \$6,000 and a coefficient of variation of 7.0. The claim frequency is assumed to be Poisson. Further assume that the contagion parameters are $c = 0.02$ and $b = 0.003$. If an insurer has \$600 million of written premium and an expected loss ratio of 105% on an undiscounted basis, this implies the following expected claim counts and aggregate claim costs:

[20] See Klugman, Panjer and Wilmot, Page 292.

[21] See Heckman and Meyers

Table 11: Aggregate Loss Distribution

Expected Claim Count	105,000
Expected Aggregate Loss	630,000,000
Std Dev of Aggregate Loss	13,771,498

These parameters could then be used to fit an appropriate distribution for the aggregate claim costs for the line of business. The model can also be readily adjusted to reflect different premium volumes, different expected loss ratios or different attachment point and limit profiles[22]. This flexibility is particularly appealing because it can be achieved in a consistent fashion across different lines of business.

In addition, aggregating the moments (mean and standard deviation) across different lines of business is also convenient using this model. If the frequency and severity distributions across different lines of business are assumed to be impacted by *common* shocks, though with different sensitivity to these common shocks, this will induce *dependency* across different lines of business.

These two features, consistency across lines of business and the ability to infer dependency across lines of business, are particularly beneficial in the application discussed here.

- Simulation – Aggregate losses can also be estimated via simulation and the simulated results can either be used directly (via the empirical distribution) or fit to a particular distribution model. While this approach has the advantage of allowing for complex policy structures to be modeled with minimal mathematical complexity, the results can be unstable without a large number of iterations. This, as well as the processing time required to run a large number of iterations, can make it difficult to test the sensitivity to the assumptions.

Inference from Reserve Risk Models

An alternative to the direct analysis of the insurance pricing risk for the new business is to infer the magnitude of the risk on new business from the reserve risk models described earlier. The reserve risk models estimate the potential variability in unpaid losses as of some date <u>after</u> the policies are written and certain previously unknown parameters have been determined, such as the premium volume, the number of catastrophe events, some portion of the total claim counts, etc.

Recognizing that the reserve risk models reflect the risk *conditional* on this particular information, *unconditional* models can be inferred from these models and applied to the current risk assumption horizon at inception. Alternatively, a rough approximation that assumes the coefficient of variation for the most recent accident year can be used for the prospective accident year can be used.

3.3.4 Property-Catastrophe Risk

Ever since the large insured hurricane and earthquake losses of the early 1990s, natural catastrophe risk modeling has become substantially more sophisticated.

Prior to the development of these models, insurers often relied upon historical loss experience to assess their potential losses from these natural catastrophes. But historical catastrophe loss experience can be a misleading indicator of potential losses for a variety of reasons, including the fact that the events are rare, the exposure changes over time, severities change over time based on building materials and designs, repair and contents replacement costs are poor indicators of current costs, etc.

In contrast, the leading catastrophe risk models rely on meteorological, seismological and engineering data to produce a probability distribution of potential catastrophe losses. Through simulation of various events, an assessment of damage to affected property is determined together with an assessment of the impact of specific insurance and reinsurance coverages. From this, probability

[22] To adjust the results to reflect different attachment points and limits, the mean and the standard deviation of the ground up and unlimited claim severity distribution would be adjusted to reflect the mean and the standard deviation of the appropriate layer.

distributions of the insurers' potential gross and net losses from earthquakes, hurricanes, severe storms and related events can be derived.

These models typically have various modules, such as:

- Stochastic Module/Hazard Module – These modules jointly determine the possible events such as earthquakes and hurricanes that can occur, as well as their location, intensity, etc.
- Damage Module (Vulnerability Module) – This uses the exposure information for the insurer to determine the damage that would occur for any given event.
- Financial Analysis Module – This applies the insurance and reinsurance policy terms to the losses to determine the financial impact to the insurer.

The specific capabilities and model features vary from vendor to vendor and will not be addressed in this paper.

3.4 Risk Aggregation

Given the risk distributions for market, credit, loss reserves, underwriting and property-catastrophe risk, the next step is to determine an aggregate risk distribution for all risk sources combined[23]. As noted earlier, in instances where the risk distributions for each category were not derived using comparable risk exposure horizons, this may be far more challenging than it appears.

Ignoring that complexity for the moment, the critical issue to address is the degree of correlation or dependency across the various risk categories.

3.4.1 Correlation vs. Dependency

These two terms are often used interchangeably, though technically there is an important difference between correlation and dependency. Mathematically, the term *correlation* refers to a specific measure of linear dependency, while *dependency* reflects a more general measure of the degree to which different random variables depend on each other.

The best way to see the distinction is through a very simple example. Assume that X represents a Standard Normal random variable and that $Y = X^3$. For any given values of X and Y, it is clear that Y_i is entirely dependent upon the value of X_i. However, if the values were simulated and the correlation were measured, the correlation would be estimated to be only approximately 0.78. In this case, perfect dependency does not imply perfect correlation.

3.4.2 Measures of Dependency

Recognizing this distinction between correlation and dependency, how would one derive measures of dependency across risk categories, or within risk categories and across different asset classes or lines of business?

Despite the importance of this question, there are currently no entirely satisfying answers. Instead, there are three common methods used, each of which suffer from various practical limitations.

1. Empirical Analysis of Historical Data – Despite the obvious appeal of this approach, the reality is that often the data required for this analysis does not exist. Worse, even when data does exist the measurement errors often produce estimates of correlations that are unreliable, inconsistent and counter-intuitive. Finally, by definition historical data contains very little insight into how much correlation or dependency exists when "tail events" occur. Seemingly independent events under normal conditions may turn out to be more highly dependent under extreme conditions. For example, when Russia defaulted on its debt in the Summer of 1998, a flight to quality and demand for liquidity caused simultaneous disruptions in a variety of sectors and led to the collapse of LTCM, a multi-billion dollar hedge fund.

[23] Some practitioners do not derive an aggregate risk distribution and instead use the stand-alone risk distributions to derive risk measures for each risk and then simply aggregate these risk measures into an overall aggregate risk measure. This simplified approach is discussed below in Section 3.4.3.

2. Subjective Estimates – Subjective estimates can be made that reflect dependency during tail events and that reflect the user's intuition, so at times these are preferred. However, this approach suffers from the fact that as the number of unique risk categories or lines of business increases, the number of pairwise correlation/dependency assumptions that must be made grows exponentially.

   Aside from the obvious burden this places on the user, the task of ensuring internal consistency becomes onerous. For instance, with three lines of business, subjective estimates of the correlation between Line A and Line B and subjective estimates of the correlation between Line A and Line C necessarily restrict the range of internally consistent subjective estimates of the correlation between Line B and Line C[24].

   It may be possible to enforce a bit more structure on this process by adopting certain explicit conventions with regard to what can or cannot cause correlation, which will help to avoid these potential inconsistencies. The results will still be subjective, but may be slightly less onerous to produce.

3. Explicit Factor Models – In many advanced applications of Value at Risk for asset portfolios, explicit factor models are used to link the variability of different assets or asset classes to common *factors*. The consequence of this is that correlations across assets or asset classes can be derived based on each asset's respective sensitivity to these common factors.

   An insightful application of this approach was alluded to earlier in the discussion of the collective risk model with common "shocks", which is described in great detail by Meyers, Klinker and Lalonde. Given the assumption of common shocks to the frequency and severity parameters, correlations across lines of business can be derived based upon the contagion parameters.

   This particular approach is intended to reflect the dependency across lines of business and across the reserve and underwriting risk categories. Separate assumptions would generally have to be made to reflect dependency across other risk categories (e.g. market, credit and property catastrophe).

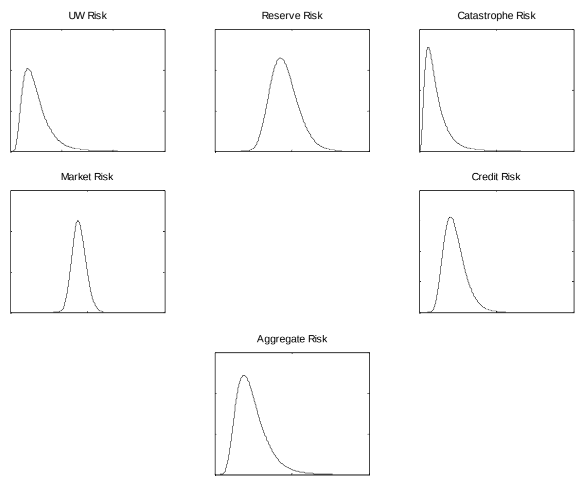
3.4.3 Aggregate Risk Distribution

Given the selected correlation or dependency measures, the next step is to create an aggregate risk distribution using each of the component risk distributions.

Ignoring, for now, the potential inconsistency of the measurement basis for the different risk types, the goal is to combine the market, credit, loss reserve, underwriting and property catastrophe distributions into a single aggregate distribution. The following diagram depicts each of the stand-alone risk distributions and the resulting aggregate risk distribution that can be obtained using any of the methods described in this section.

[24] The technical requirement is that the correlation matrix must be positive, semi-definite. See Rebonato, 1999, for a discussion of a technique that can be used to "tweak" an invalid correlation matrix so that this requirement is met.

Table 12: Aggregate Risk Distribution



To derive this aggregate distribution, a variety of mathematical techniques can be used:

- Closed Form Solutions – In highly stylized and simplified cases, it may be possible to derive closed form, analytical formulas for the aggregate risk distribution. However, in practice the wide variety of stand-alone risk distributions may make this impractical.
- Approximation Methods – Several approximation methods are available to overcome the complexity of deriving analytical formulas, including assuming that all risk distributions are Normal or Lognormal and deriving the model parameters from either the actual moments of the respective distribution or specific percentiles of the actual distributions.
- Simulation Methods – Simulation methods are often favored by practitioners because of their intuitive interpretation and the "brute force" way in which the results can be derived. However, run-time and stability concerns can make these approaches impractical and hamper the users' ability to thoroughly test the implications of the model and assumptions.

  When simulation is used, it is important to reflect the dependency across the simulated variables and the uniqueness of the stand-alone, marginal distributions. Two general approaches are common:

  1. Iman-Conover Method – This method, which is used in the commercial software package @Risk, uses a *rank correlation* measure that effectively simulates each random variable separately based on its marginal distribution and then "reshuffles" the stand-alone results in such a way as to preserve the specific rank correlations[25].
  2. Copula Method – Copula methods are similar conceptually to the Iman-Conover method and attempt to ensure, for instance, that when two variables are highly and positively dependent upon each other, a value for one variable in the far right tail of its distribution will *generally* result in a value for the second variable that is also in the far right tail of its distribution. Alternatively, a highly negative dependency will result in the second variable *generally* being in the far left tail of its distribution.

     To achieve the goal stated above, one could simulate correlated/dependent <u>percentiles</u>, which will reflect values between 0 and 1 and then use these correlated/dependent percentiles to obtain values from the respective distributions. Copulas represent multivariate distributions with values that range from 0 to 1 and therefore they can naturally be used to represent these dependent percentiles.

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[25] See Mildenhall's Chapter in the CAS Working Party on Correlation Report for a thorough discussion of this method.

Different copulas will result in different degrees of dependency, particularly in the tail.

A *normal copula* uses a multivariate standard normal distribution to generate correlated standard normal values and then maps these into their corresponding percentiles by inverting the standard normal distribution function. The resulting percentiles will themselves be dependent, in the sense that when the correlations are assumed to be high, the values of the various simulated percentiles will be generally similar.

However, normal copulas tend to induce rather low degrees of dependency in the tails of the distributions. Therefore, other copulas with more tail dependency are often used. For instance, if correlated values from a *Student's t* distribution are used in place of the multivariate normal distribution, more tail dependency results, depending on the number of degrees of freedom assumed for the *t* distributions.

Other much more complex copulas can also be used, which are conceptually the same but significantly more difficult to use.

3.4.4 Alternative Approach – Aggregate Risk Measures

The description in the last section assumed that the intent is to model the aggregate risk distribution, from which an aggregate risk measure can be calculated. It is quite common for practitioners to avoid this step of first deriving the aggregate distribution and instead aggregate the stand-alone risk measures directly using a simple formula.

This approach was adopted by the NAIC for use in their RBC formula and was referred to as the *square root rule*[26]. Under the square root rule,

$$C = \sqrt{\sum C_i^2 + \sum_i \sum_{j \neq i} \rho_{ij} C_i C_j}$$

where C represents the risk measure for the aggregate risk distribution, C_i represents the risk measure for risk source i and ρ is the assumed correlation between the risk sources.

This approximation is exact when the risk measure is proportional to the standard deviation (as it is for the relative VaR risk measure) and all of the risk distributions are normal. This is because with normally distributed risks, the aggregate risk distribution's standard deviation is calculated using the same "square root rule". The proportionality constant can therefore be factored out and the resulting aggregate risk measure is simply the proportionality constant times the aggregate standard deviation.

In symbols, if α is the proportionality constant, then $C_i = \alpha\sigma_i$ for each risk type and for the aggregate risk. In this case, the square root rule can be shown to be exact:

$$\begin{aligned} C &= \sqrt{\sum C_i^2 + \sum_i \sum_{j \neq i} \rho_{ij} C_i C_j} \\ &= \sqrt{\sum (\alpha\sigma_i)^2 + \sum_i \sum_{j \neq i} \rho_{ij} (\alpha\sigma_i)(\alpha\sigma_j)} \\ &= \alpha\sqrt{\sum (\sigma_i)^2 + \sum_i \sum_{j \neq i} \rho_{ij} (\sigma_i)(\sigma_j)} \\ &= \alpha\sigma \end{aligned}$$

This method of aggregating risk measures has become widely used, despite the fact that it represents a crude approximation in more general cases when the distributions are not normal and the risk measures are not a constant multiple of the stand-alone standard deviations[27].

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[26] See Butsic, 1993.

[27] In his 1993 paper for the NAIC RBC Task Force, Butsic argued that when using the EPD Ratio as the standard to determine stand-alone and aggregate capital requirements, this square root rule leads to a conservative estimate of the required capital.

4. Aggregate Risk Capital and Allocation to Business Units

In this section, the calculation of a firm's aggregate risk capital and the allocation of this to various business units or risk sources will be demonstrated using a highly simplified numerical example. To ensure that the focus is properly placed on the methods and their implications, the numerical examples will intentionally ignore some risk types that would have to be reflected in a more realistic application.

4.1 Assumptions

A hypothetical insurance company, Sample Insurance Company (SIC), will be used for the numerical example based on the following key assumptions:

- Invested Assets – SIC is assumed to have $19.6 million in invested assets, equal to the undiscounted value of their existing loss reserves. New premiums collected, net of operating expenses, will be invested in identical assets. The rate of return (change in value) of the invested asset portfolio is assumed to be normally distributed and uncorrelated with all other risk categories. The expected return is 5% per annum with a standard deviation of 3.75% per annum.
- Loss Reserves – SIC is assumed to have $19.6 million in *undiscounted* loss reserves associated with the premiums written and earned in the past. For convenience, the reserve risk distribution parameters will be assumed to be identical to those used in the loss reserve risk example shown in Section 3. Recalling the details from that section, the reserve risk distribution has the following lognormal parameters: $\mu = 16.703$, $\sigma = 0.126$.
- Written Premium – Over the prospective year, SIC will be assumed to write a total of $6.4 million of new premium in each of two lines of business – Line A and Line B. The premium will be assumed to be written on the first day of the year and, after paying up-front expenses, invested entirely in marketable securities identical to those in which the existing assets are invested. The premium is assumed to be fully earned during the year and the loss payment patterns for both lines of business will be assumed to be the same.
    - Line A – This line is expected to have a *present value*[28] loss ratio of 91.6%, with discounting to the end of the first year, and lognormal distribution parameters as shown in the underwriting risk example in Section 3 ($\mu = -0.1099$, $\sigma = 0.2090$). These losses will be assumed to have a correlation coefficient of 0.50 with the reserve risk and a correlation coefficient of 0.25 with the underwriting results for Line B (described below).
    - Line B – This line is also expected to have a present value loss ratio of 91.6%, with discounting to the end of the first year, and a coefficient of variation that is 50% higher than the example shown in Section 3 ($\mu = -0.1359$, $\sigma = 0.3094$). The higher coefficient of variation is assumed so that differences in risk between the lines of business can be emphasized in the examples that follow. The losses for this line are assumed to have a correlation coefficient of 0.25 with the reserve risk and a correlation coefficient of 0.25 with the Line A underwriting risk.
- Expense Ratio – Expenses are assumed to equal 5.0% of the written and earned premium, and paid entirely at the beginning of the year.

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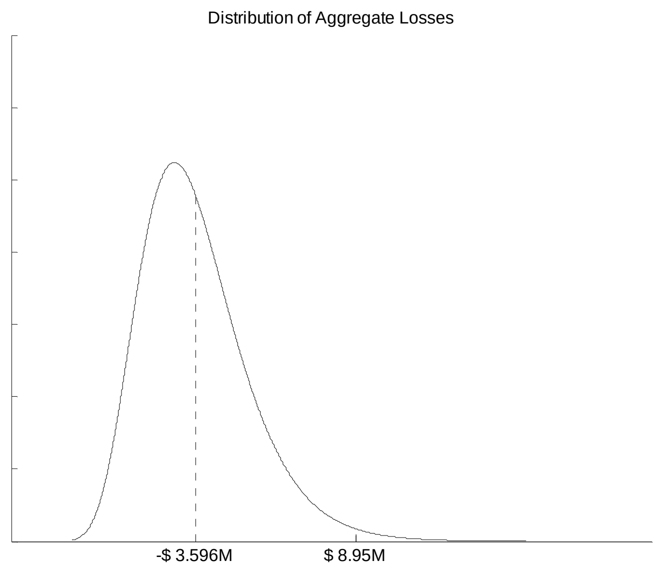
[28] The loss payments are discounted to the *end* of the first year, to mirror a simplifying assumption that all outstanding losses are paid, on a discounted basis, at the end of the year. This assumption is used to simplify the aggregation of the insurance risks with the market risk and to define "loss" consistently throughout the model.

- Key Simplifications – For convenience, the example ignores the following risk sources that would normally be included:
  - Credit Risk
  - Property-Catastrophe Risk
  - Operational and Strategic Risks

4.2 Aggregate Risk and Risk Capital

Given the assumptions described above, the aggregate risk distribution for SIC can be readily obtained by simulating from each of the stand-alone risk distributions and using a normal copula to account for the desired correlation/dependency among the risk types. For presentation purposes, a lognormal distribution was fit to the empirical aggregate distribution[29], using the method of moments. The resulting distribution is shown below:



For the purposes of this distribution, the amounts shown represent the potential "loss", again with profits depicted as negative amounts and losses as positive amounts. Because both assets and liabilities are included in this calculation, the aggregate loss includes the losses (or profits) in the investment portfolio and the insurance claim costs and expenses in excess of the premiums.

Recall that the company was assumed to have \$19.6 million of invested assets initially, collected \$12.8 million of premium for the two lines of business and paid 5% of that premium in expenses. These amounts are available to pay claims but are assumed to be contributed by the policyholders, to distinguish them from the *risk capital* that must be contributed by shareholders[30].

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[29] Because of the desire to assume a lognormal distribution, which cannot take on negative values, the distribution was shifted by the amount of the minimum loss (maximum profit) in the simulation to determine the parameters and then shifted back for the purposes of labeling the graph shown.

[30] In an actual application the company is likely to have committed capital in excess of the capital funded by policyholders, though for the purposes of the calculations that follow it is easiest to ignore the existence of this committed capital. The goal of these calculations is to assess the amount of risk capital needed, so including some of what will account for this risk capital only confuses matters. Whether the assets supporting the reserves are determined on a discounted or undiscounted basis will, of course, impact the dollar amount of risk capital determined and may need to be taken into account for different uses of the results.

From this aggregate risk distribution, the 99th percentile risk measure is $8.95 million. If this amount of risk capital were contributed to the firm the probability of having insufficient assets to pay all of the claims fully would be 1%[31].

Because of the use of the percentile risk measure and the connection this has to the amount of capital needed to reduce the risk of default to the stated probability, the $8.95 million figure can be interpreted as an amount of risk capital. It is then natural to allocate this risk capital to the various risks or business units that create the need for this risk capital. As stated earlier, it may also be appropriate to simply allocate the firm's actual capital, a point that will be discussed in Section 6.

4.3 Allocation of Risk Capital

In this section, several capital allocation approaches will be discussed and demonstrated using the SIC numerical example described above. In each case, the $8.95 million of risk capital will be allocated to some or all of the major risk sources. The methods discussed include the following:

- Proportional Allocation Based on a Risk Measure – This method simply calculates stand-alone risk measures for each risk source (market risk, reserve risk, Line A underwriting risk, Line B underwriting risk) and then allocates the total risk capital in proportion to the separate risk measures.
- Incremental Allocation – This method determines the impact that each risk source has on the aggregate risk measure and allocates the total risk capital in proportion to these incremental amounts.
- Marginal Allocation (Myers-Read Method) – This method determines the impact of a small change in the risk exposure for each risk source (e.g. amount of assets, amount of reserves, premium volume) and allocates the total risk capital in proportion to these marginal amounts. One particular method that will be demonstrated is the Myers-Read method.
- Co-Measures Approach (Kreps, Ruhm-Mango) – This method determines the contribution each risk source has to the aggregate risk measure. The method that was independently developed by Kreps and by Ruhm and Mango will be demonstrated.

Proportional Allocation Based on a Risk Measure

Using any selected risk measure, such as a percentile risk measure (VaR) or the CTE, each unit's proportional risk measure to the sum of all the risk measures is applied to the total capital. For example, if the stand-alone 99th percentile risk measure, which will be referred to here as the 99% VaR, is used for each risk source the following allocation percentages would be calculated:

Table 13: Capital Allocation – Proportional to 99% VaR

	99.00% VaR	% of Total	Allocated Capital
Market Risk	1,183,461	8%	742,665
Reserve Risk	4,440,453	31%	2,786,545
Line A UW Risk	3,243,793	23%	2,035,598
Line B UW Risk	5,394,016	38%	3,384,941
Total	14,261,723		8,949,750

Because the 99% VaR risk measure was used to determine the aggregate capital, it seems reasonable to use the same risk measure to perform the allocation. However, some practitioners choose to use a

[31] Note that this is not entirely accurate due to the fact that the risk in the marketable securities has only been measured over a 1 year horizon, whereas the claims will be paid over a longer horizon. The current simplified model assumes that the present value of the outstanding claims will all be paid at the end of the year, or equivalently that after the end of the year the invested assets will no longer generate risk. More importantly, the model assumes a run-off scenario after one year of premiums are written, which is rarely a realistic assumption. As a result, the term *default probability* should be very carefully interpreted.

different risk measure as the basis for allocating risk than is used to measure the aggregate risk capital.

For instance, if the 99.97% VaR is used to allocate risk capital but the same total amount of risk capital from the previous example is allocated, the following would be obtained:

Table 14: Capital Allocation – Proportional to 99.97% VaR

	99.97% VaR	% of Total	Allocated Capital
Market Risk	2,500,702	10%	851,813
Reserve Risk	8,035,878	31%	2,737,259
Line A UW Risk	5,666,239	22%	1,930,089
Line B UW Risk	10,071,313	38%	3,430,588
Total	26,274,131		8,949,750

Similarly, if the 99% CTE is used as the risk measure[32], the following would be obtained:

Table 15: Capital Allocation – Proportional to 99% CTE

	99.00% CTE	% of Total	Allocated Capital
Market Risk	1,593,170	9%	799,365
Reserve Risk	5,441,265	31%	2,730,126
Line A UW Risk	3,922,399	22%	1,968,043
Line B UW Risk	6,880,426	39%	3,452,217
Total	17,837,260		8,949,750

Notice that in all three cases here, the allocations are quite similar. In other applications, particularly those that include highly skewed risks such as property-catastrophe risk, this will not always be the case. In addition, as will be discussed in Section 6, there are many instances where it may be appropriate to use risk measures that are not "tail" based. In these instances, the differences that result could be more significant.

For example, the following two tables show the allocations that would result using the 80% VaR and the 80% CTE risk measures:

Table 16: Capital Allocation – Proportional to 80% VaR

	80.00% VaR	% of Total	Allocated Capital
Market Risk	-586,016	-35%	-3,132,546
Reserve Risk	335,121	20%	1,791,389
Line A UW Risk	756,744	45%	4,045,176
Line B UW Risk	1,168,409	70%	6,245,731
Total	1,674,258		8,949,750

Table 17: Capital Allocation – Proportional to 80% CTE

	80.00% CTE	% of Total	Allocated Capital
Market Risk	80,957	1%	117,902
Reserve Risk	1,809,817	29%	2,635,718
Line A UW Risk	1,622,380	26%	2,362,745
Line B UW Risk	2,632,196	43%	3,833,385
Total	6,145,350		8,949,750

[32] Given the simplicity of the model, the CTE could be calculated analytically. In this case, the CTE has been calculated using the simulated results instead. See Hardy for details of the analytical formulas.

Notice that in the 80% VaR allocation, the market risk allocation is negative. This reflects the fact that at the 80th percentile of the market risk distribution, the market returns are positive and *reduce* the aggregate capital needs. The impact of this is offset significantly in the 80% CTE allocation because the entire tail of the market risk distribution is taken into account and therefore the scenarios in which the market returns are negative impact the overall capital allocation to market risk.

Incremental Allocation

Under this approach, an aggregate risk measure is selected and calculated for the aggregate risk distribution. Then, the same risk measure is recalculated after removing one of the business units. The difference in the capital requirement with and without the selected business unit then represents the *incremental*[33] capital requirement for the business unit.

Using the incremental capital requirements for each business unit, the firm's capital can then be allocated to each unit in proportion to its respective incremental capital requirements. This is demonstrated in the following table.

Table 18: Capital Allocation – Incremental Based on 99% VaR

	Total 99.00% VaR	All-Other 99.00% VaR	Incremental 99.00% VaR	% of Total	Allocated Capital
Market Risk	8,949,750	8,661,043	288,707	3%	241,168
Reserve Risk	8,949,750	5,510,089	3,439,661	32%	2,873,285
Line A UW Risk	8,949,750	5,869,650	3,080,099	29%	2,572,929
Line B UW Risk	8,949,750	5,044,312	3,905,437	36%	3,262,367
Total			10,713,904		8,949,750

An important characteristic of this allocation method is that the incremental amounts do not add up to the total capital, even though the same risk measure was used. This is a characteristic that some practitioners find troublesome and there is disagreement over whether the "excess" amount should be allocated[34].

Marginal Allocation

The incremental allocation eliminates an entire business unit to determine its capital requirements. Instead, one could eliminate one dollar of revenue or one dollar of expected loss from each unit sequentially and use the change in the firm's total capital requirement as an estimate of the marginal capital requirement for the unit.

Applying this marginal requirement to the total revenue or total expected losses for the business unit provides an alternative measure of the capital needed for the unit. This can then be allocated in the same manner as described above for the incremental allocation method.

This approach typically results in a more appropriate result, however it may be impractical in certain circumstances where not all risk sources can be represented relative to revenue or expected loss or their marginal impacts easily determined.

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[33] Different authors have adopted different terminology for incremental and marginal methods. Throughout this document, the term *incremental method* is used to refer to calculations with and without entire business units or risk sources and the term *marginal method* is used to refer to calculations before and after a small change in the risk exposure.

[34] In an influential paper by Merton & Perold that used a different risk measure, they argued against allocation of the excess. In that paper the risk capital was defined as the cost of purchasing protection against default, which is similar to an EPD risk measure. Mango has extended the Merton & Perold approach to insurance applications and argues persuasively that capital allocation is only an intermediate step towards the real goal, which is to allocate the costs of risk capital and not the capital itself. As a result, allocation of all of the capital is not necessary.

Myers-Read Method[35]

This is a specific type of marginal allocation method, but its basis is somewhat different than those described above. Because an insurance company's total potential losses almost always exceed its assets, its owners have an option to default on the firm's obligations and *put* the claims (or some portion thereof) back to the policyholders. The value of this put option will decline as the amount of capital held increases for the same exposures. The Myers-Read method allocates capital so as to equalize the marginal impact that each business unit has on the value of this put option.

To apply this method, the value of the default option is calculated based on the firm's current capital and its current exposures. The exposure for a given business unit is then increased and the capital needed to maintain the same value of the firm's aggregate default option is determined. This capital then represents a marginal requirement per unit of expected loss for each unit that can be applied to the unit's expected losses.

The results of this method are demonstrated in the table below (see Appendix B for the technical details). For this example, the target EPD Ratio has been set arbitrarily to 0.186% so that the resulting aggregate risk capital is identical to the 99% VaR risk measure used in the other allocation method examples. In addition, the methodology takes into account the market risk in the invested assets, though it does not allocate capital to the market risk component. All capital is allocated to the lines of business for which there are liabilities, since it is only the need to pay liabilities that gives rise to the need to hold capital[36].

Table 19: Capital Allocation – Myers-Read Method (0.186% EPD Ratio)

	Capital to Loss Ratio	Expected Claims	Capital	% of Total	Allocated Capital
Reserve Risk	21.78%	18,091,233	3,939,466	44%	3,939,466
Line A UW Risk	33.92%	5,860,732	1,988,079	22%	1,988,079
Line B UW Risk	51.57%	5,860,732	3,022,205	34%	3,022,205
Total			8,949,750		8,949,750

This particular method has become popular because it produces additive capital requirements that sum to the total capital requirement for the firm when the same risk measure is used. Three points are worth noting with respect to this method:

- The method was not developed as a means for determining risk-adjusted capital requirements; it was developed as a means to allocate the frictional costs of capital to various businesses. While it may be used for the former purpose, it is not necessarily more appropriate for this purpose than the other methods discussed.
- Because this method requires the valuation of the default option, its application may require substantially more quantitative resources compared to other methods, except in certain limited circumstances.
- Significant mathematical challenges have been raised that suggest that the Myers-Read method is not appropriate for most insurance applications. The method assumes that risk exposure in a business unit can be increased or decreased without impacting the shape of the loss distribution, a property referred to as homogeneity[37]. Except when risk can be increased

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[35] The original Myers-Read paper presents the derivation of their approach. Butsic (1999) and Venter (2003) each present insightful discussions of this method and present simplified formulas that can be used to implement Myers-Read.

[36] This is a subtle point that is often overlooked because while it is true that a risk measure such as VaR can be used for an asset portfolio, in the absence of liabilities *risk capital* is not needed in the sense discussed here. There is clearly risk associated with marketable securities, though it would be odd to invest capital to protect against this risk. This is another example that highlights the need to understand the distinction between a risk measure and risk capital as it has been defined here as the amount needed to ensure that liabilities can be satisfied.

[37] See Mildenhall's discussion of the Myers-Read method for details.

or decreased through changes in *quota share* percentages, insurance loss distributions will not exhibit homogeneity when adding or removing policies from the firm's mix of business.

Co-Measures Approach[38]

This approach establishes the firmwide capital requirement using a particular conditional risk measure, such as VaR or CTE, and then calculates the Co-Measure for each business unit by calculating the comparable risk measure for the unit subject to the *condition* applied to the entire firm.

For example, consider the case where the risk measure selected is the CTE. The firmwide CTE is the average value of the losses *given that* the losses for the firm exceed the Value at Risk at a chosen percentile. To determine the Co-CTE for a given business unit, simply calculate the average losses *for each business* subject to the same firmwide condition that the total losses for the firm exceeds the chosen percentile.

This is very easy to implement in a simulation context. For example, the four key risk components for the SIC example were simulated using a normal copula method and the aggregate "loss" was determined for each of 50,000 simulation scenarios. The results were sorted in descending order based on the total loss and the worst 1% of the scenarios (the top 500 scenarios) were identified, as shown below:

Table 20: Co-CTE Calculations

Sorted Scenario	Market	Reserves	Line A	Line B	Total
1	779,323	12,180,298	3,188,429	4,994,583	21,142,632
2	494,425	8,169,822	3,734,913	8,695,665	21,094,825
3	-3,407,081	13,140,377	7,607,985	788,471	18,129,751
4	-779,922	2,587,705	5,675,660	10,386,216	17,869,658
5	-1,311,004	-1,203,142	3,238,333	16,924,158	17,648,345
6	-1,392,828	5,488,457	6,646,703	6,799,820	17,542,152
7	-255,475	4,812,487	4,018,249	7,904,885	16,480,145
8	-10,210	6,710,721	2,273,968	7,472,474	16,446,953
9	-1,896,169	4,433,724	1,652,542	12,169,231	16,359,328
10	758,494	3,132,459	2,330,630	10,003,805	16,225,388
11	-1,291,494	8,133,807	5,475,393	3,899,206	16,216,912
12	1,523,399	8,164,027	1,320,562	4,996,263	16,004,250
13	-1,507,026	8,701,922	4,941,913	3,358,494	15,495,303
14	-418,192	-390,473	1,172,596	15,112,222	15,476,153
15	348,569	4,904,846	4,173,982	6,001,026	15,428,423
:	:	:	:	:	:
490	-470,761	3,622,090	-148,615	4,519,262	7,521,976
491	-980,559	3,630,412	1,980,834	2,889,533	7,520,220
492	-2,921,510	2,906,628	-200,015	7,730,833	7,515,936
493	-1,179,044	3,552,559	2,343,631	2,794,807	7,511,953
494	-2,744,202	2,173,409	4,717,356	3,364,141	7,510,703
495	127,947	1,318,389	4,749,312	1,308,659	7,504,307
496	42,016	1,663,231	1,653,643	4,143,005	7,501,894
497	-1,062,298	2,170,695	6,366,285	27,183	7,501,865
498	-901,735	4,579,393	-124,816	3,947,145	7,499,986
499	-2,782,565	972,163	1,896,786	7,411,779	7,498,163
500	-2,959,845	6,146,281	863,894	3,441,193	7,491,523
Co-CTE	**-908,399**	**3,715,533**	**2,279,319**	**4,549,138**	**9,635,591**

The overall 99% CTE is simply the average total loss for the 500 worst scenarios, or $9.635 million. For each of these specific scenarios, the four main risk components make a different contribution to the total loss. For example, in Scenario 1, 58% of the total loss came from the reserve risk, 24% came

[38] See Kreps or Ruhm and Mango for a complete discussion of this approach.

from Line B's underwriting risk, 15% came from Line A's underwriting risk and 3% came from the market risk. Note though that, on average over these 500 scenarios, the market risk component actually *reduced* the total loss (due to profits in the investment portfolio rather than losses). Taking an average for each of these risk components, not across each of their own respective worst 1% of outcomes but rather across these specific 500 scenarios that represent the worst 1% of the total outcomes, the Co-CTE's are calculated as shown in the bottom row of the table. These reflect the average contribution each makes to the total losses.

Table 21: Capital Allocation – Proportional to 99% Co-CTE

	99.00% Co-CTE	% of Total	Allocated Capital
Market Risk	-908,399	-9%	-843,742
Reserve Risk	3,715,533	39%	3,451,069
Line A UW Risk	2,279,319	24%	2,117,082
Line B UW Risk	4,549,138	47%	4,225,340
Total	9,635,591		8,949,750

As shown in this table, on average the reserve risk contributes 39% of the total losses, Line A's underwriting risk contributes 24% of the total losses and Line B's underwriting risk contributes 47% of the total losses.

In addition, the Co-CTE's "add-up" to the total CTE as shown in the bottom row of the scenario summary. But to remain consistent with the other allocation examples and to highlight the ability to separate the allocation method from the amount allocated, the final allocation in the last column uses the Co-CTE allocation percentages applied to the 99th percentile risk measure (99% VaR) total risk capital figure used earlier.

5. Guiding Strategic Decisions

In this section, five specific applications of risk-adjusted performance metrics and the methods discussed in the previous sections are presented:

- Assessing Capital Adequacy
- Setting Risk Management Priorities
- Evaluating Alternative Risk Management Strategies
- Risk-Adjusted Performance Measurement
- Insurance Policy Pricing

5.1 Assessing Capital Adequacy

Insurers sell a promise to pay claims that, under certain conditions, could far exceed the premiums collected. As a result, in addition to carrying reserves for expected claims, they must also hold capital to provide their policyholders with reasonable assurances that their claims will be paid.

Regulators require certain minimum capital levels and various rating agencies have their own methods of assessing the adequacy of an insurer's capital base and assigning a financial strength or claims paying ability rating. Key questions that these rating agencies seek to have answered include:

- Is the firm sufficiently capitalized to meet current policyholder obligations?
- Does management understand the source of risk in the business?
- Does management actively measure and manage its exposure to risk?

The aggregate risk profile and the aggregate risk measures used to determine the firm's risk capital are useful in addressing these questions. They require the firm to develop risk models for each type of risk, select an aggregation method and choose an appropriate risk measure.

Firms capable of performing these calculations should be in a better position to demonstrate their claims paying ability and should have the tools they need to understand what drives the risk in their business.

5.2 Setting Risk Management Priorities

To assess firmwide capital adequacy, the capital allocation methods presented in Section 4 are not needed. By incorporating these allocation methods, firms can identify those business units or those activities that generate the greatest need for risk capital. Those business units may offer the greatest opportunity to reduce capital needs through effective risk management actions aimed at mitigating or transferring risk.

5.3 Evaluating Alternative Risk Management Strategies

Going further, measures of expected profitability can be incorporated and risk-adjusted return on capital (RAROC) measures can be calculated. This provides a means to test the impact of alternative strategies aimed at reducing risk, by comparing the costs and benefits of risk reduction. For example, a firm's overall RAROC or the RAROC for a particular business unit could be compared before and after a specific risk mitigation strategy to determine whether the transaction increases or decreases the return per unit of risk. Such an analysis is commonly performed to evaluate alternative reinsurance programs, for example.

5.4 Risk-Adjusted Performance Measurement

It is often desirable to evaluate actual, *ex post*, performance of different business units. Traditional measures of financial performance for insurers, such as historical loss ratios, can provide misleading indications of relative results for two business units with different levels of risk. For instance, if a business unit with a high degree of risk were to have a lower loss ratio than a business unit with a low amount of risk, the loss ratios alone may not properly identify which of the two business units performed "better". The use of a risk-adjusted performance metric such as RAROC may allow these business units to be more fairly compared. The explicit risk-adjustment may also be an improvement over judgmental premium to surplus ratios.

As an example of this process, consider the Sample Insurance Company presented in Section 4. Rather than rely upon the expected loss ratios, hypothetical values for the actual loss ratios realized over the year will be used. For this example, the actual loss ratios will be assumed to equal 92% for Line A and 86% for Line B.

Based solely on the loss ratios, it is natural to assume that Line B performed better. Calculation of an "economic profit" could also be used to show that Line B had a larger present value profit. For example, assuming that the actual market returns were 5%, then each line of business would have had economic profit at the end of the year equal to the following[39]:

Table 22: Calculation of Actual Economic Profit

	Line A	Line B	Calculations
(1) Premium	6,400,000	6,400,000	Actual
(2) Expense Ratio	5.00%	5.00%	Actual
(3) Expenses	320,000	320,000	(3) = (1) * (2)
(4) Investment Return	5.00%	5.00%	Actual
(5) Investment Income	304,000	304,000	(5) = (4) * [(1) - (3)]
(6) Discounted Loss Ratio	92.00%	86.00%	Actual
(7) Discounted Claim Costs	5,888,000	5,504,000	(7) = (6) * (1)
(8) **Economic Profit**	**496,000**	**880,000**	(8) = (1) - (3) + (5) - (7)

As shown in Section 4, Line B exposed the firm to substantially more risk than Line A and its profit per dollar of risk capital was actually lower. For instance, if the 99% Co-CTE allocation method were used, the following table shows the RAROC for these two business units:

Table 23: Comparison of RAROC – Using Co-CTE Allocation

	Economic Profit	Co-CTE (99%) Allocated Capital	RAROC
Line A	496,000	2,117,082	23.4%
Line B	880,000	4,225,340	20.8%

By rescaling the profit by the allocated capital for the underwriting risk, the risk-adjusted profitability measure shows that despite the lower loss ratio and higher economic profit, Line B required far more capital to support its operations and as a result did *not* outperform Line A.

This use of RAROC to better inform the assessment of performance shows that it is possible to take risk into consideration in a relatively simple manner. However, Section 4 showed that there were a variety of allocation methods that could be used. For instance, if the proportional allocation based on the 99th percentile (99% VaR) risk measure were used, the following alternative results would be obtained:

[39] Recall that the present value, or discounted, loss ratio reflects the value of the losses at the *end* of the year.

Table 24: Comparison of RAROC – Using Proportional 99% VaR Allocation

	Economic Profit	99% VaR Allocated Capital	RAROC
Line A	496,000	2,035,598	24.4%
Line B	880,000	3,384,941	26.0%

This comparison shows that RAROC, despite its appeal as a means to risk-adjust performance metrics, does not necessarily produce unambiguously superior performance measures. Depending upon the method used for the allocation, the RAROC for Line B could be either lower than or higher than the RAROC for Line A. These results are highly sensitive to a variety of implicit and explicit assumptions that can materially impact the allocation of capital to specific business units.

5.5 Insurance Policy Pricing

A natural extension of the RAROC analysis just demonstrated, which focused on a relative comparison of two business units, is to use RAROC directly in insurance policy pricing. The rationale would be to set the price such that the expected RAROC is above a specified target rate.

Suppose, for instance, that an acceptable RAROC target of 15% is assumed. The premium that should be charged such that Line B's expected RAROC was equal to at least 15% would then be easy to determine. One approach, albeit overly simplified and somewhat naïve, is to simply choose one of the many capital allocation methods and then solve for the additional risk margin, which will be denoted by π here, such that the RAROC equals the target rate of 15%.

For the sake of a numerical example, consider the allocation of risk capital to Line B using the Co-CTE allocation method. Based on the existing assumptions regarding Line B's *expected* loss ratio rather than the actual loss ratio used in the previous example, this produces the following expected economic profit and expected RAROC for Line B:

Table 25: Expected Economic Profit – Line B

	Line B	Calculations
(1) Premium	6,400,000	Expected
(2) Expense Ratio	5.00%	Expected
(3) Expenses	320,000	(3) = (1) * (2)
(4) Investment Return	5.00%	Expected
(5) Investment Income	304,000	(5) = (4) * [(1) - (3)]
(6) Discounted Loss Ratio	91.60%	Expected
(7) Discounted Claim Costs	5,862,400	(7) = (6) * (1)
(8) **Expected Economic Profit**	**521,600**	(8) = (1) - (3) + (5) - (7)

Table 26: Expected RAROC – Using Co-CTE Allocation

	Expected Economic Profit	Co-CTE (99%) Allocated Capital	RAROC
Line B	521,600	4,225,340	12.3%

With no additional risk margin, the RAROC is below the target rate. The following equation can be used to solve for the additional risk margin, π, that produces the target rate of 15%[40]:

$$RAROC = \frac{[\text{Original Premium} + \text{Additional Risk Margin - Expenses}] * (1 + \text{Expected Investment Income}) - \text{PV of Expected Claims}}{\text{Allocated Risk Capital}}$$

$$= \frac{[6{,}400{,}000 + \pi - 320{,}000] * (1 + 5\%) - 5{,}862{,}400}{4{,}225{,}340} = 15\%$$

$$\pi = 106{,}858$$

This solution can also be derived using what is often referred to as an Economic Value Added or EVA™ approach[41]. If the $4,255,340 is treated as the "required capital" to write Line A and the 15% RAROC target is the "per unit cost of capital", then the total dollar cost of the capital is 15% * $4,255,340 = $633,801. This is the amount of expected economic profit that would have to be incorporated into the premium. Since the original premium already accounted for $521,600 of this expected profit, only $106,858 of additional risk margin would have to be incorporated to meet the RAROC target rate.

Table 27: Calculation of Additional Risk Margin Required

	Amount	Calculations
(1) Allocated Risk Capital	4,225,340	Based on Co-CTE Allocation
(2) Target RAROC	15.0%	Assumed
(3) Required Economic Profit	633,801	(3) = (1) * (2)
(4) Current Economic Profit	521,600	Based on Assumptions
(5) Shortfall	112,201	(5) = (3) - (4)
(6) Expected Investment Income	5.00%	Based on Assumptions
(7) Additional Risk Margin Required	106,858	(7) = (5)/[1 + (6)]

Notice that in this calculation the additional risk margin is assumed to earn the same expected rate of investment income as the net premiums. An argument could be made that the additional risk margin should be assumed to be invested in risk-free assets only, to avoid the need to calculate the additional risk capital that investing these funds in risky assets might produce. But the impact of this is likely to be small and can usually be ignored.

Additional Considerations

Using RAROC for pricing, as in this example, is appealing because the steps are logical and easy to explain. However, some subtle complications can arise in practice that are not as obvious in this example due to some of the simplifications made. In this section, the consequences of three specific simplifications of importance to pricing applications will be discussed (additional complications relevant to all applications will be discussed in Section 6):

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[40] For simplicity, the additional risk margin in this section will be assumed to not affect expenses such as commissions or premium taxes that are commonly proportional to total premium. In practice, the formulas shown here would have to be adjusted for such expenses.

[41] EVA™ is the trademarked terminology used by Stern Stewart & Co. This approach is compatible with RAROC though it is expressed in dollars instead of as a ratio. See Brealey & Myers for a discussion of the advantages of using profitability measures denominated in dollars rather than profitability ratios.

- Investment Income on Allocated Capital
- Multi-Period Capital Commitment
- Cost of Capital (Target RAROC Rate)

Investment Income on Allocated Capital

In the simplified example shown above, it was assumed that the target return on the allocated risk capital was 15%. How this target return is calculated depends on how the economic profit is defined.

The definition of economic profit used in the example above did not include the investment income that can be expected to be earned on the allocated risk capital itself. As a result, the 15% target return also excludes the investment rate assumed to be earned on the allocated risk capital. The target return is technically an *excess return* over the investment rate in this case.

Alternatively, the investment income expected to be earned on the allocated risk capital could be included in the calculation of economic profit. In this case, the target return should be inclusive of the investment rate assumed to be earned on the allocated risk capital. In a single period context, the two approaches lead to the same risk margin. However, when risk capital is required over multiple periods, the approach used in the examples above is easier to apply.

Multi-Period Capital Commitment

Up until now, the allocated risk capital was assumed to be exposed to risk for only a single period. This allowed the discussion of RAROC to be somewhat simplified and did not impact any of the conclusions drawn from the previous examples, in part because both Line A and Line B had the same claim payment patterns and the comparisons were made *relative* to each other rather than on an absolute basis.

But in the context of policy pricing, it is important to recognize that the *initial* capital required to write the policy does not fully reflect the total capital costs. The risk will not be fully resolved in a single period and so some risk capital will be needed in subsequent periods as well, perhaps until the final claims are paid. It is easy to see how one might account for this in practice. One common approach is to assume an *average* pattern for the release of the risk capital and then use that pattern either to adjust the RAROC ratio or to modify the target rate.

To see how these adjustments could be made, consider an assumed claim payment pattern for Line A as follows (chosen arbitrarily for simplicity):

Table 28: Claim Payment Pattern - Line A

Year	% Paid
1	50%
2	30%
3	15%
4	5%

Further, assume that the risk capital will be released, on average, at the same rate as the claims are paid. In reality, under some scenarios more capital will be released, perhaps faster or slower than this pattern, and under some scenarios more capital may even be committed to support this line of business. But given the assumed release pattern for the allocated risk capital, the cost per unit of risk capital (15% for the sake of this example) can be applied to the outstanding risk capital each period and the aggregate cost of risk capital over the life of the exposures determined as shown below:

Table 29: Aggregate Cost of Risk Capital – Multi-Period Release of Risk Capital

Year	% Paid	Beginning Risk Capital	Cost of Risk Capital	PV Cost of Risk Capital	Risk Capital Released	Ending Risk Capital
1	50%	4,225,340	633,801	603,620	2,112,670	2,112,670
2	30%	2,112,670	316,901	287,438	1,267,602	845,068
3	15%	845,068	126,760	109,500	633,801	211,267
4	5%	211,267	31,690	26,071	211,267	0
			1,109,152	1,026,630		

Using the EVA™ approach demonstrated above for the single period case, where the premium is adjusted to ensure that the total dollar cost of risk capital is recovered through the premium charges, it is easy to see how this total cost of risk capital might be reflected in the policy pricing.

To stay within the RAROC framework, it is sometimes helpful to convert this solution that takes into account the multi-period nature of the risk capital commitment into either an adjusted RAROC metric or an adjusted target rate.

To see the adjusted target rate first, note that the EVA™ approach makes it clear that if C_i reflects the beginning risk capital each period and R is the constant cost of risk capital each period, and r is the investment income rate expected to be earned on the risk margin[42] (assumed to be 5% in the previous examples), then the policy pricing should reflect the following expected economic profit:

$$\text{Economic Profit} = \sum C_i R(1+r)^{-i}$$

From this, both sides can be divided by the initial risk capital, C_1, to determine the adjustment factor (shown in brackets below) to use to modify the target rate, R:

$$\frac{\text{Economic Profit}}{\text{Initial Risk Capital}} = \frac{\sum C_i R(1+r)^{-i}}{C_1}$$

$$\text{RAROC} = R\left[\frac{\sum C_i (1+r)^{-i}}{C_1}\right]$$

Using the example shown above, the RAROC target rate would be the original rate R = 15% adjusted by the factor given in the brackets, or 1.62. Given this average pattern for the release of capital, the RAROC target rate would have to be adjusted to 15%*1.62 = 24.3%. Then, the target economic profit needed to achieve this target RAROC would simply be $4,225,340 * 24.3% = $1,026,630.

Alternatively, some practitioners[43] suggest altering the calculation of RAROC by including the present value of the risk capital commitments in the denominator instead of simply using the *initial* capital (denoted C_1 to reflect the capital at the beginning of the first period). This is algebraically the same as the formula above:

$$\frac{\text{Economic Profit}}{C_1} = R\left[\frac{\sum C_i(1+r)^{-i}}{C_1}\right]$$

$$\frac{\text{Economic Profit}}{C_1} * \left[\frac{C_1}{\sum C_i(1+r)^{-i}}\right] = R$$

$$\frac{\text{Economic Profit}}{\sum C_i(1+r)^{-i}} = R$$

Steady State Assumption

A common simplification assumes a "steady state" and incorporates the reserve risk capital into the calculations of the initial required capital for each line of business. Then, instead of reflecting the present value of all future capital commitments from the underwriting risk alone (the $\sum C_i(1+r)^{-i}$ term in the formulas above), the initial capital requirements for *both* the underwriting and reserve risk are used.

[42] Earlier it was noted that the examples here use the same investment income assumptions used to derive the risk capital. The question of whether this rate should really just be the risk-free rate will not be addressed here. In addition, corporate income taxes are ignored. With corporate income taxes, and assuming that the risk margin is treated as taxable income, these numbers may need to be grossed up to reflect the after-tax funds contributed by policyholders.

[43] See Nakada, et al

In the example here, the Sample Insurance Company was assumed to have existing loss reserves, but the line of business was not specified. While the existing reserves also required risk capital, it was ignored in all of the numerical calculations that were aimed at assessing the pricing for the *new* business only. But under certain limited circumstances, it may be possible to include the risk capital associated with the reserves along with the underwriting risk capital as an approximation for the denominator shown above. For example, if the reserves were all for Line A and the riskiness of this line of business has not changed, then combining the total reserve and underwriting risk capital may serve as an approximation for the denominator in the previous equation. Because of differences in how the diversification impacts the different formulas, this approximation may be relatively poor in some cases.

Of course, in other cases, such an approximation will clearly not be appropriate. For instance, if Line B were an entirely new line of business, there would be no way to approximate the denominator by including any portion of the reserve risk capital into the calculations. The inability to use this simplification across all lines of business would further complicate comparisons across different lines of business.

<u>Cost of Risk Capital</u>

In the above discussion, a constant 15% cost of risk capital was assumed, without explanation or justification. It is worth exploring, particularly in the context of insurance policy pricing, how this cost of risk capital should be determined in practice.

Although the RAROC measure is intuitively appealing, it is more *ad hoc* than many practitioners often recognize. Because it is referred to as a return on capital, it is quite common for practitioners to assume that standard "rate of return" benchmarks, such as those derived from models such as Capital Asset Pricing Model (CAPM) or the Fama-French 3-Factor Model, are applicable. In reality, the rate used for the cost of risk capital must take into account the specific way in which the RAROC metric is defined.

The most significant issues include the following:

- Numerous textbook discussions of RAROC suggest using risk-adjusted return models such as CAPM to establish the cost of risk capital and to assess whether or not the RAROC exceeds this value. Despite the fact that both RAROC and CAPM produce "risk-adjusted returns", the risk adjustment in RAROC reflects a different definition of "risk" than is used in these theoretical models.

  Models such as CAPM measure the systematic risk associated with an investment, which accounts for the *marginal* contribution the investment adds to an existing portfolio of diversified investments. RAROC, even for the total firm, incorporates an entirely different measure of risk based on the relationship between a cash flow's expected value and certain values in the tail of its probability distribution[44].

- RAROC is artificially "leveraged". The denominator reflects neither the total market value of the "invested" capital (as is assumed in the theoretical return models) nor the firm's actual capital that could be exposed to loss (the committed capital).

  If the firm's shareholders desire a given target rate of return on their investment, the dollar value of their target "income" will depend on the total market value of the firm's equity. This will almost always exceed the value of the firm's book equity (the difference being attributed to their franchise value), though under certain assumptions regarding the stability of the

[44] It should be noted that the distinction being made here between systematic risk in CAPM, on the one hand, and tail measures of risk in RAROC, on the other hand, may not necessarily be as stark as implied here. Many academic researchers have begun to question the focus on systematic risk in models of return expectations and have suggested a variety of methods to also incorporate measures of non-systematic risk (see Froot & Stein and Shimko). Others have instead suggested that RAROC itself could be adapted to also incorporate measures of systematic risk (see Crouhy, Turnbull & Wakeman or Wilson).

firm's market to book value multiple, the rate of return on market value and the rate of return on book value may be equal.

Nevertheless, earning this rate of return solely on the firm's *risk capital* will not necessarily be sufficient to satisfy the income expectations of the shareholders. Using this lower base in the denominator of RAROC artificially inflates the rate of return on "capital", with only a modest offset due to the fact that the numerator also ignores a component of "income" based on changes in the franchise value of the firm.

When RAROC is measured for distinct business units within the firm, the capital allocated to those business units will depend upon the degree to which diversification effects are reflected in the amount allocated, the risk measure used and other somewhat arbitrary decisions. The business unit losses are not literally limited to the amount of risk capital allocated to it, so this leverage effect on the RAROC is even more artificial.

Taking these considerations into account is a bigger challenge than is often recognized and entirely satisfactory methods for calibrating the cost of risk capital do not exist.

One acceptable compromise is to recognize that models such as CAPM or the Fama-French 3-Factor Model are reasonable means to quantify shareholders' target return on the firm's total capital (e.g. GAAP book value). Under a conservative assumption that only the total risk capital is "at risk", the CAPM return can be adjusted upwards by the ratio of the firm's total capital to the firm's risk capital. Alternatively, rather than using the (arbitrary) risk capital in the RAROC calculation, the firm's total capital could be used along with the allocation methods discussed here. In either case, this allows the pricing model to reflect the aggregate compensation required by the shareholders for assuming systematic risk and then allocates this total amount to different business units (or policies) in a manner that reflects the relative "risk" of each.

This approach does not account for the differential degrees of leverage in each business unit. This is because after taking into account diversification benefits, it is quite difficult to quantify how much additional leverage has been introduced into the calculation.

This approach also does not address the potential for different business units to have different degrees of systematic risk. Theoretically this should be easy to deal with, though in practice adjustments to reflect differing degrees of systematic risk across segments of the total firm are quite difficult to make because of the limited ability to reliably quantify these differences[45].

Many alternative methods for quantifying the cost of risk capital have been proposed. For example, Feldblum suggests incorporating the frictional costs of holding capital, such as those that result from the double taxation of investment income. Venter points out though that this is incomplete because it doesn't address the compensation required for assuming the *risk* that would reasonably need to be included even in the absence of corporate taxes (e.g. for Bermuda-based reinsurance firms that are not subject to corporate income taxes).

Yet another approach has been suggested by Mango. Mango notes that while it is common to refer to the allocation of "capital", it is really just an allocation of underwriting *capacity* and therefore the policy or business unit must earn adequate profits to pay for this capacity. In addition, each policy or business unit also is given the ability to call upon the capital not explicitly allocated to it, if needed to pay claims, and therefore must also earn adequate profits to compensate the firm for the value of this capital call. These costs could be combined and reflected as a cost per unit of allocated risk capital.

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[45] See Cummins and Phillips for an example of how one might be able to use the *full information beta* approach to distinguish between measures of systematic risk for different lines of business.

6. Practical Considerations

In an effort to streamline the discussion and introduce the RAROC concept as fully as possible, many real-world considerations that can complicate this type of analysis have been intentionally ignored. In this section, some of these issues are highlighted and briefly described.

6.1 Time Horizons

In Section 3, it was briefly noted that inconsistent time horizons are often used to measure a firm's aggregate risk profile. For instance, market risk is typically measured based on the potential change in the value of the assets over a one-year period, while the insurance risks are measured based on the potential ultimate liability. In many instances, such as property-catastrophe or other very short-tailed insurance risks, this distinction is trivial. But in some instances, particularly those for which the ultimate liability is highly dependent upon the realization of unknown claim severity trends, this distinction could be material.

Some practitioners argue that to resolve this issue, the market and credit risks could be measured over the entire lifetime of the insurance liabilities. This significantly complicates the mechanics of the models (requiring complex DFA models) and introduces new challenges to estimate the parameters for the models. It is far more difficult to quantify equity market, interest rate and foreign exchange rate parameters over long horizons due, for instance, to limited availability of long-horizon historical data and more significant serial correlations. More importantly, over longer horizons it is far less reasonable to assume a fixed portfolio and to ignore important strategic decisions that may be made in response to market movements.

An alternative approach being explored by European insurance regulators in conjunction with Solvency II is to focus on the change in *value* of the insurance liabilities over a one-year period. Although conceptually more consistent with the methods used to measure market risks, there are serious questions that have been raised about this approach for certain classes of insurance. In many cases, information relevant to the revaluation of the liabilities is not available over a short horizon and so this approach will result in limited potential change in the value, even in cases where there is substantial risk over a longer horizon.

An example of this can be found in high layer excess general liability insurance policies. Over a short horizon, such as one year, the premium required to transfer the risk to a third party (a standard measure of the *value* of an insurance risk) is unlikely to differ materially from the premium initially charged to write the policy. But as the underlying claims are reported and settled and claim severity trends accumulate over a long horizon, there very well could be material risk associated with these policies.

There does not appear to be consensus on how to express the time horizons over which the risk is measured when market risk, credit risk and long-term insurance risks are combined. Regulatory discussions in Europe suggest a trend towards measuring insurance risks over shorter horizons[46] rather than over the lifetime of the liabilities. But practices vary and many implementation details remain unresolved.

6.2 Alternative Return Measures

The discussion to this point has assumed that the return measure reflects economic profit, though this is just one of many variations of the RAROC approach that can be used.

<u>Benefits of Accounting Measures of Income</u>

Some firms have found it challenging to introduce RAROC considerations into their organizations using a return measure that is substantially different from the GAAP or Statutory calendar year income measures with which senior management is familiar. For this reason, they prefer to use these accounting metrics in the RAROC calculation. When this is done, the result becomes something akin

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[46] See Conway and McCluskey, 2006.

to a *calendar year* RAROC calculation, though inconsistencies with the denominator of the RAROC calculation become inevitable.

Taxes

One clearly important variation of the calculations shown in this paper is to include the effect of corporate income taxes on any measures of return. Depending upon the tax jurisdiction and the specific tax position of the firm, this could prove to be a fairly complex issue. Nonetheless, corporate income taxes are a real expense that should be reflected, if applicable.

Stranded Capital

Some practitioners, in an attempt to account for the leverage effect noted in Section 5.5, reduce the return measures used in RAROC for what they describe the cost of *stranded capital*. The stranded capital is defined as the amount of capital held in excess of the (allocated) risk capital. In some cases, this amount is limited to the amount of regulatory or rating agency measures of "required" capital and in other cases it reflects the difference between all of the firm's capital and the (allocated) risk capital. Reducing the rate of return by this cost is conceptually identical to the adjustment noted in Section 5.5, where all of the firm's capital is allocated rather than just the firm's risk capital. However, depending on the rate of return used, some small differences between the two approaches may result.

Investment Income

In the numerical examples that dealt primarily with single-period capital commitments, investment income was included in the definition of economic profit because all of the calculations were assumed to occur at the end of the period. When multiple periods are reflected, it is easier to work with *present value* amounts. When this is done, including investment income becomes more complicated and the RAROC ratio is not a true "rate of return".

6.3 Risk-Based Allocation

In Section 4, all of the methods discussed to allocate risk capital relied upon "tail" measures of risk using either a percentile (VaR), CTE, EPD Ratio or Co-Measure methodology. While this produces a "risk-based" allocation, it does so using a rather limited view of what drives the risk to the firm and tends to allocate capital primarily to those businesses with the most extreme levels of skewness, such as businesses exposed to property-catastrophe risks. This may make sense in the regulatory or rating agency applications where many of these risk capital models were first developed, but this is less appropriate when these models are used to manage the interests of the firm's shareholders.

An alternative approach alluded to in Section 3 starts with the observation that the firm's shareholders could be severely impacted by less extreme events that, while not in the "tail", would materially affect the firm's credit rating, financial strength or ability to operate as a going concern. Then, the business units that most impact these measures of "risk" would be allocated more of the firm's capital and its associated cost.

Similar risk measures may still be appropriate, though the percentiles at which they are measured are less likely to be values such as 99% or 99.97%. Instead, thresholds based on a target percentage loss of surplus are more likely to be used.

6.4 Diversification Adjustments

In Section 3, numerous challenges associated with estimating correlation or dependency parameters across business units and risk sources were mentioned. What wasn't emphasized is that these difficult assumptions often drive the determination of the firm's aggregate risk profile, the allocation of risk capital to business units and the resulting RAROC calculations. In light of this, it is worth questioning the role that RAROC measures should play in important managerial decisions. They can be informative and insightful, but they should not serve as the sole metric that drives such decisions.

7. Conclusion

This paper has presented an introductory overview of risk-adjusted performance measurement. Using a simplified version of a commonly used performance metric, Risk-Adjusted Return on Capital (RAROC), new insights into common managerial decisions may be possible.

The method shown here began with the development of an aggregate, firmwide risk profile and then used various risk measures to quantify how much of the firm's capital was "at risk". Aside from highlighting the level and sources of risk in the firm, this measure of risk capital was allocated to various business units or activities and then used to compare relative performance or to guide pricing decisions.

Through the examples shown, a variety of critical challenges likely to be encountered when using this framework were presented to highlight the strengths as well as the limitations of the methodology.

Appendix A: Risk Exposure Horizon

The distinction between the lifetime of liability uncertainty and the one-year horizon uncertainty relates to the *risk exposure horizon*, or the time period over which the risk can materialize.

As alluded to in the discussion of market risk, consistency of the risk exposure horizon is particularly challenging in risk capital models. There are several schools of thought regarding how to handle this issue among practitioners:

- Use One-Year Risk Exposure Horizon for All Risk Types
- Use Multi-Period "DFA" Models
- Ignore the Inconsistencies

Each of these approaches will be discussed briefly below.

One-Year Risk Exposure Horizon for All Risk Types

Some practitioners want to ensure that the risk measures for market, credit and insurance risks can be aggregated easily and therefore they measure risk from *all* sources by estimating their potential change in value over a common time horizon, typically one year to coincide with typical accounting-based profitability measures.

While this approach appears more mathematically pure, in practice this is very difficult to achieve, for several reasons.

First, there are no reliable methods that can be used to estimate the timing of the recognition of adverse loss development for loss reserves. Limited historical data does exist, but because practices among firms vary considerably with regard to how they report their losses by line and whether they test reserve adequacy at any particular point in time on a by-line basis or in the aggregate, this data is unlikely to prove reliable.

Second, this "change in value" perspective is not entirely consistent with a market VaR measure if it only reflects the change in the *best estimate* of the reserve over the time horizon. Market VaR calculations reflect the potential change in the *market value* and therefore for the reserve risk to truly reflect the change in *value*, it would also have to reflect a risk margin. Only when a risk margin is included will the amount reflect the price at which the risk could realistically be transferred to a third party. In the absence of such adjustments, the figures cannot fairly be represented as a change in "value" in the same sense that the market VaR reflects the change in value for the invested assets.

And third, the vast majority of the "risk" inherent in loss reserves will <u>not</u> be resolved over the course of a single one-year period. As a result, methods that focus on only a single period will necessarily ignore a significant amount of the total risk for an insurer.

Multi-Period "DFA" Models

Dynamic Financial Analysis (DFA) models make it theoretically possible to account for market and credit risks throughout the lifetime of the liabilities, thereby ensuring a consistent risk exposure horizon across all risk categories. However, this advantage does not come without some associated complexity. In particular, modeling long term market and credit risk exposure is far more complex than simply extending short-term market and credit risk measurement metrics, since over long horizons the issues associated with model parameters, serial correlation and management's strategy become significant drivers of the results.

Ignore the Inconsistencies

A common approach inherent in virtually all regulatory and rating agency capital models is to measure some risks over a one-year horizon and other risks, notably the insurance risks, over a lifetime of liability horizon. This renders the aggregate risk models difficult to interpret, but it does greatly simplify the modeling effort.

Interestingly, this approach may not actually be problematic for some applications. Many applications assume the existence of an *aggregate* risk distribution and in those instances, mixing risk exposure horizons clearly leads to resulting measures that are difficult to interpret. However, one could take the view that only the insurance risks need to be aggregated and used to determine the "economic capital" needed today to satisfy current obligations (see Section 2.2 for a discussion of the distinction between economic capital and risk capital, as used here).

The market and credit risk measures do not have to be explicitly aggregated with this longer-term risk measure. Instead, they can be used to reflect a "haircut" to the current asset balances to account for the potential loss in value of the assets that could occur over the course of one year. This recognizes that the firm always has some flexibility to alter its allocation from risky assets to lower risk, or even risk-free, assets at the end of the year (or some other chosen horizon).

In many respects, this is the approach reflected in the current S&P Capital Adequacy Ratio (CAR), which does not attempt to aggregate market, credit and insurance risk. Instead, the S&P CAR merely attempts to compare the economic capital required for the insurance risks to the "adjusted" assets actually held (net of the haircut based on a one-year market VaR and a 10-year measure of expected credit losses).

Appendix B: Myers-Read Capital Allocation

Introduction

In this Appendix, the calculations used to produce the Myers-Read allocation will be shown in detail. The reader is encouraged to review the Myers-Read paper for the theoretical basis for these calculations and to the papers by Butsic and Venter for insightful discussions of the method as well as simplified calculations.

The Myers-Read methodology is also described in some detail by Cummins in a paper that appears on the CAS examination syllabus. For convenience, the calculations that follow are based on the methodology presented in the Cummins paper.

Default Option

A critical element of the Myers-Read method is the evaluation of the firm's so-called default option, which reflects the value of their right to default (in whole or in part) on their obligations to policyholders.

With fixed liabilities and risky assets, this can be evaluated as simply a put option on the assets of the firm with a strike price equal to the (fixed) value of the liabilities. When the assets are fixed and the liabilities are risky, the default option is more accurately described as a call option on the liabilities with a strike price equal to the assets. And in the most general case, when both assets and liabilities are risky, the default option is technically an option to exchange the assets for the liabilities. This option to exchange the assets for the liabilities is more complicated than a standard put or call option and cannot be easily quantified using the Black-Scholes option pricing formula.

Cummins simplifies this otherwise complex option by characterizing it as a put option on the *asset to liability ratio* rather than an option on either the assets or the liabilities. Specifically, the default option is a standard put option on the ratio of the assets to the liabilities with a strike price equal to 1.0. If the ratio of assets to liabilities is less than 1.0, then the firm is insolvent and the deficit (as a percent of the liabilities) is the amount by which the ratio is below 1.0.

The volatility parameter used in the Black-Scholes put formula therefore represents the volatility of the asset-to-liability ratio. It reflects not just the separate asset and liability standard deviations but also their correlations. In the simplifying case where the assets and liabilities are independent, the volatility of the asset to liability ratio, $\sigma_{V/L}$, is related to the asset volatility, σ_V, and the liability volatility, σ_L, by the following equation[47]:

$$\sigma_{V/L}^2 = \sigma_V^2 + \sigma_L^2$$

Myers-Read Allocation

The Myers-Read method estimates the marginal capital for a particular line of business by determining the effect on the default option of a small increase in the size of the line (based on expected loss amount). They begin with the formula for the default option as a function of the expected loss amounts for each line and calculate the partial derivatives with respect to each line. The capital needed for each line is then determined such that each line has the same marginal impact on the firm's overall default option value (as a percentage of the expected losses).

This method is designed to allocate a set amount of capital and so it does not necessarily specify what that total capital amount should be. In the Cummins example, he assumes a 5% EPD Ratio target for the firm overall, which is equivalent to assuming that the default option should be worth 5% of expected aggregate losses.

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[47] This relationship follows from Ito's Lemma when both the assets and liabilities are assumed to follow geometric Brownian motion.

The resulting formula for the capital to liability ratio, s_i, for each line is given as:

$$s_i = s - \left(\frac{\partial p}{\partial s}\right)^{-1}\left(\frac{\partial p}{\partial \sigma}\right)\left[(\sigma_{iL} - \sigma_L^2) - (\sigma_{iV} - \sigma_{LV})\right]\frac{1}{\sigma}$$

In this formula, the σ terms with the *V* subscripts reflect the covariance of the line *i* losses with the assets and the covariance of the total losses with the assets, respectively. Since the assets are assumed to be independent of the liabilities, both of those terms are zero in the case discussed in this paper. In addition, the σ parameter with no subscripts reflects the overall volatility of the assets to liability ratio and uses the formula shown above. Finally, the term σ_{iL} reflects the covariance of line *i* with the total losses for all lines. Using the expression for the total variance, the formula for this covariance term is follows:

$$\sigma_L^2 = \Sigma\, w_i^2\, \sigma_i^2 + \Sigma\Sigma_{i\gamma j}\, w_i w_j \sigma_{ij}$$

$$= \Sigma\Sigma w_i w_j \sigma_i \sigma_j \rho_{ij}$$

$$= \Sigma w_i[\Sigma w_j \sigma_i \sigma_j \rho_{ij}]$$

$$= \Sigma w_i[\text{Covariance of Line } i \text{ with Total Losses}]$$

For the two partial derivative terms, these can be derived simply by writing the formula for the default option value as a put option on the asset to liability ratio A/L = (1+*s*) with a strike price of 1.0. The notation is simplified by assuming $r = 0$ and $T = 1$.

$$p = N(-d_2) - (1+s)N(-d_1)$$

where $d_1 = \dfrac{\ln(1+s)}{\sigma} + \sigma/2$ and $d_2 = d_1 - \sigma$.

From this it is relatively easy to calculate the two partial derivatives needed for the formula for the surplus to liability ratios. The first is found by taking the derivative of *p* with respect to *s* but paying attention to the fact that there is an *s* term in the d_1 and d_2 terms that makes the derivative a bit more complicated than it first appears. When this is done, the option delta is $\partial p/\partial s = -N(-d_1)$.

Similarly, the option vega is $\partial p/\partial \sigma = N'(-d_2)$.

Surplus to Liability Ratios - Numerical Example

In the paper, the example involved three lines of business (reserve risk, Line A underwriting risk and Line B underwriting risk). The following are the calculations for each of the components of the surplus to liability ratios for these three lines of business.

Summary of Key Liability Assumptions

Expected Loss and Volatility Assumptions

	Expected Liability	CV	Volatility
Reserve	18,091,233	12.7%	12.6%
Line A UW	5,860,732	21.1%	20.9%
Line B UW	5,860,732	31.7%	30.9%
Total	29,812,697	13.6%	13.5%

Liability Correlation Assumptions

	Reserve	Line A UW	Line B UW
Reserve	1.00	0.50	0.25
Line A UW	0.50	1.00	0.25
Line B UW	0.25	0.25	1.00

Covariance of Each Liability Line with the Total Liability Distribution

Covariance of Each Liability Line with Total Liability

Line	Covariance with Total
Reserve	0.0141
Line A UW	0.0198
Line B UW	0.0279
Total Liability Volatility Squared	0.0179
Total Liability Volatility	0.1340

Volatility of the Asset to Liability Ratio

Asset to Liability Ratio Volatility

Liability Volatility	0.1340
Asset Volatility	0.0400
Asset to Liability Ratio Volatility	0.1398

Overall Capital to Liability Ratio

Total Capital	8,949,750
Total Liability	29,812,697
Capital to Liability Ratio	0.3002

Calculation of Current Insolvency Put Value and Partial Derivatives

Default Option Value

Spot Price	1.30
Strike Price	1.00
Volatility	0.1398
d1	1.9477
d2	1.8079
N(-d1)	0.0257
N(-d2)	0.0353
Put Value	0.186%
Delta	-0.0257
Vega	0.0778

Calculate Capital to Liability Ratios for Each Line

This calculation uses the formula: $s_i = s - \left(\frac{\partial p}{\partial s}\right)^{-1}\left(\frac{\partial p}{\partial \sigma}\right)\left[(\sigma_{iL} - \sigma_L^2) - (\sigma_{iV} - \sigma_{LV})\right]\frac{1}{\sigma}$

Capital to Liability Ratio

Reserve	21.78%
Line A UW	33.92%
Line B UW	51.57%

Determine Capital Allocation by Line

Capital Allocation - Myers-Read Method (0.186% EPD Ratio)

	Capital to Loss Ratio	Expected Claims	Capital
Reserve Risk	21.78%	18,091,233	3,939,466
Line A UW Risk	33.92%	5,860,732	1,988,079
Line B UW Risk	51.57%	5,860,732	3,022,205
Total			8,949,750

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RISKINESS LEVERAGE MODELS

RODNEY KREPS

Abstract

A general formulation of risk load for total cash flows is presented. It allows completely additive co-measures[1] *at any level of detail for any dependency structure between random variables constituting the total. It is founded on the intuition that some total outcomes are more risky per dollar than others, and the measure of that is a "riskiness leverage ratio." This riskiness leverage function is an essentially arbitrary choice, enabling an infinite variety of management attitudes toward risk to be expressed.*

The complete additivity makes these models useful. What makes them interesting is that attention can be turned toward asking "What is a plausible risk measure for the whole, while being prepared to use the indicated allocation technique for the pieces?" The usual measures are special cases of this form, as shown in some examples.

While the author does not particularly advocate allocating capital to do pricing, this class of models does allow pricing at the individual policy clause level, if so desired.

Further, the desirability of reinsurance or other hedges can be quantitatively evaluated from the cedant's point of view by comparing the increase in the mean cost of underwriting with the decrease in capital cost from reduction of capital required.

[1]Gary Venter coined this term, in parallel with variance and covariance.

ACKNOWLEDGEMENT

Many thanks are due to the referees, Gary Venter, John Pagliaccio, Peter Wei, and Katie Farrell for helpful discussion and suggestions. Errors are totally the author's.

1. INTRODUCTION

The generic problem is that there are a number of random liabilities and assets for a company and a single pool of shared capital to support them. Their mean is usually meant to be supported by the reserves and their variability supported by the surplus, with the total assets of the company being the sum. Frequently, it is desired that the supporting capital be allocated in considerable detail—for example, to underwriter within line of business within state. This is not an end in itself, but is usually meant to help to understand profitability (or lack of it) in a business unit by associating a target rate of return with the allocated surplus and comparing to the actual profit return distribution. Sometimes the allocation is meant to be used for creating a pricing risk load as the allocated surplus times a target rate of return. Really, it is the cost of capital that is being allocated.[2]

One would like to have a methodology that would allow allocation of an essentially arbitrary form for the total capital required, and would also like to have an interpretation of the form in terms of statistical decision theory. The total capital including surplus will usually be represented as the sum of a risk load and a mean outcome. These can be calculated for a given distribution of total results. No attempt to connect risk load to a theory of pricing will be made here, although given the shape of the distribution in the context of a given theory such a connection could be made. It is simply assumed that some appropriate mean return is needed to attract and retain capital for the total risk.

[2]Gary Venter, private communication.

There are several desirable qualities for an allocatable risk load formulation: (1) it should be able to be allocated to any desired level of definition; (2) the risk load allocated for any sum of random variables should be the sum of the risk load amounts allocated individually; (3) the same additive formula is used to calculate the risk load for any subgroup or group of groups.

This means that senior management can allocate capital to regions, and then regional management can allocate their capital to lines of business, and the allocations will add back up to the original. Further, it also means that the lines of business will add to the allocations for total lines of business as seen at the senior management level.

Ultimately, the choice of the riskiness leverage function will reflect management attitudes toward risk. The intention of this paper is to provide an interpretable framework for infinitely many choices, all of which can be appropriately allocated. It will be argued that the risk load must be considered in the context of the capital to support the risk.

Once management has experimented with various riskiness leverage functions and found a formulation with which they are comfortable, then it can be used to evaluate potential management decisions quantitatively. For example, buying reinsurance or choosing between reinsurance programs can be framed by including the variables representing the reinsurance cash flows. The general effects from a well-designed program will be to increase the mean cost—because the reinsurer needs to make a profit, on average—and to decrease the risk load and its associated cost—because the reinsurance is a good hedge against severe outcomes. If there is a net reduction in total cost, then there is an advantage to the alternative. It is worth noting that no financial information except the price is needed from the reinsurer. In particular, whatever return the reinsurer may think he will get from the contract is irrelevant to the cedant's decision to buy or not.

Section 2 introduces the framework and some practical notes; Section 3 is the development of the form and some of its properties; Section 4 is various examples, including some of the usual suspects for risk measures; Section 5 talks about what general properties might be desirable; and Section 6 is a numerical example with an accompanying spreadsheet.

2. THE FRAMEWORK

Assume n random financial variables X_k, $k = 1$ to n; and let $X = \sum_{k=1}^{n} X_k$ be their sum, the net result to the company. These variables may be from assets and/or liabilities but we will think of them for the initial exposition as liabilities. The convention used here is the actuarial view that liabilities are positive and assets are negative. This is an odd point of view for financial reports, and so in the accompanying exemplar spreadsheet, to be discussed at length in Section 6, the formulas are rephrased with the variables being net income streams and positive income being positive numbers.

Denote by μ the mean of X, C the total capital to support X, and R the risk load for X. Their relationship is

$$C = \mu + R \tag{2.1}$$

In more familiar terms, for balance sheet variables the capital would be the total assets, the mean the booked net liabilities, and the risk load the surplus.

Correspondingly, let μ_k be the mean of X_k, C_k be the capital allocated to X_k and R_k be the risk load for X_k. These satisfy the equation analogous to Equation (2.1):

$$C_k = \mu_k + R_k. \tag{2.2}$$

Using the abbreviation

$$\overline{dF} \equiv f(x_1, x_2, \ldots, x_n) dx_1 dx_2 \ldots dx_n, \tag{2.3}$$

where $f(x_1, x_2, \ldots, x_n)$ is the joint probability density function of all the variables, the individual means are defined by

$$\mu_k \equiv \int x_k \overline{dF}, \tag{2.4}$$

and the overall mean is

$$\mu \equiv \int \left[\sum_{k=1}^{n} x_k\right] \overline{dF} = \sum_{k=1}^{n} \mu_k. \tag{2.5}$$

Riskiness leverage models have the form

$$R_k \equiv \int \overline{dF}(x_k - \mu_k) L(x) \qquad \text{with} \quad x \equiv \sum_{k=1}^{n} x_k. \tag{2.6}$$

Then

$$R = \int \overline{dF}(x - \mu) L(x) = \int f(x)(x - \mu) L(x) dx. \tag{2.7}$$

The essential key to this formulation is that *the riskiness leverage L depends only on the sum of the individual variables*. In the second form of Equation (2.7), $f(x)$ is the density function for X, the sum of random variables.

It follows directly from their definitions that $R = \sum_{k=1}^{n} R_k$ and $C = \sum_{k=1}^{n} C_k$, no matter what the joint dependence of the variables may be.

In analogy with the relation of covariance to variance, the R_k will be referred to as co-measures of risk for the measure R. On occasion, the C_k will also be referred to as co-measures when the context is clear. Since additivity is automatic with these co-measures, what remains is to *find appropriate forms for the riskiness leverage $L(x)$*.

The form can be thought of as the risk load being a probability-weighted average of risk loads over outcomes of the total net loss:

$$R = \int dx\, f(x) r(x) \qquad \text{where} \quad r(x) = (x - \mu) L(x). \tag{2.8}$$

Again, the riskiness leverage reflects that not all dollars are equally risky, especially dollars that trigger analyst or regulatory tests.

Equation (2.8) is a standard decision-theoretic formulation for R. It could have been written down immediately, except that the special form for the risk load for outcomes is needed so that the co-measures have good properties. Another version of Equation (2.8) is to represent the risk load as an integral over risk load density:

$$R = \int rld(x)dx \qquad \text{where} \quad rld(x) = f(x)(x-\mu)L(x). \tag{2.9}$$

This has the advantage of showing which outcomes most contribute to the risk load. Another formulation, of note to theorists, is to say that the riskiness leverage modifies the joint density function and that the allocations are statistical expectations on a risk-adjusted density function. However, the support of L needs to be the same as the support of f to make this really work.

$$R = \int dx\, f^*(x)(x-\mu) \qquad \text{with} \quad f^*(x) = f(x)L(x). \tag{2.10}$$

A closely related useful form for thinking about the risk loads is that they are conditional expectations of a variable less its mean on the risk-adjusted measure, and that the conditions refer to the overall total variable. A typical condition might be that the total loss is greater than some specified value.

If we just want to think about co-measures without the explicit breakout into mean and risk load, we can use the generalization

$$R_k \equiv \int \overline{dF}(x_k - a\mu_k)L(x) \qquad \text{with} \quad x \equiv \sum_{k=1}^{n} x_k, \tag{2.11}$$

where any constant value can be used for a. A prime candidate is $a = 0$, and in the exemplar spreadsheet in Section 6 this is done because the variables considered there are net income variables.

It is also clear from Equation (2.6) that some variables may have negative risk loads, if they happen to be below their mean when the riskiness leverage on the total is large. This is a desirable feature, not a bug, as software developers say. Hedges in general and reinsurance variables in particular should exhibit this behavior, since when losses are large they have negative values (ceded loss) greater than their mean costs.

Practical Notes

Actual calculation of Equations (2.6) and (2.7) cannot be done analytically, except in relatively simple cases. However, in a true Monte Carlo simulation environment they are trivially evaluated. All one has to do is to accumulate the values of X_k, $L(X)$, and $X_k L(X)$ at each simulation. At the end, divide by the number of simulations and you have the building blocks[3] for a numerical evaluation of the integrals. As usual, the more simulations that are done the more accurate the evaluation will be. For companies that are already modeling with some DFA model it is easy to try out various forms for the riskiness leverage.

This numerical procedure is followed in the spreadsheet of Section 6, which has assets and two correlated lines of business. All the formulas are lognormal so that the exact calculations for moments could be done. However, the spreadsheet is set up to do simulation in parallel with the treatment on a much more complex model. It is also easy to expand the scope. If one starts at a very high level and does allocations, these allocations will not change if one later expands one variable (e.g., countrywide results) into many (results by state) so long as the total does not change.

[3]The mean for X_k is just the average over simulations, and it might be advantageous to calculate this first. The risk load is just the average over simulations of $X_k L(X)$ minus the mean of X_k times the average over simulations of $L(X)$.

Fundamentally, a risk measure should arise from economic requirements and management attitudes toward risk as part of the management business model. In this paper's class of models the risk attitude information is in the riskiness leverage function.

Gedanken[4] experiments indicate that to get the riskiness leverage it is probably desirable to start with plausible relativities between outcomes. After that is done, set the overall scale by some criterion such as probability of ruin (Value At Risk), mean policyholder deficit, Tail Value At Risk (TVAR)[5] or anything else that references the total capital and suits management's predilections. It is best if the overall level can be framed in the same terms as the relativities. In the Section 6 spreadsheet, TVAR is used.

In general, it might be good to start with simple representations, say with two parameters, and then see what consequences emerge during the course of testing. More remarks will be made later on specific forms. It will also be shown that the usual forms of risk measure can be easily framed and the differences between them interpreted in terms of different riskiness leverages.

A warning: there is no sign of time dependence in this formulation so far. Presumably the variables refer to the present or future value of future stochastic cash flows, but there is considerable work to be done to flesh this out.[6]

3. FORM DEVELOPMENT

Here we will start from a covariance formulation and proceed to the framework above by a detailed mathematical derivation.

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[4]That is, thought experiments, as contrasted with the real thing. The term is from the early days of relativity.

[5]TVAR is the average value of a variable, given that it is past some defined point in the tail. For example, one could ask for the average loss size given that the loss is excess of $10M.

[6]The work of Leigh Halliwell "The Valuation of Stochastic Cash Flows" may provide a way of looking at this problem.

Various proposed schemes[7] have utilized the fact that an allocation formula of the form

$$C_k = \alpha\mu_k + \beta\mathrm{Cov}(X_k, X) \tag{3.1}$$

will always be additive no matter what the dependency between the X_k may be. That is,

$$\begin{aligned} C &\equiv \alpha\mu + \beta\mathrm{Var}(X) \\ &= \alpha\mathrm{E}(X) + \beta\mathrm{Cov}(X, X) \\ &= \alpha\sum_{k=1}^{n}\mu_k + \beta\sum_{k=1}^{n}\mathrm{Cov}(X_k, X) \\ &= \sum_{k=1}^{n} C_k. \end{aligned} \tag{3.2}$$

A similar result will hold for the sum of any subset of the variables, thus ensuring the desired properties of the allocation. The sum of covariances of the individual variables with the total is the covariance of the total with itself. This paper generalizes this notion.

This form can be pushed further by imposing the reasonable requirement[8] that if a variable has no variation, then the capital to support it is simply its mean value with no additional capital requirement. This requires $\alpha = 1$. Then, with capital being the sum of the mean and the risk load,

$$R_k = \beta\mathrm{Cov}(X_k, X) \tag{3.3}$$

and

$$R = \beta\mathrm{Var}(X) \tag{3.4}$$

and so finally

$$R_k = R\frac{\mathrm{Cov}(X_k, X)}{\mathrm{Var}(X)}. \tag{3.5}$$

[7] For a sampling, try [6], [2], and [4]. There are no doubt others.

[8] In [6], since the company can default, a constant value carries a negative risk load. We are assuming an ongoing company.

This form is familiar from CAPM.

However, it is clear that there are many independent linearly additive statistics. Back up a little to the definitions of mean and covariance, expressed as integrals over the joint density function:

$$\begin{aligned}\mu_k \equiv \mathrm{E}(X_k) &= \int x_k f(x_1,\ldots,x_n)dx_1\ldots dx_n \\ &\equiv \int x_k \overline{dF}. \end{aligned} \tag{3.6}$$

The additivity of the mean then comes from

$$\mu \equiv \mathrm{E}(X) = \int \sum_{k=1}^{n} x_k = \sum_{k=1}^{n} \int x_k = \sum_{k=1}^{n} \mu_k. \tag{3.7}$$

The covariance of one variable with the total is defined as

$$\mathrm{Cov}(X_k, X) \equiv \int \overline{dF}(x_k - \mu_k)(x - \mu), \tag{3.8}$$

where $x \equiv \sum_{k=1}^{n} x_k$. The additivity of the covariance is from

$$\begin{aligned}\mathrm{Cov}(X,X) &= \int \overline{dF}(x-\mu)^2 \\ &= \int \overline{dF}\left[\sum_{k=1}^{n}(x_k - \mu_k)\right](x-\mu) \\ &= \sum_{k=1}^{n}\int \overline{dF}(x_k - \mu_k)(x-\mu) \\ &= \sum_{k=1}^{n}\mathrm{Cov}(X_k, X). \end{aligned} \tag{3.9}$$

We want to generalize this result, and to do so we need more independent statistics that are linear functionals in X_k. Define the moment expectations

$$\mathrm{E}_m(X_k) \equiv \int \overline{dF}[(x_k - \mu_k)(x-\mu)^m]. \tag{3.10}$$

Then, following the same argument as in Equation (3.9), for any m

$$\mathrm{E}_m(X) = \sum_{k=1}^{n} \mathrm{E}_m(X_k). \tag{3.11}$$

Notice that the moment expectation for $m = 1$ is just the covariance of X_k with the total.

The individual risk load may now be formulated as

$$R_k = \sum_{m=1}^{\infty} \beta_m \mathrm{E}_m(X_k), \tag{3.12}$$

and there are now an infinite number of arbitrary constants to play with. Since there are so many independent constants, essentially any form can be approximated arbitrarily well.

For any choice of the constants β_m, the total risk load is the sum of the individual risk loads:

$$R = \sum_{m=1}^{\infty} \beta_m \mathrm{E}_m(X) = \sum_{m=1}^{\infty} \beta_m \sum_{k=1}^{n} \mathrm{E}_m(X_k) = \sum_{k=1}^{n} R_k. \tag{3.13}$$

This risk load can be put into a more transparent form by writing it as

$$R_k = \sum_{m=1}^{\infty} \beta_m \mathrm{E}_m(X_k) = \int \overline{dF}(x_k - \mu_k) \sum_{m=1}^{\infty} \beta_m (x - \mu)^m. \tag{3.14}$$

Since the term with $m = 0$ integrates to 0 (that being the definition of the mean), what is present is a Taylor series expansion of a function of the total losses about μ. Thus, Equation (3.14) may be written as

$$R_k = \int \overline{dF}(x_k - \mu_k) L(x). \tag{3.15}$$

This is the framework described earlier.

Properties

Clearly, the allocation properties are all satisfied for any choice of $L(x)$. The risk load has no risk for constant variable

$$R(c) = 0.$$

It also will scale with a currency change

$$R(\lambda X) = \lambda R(X),$$

provided $L(x)$ is homogeneous of order zero:

$$L(\lambda x) = L(x).$$

The reason this is required is that there is already a currency dimension in the term multiplying L. This can be made to happen, for example, by making L a function of ratios of currencies such as x/μ or x/σ, where σ is the standard deviation of X.

However, a more interesting possibility is to make L also be a function of x/S, where again S is the total surplus of the company. Since asset variability is in principle included in the random variables, S should be a guaranteed-to-be-available, easily liquefiable capital. This could come, for example, by having it in risk-free instruments or by buying a put option on investments with a strike price equal to what a risk-free investment would bring, or any other means with a sure result.

It is intuitively clear that S must come into the picture. Consider the case where loss is normally distributed with mean 100 and standard deviation 5. Is this risky for ruin, from a business point of view? If the surplus is 105, it is—but if it is 200 it is not. The natural interpretation is that the riskiness leverage should be a function of the ratio of the difference of the outcome from the mean to the surplus. Since the riskiness leverage could be used (with a pre-determined leverage) to give the surplus, there is a certain recursive quality present.

This formulation of risk load may or may not produce a coherent risk measure.[9] The major reason is that subadditivity[10] $[R(X+Y) \leq R(X) + R(Y)]$ depends on the form of $L(x)$. It might be remarked that superadditivity $[R(X+Y) > R(X) + R(Y)]$ is well known in drug response interactions, where two drugs taken separately are harmless but taken together are dangerous. While axiomatic treatments may prefer one form or another, *it would seem plausible that the risk measure should emerge from the fundamental economics of the business* and the mathematical properties should emerge from the risk measure, rather than vice versa.

A riskiness leverage formulation clearly allows the entire distribution to influence the risk load, and does not prescribe any particular functional form for the risk measure. In addition, many familiar measures of risk can be obtained from simple forms for the riskiness leverage ratio.

4. EXAMPLES

Risk-Neutral

Take the riskiness leverage to be a constant; the risk load is zero.

The positive risk load balances the negative risk load. This would be appropriate for risk of ruin if the range of x where $f(x)$ is significant is small compared to the available capital, or if the capital is infinite. It would be appropriate for risk of not meeting plan if you don't care whether you meet it or not.

Variance

Take

$$L(x) = \frac{\beta}{S}(x - \mu). \qquad (4.1)$$

This riskiness leverage says that the whole distribution is relevant; that there is risk associated with good outcomes as much as

[9]In the sense of [1] the actual risk measure is mean $+R$.

[10]A requirement for coherence. See [5] or [1].

bad; and that the outcome risk load increases quadratically out to infinity.

This gives the usual

$$R = \frac{\beta}{S}\int_0^\infty dx\, f(x)(x-\mu)^2 \qquad (4.2)$$

and

$$R_k = \frac{\beta}{S}\int \overline{dF}(x_k-\mu_k)\left(\sum_{j=1}^{n} x_j - \mu\right). \qquad (4.3)$$

Note that Equation (4.1) is suggestively framed so that β is a dimensionless constant available for overall scaling. The total capital then satisfies

$$C = \mu + S, \qquad (4.4)$$

and the solution for $S = R$ is proportional to the standard deviation of the total:

$$S = \sqrt{\beta\,\mathrm{Var}(X)}. \qquad (4.5)$$

It is perfectly possible, of course, to use some other formulation of the constant, say β/μ, which would then give a different measure. Such a measure would imply that the riskiness leverage does not depend on the amount of surplus available unless it was hidden in the scaling factor β.

TVAR (Tail Value At Risk)

Take the riskiness leverage

$$L(x) = \frac{\theta(x-x_q)}{1-q}. \qquad (4.6)$$

The value q is a management-chosen percentage; for example, $q = 99\%$. The quantile x_q is the value of x where the cumulative distribution of X, the total, is equal to q. That is, $F(x_q) = q$. $\theta(x)$ is the step function: zero for negative argument and 1 for positive. See Appendix A for mathematical asides on this function.

This riskiness leverage ratio is zero up to a point, and then constant. Here the constant is chosen so as to exactly recreate TVAR, but clearly any constant will give a similar result. In fact, a riskiness leverage ratio that is constant up to a point and then jumps to another constant will give a similar result.

$$
\begin{aligned}
C &= \mu + \int dx\, f(x)(x-\mu)\frac{\theta(x-x_q)}{1-q} \\
&= \mu + \int_{x_q}^{\infty} dx\, f(x)\frac{x-\mu}{1-q} \\
&= \mu - \frac{\mu}{1-q}(1-q) + \frac{1}{1-q}\int_{x_q}^{\infty} dx\, f(x)x \\
&= \frac{1}{1-q}\int_{x_q}^{\infty} dx\, f(x)x. \qquad (4.7)
\end{aligned}
$$

This is the definition of TVAR, well known to be coherent.[11]

We see shortly that the allocated capital is just the average value of the variable of interest in the situations where the total is greater than x_q. This is one example of the conditional expectation referred to earlier.

$$
\begin{aligned}
C_k &= \mu_k + \int \overline{dF}(x_k-\mu_k)\frac{\theta(x-x_q)}{1-q} \\
&= \mu_k - \frac{\mu_k}{1-q}\int \overline{dF}\theta(x-x_q) + \frac{\int \overline{dF} x_k \theta(x-x_q)}{1-q} \\
&= \frac{\int \overline{dF} x_k \theta(x-x_q)}{1-q}. \qquad (4.8)
\end{aligned}
$$

This measure says that only the part of the distribution at the high end is relevant.

[11] [5], Op. cit.

VAR (Value At Risk)

Take the riskiness leverage

$$L(x) = \frac{\delta(x - x_q)}{f(x_q)}. \tag{4.9}$$

In Equation (4.9) $\delta(x)$ is the Dirac delta function.[12] Its salient features are that it is zero everywhere except at (well, arbitrarily close to) zero and integrates to one.[13] See Appendix A for remarks about this very useful function. Here the riskiness leverage ratio is all concentrated at one point. The constant factor has been chosen to reproduce VAR exactly, but clearly could have been anything.

$$\begin{aligned} C &= \mu + \int dx\, f(x)(x - \mu)\frac{\delta(x - x_q)}{f(x_q)} \\ &= \mu + x_q - \mu \\ &= x_q. \end{aligned} \tag{4.10}$$

This gives value at risk, known not to be coherent.[14] This measure says that only the value x_q is relevant; the shape of the loss distribution does not matter except to determine that value.

The capital co-measure is the mean of the variable over the hyperplane where the total is constant at x_q:

$$\begin{aligned} C_k &= \mu_i + \int \overline{dF}(x_k - \mu_k)\frac{\delta(x - x_q)}{f(x_q)} \\ &= \frac{1}{f(x_q)} \int \overline{dF} x_k \delta\left(\sum_{j=1}^{n} x_j - x_q\right). \end{aligned} \tag{4.11}$$

In a simulation environment one would have to take a small region rather than a plane. This could most easily be done as the

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[12]Introduced in 1926.

[13]This implies that $\int dx\, f(x)\delta(x - a) = f(a)$. See Appendix 1.

[14][5], Op. cit.

difference of two closely neighboring TVAR regions. This was done using the formulation of the exemplar spreadsheet and a 1% width of the region.

SVAR (Semi-Variance)

Take the riskiness leverage

$$L(x) = \frac{\beta}{S}(x-\mu)\theta(x-\mu). \tag{4.12}$$

The risk load is the semi-variance—the "downside" of the variance:

$$R = \frac{\beta}{S}\int_{\mu}^{\infty} dx\, f(x)(x-\mu)^2, \tag{4.13}$$

and

$$R_k = \frac{\beta}{S}\int \overline{dF}(x_k-\mu_k)(x-\mu)\theta(x-\mu). \tag{4.14}$$

This measure says that risk loads are only non-zero for results worse (greater) than the mean. This accords with the usual accountant's view that risk is only relevant for bad results, not for good ones. Further, this says the load should be quadratic to infinity.

Mean Downside Deviation

Take the riskiness leverage

$$L(x) = \beta\frac{\theta(x-\mu)}{1-F(\mu)}. \tag{4.15}$$

$F(x)$ is the cumulative distribution function for X, the total. This risk load is a multiple of the mean downside deviation, which is also TVAR with $x_q = \mu$. This riskiness leverage ratio is zero below the mean, and constant above it. Then

$$R(X) = \frac{\beta}{1-F(\mu)}\int_{\mu}^{\infty} dx\, f(x)(x-\mu), \tag{4.16}$$

and

$$R_k = \frac{\beta}{1 - F(\mu)} \int \overline{dF}(x_k - \mu_k)\theta(x - \mu). \qquad (4.17)$$

In some sense this may be the most natural naive measure, as it simply assigns capital for bad outcomes in proportion to how bad they are. Both this measure and the preceding one could be used for risks such as not achieving plan, even though ruin is not in question.

In fact, there is a heuristic argument suggesting that $\beta \approx 2$. It runs as follows: suppose the underlying distribution is uniform in the interval $\mu - \Delta \leq x \leq \mu + \Delta$. Then in the cases where the half-width Δ is small compared to μ, the natural risk load is Δ. For example, if the liability is \$95M to \$105M, then the natural risk load is \$5M. So from Equation (4.17)

$$\Delta = R(X) = \frac{\beta}{0.5} \int_{\mu}^{\mu+\Delta} \frac{dx}{2\Delta}(x - \mu) = \frac{\beta\Delta}{2}. \qquad (4.18)$$

However, for a distribution that is not uniform or tightly gathered around the mean, if one decided to use this measure, the multiplier would probably be chosen by some other test such as the probability of seriously weakening surplus.

Proportional Excess[15]

Take the riskiness leverage

$$L(x) = \frac{h(x)\theta[x - (\mu + \Delta)]}{x - \mu}, \qquad (4.19)$$

where to maintain the integrability of R_k either $h(\mu) = 0$ or $\Delta > 0$. Then

$$R = \int f(x)h(x)\theta[x - (\mu + \Delta)]dx, \qquad (4.20)$$

and

$$R_k = \int \overline{dF} \frac{x_k - \mu_k}{x - \mu} h(x)\theta[x - (\mu + \Delta)]. \qquad (4.21)$$

[15] Another contribution from Gary Venter.

The last form has the simple interpretation that the individual allocation for any given outcome is pro-rata on its contribution to the excess over the mean.

5. GENERIC MANAGEMENT RISK LOAD

Most of the world lives in a situation of finite capital. Frame the question as "given the characteristics of the business, what is an appropriate measure of risk to the business, which generates a needed surplus S?" In the spreadsheet example this is done with a simplistic riskiness leverage function.

Clearly, the question at the heart of the matter is what an appropriate measure of riskiness might be. There are many sources of risk among which are the risk of not making plan, the risk of serious deviation from plan, the risk of not meeting investor analysts' expectations, the risk of a downgrade from the rating agencies, the risk of triggering regulatory notice, the risk of going into receivership, the risk of not getting a bonus, etc.

Given the above, it seems plausible that company management's list for the properties of the riskiness leverage ratio should be that it:

1. be a downside measure (the accountant's point of view);
2. be more or less constant for excess that is small compared to capital (risk of not making plan, but also not a disaster);
3. become much larger for excess significantly impacting capital; and
4. go to zero (or at least not increase) for excess significantly exceeding capital—once you are buried, it doesn't matter how much dirt is on top.

With respect to (3), the risk function probably has steps in it, especially as regulatory triggers are hit. For (4), a regulator might want to give more attention to the extreme areas. In fact, a regulator's list of properties for the riskiness leverage might include that it

1. be zero until capital is seriously impacted, and
2. not decrease, because of the risk to the state guaranty fund.

TVAR could be used as such a risk measure if the quantile is chosen to correspond to an appropriate fraction α of surplus. This would be

$$L_{\text{Regulator}}(x) = \frac{\theta(x - \alpha S)}{1 - F(\alpha S)}. \tag{5.1}$$

However, everyone recognizes that at some level of probability management will have to bet the whole company. There is always business risk.

Management may more typically formulate its risk appetite in forms such as "For next year, we want not more than a 0.1% chance of losing all our capital, and not more than a 10% chance of losing 20% of capital." This is basically two separate VAR requirements, and can be satisfied by using the larger of the two required capital amounts. Or, as in the spreadsheet, management may choose to say something like, "We want our surplus to be $1\frac{1}{2}$ times the average bad result in the worst 2% of cases."

A (much too) simple example approximately satisfying (1) to (3) on management's list consists of linear downside riskiness leverage:

$$L(x) = \begin{cases} 0 & \text{for} \quad x < \mu \\ \beta\left[1 + \alpha\dfrac{(x-\mu)}{S}\right] & \text{for} \quad x > \mu \end{cases}. \tag{5.2}$$

The value of α is essentially the relative riskiness at the mean and at excess over mean equal to surplus. The value of β is again an overall scale factor. In the spreadsheet the allocations are nearly independent of the value of α, and TVAR is used for the example. The suggested use is to get the riskiness leverage function, and then to evaluate the effects of reinsurance (approximated by an increase in the mean and a decrease in the coefficient of variation) by seeing how the capital requirement changes for the same leverage function.

6. EXEMPLAR SPREADSHEET

The Excel workbook “Mini DFA.xls” has two lines of business with a correlation between the lines and investment income. The example is meant to be oversimplified but plausible, and takes the underwriting result for each line as a fixed premium less random draw on loss and expense. There is investment income on the surplus but no explicit consideration of it within the reserves. On the other hand, the lines of business are priced to a net positive underwriting result, so we could say that we are looking at future values including all investment income.

Cells with a blue background are input cells, and the reader is invited to change them and see how the results change. All the formulas are lognormal so that the exact calculations could be done. However, there is a “Simulate” button on the spreadsheet that will give statistics and cumulative distribution functions for whatever set of cells is selected. Simulation is used to get the overall results and the allocation ratios for different risk measures.

The sheets in the workbook are of two types: the data sheets (e.g., “basics”) and the simulations done on them (“Sim basics”). The different sheets are generally different business alternatives. We start with “basics,” which gives the basic setup of the business, and continue on: “TVARS” calculates various TVAR measures, “change volume” changes the volumes of the lines, and “reinsurance” and “reinsurance (2)” explore the effects of reinsurance. We will walk through them in detail, with commentary.

In all of them, the layout is the same. The two lines of business and the investment on surplus are laid out in columns, with blue background for user input. The financial variables are the two net underwriting results and the investment result, all of which vary randomly. F9 will recalculate to a new set of results. Below the income variables are the starting and ending surplus, and calculated mean and current (random) return. Interesting simulation results such as allocation percentages are displayed to the right of the surplus calculation.

Starting with "basics," Line A has a mean surplus of 10,000,000 and a standard deviation of 1,000,000 and Line B has a mean surplus of 8,000,000 and a standard deviation of 2,000,000. There is a correlation of about 25% between the lines (if the functions were normal rather than lognormal, it would be exactly 25%). Each line is written with a premium equal to the mean loss plus 5%. We interpret this calculation as our estimate at time zero of the value at time 1 of the underwriting cash flows, including all investment returns on reserves and premiums.

The investment income on the surplus is taken directly. The investment is at a mean rate of 4% with a standard deviation of 10%. The total of the results, on which we will define our leverage functions, is then added to the beginning surplus of 9,000,000 to get the ending surplus. As a consequence of the input values, the mean return on surplus is 14%. We would all be happy to have such a company, provided it is not too risky.

The simulation ("Sim basics") shows the actual correlation of the lines and the coefficient of variation on the return, as well as the distribution of total ending surplus and return. From the "Sim basics" sheet we can see that the probability of ruin is less than one in a thousand, and the coefficient of variation on the return is better than on the investment, which is good. We can also see from comparing the simulated means and standard deviations of the income variables to their known underlying values that the simulation is running correctly.

Management has decided that it wants to consider not just ruin, but on-going risk measures. In particular, it wants to get the TVAR values at various percentiles. It wants to formulate its risk appetite as *"For the x percent of possibilities of net income that are less than $(income corresponding to x%), we want the surplus to be a prudent multiple of the average value so that we can go on in business."* What we do not know yet is what is *x%*, and what is the "prudent multiple." Gary Venter has suggested that the prudent multiple could be such that the renewal book can still be serviced after an average bad hit.

The sheet "TVARs" has the calculations needed for TVAR simulation in cells G36:N42. Column G contains the percentage values from 10% to 0.1%, and Column H the values of the total net income corresponding to those percentages. These values come from the sheet "Sim basics." Column I answers the question if whether the income is less than the value in Column H. Columns J through M are either "FALSE" if Column I is FALSE, or contain respectively the total income, the Line A income, the Line B income, and the investment income. Column N is a variable that is 1 if Column I is TRUE, and zero if it is FALSE. Upon selecting these cells and simulating, the mean value of Column N (for each row) will be the percentage of the time that the condition was satisfied. This should be close to the percentage in Column G. During simulation, non-numeric values in the selected cells are ignored. The mean values of cells in Columns J through M are the conditional means of the income variables for different threshold values, as desired.

The result of simulation is:

	Income	Mean Value of TVAR and Allocation Percentages			
%	is Below	Total	Line A	Line B	Investment
0.1	(8,892,260)	(10,197,682)	12.30%	85.99%	1.71%
0.2	(7,967,851)	(9,326,936)	12.49%	85.73%	1.78%
0.4	(7,024,056)	(8,380,265)	12.89%	85.09%	2.02%
1	(5,749,362)	(7,129,796)	13.38%	84.67%	1.95%
2	*(4,732,795)*	*(6,159,564)*	*13.60%*	*84.30%*	*2.10%*
5	(3,309,641)	(4,811,947)	13.60%	84.20%	2.20%
10	(2,143,327)	(3,734,177)	13.26%	84.94%	1.80%

The allocation percentages are just the ratios of the means for the pieces to the mean for the total; they automatically will add to 100%. What is noticeable here is that the allocation percentages change very little with the TVAR level, and that Line B needs some six times the surplus of Line A. That it needs more is not surprising; that it needs so much more perhaps is. What these allocations say is that when the total result is in the worst 10% of cases, about 5/6 of it is from Line B.

Management decides to adopt the rule *"We want our surplus to be $1\frac{1}{2}$ times the average negative income in the cases where it is below the 2% level."* That row is in *italic*, and this rule means that the 9,000,000 surplus is sufficient.

Using those allocation percentages, the mean returns on allocated surplus are Total: 14%; Line A: 40.9%; Line B: 5.3%; Investment: 190.6%. The total is a weighted average of the pieces. One needs to be careful in interpreting these return numbers, because they are dependent on both the relative volume of the lines and on the allocation method. But in any case, because Line B needs so much of the surplus, its return is depressed and the other returns are enhanced.

The next sheet, "change volume," looks at the case where we can change the underwriting volumes of Lines A and B. Clearly we want to reduce Line B and increase Column A, so the example has Column A increased by 60% and Column B decreased by 75%. This keeps the same mean net income. The standard deviations have been taken as proportional to volume, thinking of each line as a sum of independent policies.

Running the simulations, the allocations for Line A, Line B, and Investments now are respectively 32.8%, 60.9%, and 6.4%. Their implied returns change to 27.1%, 1.8%, and 62.8%. Line B is still bad, but because there is less of it, there is not such a large contribution at the 2% level. The 2% level, which was (4,732,795), is now better at about (3,250,000).

We also see that according to the management rule, we can release surplus of about 2,500,000. Alternatively, we can keep the same surplus and have a more conservative rule, with the prudent ratio being 2 instead of $1\frac{1}{2}$.

However, it may not be possible to change line volume, for various reasons. For example, these may be two parts of an indivisible policy, like property and liability from homeowners. Regulatory requirements may make it difficult to exit Line B. In addition, it takes time to switch the portfolio and requires a major underwriting effort. Management may decide to look at the possibility of buying reinsurance to improve the picture, since that is a decision that can be implemented quickly and easily changed next year.

The sheet "reinsurance" has an excess reinsurance contract on Line B, with a limit of 5,000,000 and an attachment of 10,000,000. It is priced with a load of 25% of its standard deviation. Once again, note that in the spreadsheet the results are calculated because we used easy forms, but that we could have complex forms and just simulate. The reinsurance results flow into the total net income.

Running the simulations, the allocations for Lines A and B, Investments, and now Reinsurance are respectively 36.3%, 73.9%, 14.2%, and −24.4%. The negative value for the reinsurance allocation reflects that the hedge is working, effectively supplying capital in these events. However, because of the positive net average cost of reinsurance, the return on the total is reduced to 12.1%. The implied returns on the pieces are 15.3%, 6.0%, 28.3%, and 7.9%. Line B is still bad, but because of the reinsurance there is not such a large contribution at the 2% level. Again, the 2% level has gone from (4,732,795) to (3,300,000). If we were to combine the reinsurance into Line B the combined allocation would be 49.5% and the return would be 5.1%.

There is also some 3,000,000 in surplus that the management rule would allow to be released. In the sheet "reinsurance (2)" the starting surplus has been reduced to 7,250,000 in order to bring the mean return on the total back up to 14%. Running the simulations, the 2% level on income is actually (3,237,000) but we ran the TVAR at (3,300,000). The essential point is that the results look reasonable, and the rule would allow release of still more surplus.

What is omitted in the calculation is the value of the 1,750,000 already released from the original 9,000,000 surplus. What this is worth depends on how the released surplus is going to be used. At the very least, this should be worth the risk-free income from it. Classical financial theory would suggest that it should be evaluated at the firm's cost of borrowing.

Measures other than TVAR were also run on the same basic situation, but are not shown in the spreadsheet. They were of two types. One was VAR measures, using a 1% interval around the VAR values. This measure says, given that the total loss is *at* a particular level, how much of it is from the different contributions. The other class of measures is the power measures, as in Equation (3.10). Each measure is a power of $(\mu - x)$ for $\mu > x$, and zero otherwise. In other words, these are downside measures.[16] The powers 0 and 1 are respectively the mean downside deviation and the semivariance. The others could be called "semiskewness," "semikurtosis," and so on—but why bother?

The results for VAR are quite similar to TVAR, except at the 10% level. This is because of the particular conditions we have for variability and correlation, and will not be true in general.

[16]Note that in contrast to the earlier discussion on losses where the downside is outcomes greater than the mean, here on return to surplus the downside is outcomes less than the mean.

	Mean Value and Allocation Percentages			
%	Total	Line A	Line B	Investment
0.1	(8,892,557)	13.51%	84.01%	2.48%
0.2	(7,969,738)	13.41%	84.74%	1.85%
0.4	(7,021,936)	15.32%	83.22%	1.46%
1	(5,746,279)	13.94%	84.18%	1.88%
2	(4,731,425)	14.20%	83.43%	2.38%
5	(3,308,824)	13.38%	83.64%	2.98%
10	(2,143,340)	11.16%	88.07%	0.76%

The downside power measure simulation results are:

	Mean Values ^(1/(N + 1)) and Allocations from Simulation			
Power	Total	Line A	Line B	Investment
0	2,183,834	22.44%	65.52%	12.04%
1	2,839,130	20.63%	69.79%	9.58%
2	3,424,465	19.42%	72.30%	8.28%
3	3,985,058	18.35%	74.30%	7.35%
4	4,510,337	17.43%	75.97%	6.60%
5	5,018,663	16.55%	77.45%	6.00%
6	5,514,616	15.69%	78.79%	5.51%

As the power increases and the measure is increasingly sensitive to the extreme values, the allocations move toward the TVAR allocations. This is probably not surprising.

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APPENDIX A

SOME MATHEMATICAL ASIDES

$\theta(x)$ is the step function: zero for negative argument and 1 for positive. It is also referred to as the index function.

$\delta(x)$ is the Dirac delta function. It can be heuristically thought of as the density function of a normal distribution with mean zero and standard deviation arbitrarily small compared to anything else in the problem. This makes it essentially zero everywhere except at zero but it still integrates to 1.

The index function can also be thought of as the cumulative distribution function of the same normal distribution, and it is in this sense that the delta function can be thought of as the derivative of the index function. All the usual calculus rules about derivatives apply without modification.

Always, we are implicitly taking the limit as the standard deviation of this distribution goes to zero. This whole usage can be justified in the theory of linear functionals, but the author has no idea where.

These notions lead to some fundamental properties of the delta function. For any continuous function $f(x)$

$$f(a) = \int f(x)\delta(x-a)dx, \tag{A.1}$$

and for $c > b$

$$\int_b^c f(x)\delta(x-a)dx = \theta(c-a)\theta(a-b)f(a). \tag{A.2}$$

If $h(a) = 0$ then

$$\int f(x)\delta(h(x))dx = \frac{f(a)}{|h'(a)|}. \tag{A.3}$$

The density function $f(x)$ for the total sum of variables can most easily be written as

$$f(x) = \int \overline{dF}\,\delta\left(x - \sum_{k=1}^{n} x_k\right)$$
$$\equiv \int dx_1 \ldots dx_n f(x_1,\ldots,x_n)\delta\left(x - \sum_{k=1}^{n} x_k\right). \qquad \text{(A.4)}$$

For calculation this is often a convenient form, as in the derivation of Equation (2.7):

$$\int \overline{dF}\left(\sum_{k=1}^{n} x_i - \mu\right) g\left(\sum_{k=1}^{n} x_k\right)$$
$$= \int dx \int \overline{dF}\,\delta\left(x - \sum_{k=1}^{n} x_k\right)(x-\mu)g(x)$$
$$= \int f(x)(x-\mu)g(x)dx. \qquad \text{(A.5)}$$

Similarly, the marginal density for any variable can be written

$$f_k(y) = \int \overline{dF}\,\delta(y - x_k). \qquad \text{(A.6)}$$

The cumulative distribution function for the total is

$$F(x) = \int \overline{dF}\,\theta\left(x - \sum_{k=1}^{n} x_k\right)$$
$$\equiv \int dx_1 \ldots dx_n f(x_1,\ldots,x_n)\theta\left(x - \sum_{k=1}^{n} x_k\right), \qquad \text{(A.7)}$$

and

$$f(x) = \frac{d}{dx}F(x) \qquad \text{(A.8)}$$

emerges from simple differentiation rules.

Kreps, R.E., “Riskiness Leverage Models,” PCAS XCII, 2005, pp. 31-60.

For candidates attempting to replicate the exhibits in this paper, a spreadsheet developed by the author can be downloaded below:

https://www.casact.org/research/dare/documents/Kreps-Mini_Dfa_final.xls

AN APPLICATION OF GAME THEORY: PROPERTY CATASTROPHE RISK LOAD

DONALD F. MANGO

Abstract

Two well known methods for calculating risk load—Marginal Surplus and Marginal Variance—are applied to output from catastrophe modeling software. Risk loads for these marginal methods are calculated for sample new and renewal accounts. Differences between new and renewal pricing are examined. For new situations, both current methods allocate the full marginal impact of the addition of a new account to that new account. For renewal situations, a new concept is introduced which we call "renewal additivity."

Neither marginal method is renewal additive. A new method is introduced, inspired by game theory, which splits the mutual covariance between any two accounts evenly between those accounts. The new method is extended and generalized to a proportional sharing of mutual covariance between any two accounts. Both new approaches are tested in new and renewal situations.

ACKNOWLDGEMENTS

The author would like to thank Sean Ringsted, Eric Lemieux and Paul Kneuer for their reviews of earlier versions of the paper, and the Committee on Review of Papers for their helpful commentary, all of which greatly improved the paper.

1. INTRODUCTION

The calculation of risk load continues to be a topic of interest in the actuarial community—see Bault [1] for a recent survey of well known alternatives. One area of great need, where the

CAS literature is somewhat scarce, is calculation of risk loads for property catastrophe insurance.

Many of the new catastrophe modeling products produce occurrence size-of-loss distributions for a series of simulated events. These output files might contain an event identifier, event probability, and modeled loss amount for that event for the selected portfolio of exposures. Given such output files for a portfolio before and after the addition of a new account, one could calculate the before and after portfolio variance and standard deviation. The difference will be called the *marginal impact* of that new account on the portfolio variance or standard deviation.[1]

Two of the more well known risk load methods from the CAS literature—what shall be called "Marginal Surplus" (MS) from Kreps [3] and "Marginal Variance" (MV) from Meyers [6]—use the marginal change in portfolio standard deviation (variance) due to the addition of a new account to calculate the risk load for that new account. However, problems arise when these marginal methods are used to calculate risk loads for the renewal of accounts in a portfolio. These problems can be traced to the *order dependency* of the marginal risk load methods.

Order dependency is a perplexing issue. Many marginal risk load methods—whether based on variance, standard deviation, or even a selected percentile of the loss distribution—suffer from it. It is also not just an actuarial issue; even the financial community struggles with it. "Value at Risk" (VAR) is an attempt by investment firms to capture their risk in a single number. It is a selected percentile of the return distribution (e.g., 95th) for a portfolio of financial instruments over a selected time frame (e.g., 30 days). VAR can be calculated for the entire portfolio or for a desired subset (e.g., asset class). But so-called "marginal"

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[1]The variance and standard deviation are "between account" and "between event," and ignore any parameter uncertainty associated with the modeled loss amount for a given event and account.

or "component" VAR has, to this point, eluded satisfactory solution in the finance community precisely because of what will be termed *renewal additivity*. Finance professionals charged with assessing how much VAR a certain financial instrument or asset class contributes to the total VAR are dealing with the same unresolved order dependency issue. As the finance and insurance worlds blend more and more, perhaps actuaries will combine forces with quantitative analysts and Certified Financial Analysts (CFAs) to determine a solution.

The remainder of this paper is organized as follows. Section 2 describes the basic characteristics of a catastrophe occurrence size-of-loss distribution. Sections 3 and 4 describe the application of the MV and MS methods to a simplified occurrence size-of-loss distribution. Sections 5 and 6 calculate risk loads both in assembling or building up a portfolio of risks and in subsequently renewing that portfolio. Section 7 discusses the differences between build-up and renewal results.

Section 8 introduces a new concept to the theory of property catastrophe risk loads—renewal additivity. However, the concept is not new to the field of game theory. Section 9 introduces game theory concepts underlying a new approach. Section 10 extends and generalizes the effect of the new approach to sharing of covariance between accounts. Section 11 concludes by applying the new approach to some examples.

2. THE CATASTROPHE OCCURRENCE SIZE-OF-LOSS DISTRIBUTION

For demonstration purposes throughout the paper, a simplified version of an occurrence size-of-loss distribution will be used. It captures the essence of typical catastrophe modeling software output, while keeping the examples understandable.[2]

[2]In particular, only single event or occurrence size-of-loss distributions will be considered. Many models also produce multi-event or aggregate loss distributions. Occurrence

A modeled event denoted by identifier i is considered an independent Poisson process with occurrence rate[3] λ_i. To simplify the mathematics, following Meyers [6], the binomial approximation with probability of occurrence p_i [where $\lambda_i = -\ln(1-p_i)$] will be employed. This is a satisfactory approximation for small[4] λ_i.

For an individual account or portfolio of accounts, the model produces a modeled loss for each event L_i. A table containing the event identifiers i, the event probabilities p_i and modeled losses L_i will be referred to as an "occurrence size-of-loss distribution."

From Meyers [6], the formulas for expected loss and variance are

$$\mathrm{E}(L) = \Sigma_i[L_i \times p_i], \qquad \text{and} \tag{2.1}$$

$$\mathrm{Var}(L) = \Sigma_i[L_i^2 \times p_i \times (1-p_i)], \tag{2.2}$$

where Σ_i = sum over all events.

The formula for covariance of an existing portfolio L (with losses L_i) and a new account n (with losses n_i) is

$$\mathrm{Cov}(L,n) = \Sigma_i[L_i \times n_i \times p_i \times (1-p_i)]. \tag{2.3}$$

Note that $\mathrm{Cov}(L,n)$ is always greater than zero when each of L_i, n_i, p_i, and $(1-p_i)$ are greater than zero.

The total variance of the combined portfolio $(L+n)$ is then

$$\mathrm{Var}(L+n) = \mathrm{Var}(L) + \mathrm{Var}(n) + 2\,\mathrm{Cov}(L,n). \tag{2.4}$$

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size-of-loss distributions reflect only the *largest* event that occurs in a given year. Aggregate loss distributions reflect the sum of losses for all events in a given year. Clearly, the aggregate distribution would provide a more complete picture, but for purposes of the exposition here, the occurrence distribution works well and the formulas are substantially less complex.

[3]This implies that the loss for a given event and account is fixed and known.

[4]An event with a probability of 0.001 (typical of the more severe modeled events) would have $\lambda = 0.0010005$.

3. THE MARGINAL SURPLUS (MS) METHOD

This is a translation of the method described in Rodney Kreps's paper, "Reinsurer Risk Loads from Marginal Surplus Requirements" [3] to property catastrophe calculations.

Consider:

L_0 = losses from a portfolio before a new account is added,

L_1 = losses from a portfolio after a new account is added,

S_0 = Standard deviation of L_0,

S_1 = Standard deviation of L_1,

R_0 = Return on the portfolio before new account is added, and

R_1 = Return on the portfolio after new account is added.

Borrowing from Mr. Kreps, assume that needed surplus, V, is given by[5]

$$V = z \times \text{standard deviation of loss} - \text{expected return}, \quad (3.1)$$

where z is, to cite Mr. Kreps [3, p. 197], "a distribution percentage point corresponding to the acceptable probability that the actual result will require even more surplus than allocated." Then

$$\begin{aligned} V_0 &= z \times S_0 - R_0, \qquad \text{and} \\ V_1 &= z \times S_1 - R_1. \end{aligned} \quad (3.2)$$

The difference in returns $R_1 - R_0 = r$, the risk load charged to the new account. The marginal surplus requirement is then

$$V_1 - V_0 = z \times [S_1 - S_0] - r. \quad (3.3)$$

Based on the required return, y, on that marginal surplus (which is based on management goals, market forces and risk appetite),

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[5]Mr. Kreps sets needed surplus equal to $z \times$ standard deviation of return − expected return. If one assumes premiums and expenses are invariant, then Var(Return) = Var($P - E - L$) = Var(L).

the MS risk load would be

$$r = [yz/(1+y)][S_1 - S_0]. \quad (3.4)$$

4. THE MARGINAL VARIANCE (MV) METHOD

The Marginal Variance Method is based on Glenn Meyers's paper, "The Competitive Market Equilibrium Risk Load Formula for Catastrophe Ratemaking" [6].

For an existing portfolio L and a new account n, the MV risk load r would be

$$\begin{aligned} r &= \lambda \times \text{Marginal Variance of adding } n \text{ to } L \\ &= \lambda \times [\mathrm{Var}(n) + 2\,\mathrm{Cov}(L,n)], \end{aligned} \quad (4.1)$$

where λ is a multiplier similar to $yz/(1+y)$ from the MS method although dimensioned to apply to variance rather than standard deviation.[6]

5. BUILDING UP A PORTFOLIO OF TWO ACCOUNTS

Exhibit 1 shows the occurrence size-of-loss distribution and risk load calculations for building up (assembling) a portfolio of two accounts, X and Y. It is assumed X is written first and is the only risk in the portfolio until Y is written.

5.1. MS Method

Pertinent values from Exhibit 1 for the Marginal Surplus method are summarized in Table 1.

Item 1 is the change in portfolio standard deviation from adding each account, or *marginal* standard deviation.

[6]Mr. Meyers develops a variance-based risk load multiplier by converting a standard deviation-based multiplier using the following formula [6, p. 573]: λ = (Rate of Return $\times$ Std Dev Mult^2)/(2 $\times$ Avg Capital of Competitors).

TABLE 1

BUILDING UP X AND Y: MARGINAL SURPLUS METHOD

	Account X	Account Y	Account X +Account Y	Portfolio $(X+Y)$
(1) Change in Standard Deviation	$4,429.00	$356.00	$4,785.00	$4,785.00
(2) Risk Load Multiplier	0.33	0.33	—	0.33
(3) Risk Load = (1) × (2)	$1,461.71	$117.43	$1,579.14	$1,579.14

Item 2 is the Risk Load multiplier of 0.33. Using Kreps's formula, a return on marginal surplus y of 20% and a standard normal multiplier z of 2.0 (2 standard deviations, corresponding to a cumulative non-exceedance probability of 97.725%) would produce a risk load multiplier of

$$yz/(1+y) = 0.20 \times 2/1.20 = 0.33 \text{ (rounded).} \qquad (5.1)$$

Item 3 is the Risk Load, the product of Items 1 and 2.

Since X is the first account, the marginal standard deviation from adding X equals the standard deviation of X, Std Dev (X) = \$4,429. This gives a risk load of \$1,461.71.

The marginal standard deviation from writing Y equals Std Dev $(X+Y)$ − Std Dev (X) or \$356, implying a risk load of \$117.43.

The sum of these two risk loads $X+Y$ is \$1,461.71 + \$117.43 = \$1,579.14. This equals the risk load that this method would calculate for the combined account $(X+Y)$.

5.2. *MV Method*

Pertinent values from Exhibit 1 for the Marginal Variance method are summarized in Table 2.

TABLE 2

BUILDING UP X AND Y: MARGINAL VARIANCE METHOD

	Account X	Account Y	Account X +Account Y	Portfolio $(X+Y)$
(1) Change in Variance	19,619,900	3,279,059	22,898,959	22,898,959
(2) Risk Load Multiplier	0.000069	0.000069	—	0.000069
(3) Risk Load = (1) × (2)	\$1,353.02	\$226.13	\$1,579.14	\$1,579.14

Item 1 is the change in portfolio variance from adding each account, or *marginal* variance.

Item 2 is the Variance Risk Load multiplier λ of 0.000069. To simplify comparisons between the two methods (recognizing the difficulty of selecting a MV-based multiplier[7]), the MS multiplier was converted to a MV basis by dividing by Std Dev $(X+Y)$:

$$\lambda = 0.33/4{,}785 = 0.000069. \tag{5.2}$$

This means the total risk load calculated for the portfolio by the two methods will be the same, although the individual risk loads for X and Y will differ between the methods.

Item 3 is the Risk Load, the product of Items 1 and 2.

Since X is the first account, the marginal variance from adding X equals the variance of X, Var(X) = \$19,619,900. This gives a risk load of \$1,353.02.

The marginal variance from writing Y equals Var($X+Y$) – Var(X), or \$3,279,059, implying a risk load of \$226.13.

The sum of these two risk loads is \$1,353.02 + \$226.13 = \$1,579.14. This equals the risk load which this method would calculate for the combined account $(X+Y)$.

[7] Mr. Meyers [6, p. 572] admits that in practice "it might be difficult for an insurer to obtain the (lambdas) of each of its competitors." He goes on to suggest an approximate method to arrive at a usable lambda based on required capital being "Z standard deviations of the total loss distribution" [6, p. 574].

TABLE 3

RENEWING X AND Y: MARGINAL SURPLUS METHOD

	Account X	Account Y	Account X +Account Y	Portfolio ($X+Y$)
(1) Change in Standard Deviation	$4,171.00	$356.00	$4,526.00	$4,785.00
(2) Risk Load Multiplier	0.33	0.33	—	0.33
(3) Risk Load = (1) × (2)	$1,376.27	$117.43	$1,493.70	$1,579.14
(4) Build-Up Risk Load	$1,461.71	$117.43	$1,579.14	$1,579.14
(5) Difference	($85.45)	$0.00	($85.45)	$0.00

6. RENEWING THE PORTFOLIO OF TWO ACCOUNTS

Exhibit 2 shows the natural extension of the build-up scenario—renewal of the two accounts, in what could be termed a "static" or "steady state" portfolio (one with no new entrants).

As for applying these methods in the renewal scenario, renewing policy X is assumed equivalent to adding X to a portfolio of Y; renewing Y is assumed equivalent to adding Y to a portfolio of X.

6.1. MS Method

Pertinent values from Exhibit 2 for the Marginal Surplus method are summarized in Table 3.

The marginal standard deviation for adding Y to X is $356.00, same as it was during build-up—see Section 5.1. The risk load of $117.43 is also the same.

However, adding X to Y gives a marginal standard deviation of Std Dev $(X+Y)$ – Std Dev (Y) = $4,171.00. This gives a risk load for X of $1,376.27, which is $85.45 less than $1,461.71, the risk load for X calculated in Section 5.1.

TABLE 4

RENEWING X AND Y: MARGINAL VARIANCE METHOD

	Account X	Account Y	Account X +Account Y	Portfolio $(X+Y)$
(1) Change in Variance	22,521,000	3,279,059	25,800,059	22,898,959
(2) Risk Load Multiplier	0.000069	0.000069	—	0.000069
(3) Risk Load = (1) × (2)	\$1,553.08	\$226.13	\$1,779.21	\$1,579.14
(4) Build-Up Risk Load	\$1,353.02	\$226.13	\$1,579.14	\$1,579.14
(5) Difference	\$200.06	\$0.00	\$200.06	\$0.00

The sum of these two risk loads in Table 3 is \$1,376.27 + \$117.43 = \$1,493.70. This is also \$85.45 less than the total risk load from Section 5.1.

6.2. MV Method

Pertinent values from Exhibit 2 for the Marginal Variance method are summarized in Table 4.

The marginal variance for adding Y to X is 3,279,059, same as it was during build-up—see Section 5.2. The risk load of \$226.13 is also the same.

However, adding X to Y gives a marginal variance of $\mathrm{Var}(X+Y) - \mathrm{Var}(Y)$, or 22,521,000. The risk load is now \$1,553.08, which is \$200.06 more than the \$1,353.02 calculated in Section 5.2.

The sum of these two risk loads is \$1,553.08 + \$226.13 = \$1,779.21. This is also \$200.06 more than the total risk load from Section 5.2.

7. EXPLORING THE DIFFERENCES BETWEEN NEW AND RENEWAL

Why are the total Renewal risk loads different from the total Build-Up risk loads?

In Section 5.1 (Build-Up), the marginal standard deviation for X, ΔStd Dev(X), was

$$\begin{aligned}\Delta\,\text{Std Dev}(X) &= \text{Std Dev }(X)\\ &= \sqrt{\Sigma_i[X_i^2 \times p_i \times (1-p_i)]}\,, \qquad (7.1)\end{aligned}$$

where X_i = modeled losses for X for event i, while in Section 6.1 (Renewal), the marginal standard deviation was

$$\begin{aligned}\Delta\,\text{Std Dev}(X) &= \text{Std Dev}(X+Y) - \text{Std Dev}(Y)\\ &= \sqrt{\Sigma_i[(X_i+Y_i)^2 \times p_i \times (1-p_i)]}\\ &\quad - \sqrt{\Sigma_i[Y_i^2 \times p_i \times (1-p_i)]}\,. \qquad (7.2)\end{aligned}$$

For positive Y_i, this value is less than Std Dev (X). Therefore, one *would* expect the Renewal risk load to be less than the Build-Up.[8]

Unfortunately, when the MS method is applied in the renewal of all the accounts in a portfolio, the sum of the individual risk loads will be less than the total portfolio standard deviation times the multiplier. This is because the sum of the marginal standard deviations (found by taking the difference in portfolio standard deviation with and without each account in the portfolio) is less than the total portfolio standard deviation.[9] Please recall that the square root operator is *sub-additive*: the square root of a sum is less than the sum of the square roots.[10]

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[8]For example, assume Var(X) = 9, Var(Y) = 4, Cov(X,Y) = 1.5; then

$$\Delta\,\text{Std Dev}(X) = \sqrt{\text{Var}(X)} = \sqrt{9} = 3, \qquad \text{for} \quad X \text{ alone},$$

$$\Delta\,\text{Std Dev}(X) = \sqrt{9+4+2\times 1.5} - \sqrt{4} = 4 - 2 = 2 < 3, \qquad \text{for} \quad X \text{ added to } Y.$$

[9]The same issue is raised in Mr. Gogol's discussion. He suggests correcting for this sub-additivity by using a weighted average of the contract's own standard deviation and its last-in marginal standard deviation. The weight is chosen so the sum of these redefined marginal impacts equals the total portfolio standard deviation [2, p. 363].

[10]For example, $\sqrt{9+16} < \sqrt{9} + \sqrt{16}$.

What about marginal variance? In Section 5.2 (Build-Up), the marginal variance $\Delta\,\mathrm{Var}(X)$ was

$$\begin{aligned}\Delta\,\mathrm{Var}(X) &= \mathrm{Var}(X)\\ &= \Sigma_i[X_i^2 \times p_i \times (1-p_i)],\end{aligned} \tag{7.3}$$

while in Section 6.2 (Renewal) the marginal variance was

$$\begin{aligned}\Delta\,\mathrm{Var}(X) &= \mathrm{Var}(X+Y) - \mathrm{Var}(Y)\\ &= [\mathrm{Var}(X) + 2\,\mathrm{Cov}(X,Y) + \mathrm{Var}(Y)] - \mathrm{Var}(Y)\\ &= \mathrm{Var}(X) + 2\,\mathrm{Cov}(X,Y)\\ &> \mathrm{Var}(X).\end{aligned} \tag{7.4}$$

Since $2\,\mathrm{Cov}(X,Y)$ is greater than zero, one would expect the Renewal risk load to be greater than the Build-Up.

However, when the MV method is applied in the renewal of all the accounts in a portfolio, the sum of the individual risk loads will be more than the total portfolio variance times the multiplier. This is because the sum of the marginal variances (found by taking the difference in portfolio variance with and without each account in the portfolio) is greater than the total portfolio variance. The covariance between any two risks in the portfolio is double counted: when each account renews, it is allocated the full amount of its shared covariance with all the other accounts.

8. A NEW CONCEPT: RENEWAL ADDITIVITY

The renewal scenarios point out that these two methods are not what I call *renewal additive*, defined as follows:

> For a given portfolio of accounts, a risk load method is *renewal additive* if the sum of the renewal risk loads calculated for each account equals the risk load calculated when the entire portfolio is treated as a single account.

Neither the MS nor the MV method is renewal additive: MS because the square root operator is sub-additive; MV because the covariance is double counted. So why should renewal additivity matter? Consider what happens when either of these non-renewal -additive methods is used to renew the portfolio. The MV method would result in quoted renewal premiums the sum of whose risk loads would be greater than the required total risk load of ($\lambda \times$ total portfolio variance). One would in essence overcharge every account. The opposite is true for the MS case, where one would undercharge every account.

In order for the MS or MV methods to be renewal additive, one must assume an entry order for the accounts. Since the marginal impacts depend on the size of the existing portfolio, the entry order selected for an account could determine whether it is written or declined.

Renewal additivity reduces the renewal risk load calculation to an allocation of the total portfolio amount back to the individual accounts. An objective, systematic allocation methodology for renewals would be desirable. Examples of many such allocation methodologies can be found in the field of game theory.

9. A NEW APPROACH FROM GAME THEORY

Two ASTIN papers by Jean Lemaire—"An Application of Game Theory: Cost Allocation" [4], and "Cooperative Game Theory and Its Insurance Applications" [5]—focus on general insurance applications of game theory. Lemaire also provides an extensive list of real world applications of game theory [4, p. 77], including tax allocation among operating divisions of McDonnell-Douglas, maintenance costs of the Houston medical library, financing of large water resource development projects in Tennessee, construction costs of multi-purpose reservoirs in the U.S., and landing fees at Birmingham airport. Consider this example from [5]:

> "The Treasurer of ASTIN (player 1) wishes to invest the amount of 1,800,000 Belgian francs on a short term

(3 months) basis. In Belgium, the annual interest rate is a function of the sum invested.

Deposit (in Belgian Francs)	Annual Interest Rate
0–1,000,000	7.75%
1,000,000–3,000,000	10.25%
3,000,000–5,000,000	12.00%

The ASTIN Treasurer contacts the Treasurers of the International Actuarial Association (I.A.A.–player 2) and of the Brussels Association of Actuaries (A.A.Br.–player 3). I.A.A. agrees to deposit 900,000 francs in the common fund, A.A.Br. 300,000 francs. Hence the 3-million mark is reached and the interest rate will be 12%. How should the interests be split among the three associations?" [5, p. 18]

Games such as this are referred to as "cooperative games with transferable utilities." They typically feature:

1. participants (players) that have some benefits (or costs) to share (political power, savings, or money),
2. the opportunity to share benefits (costs) results from cooperation of all participants or a sub-group of participants,
3. freedom for players to engage in negotiations, bargaining, and coalition formation, and
4. conflicting player objectives: each wants to secure the largest part of the benefits (smallest share of the costs) for himself. (See [5, p. 20].)

Cooperative games can be used as models for situations where participants must share or allocate an amount of money. Players may want to maximize or minimize their allocation depending on the nature of the problem. If the group is deciding who pays what share of the total tax bill, players will want to minimize their share. If on the other hand the group is deciding how to split a pot of bonus money, players will want to maximize their share.

The total amount to be allocated is determined by the *characteristic function*, which associates a real number $v(S)$ to each coalition (group) S of players. It can be either sub-additive or super-additive, defined as follows:

Sub-Additive

$v(S) + v(T) > v(S \text{ union } T)$ for every disjoint S and T.

Super-Additive

$v(S) + v(T) < v(S \text{ union } T)$ for every disjoint S and T.

In the actuarial association example above, the characteristic function would be the money earned by each coalition (combination) of associations. It is an example of a super-additive characteristic function where the players seek to maximize their allocation. An example of a sub-additive characteristic function would be the insurance premium for a risk purchasing group: the sum of the individual members' insurance premiums is more than the insurance premium for the risk purchasing group as a whole. These players would seek to minimize their allocations, since they want to be charged the lowest premium. (Equivalently, these players want to maximize their savings as a result of joining the group—savings being the difference between their allocation from the group and their stand-alone premium.)

Allocation Rules

A player's marginal impact depends on its entry order. In the example, the "allocation [to the three associations] of course depends on the order of formation of the grand coalition" [5, p. 27]. In the interests of fairness and stability, a new member should probably receive an allocation amount somewhere between its stand-alone value and its full marginal impact on the coalition characteristic function—but where in between? How much is fair? These questions must be answered simultaneously for all the players, balancing questions of stability, incentives to split from the group, bargaining power, and marginal impact to the coalition characteristic function value.

To help answer the allocation question, game theory has developed a set of standards or rules for allocations. First, legitimate allocation methods must be additive—the sum of the players' allocations must equal the total amount to be allocated. The MV and MS methods are not (renewal) additive: they either allocate too much (MV) or too little (MS) in the renewal case.

Second, a coalition should be stable, which roughly translates to fair. There must not be incentives for either a single player or a sub-group of players to split from the group and form a faction. These "rules of fairness" are referred to as the conditions of individual and collective rationality (see [4, p. 66–68]). Individual rationality means a player is no worse off for having joined the coalition. Collective rationality means no subgroup would be better off on its own.

These rules can be formalized into a set of acceptable ranges of allocations for each player. This set defines what is known as the *core* of the game. It consists of all allocations satisfying these fairness and stability conditions.

Consider the Brussels Association of Actuaries (A.A.Br.–player 3) from the example. They have 300,000 francs, and on their own could earn 7.75%. If they join as the third player, they will push the coalition rate of return from 10.25% to 12.00%. How much should they earn? Certainly not less than 7.75%—it would not be individually rational for them to join. Conversely, they should not earn so much that players 1 and 2 end up earning less than 10.25%—that would not be collectively rational for them. In that case, players 1 and 2 would be better off forming their own faction. Similar exercises can be performed from the perspective of players 1 and 2. The resulting set of acceptable allocations defines the boundaries of the core (see [5, p. 26]).

TABLE 5

TRANSLATION FROM GAME THEORY TO PROPERTY CAT RISK LOAD

Game Theory	Property Cat Risk Load
Player	Account
Coalition	Portfolio
Characteristic Function	Portfolio Variance or Standard Deviation

Translating to Property Cat Risk Load

Given this brief introduction, a reasonable first attempt at translating from the game theory context might be as shown in Table 5.

Because of the covariance component, portfolio variance is a super-additive characteristic function: the variance of a portfolio is greater than the sum of the individual account variances. Standard deviation, on the other hand, is a sub-additive characteristic function because of the sub-additivity of the square root operator: the standard deviation of a portfolio is less than the sum of the individual account standard deviations.

This means, from the game theory perspective at least, that the choice between variance and standard deviation is material. It determines whether the characteristic function is sub-additive or super-additive. This is a fundamental paradox of the game theory translation of the risk load problem, and will require further research to resolve.

Setting aside this paradox for the moment, however, the risk load problem fits remarkably well into the game theory framework. The "players" want to minimize their allocations of the portfolio total risk load. The allocation should fairly and objectively assign risk loads to accounts in proportion to their contribution to the total. Using the current definition of marginal impact of a renewal account, however, an entry order would have to be assumed in order to make the allocation additive. The results of that allocation would be heavily dependent on the selected order, however.

How can one choose the entry order of a renewal? A well known allocation method from game theory may provide the answer.

The Shapley Value

The Shapley value (named for Lloyd Shapley, one of the early leaders of the game theory field) is an allocation method that is:

1. additive,
2. at the centroid of the core, and
3. order independent.

It equals the average of the marginal impacts taken over all possible entrance permutations—the different orders in which a new member could have been added to the coalition[11] (i.e., a new account could have been added to a portfolio).

For example, consider a portfolio of accounts A and B to which a new account C is added. Shown in Table 6 are the marginal variances for adding C in the 6 possible entrance permutations ("*ABC*" in Column 1 below means A enters first, then B, then C).

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[11]Lemaire provides this more complete definition of the Shapley value [5, p. 29]: "The Shapley value can be *interpreted* as the mathematical expectation of the admission value, when all orders of formation of the grand coalition are equiprobable. In computing the value, one can assume, for convenience, that all players enter the grand coalition one by one, each of them receiving the entire benefits he brings to the coalition formed just before him. All orders of formation of N are considered and intervene with the same weight $1/n!$ in the computation. The combinatorial coefficient results from the fact that there are $(s-1)!(n-s)!$ ways for a player to be the last to enter coalition S: the $(s-1)$ other players of S and the $(n-s)$ players of $N \backslash S$ (those players in N which are not in S) can be permuted without affecting i's position."

TABLE 6

ENTRY PERMUTATIONS FOR ACCOUNT C

(1) Permutation	(2) C Enters …	(3) Marginal Variance
ABC	After A & B	$\mathrm{Var}(C) + 2 \times \mathrm{Cov}(C,A) + 2 \times \mathrm{Cov}(C,B)$
ACB	After A	$\mathrm{Var}(C) + 2 \times \mathrm{Cov}(C,A)$
BAC	After B & A	$\mathrm{Var}(C) + 2 \times \mathrm{Cov}(C,A) + 2 \times \mathrm{Cov}(C,B)$
BCA	After B	$\mathrm{Var}(C) + 2 \times \mathrm{Cov}(C,B)$
CAB	First	$\mathrm{Var}(C)$
CBA	First	$\mathrm{Var}(C)$

The Shapley value is the straight average of Column 3, Marginal Variance, over the six permutations:

$$\begin{aligned}\text{Shapley Value} &= [\text{Sum(Column 3)}]/6\\ &= [6\,\mathrm{Var}(C) + 6\,\mathrm{Cov}(C,A) + 6\,\mathrm{Cov}(C,B)]/6\\ &= \mathrm{Var}(C) + \mathrm{Cov}(C,A) + \mathrm{Cov}(C,B). \qquad (9.1)\end{aligned}$$

Or, to generalize, given

$$\begin{aligned}L &= \text{existing portfolio} \qquad \text{and}\\ n &= \text{new account}, \qquad (9.2)\end{aligned}$$

$$\text{Shapley Value} = \mathrm{Var}(n) + \mathrm{Cov}(L,n).$$

Before seeing this result, there might have been concerns about the practicality of this approach—how much computational time might be required to calculate all the possible entrance permutations for a portfolio of thousands of accounts? This simple reduction formula eliminates those concerns. The Shapley value is as simple to calculate as the marginal variance.

Comparing the Shapley value to the marginal variance formula from Section 4:

$$\text{Marginal Variance} = \mathrm{Var}(n) + 2\,\mathrm{Cov}(L,n), \qquad (9.3)$$

whereas the Shapley value only takes 1 times the covariance of the new account and the existing portfolio.

One can also calculate the Shapley value under the marginal standard deviation method. However, due to the complex nature of the mathematics—differences of square roots of sums of products—no simplifying reduction formula was immediately apparent.[12]

Therefore, the remainder of the paper will focus on the MV method and the variance-based Shapley value. Life will be much easier (mathematically) working with the variances, and very little is lost by choosing variance. Citing Mr. Bault [1, p. 82], from a risk load perspective, "both (variance and standard deviation) are simply special cases of a unifying covariance framework." In fact, Bault goes on to suggest "in most cases, the 'correct' answer is a marginal risk approach that incorporates covariance."[13]

10. SHARING THE COVARIANCE

The risk load question, framed in a game-theoretical light, has now become:

> How do accounts share their mutual covariance for purposes of calculating risk load?

The Shapley method answers, "Accounts split their mutual covariance equally." At first glance this appears reasonable, but consider the following example.

Assume two accounts, E and F. F has 100 times the losses of E for each event. Their total shared covariance is

$$\begin{aligned} 2\mathrm{Cov}(E,F) &= 2\Sigma_i[E_i \times F_i \times p_i \times (1-p_i)] \\ &= 2\Sigma_i[E_i \times 100E_i \times p_i \times (1-p_i)]. \end{aligned} \quad (10.1)$$

[12] Those wishing to employ standard deviation can use approximate methods to calculate the Shapley value. Two approaches suggested by John Major are (i) taking the average of marginal value if first in and last in; and (ii) employing Monte Carlo simulation to sample a subset of the possible entrance permutations, presumably large enough to achieve satisfactory convergence while being much more computationally efficient.

[13] Kreps also incorporates covariance in his "Reluctance" R [3, p. 198], which has the formula $R = [yz/(1+y)]/(2SC+\sigma)/(S'+S)$, where C is the correlation of the contract with the existing book. The Risk Load is then equal to $R\sigma$.

The Shapley value would equally divide this total covariance between E and F, even though their relative contributions to the total are clearly not equal. There is no question that E should be assessed some share of the covariance. The issue is whether there is a more equitable share than simply half.

One could develop a generalized covariance sharing (GCS) method which uses a weight $W_i^X(X,Y)$ to determine an account X's share of the mutual covariance between itself and another account Y for event i:

$$\text{CovShare}_i^X(X,Y) = W_i^X(X,Y) \times 2 \times X_i \times Y_i \times p_i \times (1 - p_i). \tag{10.2}$$

Then Y's share of that mutual covariance would simply be

$$\text{CovShare}_i^Y(X,Y) = [1 - W_i^X(X,Y)] \times 2 \times X_i \times Y_i \times p_i \times (1 - p_i). \tag{10.3}$$

The total covariance share allocation for account X over all events would be:

$$\text{CovShare}_{\text{Tot}}^X = \Sigma_Y \Sigma_i [\text{CovShare}_i^X(X,Y)], \tag{10.4}$$

where Σ_Y = sum over every other account Y in the portfolio.

The Shapley method is in fact an example of the generalized covariance sharing method with $W_i^X(X,Y) = 50\%$ for all X, Y and i.

Returning to the example with E and F, one could develop an example of a weighting scheme that assigns the shared covariance by event to each account in proportion to their loss for that event. $W_i^E(E,F)$, account E's share of the mutual covariance between itself and account F for event i, equals

$$\begin{aligned} W_i^E(E,F) &= [E_i/(E_i + F_i)] \\ &= [E_i/(E_i + 100E_i)] = (1/101) \\ &= \text{roughly } 1\% \text{ of their mutual covariance for event } i. \end{aligned}$$

This shall be called the "Covariance Share" (CS) method.

TABLE 7

BUILDING UP X AND Y: SHAPLEY VALUE METHOD

	Account X	Account Y	Account X +Account Y	Portfolio $(X+Y)$
(1) Change in Variance	19,619,900	1,828,509	21,448,409	22,898,959
(2) Risk Load Multiplier	0.000069	0.000069	—	0.000069
(3) Risk Load = (1) × (2)	\$1,353.02	\$126.10	\$1,479.11	\$1,579.14

TABLE 8

BUILDING UP X AND Y: COVARIANCE SHARE METHOD

	Account X	Account Y	Account X +Account Y	Portfolio $(X+Y)$
(1) Change in Variance	19,619,000	950,658	20,570,558	22,898,959
(2) Risk Load Multiplier	0.000069	0.000069	—	0.000069
(3) Risk Load = (1) × (2)	\$1,353.02	\$65.56	\$1,418.57	\$1,579.14

11. APPLYING THE SHAPLEY AND CS METHODS TO THE EXAMPLE

Consider the Shapley and CS methods applied to the two Account example for both Build-Up and Renewal.

11.1. Portfolio Build-up

Exhibit 3 shows the Build-Up of accounts X and Y from Section 5, but for the Shapley and CS methods. Pertinent values for the Shapley value are summarized in Table 7.

Pertinent values for the Covariance Share are summarized in Table 8.

Both Shapley and CS produce the same risk load for X as the MV method on Build-Up: \$1,353.02. This is because there is no covariance to share: X is the entire portfolio at this point. How-

TABLE 9

COMPARISON OF BUILD-UP RISK LOADS FOR ACCOUNT Y

Marginal Variance (MV)—Section 5.2	\$226.13
Shapley Value	\$126.10
Difference from MV	**\$100.03**
Covariance Share (CS)	\$ 65.56
Difference from MV	**\$160.57**

TABLE 10

RENEWING X AND Y: SHAPLEY VALUE METHOD

	Account X	Account Y	Account X +Account Y	Portfolio ($X+Y$)
(1) Change in Variance	21,070,450	1,828,509	22,898,959	22,898,959
(2) Risk Load Multiplier	0.000069	0.000069	—	0.000069
(3) Risk Load = (1) × (2)	\$1,453.05	\$126.10	\$1,579.14	\$1,579.14
(4) Build-Up Risk Load	\$1,353.02	\$126.10	\$1,479.11	\$1,579.14
(5) Difference	\$100.03	\$0	\$100.03	\$0

ever, compare the results of the three variance-based methods for account Y (see Table 9).

Compared to MV, which charges account Y for the full increase in variance $\mathrm{Var}(Y)+2\mathrm{Cov}(X,Y)$, the Shapley method only charges Y for $\mathrm{Var}(Y)+\mathrm{Cov}(X,Y)$. The same can be said for the CS method, although the share of the mutual covariance depends on each account's relative contribution by event, weighted and summed over all events. Now consider what happens to that *difference from MV* upon renewal.

11.2. Renewal

Exhibit 4 shows the renewal of X and Y for the Shapley and CS methods. Pertinent values for the Shapley method are summarized in Table 10.

TABLE 11

RENEWING X AND Y: COVARIANCE SHARE METHOD

	Account X	Account Y	Account X +Account Y	Portfolio $(X+Y)$
(1) Change in Variance	21,948,301	950,658	22,898,959	22,898,959
(2) Risk Load Multiplier	0.000069	0.000069	—	0.000069
(3) Risk Load = (1) × (2)	\$1,513.59	\$65.56	\$1,579.14	\$1,579.14
(4) Build-Up Risk Load	\$1,353.02	\$65.56	\$1,418.57	\$1,579.14
(5) Difference	\$160.57	\$0	\$160.57	\$0

TABLE 12

COMPARISON OF BUILD-UP AND RENEWAL RISK LOADS FOR ACCOUNT X

	Shapley	**Cov Share**
Renewal	\$1,453.05	\$1,513.59
Build-Up	\$1,353.02	\$1,353.02
Additional Renewal Risk Load over Build-Up	**\$100.03**	**\$160.57**
Difference from MV	**\$100.03**	**\$160.57**

Pertinent values for the Covariance Share method are summarized in Table 11.

With both the Shapley and CS methods, the sum of the risk loads for Account X and Account Y equals the risk load for Account $(X+Y)$, namely \$1,579.14. This means that both new methods are renewal additive.

To see what happened to the *difference from MV*, compare the risk loads calculated at Renewal for X with those at Build-Up (see Table 12).

The difference from MV during Build-Up is simply the portion of X's risk load attributable to its share of covariance with Y. It was missed during Build-Up because it was unknown—account Y had not been written.

12. CONCLUSION

This paper introduces two new approaches to determination of renewal risk load that address concerns with renewal additivity and point out the issue of covariance sharing between accounts. The ideal solution in practice might involve using a marginal method for the pricing of new accounts, and a renewal additive method for renewals.

This paper also represents a first step in addressing the perplexing question of order dependency. As mentioned in the introduction, order dependency stretches beyond the confines of actuarial pricing to the finance community at large. It will likely take a joint effort between finance professionals and actuaries to reach a satisfactory solution.

Finally, this paper brings important information from game theory to the *Proceedings*. Game theory is a rich field for actuaries to find new ideas on cost allocation, fairness, and order dependency. Many sticky social issues (taxation, voting rights, utility costs) have been resolved using ideas from game theory. Further research can be done on several questions raised during the review of this paper, including the relative bargaining power of accounts, portfolio departure rules, lack of account information, and the unresolved paradox of the sub-additive MS characteristic function versus the super-additive MV characteristic function.

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EXHIBIT 1

BUILD UP PORTFOLIO OF TWO ACCOUNTS

			Loss for Account		
Event *i*	**Prob *p*(*i*)**	**1-*p*(*i*)**	***X***	***Y***	**Portfolio (*X* + *Y*)**
1	2.0%	98.0%	25,000	200	25,200
2	1.0%	99.0%	15,000	500	15,500
3	3.0%	97.0%	10,000	3,000	13,000
4	3.0%	97.0%	8,000	1,000	9,000
5	1.0%	99.0%	5,000	2,000	7,000
6	2.0%	98.0%	2,500	1,500	4,000

E (*L*)	1,290	179	1,469
Var (*L*)	**19,619,900**	**377,959**	**22,898,959**
Std Dev (*L*)	4,429	615	4,785

Covariances	***X***	***Y***
X	**19,619,900**	1,450,550
Y	1,450,550	**377,959**

		X	***Y***	***X* + *Y***
Change in Std Deviation		4,429	356	4,785
Risk Load (Std Dev)		1,461.71	117.43	1,579.14
Multiplier :	**0.33**	**Risk Load for *X* + *Y* :**		1,579.14

Change in Variance		19,619,900	**3,279,059**	22,898,959
Risk Load (Variance)		1,353.02	226.13	1,579.14
Multiplier :	**0.000069**	**Risk Load for *X* + *Y* :**		1,579.14

EXHIBIT 2

RENEW THE PORTFOLIO OF TWO ACCOUNTS

			Loss for Account		
Event *i*	**Prob *p*(*i*)**	**1-*p*(*i*)**	***X***	***Y***	**Portfolio (*X* + *Y*)**
1	2.0%	98.0%	25,000	200	25,200
2	1.0%	99.0%	15,000	500	15,500
3	3.0%	97.0%	10,000	3,000	13,000
4	3.0%	97.0%	8,000	1,000	9,000
5	1.0%	99.0%	5,000	2,000	7,000
6	2.0%	98.0%	2,500	1,500	4,000

E (*L*)	1,290	179	1,469
Var (*L*)	**19,619,900**	**377,959**	**22,898,959**
Std Dev (*L*)	4,429	615	4,785

Covariances	***X***	***Y***
X	**19,619,900**	1,450,550
Y	1,450,550	**377,959**

		X	***Y***	***X* + *Y***
Change in Std Deviation		4,171	356	4,526
Risk Load (Std Dev)		**1,376.27**	117.43	1,493.70
0.33	*Build Up Risk Load*	1,461.71	117.43	1,579.14
	Difference	**(85.45)**		**(85.45)**

Change in Variance		22,521,000	3,279,059	25,800,059
Risk Load (Variance)		**1,553.08**	226.13	1,779.21
0.000069	*Build Up Risk Load*	1,353.02	226.13	1,579.14
	Difference	**200.06**		**200.06**

EXHIBIT 3

BUILD UP A PORTFOLIO OF TWO ACCOUNTS—ALTERNATIVES

			Covariance Share $	
Event *i*	**Prob *p*(*i*)**	**1-*p*(*i*)**	***X***	***Y***
1	2.0%	98.0%	9,920,635	79,365
2	1.0%	99.0%	14,516,129	483,871
3	3.0%	97.0%	46,153,846	13,846,154
4	3.0%	97.0%	14,222,222	1,777,778
5	1.0%	99.0%	14,285,714	5,714,286
6	2.0%	98.0%	4,687,500	2,812,500

		Total
2,328,401	572,699	2,901,100

Chg in Variance		***X***	***Y***
If added 1st		19,619,900	377,959
If added 2nd	after 1		3,279,059
	after 2	22,521,000	
Average	*(Shapley Value)*	21,070,450	1,828,509

Shapley Value	19,619,900	1,828,509	21,448,409
Risk Load (Shapley)	**1,353.02**	**126.10**	**1,479.11**
0.000069			**1,579.14**
	Deferred Risk Load		**100.03**

Covariance Share	19,619,900	950,658	20,570,558
Risk Load (Cov Share)	**1,353.02**	**65.56**	**1,418.57**
0.000069			**1,579.14**
	Deferred Risk Load		**160.57**

EXHIBIT 4

Renew the Portfolio of Two Accounts—Alternatives

			Covariance Share $	
Event *i*	**Prob *p*(*i*)**	**1-*p*(*i*)**	***X***	***Y***
1	2.0%	98.0%	9,920,635	79,365
2	1.0%	99.0%	14,516,129	483,871
3	3.0%	97.0%	46,153,846	13,846,154
4	3.0%	97.0%	14,222,222	1,777,778
5	1.0%	99.0%	14,285,714	5,714,286
6	2.0%	98.0%	4,687,500	2,812,500

		Total
2,328,401	572,699	2,901,100

Chg in Variance		***X***	***Y***
If added 1st		19,619,900	377,959
If added 2nd	after 1		3,279,059
	after 2	22,521,000	
Average *(Shapley Value)*		21,070,450	1,828,509

Shapley Value	21,070,450	1,828,509	22,898,959
Risk Load (Shapley)	1,453.05	126.10	**1,579.14**
0.000069	**Risk Load for Portfolio (*X* + *Y*)**		**1,579.14**

Covariance Share	21,948,301	950,658	22,898,959
Risk Load (Cov Share)	1,513.59	65.56	**1,579.14**
0.000069	**Risk Load for Portfolio (*X* + *Y*)**		**1,579.14**

CHAPTER EIGHT
INSURANCE PROFITABILITY

By Charles L. McClenahan, FCAS, ASA, MAAA

INTRODUCTION

Measurement of profitability is to some extent, like beauty, in the eye of the beholder. The connotation of the word *profitability* is highly dependent upon who is assessing profitability and to what purpose. To investors and insurers, *profitability* has a golden ring to it. To policyholders of a stock insurer it sounds like *markup*, while to those insured by a mutual company it is neutral. Insurance regulators either encourage profitability, when concerned with solvency, or seek to curtail it, when regulating rates. The IRS seeks to inflate it and consumer groups seek to minimize it.

In most businesses there is a clear distinction between historical profitability, which within a given set of accounting rules and conventions is relatively well established, and prospective profitability. In the property-casualty insurance business, however, there is no such clear-cut demarcation. At the end of a year only about 40% of the incurred losses for that year will have been paid by the typical property-casualty insurer. It is several years before an insurer knows with relative certainty how much money it made or lost in a given period. When *history* depends upon the *future*, things have a tendency to become confusing.

The extent to which reported profits depend upon estimated liabilities for unpaid losses provides property-casualty insurers with some opportunity to manage reported results by strengthening or weakening loss reserves. Because deficient reserves must ultimately be strengthened and redundancies must ultimately be recognized, the interplay between current reserving decisions and the amortization of past reserving decisions adds an additional level of complexity to the problem of measuring property-casualty insurance profitability.

In this paper I will attempt to avoid staking out any position regarding the qualitative assessment of profitability. Hopefully both pro-profit readers and anti-profit readers will find my positions overwhelmingly convincing. Nor will I address the convolutions of potential reserve strengthening and weakening and the associated amortization of redundancies and deficiencies. For the sake of understanding, I will simply pretend that profitability is subject to consistent and accurate determination under a given set of accounting rules and conventions.

PROFIT V. RATE-OF-RETURN

It is important at the outset to distinguish between *profit* - the excess of revenues over expenditures - and *rate-of-return* - the ratio of profit to equity, assets, sales or some other base. Profit, no matter how uncertain, is a monetary value representing the reward to owners for putting their assets at risk and has an absolute meaning in the context of currency values. Rate-of-return is a measure of efficiency which has meaning only relative to alternative real or assumed rates-of-return.

Profit is important to investors and management as sources of dividends and growth. To insureds and regulators profits provide additional security against insolvency. Rate-of-return is important to a prospective investor as a means to compare alternative investments and to an economist as an assessment of economic efficacy. These are valid and useful functions and I do not wish to minimize their importance. But the arena in which property-casualty insurance company profitability measurement is most discussed is that of rate regulation, and this paper is written in the context of what I consider appropriate in a ratemaking or rate regulatory environment.

Since rate-of-return, however expressed, begins with profit in the numerator, it seems appropriate to begin with a discussion of the measurement of property and casualty insurance company profit.

PROFIT - RATEMAKING BASIS

While it has long been realized that the investment of policyholder-provided funds is a source of income to a property and casualty insurance company, it was not until the 1970s that such income actually constituted an important part of insurance company profit. Even today it is common to hear references to *underwriting profit*, while the investment counterpart is generally termed investment *income*, not investment *profit*. In Lewis E. David's' *Dictionary of Insurance* (Littlefield, Adams & Co., 1962) there is a definition for *Underwriting Profit* but not for *Profit*, *Investment Income*, or *Interest Income*. The International Risk Management Institute's *Glossary of Insurance and Risk Management Terms* (RCI Communications, Inc., 1980) includes both *Underwriting Profit* and *Investment Income* but continues the distinction between profit and income.

Common usage notwithstanding, there are few who would contend today that investment activities should be separate from underwriting activities in the measurement of insurance company profit. And were it not for rate regulation, statutory and GAAP accounting procedures would probably suffice for the vast majority of profit calculations. Rate regulation, however, has forced property and casualty insurers to make a somewhat artificial distinction between investment income arising from the investment of policyholder funds and that arising from the investment of shareholder funds. Even in the case of mutual companies which are owned by their policyholders, the distinction is necessitated by the fact that last year's policyholder-owners may not be this year's policyholder-insureds.

When an insured purchases a policy of insurance, and pays for it up front, he or she suffers what is known as an *opportunity cost* by virtue of paying out the premium funds in advance of losses and expenses actually being paid. In theory, the policyholder could have invested the funds in some alternative until they were actually needed by the insurer. Where insurance rates are regulated for excessiveness, it is appropriate that this opportunity cost be recognized.

The opportunity cost should be calculated based upon the cash flows associated with the line of business, and should reflect the fact that not all cash flows go through invested assets - some portion being required for the infrastructure of the insurer. The buildings and desks and computer software which were originally purchased with someone else's premium dollars are now dedicated to providing service to current policyholders and should be viewed as being purchased at the beginning of the policy period and sold at the end.

Most importantly, the calculation should be made at a risk-free rate of return. It must be understood that the insured has not purchased shares in a mutual fund. The existence of an opportunity cost does not give the policyholder a claim on some part of the actual earnings of the insurer. Should the insurer engage in speculative investments resulting in the loss of policyholder supplied funds, the company cannot assess the insureds to make up the shortfall. By the same token, investment income over and above risk-free yields should not be credited to the policyholders in the ratemaking process.

Finally, investment income on surplus should be excluded from the ratemaking process. Policyholders' surplus represents owners' equity which is placed at risk in order to provide the opportunity for reward. While it provides protection to policyholders and claimants, the surplus does not **belong** to them. In fact, the inclusion of investment income on surplus creates a situation in which an insurer with a large surplus relative to premium must charge lower rates than an otherwise equivalent insurer with less surplus. In other words, lower cost for more protection. This, in my opinion, does not represent equitable or reasonable rate regulation.

One final distinction needs to be made. Rate regulation is generally a prospective process, and the methods and procedures recommended herein are designed to be efficacious on a prospective basis. When applied retrospectively, as in the case of excess profits regulations, it must be remembered that a single year of experience is rarely sufficient to assess the true profitability of a line of property and casualty business. In the case of low-frequency, high-severity lines such as earthquake, it may require scores, or even hundreds, of years to determine average profit on a retrospective basis.

RATEMAKING BASIS - NUMERICAL EXAMPLES

Consider a property and casualty insurer which writes only private passenger automobile insurance with the following expectations:

TABLE 1

PRIVATE PASSENGER AUTOMOBILE ASSUMPTIONS

(THOUSANDS OF DOLLARS)

Premium		$100,000
Loss Ratio		0.65
Expense Ratio		0.35
Loss Payout		
	Year 1	0.25
	Year 2	0.35
	Year 3	0.20
	Year 4	0.12
	Year 5	0.08

For purposes of this example, no distinction is made between pure losses and loss adjustment expenses. Premiums are assumed to be paid at policy inception, expenses at mid-term and losses at the midpoint of each year. Assume further that the risk-free rate of return is 6% per year and that 100% of underwriting cash flows are invested.

Shown below are the assumed cash flows along with the present value of those flows at 6% per year. The indicated profit—that is, the 6% present value of the underwriting cash flows—is $7,776 or 7.78% of premium.

TABLE 2

PRIVATE PASSENGER AUTOMOBILE RESULTS

(THOUSANDS)

Time	Premium	Loss	Expense	Total Cash Flow	6.0% Present Value
0.0	$100,000			$100,000	$100,000
0.5		$(16,250)	$(35,000)	(51,250)	(49,778)
1.5		(22,750)		(22,750)	(20,846)
2.5		(13,000)		(13,000)	(11,238)
3.5		(7,800)		(7,800)	(6,361)
4.5		(5,200)		(5,200)	(4,001)
Total	$100,000	$(65,000)	$(35,000)		$7,776

It is imperative that it be understood what this represents. This is the *a priori* expected net present value of the underwriting cash flows. It reflects the opportunity cost expected to be suffered by the average policyholder for the risk-free income lost through the advance payment of funds not yet required for infrastructure, loss payment or expense payment.

It is equally important to understand what this does not represent. It is not the money expected to be earned by the insurer from writing private passenger automobile insurance for one year. The insurer should expect to earn something greater than the risk-free rate of return in exchange for taking the risk that losses and expenses may exceed expectations. Nor is it the expected profit arising to owners for the year as it excludes funds generated from the investment of retained earnings and other income.

Note that this methodology is independent of level of surplus, actual investment results and past underwriting experience. It can be equitably applied to all companies and it is firmly grounded in both the substance of the insurance transaction and fundamental economic realities.

RATE-OF-RETURN—THE APPROPRIATE DENOMINATOR

As the examples above indicate, while it is fairly easy to calculate the dollar value of the *a priori* expected net present value of the underwriting cash flows associated with a given book of business under a given set of assumptions, the dollar value itself is of little value to a rate regulator charged with the assessment of whether proposed rates are inadequate or excessive.

Now it is imperative that we understand that it is the *rates* which are being regulated, not the rates-of-return. I am unaware of any rating law which states that "rates-of-return must not be excessive ..." Rate regulatory attention focused upon rate-of-return must be within the context of determination of what might constitute a reasonable profit loading in the rates, not as an attempt to equalize rates-of-return across insurers.

Two candidates for the denominator seem to be common - sales and equity. Assets might be an appropriate denominator from the standpoint of measuring economic efficiency, but equity is clearly the favorite of those seeking to measure relative values of investments while sales is favored by those who view profit provisions in the context of insurance rates themselves.

RETURN ON EQUITY

While there is little doubt that equity is an appropriate basis against which to measure company-wide financial performance of a property and casualty insurer, as I see it there are two basic problems with return-on-equity as a basis for measuring rate-of-return in rate regulation.

The first problem with return on equity is that it forces the regulator to forgo rate equity for rate-of-return equity.

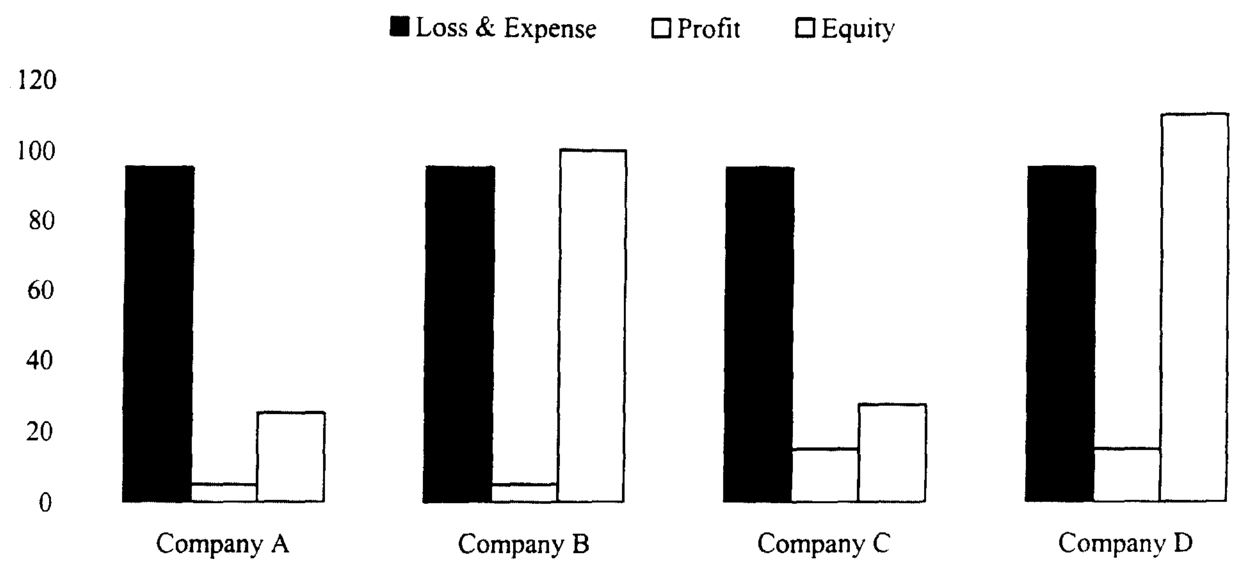


FIGURE 1. FOUR COMPANIES

Consider the example above. Here we have four companies, each writing the same coverage in the same market and providing the same level of service with an expected pure premium and expense component of $95. Companies A and B propose rates of $100 while companies C and D request approval of $110. Companies A and C are leveraged at a writings-to-surplus ratio of 4:1 while companies B and D are at 1:1.

The concept of rate equity would seem to require that companies A and B be treated identically as would C and D. But if we attempt to use equity as a base for rate-of-return this becomes a problem. Assume that the regulator has determined that a 15% return on equity is the appropriate benchmark for excessiveness. Our two highly-leveraged companies, A and C, project returns-on-equity of 20% and 55% respectively, while B and D are at 5% and 13.6%, respectively. If we use the return-on-equity benchmark we are forced to conclude that one $100 rate and one $110 rate should be disapproved as excessive while one $100 rate and one $110 rate are approved. We have subordinated rate equity to rate-of-return equity.

The second problem with return-on-equity in rate regulation is that it requires that equity be allocated to line of business and jurisdiction. And, no matter how much the rate-of-return advocate may wish to ignore the fact, there is no such thing as North Dakota Private Passenger Automobile Surplus - unless, of course, we are dealing with a company which writes North Dakota private passenger automobile insurance exclusively.

The fact is that the entire surplus of an insurer stands behind each and every risk. It supports all of the reserves related to all of the claims and policies issued by the company. And any artificial allocation of that surplus in no way limits the liability of the company to pay claims or honor other financial commitments.

By requiring the allocation of surplus to line and jurisdiction, the return-on-equity basis ignores the value inherent in unallocated surplus. In essence the method treats a multiline national company with $100 million of surplus, $1 million of which is allocated to North Dakota private passenger automobile, identically with a North Dakota automobile insurer capitalized at $1 million. While the $99 million of "unallocated" surplus provides protection to the insured which would not be available from the small monoline insurer, this additional protection is assigned zero value where surplus is allocated.

There is also the problem of an equitable allocation basis. Just how should surplus be allocated to jurisdiction and line? How should the investment portfolio be assigned in order to track incremental gains and losses in allocated surplus? What do you do in the case of surplus exhaustion? Can any return be excessive when measured against an equity deficit? Or should the surplus simply be reallocated each year without regard to actual results? These are tough questions which must be answered by those seeking to allocate surplus.

"BENCHMARK" PREMIUM-TO-SURPLUS RATIOS AS A METHOD FOR SURPLUS ALLOCATION

Some regulators, when faced with the questions raised in the previous section, have proposed using average or target ratios of premium to surplus as "benchmarks" or "normative" ratios.

In the chart below, return on equity is assumed to be 12.5%. This corresponds to a return on sales of 25% where writings are 50% of surplus and 2.5% where the risk ratio is 5:1.

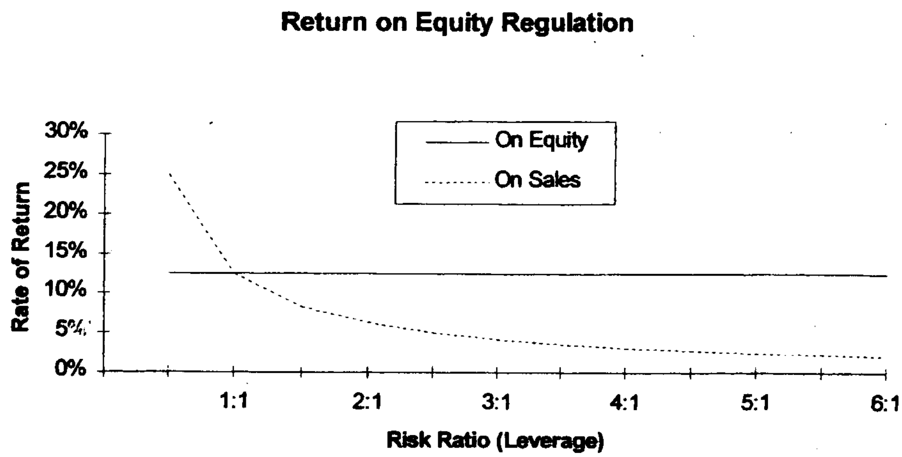


FIGURE 2

But what happens if we decide to use a benchmark risk ratio of 3:1 to allocate surplus to this particular line for this particular jurisdiction? As shown below, the return on equity will equal 12.5% only in the case where the risk ratio is actually 3:1. Where the risk ratio is lower, the return on equity will be lower. Where the risk ratio is above 3:1, the return on equity will exceed 12.5%.

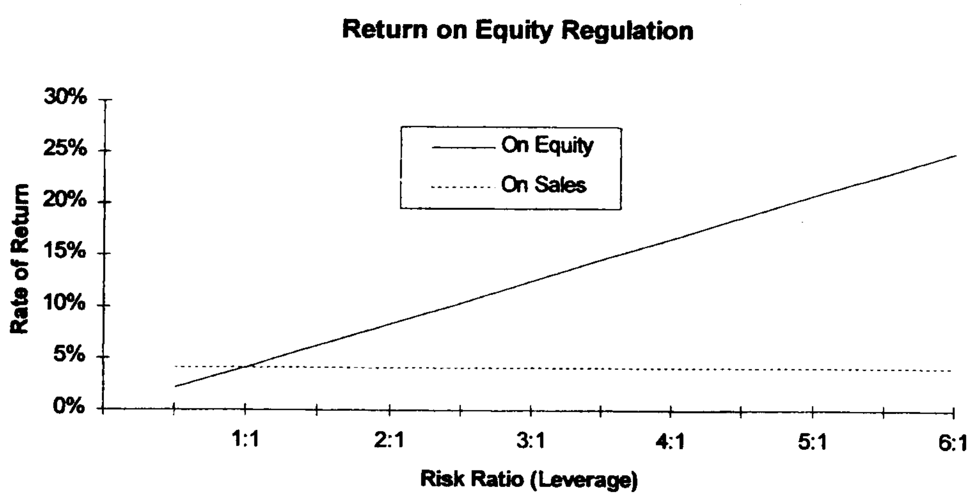


FIGURE 3

But the return on sales is now a constant (12.5%/3) = 4.1667% regardless of the actual risk ratio.

While the use of the benchmark writings-to-surplus ratio has eliminated the surplus allocation problem, the result is not return-on-equity regulation but return-on-sales regulation. And while there is nothing wrong with return-on-sales as a regulatory basis, this represents an excruciatingly complex method for return-on-sales regulation.

RETURN ON SALES

Return-on-sales relates the profit provision in the premium to the premium itself. For anyone who is familiar with the concept of *markup*, it is a natural way to view the profit component. It provides meaningful and useful information to the consumer. If you tell someone that 5% of the price of a loaf of bread represents profit to the grocer, that is helpful in the assessment of the "value" of the bread. If, on the other hand, you tell that someone that the price of the bread contains a 12.5% provision for return-on-equity to the grocer, the information is next-to-useless.

Return-on-sales based rate regulation is simply the establishment of benchmarks for what constitutes excessive or inadequate profit provisions as percentages of premium. It can be as simple as the 1921 NAIC Profit Formula which allowed 5% of premium for underwriting profit (and an additional 3% for conflagrations) or it can be as complicated as the use of benchmark writings-to-surplus ratios applied to permitted return-on-equity provisions. But however the allowable provisions are established, the application is premium-based, and independent of the relationship between premium and equity. As such, return-on-sales results in true rate regulation, not rate-of-return regulation.

PROFITABILITY STANDARDS

Whether rate-of-return is measured against sales or equity, the rate regulator must make a determination as to what constitutes a reasonable, not excessive, not inadequate, provision for profit in insurance rates. In order to keep the various components of the typical rate filing in perspective, I have prepared the following chart which represents an approximation of the composition of a typical private passenger automobile rate filing.

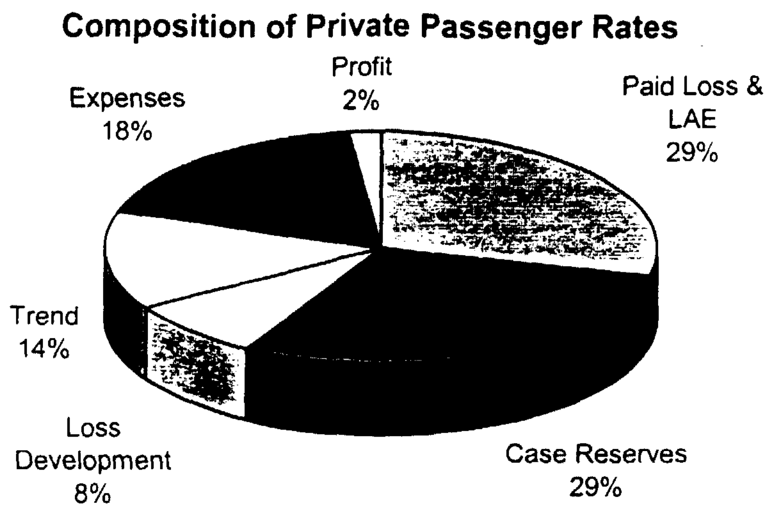


FIGURE 4

It is important to understand that there is typically a great deal of uncertainty in the calculation of indicated property and casualty insurance rates. In the private passenger example above, over 50% of the rate is comprised of estimated unpaid losses and trend. With a profit provision of approximately 2%, a small underestimation can eliminate the profit altogether. (On the other hand, a small overestimation can effectively double the profit.)

While the CAS *Statement of Principles Regarding Property and Casualty Insurance Ratemaking* states that "the underwriting profit and contingencies provisions are the amounts that, when considered with net investment and other income, provide an appropriate total after-tax return" there is no universally-accepted view of what constitutes an *appropriate* return. The application of rate regulatory authority in the U. S. evidences wide disparity. It is quite possible that a profit provision which might be viewed as excessive in one jurisdiction might be deemed inadequate in another.

There is, however, a relationship between the benchmark for excessiveness adopted within a jurisdiction and the resultant market conditions. Unlike public utilities, which are generally monopolistic and which have customer bases which are considerably more homogeneous than are insurance risks, property and casualty insurers can react to inadequate rates by tightening underwriting and/or reducing volume. In any given jurisdiction, the size and composition of the residual market, the number of insurers in the voluntary market, and the degree of product diversity and innovation are all related to the insurance industry perception of the opportunity to earn a reasonable return from the risk transfer.

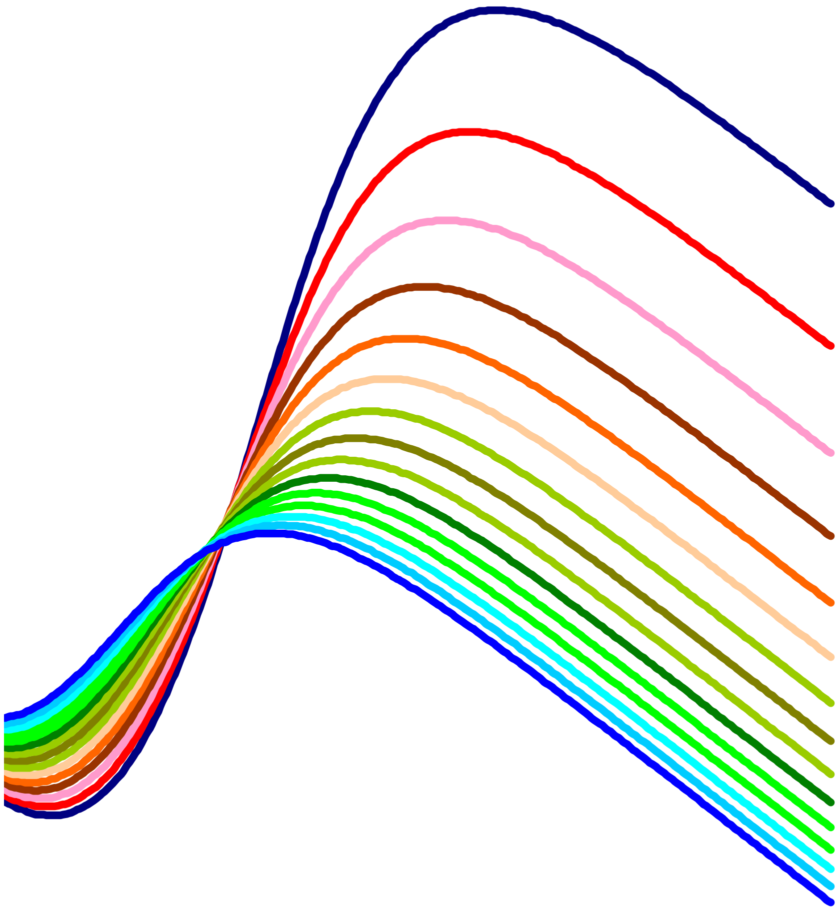
Given the relationship between rate adequacy and market conditions, the proper benchmark for excessiveness for a regulator is that which will produce the desired market characteristics. And any regulator who believes that this relationship is less powerful than a well-crafted econometric argument for a given maximum profit provision is destined to learn a lesson about the distinction between theory and practice.

CONCLUSION

This discussion has focused on the measurement of profitability in the rate regulatory environment. It must be understood that insurance company management and owners will necessarily have different, and not necessarily consistent, needs when it comes to the measurement of profitability. Management will be primarily concerned with the relative risk and return expectations associated with alternative lines of business and jurisdictions. Shareholders will be more interested in returns relative to alternative investments while policyholder-owners of mutual companies will focus on premium savings and dividends. No single basis for the measurement of profitability will adequately meet the needs of all of these interests.

Where rate regulation is concerned, however, it is clear that there must be a consistent basis for the assessment of what constitutes excessiveness in a rate which can be equitably applied to all insurers and which will facilitate fair treatment of policyholders. Such a basis is the return-on-sales approach.

It has been alleged that actuaries have made a profession out of taking something simple and making it complex. While I certainly do not agree with that allegation, William of Ockham pointed out in the fourteenth century that simplicity is to be preferred over complexity. There are simple ways to measure profit and there are very complex ways. Similarly, there are complex ways to assess rate-of-return by jurisdiction and line of business and there are simple ways. Let us not assume that the complex ways are preferable solely because they are not simple.



Managing Interest Rate Risk:

ALM, Franchise Value, and Strategy

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I am indebted to Richard Goldfarb, of Ernst & Young, and Yves Provencher, of Willis Re, for providing detailed comments on an earlier draft of this paper. The ideas and opinions expressed in this paper are those of the author, and do not necessarily represent the views of Willis Re.

Executive Summary

The objective of asset-liability management (ALM) is to measure and manage the degree to which the economic value of an insurer is adversely exposed to changes in interest rates. ALM is therefore a component of Enterprise Risk Management, which considers the impact of changes in other variables as well. As practiced by most insurers, ALM fails to take into account the existence of franchise value – the economic value to the firm of future renewals. Franchise value is not recognized by accounting rules, but can be a significant portion of an insurer's total economic value, which is reflected in its market value.

Incorporating franchise value into ALM is certainly essential, but it also poses a problem. For firms that have substantial franchise value, strategies that limit or minimize economic risk from changes in interest rates can create rating agency or regulatory problems, since these entities view the firm from an accounting point of view. The problem, then, is to identify a strategy that limits a firm's exposure to interest rate risk while simultaneously limiting its exposure to accounting rules that could jeopardize its solvency or its ratings. The solution presented here lies in adopting a pricing strategy that controls the interest rate exposure of future cash flows from new business. This solution substantially extends the analysis first presented in Panning (1994).

Keywords: *Asset liability management, ALM, solvency, franchise value, interest rate risk, hedging, Enterprise Risk Management, ERM, duration, pricing strategy*

1 Introduction

In the property-casualty industry there is a fundamental gulf between what CEO's and CFO's believe they are doing and what they actually are doing in managing their firm's exposure to interest rate risk. They believe that they are managing the value of the firm. What they actually are doing is managing the portion of their firm's value that is visible to them. For many firms – although not all of them – a considerable portion of their firm's value is invisible or only dimly visible to the firm's senior officers, because that value is not included in the accounting numbers upon which they rely. The discipline of Asset Liability Management (ALM) can achieve its stated objective of protecting the value of the firm only if it recognizes this invisible portion of a firm's value, makes it visible to senior management, and helps them to understand how to manage it effectively. Adopting this more sophisticated ALM, which succeeds in accomplishing these objectives, will distinguish successful insurers and reinsurers from unsuccessful ones.[1]

I can demonstrate this thesis with an example from my own experience. More than two decades ago I left the academic world to join a large property-casualty insurer as a quantitative analyst. At that time the CEO was establishing a direct-marketing subsidiary to sell automobile and homeowners insurance as an affiliate of a major national organization. Unfortunately, the business plan numbers – and this was a firm that lives and dies by accounting numbers – were stubbornly inconsistent with the return on equity that the CEO had promised to the board. Although the plan had been revised several times in an attempt to increase forecast returns, in each case the results were worse than before. And time was running short – the new facilities would soon be opening, and the overall plan numbers soon had to be reported to the board.

Because of these tight deadlines, I was assigned to the project, with the expectation that I would be able to speed up the recalculations of projected financial results. This was because I had a "portable" computer (it weighted 25 pounds) and knew how to use a then-new invention called a spreadsheet, which permitted complex business plans to be revised and recalculated quickly. Fortunately, I was assigned to report to the CFO for this new operation, who appreciated the difference between economics, with which I was familiar, and accounting, about which I knew very little. He assigned me to create an economic model of the new business, which his accounting team would then translate into accounting results, in accordance with Generally Accepted Accounting Principles (GAAP). This focus on economic realities ultimately led to a solution of the CEO's problem of insufficient returns on a massive investment.

The economic realities are as follows. Selling an insurance policy by direct marketing, as in this new business venture, costs considerably more than selling that same policy through an agent, who receives a sales commission – so much more, in fact, that the expected profit for the directly-marketed policy is negative. But by selling that first policy at a loss, one obtains a customer who is highly likely to repeatedly renew his or her policy. This resulting series of renewals will be highly profitable, since they will require no marketing costs or agent commissions at all. As a consequence, the loss incurred in the selling the first policy will be more than offset by the future profits from subsequent renewals.

From an economic standpoint, the prospective renewals obtained by selling the initial policy are a valuable asset. In Panning (1994) I call this asset ***franchise value***, a term subsequently utilized by Babbel and Merrill (2005). But accounting rules

[1] This and the next several paragraphs rely heavily on remarks previous published in Panning (2003).

typically do not recognize franchise value,[2] so that this valuable asset and its link to the initial sale are hidden. Instead, the accounting focus was on the overall financial results, which consisted of losses in the early years of the plan and profits later on. To accountants, then, it seemed obvious that the program's return would be increased by reducing sales volume during the loss-producing early years of the plan and increasing sales during the profitable later years of the plan. But implementing this change turned out to simply make matters worse.

By contrast, the newly created economic model demonstrated conclusively that a far better strategy was to grow the business as quickly as possible in the early years, despite the accounting losses, so as to maximize profits from renewals later on. The result was a revised strategic plan that met the CEO's promises to the board. More than two decades later, this economic planning model – now vastly elaborated and improved -- still remains the foundation of strategic planning for that business division.

This experience has several crucial implications for ALM. First, it demonstrates that franchise value is real. Policy renewals have real economic value even though accounting rules refuse to recognize that fact. The existence of such franchise value is typically, although imperfectly, reflected in an insurer's stock price. One American automobile insurer, for example, has some $12 billion in high quality short-term assets, and $9 billion in short-duration liabilities, so that its book-valued surplus of $3 billion is roughly equivalent to the economic value of its current balance sheet. But the stock of this highly profitable and rapidly growing firm is worth $14 billion, or some $2 billion more than its total assets – a remarkable example of franchise value!

[2] An important exception occurs when a firm is sold for more than its book value, in which case the excess is booked by the purchaser as an asset called goodwill.

Second, because franchise value consists of the present value of expected future cash flows from renewal business, it is exposed to interest rate risk. Despite this fact, ALM as typically practiced ignores franchise value and focuses instead on assets and liabilities recognized by accounting rules.

Third, despite its potential importance to many firms, franchise value is typically invisible to the senior executives of most firms, and therefore remains unmeasured, unreported, and consequently unmanaged. Until franchise value is recognized, measured and reported, ALM will remain incomplete because it fails to assist firms in managing this significant but invisible component of their total economic value.

My objective in this paper is to quantify franchise value and demonstrate how it can be effectively managed. Here I extend the results of an earlier paper (Panning, 1994) by quantifying the economic significance of franchise value, measuring its sensitivity to changes in interest rates, demonstrating a significant challenge to the effective management of this interest rate risk, and then showing how firms can solve that problem by adopting an appropriate pricing strategy.[3]

2 A simplified insurance firm

Financial models are indispensable both for creating understanding and for applying that understanding to actual situations. But the models we build for these two purposes differ. For creating understanding, simple models are best. Their transparency enables us to readily appreciate their virtues as well as their flaws. By contrast, the models we build to apply this

[3] The notation and assumptions used here differ slightly from those used in that earlier paper.

understanding to actual situations are necessarily far more complex, for they incorporate many more aspects of reality. For example, one can most easily understand interest rate risk from examples using zero-coupon bonds with annual compounding. But applying that understanding to actual bonds must take into account such inescapable realities as coupons, semi-annual compounding, and a wide variety of other details. In this paper our purpose is understanding rather than application, and so I shall provide a rather simple model that legitimately ignores many realistic but irrelevant complexities of an insurance firm.

The questions I address with this simplified model of an insurance firm are the following:

(a) how significant is franchise value as a component of a firm's overall value?

(b) how sensitive is franchise value to interest rate risk?

(c) what effective strategies can firms use to protect franchise value from interest rate risk?

(d) which of these strategies is best?

This model has the following characteristics:

(a) The firm writes all of its business on January 1 of each year.

(b) It pays all expenses for the year on that same day.

(c) On December 31 of each year it learns the true value of the losses (and associated loss adjustment expenses) on the policies it wrote in January, and it pays those losses that same day. Note that the first and third assumptions imply that accident year, policy year, and calendar year are identical for this company.

(d) The firm's expenses and expected losses are identical every year.

(e) If the firm has made a profit, it immediately dividends that amount to its shareholders. If, by contrast, it has incurred a loss for the year, it immediately raises equity to restore its surplus to the amount it held initially. Consequently, its surplus is identical every year. Since my concern here is ALM rather than solvency, I will ignore the possibility of losses sufficient to make the firm insolvent (a subject treated in a forthcoming paper). Similarly, I will ignore taxes, the fact that loss payments typically occur over multiple years, and the potential costs of raising capital. The model can easily be elaborated to take these realities into account, but doing so here would make the results more realistic and complex without adding insight.

(f) The model assumes, for convenience, that the term structure of interest rates is flat.

(g) The model assumes that all calculations described below occur on January 1, right after the firm has written its new business for the year.

The model incorporates the following notation:

P = the written premiums on policies that the firm writes every year; P can vary yearly;
E = the expenses, ***in dollars***, that the firm pays each year; E is constant;

L = the loss and loss adjustment expenses, ***in dollars***, that the firm expects to pay each year; it is constant;

y = the risk-free interest rate, applicable to calculating the income from the firm's assets and for discounting the firm's future cash flows; default risk is considered in a separate forthcoming paper;

S = the firm's surplus, which is the same every year (due to dividends and recapitalization);

k = the firm's target return on surplus;

cr = client retention, the percentage of clients who renew their policies from one year to the next;[4]

F = the firm's franchise value, the present value of cash flows from future renewals;

C = the firm's current economic value, the present value of surplus and business already written.

The untaxed net income for this simplified firm is

$$P - L - E + (S + P - E)*y = k*S.$$

The first three terms reflect underwriting income, and the remaining terms represent the fact that interest income is earned during the year on the firm's assets, which consist of surplus and premiums less expenses, which are paid immediately when premiums are written. Although expected losses and expenses are constant from year to year, the firm varies its premiums from year to year (if necessary) so as to achieve a target dollar return on surplus, represented by $k*S$, where k is the target percentage rate of return. The fact that premiums can vary plays an important role in our analysis of the firm's exposure to interest rate risk.

The firm will achieve its target return on surplus by setting the premiums it charges to

$$P = [S*(k-y) + L]/(1+y) + E.$$

3 The value of the firm

The firm's ***current economic value***, *C*, the economic value of its current balance sheet on January 1, is the value of its current assets, consisting of surplus plus premiums less expenses, less the discounted value of its expected losses. If $S = 50$, $L = 75$, $E = 25$, and $y = 5\%$, the premium required to achieve an expected year-end return on surplus of $k = 15\%$ is 101.19. Then *C*, the current value of the firm is

$$S+P-E-L/(1+y),$$

which equals 54.76. (This representation ignores a number of issues concerning risk premiums, which are important but more appropriately treated on another occasion.)

[4] My intention here is to focus on franchise value and its implications for managing interest rate risk. I explicitly contrast franchise value, which is the present value of future renewals, from the firm's current economic value, consisting of the economically adjusted values on its current balance sheet, as defined by accounting rules. I refer to the combination of current economic value and franchise value as the firm's total economic value. This is slightly misleading, however, since the firm's total economic value, as imperfectly represented by its market value, includes a third component that consists of the present value of its growth prospects. This third component is in fact recognized in Panning (1994), and in a forthcoming paper on Enterprise Risk Management. But demonstrating the conclusions presented here did not require that growth prospects be explicitly considered as well. Introducing them here would have added complexity at the potential expense of clarity. This footnote is simply a warning that the model presented here may need some elaboration if applied to a firm that is growing rapidly.

Now suppose that we calculate the firm's ***franchise value***, *F*, the present value of cash flows from its future renewals, taking into account both the time value of money and the firm's client retention rate *cr*. If interest rates and the target return on surplus remain unchanged, then the values of *P*, *L*, and *E* in a given year will be followed by the values *P*cr, L*cr,* and *E*cr* in the subsequent year. So to find the present values of these cash flows we must take customer retention into account as well as the time value of money. To do this we create a multiplier $d = cr/(1+y)$. Provided that interest rates remain unchanged, the present value of future premiums equals $P*(d+d^2+ \ldots + d^n)$.

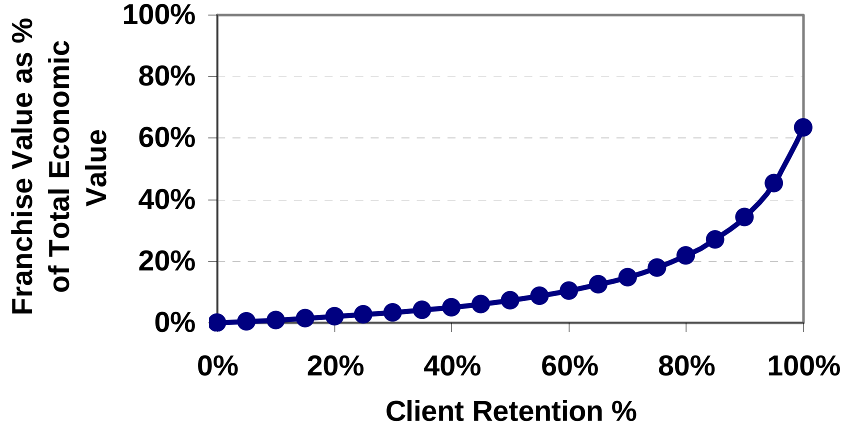
As $n \rightarrow \infty$, the present value of future premiums converges to $P*d/(1-d)$, or, equivalently, $P*cr/(1+y-cr)$. Note that when $cr = 1$, this is identical to the formula for the present value of a perpetuity. Similarly, the present value of future expenses associated with retained business is $E*d/(1-d)$. Losses are paid a year later than premiums and expenses, so their present value is $[L*d/(1-d)]/(1+y)$. These three components of future renewals can be combined to give the firm's franchise value as

$$F = [P-E-L/(1+y)]*d/(1-d).$$

For the parameter values given above, and with a client retention *cr* = 90%, the firm's franchise value *F* is 28.57. If we add franchise value to the firm's current economic value we obtain the ***total economic value*** of the firm, 83.33. Here and throughout, we consider this to be identical to the firm's ***total market value*** or ***market capitalization*** provided that it is publicly traded. If this firm's stock price fully reflected its total market value, then its ratio of market value to (economically adjusted) book value would be approximately 1.5. As one would expect, this ratio is sensitive to several underlying assumptions. Figure 1 shows how the market-to-book ratio varies with client retention, with all other variables kept constant. Note that t high retention levels the ratio climbs rapidly.

Figure 1: Effect of Client Retention on a Firm's Market-to-Book Ratio

Figure 2: The Effect of Client Retention on Franchise Value



Another way to view the potential importance of franchise value is to illustrate it as a percent of the firm's total market value, $F + C$, as in Figure 2. Note that when client retention is 80% or greater, franchise value comprises a significant percentage (20% or more) of the firm's total market value.

The numbers shown in Figures 1 and 2 are illustrative, since they assume that variables other than client retention remain constant. In fact, as we shall see, franchise value is significantly affected by the level of interest rates, by the firm's target return on surplus, and, most important, by its pricing strategy.

4 The interest rate sensitivity of franchise value

We have now established that franchise value is significant, and that at high levels of client retention it can comprise a considerable percentage of a firm's total economic value. Next we demonstrate that franchise value is sensitive to interest rate risk, by calculating the duration of the firm's franchise value. However, because the premium component of a firm's franchise value depends on the firm's pricing policy, which in turn can depend on the level of interest rates, we must first describe how the firm's target return on surplus, k, is determined.

The firm modeled here sets its premiums so that its expected net income equals $k*S$, where k is the firm's target return on surplus. The model assumes that the firm has rationally chosen its surplus amount S, and prices its business according to a fixed rule. The return on surplus k may be fixed or may depend on current interest rates. Here we assume that $k = a + b*y$, where a and b are constants for a given firm but may differ from one firm to another, and y is the spot interest rate corresponding to the maturity of the firm's liabilities (in this case one year). If $b = 0$, then the target dollar return on surplus is simply $a*S$, where a is some constant percentage. For example, a number of CEOs simply set their target return on surplus at 15%. Their policy can be represented by setting $b = 0$ and $a = 15\%$.

Setting a fixed target return can be problematic, however, since interest rates may rise to exceed that level (as they briefly did in the early 1980's). A more pragmatic pricing policy may therefore be to set the target return as a risk-free rate of interest plus some risk premium, so that $b = 1$ and a is the risk premium, say, 10%, so that with y = 5% the target return on surplus is again 15%.

The point of representing the firm's pricing policy in this way is that the premiums it charges may not be fixed but may instead, with pricing policies where $b \neq 0$, vary with the level of interest rates. This relationship must be specified so that it can be taken into account when we calculate the duration of the firm's franchise value. Note that the values of the parameters a and b are behavioral assumptions intended to reflect what the firm actually does, and not necessarily what it professes to do (since these may differ materially).

Given this specification of k, the firm's target return on surplus, we can now restate the value of the firm's franchise value, as follows:

$$F = \frac{cr * S * (a + (b-1) * y)}{(1+y) * (1+y-cr)}.$$

Note that for combinations of a and b that give a target return on surplus of 15%, this equation produces the same franchise value as that given earlier.

By definition, the *duration* of F with respect to changes in interest rates is $D = -1*(dF/dy)/F$, which is the negative of the first derivative of F with respect to y, as a percentage of the current value of *F*. A series of tedious calculations produces the following result:

$$D = \frac{a-b+1}{(1+y)*(a+by-y)} + \frac{1}{1+y-cr}.$$

For the parameters *P*, *E*, *L*, and *S* used earlier, and for a = 15% and $b = 0$, $D = 17.6$.

To see why the duration of franchise value is so high, it is helpful to see the components from which it is calculated, as shown in Table 1. The dollar duration of franchise value is the product of premium present value and its duration, less the comparable products for losses and expenses, which is equal to 607.14*7.85 - 428.57*7.62 – 150.00*6.67, or 503.40. Finally, the duration of franchise value is equal to its dollar duration divided by its present value, or 17.62. (The key here is to first calculate the PV and Dollar Duration of the total, and to divide the latter by the former to obtain the Duration of the total.)

Table 1: PV and Duration of Franchise Value

	Annual Value	Present Value (PV)	Duration (D)	Dollar Duration (PV*D)
Premiums	101.19	607.14	7.85	4,768.71
Losses	-75.00	-428.57	7.62	-3,265.31
Expenses	-25.00	-150.00	6.67	-1,000.00
Total		**28.57**	**17.62**	**503.40**

We see from these calculations that the duration of future premiums is significantly higher than the duration of losses and expenses. How is this possible when we know that premiums and expenses are received and paid simultaneously, at the beginning of each year, and losses are paid a year later?

The explanation for this is that premium cash flows are interest-sensitive. When interest rates rise, premium cash flows become smaller due to the particular pricing policy we have assumed in our example (where the target return on surplus is a constant 15%). When premiums are interest-sensitive, a rise in interest rates has a double impact. Not only does the present value of each dollar of future premiums decline, but the number of dollars of future premiums also declines. ***The first of these two effects is unavoidable when interest rates change. But the magnitude of the second effect can be changed by adopting a different pricing strategy.*** As we shall see, this last point is crucial for effectively managing the interest rate risk attributable to a firm's franchise value.

5 Managing the interest rate risk of franchise value

A principal goal of ALM is to measure and manage the sensitivity of a firm's total economic value to changes in interest rates. To continue the example used here, let us assume that the modeled firm has invested its current assets, consisting of surplus plus written premiums less expenses, in a portfolio with a duration of one year. Its liabilities also have a maturity of one year and a duration just less than one year. Consequently, the duration of its current economic value (54.76) is one year.

But taking its franchise value of 28.57 into account means that the firm's <u>total</u> economic value is 83.33, or 52% larger than its <u>current</u> economic value, and this additional component has a duration of

17.62. The firm's total economic value therefore has a duration of (54.76*1+28.57*17.62)/83.33, or 6.70.

Suppose that the firm believes that this duration of its total economic value is too large. How can it go about reducing that duration? One way, the traditional approach, would be to reduce the duration of its invested assets. This could be done either by changing the composition of the firm's investment portfolio, or by purchasing derivative securities that modify the firm's asset duration. Let's suppose that our example firm chooses the first alternative, and reduces the duration of its invested assets to zero. This would reduce the duration of its total economic value to 5.18, which the firm may still consider unacceptably high. If the firm had a higher client retention percentage, 95% rather than the 90% assumed here, the problem would be even greater. Franchise value would comprise an even greater portion of its total economic value, and reducing the duration of its invested assets to zero would reduce the duration of its total economic value from an initial 10.03 to 8.76, a value that many executives would still regard as too high.

These results present a practical dilemma that has two aspects. First, the greater the franchise value of a firm, the more difficult it is for that firm to manage the interest rate risk of its total economic value by reducing the duration of its investment portfolio. A firm with significant franchise value would have to reduce the duration of its invested assets to zero or even below zero, which is infeasible in practice although possible in principle. Second, a further problem with such a strategy is that the potential benefits of implementing it would be totally invisible to regulatory authorities and rating agencies, who see only the accounting numbers of a firm. Indeed, given their information, regulators and rating agencies might well see actions intended to protect total economic value as increasing a firm's risk rather than reducing it, or, even worse, as jeopardizing the firm's solvency and financial ratings. The key fact here is that managing the interest rate risk of franchise value and total economic value can be quite problematic if the rationale for doing so remains invisible to rating agencies and regulatory authorities.[5]

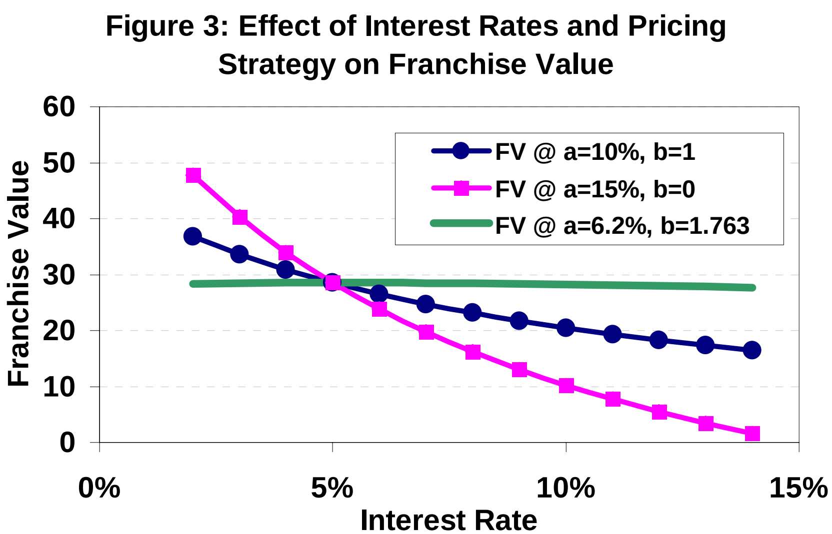
6 Using pricing strategy to manage total economic value

Fortunately, there is a solution to the dilemma just posed. It consists in adopting a pricing strategy that substantially alters the sensitivity of a firm's total economic value to changes in interest rates. In the example give earlier, where a = 15% and b = 0, the duration of the firm's franchise value and total economic value are 17.62 and 6.70, respectively. But suppose we alter the firm's pricing policy by changing these parameters to a = 10% and b = 1. In this case the target return on surplus remains at 15% (given that the risk-free yield remains at 5%), but the durations change from 17.62 to 7.62 for franchise value, and from 6.70 to 3.27 for total economic value. The key insight here is that **a firm's pricing strategy can significantly affect the duration of its franchise value and, consequently, the duration of its total economic value.**

This insight suggests a more systematic approach to managing the duration of total economic value: find a combination of the strategy parameters a and b such that the return on surplus and the duration of total economic value are both acceptable. This can be

[5] This problem could in fact be avoided if the information that regulators require was more thorough, more consistent, and more focused on economic values. The reality is that statutory information is woefully incomplete and incredibly inconsistent, so that one cannot reliably reconstruct even the simplest relationships between an insurer's income statement, balance sheet, and cash flow statement.

done either by systematic numerical search or by constrained optimization procedures. For example, if the firm in our example wanted a target return on equity of 15% but a total economic value with a duration of zero, it should implement a pricing strategy with the parameters *a* = 6.2% and *b* = 1.763 to achieve those objectives. The consequences of this and the two previously mentioned pricing strategies are shown in Figure 3 for the three different pricing strategies just described.



Managing the duration of total economic value by choosing appropriate pricing policies has limitations as well as advantages. An important limitation is that any desired combination of a target return on surplus and target duration of total economic value can rigidly be maintained only for a rather narrow range of interest rates. Large changes in interest rates will necessarily disrupt the combination initially established. But this same limitation is virtually ubiquitous in ALM due to the nonlinearity of prices relative to interest rates. For example, the duration of a bond portfolio will change as interest rates change. In managing franchise value as in managing bond portfolios, achieving very ambitious ALM goals requires more complex strategies than the relatively simple duration management strategies considered here.

But despite this limitation, the strategy identified and evaluated here has a very important virtue: it avoids the potential rating agency and regulatory risk associated with strategies that focus on managing the duration of the firm's invested assets as a means of managing the risk to its franchise value and total economic value. This key advantage results from the fact that implementing a pricing strategy is nearly as invisible to these external audiences as the franchise value it is intended to protect.

A key problem here is that financial service firms are not very transparent to outsiders such as rating agencies, stock analysts, and regulators, who tend to rely heavily upon rules of thumb and sometimes innocent but, in their view, alarming details of the almost idiosyncratic data available to them through statutory reports. I vividly recall a meeting with state regulators where I was grilled at length about a single municipal bond that had been downgraded to junk status. This bond in fact comprised about 0.1% of the total bond portfolio, but was treated as if it were somehow crucial to the firm's solvency. Under the circumstances I simply promised that we would sell that bond and reinvest the proceeds in an investment-grade security. Making that commitment totally changed the atmosphere of the meeting, which proceeded absolutely smoothly from then on. Nonetheless, I was disturbed by the fact that none of the crucial questions for which I had prepared thorough answers was asked. I left that meeting wondering whether some of our competitors might in fact be taking actions that could jeopardize their solvency -- thus exposing our firm to potential guarantee fund assessments -- but nonetheless be undetected by regulators.

7 Conclusion

It has long been recognized that a firm's exposure to interest rate risk depends on the assets and liabilities on its balance sheet and the volatility of interest rates. In this paper I have attempted to broaden our understanding of interest rate risk and of asset-liability management by providing two additional insights. The first is that relying on traditional accounting rules to identify a firm's economic assets and liabilities can blind us to the importance of franchise value, the present value of an insurer's future renewals. Here I have demonstrated the importance of franchise value by showing that it is an essential factor in the direct marketing of personal lines insurance, and by quantifying its economic value for plausible combinations of parameters. I also quantified the sensitivity of franchise value and of total economic value to changes in interest rates and identified some of the potential difficulties in attempting to protect franchise value by changing the duration of invested assets.

Conventional understanding also recognizes that the way to manage a firm's exposure to interest rate risk is to select or alter the composition of its assets and liabilities. Here I have provided a second new insight, namely, that the firm's exposure to interest rate risk can also depend on a third variable, the pricing strategy adopted by the firm. An appropriately chosen pricing strategy can avoid the potential difficulties in protecting franchise value, and can likewise be flexible in achieving a targeted duration and a targeted return on surplus. Although pricing strategy has its limitations as a tool for asset-liability management, these limitations arise from pricing nonlinearities that likewise afflict the more conventional methods typically employed. In an earlier paper (Panning, 1999) I demonstrated that the risk of an equity portfolio can be substantially altered by the use of an appropriate dynamic investment strategy. The second insight presented here is a generalization of that conclusion to the potential use of a dynamic pricing strategy to manage the interest rate risk of a firm's franchise value.

My hope is that these two insights will enable insurers to recognize, quantify, and begin to manage an important but invisible asset – franchise value. Franchise value comprises a potentially significant portion of a firm's total economic value and, if the firm is publicly traded, its market valuation. But managers can manage only what is visible to them. I hope that this analysis will make franchise value more visible and ultimately enable insurers to manage what is now invisible.

8 References

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IRR, ROE, and PVI/PVE

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Abstract:

This paper presents three related measures of the return on a Property-Casualty insurance policy. These measures are based on a hypothetical Single Policy Company model. Accounting rules are applied to project the Income and Equity of the company and the flows of money between the company and its equity investors. These are called Equity Flows. The three measures are: i) the Internal Rate of Return (IRR) on Equity Flows, ii) the Return on Equity (ROE), and iii) the Present Value of Income over the Present Value of Equity (PVI/PVE). The IRR is the yield achieved by an equity investor in the Single Policy Company. The ROE is the Growth Model Calendar Year ROE computed on a book of steadily growing Single Policy business. The PVI/PVE is computed by taking present values of the projected Income and Equity of the Single Policy Company. The paper includes new results relating the PVI/PVE and ROE to the IRR. Beyond developing the foundation and theory of these return measures, the other main goal of the paper is to demonstrate how to use the measures to obtain risk-sensitive prices. To do this, Surplus during each calendar period is set to a theoretically required amount based on the risk of the venture. The main source of risk arises from uncertainty about the amount and timing of subsequent loss payments. With the IRR and PVI/PVE, the indicated prices are those needed to achieve a fixed target return. The indicated price using the Growth Model is that needed to hit the target return at a specified growth rate. With the Growth Model, one can also compute the premium-to-surplus leverage ratio for the Book of Business when it achieves equilibrium. The ability to relate indicated pricing to a leverage ratio, growth rate, and return is an advantage of Growth Model and could lead to greater acceptance of its results. The paper includes sensitivity analysis on the returns and on the indicated profit provisions. In the presentation, the analysis of return is initially done for a single loss scenario. Later, there is discussion on how to model the return when losses are a random variable instead of a single point estimate. Finally, there is a comparison of the approach in this paper versus that of the Discounted Cash Flow model.

Keywords: ROE, IRR, PVI/PVE

1. INTRODUCTION

In this paper, we will present three related ways to measure the return on an insurance policy. The three measures are:

- The Internal Rate of Return on Equity Flows (IRR)

- The Growth Model Calendar Year Return on Equity (ROE)
- The Present Value of Income Over Present Value of Equity (PVI/PVE)

Then we will demonstrate how to use these measures to price Property-Casualty insurance products. We will do this from the perspective of a pricing actuary conducting analysis for a stock insurance company. Whether any of these methods is appropriate in another context is a subject outside the scope of our discussion.

There is nothing novel about using measures of return to price products. The idea is simple enough: any venture with return above a given target hurdle rate is presumably profitable enough to be undertaken. The indicated price for a product can then be defined as the one at which its expected return hits the target. Within the context of internal corporate pricing analysis, corporate management usually sets the target return and a common target is generally used for all insurance ventures.

A significant problem in Property and Casualty insurance pricing applications is that there is no one universally accepted measure of return. The sale of an insurance policy leads to cash flows, underwriting income, investment income, income taxes, and equity commitments that may span several years. How do we distill all this into one number, the return on the policy?

Our three measures are based on two related, but distinct, notions of return on a policy. The first idea is to define return from the perspective of an equity investor who supplies all the capital required to support the policy and who in return receives all the profits it generates. The other idea is to generalize the return achieved by a corporation so that it can be applied to a policy. GAAP ROE (Return on Equity) is a commonly accepted measure of corporate calendar year return. We have two ways to adapt this to a single policy. One is to extend GAAP ROE beyond a single calendar year so that it can handle multi-year ventures. The other is to generate a hypothetical book of business and then measure its ROE. Thus we will end up with three measures of return.

To ensure necessary precision in our analysis, we will define our measures of return by modeling a hypothetical company, the Single Policy Company, which writes a particular policy, the Single Policy. The Single Policy Company writes no other business and is liquidated when the last loss and expense payment is made. Suppose we consider a particular loss scenario and have a model for its anticipated premium, loss, and expense cash flows. We can then apply accounting rules to derive the underwriting income for the Single Policy Company. With other assumptions about investment returns, Statutory Surplus requirements, and taxes, we can derive the company's Investment Income, Income Tax, GAAP After-Tax-Income and GAAP Equity for each accounting period. We will also model a related hypothetical company, the Book of Business Company. This company has a portfolio consisting entirely of Single Policy business. Each period it writes a policy that is a scaled version of the Single Policy. The Book of Business Company begins operations when it writes its first policy and is liquidated after the last loss and expense payment is made on the last policy. We can project the Income Statement and Balance Sheet for the Book of Business Company. Our three profitability measures are defined from the Single Policy and Book of Business Company constructs.

The IRR on Equity Flows is the return that would be achieved by an equity investor in the Single Policy Company. It is a total return measure that reflects the equity requirements, underwriting income, investment income, and taxes associated with the policy by accounting period over time.

PVI/PVE is another measure of profitability based on the Single Policy Company model. It is a generalization of GAAP ROE defined as the ratio of the present value of income valued as of the end of year 1 over the present value of equity. We will show that PVI/PVE will also equal IRR if the present values are computed using a rate equal to the IRR.

Growth Model Return on Equity (ROE) is defined as the Calendar Year ROE that will eventually be achieved by the Book of Business Company if it grows at a constant rate. Under the constant growth assumption, the company will attain an equilibrium in which its Calendar

Year ROE stays constant. We will show that Growth Model ROE equals IRR if the growth rate is also the IRR.

We will derive indicated prices from our return measures. We want these indicated prices to be consistent and sensitive to risk. We also want them to reasonably reflect management's risk-return preferences. To achieve this, we will set Surplus in our model based on a theoretical requirement, and not on an allocation of actual Surplus. Since each of our return measures is sensitive to the effects of leverage, the resulting prices will vary with risk. There are several ways to derive theoretical Surplus requirements and we will not advocate any particular method. We will assume that one has been chosen and that it incorporates any necessary portfolio correlation and order adjustments.

We have said Surplus in our model is a theoretically required amount based on the risk of the venture. But what risk are we talking about? While there is some risk related to the investment of assets, the principal risk in Property and Casualty insurance ventures stems from uncertainty about the timing and amount of loss payments[1]. That is the sole risk we will consider in setting Surplus for our model.

Our initial Surplus is based on the distribution of the present value of ultimate losses. This seemingly innocuous statement has major implications in pricing analysis. For if we vary the premium, we do not change the losses and therefore do not change the amount of surplus. The conclusion is that variations in pricing should lead to variations in the premium-to-surplus

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[1] Robbin and DeCouto[15] argue that the risk measure should act on the present value of underwriting cash outflow, where underwriting cash outflow is loss plus expense less premium. This allows consistent treatment of swing rating plans and contingent commissions, where the premium or expense may be functions of the loss. We will simplify matters in this discussion and assume premium and expense are not adjusted retrospectively.

ratio. In order to see this, consider an example in which the required surplus is derived from the loss distribution and is equal to $50. Suppose the initial premium is $100, so that the initial premium-to-surplus ratio is 2.00. Now consider the situation when the premium is changed to $110. Since the loss distribution is unchanged while the premium has been increased, the required initial surplus should still suffice[2]. Let us suppose it stays at $50. Even though the required surplus has not changed, the leverage ratio is now 2.20 (2.20= 110/50).

The situation is even more complicated when we consider the duration of surplus commitments. Following our logic one step further, we should set surplus at each point in time based on the risk associated with unpaid losses. Since it may take many years for all loss to be paid on a policy, the surplus will evolve over several years. This underscores the conclusion that when pricing analysis is being conducted the proper way to set surplus is not with a fixed premium-to-surplus ratio. This does not mean that in a different context, such as in solvency regulation or rating agency analysis, that comparisons against fixed premium-to-surplus ratios would not be appropriate.

As a caution we should note that our discussion has not addressed the question of comparability between insurance ventures and alternative non-insurance ventures. Since delving into this larger question would take us too far afield from our main topic, we will not consider it further. Also, we should note that in the modeling examples in this paper, Surplus is set simply as a fixed percentage of the expectation of the present value of unpaid losses. This is done in order to clarify the presentation. In any actual application, this loading percentage should vary with the risk by policy and development age.

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[2] Robbin and DeCouto [15] discuss two sorts of capital requirements. One is called Level Sensitive and it declines as the premium rate is increased. The other is called Deviation Sensitive and it stays invariant when the premium rate changes. The approach in this paper is equivalent to the Deviation Sensitive approach.

An equivalent, but different, approach to pricing can likely be obtained by using a fixed and common Surplus requirement for all insurance ventures in conjunction with target returns that vary with risk. In order to avoid debate on which approach is better, we will allow that our preference for using a fixed target return on risk-sensitive capital may be largely aesthetic.

The IRR on Equity Flows has already been presented in the Robbin [13] and Feldblum [8] Study Notes. It has also been used in NCCI rate filings. Appel and Butler [1] have previously addressed some criticisms of the IRR approach. The PVI/PVE has also been presented by Robbin [13] and it appears to be equivalent to the NVP Return developed by Bingham [2].

The Growth Model ROE has some connection to previous work done by Roth [16]. In it, he showed how to convert calendar year figures into a true measure of current year return. He also advocated a target return that includes provision for growth as well as the current return needed for shareholders. The Growth Model ROE provides a way to implement these ideas in a pricing context. With it, the actuary can relate indicated pricing with a calendar year ROE, growth rate, and leverage ratio. These are metrics of interest to insurance company executives and could lead to greater acceptance of the results.

Our analysis will also touch on some of the differences between alternative approaches. First it is important to clarify differences between different IRR models. Some authors have discussed an IRR that is an IRR on underwriting cash flows (paid premium less paid loss and paid expense). There has rightly been criticism that this IRR may not even be defined when the flows switch sign more than once. This may not happen frequently in such models, but the counterexamples given by critics are not unduly atypical.[3] However, as we shall later see, it would be very unusual for the Equity Flows we define to change sign more than once. So this criticism generally does not apply to our IRR on Equity Flows.

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[3] See D'Arcy [5] p525.

Discounted Cash Flow models have many features in common with our three models, but there are important differences. Perhaps most notable is the tautological point that they are focused on underwriting cash flows. As a consequence, they either omit or need to graft on factors such as the accounting treatment of expenses and Surplus requirements. Consider that these methods have no direct way to reflect the conservative treatment of expenses under Statutory Accounting or, equivalently, no direct way to reflect the Deferred Acquisition Balance under GAAP. While some DCF methods do account for taxes on investment income related to Surplus, their results are relatively insensitive to the leverage effects of Surplus. As well, there is no way to study the impact on return from holding discounted loss reserves.

In Section 2, we will present the Single Policy Model. We will use it to define the IRR on Equity Flows in Section 3 and the PVI/PVE Measure in Section 4. In Section 5 we will construct the Book of Business Growth model and define the Growth Model Equilibrium Calendar Year ROE. In Section 6, we will consider modeling returns when the loss can be a random variable instead of a single point estimate. In Section 7, we will study the sensitivity of our return measures to the premium, Surplus level, the interest rate, and the loss payout pattern. We will do this with reserves held at full value or discounted. Then, in Section 8, we will show how to use these measures to derive profit provisions. We will examine the sensitivity of these profit provisions to the Surplus level, the interest rate, and the loss payout pattern. In Section 9 we will compare our approach against the Risk-Adjusted Discounted Cash Flow procedure.

2. THE SINGLE POLICY COMPANY MODEL

Our objective here is to show how to model the accounts of the Single Policy Company based on assumptions about the underwriting results and cash flows of the Single Policy. Our specific goal is to derive the Income and Equity of the Single Policy Company. We will often make simplifying assumptions as this will make it easier to understand the procedure[4]. When modeling actual policies for business analysis, sufficient detail should be incorporated.

An initial assumption we will make is that results are exactly as anticipated. Thus, we will derive a return that is really a return "if all goes just as planned". Later, we will discuss modeling when there is a distribution of possible outcomes.

Before modeling the various income statement, cash flow, and balance sheet accounts, we need to carefully state our indexing conventions. We will use a subscript, j, to denote the value of an income item or cash flow occurring at the end of the j^{th} accounting period. Similarly, a balance sheet account with a subscript, j, denotes its value as of the end of the j^{th} accounting period. We use the subscript, $j=0$, for a cash flow to indicate the flow takes place at policy inception. As well we use the $j=0$ subscript for a balance sheet account to denote its initial value. However, we will assume that income can only be declared at the end of an accounting period so that any income item with a $j=0$ subscript is automatically zero. This is an important assumption. If we were working with an accounting system with some income or loss declared at inception, we would adopt a modified accounting system that would defer that income to the end of the first period and post the appropriate deferred balance as a debit or credit to surplus. To simplify the analysis, we will also assume that no cash flows take place at intermediate times and that the value of a balance sheet account stays constant during a period. This implies the average value of a balance sheet account <u>during</u> the $(j+1)^{st}$ period is equal to its value as of the end of the $(j)^{th}$ period. We will use annual accounting in presenting our model. We will later add a few comments on refining the accounting to a quarterly or monthly basis. Finally, we will assume that the last loss payment is made exactly "n" periods after policy inception and that the Single Policy Company is then liquidated.

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[4] See Feldblum [8] for a more extensive discussion of modeling details.

As regards accounting conventions, our general approach will be to use Statutory Accounting and make some of the adjustments needed to derive GAAP Income and GAAP Equity. Our Income and Equity will also reflect some simplifications. Nonetheless, unless there is a need to make a distinction, we will refer to our Income and Equity as "GAAP".

With these conventions we define booked underwriting income for the j^{th} accounting period:

(2.1

$$U_j = EP_j - IL_j - IX_j$$
$$\text{for } j = 1, 2, \ldots, n$$

Here U is underwriting gain, EP is earned premium, IL is incurred loss, and IX is incurred underwriting and general expense. The loss includes loss adjustment expense. The incurred loss is calculated on a calendar period accounting basis so that it reflects posted IBNR adjustments as well as case incurred losses. However, the loss reserve is not necessarily held at full value, but could be discounted. In the examples in the Exhibits, we compute expense as the sum of a fixed amount plus a component that varies with premium. We assume the Statutory Incurred Expenses are incurred according to a fixed pattern, while the GAAP Expenses are incurred as premium is earned. The difference between Statutory and GAAP Incurred Expense to date is called the Deferred Acquisition Cost Balance (DAC). To keep matters simple, we ignore policyholder dividends.

Next we turn to the very critical question of how Equity is handled in our model. Our assumption is that Equity will be derived from Statutory Surplus and that the Statutory Surplus will adhere to pre-set requirements. We define S_j as the Required Surplus as of the end of the j^{th} period. In later examples, we will always set Required Surplus as a fixed percentage of the expected discounted unpaid loss. However, for our initial purposes, it is not so important how it is set, as the fact that it is set in advance. We can then derive Q_j, the required GAAP Equity.

We make the simplifying assumption that the only difference between GAAP and Statutory Accounting is in the treatment of initial expenses. Thus, we only need to adjust Q_0 for Deferred Acquisition Costs (DAC). Under this hypothesis we have:

(2.2

$$Q_0 = S_0 + DAC$$
$$Q_j = S_j \text{ for } j = 1,2,\ldots,n$$

Note that $Q_n = 0$ since that is the time when the last loss is paid.

Next we define assets as the sum of Statutory Reserves and Statutory Surplus:

(2.3

$$A_j = UEPR_j + XRSV_j + LRSV_j + S_j$$
$$\text{for } j = 0, 1, 2, \ldots n$$

This equation embodies the fundamental accounting principle that the balance sheet must balance. Here UEPR is the Unearned Premium, XRSV is the Statutory Expense Reserve, LRSV is the Loss Reserve, and S is the Surplus. The Loss Reserve is the calendar period loss reserve, inclusive of IBNR as well as case reserves. We could write a similar equation under GAAP. While the Equity would differ from Statutory Surplus and the expense reserves would be different, the resulting assets would be the same under the simplifying assumptions we have made[5]. Note the basic balance sheet formula is used here to define the assets. In contrast, when evaluating real companies, the assets are given and it is the surplus that is then derived by subtracting the liabilities.

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[5] As long as there are no GAAP assets such as Goodwill that do not exist in Statutory Accounting, we will have equality between GAAP and Statutory Assets even though the liabilities may differ.

Next we derive invested assets:

(2.4

$$IA_j = A_j - RECV_j$$

$$\text{for } j = 0, 1, 2, \ldots, n$$

In this formula, we use RECV to denote receivables and amounts recoverable.

With invested assets we can compute investment income for each accounting period. Letting "i" denote the risk-free return on invested assets, we have:

(2.5

$$II_j = i \cdot IA_{j-1}$$

$$\text{for } j = 1, 2, \ldots, n$$

We define pre-tax income as the sum of investment income and underwriting income:

(2.6

$$INCPTX_j = U_j + II_j$$

$$\text{for } j = 1, 2, \ldots, n$$

To handle taxes, we define taxable underwriting income, UITX, and taxable investment income IITX by period. We let t_U denote the tax rate on underwriting income and t_I the tax rate on the taxable investment income. We then compute the tax each period via:

(2.7

$$TAX_j = t_U UITX_j + t_I IITX_j$$
$$\text{for } j = 1, 2, \ldots, n$$

Note we are allowing income taxes to be negative. Also note that taxes in our simplified model are paid when the income is declared. A more realistic approach might utilize carry-forwards and carry-backs in the tax calculation. We would also apply the reserve discounting, unearned premium disallowance, and other provisions of the current US tax code. As well, we would model GAAP in more detail by setting up a deferred tax balance to reflect differences between tax basis and accounting basis income. While the model could be made more elaborate and realistic along these lines, we will avoid complications by using our simplified approach in this paper. In any real-world application, the actual tax code should be modeled in detail. A final note on taxes is that in our examples we will simplify matters by using a common tax rate for underwriting and investment income.

Finally, we define after-tax income:

(2.8

$$I_j = INCPTX_j - ITAX_j$$
$$\text{for } j = 1, 2, \ldots, n$$

Now that we have the Income and Equity accounts of the Single Policy Company, we are ready to define the return on the Single Policy.

3. THE IRR ON EQUITY FLOWS

We now define equity flows as the flows of money between an equity investor and a company. The flows of money could be due to the purchase of stock, the payment of dividends, or the repurchase of stock. We suppose the equity flows are given by the reconciliation formula: equity flow equals income less the change in the equity balance[6]. This presumes any capital shortfall will by corrected by using equity capital[7]. Under this definition, flows of investor capital into the company carry a negative sign, while payments from the company to the investors carry a positive sign.

To compute the Equity Flow, F, we add the Income and subtract the increase in the Single Policy Company's Equity:

For j= 0, we set

(3.1

$$F_0 = I_0 - Q_0 = -Q_0$$

For j= 1,2, ..., n, we set:

(3.2

$$F_j = I_j - (Q_j - Q_{j-1}) = I_j - \Delta Q_{j-1}$$

Figure 1 depicts this general construction.

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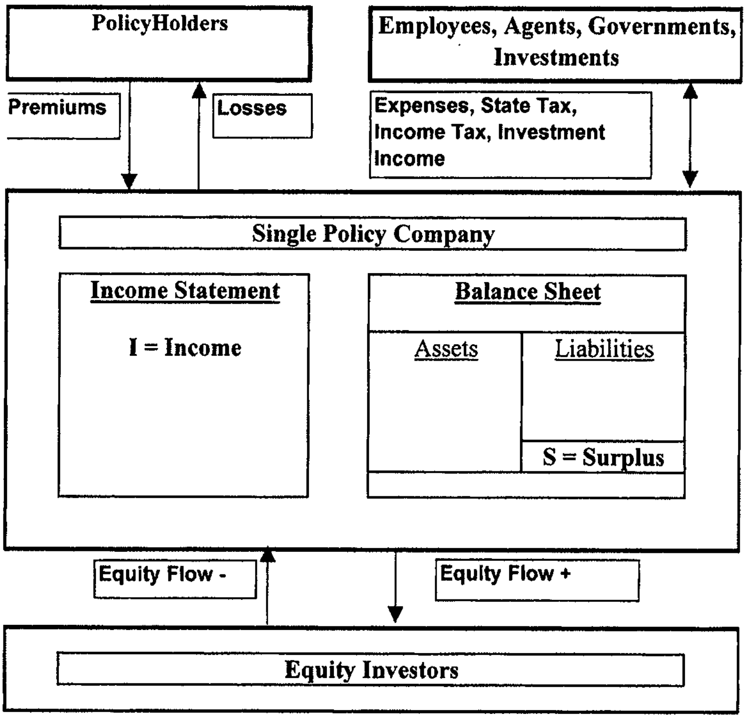
[6] This is a simplified version of the formula in Roth[15].

[7] In other words, we will not consider the use of debt and other non-equity capital in meeting the Surplus requirements.

Figure 1



For the insurance applications we are considering, the initial equity flow, F_0, will always be negative. There are two reasons for this. First, the initial commitment of equity needed to fund the Surplus, S_0, contributes the amount $-S_0$, to the initial equity flow. Second, there is a commitment of equity associated with the Deferred Acquisition Cost balance. This is also called the "Equity in the Unearned Premium Reserve". It arises from the conservative treatment of expenses in Statutory Accounting under which acquisition expenses are incurred up-front rather than as the premium is earned.

The IRR on Equity Flows, y, solves the IRR equation:

(3.3

$$\sum_{j=0} F_j \cdot (1+y)^{-j} = \sum_{j=0} F_j \cdot w^j = 0 \quad \text{where } w = (1+y)^{-1}$$

The IRR, if it exists and is unique, is comparable to the interest rate on a loan or the yield rate on a bond. However, since IRR is in general the solution to a nth degree polynomial, there might be multiple real roots. In that case, for each real root, the equity flows can be decomposed into a series of lending and borrowing transactions at the rate of interest equal to that root. For example, if the flows are (–200, +420, -220), the roots are 0% and 10%. With 0%, a loan of 200 is made from A to B and paid back after one year, and then a loan of 220 is made from B to A and it is paid back a year later. The decomposition is: (-200, 420, -220) = (-200, 200, 0) + (0, 220, -220). For the 10% interest rate, the decomposition is (-200,420, -220) = (-200, +220, 0) +(0, 200, -220). This is shown in the following chart.

Figure 2

	Combined Flow from A to B	**Loan From A to B**		**Loan From B to A**	
Time	**FV Flows**	**FV Flows**	**PV @ 10%**	**FV Flows**	**PV @ 10%**
0	-200	-200	-200	0	0
1	420	220	200	200	181.82
2	-220	0	0	-220	-181.82

While multiple roots are a general problem for IRR analysis[8], they do not arise, except in pathological cases, when computing the IRR on the anticipated Equity Flows for a Single Policy. This is because the Equity Flows in our model only switch signs once. As previously noted, the initial Equity Flow is negative due to the up-front commitment of Surplus and the posting of the Deferred Acquisition Cost balance. After that, during the period the premium is earned, the Equity Flows could be negative or positive depending on the amount of underwriting loss and expense in relation to premium and on whether reserves are held at full value or are discounted. Thereafter, the Equity Flows are all positive. This is due to the earning of investment income and the takedown of Surplus[9]. Also, note that anticipated deferred premium payments or salvage and subrogation loss recoveries and other factors that could lead to reversals in the sign of the net underwriting cash flows do not lead to reversals in the sign of the Equity Flows we have defined. This is true because such payments do not impact the booked underwriting gain. With only one sign change in the Equity Flows, there will be only one root to the polynomial equation and the IRR on the Equity Flows will thus be unique.

On the first sheets of Exhibits 2 and 3 are examples showing the accounts of the Single Policy Company for a hypothetical policy. In each case, the resulting equity flows switch signs once and as a result the IRR is unique. Exhibit 2 is the base case. In Exhibit 3 we show results when loss reserves are discounted. Our ability to do this stems from having an underlying corporate structure with balance sheets and income statements. With Discounted Cash Flow models, there is no natural way to model the distinction.

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[8] Sign reversals are a problem for single policy cash flow analysis as shown in D'Arcy [5].

[9] There is also an implicit assumption that reserves, if discounted, will be discounted at a consistent rate that is less than the anticipated risk-free immunized investment rate. Pathological examples can be constructed by abruptly altering the reserve discount rate from one period to the next. This could lead to reversals in the sign of the Equity Flows.

Two objections that have been raised against IRR are, first, it may not exist due to multiple roots to the IRR equation, and, second, it has an implicit reinvestment assumption at a rate different from the market rate. Appel and Butler [1] have already answered these on general grounds. To eliminate the sign changes that lead to multiple roots, they introduced preferential borrowing and lending rules between a firm and a project under the assumption that "...a transfer of a loan to a future date must be accomplished at the market rate of interest". While we agree with Appel and Butler on general grounds, we do not need such a sweeping argument. We may grant there are general problems with IRR analysis when the flows change sign more than once, but the Equity Flows we are analyzing only experience one sign change. So, for our particular application, that is not an issue.

4. THE PVI/PVE MEASURE

While the IRR on the Equity Flows is an intuitive measure comparable to the interest rate on a loan, we would also like to define a single policy ROE, a measure expressed as the ratio of income over equity. In calendar year accounting it makes perfect sense to take the ratio of income for the year over the initial (or average) equity for the year. However, the Single Policy generates Income over many years and it has Equity requirements that may span more than one year. To summarize the multi-year Income and Equity associated with the Single Policy, we will take present values. The result is a measure of return, PVI/PVE, the ratio of the present value of income over the present value of equity.

Let r_I be the interest rate we will use to discount Income and let r_Q be the interest rate we will use to discount Equity. We set $w_I = (1+r_I)^{-1}$ and $w_Q = (1+r_Q)^{-1}$. Assume the last loss payment for the Single Policy is made at the end of "n" years. Then PVI/PVE is given as:

(4.1

$$\text{PVI / PVE} = \frac{(1 + r_I) \cdot \sum_{j=1}^{n} I_j \cdot w_I^{-j}}{\sum_{j=0}^{n-1} Q_j \cdot w_Q^{-j}}$$

Note the formula is effectively discounting income to the end of the first year. This is done to make the definition of return consistent with the usual definitions of ROE and interest rate. In those definitions, income is taken at the end of the year and is not discounted. Note that under our definition a one-year venture has PVI/PVE equal to the interest rate and is independent of the rates used for discounting[10].

We have allowed for possibly different rates to be used for discounting numerator and denominator. However, our favored approach is to discount both at the same rate and we will henceforth assume a common rate is used unless otherwise stated. Also, we believe that in the PVI/PVE context, the appropriate rate for discounting is the cost of capital. We favor the cost of capital over the risk-free rate because the Single Policy Company can borrow at the cost of capital. The thought is that the Single Policy Company could use borrowed money to give its equity investors the PVI/PVE return each year. The income generated by the Single Policy Company in subsequent years would be used to repay the loans. We have previously mentioned a criticism against IRR: that it uses implicit rates of reinvestment at non-market rates of interest. It is hard to raise a similar criticism against PVI/PVE when the discounting is done using the cost of capital. The rate is explicit and it is the market rate for the company.

For a numerical example, suppose the Single Policy has a two-year payout pattern and assume the Single Policy Company will have Equity of 40.0 for year one, and 22.0 for year two.

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[10] If \$100 is put in a bank account at the start of the year and earns \$10 of interest paid at the end of the year, the return is 10%. The \$10 is not discounted.

Using our indexing notation, we would have $Q_0 = 40.0$, $Q_1 = 22.0$, and $Q_3 = 0$. Now assume income of 5.0 for year one and 4.4 for year two. With our notation, this would translate to $I_0 = 0.0$, $I_1 = 5.0$ and $I_2 = 4.4$. Using a 10.0% rate for discounting, the present value of the income at the end of year one would be 9.0 (5.0 + 4.4/1.1). The present value of the equity would be 60.0 (40.0 + 22.0/1.1). Thus the resulting PVI/PVE would be 15.0% (9.0/60.0).

Next we will show that PVI/PVE is equal to the IRR if the rates for discounting are set equal to the IRR.

(4.2

Result Relating PVI/PVE and IRR: If $r_I = r_Q$ = IRR, then PVI/PVE = IRR.

Proof: Let y = IRR and $w = (1+y)^{-1}$. Then from the IRR Equation we have

(4.3

$$0 = \sum_{j=1}^{n} I_j \cdot w^{-j} - Q_0 - \sum_{j=1}^{n-1} (Q_j - Q_{j-1}) \cdot w^{-j} + Q_{n-1} w^{n}$$

It follows that:

(4.4

$$\sum_{j=1}^{n} I_j \cdot w^{-j} = Q_0 + (Q_1 - Q_0)w + (Q_2 - Q_1)w^2 + \ldots + (Q_{n-1} - Q_{n-2})w^{n-1} - Q_{n-1}w^n$$

$$= (1-w) \cdot \sum_{j=0}^{n-1} Q_j \cdot w^{-j}$$

Dividing both sides by the present value of the equity, we obtain:

(4.5

$$1 - w = \frac{\sum_{j=1}^{n} I_j \cdot w^{-j}}{\sum_{j=1}^{n} Q_j \cdot w^{-j}}$$

and multiplying by $(1+y)$ leads to the desired result.

This result can be viewed as a way to interpret IRR. Under this interpretation, IRR is a PVI/PVE measure in which the rates for discounting change with the profitability of the policy. Note the idea that these rates should change is antithetical to the PVI/PVE approach. Under the PVI/PVE approach, these rates are, in principle, fixed before modeling the particular result for a policy. In Exhibits 2 and 3 we show the two PVI/PVE that result from use of two different discount rates. The first is based on a common rate of 12.0% and the second is based on a rate equal to the IRR.

Now, suppose we set the target IRR, target PVI/PVE, and the PVI/PVE discounting rates equal to the cost of capital and derive the resulting profit provisions. According to our theory, the two measures will generate identical profit provisions. So in the end, as far as indicated profit provisions are concerned, we arrive at the same answer whether we use IRR or PVI/PVE. In that situation, PVI/PVE does not provide an alternative to IRR, but rather another justification for the validity of an indicated IRR-derived profit provision.

5. BOOK OF BUSINESS GROWTH MODEL

We will construct a book of Single Policy business by writing a policy at the start of each accounting period. Each policy is a scaled version of the Single Policy. By summing contributions from all prior policies we can derive the income statement items, cash flows, and balances for the Book of Business Growth Company. If the scaling factors are generated from a uniform growth rate, we can express the accounts for the Book of Business Company as polynomial functions of the growth rate. We will see that the company goes through a start-up phase during which its reserves, assets, surplus and investment income all increase at a rate higher than the generating growth rate. Eventually, the company reaches an equilibrium growth phase at which point all accounts increase at the generating growth rate. We will measure the calendar period return for the Book of Business Growth Company.

Before we can properly analyze the Book of Business Company, we need to convert our indexing notation from one that refers to timing to one that refers to accounting period. We do this by introducing beginning of period (BOP) and end of period (EOP) suffix notation. The conversion is straightforward. Balance sheet accounts having a subscript, "0", get converted to accounts with a suffix BOP and a subscript "1". In other words, the balance at time t=0 is viewed as the balance for the beginning of period 1. For a balance sheet account, B_t, with time value index, t, strictly larger than zero, we define the ending balance at the end of period "t", $BEOP_t$, to be equal to B_t. Under our assumptions, this is the starting value for the next period, so that we have: $BBOP_{t+1} = B_t$. Also, since we have assumed that income is only declared at the end of time periods, the translation is very easy for income accounts: an account with a timing subscript t becomes an end of period account for period t. Figure 3 provides a simple numerical example of the conversion to accounting period notation.

Figure 3

Single Policy - Timing Notation		
	Equity	Income
t	Q	I
0	40.0	0.0
1	22.0	5.0
2	0.0	4.4

Single Policy - Accounting Notation			
	Equity		Income
Period	QBOP	QEOP	IEOP
1	40.0	22.0	5.0
2	22.0	0.0	4.4

Next, we will extend this notation to the Book of Business Growth Company, by adding a prefix G in front of a Single Policy Company variable. We assume the business premium volume is growing at a fixed rate of growth, g, and that a new scaled version of the Single Policy is added to the Growth Company at the start of each period. We let "n" denote the number of periods till all loss is paid for the Single Policy. We can then translate a Single Policy Balance Sheet account, B, to the corresponding beginning of period and end of period balances for the Book of Business Growth Company using the following formulas:

(5.1

$$GBBOP_k = \sum_{i=0}^{k-1} B_i \cdot (1+g)^{k-1-i}$$

(5.2

$$GBEOP_k = \sum_{i=1}^{k} B_i \cdot (1+g)^{k-i}$$

For example, the Equity at the beginning of year two would be $GQBOY_2 = Q_0(1+g)+Q_1$ and the Equity at the end of year two would be $GQEOY_2=Q_1(1+g)+Q_2$.

The summations in formulas 5.1 and 5.2 can be readily understood with a policy contribution diagram:

Figure 4

Book of Business with n=2
Balance Sheet Account Growth - Policy Contribution Diagram

	Year 1		Year 2		Year 3		Year 4	
Policy	BOY	EOY	BOY	EOY	BOY	EOY	BOY	EOY
1	B_0	B_1	B_1					
2			$(1+g)*B_0$	$(1+g)*B_1$	$(1+g)*B_1$			
3					$(1+g)^2*B_0$	$(1+g)^2*B_1$	$(1+g)^2*B_1$	
4							$(1+g)^3*B_0$	$(1+g)^3*B_1$

To provide a numerical example, suppose the Single Policy had Equity balances: $Q_0 = 40.0$ and $Q_1 = 22.0$, and $Q_2 = 0$.. Assume the Growth Company writes the Single Policy at the beginning of year one and writes a 10% larger version of the Single Policy at the start of year two. Using 5.1 and 5.2, the total Equity for the two policies at the beginning of year two would be 66.0 (40.0*1.1+ 22.0). The total Equity would then drop to 24.2 (22.0*1.1) at the end of year two. Using our growth model notation, we would write $GQBOY_1 = 40.0$, $GQEOY_1 = 22.0$, $GQBOY_2 = 66.0$, and $GQEOY_2 = 24.2$.

It is important to note that, even though we have assumed end of period balances for one period are identical to the starting balances for the next period for the Single Policy, the same is not true for the Growth Company. This is true because a new policy is added to the Growth Company portfolio at the start of the next period. The balances from the new policy show up in beginning of period balances for that next period.[11].

[11] For example, since a new policy is written on 1/1/(y+1), the unearned premium balance on 12/31/y is different from the unearned premium balance on 1/1/(y+1).

We will next write a formula for Growth Company income statement accounts. However, under our assumptions, the beginning of period income will always be zero. So we only need supply a formula for "end of period" income items:

(5.3

$$GIEOP_k = \sum_{j=1}^{k} I_j \cdot (1+g)^{k-j}$$

Again, a policy contribution diagram can be useful in understanding the summation:

Figure 5

Book of Business with n=2
Income Account Growth - Policy Contribution Diagram

	Year 1		Year 2		Year 3		Year 4	
Policy	BOY	EOY	BOY	EOY	BOY	EOY	BOY	EOY
1		I_1		I_2				
2				$(1+g)*I_1$		$(1+g)*I_2$		
3						$(1+g)^2*I_1$		$(1+g)^2*I_2$
4								$(1+g)^3*I_1$

To continue with our numerical example, suppose the Single Policy had income of 5.0 at time t=1 and income of 4.4 at time t=2. Under the Growth Model, this would translate to income of 5.0 at the end of year one and 4.4 at the end of year two. Again supposing a 10% larger version of the policy was written at the start of year two, the total income for the Book of Business Company would be 5.0 at the end of year one and 9.9 (9.9 = 5.0*1.1 + 4.4) at the end of year two. The ROE for year two would be 15.0% (.15 = 9.9/66.0).

Now we consider what happens when the Growth Company has been growing for "n" periods. After that, all income statement and balance sheet accounts will be increasing at the growth rate and we say the business is in the Equilibrium Growth Phase. When this equilibrium has been reached, the formulas can be written as:

(5.4

$$GBBOP_{n+k} = (1+g)^k \sum_{j=0}^{n-1} B_j \cdot (1+g)^{n-1-j} = (1+g)^{k+n-1} \sum_{j=0}^{n-1} B_j \cdot (1+g)^{-j}$$

(5.5

$$GBEOP_{n+k} = (1+g)^k \sum_{j=1}^{n-1} B_j \cdot (1+g)^{n-j} = (1+g)^{k+n-1} \sum_{j=1}^{n-1} B_j \cdot (1+g)^{-(j-1)}$$

So, for example, if n=2, the Equity at the beginning of the fourth year would be given as:

(5.6

$$GQBOY_4 = (1+g)^3 (Q_0 + Q_1 (1+g)^{-1})$$

The Equity at the end of the fourth year would be:

(5.7

$$GQEOY_4 = (1+g)^3 (Q_1)$$

The general formula for income in the k[th] year of equilibrium is:

(5.8

$$GIEOP_{n+k} = (1+g)^k \sum_{j=1}^{n} I_j \cdot (1+g)^{n-j} = (1+g)^{k+n-1} \sum_{j=1}^{n} I_j \cdot (1+g)^{-(j-1)}$$

We can now compute ROE when the Book of Business Growth Company is in the Equilibrium Growth Phase. Our ROE will be defined as the ratio of end of period Income over beginning of period Equity. For any year in the Equilibrium Growth Phase, the ratio will be:

(5.9

$$\text{ROE} = \frac{\sum_{j=1}^{n} I_j \cdot (1+g)^{-(j-1)}}{\sum_{j=0}^{n-1} Q_j \cdot (1+g)^{-j}}$$

A key observation is that Equilibrium Growth ROE is a function of the growth rate. We are now ready to show that if the growth rate is equal to the IRR on Equity Flows, then the ROE will also equal that IRR.

(5.10

Result Relating IRR and CY Growth ROE: Calendar Year ROE in the Equilibrium Growth phase will equal IRR if the Book of Business is growing at a uniform growth rate equal to the IRR..

Proof. Let g= IRR and set w = $(1+g)^{-1}$. We rewrite the IRR defining equation 2.11 as follows

(5.11

$$\sum_{j=1}^{n} I_j \cdot w^j = Q_0 + \sum_{j=1}^{n-1} (Q_j - Q_{j-1}) \cdot w^j - Q_{n-1} w^n$$

Expanding the right hand side and regrouping, we have

(5.12

$$\sum_{j=1}^{n} I_j \cdot w^j = (1-w)\cdot Q_0 + (1-w)Q_1 w^1 + \ldots$$
$$= \frac{g}{1+g}\sum_{j=0}^{n-1} Q_j \cdot w^j$$

Therefore it follows that:

(5.13

$$\frac{\sum_{j=1}^{n} I_j \cdot w^j}{\sum_{j=0}^{n-1} Q_j \cdot w^j} = \frac{g}{1+g}$$

From that we derive:

(5.14

$$g = \frac{\sum_{j=1}^{n} I_j \cdot (1+g)^{-(j-1)}}{\sum_{j=0}^{n-1} Q_j \cdot (1+g)^{-j}} = \text{ROE}$$

Thus we have proved our desired result.

The reader may note that this proof is essentially the same as the proof for the PVI/PVE result, with the growth rate playing the role of the rate used for discounting. The Growth Model ROE also provides another interpretation of IRR. Consider that once in the Equilibrium Growth Phase the Equity increases from one year to the next by the factor, (1+g). When g equals the IRR, our result says that ROE is equal to the growth rate g. The conclusion is that all the Income is being used to support growth and that the Income generated is all that is needed to support growth at that rate. In other words the end of period Income from one period equals the increase in beginning of period Equity for the next period. So, when we find IRR we are finding the maximal self-sustaining growth rate. It is self-sustaining in the sense that equity investors need supply no more capital once the Equilibrium Growth Phase is reached.

In Exhibits 2 and 3 we show Growth Model accounts for our example. We do this in two stages. First in Sheet 2 of these exhibits, we restate the Single Policy Model accounts using our Beginning of Year (BOY) and End of Year (EOY) accounting conventions. Then, we show growth results in Sheet 3, all at a common growth rate of 5.0%. We compute ROE for each year in the Growth Model. A summary table displays IRR and ROE results. The ROE summary results are for the Equilibrium Growth Phase. In Exhibits 2 and 3, we also have a Sheet 4 that displays accounts where the calculations have been done using a growth rate equal to the IRR. For those scenarios, the ROE equals the IRR, thus demonstrating our theoretical result. For the Sheet 3 scenarios, the two measures are not equal.

If we compare Sheet 3 ROE results by year in Exhibit 2, which is based on full value reserves, versus the comparable ones in Exhibit 3, which is based on discounted reserves, we find that they are nearly identical in equilibrium. However, during the start-up years, the ROE based on discounted reserves is quite a bit higher. This is true even though leverage ratios are unrealistically high in the initial years in both models. Were the leverage ratios reduced in those initial years, the ROEs would decline in both cases. So, in the case when reserves are held full value, the pattern of low ROEs in the initial years rising up to the equilibrium value would be even more pronounced. This leads us by example to a general observation: rapid growth tends to depress ROE, but this can be countered by discounting reserves. Thus, our theory tends to make us more apt to scrutinize the adequacy of reserves and capital in a rapidly growing firm that posts a high ROE and has a heavy concentration in long-tailed lines of business.

We have presented models constructed on an annual basis. It is straightforward to build comparable models on a quarterly or monthly basis, because the accounting rules allow us to do so. Quarterly equity flows can thus be computed and a quarterly effective IRR can be derived from them. PVI/PVE presents a little bit of a problem. Because we have four equity values each year instead of one, our PVE denominator will be roughly four times as large as the PVE from the annual model. On the other hand, the PVI numerator does not necessarily increase or decrease in moving from an annual model to a quarterly one. Two alternatives that have been proposed to deal with this are: i) view the return as a quarterly effective return or ii) annualize the return by dividing the Equity roughly by 4.[12] For ROE we have comparable choices. We could take income for a quarter and divide it by the equity for that quarter. The result would be a quarterly return. The alternative is to take a full year's income and divide it by the average equity for the four quarters. We will not do that in our demonstration. Our point is simply that it is not terribly difficult to extend our models to a quarterly basis. That would allow us to achieve greater accuracy.

6. RETURNS WHEN LOSS IS A RANDOM VARIABLE

We have derived our return measures by modeling results of hypothetical corporations under the assumption all goes as planned. In particular, we have modeled loss as a single point estimate. We now explore how to compute the returns when loss is a random variable. Assume we have a loss distribution consisting of a finite number of loss scenarios and associated probabilities. To be complete, we could also have a more complicated set of scenarios, each consisting of a loss amount and a loss payout pattern. But, for our current work, we will assume it is only the loss amount that varies.

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[12] See Robbin [13] for a more in-depth discussion of annualization.

Our plan is to model the Income, Surplus, and Equity Flows of each scenario. At first this would seem to be easy. We could just plug the loss amount for each scenario into our model and let it run. However, the problem is a bit harder than that. We can identify at least three major related issues that need to be resolved. The first is whether to let our Single Policy Company go bankrupt in adverse scenarios. The second is the related issue of how to set Surplus. The third is how to model the timing of when the actual ultimate loss is recognized.

We could let our Single Policy go bankrupt in very unprofitable scenarios. The opposite approach is to keep it afloat by implicitly assuming the equity investors will pump in as much money as is needed. This is over and above the initial or planned commitment of Capital. A compromise position is to assume the equity investors post some fixed amount of extra money that could be tapped if needed. The rental of this extra capital should carry a charge. In a setup suggestive of the shared assets paradigm for insurance developed by Mango [10], we could model a Holding Company that would back a portfolio of different Single Policy Company subsidiaries. The Holding Company would assess a "use of extra equity" charge against each Single Policy Company and would be an intermediary between the equity investors and these subsidiary companies. The required segregated Holding Company capital would then depend on the amount of capital in each Single Policy Company subsidiary, the odds each subsidiary would need to draw on Holding Company funds, and the covariance between results of the subsidiaries. While this is conceptually attractive as well as more realistic, it is complicated. We will leave implementation of this approach as a topic for future research. Instead, we will model a company that does not go bankrupt. While this approach has some conceptually debatable underpinnings, it is the easiest to implement. Further, as we will later argue, it provides a conservative estimate of what would result from a more complete model.

In regard to what Surplus requirement should be used, we believe, on theoretical grounds, that all scenarios should start with the same initial Surplus. The reason is simple: at the outset there is no way to know what scenario will ensue. Under our procedure, the initial Surplus would thus be set as a percentage of the expected present value of unpaid losses. The expectation would be taken with respect to all scenarios. After that the situation gets more complicated. As results are posted for the first accounting period, company management may have a better idea than at the start which scenarios are more likely than others. In theory it would then set the Surplus based on its revised estimate of present value of unpaid losses. While this is in some sense realistic as well as conceptually appealing, it is complicated. For our current purposes, we will opt again for the simplest approach and assume a common amount of Surplus at each point in time for all scenarios. The common amount of Surplus would be set at a given point in time as a percentage of the expected present value of unpaid losses. In concept, the percentage would be based on a risk measure operating on the distribution of the present value of unpaid losses. In the examples we use the same percentage for all evaluations.

Now we turn to the question of when to recognize the ultimate loss in a given scenario. Initially, we know only the expected loss over all scenarios. Within any particular scenario, the discrepancy must eventually be recognized on the books of the Single Policy Company. The timing of this recognition will impact underwriting income, loss reserves, investment income, income taxes, and equity flows. Our approach is to recognize the difference at the end of the first accounting period.[13] An alternative is to set reserves equal to the expected ultimate loss times the percent of loss unpaid. The expectation is over all scenarios. Under this approach, the difference between the ultimate loss in the particular scenario and the expected ultimate over all scenarios would be recognized piecemeal as the losses are paid. Various intermediate recognition algorithms could also be used and all the methods could be adjusted to handle reserve discounting. While it is somewhat unrealistic to assume complete recognition of the ultimate loss at the first evaluation, this leads to the simplest algorithm. As well, we will argue that it is the most conservative approach.

Use of our simplest solutions to each of these problems leads to a very convenient modeling result: the average income, average equity, and average equity flow over all scenarios

[13] In a quarterly model, we would recognize one fourth of the difference at the end of each of the first four quarters.

are the same as those resulting when the model is run on the average scenario. In Exhibit 5, we illustrate this with a three-point loss distribution. What this means is that we do not need to separately model all the scenarios to find the returns. Our results for the average scenario will suffice.

An important caveat is that this observation only applies when the premium and expenses are fixed and do not vary with the loss. With Retrospective Rating plans, for example, the premium varies with the loss, and is further subject to Maximum and Minimum Retro Premium restrictions. The average underwriting loss for such a plan does not in general equal the underwriting loss that results from the average loss scenario. So we would need to model the full distribution when dealing with a Retro Plan. However, when complications of that sort are not present, we have found that our simplifying assumptions will allow us to legitimately reduce the distribution of losses to a single scenario.

What have we lost by adopting these simplifications? The answer is that the major factor we are missing is consideration of the default scenarios in which the Single Policy Company fails to meet its obligations to policyholders. We have incorrectly assumed the equity investors would keep the company afloat rather than letting it become insolvent. In effect, we have neglected to put a cap on the downside risk to the equity investors. Because we have not done so, the amounts lost by the investors in adverse scenarios are greater in our model than those that would be indicated in a more sophisticated model. The conclusion is that our model leads to a more conservative average result. In other words, our returns are lower than what they would be if we had modeled the default option. Though our simplified approach would thus be inappropriate for some applications, such as modeling Guarantee Fund assessments, its conservative answers are arguably the answers that are most useful in internal corporate pricing analysis. In that context, the more complete models can exhibit inadequate sensitivity to the tail of the loss distribution. While increasing the relative weight of the tail does increase the risk measure and thus the required Surplus, this is partly offset by the assumption that the equity investors can walk away from the big events. With our simplified approach, there is no walking away and, therefore, no offset. Thus the returns we derive are sensitive to tail events. We feel this is more appropriate in the pricing context of our discussion.

7. SENSITIVITY OF RETURNS

Before going further, it is useful to study how our three measures of return respond to changes in premium, Surplus, interest rate, and payout pattern. We will do this with a simple example. Base case assumptions are shown in Exhibit 1.

The sensitivity of return with respect to premium is of interest when pricing a particular product or policy. Perhaps the return on a product is initially below target at the premium suggested by an agent or broker. Knowing the sensitivity to premium will provide us an intuition about much more premium it will it take to get to the target. Summary premium sensitivity results for our example are shown in Exhibit 4 on Sheets 1 and 2. Reserves are held at full value for Sheet 1 and are discounted in Sheet 2. All Growth Model results assume a 5.0% growth rate and all PVI/PVE results assume discounting at 12.0%. These selections would be appropriate if we suppose that corporate management has targeted a 5.0% growth rate and a 12.0% calendar year ROE. As might be expected, due to the fact that all three models share a common foundation, there is not much difference in the results. Only when returns are negative in the low premium scenario do we see any real difference and even that is fairly modest. In that scenario, the IRR is not quite as negative as the PVI/PVE.

As premiums increase by a constant increment, the returns increase, but in a slightly nonlinear fashion. The IRR goes up at a slightly increasing rate, while the PVI/PVE and ROE rise at a slightly decreasing rate. While a full explanation of the nonlinearities would require detailed analysis, we can at least indicate that our assumptions regarding Deferred Acquisition are part of the explanation as regards PVI/PVE and Growth Model ROE. According to these assumptions, an increase in premium leads to an increase in DAC and thus to an increase in PVE and GAAP Equity in the respective models. The increase in the DAC component of Equity slightly moderates the increase in returns caused by the premium increase. Another consequence of our modeling assumptions is that, counterintuitively, an increase in premium can lead to a reduction in investment income in the second year of the policy. This happens since we have supposed some premium is not paid till the second year. The assets in that year are equal to Reserves plus Surplus and do not change when premium is increased. However the rise in premium boosts the Receivables and thus decreases the investible assets.

Note that the different premium scenarios have different premium-to-surplus leverage ratios. This is in accord with our assumption that the Surplus requirement is driven by the loss distribution. Since all the premium sensitivity scenarios thus have the same amount of Surplus and differing amounts of premium, they end up with different leverage ratios. Another observation is that the change in Equilibrium Growth Model ROE as the result of a change in premium is the same whether reserves are held at full value or are discounted. This makes intuitive sense since the amount of Equity in our model is independent of whether actual reserves are held at full value or are discounted.

Now we examine the sensitivity of our returns to changes in the level of Surplus. This might be of interest when comparing products with different levels of risk. The different levels of risk would translate into different Surplus loading factors for the products. The results for our example are shown in Exhibit 4, Sheet 3. There is nothing surprising: more Surplus produces returns closer to the after-tax yield on investment, no matter which of our return measures is used. However, the sensitivity is perhaps lower than might be guessed in advance. As we increase our loading factor for Surplus so that the Growth Model premium-to-surplus ratio drops from around 3.0 to around 2.0, the returns drop by a bit less than 2 points. The major reason for this is that the after-tax return on investment of the Surplus is fixed and immune to the effects of leverage. So, of the roughly 11.7% returns we get in our low Surplus scenario, nearly 4.0% is achieved on the Surplus itself and only the increment of 7.7% is due to the insurance venture. To get a rough estimate of the Surplus sensitivity in moving from leverage of 3.0 to leverage of 2.0, we would multiply the 7.7% by 2/3 to get 5.1%. The difference of 2.6% is higher than our observed difference of nearly 2.0%, but it suggests that the observed sensitivity is plausible.

We next look at the sensitivity of our returns to changes in the interest rate. As is to be expected, the higher interest rates yield higher returns. They are even a bit higher than one might initially have guessed. This is due to our method of setting Surplus values as a percentage of the present value of unpaid loss. As the interest rate increases, these present values decline. This reduces the amount of Surplus, and so the Growth Model leverage ratios increase.

Finally we turn to examine sensitivity due to changes in the payout pattern. To make the analysis cleaner, we changed our Surplus-loading factor between scenarios so that all scenarios would have the same Growth Model leverage ratio. Implicitly we are assuming that the longer tailed scenarios have lower risk that just offsets the larger commitment of Surplus due to their longer duration. The results are just as expected: longer payout patterns lead to higher returns. The effects are significant. We see that a change in duration of half a year can change the return by over 2 points. This result is sensitive to the interest rate assumption of 6.0% used in our analysis. With a higher rate, we would see even greater sensitivity.

To summarize, the returns exhibit appropriate sensitivities that we can intuitively explain after the fact, even if we did not entirely foresee them beforehand. We should caution that the particular results we have presented are critically dependent on our modeling assumptions. The results would differ if the required Surplus or the Deferred Acquisition balance were computed differently.

8. INDICATED PROFIT PROVISIONS

We define the Indicated Profit Provisions and Indicated Premiums for each of our measures by solving for the profit provision and resulting premium that yields a return equal to the selected target return. Results are shown in Exhibit 5 assuming a target of 12.0%. All results assume reserves are held at full value. Recall that for PVI/PVE we also need to choose a rate for discounting income and equity. We again chose 12.0% under our logic that the cost of capital is a natural target and the natural rate to use for such discounting. However, according to our result relating IRR and PVI/PVE, when the same rate is used for the target and for discounting, we will end up with a PVI/PVE equal to the IRR. Thus our indicated profit provisions for IRR and PVI/PVE are identical. With the Growth Model ROE, we used a growth rate target of 5.0%. If we had used a growth target of 12.0%, results for ROE would have also been the same as for IRR. However, we have no logic that compels such a choice. Rather, we have assumed that management has specified a long-term growth target of 5.0% and a target calendar year return of 12.0%.

In Sheet 1, we examine sensitivity of the Indicated Profit Provisions to changes in the level of Surplus. We change the level of Surplus by varying the Surplus-loading factor. As we would anticipate, higher Surplus loading factors give rise to higher profit provisions. However, the leverage ratios do not follow a direct inverse relation with the loading factors. The divergence arises because the premium is also changing between scenarios. As shown in Exhibit 5, the ROE profit provision moves from –1.97% to –0.13% in response to a change in Surplus loading factors that reduces the Growth Model leverage ratio from 3.09 to 2.15.

Next we examine sensitivity of indicated premiums to a changes interest rates while keeping the target return fixed. Results are shown in Sheet 2. Raising the interest rate leads to a reduction in the profit provision. This is in accord with our intuition. With more investment income we need less underwriting income to achieve the target. The IRR and ROE results are similar, but not identical. With our loss payout pattern duration of only 2.0 years, moving the interest rate up one point reduces the indicated profit provision by a bit less than 2.0 points. The result also depends on our Surplus-loading factor. With a higher loading factor, we could drive sensitivity down. The results can also be explained by noting that interest rates impact the leverage ratio in our model. On the one hand, increasing the interest rate reduces the present value of unpaid loss. That reduces the Surplus. On the other hand, higher interest rates reduce the indicated premium, assuming the target return stays fixed. This happens because they reduce the difference between that target return and the after-tax investment return as well as increase the investment income on our full value reserves. The net tradeoff between the reduction in Surplus and the reduction in Premium as seen in our results is that the leverage ratios decrease modestly with an increase in the interest rate.

Finally, we turn to sensitivity analysis of the indicated profit provisions with respect to changes in the loss payout pattern. Results are shown in Sheet 3. To facilitate comparisons, we adjust our loading factors for Surplus in order to achieve a constant Equilibrium Growth Model leverage ratio in all scenarios. We see, as expected, that the results show significant response to the duration of the payout pattern. Increasing the duration by half a year moves the profit provision down by just over 2.0 points when the interest rate is 6.0%.

To summarize, despite a few subtleties, the models produce Indicated Premiums that are appropriately responsive to changes in key inputs. Next, we will compare our corporate structure approach with the Risk-Adjusted Cash Flow Model.

9. COMPARISON TO THE RISK-ADJUSTED DISCOUNTED CASH FLOW MODEL

The Risk-Adjusted Discounted Cash Flow Model (RA DCF) has often been used in pricing. However, it takes a different approach to pricing than the one we have taken. Instead of finding the Indicated Premium needed to hit a fixed target return on Risk-sensitive Surplus, the RA DCF approach is to find the Fair Premium directly. The Fair Premium is defined as the sum of loss, expense, and income tax cost components. Each component is discounted. However, since losses are a risky cash flow, they are discounted at a risk-adjusted rate.

In words, the formula is

(9.1

Fair Premium =
PV of Loss at the Risk - Adjusted Rate + PV of Expense
+ PV of Tax on Investment Income on Surplus and Premium net of Expense
+ PV of Tax on Underwriting Income from Premium less Expense
- PV of Tax Reduction for Losses at the Risk - Adjusted Rate

For a single period example, we can write the formula in mathematical symbols as follows:

(9.2

$$P = \frac{L}{(1+r_A)} + \frac{X}{(1+r_f)} + \frac{T_I \cdot r_f \cdot (P - X + S)}{(1+r_f)} + \frac{T_U \cdot (P - X)}{(1+r_f)} - \frac{T_U \cdot L}{(1+r_A)}$$

Here P stands for premium, L is loss, and X is expense. The losses are discounted at a risk adjusted rate, r_A, which is less than or equal to the risk –free rate, r_f. The tax rate on investment income is T_I and the tax rate on underwriting income is T_U. Here S stands for Surplus. Note that the Fair Premium includes a provision for the tax on the investment income from both the Surplus and the balance of underwriting cash flows.

The risk-adjusted rate is a key parameter in the RA DCF model. As D'Arcy and Dyer [6] note, determination of this rate is a "thorny issue"[14]. They describe two approaches. One is to view the adjustment "as a form of compensation to the insurer for placing its capital at risk in the insurance contract"[15]. The second is to derive the risk-adjustment from principles of the Capital Asset Pricing Model (CAPM). This is the approach used by Myers and Cohn [12] in

[14] D'Arcy and Dyer [6], p.342.

[15] D'Arcy and Dyer [6], p.342.

their original paper introducing the model. Under CAPM, there should be no charge for process risk, only for systematic risk related to the covariance of insurance losses with returns on the stock market. This covariance is known as "beta. The determination of beta has been the subject of some disagreement. Some believe beta is close to zero. For example, Vaughn [17] notes:: " For many P/L lines, indemnity losses possess very little systematic risk. As such, the risk-free rate is often used as an acceptable approximation ..."[16]. However, Derrig [7] and others have used a non-zero, CAPM-based beta in rate filings.

This short introduction to the RA DCF model is necessarily incomplete, but it will suffice to allow us to reasonably compare that model against the procedure we have presented. The most obvious distinction is that the RA DCF is a method to determine premium without need to assume a target return. In our models, the Indicated Premium is that needed to achieve a given target return (or target return at a given target growth rate for the Growth Model).

The next major distinction is that the RA DCF model has no underlying corporate or accounting structure, while such a framework is the basis for defining our returns. Because of this, the RA DCF has no natural way to reflect the conservative treatment of expenses under Statutory Accounting. In our corporate model, this was handled by making an adjustment to GAAP Equity for Deferred Acquisition Costs. As well, there is no natural way in the RA DCF framework to reflect reserve discounting. While reserve discounting does not impact underwriting cash flows, it does impact the flow of funds to equity investors. Our corporate model of Equity Flows takes this into account.[17]

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[16] Vaughn [17], p. 406

[17] Another anomaly caused by lack of an accounting substructure is that the balance of investible assets does not automatically decay to zero. However, since it usually decays to a positive or negative balance close to zero and the RADCF provision is for the present value of taxes on the investment income on the balance, the practical impact of the non-disappearing balance is usually negligible.

The next point of distinction concerns the role of Surplus. In the RA DCF, it plays no direct major role. There is a provision in the Fair Premium for the present value of the tax on the investment of the Surplus, but this is usually small. Consider a one-year example assuming a 3.0 leverage ratio, 6.0% interest rate, and a 35% tax rate. The full value tax in that case would come to around 0.69%. Not only is the effect small, the sensitivity to changes in Surplus is even smaller. Reducing the leverage ratio to 2.0 in our example produces a full value tax of 1.05%. The difference of 0.36% is significantly smaller than the 1.84% difference (-0.13% -(-1.97%)) seen in our Growth Model ROE results. Further, if the tax rate were zero, the Fair Premium would be independent of Surplus. In contrast, in our models the leverage effect of Surplus has a critical impact on the results. It is revealing that in some RA DCF models[18], Surplus is assumed to be larger than the amount needed to ensure that there is essentially no chance of insolvency. This view of Surplus is effectively tantamount to regarding it as a "free" good; there is more than enough of it to go around. However, in the corporate context of our models, Surplus is in scarce supply.

Another major difference between the models concerns their sensitivity to risk. As we previously noted, risk sensitivity in the RA DCF model depends on how beta is selected. Yet, that selection is problematic. If we follow Vaughn and use no risk-adjustment, RA DCF pricing would have no sensitivity to risk. Since we believe pricing ought to be risk-sensitive, we would disagree with this implementation of the RADCF: it is an RADCF without the "RA". If we follow others who use CAPM to derive a non-zero beta, we would have some risk sensitivity. However, those methods have typically been applied at a line of business level for the industry. It is not obvious how to extend them to pricing different products within a line for a single company.

Finally, we could follow those who set the beta so as to provide an adequate return on risk-sensitive capital. In that case, we would look to our approach to arrive at the Indicated

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[18] See Vaughn [17].

Premium and solve for the beta that leads to the same answer. While the presentation of that result as a RA DCF calculation might be useful in some situations, it forces us to think about risk sensitivity in terms of changes in beta. Within our framework, risk sensitivity depends on the Surplus requirement formula and the spread between the target return and the after-tax yield on investment. We believe actuaries and insurance company management find it more intuitive to think in those terms. Further, though there are disagreements about how to set theoretical Surplus, they are not as severe as the disagreements over beta.

Ultimately we feel the methods arise in different contexts and reflect different perspectives in pricing. Others have noted these differences[19]. Management, we believe, will be far less interested in knowing the Fair Premium for a product than it will be in knowing the Indicated Premium needed to attain its risk-return objectives. One the other hand, as the title of the Myers and Cohn paper [12], " A Discounted Cash Flow Approach to Property-Liability Insurance Rate Regulation" makes clear, that model was originally developed to handle pricing in a regulatory arena. From a policyholder or regulatory perspective, there may be much greater concern with finding the Fair Premium than knowing whether the premium is adequate for shareholders to achieve the expected return they desire. While the Fair Premium may contain some compensation for the equity investors of the insurance company, those investors may or may not find that compensation acceptable.

One other issue that must be clarified is that there are discounting methods, such as the one developed by Butsic [4], in which the losses are discounted at a risk-adjusted rate, yet which are closer to our method than to the RA DCF approach. In Butsic's model, the rate adjustment depends explicitly on the equity requirement and a given target return. Butsic sets the equity requirement as a percentage of the discounted loss reserve. He also computes an IRR that is conceptually the same as our IRR on Equity Flows. He finds the premium needed to hit a given target return. What Butsic shows is that if reserves are discounted at just the right rate,

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[19] See Bingham [2].

then the ROE for each year is equal to the IRR and the target return. His rate for discounting losses is given as:

(9.3

$$r_A = i - e(R - i)$$

Here i is the risk-free rate, R is the target return, and e is the equity loading factor relative to the discounted reserve.

What Butsic has done is to show how to modify the accounting system to bring it into accord with economic reality so the anticipated calendar year returns each year would be the same as the IRR. If we were to discount reserves in our model according to Butsic's formula, we would obtain the same results.

10. CONCLUSION

We have covered many topics and now it is time to summarize what has been accomplished. The first step in our journey was to define our three measures based on a hypothetical corporate structure. Looking back we can see that this structure enforced a certain discipline in our analysis. We had to be precise about the amount of Surplus being held and about the flows of money to and from equity investors. The structure allowed us to reflect the impact of the DAC adjustment in GAAP and the effect of reserve discounting. Having a corporate structure that incorporates accounting rules is a critical aspect of our approach. Further we can conclude that models without sufficient corporate structure cannot fully capture key aspects of the return on an insurance venture, at least not the return to an equity investor or to the insurance company.

We proved results relating PVI/PVE and ROE to IRR and used these to provide new interpretations of IRR. We found that, with some simplifying assumptions, we could conveniently use a single average loss scenario to obtain the average return when the loss is a random variable. We then argued that these simplifying assumptions led to a conservative answer that was appropriate in the internal corporate context of our pricing analysis. With examples, we explored the sensitivity of our returns to changes in premium adequacy, Surplus level, interest rate, and payout pattern.

Our examination of the sensitivity of indicated profit provisions showed that these models should lead to reasonably responsive risk-sensitive prices for insurance products. The risk-sensitive pricing was obtained by using risk-sensitive Surplus requirements in conjunction with a fixed target return.

We have seen the Growth Model ROE emerge as a very strong contender to the IRR on Equity Flows. While there was not much of a difference in the results, the Growth Model allows us to directly relate product pricing to long-term calendar year ROE and growth rate targets. It also produces a calendar year premium-to-surplus leverage ratio for the Book of Business in equilibrium. This could be compared against industry benchmarks.

We have discussed why results from our models would differ from those of others such as the Risk-Adjusted Discounted Cash Flow model. This was done in an attempt to increase understanding. While some of our comments could be taken as critical, we have not gone so far as to say there is anything inappropriate about using other approaches in other contexts. In some regulatory situations, it may well be better to use the RA DCF model than any of the three we have presented.

There already is a significant body of literature on other ways of pricing in general[20] and on other ways of pricing insurance products in particular[21]. However, we feel we have demonstrated a methodology for deriving indicated prices that should be appropriate for internal corporate pricing analysis. We believe each of our three measures of return could reasonably be used in that context. Methods similar to ours are in common use and we hope our work furthers their acceptance. In conclusion, while we have left some theoretical questions unresolved and frequently adopted simplifying assumptions, we believe we have nonetheless demonstrated three variants of an approach to pricing that is both sound and practical.

[20] For example, the Black-Scholes formula for pricing options does not use a target return.

[21] See D'Arcy and Dyer [6], Derrig [7], and Robbin [13] for various alternative approaches to pricing property and casualty insurance products.

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Acknowledgement

The author gratefully acknowledges the contributions of C. W. (Walt) Stewart. It was Walt who first introduced the author to the key ideas that are the foundation of this paper.

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Exhibit 2	Sheet 2	Single Policy Model -Base Case Premium and FV Reserve-BOY/EOY Accounting
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Exhibit 6	Sheet 4	Single Policy Company Accounts Under Loss Scenario 3

Exhibit 1

Profit Measure Examples
Assumptions

Rates	
Investment Return	6.00%
Tax Rate	35.00%
PVI/PVE Discount Rate Selection	12.00%
Growth Rate Target	5.00%

Surplus Requirements	
Ratio to PV Unpaid Loss	31.5%
Rate for Discounting Unpaid Loss	6.00%

Patterns

Earning and Incurral

Year	Earned Premium	Full Value Incurred Loss	Stat Incurred Expense	GAAP Incurred Expense
0	0.0%	0.0%	60.0%	0.0%
1	100.0%	100.0%	40.0%	100.0%
2	0.0%	0.0%	0.0%	0.0%
3	0.0%	0.0%	0.0%	0.0%
4	0.0%	0.0%	0.0%	0.0%

Payment Patterns

Year	Paid Premium	Paid Loss	Paid Expense
0	75.0%	0.0%	30.0%
1	20.0%	25.0%	45.0%
2	5.0%	50.0%	20.0%
3	0.0%	25.0%	5.0%
4	0.0%	0.0%	0.0%
Total	100.0%	100.0%	100.0%
PV Factor (t=0)	0.9832	0.8908	0.9445

Underwriting

		Loss	Expense
Fixed		72.00	10.00
Variable		0.0%	20.0%

Exhibit 2
Sheet 1

Single Policy Company

UW Assumptions	Amount	Ratio	Financial Assumptions		IRR and PVI/PVE Results		
Premium	100.0	100.0%	Interest Rate	6.00%	IRR	10.74%	10.74%
Loss	72.0	72.0%	Tax Rate	35.00%	PVI/PVE Discount Rate	12.00%	10.74%
Expense	30.0	30.0%	Rsv Discount Rate	0.00%	PVI	6.05	6.10
Combined	102.0	102.0%	S as % of PV Unpaid Loss	31.50%	PVE	56.52	56.78
			PV Loss Discount for S Calc	6.00%	PVI/PVE	10.71%	10.74%

Year	Earned Premium	Incurred Loss	Stat Incurred Expense	Stat UW Income	Paid Premium	Paid Loss	Paid Expense	UW Cash Flow
0	0.0	0.0	18.0	-18.0	75.0	0.0	9.0	66.0
1	100.0	72.0	12.0	16.0	20.0	18.0	13.5	-11.5
2	0.0	0.0	0.0	0.0	5.0	36.0	6.0	-37.0
3	0.0	0.0	0.0	0.0	0.0	18.0	1.5	-19.5
4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Total	100.0	72.0	30.0	-2.0	100.0	72.0	30.0	-2.0

Year	Unearned Premium	Loss Reserve	PV Unpaid Loss	Stat Expense Reserve	Total Stat Reserves	Surplus	Assets	Receivables	Invested Assets	Inv Income
0	100.0	0.0	64.1	9.0	109.0	20.2	129.2	25.0	104.2	
1	0.0	54.0	50.0	7.5	61.5	15.7	77.2	5.0	72.2	6.3
2	0.0	18.0	17.0	1.5	19.5	5.3	24.8	0.0	24.8	4.3
3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.5
4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Year	DAC	GAAP Equity	GAAP Incurred Expense	GAAP UW Income	GAAP Pre-tax Income	Income Tax	Income	Change in Equity	Equity Flow
0	18.0	38.2	0.0	0.0	0.0	0.0	0.0	38.2	-38.2
1	0.0	15.7	30.0	-2.0	4.3	1.5	2.8	-22.5	25.2
2	0.0	5.3	0.0	0.0	4.3	1.5	2.8	-10.4	13.2
3	0.0	0.0	0.0	0.0	1.5	0.5	1.0	-5.3	6.3
4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Total			30.0	-2.0	10.1	3.5	6.6	0.0	6.6

Exhibit 2
Sheet 2

Single Policy Company- BOY and EOY Accounting

UW Assumptions	Amount	Ratio	Financial Assumptions		IRR and PVI/PVE Results		
Premium	100.0	100.0%	Interest Rate	6.00%	IRR	10.74%	10.74%
Loss	72.0	72.0%	Tax Rate	35.00%	PVI/PVE Discount Rate	12.00%	10.74%
Expense	30.0	30.0%	Rsv Discount Rate	0.00%	PVI	6.05	6.10
Combined	102.0	102.0%	S as % of PV Unpaid Loss	31.50%	PVE	56.52	56.78
			PV Loss Discount for S Calc	6.00%	PVI/PVE	10.71%	10.74%

Year	Earned Premium EOY	Incurred Loss EOY	GAAP Incurred Expense EOY	GAAP UW Income EOY	Paid Premium BOY	Paid Premium EOY	Paid Loss EOY	UW Cash Flow BOY	UW Cash Flow EOY
1	100.0	72.0	30.0	-2.0	75.0	20.0	18.0	66.0	-11.5
2	0.0	0.0	0.0	0.0	0.0	5.0	36.0	0.0	-37.0
3	0.0	0.0	0.0	0.0	0.0	0.0	18.0	0.0	-19.5
4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Year	Unearned Premium BOY	Unearned Premium EOY	Loss Reserve BOY	Loss Reserve EOY	Stat Expense Reserve BOY	Stat Expense Reserve EOY	Total Stat Reserves BOY	Total Stat Reserves EOY
1	100.0	0.0	0.0	54.0	9.0	7.5	109.0	61.5
2	0.0	0.0	54.0	18.0	7.5	1.5	61.5	19.5
3	0.0	0.0	18.0	0.0	1.5	0.0	19.5	0.0
4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Year	Surplus BOY	Surplus EOY	Assets BOY	Assets EOY	Receivables BOY	Receivables EOY	Invested Assets BOY	Investment Income EOY
1	20.2	15.7	129.2	77.2	25.0	5.0	104.2	6.3
2	15.7	5.3	77.2	24.8	5.0	0.0	72.2	4.3
3	5.3	0.0	24.8	0.0	0.0	0.0	24.8	1.5
4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Year	DAC BOY	DAC EOY	GAAP Equity BOY	GAAP Equity EOY	GAAP Pre-tax Income EOY	Income Tax EOY	GAAP Income EOY
1	18.0	0.0	38.2	15.7	4.3	1.5	2.8
2	0.0	0.0	15.7	5.3	4.3	1.5	2.8
3	0.0	0.0	5.3	0.0	1.5	0.5	1.0
4	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Exhibit 2
Sheet 3

Book of Business Growth Company

UW Assumptions	Amount	Ratio	Financial Assumptions		IRR and ROE Results	
Premium	100.0	100.0%	Interest Rate	6.00%	IRR	10.74%
Loss	72.0	72.0%	Tax Rate	35.00%	EQ Growth ROE	10.90%
Expense	30.0	30.0%	Rsv Discount Rate	0.00%	EQ Growth P/S	2.50
Combined	102.0	102.0%	S as % of PV Unpaid Loss	31.50%	Growth Rate	5.00%
			PV Loss Discount for S Calc	6.00%		

Year	Earned Premium EOY	Incurred Loss EOY	GAAP Incurred Expense EOY	GAAP UW Income EOY	Paid Premium BOY	Paid Premium EOY	Paid Loss EOY	UW Cash Flow BOY	UW Cash Flow EOY
1	100.0	72.0	30.0	-2.0	75.0	20.0	18.0	66.0	-11.5
2	105.0	75.6	31.5	-2.1	78.8	26.0	54.9	69.3	-49.1
3	110.3	79.4	33.1	-2.2	82.7	27.3	75.6	72.8	-71.0
4	115.8	83.3	34.7	-2.3	86.8	28.7	79.4	76.4	-74.6

Year	Unearned Premium BOY	Unearned Premium EOY	Loss Reserve BOY	Loss Reserve EOY	Stat Expense Reserve BOY	Stat Expense Reserve EOY	Total Stat Reserves BOY	Total Stat Reserves EOY
1	100.0	0.0	0.0	54.0	9.0	7.5	109.0	61.5
2	105.0	0.0	54.0	74.7	17.0	9.4	176.0	84.1
3	110.3	0.0	74.7	78.4	19.3	9.8	204.2	88.3
4	115.8	0.0	78.4	82.4	20.3	10.3	214.5	92.7

Year	Surplus BOY	Surplus EOY	Assets BOY	Assets EOY	Receivables BOY	Receivables EOY	Invested Assets BOY	Investment Income EOY	P/S
1	20.2	15.7	129.2	77.2	25.0	5.0	104.2	6.3	4.95
2	37.0	21.9	212.9	106.0	31.3	5.3	181.7	10.9	2.84
3	44.2	23.0	248.4	111.3	32.8	5.5	215.6	12.9	2.50
4	46.4	24.1	260.8	116.8	34.5	5.8	226.4	13.6	2.50

Year	DAC BOY	DAC EOY	GAAP Equity BOY	GAAP Equity EOY	GAAP Pre-tax Income EOY	Income Tax EOY	GAAP Income EOY	GAAP ROE
1	18.0	0.0	38.2	15.7	4.3	1.5	2.8	7.23%
2	18.9	0.0	55.9	21.9	8.8	3.1	5.7	10.24%
3	19.8	0.0	64.0	23.0	10.7	3.8	7.0	10.90%
4	20.8	0.0	67.2	24.1	11.3	3.9	7.3	10.90%

Exhibit 2
Sheet 4

Book of Business Growth Company

UW Assumptions	Amount	Ratio	Financial Assumptions		IRR and ROE Results	
Premium	100.0	100.0%	Interest Rate	6.00%	IRR	10.74%
Loss	72.0	72.0%	Tax Rate	35.00%	EQ Growth ROE	10.74%
Expense	30.0	30.0%	Rsv Discount Rate	0.00%	EQ Growth P/S	2.58
Combined	102.0	102.0%	S as % of PV Unpaid Loss	31.50%	Growth Rate	10.74%
			PV Loss Discount for S Calc	6.00%		

Year	Earned Premium EOY	Incurred Loss EOY	GAAP Incurred Expense EOY	GAAP UW Income EOY	Paid Premium BOY	Paid Premium EOY	Paid Loss EOY	UW Cash Flow BOY	UW Cash Flow EOY
1	100.0	72.0	30.0	-2.0	75.0	20.0	18.0	66.0	-11.5
2	110.7	79.7	33.2	-2.2	83.1	27.1	55.9	73.1	-49.7
3	122.6	88.3	36.8	-2.5	92.0	30.1	79.9	80.9	-74.6
4	135.8	97.8	40.7	-2.7	101.9	33.3	88.5	89.6	-82.6

Year	Unearned Premium BOY	Unearned Premium EOY	Loss Reserve BOY	Loss Reserve EOY	Stat Expense Reserve BOY	Stat Expense Reserve EOY	Total Stat Reserves BOY	Total Stat Reserves EOY
1	100.0	0.0	0.0	54.0	9.0	7.5	109.0	61.5
2	110.7	0.0	54.0	77.8	17.5	9.8	182.2	87.6
3	122.6	0.0	77.8	86.2	20.8	10.9	221.3	97.0
4	135.8	0.0	86.2	95.4	23.1	12.0	245.0	107.4

Year	Surplus BOY	Surplus EOY	Assets BOY	Assets EOY	Receivables BOY	Receivables EOY	Invested Assets BOY	Investment Income EOY	P/S
1	20.2	15.7	129.2	77.2	25.0	5.0	104.2	6.3	4.95
2	38.1	22.8	220.3	110.4	32.7	5.5	187.6	11.3	2.91
3	47.6	25.2	268.8	122.2	36.2	6.1	232.6	14.0	2.58
4	52.7	27.9	297.7	135.4	40.1	6.8	257.6	15.5	2.58

Year	DAC BOY	DAC EOY	GAAP Equity BOY	GAAP Equity EOY	GAAP Pre-tax Income EOY	Income Tax EOY	GAAP Income EOY	GAAP ROE
1	18.0	0.0	38.2	15.7	4.3	1.5	2.8	7.23%
2	19.9	0.0	58.0	22.8	9.0	3.2	5.9	10.13%
3	22.1	0.0	69.6	25.2	11.5	4.0	7.5	10.74%
4	24.4	0.0	77.1	27.9	12.7	4.5	8.3	10.74%

Exhibit 3
Sheet 1

Single Policy Company

UW Assumptions			Financial Assumptions		IRR and PVI/PVE Results		
	Amount	Ratio	Interest Rate	6.00%	IRR	10.99%	10.99%
Premium	100.0	100.0%	Tax Rate	35.00%	PVI/PVE Discount Rate	12.00%	10.99%
Loss	72.0	72.0%	Rsv Discount Rate	6.00%	PVI	6.22	6.23
Expense	30.0	30.0%	S as % of PV Unpaid Loss	31.50%	PVE	56.52	56.73
Combined	102.0	102.0%	PV Loss Discount for S Calc	6.00%	PVI/PVE	11.01%	10.99%

Year	Earned Premium	Incurred Loss	Stat Incurred Expense	Stat UW Income	Paid Premium	Paid Loss	Paid Expense	UW Cash Flow
0	0.0	0.0	18.0	-18.0	75.0	0.0	9.0	66.0
1	100.0	68.0	12.0	20.0	20.0	18.0	13.5	-11.5
2	0.0	3.0	0.0	-3.0	5.0	36.0	6.0	-37.0
3	0.0	1.0	0.0	-1.0	0.0	18.0	1.5	-19.5
4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Total	100.0	72.0	30.0	-2.0	100.0	72.0	30.0	-2.0

Year	Unearned Premium	Loss Reserve	PV Unpaid Loss	Stat Expense Reserve	Total Stat Reserves	Surplus	Assets	Receivables	Invested Assets	Inv Income
0	100.0	0.0	64.1	9.0	109.0	20.2	129.2	25.0	104.2	
1	0.0	50.0	50.0	7.5	57.5	15.7	73.2	5.0	68.2	6.3
2	0.0	17.0	17.0	1.5	18.5	5.3	23.8	0.0	23.8	4.1
3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.4
4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Year	DAC	GAAP Equity	GAAP Incurred Expense	GAAP UW Income	GAAP Pre-tax Income	Income Tax	Income	Change in Equity	Equity Flow
0	18.0	38.2	0.0	0.0	0.0	0.0	0.0	38.2	-38.2
1	0.0	15.7	30.0	2.0	8.3	2.9	5.4	-22.5	27.8
2	0.0	5.3	0.0	-3.0	1.1	0.4	0.7	-10.4	11.1
3	0.0	0.0	0.0	-1.0	0.4	0.1	0.3	-5.3	5.6
4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Total			30.0	-2.0	9.8	3.4	6.4	0.0	6.4

Exhibit 3
Sheet 2

Single Policy Company- BOY and EOY Accounting

UW Assumptions			Financial Assumptions		IRR and PVI/PVE Results		
	Amount	Ratio	Interest Rate	6.00%	IRR	10.99%	10.99%
Premium	100.0	100.0%	Tax Rate	35.00%	PVI/PVE Discount Rate	12.00%	10.99%
Loss	72.0	72.0%	Rsv Discount Rate	6.00%	PVI	6.22	6.23
Expense	30.0	30.0%	S as % of PV Unpaid Loss	31.50%	PVE	56.52	56.73
Combined	102.0	102.0%	PV Loss Discount for S Calc	6.00%	PVI/PVE	11.01%	10.99%

Year	Earned Premium EOY	Incurred Loss EOY	GAAP Incurred Expense EOY	GAAP UW Income EOY	Paid Premium BOY	Paid Premium EOY	Paid Loss EOY	UW Cash Flow BOY	UW Cash Flow EOY
1	100.0	68.0	30.0	2.0	75.0	20.0	18.0	66.0	-11.5
2	0.0	3.0	0.0	-3.0	0.0	5.0	36.0	0.0	-37.0
3	0.0	1.0	0.0	-1.0	0.0	0.0	18.0	0.0	-19.5
4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Year	Unearned Premium BOY	Unearned Premium EOY	Loss Reserve BOY	Loss Reserve EOY	Stat Expense Reserve BOY	Stat Expense Reserve EOY	Total Stat Reserves BOY	Total Stat Reserves EOY
1	100.0	0.0	0.0	50.0	9.0	7.5	109.0	57.5
2	0.0	0.0	50.0	17.0	7.5	1.5	57.5	18.5
3	0.0	0.0	17.0	0.0	1.5	0.0	18.5	0.0
4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Year	Surplus BOY	Surplus EOY	Assets BOY	Assets EOY	Receivables BOY	Receivables EOY	Invested Assets BOY	Investment Income EOY
1	20.2	15.7	129.2	73.2	25.0	5.0	104.2	6.3
2	15.7	5.3	73.2	23.8	5.0	0.0	68.2	4.1
3	5.3	0.0	23.8	0.0	0.0	0.0	23.8	1.4
4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Year	DAC BOY	DAC EOY	GAAP Equity BOY	GAAP Equity EOY	GAAP Pre-tax Income EOY	Income Tax EOY	GAAP Income EOY
1	18.0	0.0	38.2	15.7	8.3	2.9	5.4
2	0.0	0.0	15.7	5.3	1.1	0.4	0.7
3	0.0	0.0	5.3	0.0	0.4	0.1	0.3
4	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Exhibit 3
Sheet 3

Book of Business Growth Company

UW Assumptions	Amount	Ratio	Financial Assumptions		IRR and ROE Results	
Premium	100.0	100.0%	Interest Rate	6.00%	IRR	10.99%
Loss	72.0	72.0%	Tax Rate	35.00%	EQ Growth ROE	10.85%
Expense	30.0	30.0%	Rsv Discount Rate	6.00%	EQ Growth P/S	2.50
Combined	102.0	102.0%	S as % of PV Unpaid Loss	31.50%	Growth Rate	5.00%
			PV Loss Discount for S Calc	6.00%		

Year	Earned Premium EOY	Incurred Loss EOY	GAAP Incurred Expense EOY	GAAP UW Income EOY	Paid Premium BOY	Paid Premium EOY	Paid Loss EOY	UW Cash Flow BOY	UW Cash Flow EOY
1	100.0	68.0	30.0	2.0	75.0	20.0	18.0	66.0	-11.5
2	105.0	74.4	31.5	-0.9	78.8	26.0	54.9	69.3	-49.1
3	110.3	79.1	33.1	-1.9	82.7	27.3	75.6	72.8	-71.0
4	115.8	83.1	34.7	-2.0	86.8	28.7	79.4	76.4	-74.6

Year	Unearned Premium BOY	Unearned Premium EOY	Loss Reserve BOY	Loss Reserve EOY	Stat Expense Reserve BOY	Stat Expense Reserve EOY	Total Stat Reserves BOY	Total Stat Reserves EOY
1	100.0	0.0	0.0	50.0	9.0	7.5	109.0	57.5
2	105.0	0.0	50.0	69.5	17.0	9.4	171.9	78.8
3	110.3	0.0	69.5	72.9	19.3	9.8	199.0	82.8
4	115.8	0.0	72.9	76.6	20.3	10.3	209.0	86.9

Year	Surplus BOY	Surplus EOY	Assets BOY	Assets EOY	Receivables BOY	Receivables EOY	Invested Assets BOY	Investment Income EOY	P/S
1	20.2	15.7	129.2	73.2	25.0	5.0	104.2	6.3	4.95
2	37.0	21.9	208.9	100.7	31.3	5.3	177.6	10.7	2.84
3	44.2	23.0	243.2	105.8	32.8	5.5	210.4	12.6	2.50
4	46.4	24.1	255.3	111.0	34.5	5.8	220.9	13.3	2.50

Year	DAC BOY	DAC EOY	GAAP Equity BOY	GAAP Equity EOY	GAAP Pre-tax Income EOY	Income Tax EOY	GAAP Income EOY	GAAP ROE
1	18.0	0.0	38.2	15.7	8.3	2.9	5.4	14.07%
2	18.9	0.0	55.9	21.9	9.8	3.4	6.4	11.38%
3	19.8	0.0	64.0	23.0	10.7	3.7	6.9	10.85%
4	20.8	0.0	67.2	24.1	11.2	3.9	7.3	10.85%

Exhibit 3
Sheet 4

Book of Business Growth Company

UW Assumptions	Amount	Ratio	Financial Assumptions		IRR and ROE Results	
			Interest Rate	6.00%	IRR	10.99%
Premium	100.0	100.0%	Tax Rate	35.00%	EQ Growth ROE	10.99%
Loss	72.0	72.0%	Rsv Discount Rate	6.00%	EQ Growth P/S	2.58
Expense	30.0	30.0%	S as % of PV Unpaid Loss	31.50%	Growth Rate	10.99%
Combined	102.0	102.0%	PV Loss Discount for S Calc	6.00%		

Year	Earned Premium EOY	Incurred Loss EOY	GAAP Incurred Expense EOY	GAAP UW Income EOY	Paid Premium BOY	Paid Premium EOY	Paid Loss EOY	UW Cash Flow BOY	UW Cash Flow EOY
1	100.0	68.0	30.0	2.0	75.0	20.0	18.0	66.0	-11.5
2	111.0	78.5	33.3	-0.8	83.2	27.2	56.0	73.3	-49.8
3	123.2	88.1	37.0	-1.9	92.4	30.2	80.1	81.3	-74.7
4	136.7	97.8	41.0	-2.1	102.5	33.5	88.9	90.2	-82.9

Year	Unearned Premium BOY	Unearned Premium EOY	Loss Reserve BOY	Loss Reserve EOY	Stat Expense Reserve BOY	Stat Expense Reserve EOY	Total Stat Reserves BOY	Total Stat Reserves EOY
1	100.0	0.0	0.0	50.0	9.0	7.5	109.0	57.5
2	111.0	0.0	50.0	72.5	17.5	9.8	178.5	82.3
3	123.2	0.0	72.5	80.4	20.9	10.9	216.5	91.3
4	136.7	0.0	80.4	89.3	23.2	12.1	240.3	101.4

Year	Surplus BOY	Surplus EOY	Assets BOY	Assets EOY	Receivables BOY	Receivables EOY	Invested Assets BOY	Investment Income EOY	P/S
1	20.2	15.7	129.2	73.2	25.0	5.0	104.2	6.3	4.95
2	38.2	22.8	216.6	105.1	32.7	5.5	183.9	11.0	2.91
3	47.7	25.3	264.3	116.7	36.3	6.2	227.9	13.7	2.58
4	53.0	28.1	293.3	129.5	40.3	6.8	253.0	15.2	2.58

Year	DAC BOY	DAC EOY	GAAP Equity BOY	GAAP Equity EOY	GAAP Pre-tax Income EOY	Income Tax EOY	GAAP Income EOY	GAAP ROE
1	18.0	0.0	38.2	15.7	8.3	2.9	5.4	14.07%
2	20.0	0.0	58.1	22.8	10.3	3.6	6.7	11.48%
3	22.2	0.0	69.9	25.3	11.8	4.1	7.7	10.99%
4	24.6	0.0	77.6	28.1	13.1	4.6	8.5	10.99%

Exhibit 4
Sheet 1

Return Measures
Sensitivity to Premium **Full Value Reserve**

Scenario	1	2	3	4	5	6	7
Premium	80.00	85.00	90.00	95.00	100.00	105.00	110.00
Combined Ratio	122.50%	116.47%	111.11%	106.32%	102.00%	98.10%	94.55%
Resulting Growth Model P/S	2.00	2.12	2.25	2.37	2.50	2.62	2.75
Returns							
IRR	-7.00%	-2.74%	1.65%	6.15%	10.74%	15.40%	20.10%
PVI/PVE	-9.21%	-4.07%	0.96%	5.89%	10.71%	15.43%	20.05%
ROE	-8.47%	-3.47%	1.42%	6.21%	10.90%	15.49%	19.99%
Change in Returns							
IRR		4.27%	4.39%	4.50%	4.59%	4.66%	4.70%
PVI/PVE		5.14%	5.03%	4.92%	4.82%	4.72%	4.62%
ROE		5.00%	4.89%	4.79%	4.69%	4.59%	4.50%

Assumptions for All Scenarios

Financial

Interest Rate	6.00%
Tax Rate	35.00%
Reserve Discount Rate	0.00%
Surplus as % of PV Unpaid Loss	31.50%
Rate for PV Calculation	6.00%
Rate for PVI/PVE Discounting	12.00%
ROE Growth Rate	5.00%

Assumptions for All Scenarios

Underwriting

Premium		Loss		Expense	
	varies	Fixed	72.00	Fixed	10.00
				Variable	20.0%
Premium Payment		Loss Payout		Expense Payout	
Year	%	Year	%	Year	%
0	75.0%	0	0.0%	0	30.0%
1	20.0%	1	25.0%	1	50.0%
2	5.0%	2	50.0%	2	25.0%
3	0.0%	3	25.0%	3	0.0%
4	0.0%	4	0.0%	4	0.0%

Exhibit 4
Sheet 2

Return Measures
Sensitivity to Premium **Discounted Reserve**

Scenario	1	2	3	4	5	6	7
Premium	80.00	85.00	90.00	95.00	100.00	105.00	110.00
Combined Ratio	122.50%	116.47%	111.11%	106.32%	102.00%	98.10%	94.55%
Resulting Growth Model P/S	2.00	2.12	2.25	2.37	2.50	2.62	2.75
Returns							
IRR	-7.74%	-3.23%	1.42%	6.16%	10.99%	15.87%	20.79%
PVI/PVE	-8.89%	-3.75%	1.27%	6.19%	11.01%	15.73%	20.34%
ROE	-8.52%	-3.53%	1.36%	6.15%	10.85%	15.44%	19.94%
Change in Returns							
IRR		4.52%	4.64%	4.75%	4.83%	4.88%	4.92%
PVI/PVE		5.14%	5.03%	4.92%	4.82%	4.72%	4.62%
ROE		5.00%	4.89%	4.79%	4.69%	4.59%	4.50%

Assumptions for All Scenarios

Financial

Interest Rate	6.00%
Tax Rate	35.00%
Reserve Discount Rate	6.00%
Surplus as % of PV Unpaid Loss	31.50%
Rate for PV Calculation	6.00%
Rate for PVI/PVE Discounting	12.00%
ROE Growth Rate	5.00%

Assumptions for All Scenarios

Underwriting

Premium		Loss		Expense	
	varies	Fixed	72.00	Fixed	10.00
				Variable	20.0%
Premium Payment		Loss Payout		Expense Payout	
Year	%	Year	%	Year	%
0	75.0%	0	0.0%	0	30.0%
1	20.0%	1	25.0%	1	50.0%
2	5.0%	2	50.0%	2	25.0%
3	0.0%	3	25.0%	3	0.0%
4	0.0%	4	0.0%	4	0.0%

Exhibit 4
Sheet 3

Return Measures
Sensitivity to Surplus

Scenario	1	2	3	4	5	6	7
Surplus as % of PV Unpaid Loss	25.50%	27.50%	29.50%	31.50%	33.50%	35.50%	37.50%
Resulting Growth Model P/S	3.08	2.86	2.67	2.50	2.35	2.22	2.10
Returns							
IRR	11.73%	11.37%	11.04%	10.74%	10.46%	10.21%	9.97%
PVI/PVE	11.72%	11.35%	11.02%	10.71%	10.42%	10.16%	9.92%
ROE	11.96%	11.57%	11.22%	10.90%	10.60%	10.33%	10.09%

Assumptions for All Scenarios

Financial

Interest Rate	6.00%
Tax Rate	35.00%
Reserve Discount Rate	0.00%
Surplus as % of PV Unpaid Loss	varies
Rate for PV Calculation	6.00%
Rate for PVI/PVE Discounting	12.00%
ROE Growth Rate	5.00%

Assumptions for All Scenarios

Underwriting

Premium		Loss		Expense	
Fixed	100.00	Fixed	72.00	Fixed	10.00
				Variable	20.0%
Premium Payment		Loss Payout		Expense Payout	
Year	%	Year	%	Year	%
0	75.0%	0	0.0%	0	30.0%
1	20.0%	1	25.0%	1	50.0%
2	5.0%	2	50.0%	2	25.0%
3	0.0%	3	25.0%	3	0.0%
4	0.0%	4	0.0%	4	0.0%

Exhibit 4
Sheet 4

Return Measures
Sensitivity to Interest Rate

Scenario	1	2	3	4	5	6	7
Interest Rate	4.50%	5.00%	5.50%	6.00%	6.50%	7.00%	7.50%
Resulting Growth Model P/S	2.44	2.46	2.48	2.50	2.52	2.53	2.55
Returns							
IRR	7.48%	8.56%	9.65%	10.74%	11.84%	12.93%	14.04%
PVI/PVE	7.38%	8.48%	9.59%	10.71%	11.83%	12.96%	14.10%
ROE	7.54%	8.65%	9.77%	10.90%	12.03%	13.18%	14.33%

Assumptions for All Scenarios

Financial

Interest Rate	varies
Tax Rate	35.00%
Reserve Discount Rate	0.00%
Surplus as % of PV Unpaid Loss	31.50%
Rate for PV Calculation	varies
Rate for PVI/PVE Discounting	12.00%
ROE Growth Rate	5.00%

Assumptions for All Scenarios

Underwriting

Premium		Loss		Expense	
Fixed	100.00	Fixed	72.00	Fixed	10.00
				Variable	20.0%
Premium Payment		**Loss Payout**		**Expense Payout**	
Year	%	Year	%	Year	%
0	75.0%	0	0.0%	0	30.0%
1	20.0%	1	25.0%	1	50.0%
2	5.0%	2	50.0%	2	25.0%
3	0.0%	3	25.0%	3	0.0%
4	0.0%	4	0.0%	4	0.0%

Exhibit 4
Sheet 5

Return Measures
Sensitivity to Payout Pattern

Scenario		1=Base	2	3	4	5	6	7
Loss Pattern	Year							
	0	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	1	25.00%	100.00%	50.00%	0.00%	0.00%	0.00%	0.00%
	2	50.00%	0.00%	50.00%	100.00%	50.00%	0.00%	0.00%
	3	25.00%	0.00%	0.00%	0.00%	50.00%	100.00%	50.00%
	4	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	50.00%
Surplus % of PV Unpaid Loss		31.50%	58.96%	40.72%	31.10%	25.68%	21.87%	19.32%
Resulting Growth Model P/S		2.50	2.50	2.50	2.50	2.50	2.50	2.50
Indicated Profit Margins								
IRR Method		10.74%	6.34%	8.60%	10.82%	12.85%	14.83%	16.61%
PVI/PVE Method		10.71%	6.33%	8.55%	10.79%	12.88%	14.97%	16.92%
ROE Method		10.90%	6.35%	8.65%	10.95%	13.15%	15.34%	17.43%

Assumptions for All Scenarios

Financial

Interest Rate	6.00%
Tax Rate	35.00%
Reserve Discount Rate	0.00%
Surplus as % of PV Unpaid Loss	varies
Rate for PV Calculation	12.00%
Rate for PVI/PVE Discounting	12.00%
Growth Rate	5.00%

Assumptions for All Scenarios

Underwriting

Premium		Loss		Expense	
		Fixed	72.00	Fixed	10.00
				Variable	20.0%
Premium Payment		Loss Payout		Expense Payout	
Year	%	Year	%	Year	%
0	75.0%	0	varies	0	30.0%
1	20.0%	1	varies	1	45.0%
2	5.0%	2	varies	2	20.0%
3	0.0%	3	varies	3	5.0%
4	0.0%	4	varies	4	0.0%

Exhibit 5
Sheet 1

Indicated Profit
Sensitivity to Surplus

Scenario	1	2	3	4	5	6	7
Surplus as % of PV Unpaid Loss	25.50%	27.50%	29.50%	31.50%	33.50%	35.50%	37.50%
Resulting Growth Model P/S	3.09	2.87	2.69	2.53	2.38	2.26	2.15
Indicated Profit Margins							
IRR Method	-1.79%	-1.49%	-1.20%	-0.90%	-0.61%	-0.32%	-0.03%
PVI/PVE Method	-1.79%	-1.49%	-1.20%	-0.90%	-0.61%	-0.32%	-0.03%
ROE Method	-1.97%	-1.65%	-1.34%	-1.04%	-0.73%	-0.43%	-0.13%

Assumptions for All Scenarios

Financial

Interest Rate	6.00%
Tax Rate	35.00%
Reserve Discount Rate	0.00%
Surplus as % of PV Unpaid Loss	varies
Rate for PV Calculation	6.00%
IRR Target Return	12.00%
PVI/PVE Target Return	12.00%
Rate for PVI/PVE Discounting	12.00%
ROE Target Return	12.00%
ROE Target Growth Rate	5.00%

Assumptions for All Scenarios

Underwriting

Premium		Loss		Expense	
		Fixed	72.00	Fixed	10.00
				Variable	20.0%
Premium Payment		Loss Payout		Expense Payout	
Year	%	Year	%	Year	%
0	75.0%	0	0.0%	0	30.0%
1	20.0%	1	25.0%	1	25.0%
2	5.0%	2	50.0%	2	50.0%
3	0.0%	3	25.0%	3	25.0%
4	0.0%	4	0.0%	4	0.0%

Exhibit 5
Sheet 2

Indicated Profit
Sensitivity to Interest Rate

Scenario	1	2	3	4	5	6	7
Interest Rate	4.50%	5.00%	5.50%	6.00%	6.50%	7.00%	7.50%
Resulting Growth Model P/S	2.56	2.55	2.54	2.53	2.52	2.50	2.49
Indicated Profit Margins							
IRR Method	1.91%	0.98%	0.05%	-0.90%	-1.86%	-2.82%	-3.80%
PVI/PVE Method	1.91%	0.98%	0.05%	-0.90%	-1.86%	-2.82%	-3.80%
ROE Method	1.88%	0.92%	-0.05%	-1.04%	-2.03%	-3.03%	-4.05%

Assumptions for All Scenarios

Financial

Interest Rate	varies
Tax Rate	35.00%
Reserve Discount Rate	0.00%
Surplus as % of PV Unpaid Loss	31.50%
Rate for PV Calculation	varies
IRR Target Return	12.00%
PVI/PVE Target Return	12.00%
Rate for PVI/PVE Discounting	12.00%
ROE Target Return	12.00%
ROE Target Growth Rate	5.00%

Assumptions for All Scenarios

Underwriting

Premium		Loss		Expense	
		Fixed	72.00	Fixed	10.00
				Variable	20.0%
Premium Payment		Loss Payout		Expense Payout	
Year	%	Year	%	Year	%
0	75.0%	0	0.0%	0	30.0%
1	20.0%	1	25.0%	1	25.0%
2	5.0%	2	50.0%	2	50.0%
3	0.0%	3	25.0%	3	25.0%
4	0.0%	4	0.0%	4	0.0%

Exhibit 5
Sheet 3

Indicated Profit
Sensitivity to Payout Pattern

Scenario		1=Base	2	3	4	5	6	7
Loss Pattern	Year							
	0	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	1	25.00%	100.00%	50.00%	0.00%	0.00%	0.00%	0.00%
	2	50.00%	0.00%	50.00%	100.00%	50.00%	0.00%	0.00%
	3	25.00%	0.00%	0.00%	0.00%	50.00%	100.00%	50.00%
	4	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	50.00%
Surplus % of PV Unpaid Loss		31.50%	62.00%	41.76%	31.08%	25.03%	20.77%	17.90%
Resulting Growth Model P/S		2.53	2.53	2.53	2.53	2.53	2.53	2.53
Indicated Profit Margins								
IRR Method		-0.90%	2.96%	1.02%	-0.97%	-2.88%	-4.85%	-6.72%
PVI/PVE Method		-0.90%	2.96%	1.02%	-0.97%	-2.88%	-4.85%	-6.72%
ROE Method		-1.04%	2.94%	0.98%	-1.09%	-3.16%	-5.34%	-7.52%

Assumptions for All Scenarios

Financial

Interest Rate	6.00%
Tax Rate	35.00%
Reserve Discount Rate	0.00%
Surplus as % of PV Unpaid Loss	varies
Rate for PV Calculation	6.00%
IRR Target Return	12.00%
PVI/PVE Target Return	12.00%
Rate for PVI/PVE Discounting	12.00%
ROE Target Return	12.00%
ROE Target Growth Rate	5.00%

Assumptions for All Scenarios

Underwriting

Premium		Loss		Expense	
		Fixed	72.00	Fixed	10.00
				Variable	20.0%
Premium Payment		Loss Payout		Expense Payout	
Year	%	Year	%	Year	%
0	75.0%	0	varies	0	30.0%
1	20.0%	1	varies	1	45.0%
2	5.0%	2	varies	2	20.0%
3	0.0%	3	varies	3	5.0%
4	0.0%	4	varies	4	0.0%

Exhibit 6
Sheet 1

Results for Three Point Loss Distribution
Sensitivity to Premium   Full Value Reserve

Scenario	1			2			3			Average over All Scenarios		
Probability	40.00%			40.00%			20.00%					
Premium	100.00			100.00			100.00			100.00		
Loss	60.00			72.00			96.00			72.00		
Combined Ratio	90.00%			102.00%			126.00%			102.00%		
Returns												
IRR	24.11%			10.74%			-11.63%			10.74%		
PVI/PVE	23.79%			10.71%			-15.45%			10.71%		
Results by Year	Equity	Income	Equity Flow	Equity	Income	Equity Flow	Equity	Income	Equity Flow	Equity	Income	Equity Flow
Year												
0	38.20	0.00	-38.20	38.20	0.00	-38.20	38.20	0.00	-38.20	38.20	0.00	-38.20
1	15.74	10.56	33.02	15.74	2.76	25.22	15.74	-12.84	9.62	15.74	2.76	25.22
2	5.35	2.47	12.86	5.35	2.82	13.21	5.35	3.52	13.91	5.35	2.82	13.21
3	0.00	0.85	6.20	0.00	0.97	6.32	0.00	1.20	6.55	0.00	0.97	6.32

Exhibit 6
Sheet 2

Single Policy Company **Loss Scenario 1**

E[Loss]	72.0

UW Assumptions	Amount	Ratio	Financial Assumptions		IRR and PVI/PVE Results	
Premium	100.0	100.0%	Interest Rate	6.00%	IRR	24.11%
Loss	60.0	60.0%	Tax Rate	35.00%	PVI/PVE Discount Rate	12.00%
Expense	30.0	30.0%	Rsv Discount Rate	0.00%	PVI	13.45
Combined	90.0	90.0%	S as % of E[PV Unpaid Loss]	31.50%	PVE	56.52
			PV Loss Discount for S Calc	6.00%	PVI/PVE	23.79%

Year	Earned Premium	Incurred Loss	Stat Incurred Expense	Stat UW Income	Paid Premium	Paid Loss	Paid Expense	UW Cash Flow
0	0.0	0.0	18.0	-18.0	75.0	0.0	9.0	66.0
1	100.0	60.0	12.0	28.0	20.0	15.0	13.5	-8.5
2	0.0	0.0	0.0	0.0	5.0	30.0	6.0	-31.0
3	0.0	0.0	0.0	0.0	0.0	15.0	1.5	-16.5
4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Total	100.0	60.0	30.0	10.0	100.0	60.0	30.0	10.0

Year	Unearned Premium	Loss Reserve	Expected PV Unpaid Loss	Stat Expense Reserve	Total Stat Reserves	Surplus	Assets	Receivables	Invested Assets	Inv Income
0	100.0	0.0	64.1	9.0	109.0	20.2	129.2	25.0	104.2	
1	0.0	45.0	50.0	7.5	52.5	15.7	68.2	5.0	63.2	6.3
2	0.0	15.0	17.0	1.5	16.5	5.3	21.8	0.0	21.8	3.8
3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.3
4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Year	DAC	GAAP Equity	GAAP Incurred Expense	GAAP UW Income	GAAP Pre-tax Income	Income Tax	Income	Change in Equity	Equity Flow
0	18.0	38.2	0.0	0.0	0.0	0.0	0.0	38.2	-38.2
1	0.0	15.7	30.0	10.0	16.3	5.7	10.6	-22.5	33.0
2	0.0	5.3	0.0	0.0	3.8	1.3	2.5	-10.4	12.9
3	0.0	0.0	0.0	0.0	1.3	0.5	0.9	-5.3	6.2
4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Total			30.0	10.0	21.4	7.5	13.9	0.0	13.9

Exhibit 6
Sheet 3

Single Policy Company **Loss Scenario 2**

E[Loss]	72.0

UW Assumptions			Financial Assumptions		IRR and PVI/PVE Results	
	Amount	Ratio	Interest Rate	6.00%	IRR	10.74%
Premium	100.0	100.0%	Tax Rate	35.00%	PVI/PVE Discount Rate	12.00%
Loss	72.0	72.0%	Rsv Discount Rate	0.00%	PVI	6.05
Expense	30.0	30.0%	S as % of E[PV Unpaid Loss]	31.50%	PVE	56.52
Combined	102.0	102.0%	PV Loss Discount for S Calc	6.00%	PVI/PVE	10.71%

Year	Earned Premium	Incurred Loss	Stat Incurred Expense	Stat UW Income	Paid Premium	Paid Loss	Paid Expense	UW Cash Flow
0	0.0	0.0	18.0	-18.0	75.0	0.0	9.0	66.0
1	100.0	72.0	12.0	16.0	20.0	18.0	13.5	-11.5
2	0.0	0.0	0.0	0.0	5.0	36.0	6.0	-37.0
3	0.0	0.0	0.0	0.0	0.0	18.0	1.5	-19.5
4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Total	100.0	72.0	30.0	-2.0	100.0	72.0	30.0	-2.0

Year	Unearned Premium	Loss Reserve	Expected PV Unpaid Loss	Stat Expense Reserve	Total Stat Reserves	Surplus	Assets	Receivables	Invested Assets	Inv Income
0	100.0	0.0	64.1	9.0	109.0	20.2	129.2	25.0	104.2	
1	0.0	54.0	50.0	7.5	61.5	15.7	77.2	5.0	72.2	6.3
2	0.0	18.0	17.0	1.5	19.5	5.3	24.8	0.0	24.8	4.3
3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.5
4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Year	DAC	GAAP Equity	GAAP Incurred Expense	GAAP UW Income	GAAP Pre-tax Income	Income Tax	Income	Change in Equity	Equity Flow
0	18.0	38.2	0.0	0.0	0.0	0.0	0.0	38.2	-38.2
1	0.0	15.7	30.0	-2.0	4.3	1.5	2.8	-22.5	25.2
2	0.0	5.3	0.0	0.0	4.3	1.5	2.8	-10.4	13.2
3	0.0	0.0	0.0	0.0	1.5	0.5	1.0	-5.3	6.3
4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Total			30.0	-2.0	10.1	3.5	6.6	0.0	6.6

Exhibit 6
Sheet 4

Single Policy Company **Loss Scenario 3**

E[Loss]	72.0

UW Assumptions			Financial Assumptions		IRR and PVI/PVE Results	
	Amount	Ratio	Interest Rate	6.00%	IRR	-11.63%
Premium	100.0	100.0%	Tax Rate	35.00%	PVI/PVE Discount Rate	12.00%
Loss	96.0	96.0%	Rsv Discount Rate	0.00%	PVI	-8.73
Expense	30.0	30.0%	S as % of E[PV Unpaid Loss]	31.50%	PVE	56.52
Combined	126.0	126.0%	PV Loss Discount for S Calc	6.00%	PVI/PVE	-15.45%

Year	Earned Premium	Incurred Loss	Stat Incurred Expense	Stat UW Income	Paid Premium	Paid Loss	Paid Expense	UW Cash Flow
0	0.0	0.0	18.0	-18.0	75.0	0.0	9.0	66.0
1	100.0	96.0	12.0	-8.0	20.0	24.0	13.5	-17.5
2	0.0	0.0	0.0	0.0	5.0	48.0	6.0	-49.0
3	0.0	0.0	0.0	0.0	0.0	24.0	1.5	-25.5
4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Total	100.0	96.0	30.0	-26.0	100.0	96.0	30.0	-26.0

Year	Unearned Premium	Loss Reserve	Expected PV Unpaid Loss	Stat Expense Reserve	Total Stat Reserves	Surplus	Assets	Receivables	Invested Assets	Inv Income
0	100.0	0.0	64.1	9.0	109.0	20.2	129.2	25.0	104.2	
1	0.0	72.0	50.0	7.5	79.5	15.7	95.2	5.0	90.2	6.3
2	0.0	24.0	17.0	1.5	25.5	5.3	30.8	0.0	30.8	5.4
3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.9
4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Year	DAC	GAAP Equity	GAAP Incurred Expense	GAAP UW Income	GAAP Pre-tax Income	Income Tax	Income	Change in Equity	Equity Flow
0	18.0	38.2	0.0	0.0	0.0	0.0	0.0	38.2	-38.2
1	0.0	15.7	30.0	-26.0	-19.7	-6.9	-12.8	-22.5	9.6
2	0.0	5.3	0.0	0.0	5.4	1.9	3.5	-10.4	13.9
3	0.0	0.0	0.0	0.0	1.9	0.6	1.2	-5.3	6.6
4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Total			30.0	-26.0	-12.5	-4.4	-8.1	0.0	-8.1

THE UNDERWRITING PROFIT PROVISION

by

Dr. Ira Robbin

THE UNDERWRITING PROFIT PROVISION

By Dr. Ira Robbin

OUTLINE

ACKNOWLEDGEMENT

Much credit goes to David Ruhm, David Russell, and Allison Bizardi for the invaluable support they gave in constructing spreadsheets, devising examples, and fashioning output exhibits. They also served as able editors and ghost writers. Their many hours of effort are much appreciated.

DISCLAIMER

The views expressed in this paper are solely those of the author. They are not necessarily in accord with the positions or practices of CIGNA Corporation.

THE UNDERWRITING PROFIT PROVISION

I. INTRODUCTION

A. Underwriting Profit Provisions and Actual Underwriting Profits

This paper is a presentation of methods used for computing the provision for underwriting profit in property and casualty insurance rates. The provision for underwriting profit is one component of an actuarially derived premium rate. Adding the provision for underwriting profit to the sum of provisions for losses and expenses yields the total premium rate. Here the loss provision for property lines is assumed to include an adequate load for the long-run expectation of catastrophe losses. If the premium rate is greater than the sum of the loss and expense provisions, then the underwriting profit provision is positive. The underwriting profit provision could also be negative.

Actual underwriting profit is the difference between the premium and the sum of losses and expenses. Since the rating provisions for losses and expenses may differ from the actual losses and expenses, the underwriting profit provision may differ from the actual underwriting profit.

B. Historical Overview

There are a variety of methods in use for determining profit provisions in state rate filings and for calculating profit targets. The methods are often based on different fundamental approaches and employ different sorts of data. It is instructive to review the history to see how such a situation developed. From 1921 until the 1960's, a 5% underwriting profit provision was accepted as appropriate for most lines of insurance, with the exception of Workers' Compensation which had a 2.5% load. Though they had no strong theoretical justification, these loads were used for many years without serious challenge. However, this changed in the 70's and early 80's. The impact of investment income and income taxes on the bottom line became increasingly important. Some companies apparently prospered even though actual underwriting profits were less than those provided for in the manual rates. Others even operated at an underwriting loss.

Meanwhile the rising price of insurance made the affordability of insurance a contentious issue in state politics. In some jurisdictions, the controversy led to changes in the calculation of the underwriting profit provision. The changes were often intended to simply lower the price of insurance by lowering the rates. Despite the evidence of the early 80's that some insurance companies would compete on price to the point of insolvency, the tide of increased profit provision regulation continued unabated.

As a result, the ratemaking actuary now faces a variety of algorithms for computing the underwriting profit provision. In some states, an investment income percentage is subtracted from a more traditional provision to yield the final profit figure. In others, the provision is set so that it would theoretically give the insurance company a total rate of return deemed proper by regulators.

C. Underwriting Profit and Total Profit

To understand the debate about the underwriting profit provision, one needs a clear understanding of the distinction between total profit and underwriting profit. The total profit for an insurance company is roughly the sum of underwriting gains plus investment gains less income taxes. The investment gains include the sum of interest, dividends, and real estate income plus realized capital gains. If the investment income is large enough, the company can make an overall profit, even if underwriting profits are negative. Negative underwriting profits can also reduce income taxes by offsetting otherwise taxable investment income.

In principle, one might want to define an underwriting profit provision as adequate if it would lead to an adequate total profit net of investment income and income taxes. Though this total return concept sounds straightforward, it is not obvious how to apply it in ratemaking. It also raises the question of what is an adequate total return.

Perhaps the major problem in applying a total return approach to ratemaking is that ratemaking is done on a prospective policy year basis, while total return is most commonly measured on a calendar year basis. The two perspectives are not synonymous. Generally Accepted Accounting Principles Return on Equity (GAAP ROE) is the usual measure of return for a stock insurance company. GAAP ROE is a calendar year return defined as the ratio of calendar year GAAP income over GAAP equity. A key point about calendar year return is that it is partly dependent on the past; the return in any calendar year includes income attributable to prior writings. However, rates are made looking to the future and rates made today will influence earnings in many subsequent calendar years. How should one define a policy year return and how should that return relate to calendar year return on equity ? What is the income and equity for a policy year (or a policy) and how does one account for the time value of money in figuring out the return? Even if such a return could be defined, how does one select an appropriate target return?

A different but related approach to selecting an appropriate underwriting profit provision is to set rates with an explicit offset for investment income on policyholder-supplied funds. Policyholder-supplied funds could be conceptually viewed as the accumulated balance of paid premiums less the sum of loss and expense payments. (One should also subtract any declared profits, as these have presumably been paid out as stockholder dividends or put into surplus used to back further writings.) The relative amount of policyholder-supplied funds by line is related to the time lag between the receipt of premiums and the payout of losses and expenses. The investment income offset due to policyholder-supplied funds could be estimated by projecting underwriting cash flows prospectively <u>or</u> by looking at calendar year data. When taking a calendar year approach, one must estimate what portion of the total investment income is generated by policyholder-supplied funds. The remaining portion is earned on stockholder-supplied funds, and some argue it therefore should not be credited to the policyholders.

The stockholder-supplied funds include the insurance company's surplus plus amounts needed to offset the portion of the loss and expense reserves not covered by premiums. The stockholders may have also supplied some cash to cover pre-paid expenses, but these sums, being already paid out, are not available for investment.

The conceptual split between policyholder and stockholder supplied funds also helps in gauging the adequacy of a given total return. As previously mentioned, it is possible for a company to have a positive total profit even if underwriting profits are negative. However, a positive total profit does not necessarily mean a stock company has generated a fair return, or even a return above breakeven, for its stockholders. For example, if the company must use some of the investment income on stockholder-supplied funds to cover underwriting losses, the total profit might still be positive but the return would be below breakeven.

D. Five Types of Underwriting Profit

To clarify matters, distinctions should be made between various types of underwriting profit.

(1) There are **underwriting profit provisions** included in **manual rates** and in rate filings to change manual rates.

(2) There are **corporate target underwriting profit provisions** which insurance company management may request from the actuary as information when deciding how much to charge. For a stock company, these are presumably sufficient to yield an expected return to stockholders comparable to the market yield on alternative investments of similar risk.

(3) Related to target profit provisions are **breakeven underwriting profit provisions**, defined to generate an expected return to stockholders equal to the rate of return on risk-free investments such as U.S government bonds. A breakeven profit provision provides the stockholder no compensation for putting his/her funds at risk of having to cover underwriting losses. This ignores the risk-return trade-off (higher return for higher risk) central to modern financial theory.

(4) The **charged underwriting profit provision** may differ from the provision in the manual rate as well as from the target and the breakeven provisions. The charged rate is obtained by applying experience and schedule rating modifications and other adjustments (eg. premium discount in Workers' Compensation) to the manual rate.

(5) **Actual underwriting profits** may differ from the provisions charged because the provisions for losses and expenses will seldom be exactly accurate and because actual catastrophe losses rarely match catastrophe provisions in any one year. If actual underwriting profits are consistently below the provisions charged, then the methods used to estimate losses and expenses should be examined for bias.

The focus of this paper will be on methods used for determining profit provisions in state rate filings or for calculating profit targets and breakeven profit provisions. However, it is beyond the scope of this paper to give any advice on what should be charged or to provide detailed analysis of actual profits.

E. Regulation

Underlying the controversy about the profit load provision is a more profound disagreement about the theory under which rates and profit provisions should be regulated. Under a "rate of return regulation" approach, the state ought to stringently regulate premium rates so that companies should be able to anticipate adequate, but not excessive, total returns. The definition of an adequate target return presumably reflects relative risk by line of business. This approach has been justified by arguing that insurance company rates should be regulated in a fashion similar to the way that utility company rates are regulated. Utility companies enjoy a natural monopoly position and their rates are rather strictly controlled. Under the Hope Decision, utility rates are supposed to be set so that the utility company should be able to achieve a rate of return commensurate with the risk involved.

Under a constrained free market theory, if the manual rate is reasonably adequate and if there is some limited flexibility to price insurance at a level different from the manual rate, then the competitive market should force the price of insurance to the optimal level. Within this context, the manual rates can be thought of as a starting point against which adjustments can be made by individual companies responding to the dynamics of the market. If the manual rate is set too low and overly stringent state regulation of prices prevents competitive adjustments, market disruptions could theoretically result.

The analogy between utilities and insurance companies has also been criticized. The insurance business is not a natural monopoly; nor is it a highly concentrated oligopoly. The large number of firms and the relatively low market share of even the biggest insurers are cited as evidence that a competitive market model is the appropriate one.

This brief and incomplete discussion of profit provision regulation theories is only intended to make one aware that there are fundamental differences in approaches to the issue.

F. Different Models

No matter what side of the issue one takes, the actuary should still be versed in the calculation of underwriting profit provisions. In a rate filing context, the calculation of the profit provision should conform with applicable state regulation. In a competitive environment, the actuary should be able to provide company management with the target profit provision needed to achieve an adequate total return.

After some preliminaries, seven models will be presented. Variants of some of these models have been used or mandated for use in state rate filings, but the specifics of state regulation will not be discussed. Simple examples will be worked out with each of these models, and some limited comparisons will be made. However, since not all of the models use the same type of data, and since the choice of parameters is often crucial, one should take extreme care in generalizing from these examples.

The first algorithm is a **Calendar Year Investment Offset Procedure**. In this procedure, the traditional profit load is reduced by subtracting an investment figure derived from calendar year data. The second algorithm, the **Present Value Offset Procedure**, also features an offset to the traditional profit load. Here the offset reflects the difference in the present values of the loss payment patterns of the line of business under review and some short-tailed reference line. The third algorithm, the **Calendar Year Return Method**, does not use the traditional profit load. Instead, the profit provision is set so as to achieve a selected target return, where return is calculated with investment income figures derived from calendar year data.

In the fourth model, the **Present Value of Income over the Present Value of Equity** method, a present value return is defined as the ratio of the present value of accounting income over the annualized present value of equity. The underwriting profit is adjusted until the present value return is equal to a target return.

In the fifth algorithm, the **Present Value Return on Cash Flow** model, the underwriting profit provision is set so that the present value of the underwriting cash flows and investment income on investible equity less income tax payments is equal to the present value of the changes in equity. In this algorithm, the underwriting cash flows, investment income on investible equity, and tax payments are discounted using the rate of return on investments, while the changes in equity are discounted at the target rate of return.

The sixth algorithm, the **Risk-Adjusted Discounted Cash Flow Method**, does not use the traditional profit load nor a target return. Rather, the premium is calculated so that the present value of premium payments will equal the present value of paid losses, expenses, and income taxes. However, since the loss payments are not known with certainty, they are discounted at a new money risk-adjusted rate which is lower than the new money risk-free rate.

Finally, an **Internal Rate of Return on Equity Flows** algorithm will be presented. The strategy in this algorithm is to estimate flows of money between a hypothetical stockholder and a hypothetical company formed to write one policy. These "equity flows" are related to the projected stream of total income and the assumed surplus requirements associated with the writing of the policy. The profit provision is found by adjusting the premium until the yield on the equity flows is equal to a target return.

Which, if any, of the models is "right" is not a question that can be easily decided. Each method has its adherents. Implicit in the use of any model in a rate regulatory context is a theory on the role of the state in controlling insurance prices and assumptions about the competitiveness of the market. Once beyond such overall contextual issues, there are a multitude of modelling questions to consider. Should surplus should be in the model, and if so, how should the surplus requirement be determined? Should risk be reflected in the model, and if so, how ? Is it better to use cash flows or income flows? How does one reflect income taxes? After questions of model construction are resolved, there are parameter selection issues. What is the appropriate rate for discounting the flows? Should new money or embedded yields be used? What is the right target return?

Even if one is only considering profit targets so that the role of the state can be momentarily ignored, there is no simple empirical test of which model is better. While one can compare profit targets with actual profits, a judgement on the adequacy of actual profits depends on some underlying economic theory. In other words, a model that accurately estimates what underwriting profits were does not necessarily say what they should have been. This is conceptually different from the situation encountered when evaluating the performance of algorithms for estimating losses and expenses. With loss and expense estimation procedures, one could, in principle, compare the differing prior estimates against subsequent results. No external theory would be required.

In summary, each actuary must make his or her own decision about which underwriting profit provision model best captures the situation and is most appropriate in a given context.

II. UNDERWRITING PROFIT PROVISION IN THE PREMIUM FORMULA

Let L be the prospective estimate of loss and let c denote the ratio against loss of expenses proportional to loss. Let FX denote the provision for fixed expenses. Assume unallocated and allocated loss adjustment expenses are contained in some fashion in the loss, the expenses proportional to loss, or the fixed expenses. Write VR to stand for the variable expense ratio where variable expense encompasses all expenses such as commissions and premium taxes that are proportional to premium. Write U for the underwriting profit provision and let CR stand for the combined ratio. To summarize the notation:

(II.1)

Symbol	Description
L	**Loss**
c	**Loss Proportional Expense Ratio**
FX	**Fixed Expense**
VR	**Variable Expense Ratio**
CR	**Combined Ratio**
U	**Underwriting Profit Provision**

The premium, P, is given as:

(II.2) $P = (1+c)L + FX + P*VR + P*U$, which can be rewritten as:

(II.3) $P = ((1+c)L + FX)/(1-(VR+U))$

The combined ratio is the ratio of losses and expenses to premium:

(II.4) $CR = VR + ((1+c)L+FX)/P$

The combined ratio is related to the underwriting profit provision as follows:

(II.5) $U = (1-CR)$

Note that a profit provision of -100% does not correspond to a $0 premium, but rather a combined ratio of 200%. In the following chapters, the expenses proportional to loss will be omitted in the interest of simplifying matters.

III. CALENDAR YEAR INVESTMENT INCOME OFFSET PROCEDURE

Under the procedure to be discussed in this section, an offset is made against the traditional underwriting profit provision in order to arrive at a final number. The offset is an investment income figure which is expressed as a percentage of premium. It is obtained by looking at calendar year investment returns and allocating them by line. Adjustments are made to reflect income taxes and investment income earned from stockholder-supplied funds.

A defining feature of this method is its reliance on calendar year figures. Since calendar year results are reported by companies in their Annual Statements, the data supporting the calculations is easily obtained and verified. This is a distinct advantage. Use of reported figures can dispel criticism that the insurance company is making unduly pessimistic investment projections in its rate filings while earning record investment gains. Finally, in most cases, calendar year investment portfolio yields tend to be relatively stable.

However, since calendar year results are an inherently retrospective summary of contributions from current and prior policy years, their applicability in prospective ratemaking could be challenged. In particular, the prior growth history and loss experience of the line could distort the answers. However, unless the line has experienced rapid growth or decline, the figures, which might not be exactly correct for any specific year, will tend to balance out over several years of reviews. Also, certain adjustments can be made to partly correct the problem. For instance, the portion of the investment income offset due to loss reserves can be stated relative to incurred loss and then be put at a level consistent with the permissible loss ratio in the rates.

Next, a particular Calendar Year Investment Offset Procedure will be described. It is a streamlined version of procedures, such as the Insurance Services Office (ISO) State "X" Method, which have been actually used in rate filings. The simplified version is shown in Exhibit 2.

The first step in the algorithm is to find the ratio of pre-tax investment income relative to average invested assets. By applying appropriate tax rates to the various classes of investment income, one can calculate an after-tax portfolio yield. One should exercise care in making sure the right tax rates are used in this step of the calculation. The interest on taxable bonds is fully taxable at a 34% rate, while stock dividends are subject to an 80% exclusion. Under the proration provisions of the 1986 Tax Reform Act, the interest on "tax exempt" bonds acquired after August 1986 is subject to tax after an 85% exclusion. The actuary should review current tax law to ensure that investment taxes are correctly reflected.

One issue to face in this phase of the calculation is how to treat capital gains, both realized and unrealized. Some argue that capital gains should not be included. Others include only realized capital gains, perhaps using a multi-year rolling average of realized capital gains to invested assets to enhance stability. Others reflect all capital gains in computing the portfolio yield, again with some averaging over several years to avoid excessive volatility of results.

Once the after-tax portfolio yield is computed, the next goal of the calculation is to relate invested assets to premiums. Since policyholder premiums only generate part of the invested assets, a algorithm is needed to estimate the policyholder-supplied funds. The estimate is done indirectly by looking at the unearned premium reserve and loss reserve. These liabilities are offset by assets, only some of which are investible, and only some of which were supplied by the policyholder. A somewhat detailed series of computations is needed to estimate the investible balance due to policyholder supplied funds.

With respect to the unearned premium reserve, the first step is to obtain the ratio of average direct unearned premiums to direct earned premiums from the Annual Statement. Then this ratio is reduced to account for prepaid expenses and premium balances owed to the company.

The prepaid expenses include all or part of the commissions, premium taxes, other acquisitions expenses, and company overhead. The point here is that the company has already paid out these expenses, so the premiums paid to cover them are not generating investment income. For example, if the prepaid expense ratio against premiums was 20%, then one would reduce the unearned premium to 80% of its original value.

The premium balances cover all premiums owed (and less than 90 days late) including those deferred and not yet due. Again, the point is that the company does not have the cash to invest; it merely has an IOU. The appropriate unpaid premium balances are obtained from the Annual Statement, ratioed against direct earned premium, and then the ratio is subtracted from the previously calculated unearned premium ratio (to direct earned premium) net of pre-paid expense.

With respect to the calculation of the contribution associated with the loss reserve, one first computes the ratio of calendar year loss reserves to calendar year incurred losses. To obtain more stable results , one might calculate a multi-year average for this factor. This reserves-to-incurred loss factor is then multiplied by the permissible loss ratio. Thus we arrive at a ratio of reserves to premiums.

Note that actual ratios of reserves-to-premiums are not used in the calculation. Instead, a reserves-to-premium ratio consistent with the permissible loss ratio in the prospective rate is calculated as the product of the permissible loss ratio times the historic average reserves-to-incurred loss ratio. While the reserves-to-incurred ratio could be distorted by rapid growth or decline and changes in reserve adequacy, a reserves-to-premium ratio would also be subject to distortion from changes in premium adequacy. Using the reserves-to-incurred ratio along with the permissible loss ratio provides the policyholder credit for the investment income associated with the loss provision of the premium.

To summarize, the equation for policyholder-supplied funds is:

(III.1)

$$PHSF = \left(\frac{UEPR}{PREM}\cdot(1-PPACQ)-\frac{RECV}{PREM}\right) + PLR\cdot\frac{LRES}{INCL}$$

where:

PHSF = Policyholder-Supplied Funds (as a ratio to premium)

UEPR = Unearned Premium Reserves

PPACQ = Prepaid Acquisition Expense Ratio

RECV = Premiums Receivable

LRES = Loss Reserves

INCL = Incurred Loss

PLR = Permissible Loss Ratio

The final underwriting profit provision is then given as:

(III.2) $$\mathbf{U = U^0 - i_{AFIT} \cdot PHSF}$$

where U^0 is the traditional underwriting profit provision and i_{AFIT} is the after-tax portfolio yield.

There is a slight problem in determining the permissible loss ratio. Note the permissible loss ratio is used to calculate the profit provision which, in turn, is used to calculate the permissible loss ratio. To be technically correct, the calculation should be done iteratively until the permissible loss ratio is consistent with the offset and vice versa.

The key advantage of this method is its practicality. Figures come from documents already filed with statutory authorities, and the numbers are generally stable, especially if capital gains are excluded. The calculation is fairly short and the logic behind it is not too difficult. The key disadvantage is the lack of an underlying general economic theory to support the calculation. Other problems include the possibility of distortion when there is rapid growth or decline in loss volume or when there are significant changes in loss reserve adequacy. However, overall, this algorithm does account for investment income in a fairly straightforward manner. In stable growth scenarios with stable patterns of reserving, it seems appropriate for use in state rate filings.

IV. PRESENT VALUE OFFSET METHOD

In this method, as with the Calendar Year Investment Offset Procedure, an offset is applied against the traditional underwriting profit provision to get a final number. The offset is based on the difference between the present value of losses for a short-tailed reference line and the present value of losses for the line under review. The traditional provision is assumed to be correct for the reference line. Suppose $\vec{x}$ = (x(1), x(2), ... , x(n)) is a loss payout pattern, where x(j) is the fraction of ultimate loss paid out at the end of the jth year. Thus, the sum of the x(j) is unity. If the ultimate loss is L, then the amount paid out at the end of the jth year is L(j), where L(j) = L·x(j). Given an interest rate, i, let v = 1/(1+i). The present value of losses would be given as:

(IV.1)

$$PV(\vec{L};i) = L \cdot PV(\vec{x};i) = L \cdot \sum_{j=1}^{n} x(j) \cdot v^j$$

Suppose $\vec{x}^0$ is a reference loss payout pattern and $\vec{x}$ is the loss payout pattern for the line under review. The present value loss differential is defined as the difference in present values of loss under these two patterns. Letting the total amount of loss be given by a permissible loss ratio and expressing the present value loss differential as a ratio to premium, one obtains:

(IV.2)

$$DELPVLR = PLR \cdot (PV(\vec{x}^0;i) - PV(\vec{x};i))$$

The underwriting profit provision is then given as:

(IV.3) $$U = U^0 - DELPVLR$$

where U^0 is the traditional underwriting profit provision.

One version of this method is demonstrated in Exhibit 3. It is similar, though not identical, to the algorithm used in Florida filings.

The intent of the calculation is to adjust for the differences in investment income potential between lines of business. To understand how this is accomplished, one could ignore loss proportional expenses and rewrite the premium formula as:

(IV.4) $$P = PVL + FX + VR \cdot P + U^0 \cdot P + L(1-PVx^0)$$

where:

PVL = present value of losses for the line

L = full value of losses for the line

PVx^0 = the present value loss pattern for the reference line.

From this perspective, the loss provision is now being counted only at its present value, while the profit provision is effectively the sum of the traditional underwriting profit provision plus the scant investment income potential of the reference line. The thought is that if the present value loss provision were collected at policy inception, it would, on average, generate enough investment income so that the (full value) loss payouts could be covered.

The selection of the interest rate for discounting losses is critical in this method. One could use the portfolio yield from a recent year, the current embedded portfolio yield, an estimate of the portfolio yield for the year the rates will be in effect, or a new money yield. In keeping with the prospective nature of ratemaking, new money yields are theoretically preferable. However, portfolio yields are more stable and more easily verifiable. Also, their use may eliminate concern that the company is using low yields in its rate filings, while reporting high yields in its financial statements.

One way to partially account for income taxes within the context of this method is to use an after-tax interest rate for discounting the loss payout patterns. If the present value loss provision is computed with the after-tax interest rate, the after-tax accumulation of the present value loss provision will be sufficient to cover the expected full value loss payments. Note that the reflection of income taxes is only accounting for the income tax on investment income associated with the present value loss provision. Since this is only part of overall income tax, the tax adjustment is, in some sense, incomplete. A more elaborate model could be constructed within the same general framework by computing projected income taxes for the reference line and the line under review. An adjustment could then be made for the difference in the present values of the projected income tax payments. This will not be done here.

One could take a prospective or retrospective approach in putting the interest rate on an after-tax basis. Under the prospective approach, one applies the appropriate prospective tax rates to the pre-tax yields of each type of investment to get after-tax yields. The after-tax yield of the whole portfolio is calculated by weighting the after-tax yields by the assumed mix of assets.

Under the retrospective approach, the tax rate is derived from a company's actual income tax rate on all income for the prior calendar year. The implicit assumption is that the previous calendar year's actual income tax on all income for all lines and all states is a good indicator of the projected income tax on investment income associated with the line and state under review. This assumption is debateable. For instance, the tax paid in the past reflects past underwriting profits which may or may not be consistent with the underwriting profit provision in the prospective rates. As well, one could raise questions about implicit interstate and interline rate subsidies. Finally, one could argue that use of the prior year's actual tax rate operates perversely in that it effectively penalizes companies that lost money and rewards those that were profitable.

However the tax issue is settled, this method does account for investment income in a direct and simple fashion. While calendar year investment returns may be used in arriving at an appropriate discount rate, the method is not distorted by rapid growth or decline as was the Calendar Year Investment Offset procedure of Chapter III. Also, there is no need to select a target return or estimate a surplus requirement as is needed in some of the methods that follow.

V. CALENDAR YEAR RETURN ON EQUITY (ROE) METHOD

This method utilizes calendar year figures to calculate a rate of return on equity. The underwriting profit provision is chosen so that the rate of return is equal to a specified target return.

Return on equity is related to underwriting profit by the general formula:

(V.1)

$$ROE = \frac{INC}{EQ} = \frac{U \cdot P + II - FIT}{EQ}$$

where:

ROE	=	Return on Equity
INC	=	Total Income
U	=	Underwriting Profit Provision
P	=	Premium
II	=	Investment Income
FIT	=	Federal Income Tax
EQ	=	Equity

Equity is often set by first choosing a Statutory Surplus requirement. This is usually done by selecting a premium-to-surplus ratio that may vary by line of insurance. GAAP Equity can then be estimated by applying an historic equity-to-surplus ratio. A more complicated alternative is to add in the deferred acquisition expense balance and make other GAAP adjustments.

Taxes apply to both underwriting income and investment income. Whereas one uniform tax rate generally applies to all underwriting income, the investment portfolio may contain a variety of securities with effective tax rates that differ from each other and from the rate applicable to underwriting income. For this reason, it may be useful to split the tax into its two components: tax on underwriting income, and tax on investment income. Then, the total income numerator may be rewritten as the sum of after-tax investment income and after-tax underwriting income. Now suppose the tax on underwriting income is simply computed by applying a tax rate. This, of course, is incorrect as it ignores the 1986 Tax Reform Act, but under this assumption, total income may be conveniently expressed as:

(V.2)

$$INC = (1-t_u)\cdot U\cdot P + II_{AFIT}$$

where:

t_u = Income tax rate on underwriting income

II_{AFIT} = After-tax investment income

After-tax investment income is computed using an after-tax yield calculated in the same fashion as the after-tax yield obtained in the Calendar Year Investment Offset Method (See Chapter III). Recall that in that calculation each class of investment was subject to the appropriate prospective tax rate. Alternatively one could use a tax rate based on the company's recent tax returns. Counterarguments made in Chapter IV still apply here.

The after-tax yield should be multiplied against all investible funds no matter whether they were provided by policyholders or stockholders. Thus, the after-tax investment income may be expressed as:

(V.3)

$$II_{AFIT} = i_{AFIT} \cdot (PHSF \cdot P + S)$$

where:

i_{AFIT} = After-tax return on invested assets

PHSF = Policyholder-supplied funds (as a ratio to premium)

S = Surplus

The method is illustrated in the Exhibit 4. Note that the policyholder-supplied funds and investment yields from the Calendar Year Investment Offset example were used in this Calendar Year ROE example. The Calendar Year ROE Method shown here is roughly similar to methods promulgated in California under Proposition 103 regulation.

If the target return and necessary leverage ratios are given then one can solve for the corresponding underwriting profit provision to obtain the formula:

(V.4)

$$U=\frac{1}{1-t_u}[\ r\cdot(\frac{QSR}{PSR})\ -\ i_{AFIT}\cdot(PHSF+\frac{1}{PSR})]$$

where

r	=	Target Return on Equity
PSR	=	Premium-to-Surplus Ratio
QSR	=	Equity-to-Surplus Ratio

Using a calendar year return method raises several issues. First, as a calendar year method, it is subject to biases due to rapid growth or changing reserve adequacy similar to those discussed earlier. Second, there is the question of how to select and defend a target return. Related to this is the issue of whether one should divide by GAAP Equity instead of Statutory Surplus. If one uses a Statutory Surplus denominator, the resulting return will not be comparable to GAAP ROE. How to pick an appropriate leverage ratio (premium-to-surplus ratio) is another question. Should it balance against a company's actual surplus or should a target leverage ratio be assumed? Should the leverage ratios vary by line?

However, if one can address these issues, a calendar year return methodology does have some positive features. Figures used in the calculation are published in the Insurance Expense Exhibit and the Annual Statement, making verification relatively simple. Moreover, the key attraction of the method is that it produces a return on equity which is in some sense comparable to the GAAP ROE commonly used to measure profitability in many industries.

VI. PRESENT VALUE OF INCOME OVER PRESENT VALUE OF EQUITY (PVI/PVE) MODEL

Under this method, the underwriting profit provision is set so as to achieve a target "present value return" called the "PVI/PVE". As the name suggests, the PVI/PVE is the ratio of the present value of accounting income over the present value of equity. It is calculated with a single policy model constructed so that income is earned and equity is evaluated in accord with accounting (usually GAAP) conventions. The income is the sum of underwriting income plus investment income less income taxes.

The fundamental equation is:

(VI.1)

$$r = \frac{PV(INC)}{PV(EQ)}$$

where:

r = target return

PV(INC) = present value of income

PV(EQ) = present value of equity

The PVI/PVE is calculated by first constructing a model to produce Statutory and GAAP balance sheets and income statements for a company hypothetically writing a single policy. In the model, premium is earned evenly over the policy term and the unearned premium reserve is calculated as the difference between written premium less premium earned to date. Premiums receivable are the difference between written premiums less premiums paid to date. Loss and loss adjustment expense is incurred uniformly over the policy term, and the loss and loss adjustment expense reserve is equal to the difference between amounts incurred to date less amounts paid to date. Thus, the reserve in the model is always exactly adequate. Implicitly, it includes claims incurred but not reported (IBNR) as well as case reserves on known claims. Following Statutory accounting, expenses such as premium tax, commission, and acquisition expense are incurred up-front when the policy is written. General expenses, on the other hand, need not all be incurred at policy inception. Statutory expense reserves are posted as the difference between expenses incurred to date less expenses paid to date. Under GAAP, the premium tax and commission are incurred uniformly over the policy term instead of up-front. The incurral of some other acquisition expenses may also be deferred. The difference between Statutory incurred expenses to date less GAAP incurred expenses to date is sometimes called the Deferred Acquisition Balance and is counted as a GAAP asset. It is also sometimes referred to as the "equity in the unearned premium reserve" since it may be calculated by multiplying the unearned premium reserve by the ratio of deferrable expense relative to premium.

In the model, investment income for each accounting period is approximated by applying the pre-tax investment yield to the average balance of invested assets during the period. The invested assets can be computed by taking total assets and subtracting non-investible assets such as premiums receivable. The assets for the hypothetical company are equal to the sum of Statutory Reserves plus Statutory Surplus. This follows because any balance sheet must be in balance.

Income taxes should be based on relevant provisions of the tax code as applied to the hypothetical company. A detailed calculation of taxable income would require consideration of reserve discounting, the 20% disallowance for unearned premium, and other provisions of the Tax Reform Act of 1986. An issue that arises in modelling taxes is what to do if tax accounting income is negative. The usual practice is to allow a negative tax (i.e. a payment from the government to the company) when taxable income is negative. Another option is to allow a negative tax in any period only if it offsets positive taxes paid in previous periods.

The equity in the model is often set to be level for one year, and the amount of equity is determined by selecting a premium-to-equity ratio. A ratio of expected loss-to-equity can also be used for this purpose. More general assumptions are often made. For example, some equity may be held over several years in proportion to loss reserves. Also, in many models, the amount of equity may vary during a calendar period. For instance, if one starts with a "block" surplus assumption that fixes the level of Statutory Surplus, the corresponding level of GAAP Equity for a single policy will generally vary over the year. Note that some of the GAAP Equity is non-investible; in particular, the "equity in the unearned premium reserve" which is associated with pre-paid expense.

Since present values are commonly taken of a series of flows rather than a series of balances, the calculation of the present value of equity is a somewhat unique feature of the model. Further, the present value of equity must be defined with some care since one needs a definition that yields sensible results when equity is held over several years or when equity is evaluated more frequently than at the end of each year. One way to fashion such a definition is to first define average equity balances during each accounting period based on the equity balances at the end of each time period. Let EQ_j be the equity balance as of the end of the j^{th} time period where each time period lasts an m^{th} of a year. Write EQ_0 for the balance at policy inception. Next, let EQB_j denote the average equity balance during the j^{th} time period. The average equity balance during the j^{th} period may usually be approximated as the numerical average of consecutive quarter ending equity values. However, when equity is based on a "block" surplus which is taken down immediately after the end of a year, this rule is violated and one must take more care in defining the average equity balance during the period. In a quarterly model with a "block" equity requirement, the balances would be :

(VI.4)

Quarter	Qtr Ending Equity Balance (EQ_j)	Average Equity Balance During Qtr (EQB_j)
0	EQ_0	-----
1	EQ_1	$EQB_1 = (EQ_0+EQ_1)/2$
2	EQ_2	$EQB_2 = (EQ_1+EQ_2)/2$
3	EQ_3	$EQB_3 = (EQ_2+EQ_3)/2$
4	EQ_4	$EQB_4 = (EQ_3+EQ_4)/2$
5	$EQ_5 = 0$	$EQB_5 = 0$

Consider how the use of the average equity balances during each quarter eliminates the problem of having too many quarter ending balances and the problem of how to handle the takedown of equity just after the end of the fourth quarter.

One way of defining the PVI/PVE is to calculate an m^{th}ly effective PVI/PVE return as follows:

(VI.5)

$$PVI/PVE = \frac{\sum_{j=0} INC_j \cdot v_{(m)}^j}{\sum_{j=1} EQB_j \cdot v_{(m)}^j}$$

Here $v_{(m)} = (1+i)^{-1/m}$ where i is the interest rate used for discounting.

Now consider this definition as applied to a simple example in which a balance of \$100 is maintained in a bank account for n quarters and \$2 of interest is received at the end of each quarter. The present value of the income is $\$2 \cdot (v_{(4)} + v_{(4)}^2 + \ldots + v_{(4)}^n)$, while the present value of equity is $\$100 \cdot (v_{(4)} + v_{(4)}^2 + \ldots + v_{(4)}^n)$. Thus, the PVI/PVE is 2% effective quarterly. More generally, if one has a venture yielding interest of $y_{(m)} \cdot B_j$ paid at the end of the j^{th} period on a balance of B_j, the PVI/PVE will be $y_{(m)}$ effective m^{th}ly. This will be true irrespective of the interest rate used in discounting the income and the equity, as long as the same rate is used in discounting both.

In more general cases, when income in each period is not a fixed multiple of the equity balances, the PVI/PVE as defined by VI.5 will depend on the interest rate (or rates) used in discounting the income and the equity. This leads to some disagreeable results even in some very simple scenarios. For example, if \$8 is paid at the end of the year on a fixed balance of \$100 and one uses a quarterly model with a 10% rate for discounting, the effective annual return would be calculated as 7.942% $(\{1+ .08*1.1^{-1}/(1.1^{-.25}+1.1^{-.50}+1.1^{-.75}+1.1^{-1})\}^4)$ rather than 8%. A related problem with VI.5 is that the present value of equity undergoes drastic changes if one shifts from an annual model to, say, a quarterly model. For instance, if a balance of \$100 is held constant for one year, its present value is $100 in an annual model. Using a 10% rate for discounting, the present value balance becomes $377 in a quarterly model.

To address these concerns, one can use an alternate definition of PVI/PVE in which all income is evaluated as of the end of the first year and in which the present value of equity is put on an "annualized" basis as follows:

(VI.6)

$$PVE_{Ann} = \frac{\sum_{j=1} EQB_j \cdot v_{(m)}^{j-1}}{\sum_{j=1}^{m} v_{(m)}^{j-1}}$$

Note that when equity is held fixed for one year, its annualized present value equals the "block" amount under any m^{th}ly model irrespective of "m" or the interest rate used in discounting.

Under this approach, the annual effective PVI/PVE is given as:

(VI.7)

$$PVI/PVE_{Ann} = \frac{(1+i) \cdot \sum_{j=0} INC_j \cdot v_{(m)}^{j}}{PVE_{Ann}}$$

The PVI/PVE model is demonstrated in the Exhibit 5 using the annualized definitions. In the interest of simplicity, the only GAAP adjustment is for deferred acquisition costs, and income taxes are incorrectly modelled as a percent of GAAP income. Actual tax law should be followed in any business applications.

The rate used in discounting the income numerator is often selected to be the pre-tax, risk-free, new money yield on taxable investments. Usually, this same rate is used to discount the equity denominator, though sometimes the target return is also employed for this purpose. The rationale for using the pre-tax yield is that income taxes are explicitly included in the income. The target return is set a few points above the pre-tax, risk-free rate on taxables. The spread would be dependent on the risk involved. While measurement of risk is problematic, if the rate is risk-free, there is no need to worry about the investment default risk. Another justification for using the risk-free rate is that the resulting underwriting profit provision will not depend on the particular investment strategy of a company.

An alternative philosophy in selecting investment yields, discount rates, and target returns is to use actual portfolio yields in conjunction with discount rates and target returns comparable to historically acceptable GAAP ROE targets.

While one common rate is generally used in discounting both income and equity, one could conceive of using different rates. In particular, an argument could be made for discounting the equity using the target rate of return. The general dependence of results on the interest rate selection can be deduced as follows. In practice, since income is usually negative for the first few periods and then subsequently positive, it follows that an increase in the rate for discounting income will typically result in a decrease of the PVI/PVE. With respect to the denominator, a boost in the discount rate will always reduce the "PVE". This will enhance "leverage" so that positive PVI/PVE grow more positive and negative PVI/PVE become more negative. When using PVI/PVE to find an underwriting profit provision consistent with a selected (and presumably positive) target return, the leverage effect will boost PVI/PVE return. This will lead to a reduction in the indicated underwriting profit provision, assuming the target return stays fixed.

The key advantage of this method is that it is based on a measure of return that is both comparable to GAAP ROE and a generalization of the standard definition of the rate of interest. However, it does require selection of rates for calculating investment income and taking present values. Also, one must choose a target return. As with other methods that have a target return and an explicit surplus requirement, one may encounter some debate on the choice of these parameters. For instance, a regulator may lean toward a relatively low target return and a relatively small amount of surplus. In such a situation, a company actuary should be ready to support his or her selections with data on the returns achieved by the insurance industry, financial services industries, and other industries. The actuary can also buttress his or her case by citing surplus benchmarks accepted by the industry or promulgated by regulators.

VII. PRESENT VALUE CASH FLOW RETURN MODEL

With this method the underwriting profit provision is set so that the present value of total cash flow equals the present value of the changes in equity. Here an investment rate of return is used to calculate the present value of the total cash flow while a target rate of return is used in computing the present value of the changes in equity. The total cash flow is defined to be the sum of the underwriting cash flow plus the investment income on investible equity less income taxes. Underwriting cash flow consists of paid premiums less paid losses and paid expenses. The flows are derived from a single policy (or policy year) model based on assumptions about the ratios and payment patterns for losses and expenses. Note the present value of total cash flow has no distinct term for investment income on policyholder-supplied funds, while there is a term for the present value of investment income on assets that offset equity. This disparate treatment of investment income arises because the present value of investment income on policyholder-supplied funds is already taken into account in computing the present value of the underwriting cash flows.

The basic relation may be expressed in mathematical terms as:

(VII.1)

$$PV(\Delta EQ; r) = PV(TCF; i)$$

where

EQ = Equity

r = Target Rate of Return

TCF = Total Cash Flow

i = Investment rate used for discounting cash flows

The idea behind this method is that the target rate of return on equity can be used along with assumptions about the level of equity to arrive at a target value for the present value of the total cash flow. To see how this works, consider a two year venture in which $100 of equity is needed for the first year and $40 for the second. The equity changes are +100, -60, and -40. With a 20% target return, the target present value "profit" (evaluated at the end of the first year) comes to $26.67 (100·1.2 + -60 + -40/1.2). Note that the investment of $100 for the first year and $40 for the second year requires the same present value profit as a one year investment of $133 (20%·$133 = $26.67). It may also prove insightful to observe that $133 = 100 + 40/1.2.

The equation for total cash flow is:

(VII.2)

$$TCF = UWCF + INVIEQ - FIT$$

where:

UWCF = Underwriting Cash Flow

INVIEQ = Investment Income on Investible Equity

FIT = Federal Income Tax

"Investible Equity" is somewhat of a misnomer as equity is merely the difference between assets and liabilities. The term is used here to refer to that portion of the equity which can be associated with investible assets. Usually this consists of roughly the Statutory Surplus. Various ways of modelling Statutory Surplus were mentioned in VI.

Income taxes can be modelled in a simple, though inaccurate, fashion by applying the appropriate tax rate to the present value of the underwriting cash flows and to the investment yield on investible equity. The tax rate would not be applied to the interest rate used in discounting flows to present value. To get a more accurate treatment of taxes, one would have to compute income according to tax accounting conventions.

A simple example of the Present Value Cash Flow Return is given in Exhibit 6. It is similar to the fifth model presented by Mahler in his paper.

Perhaps the biggest criticism of the method is that it is not exactly clear just what sort of profit is being measured. To get a return comparable to GAAP ROE, then the present value of GAAP income should equal the present value of the total cash flow. Yet, the present value of total cash flow cannot be easily reconciled with GAAP accounting since the timing of underwriting cash flows is not generally equal to the timing of GAAP underwriting income. Nonetheless, the method has great appeal, perhaps because the present value of underwriting cash flows is precisely what most people first think of when trying to measure the underwriting profit net of associated investment income.

VIII. RISK-ADJUSTED DISCOUNTED CASH FLOW MODEL

Under this method, the strategy is to compute a "fair" premium directly. The underwriting profit provision is then calculated after the fact by comparing the fair premium against the sum of loss and expense provisions. Of course, the key question is: what is the fair premium? The answer is that the fair premium is the sum of the risk-adjusted present value of the underwriting cash flows plus an amount to cover the present value of income taxes. The risk adjustment is accomplished by discounting flows at a risk-adjusted rate that is usually less than the risk-free rate. Though one could, in principle, calculate a separate risk-adjusted discount rate for each flow, in what follows, only the paid losses will be subject to risk-adjusted discounting. The premium, expense, and income tax flows will be discounted at the risk-free rate. While surplus plays no critical role in this model, one must not forget to reflect income taxes on the investment income associated with the surplus.

The basic equation is:

(VIII.1)

$$PV(P;i_f) = PV(L;i_r) + PV(FX+VX;i_f) + PV(FIT;i_f)$$

where:

P = Premium

L = Loss

FX = Fixed Expense

VX = Variable Expense

FIT = Federal Income Tax

i_f = Risk-free rate

i_r = Risk-adjusted rate

The method is demonstrated in Exhibit 7. The demonstration is a simplified version of the Myers-Cohn model that has been used in Massachusetts rates filings.

The risk-adjusted rate is key to the method. It is calculated with the following formula:

(VIII.2)

$$i_r = i_f + \beta \cdot (i_m - i_f)$$

This formula quantifies the risk-return trade-off and is a simplified expression of the concepts underlying the Capital Asset Pricing Model (CAPM). CAPM was originally formulated to describe market prices of stocks and bonds. The idea is that the market demands higher expected returns for riskier investments. The term, $(i_m - i_f)$, is the market "premium" for an investment of average risk, or, in other words, i_m is the return on an average market portfolio.

The "beta" coefficient relates to the relative systematic risk of the particular investment under consideration. It is the ratio of the covariance between the market return and the return of the particular asset, divided by the variance of the market return. Under CAPM, a critical distinction is made between non-diversifiable, systematic risk and diversifiable, non-systematic risk. A higher return is warranted only for the systematic risk component. To get a stronger intuition, consider a stock that moves with the market, plus or minus some random fluctuations. In this case, the beta is unity. If a stock moves in the same direction as the market but with swings always twice as large, then the beta is two. The same beta results if some random fluctuations are thrown on top of the systematic double amplitude, market-following swings.

In principle, it is possible to have a negative beta, even for an asset. For example, suppose the return on a particular stock deviated from the risk-free return in the same magnitude but opposite direction as the market return. Then, an investor could theoretically achieve the risk-free rate with a portfolio half in that stock and half in a diversified market basket of stocks. Since the expected market return is always above the risk-free rate, the expected return on the hypothetical stock should be below the risk-free rate. Thus beta in this pathological case would be negative. As far as the author knows, the real world provides no good example of a stock with a negative beta, though some suggest one might be found among the stocks of gold mining companies.

Applying the CAPM concept to liabilities is a bit tricky. With stocks and bonds, the theory leads to predictions that can be checked against actual market prices. With liabilities such as loss reserves, there is no open market with publicly disclosed prices to provide empirical validation for the theory. Anecdotal evidence suggests that the price for loss portfolio transfers is usually greater than the present value of the losses transferred when present values are computed at the risk-free rate. This implies that the beta for liabilities is negative, but supplies no data for estimating "beta" magnitudes or how beta varies between lines of business.

An indirect approach to estimating a liability beta is to first calculate the beta for stocks of insurance companies and then the beta for the investment portfolios held by these companies. The difference between the market value of insurance stocks and the market value of insurer asset portfolios must be due to an implicit market valuation of insurer liabilities. With sophisticated techniques that will not be presented here, a beta for liabilities can be derived. The trouble is that most insurance companies are multi-line companies so that this method has not, in practice, been able to distinguish betas by line of business.

Because the method has great intuitive appeal and is directly grounded in modern financial theory, one might be able to accept some uncertainty about the appropriate beta. By defining a fair premium without resort to a total rate of return measure, the method neatly sidesteps the question of what is the right target rate of return. Also, since there no return relative to equity, the selection of a surplus requirement is not as critical as with other models.

IX. INTERNAL RATE OF RETURN ON EQUITY FLOW MODEL

Under this method, the underwriting profit provision is set so as to achieve a selected target return. The return is calculated by modelling a single policy. As with the PVI/PVE (Present Value of Income over Present Value of Equity) return discussed in Chapter VI, one constructs a model to calculate income statements and balance sheets for a fictitious company writing the single policy. Then, using basic accounting relations, one calculates flows of money between this company and its hypothetical stockholders. These flows are called equity flows. The return on the policy is defined to be the internal rate of return (IRR) on the equity flows. If some typical restrictions prevail, the IRR can be interpreted as the interest rate paid to stockholders on a series of equity "loans" made to the insurance company.

In general, the IRR on a sequence of flows is the rate, y, if it exists and is unique, for which the present value of the flows is zero:

(IX.1)

$$0 = PV(\vec{x};y) = \sum_{j=0}^{n} x_j \cdot (1+y)^{-j}$$

Thus, the IRR is the solution to an n^{th} degree polynomial equation. If, for example, the flows are (-200, +110, +121), then the IRR is 10% (assuming the flows are at annual intervals), since $-200 + 110*(1.1^{-1}) + 121*(1.1^{-2}) = 0$. IRR is a generalization of the interest rate on a loan. It can be shown that if the IRR exists, then the flows can be expressed as a sum of overlaid simple loans all carrying an interest rate equal to the IRR. For instance, in the previous example, the simple loans are:

(-100, +110, 0) and (-100, 0, +121),

both of which are simple loans at a 10% rate. Adopting the convention that negatives represent cash outflows and positives denote cash inflows, one can interpret the flows from the point of view of a lender obtaining 10% on the loans made. However, if all signs of the flows were reversed, then one would still obtain the same IRR. Only in this case, one would be viewing matters as a borrower paying 10% on a series of loans. More generally, one should take care with IRR to be sure to identify what side of the table one is sitting on. The IRR only measures the interest rate on the loans; not whether one is doing the borrowing or the lending.

Of course, an n^{th} degree polynomial could have as many as n real roots. If the defining equation has more than one real root, then the IRR fails to exist. In such cases, the flows can be decomposed in more than one way into a sum of simple loans such that, for each different decomposition, the simple loans carry a different interest rate. For example, the flows:

(-100, +230, -132)

have a zero present value at rates of both +10% and +20%. These flows can be expressed either as:

(-100, +110, 0) + (0, +120, -132) or

(-100, +120, 0) + (0, +110, -132) .

The first decomposition is the sum of 10% simple loans and the second is a sum of 20% loans. Not coincidentally, in each of these decompositions, one of the simple loans is a borrowing transaction and one is a lending transaction.

While the problem of multiple roots could in principle be fatal, in practice it almost never arises in modelling property and casualty insurance ventures. Thus, the previous discussion is merely a caution about the uncritical use of IRR in all situations and not a telling objection to the use of IRR for the purpose at hand.

The equation for equity flow is:

(IX.2)

$$EF_j = INC_j - (SCHNG)_j$$
$$for\ j = 0,1,2,\ldots,n$$

where:

EF = Equity Flow

INC = Total Statutory Income

SCHNG = Change in Statutory Surplus

Here a variable with a subscript, j, connotes a flow occurring or income accruing as of the end of the j^{th} time period and a balance sheet evaluated as of the end of the j^{th} time period. The "0" subscript refers to the start of the first time period. A possible confusion arises under these conventions with respect to the Change in Surplus variable. The Change in Statutory Surplus is usually equal to the difference between consecutive Statutory Surplus balances. However, an exception occurs when surplus is taken down at the start of an accounting period. For example, in a quarterly model run with a one year, block surplus of $100, the Statutory Surplus account will be $100 for indices zero through four and then become zero at index five. The Change in Statutory Surplus will be $100 at index zero and then be -$100 at index four (and not five).

An simple example of the method is shown in Exhibit 8. It is similar to the procedure used by the National Council of Compensation Insurance in several states.

The accounting principle behind Equation IX.2 is that the surplus of the company can change only if it declares income (either a gain or loss) or if it receives or distributes funds to stockholders. If one ignores unrealized capital gains, changes in non-admitted assets, and other sundry adjustments, this is exactly true. Note that equity flows are defined here so that a positive equity flow denotes a flow of money to stockholders.

Inherent in the equation is a critical distinction between the surplus of the company and the stockholder investment in the company. From the equity flow perspective, what is most important is what the stockholders will put into the company and what they will get back. Statutory and GAAP accounting rules and requirements are important, not because Statutory Surplus or GAAP Equity is in the denominator of an ROE measure, but because they can affect the equity flows. Statutory Surplus requirements and the conservative bias of Statutory accounting rules tend to increase the amount of money required from stockholders and delay or decrease the flow of funds to them. GAAP accounting rules can also impact the flow of equity, because, in a few cases, GAAP is more conservative than Statutory accounting. One example of such a "concept violation" is in the accrual of a GAAP policyholder dividend reserve liability not recognized under Statutory accounting.

While the defining equity flow equation has been given using Statutory Income and Statutory Surplus, one should more properly determine each equity flow so that the company maintains a sound balance sheet under both accounting systems. Further, once each equity flow is calculated as the difference between Statutory Income and the change in the Statutory Surplus (during period) balance, one could restate it as the difference between GAAP Income and the change in the (during period) GAAP Equity balance. Though the company may have different balance sheets and income statements under GAAP, the equity flows stay the same. Thus the IRR on Equity Flows is not a GAAP return or a Statutory return; rather it is the return to stockholders reflecting the constraints of both accounting systems.

Following the rules of Statutory accounting, expenses such as commissions, premium taxes, and acquisitions expenses are incurred up-front at policy inception. Other expenses are incurred as paid or incurred uniformly over the policy term. Expense reserves are held so that incurred equals paid plus the change in reserves. Premiums are earned and loss and loss expenses are incurred evenly over the policy term. Earned premium equals written less the change in the unearned premium reserve. Appropriate loss reserves are posted so that incurred loss is equal to paid plus the change in loss reserves.

One can now see why the "multiple roots" problem seldom arises in property and casualty insurance models. Consider that the up-front expense declaration rules of Statutory accounting and the need to fund the initial required surplus usually force the first equity flow to be negative. If underwriting losses are sufficiently large, equity flows could remain negative for the policy term. This happens, since, in the model, the stockholders must fund reserves when premiums are inadequate to do so. After the first year, a property casualty venture usually generates investment income and hence positive equity flows. The repatriation of surplus also leads to positive equity flows. Note that in the model the stockholders are a "bottomless well" providing funds as needed to maintain the pre-set surplus requirement. For its part, the company immediately returns excess funds to stockholders. It does not build up profits or retain surplus above the predetermined surplus requirement level.

The main virtue of this model is that it calculates a return to stockholders that is directly analogous to the interest rate on a loan. It is clear what return is being measured. Another possible strength of the model is that it reflects the rules of accounting insofar as they impact the flow of funds to stockholders. In other words, the cost of conservatism in Statutory accounting is included. The unified treatment of all income is an aesthetic plus.

Drawbacks include the need to select a target return and a surplus requirement. This again forces one to address the knotty issues of how to "price" the elements of risk in terms of increasing the required surplus, increasing the target return, or both.

X. CONCLUSION

The reader has been introduced to a profusion of models and may find the whole subject a bit overwhelming. Indeed, perhaps the major thesis of this paper is that the methods differ not just in their details but, more importantly, in their basic foundations. It is thus time to step back, survey the field, and pick out key characteristics. What makes one method different from another?

The first distinction is whether the traditional underwriting profit provision is used as a starting point to make adjustments or whether an attempt is made to define a "correct" profit provision from first principles. The Calendar Year Investment Offset and the Present Value Loss Offset Methods are procedures of the former type. The other methods are independent of the traditional load. Except for the Risk-Adjusted Discounted Cash Flow method in which a "fair" premium is obtained directly, a notion of total return is defined in these other methods and the profit provision is set to hit a selected return target.

The methods can also be classified by their degree of reliance on calendar year data. Calendar year investment figures play a key role in the Calendar Year Investment Offset and Calendar Year Return algorithms. At the other extreme, calendar year numbers play no role at all in the Risk-Adjusted Discounting method. With the Present Value Offset, PVI/PVE, Present Value Cash Flow Return, and IRR on Equity Flows, calendar year figures might be used in selecting interest rates or rates used for discounting, but market risk-free rates are preferred.

Another defining characteristic of a model is how "risk" is reflected. In the Risk-Adjusted

Discounted Cash Flow method, pricing for risk is central to the model. With all the total return models, risk is reflected in the selection of the surplus requirement and the target return. Risk was not considered in the models that use the traditional profit load as a starting point, though one might be able to put "risk" in after the fact.

When looking at a model, one should also understand the role of accounting conventions. Accounting constraints are central to the IRR on Equity Flows method because these constraints impact the modelled flows of stockholder equity. Whether one should be measuring GAAP ROE or Statutory ROS (Return on Surplus) is a major issue in the Calendar Year Return Procedure.

There are a host of other concerns. How are income taxes handled, if at all ? How stable is the data and the resulting profit provision? How defensible is the method if senior corporate managers or state regulators question it?

To conclude, no "one true" method has been espoused in this paper. While each actuary may prefer one method above all others or feel that some are outright nonsense, it was not the purpose here to spark vituperative debate. Rather, the intent was to impart sufficient enlightenment so the reader could judge what models are appropriate in a given situation and to make some sense of models encountered in practice. The reader should also have enough information to construct simple versions of various models. The underwriting profit provision will likely be the subject of contention for years to come, and it is hoped that the reader now has sufficient background to understand the confusion.

ACCOUNTING GLOSSARY

GAAP (Generally Accepted Accounting Principles) Vs. Statutory Accounting Principles

All insurance companies must report their results to the regulators in states in which they are licensed using the prescribed format of the NAIC Annual Statement. The rules for preparing the NAIC Annual Statement constitute Statutory accounting principles. They are balance sheet oriented and emphasize the valuation of assets and liabilities on a "liquidation basis" rather than on the "going-concern basis" used for GAAP financial statements. Generally, Statutory financial statements display a more conservative financial position and results of operations than the results reported under GAAP. The Internal Revenue Service Form which Property-Casualty insurance companies use to report their taxable income follows neither GAAP nor Statutory insurance accounting principles entirely.

Capital Gains

The sale of a capital asset at more (or less) than its cost basis for tax purposes gives rise to a realized gain (or loss). Unrealized gains reflect the excess of Market Value over Book Value. Unrealized gains and losses in the investment portfolio are not included in investment income.

GAAP Equity Vs. Statutory Surplus

Surplus is a Statutory accounting term representing the excess of assets over liabilities. Statutory accounting conservative valuation rules exclude some assets from the balances sheet (non-admitted assets). Another area of distinction is in the treatment of prepaid acquisition expenses for which Statutory accounting requires immediate expensing and for which GAAP allows amortization. Both of these differences tend to make the GAAP equity larger that the Statutory Surplus.

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GLOSSARY OF EXHIBITS

EXHIBIT	TITLE
1A	Loss and Expense Parameters Used for All Methods
1B	Payment Patterns
1C	Investment Portfolio Yield Calculation
1D	Financial Assumptions by Model
2	**Calendar Year Investment Offset Method**
3	**Present Value Offset Method**
4	**Calendar Year ROE Method: Summary**
5A	**PVI/PVE Model: Summary**
5B	PVI/PVE Model: Inputs and Assumptions
5C	PVI/PVE Model: Underwriting Cash Flows
5D	PVI/PVE Model: Underwriting Income
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6A	**Present Value Cash Flow Return Model: Summary**
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6D	Present Value Cash Flow Return Model: Underwriting Cash Flows
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7B	Risk–Adjusted Discounted Cash Flow: Inputs and Assumptions
7C	Risk–Adjusted Discounted Cash Flow: Underwriting Cash Flow Patterns
7D	Risk–Adjusted Discounted Cash Flow: Tax on Investment Income on Surplus
7E	Risk–Adjusted Discounted Cash Flow: Discount Factors
8A	**IRR Model: Summary**
8B	IRR Model: Inputs and Assumptions
8C	IRR Model: SAP Underwriting Income
8D	IRR Model: SAP Balance Sheeet
8E	IRR Model: SAP Income
8F	IRR Model: GAAP Income

EXHIBIT 1A

LOSS AND EXPENSE PARAMETERS USED FOR ALL METHODS

Loss	$65
Fixed Expense	$15
Variable Expense Ratio	25%

EXHIBIT 1B

Payment Patterns

PREMIUM			LOSS				EXPENSE		
Qtr	Percent of Ultimate	Percentage Increments	Qtr	Dollars by Quarter *	Percent of Ultimate	Percentage Increments	Qtr	Percent of Ultimate	Percentage Increments
0	40.0%	40.0%	0	$0.00	0.0%	0.0%	0	30.0%	30.0%
1	55.0%	15.0%	1	$2.00	3.1%	3.1%	1	47.5%	17.5%
2	70.0%	15.0%	2	$4.00	9.2%	6.2%	2	65.0%	17.5%
3	85.0%	15.0%	3	$7.00	20.0%	10.8%	3	82.5%	17.5%
4	100.0%	15.0%	4	$8.00	32.3%	12.3%	4	100.0%	17.5%
5	100.0%	0.0%	5	$8.50	45.4%	13.1%	5	100.0%	0.0%
6	100.0%	0.0%	6	$8.00	57.7%	12.3%	6	100.0%	0.0%
7	100.0%	0.0%	7	$6.00	66.9%	9.2%	7	100.0%	0.0%
8	100.0%	0.0%	8	$5.00	74.6%	7.7%	8	100.0%	0.0%
9	100.0%	0.0%	9	$4.00	80.8%	6.2%	9	100.0%	0.0%
10	100.0%	0.0%	10	$3.00	85.4%	4.6%	10	100.0%	0.0%
11	100.0%	0.0%	11	$2.00	88.5%	3.1%	11	100.0%	0.0%
12	100.0%	0.0%	12	$2.00	91.5%	3.1%	12	100.0%	0.0%
13	100.0%	0.0%	13	$1.00	93.1%	1.5%	13	100.0%	0.0%
14	100.0%	0.0%	14	$1.00	94.6%	1.5%	14	100.0%	0.0%
15	100.0%	0.0%	15	$1.00	96.2%	1.5%	15	100.0%	0.0%
16	100.0%	0.0%	16	$1.00	97.7%	1.5%	16	100.0%	0.0%
17	100.0%	0.0%	17	$0.50	98.5%	0.8%	17	100.0%	0.0%
18	100.0%	0.0%	18	$0.50	99.2%	0.8%	18	100.0%	0.0%
19	100.0%	0.0%	19	$0.50	100.0%	0.8%	19	100.0%	0.0%
20	100.0%	0.0%	20	$0.00	100.0%	0.0%	20	100.0%	0.0%
Total	100.0%	100.0%	Total	$65.00	100.0%	100.0%	Total	100.0%	100.0%

* Total losses fixed at $65.00 .

Used in the following models: Present Value Offset, Present Value Cash Flow, PVI/PVE, Risk–Adjusted Discounted Cash Flow, IRR

INVESTMENT PORTFOLIO YIELD CALCULATION

INVESTMENT	Average Asset Value	Interest, Dividends, and Rents Earned	Pre Tax Yield	Average Federal Tax Rate	Post Tax Yield	Net Realized Capital Gains	Pre Tax Yield	Average Federal Tax Rate	Post Tax Yield	Total Pre Tax Yield	Total Post Tax Yield
BONDS											
Taxable	565,000	54,000	9.56%	34.00%	6.31%	0	0.00%	34.00%	0.00%	9.56%	6.31%
Tax–Exempt	155,000	10,000	6.45%	0.62%	6.41%	0	0.00%	34.00%	0.00%	6.45%	6.41%
Total	720,000	64,000	8.89%	28.78%	6.33%	0	0.00%	34.00%	0.00%	8.89%	6.33%
STOCKS (Unaffiliated)											
Common	90,000	8,000	8.89%	12.67%	7.76%	870	0.97%	34.00%	0.64%	9.86%	8.40%
Preferred	25,000	3,000	12.00%	12.67%	10.48%	0	0.00%	34.00%	0.00%	12.00%	10.48%
Total	115,000	11,000	9.57%	12.67%	8.35%	870	0.76%	34.00%	0.50%	10.32%	8.85%
MORTGAGES	86,500	8,600	9.94%	34.00%	6.56%	0	0.00%	34.00%	0.00%	9.94%	6.56%
REAL ESTATE	5,300	1,300	24.53%	34.00%	16.19%	340	6.42%	34.00%	4.23%	30.94%	20.42%
SHORT TERM	70,100	6,900	9.84%	34.00%	6.50%	0	0.00%	34.00%	0.00%	9.84%	6.50%
CASH	19,200	1,200	6.25%	34.00%	4.13%	0	0.00%	34.00%	0.00%	6.25%	4.13%
TOTAL	1,016,100	93,000	9.15%	27.89%	6.60%	1,210	0.12%	34.00%	0.08%	9.27%	6.68%

Portfolio yields are used in : **Calendar Year Investment Offset Method**
Present Value Offset Procedure
Calendar Year ROE Method

FINANCIAL ASSUMPTIONS BY MODEL

Assumption Type	Calendar Year Investment Income Offset Method	Present Value Offset Procedure	Calendar Year ROE Method	Present Value Cash Flow Return Method	Present Value of Income over Present Value of Equity Model	Risk-Adjusted Discounted Cash Flow Model	Internal Rate of Return on Equity Flow Model
Surplus & Equity	N/A	N/A	"Block" held one yr	"Block" held one yr	"Block" held one yr	"Block" held one yr	"Block" held one yr
Premium to Surplus Ratio	N/A	N/A	3.0	3.0	3.0	3.0	3.0
Premium to Equity Ratio	N/A	N/A	2.5	2.5	N/A	N/A	N/A
Investment Yield, Pre-Tax Post-Tax	Portfolio Yield 9.27% 6.68%	N/A	Portfolio Yield 9.27% 6.68%	New Money 8.00%	New Money 8.00%	New Money 8.00%	New Money 8.00%
Discount Rate	N/A	5.28% (Post-Tax)	N/A	8.00%	8.00%	Losses: Risk-Adj. Rate Other: Risk-Free Rate	8.00%
Income Tax Rate	N/A	34%	34.0%	34.0%	34.0%	34.0%	34.0%
Target Return	N/A	N/A	15.0%	15.0%	15.0%	N/A	15.0%

CALENDAR YEAR INVESTMENT OFFSET METHOD

Policyholder–supplied funds on Unearned Premiums		
Average Direct Unearned Premium	$50,000	(1)
Pre–paid Expense Ratio	18%	(2)
Average Premiums Receivable	$28,000	(3)
Average Portfolio Balance due to UEPR	$13,000	(4)=(1)x[1.0–(2)]–(3)
Direct Earned Premium	$160,000	(5)
Balance Relative to Earned Premium	8.13%	(6)=(4)/(5)
Policyholder–supplied funds on Loss Reserves		
Loss Reserves to Loss Incurred (average ratio over three years)	1.20	(7)
Permissible Loss Ratio	60%	(8)
Reserves Relative to Premiums	72%	(9)=(7)x(8)
Investment Offset and Final U/W Profit		
Policyholder–supplied Funds (as a percent of premiums)	80.13%	(10)=(6)+(9)
After–tax Portfolio Yield	6.68%	(11) (see Exh. 1D)
Investment Income Offset	5.35%	(12)=(10)x(11)
Traditional U/W Profit Provision	5.00%	(13)
Final U/W Profit Provision	–0.35%	(14)=(13)–(12)

PRESENT VALUE OFFSET METHOD
LOSS PAYOUT PATTERNS

Qtr.	Discount Factor	Reference Line Pattern	Present Values at Year Zero	Evaluated Line Pattern	Present Values at Year Zero
1	0.987	10.0%	9.9%	3.1%	3.0%
2	0.975	15.0%	14.6%	6.2%	6.0%
3	0.962	20.0%	19.2%	10.8%	10.4%
4	0.950	25.0%	23.7%	12.3%	11.7%
5	0.938	15.0%	14.1%	13.1%	12.3%
6	0.926	10.0%	9.3%	12.3%	11.4%
7	0.914	5.0%	4.6%	9.2%	8.4%
8	0.902	0.0%	0.0%	7.7%	6.9%
9	0.891	0.0%	0.0%	6.2%	5.5%
10	0.879	0.0%	0.0%	4.6%	4.1%
11	0.868	0.0%	0.0%	3.1%	2.7%
12	0.857	0.0%	0.0%	3.1%	2.6%
13	0.846	0.0%	0.0%	1.5%	1.3%
14	0.835	0.0%	0.0%	1.5%	1.3%
15	0.825	0.0%	0.0%	1.5%	1.3%
16	0.814	0.0%	0.0%	1.5%	1.3%
17	0.804	0.0%	0.0%	0.8%	0.6%
18	0.793	0.0%	0.0%	0.8%	0.6%
19	0.783	0.0%	0.0%	0.8%	0.6%
20	0.773	0.0%	0.0%	0.0%	0.0%
Totals		100.0%	95.4%	100.0%	91.9%

Difference in Present Value Loss Patterns =	3.5%	(1)=[95.4%−91.9%]
Projected Loss Ratio =	65.0%	(2)
Underwriting Profit Prov. Offset =	2.3%	(3)=(1)x(2)
Standard U/W Profit Provision =	5.0%	(4)
U/W Profit Provision with Offset =	2.7%	(5)=(4)−(3)

Rate Used for Discounting = 5.28%

CALENDAR YEAR ROE METHOD
SUMMARY

	$	% of Prem	
Premium	$103.35	100.00%	(1)
Loss	$65.00	62.89%	(2)
Fixed Expense	$15.00	14.51%	(3)
Variable Expense	$25.84	25.00%	(4)=(1) x 25%
U/W Gain	($2.49)	−2.41%	(5)=(1) − [(2)+(3)+(4)]
U/W Gain After−Tax	($1.64)	−1.59%	(6)=(5) x (1−34%)
Policyholder Supplied Funds	$82.81	80.13%	(7) (See Exh.2 Line 10)
Surplus	$34.45	33.33%	(8)=(1)/(P/S Ratio) = (1)/3.0
Investible Funds	$117.26	113.46%	(9)=(7)+(8)
Investment Income	$10.87	10.52%	(10)=(9)x(Pre−Tax Yld)=(9)x9.27%
Investment Income AFIT	$7.83	7.58%	(11)=(9)x(After−Tax Yld)=(9)x6.68%
Total Net Income	$6.19	5.99%	(12)=(6)+(11)
Equity	$41.34	40.00%	(13)=(1)/(P/E Ratio)=(1)/2.5

Return on Equity (ROE)	15.0%	(14)=(12)/(13)

PVI/PVE MODEL
SUMMARY

	PRESENT VALUE *	FULL VALUE $	FULL VALUE % of Prem	
Earned Premium	$111.07	$107.89	100.00%	(1)
Incurred Loss	$66.92	$65.00	60.25%	(2)
Incurred Expense	$43.74	$41.97	38.90%	(3)
Underwriting Income	$0.41	$0.92	0.85%	(4)=(1)−(2)−(3)
Investment Income	$10.77	$10.80	10.01%	(5) (see Exh. 6F)
Income Tax	$3.80	$3.98	3.69%	(6) (See Exh. 6F)
Total Income	$7.38	$7.73	7.17%	(7)=(4)+(5)−(6)
Annualized Present Value Equity**	$49.21	----	----	(8) (see Exh. 6E)
Annual Return	15.0%	----	----	(9)=(7)/(8)
Combined	----	$106.97	99.15%	(10)=(2)+(3)
Underwriting Profit Provision	----	----	0.85%	(11)=1−(10)

* Evaluated at the end of the first year
** Evaluated at the beginning of the first year

PVI/PVE MODEL
INPUTS AND ASSUMPTIONS

Loss	$65
Fixed Expense	$15
Variable Expense Ratio	25%
Total Expense Incurral Assumption:	
GAAP	25% up front; 75% evenly for next 4 qtrs.
SAP	75% up front; 25% evenly for next 4 qtrs.
Surplus	"Block" held one year
Premium–to–Surplus Ratio	3.0
Yield on Investments	8.00%
Rate Used for Discounting	8.00%
Income Tax Rate	34%
Target Return	15.0%

PVI/PVE MODEL
UNDERWRITING CASH FLOWS

Qtr	Paid Premium	Paid Loss	Paid Expense
0	43.2	0.0	12.6
1	16.2	2.0	7.3
2	16.2	4.0	7.3
3	16.2	7.0	7.3
4	16.2	8.0	7.3
5	0.0	8.5	0.0
6	0.0	8.0	0.0
7	0.0	6.0	0.0
8	0.0	5.0	0.0
9	0.0	4.0	0.0
10	0.0	3.0	0.0
11	0.0	2.0	0.0
12	0.0	2.0	0.0
13	0.0	1.0	0.0
14	0.0	1.0	0.0
15	0.0	1.0	0.0
16	0.0	1.0	0.0
17	0.0	0.5	0.0
18	0.0	0.5	0.0
19	0.0	0.5	0.0
20	0.0	0.0	0.0
Totals	107.9	65.0	42.0

EXHIBIT 5D

PVI/PVE MODEL
UNDERWRITING INCOME

Qtr	Earned Premium	Incurred Loss	Statutory Incurred Expense	GAAP Incurred Expense	Statutory U/W Income	GAAP U/W Income
0	0.0	0.0	31.5	10.5	−31.5	−10.5
1	27.0	16.3	2.6	7.9	8.1	2.9
2	27.0	16.3	2.6	7.9	8.1	2.9
3	27.0	16.3	2.6	7.9	8.1	2.9
4	27.0	16.3	2.6	7.9	8.1	2.9
5	0.0	0.0	0.0	0.0	0.0	0.0
6	0.0	0.0	0.0	0.0	0.0	0.0
7	0.0	0.0	0.0	0.0	0.0	0.0
8	0.0	0.0	0.0	0.0	0.0	0.0
9	0.0	0.0	0.0	0.0	0.0	0.0
10	0.0	0.0	0.0	0.0	0.0	0.0
11	0.0	0.0	0.0	0.0	0.0	0.0
12	0.0	0.0	0.0	0.0	0.0	0.0
13	0.0	0.0	0.0	0.0	0.0	0.0
14	0.0	0.0	0.0	0.0	0.0	0.0
15	0.0	0.0	0.0	0.0	0.0	0.0
16	0.0	0.0	0.0	0.0	0.0	0.0
17	0.0	0.0	0.0	0.0	0.0	0.0
18	0.0	0.0	0.0	0.0	0.0	0.0
19	0.0	0.0	0.0	0.0	0.0	0.0
20	0.0	0.0	0.0	0.0	0.0	0.0
Totals	107.9	65.0	42.0	42.0	0.9	0.9

PVI/PVE MODEL
BALANCE SHEET

Qtr	Unearned Premium Reserve	Statutory Expense Reserve	Loss Reserve	End of Qtr Surplus	During Qtr Surplus	Assets	Premium Receivable	Investable Assets	Average Investable Assets	Deferred Acquisition	End–Qtr GAAP Equity	During–Qtr GAAP Equity	Present Value GAAP Equity
0	107.9	18.9	0.0	36.0	0.0	162.7	64.7	98.0	0	21.0	56.9	0.0	0.0
1	80.9	14.2	14.3	36.0	36.0	145.3	48.6	96.7	97.4	15.7	51.7	56.9	55.9
2	53.9	9.4	26.5	36.0	36.0	125.9	32.4	93.5	95.1	10.5	46.5	51.7	49.8
3	27.0	4.7	35.8	36.0	36.0	103.4	16.2	87.2	90.4	5.2	41.2	46.5	43.9
4	0.0	0.0	44.0	36.0	36.0	80.0	0.0	80.0	83.6	0.0	36.0	41.2	38.2
5	0.0	0.0	35.5	0.0	0.0	35.5	0.0	35.5	57.7	0.0	0.0	0.0	0.0
6	0.0	0.0	27.5	0.0	0.0	27.5	0.0	27.5	31.5	0.0	0.0	0.0	0.0
7	0.0	0.0	21.5	0.0	0.0	21.5	0.0	21.5	24.5	0.0	0.0	0.0	0.0
8	0.0	0.0	16.5	0.0	0.0	16.5	0.0	16.5	19.0	0.0	0.0	0.0	0.0
9	0.0	0.0	12.5	0.0	0.0	12.5	0.0	12.5	14.5	0.0	0.0	0.0	0.0
10	0.0	0.0	9.5	0.0	0.0	9.5	0.0	9.5	11.0	0.0	0.0	0.0	0.0
11	0.0	0.0	7.5	0.0	0.0	7.5	0.0	7.5	8.5	0.0	0.0	0.0	0.0
12	0.0	0.0	5.5	0.0	0.0	5.5	0.0	5.5	6.5	0.0	0.0	0.0	0.0
13	0.0	0.0	4.5	0.0	0.0	4.5	0.0	4.5	5.0	0.0	0.0	0.0	0.0
14	0.0	0.0	3.5	0.0	0.0	3.5	0.0	3.5	4.0	0.0	0.0	0.0	0.0
15	0.0	0.0	2.5	0.0	0.0	2.5	0.0	2.5	3.0	0.0	0.0	0.0	0.0
16	0.0	0.0	1.5	0.0	0.0	1.5	0.0	1.5	2.0	0.0	0.0	0.0	0.0
17	0.0	0.0	1.0	0.0	0.0	1.0	0.0	1.0	1.3	0.0	0.0	0.0	0.0
18	0.0	0.0	0.5	0.0	0.0	0.5	0.0	0.5	0.8	0.0	0.0	0.0	0.0
19	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.0
20	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
													49.2 (Annualized)*

* Present Value GAAP Equity is annualized via dividing the sum of the discounted quarterly equity values by the sum of the discount factors.

PVI/PVE MODEL
INCOME

Qtr	Investment Income	Statutory U/W Income	Statutory Pre–tax Income	GAAP U/W Income	GAAP Pre–tax Income	Income Tax*	Statutory Income	GAAP Income
0	0.0	–31.5	–31.5	–10.5	–10.5	–3.6	–27.9	–6.9
1	1.9	8.1	10.0	2.9	4.7	1.6	8.4	3.1
2	1.8	8.1	9.9	2.9	4.7	1.6	8.3	3.1
3	1.8	8.1	9.9	2.9	4.6	1.6	8.3	3.0
4	1.6	8.1	9.7	2.9	4.5	1.5	8.2	3.0
5	1.1	0.0	1.1	0.0	1.1	0.4	0.7	0.7
6	0.6	0.0	0.6	0.0	0.6	0.2	0.4	0.4
7	0.5	0.0	0.5	0.0	0.5	0.2	0.3	0.3
8	0.4	0.0	0.4	0.0	0.4	0.1	0.2	0.2
9	0.3	0.0	0.3	0.0	0.3	0.1	0.2	0.2
10	0.2	0.0	0.2	0.0	0.2	0.1	0.1	0.1
11	0.2	0.0	0.2	0.0	0.2	0.1	0.1	0.1
12	0.1	0.0	0.1	0.0	0.1	0.0	0.1	0.1
13	0.1	0.0	0.1	0.0	0.1	0.0	0.1	0.1
14	0.1	0.0	0.1	0.0	0.1	0.0	0.1	0.1
15	0.1	0.0	0.1	0.0	0.1	0.0	0.0	0.0
16	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
17	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
18	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
19	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
20	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Totals	10.8	0.9	11.7	0.9	11.7	4.0	7.7	7.7

* Income Tax is obtained by applying tax rate to GAAP Pre–Tax Income.
This simplified treatment is not in accord with current tax code provisions.

PRESENT VALUE CASH FLOW RETURN MODEL
SUMMARY

	PRESENT VALUE *	FULL VALUE $	FULL VALUE % of Prem	
Premium	$103.22	$106.20	100.00%	(1)
Loss	$57.34	$65.00	61.21%	(2)
Expense	$40.19	$41.55	39.12%	(3)
Underwriting Cash Flow	$5.69	($0.35)	−0.33%	(4)=(1)−(2)−(3)
After−tax U/W Cash Flow	$3.76	($0.23)	−0.22%	(5)=(4)x(1−34%)
Investment Income on Surplus	$2.70	$2.83	2.67%	(6)
After−tax Inv Income on Surplus	$1.78	$1.87	1.76%	(7)=(6)x(1−34%)
Total Cash Flow	$5.54	$1.64	1.54%	(8)=(5)+(7)
Changes in Equity	$5.54 **	$0.00	----	(9)
Combined	----	----	100.33%	(10)=(2)+(3)
Underwriting Profit Provision	----	----	−0.33%	(11)=(1)−(2)−(3)

* Evaluated at the start of the first year

** Evaluated at the start of the first year at target rate of return

PRESENT VALUE CASH FLOW RETURN MODEL
INPUTS AND ASSUMPTIONS

Loss	$65
Fixed Expense	$15
Variable Expense Ratio	25%
Surplus	"Block" held one year
Premium–to–Surplus Ratio	3.0
Equity	"Block" held one year
Equity–to–Surplus Ratio	1.2
Yield on Investments	8.00%
Rate Used for Discounting	8.00%
Income Tax Rate	34%
Target Return	15.0%

PRESENT VALUE CASH FLOW RETURN MODEL
INVESTMENT INCOME ON 'INVESTIBLE EQUITY' **

Qtr	Surplus	Investment Income on Surplus
0	$35.40 *	$0.00
1	$35.40	$0.71
2	$35.40	$0.71
3	$35.40	$0.71
4	$35.40	$0.71
5	$0.00	$0.00
6	$0.00	$0.00
7	$0.00	$0.00
8	$0.00	$0.00
9	$0.00	$0.00
10	$0.00	$0.00
11	$0.00	$0.00
12	$0.00	$0.00
13	$0.00	$0.00
14	$0.00	$0.00
15	$0.00	$0.00
16	$0.00	$0.00
17	$0.00	$0.00
18	$0.00	$0.00
19	$0.00	$0.00
20	$0.00	$0.00
Total		$2.83
Present Value Factor (discounting to start of 1st quarter)		0.953

* $106.20 /3.00= $35.40
** Assumes Surplus equals Investible Equity

PRESENT VALUE CASH FLOW RETURN MODEL
UNDERWRITING CASH FLOWS

Qtr	Paid Premium	Paid Loss	Paid Expense	Net U/W Cash Flow
0	42.5	0.0	12.5	30.0
1	15.9	2.0	7.3	6.7
2	15.9	4.0	7.3	4.7
3	15.9	7.0	7.3	1.7
4	15.9	8.0	7.3	0.7
5	0.0	8.5	0.0	−8.5
6	0.0	8.0	0.0	−8.0
7	0.0	6.0	0.0	−6.0
8	0.0	5.0	0.0	−5.0
9	0.0	4.0	0.0	−4.0
10	0.0	3.0	0.0	−3.0
11	0.0	2.0	0.0	−2.0
12	0.0	2.0	0.0	−2.0
13	0.0	1.0	0.0	−1.0
14	0.0	1.0	0.0	−1.0
15	0.0	1.0	0.0	−1.0
16	0.0	1.0	0.0	−1.0
17	0.0	0.5	0.0	−0.5
18	0.0	0.5	0.0	−0.5
19	0.0	0.5	0.0	−0.5
20	0.0	0.0	0.0	0.0
Totals	106.2	65.0	41.5	−0.3
Present Value Factors (discounting to start of 1st quarter)	0.972	0.882	0.967	

PRESENT VALUE CASH FLOW RETURN MODEL
CHANGES IN EQUITY

Qtr	Equity	Change in Equity	PV Change in Equity
0	42.5	42.5	42.5
1	42.5	0.0	0.0
2	42.5	0.0	0.0
3	42.5	0.0	0.0
4	42.5	−42.5	−36.9
5	0.0	0.0	0.0
6	0.0	0.0	0.0
7	0.0	0.0	0.0
8	0.0	0.0	0.0
9	0.0	0.0	0.0
10	0.0	0.0	0.0
11	0.0	0.0	0.0
12	0.0	0.0	0.0
13	0.0	0.0	0.0
14	0.0	0.0	0.0
15	0.0	0.0	0.0
16	0.0	0.0	0.0
17	0.0	0.0	0.0
18	0.0	0.0	0.0
19	0.0	0.0	0.0
20	0.0	0.0	0.0
Totals		0.0	5.5

PRESENT VALUE CASH FLOW RETURN MODEL

Discount Factors (discounting to start of first year)

Item	Rate Used in Discounting	Discount Factor
Premium	8.00%	0.972
Losses	8.00%	0.882
Expenses	8.00%	0.967
Inv Income on Inv Equity	8.00%	0.953
Changes in Equity	15.00%	

RISK–ADJUSTED DISCOUNTED CASH FLOW MODEL
SUMMARY

	PRESENT VALUE *	FULL VALUE $	FULL VALUE % of Prem	
Premium	$106.84	$101.78	100.00%	(1)
Loss	$62.58	$65.00	63.86%	(2)
Expense	$42.25	$40.45	39.74%	(3)
FIT on U/W Cash Flow	$0.68	($1.25)	−1.22%	(4)=[(1)−(2)−(3)]x34%
FIT on Inv Income on Surplus	$0.95	$0.92	0.91%	(5) (see Exh 7D)
Total	$106.47	----	----	(6)=(2)+(3)+(4)+(5)=(1)
Combined	----	$105.45	103.60%	(7)=(2)+(3)
Underwriting Profit Provision	----	----	−3.60%	(8)=1−(7)

* Evaluated at the end of the first year

EXHIBIT 7B

RISK–ADJUSTED DISCOUNTED CASH FLOW MODEL
INPUTS AND ASSUMPTIONS

Loss	$65
Fixed Expense	$15
Variable Expense Ratio	25%
Surplus	"Block" held one Year
Premium–to–Surplus Ratio	3.0
Risk–Free Rate	8.00%
Average Market Return	10.50%
Beta Coefficient	–0.750
Risk–Adjusted Rate	6.13%
Income Tax Rate	34%

RISK–ADJUSTED DISCOUNTED CASH FLOW MODEL
UNDERWRITING CASH FLOW PATTERNS

	Qtr	Premium Payment Pattern	Paid Loss Pattern	Paid Expense Pattern
	0	0.400	0.000	0.300
	1	0.150	0.031	0.175
	2	0.150	0.062	0.175
	3	0.150	0.108	0.175
	4	0.150	0.123	0.175
	5	0.000	0.131	0.000
	6	0.000	0.123	0.000
	7	0.000	0.092	0.000
	8	0.000	0.077	0.000
	9	0.000	0.062	0.000
	10	0.000	0.046	0.000
	11	0.000	0.031	0.000
	12	0.000	0.031	0.000
	13	0.000	0.015	0.000
	14	0.000	0.015	0.000
	15	0.000	0.015	0.000
	16	0.000	0.015	0.000
	17	0.000	0.008	0.000
	18	0.000	0.008	0.000
	19	0.000	0.008	0.000
	20	0.000	0.000	0.000
	Totals	1.000	1.000	1.000
Rates Used in Discounting		8.00%	6.13%	8.00%
Discount Factors (discounting to end of first year)		1.050	0.963	1.045

EXHIBIT 7D

RISK–ADJUSTED DISCOUNTED CASH FLOW MODEL
TAX ON INVESTMENT INCOME ON SURPLUS

Qtr	Surplus	Investment Income on Surplus	Federal Income Tax
0	33.927	0.000	0.000
1	33.927	0.679	0.231
2	33.927	0.679	0.231
3	33.927	0.679	0.231
4	33.927	0.679	0.231
5	0.000	0.000	0.000
6	0.000	0.000	0.000
7	0.000	0.000	0.000
8	0.000	0.000	0.000
9	0.000	0.000	0.000
10	0.000	0.000	0.000
11	0.000	0.000	0.000
12	0.000	0.000	0.000
13	0.000	0.000	0.000
14	0.000	0.000	0.000
15	0.000	0.000	0.000
16	0.000	0.000	0.000
17	0.000	0.000	0.000
18	0.000	0.000	0.000
19	0.000	0.000	0.000
20	0.000	0.000	0.000
Totals		2.714	0.923

Discount Factor 1.030

RISK–ADJUSTED DISCOUNTED CASH FLOW MODEL
DISCOUNT FACTORS (discounting to end of first year)

Item	Rate Used in Discounting	Discount Factor
Premium	8.00%	1.050
Losses	6.13%	0.963
Expenses	8.00%	1.045
Tax on Inv Inc on Surplus	8.00%	1.030

Calculation of Risk–Adjusted Rate

Risk–Free Rate	8.00%
Average Market Rate	10.50%
Beta Coefficient	–0.750
Risk–Adjusted Rate	6.13%

IRR MODEL
SUMMARY

	FULL VALUE		
	$	% of Prem	
Premium*	$108.51	100.00%	(1)
Loss	$65.00	59.90%	(2)
Expense	$42.13	38.82%	(3)
Combined	$107.13	98.73%	(4)=(2)+(3)
Underwriting Profit Provision	----	1.27%	(5)=(1)−(4)

* Premium at which IRR on Equity Flows is equal to the Target Return

IRR MODEL
INPUTS AND ASSUMPTIONS

Loss	$65
Fixed Expense	$15
Variable Expense Ratio	25%
Total Expense Incurral Assumption:	
GAAP	25% up front; 75% evenly for next 4 qtrs.
SAP	75% up front; 25% evenly for next 4 qtrs.
Surplus	"Block" held one year
Premium–to–Surplus Ratio	3.0
Yield on Investments	8.00%
Rate Used for Discounting	8.00%
Income Tax Rate	34%
Target Return	15.0%

IRR MODEL
SAP UNDERWRITING INCOME

Qtr	Paid Premium	Paid Loss	Paid Expense	Earned Premium	Incurred Loss	Statutory Incurred Expense	Statutory U/W Income
0	43.4	0.0	12.6	0.0	0.0	31.6	−31.6
1	16.3	2.0	7.4	27.1	16.3	2.6	8.2
2	16.3	4.0	7.4	27.1	16.3	2.6	8.2
3	16.3	7.0	7.4	27.1	16.3	2.6	8.2
4	16.3	8.0	7.4	27.1	16.3	2.6	8.2
5	0.0	8.5	0.0	0.0	0.0	0.0	0.0
6	0.0	8.0	0.0	0.0	0.0	0.0	0.0
7	0.0	6.0	0.0	0.0	0.0	0.0	0.0
8	0.0	5.0	0.0	0.0	0.0	0.0	0.0
9	0.0	4.0	0.0	0.0	0.0	0.0	0.0
10	0.0	3.0	0.0	0.0	0.0	0.0	0.0
11	0.0	2.0	0.0	0.0	0.0	0.0	0.0
12	0.0	2.0	0.0	0.0	0.0	0.0	0.0
13	0.0	1.0	0.0	0.0	0.0	0.0	0.0
14	0.0	1.0	0.0	0.0	0.0	0.0	0.0
15	0.0	1.0	0.0	0.0	0.0	0.0	0.0
16	0.0	1.0	0.0	0.0	0.0	0.0	0.0
17	0.0	0.5	0.0	0.0	0.0	0.0	0.0
18	0.0	0.5	0.0	0.0	0.0	0.0	0.0
19	0.0	0.5	0.0	0.0	0.0	0.0	0.0
20	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Totals	108.5	65.0	42.1	108.5	65.0	42.1	1.4

EXHIBIT 8D

IRR MODEL
SAP BALANCE SHEET

Qtr	Unearned Premium Reserve	Statutory Expense Reserve	Loss Reserve	SAP Surplus	Assets	Premium Receivable	Average Investable Assets
0	108.5	19.0	0.0	36.2	163.6	65.1	0.0
1	81.4	14.2	14.3	36.2	146.0	48.8	97.9
2	54.3	9.5	26.5	36.2	126.4	32.6	95.5
3	27.1	4.7	35.8	36.2	103.8	16.3	90.7
4	0.0	0.0	44.0	36.2	80.2	0.0	83.8
5	0.0	0.0	35.5	0.0	35.5	0.0	57.8
6	0.0	0.0	27.5	0.0	27.5	0.0	31.5
7	0.0	0.0	21.5	0.0	21.5	0.0	24.5
8	0.0	0.0	16.5	0.0	16.5	0.0	19.0
9	0.0	0.0	12.5	0.0	12.5	0.0	14.5
10	0.0	0.0	9.5	0.0	9.5	0.0	11.0
11	0.0	0.0	7.5	0.0	7.5	0.0	8.5
12	0.0	0.0	5.5	0.0	5.5	0.0	6.5
13	0.0	0.0	4.5	0.0	4.5	0.0	5.0
14	0.0	0.0	3.5	0.0	3.5	0.0	4.0
15	0.0	0.0	2.5	0.0	2.5	0.0	3.0
16	0.0	0.0	1.5	0.0	1.5	0.0	2.0
17	0.0	0.0	1.0	0.0	1.0	0.0	1.3
18	0.0	0.0	0.5	0.0	0.5	0.0	0.8
19	0.0	0.0	0.0	0.0	0.0	0.0	0.3
20	0.0	0.0	0.0	0.0	0.0	0.0	0.0

EXHIBIT 8E

IRR MODEL
SAP INCOME

Qtr	Investment Income	Statutory U/W Income	Statutory Pre−tax Income	Income Tax*	Statutory Income	Change in Surplus	Equity Flow
0	0.0	−31.6	−31.6	−3.6	−28.0	36.2	−64.2
1	1.9	8.2	10.1	1.7	8.5	0.0	8.5
2	1.9	8.2	10.1	1.6	8.5	0.0	8.5
3	1.8	8.2	10.0	1.6	8.4	0.0	8.4
4	1.6	8.2	9.9	1.6	8.3	−36.2	44.5
5	1.1	0.0	1.1	0.4	0.7	0.0	0.7
6	0.6	0.0	0.6	0.2	0.4	0.0	0.4
7	0.5	0.0	0.5	0.2	0.3	0.0	0.3
8	0.4	0.0	0.4	0.1	0.2	0.0	0.2
9	0.3	0.0	0.3	0.1	0.2	0.0	0.2
10	0.2	0.0	0.2	0.1	0.1	0.0	0.1
11	0.2	0.0	0.2	0.1	0.1	0.0	0.1
12	0.1	0.0	0.1	0.0	0.1	0.0	0.1
13	0.1	0.0	0.1	0.0	0.1	0.0	0.1
14	0.1	0.0	0.1	0.0	0.1	0.0	0.1
15	0.1	0.0	0.1	0.0	0.0	0.0	0.0
16	0.0	0.0	0.0	0.0	0.0	0.0	0.0
17	0.0	0.0	0.0	0.0	0.0	0.0	0.0
18	0.0	0.0	0.0	0.0	0.0	0.0	0.0
19	0.0	0.0	0.0	0.0	0.0	0.0	0.0
20	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Totals	10.8	1.4	12.2	4.2	8.1	0.0	8.1

* Income Tax is obtained by applying the tax rate to Statutory Pre−tax Income.
This simplified treatment is not in accord with current tax code provisions.

IRR MODEL
GAAP INCOME

Qtr	Statutory Incurred Expense	GAAP Incurred Expense	Deferred Acquisition	GAAP U/W Income	GAAP Pre–tax Income	GAAP Income	Change in Surplus	Change in Deferred Acquisition	Equity Change	Equity Flow
0	31.6	10.5	21.1	−10.5	−10.5	−7.0	36.2	21.1	57.2	−64.2
1	2.6	7.9	15.8	3.0	4.9	3.2	0.0	−5.3	−5.3	8.5
2	2.6	7.9	10.5	3.0	4.8	3.2	0.0	−5.3	−5.3	8.5
3	2.6	7.9	5.3	3.0	4.7	3.1	0.0	−5.3	−5.3	8.4
4	2.6	7.9	0.0	3.0	4.6	3.0	−36.2	−5.3	−41.4	44.5
5	0.0	0.0	0.0	0.0	1.1	0.7	0.0	0.0	0.0	0.7
6	0.0	0.0	0.0	0.0	0.6	0.4	0.0	0.0	0.0	0.4
7	0.0	0.0	0.0	0.0	0.5	0.3	0.0	0.0	0.0	0.3
8	0.0	0.0	0.0	0.0	0.4	0.2	0.0	0.0	0.0	0.2
9	0.0	0.0	0.0	0.0	0.3	0.2	0.0	0.0	0.0	0.2
10	0.0	0.0	0.0	0.0	0.2	0.1	0.0	0.0	0.0	0.1
11	0.0	0.0	0.0	0.0	0.2	0.1	0.0	0.0	0.0	0.1
12	0.0	0.0	0.0	0.0	0.1	0.1	0.0	0.0	0.0	0.1
13	0.0	0.0	0.0	0.0	0.1	0.1	0.0	0.0	0.0	0.1
14	0.0	0.0	0.0	0.0	0.1	0.1	0.0	0.0	0.0	0.1
15	0.0	0.0	0.0	0.0	0.1	0.0	0.0	0.0	0.0	0.0
16	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
17	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
18	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
19	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
20	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Totals	42.1	42.1		1.4	12.2	8.1				8.1