Tables for

CAS Exam MAS-I

Version date 01/15/2018

The following tables will be provided to the candidate with the exam. The tables on pages 3 through 15 are reprinted with the permission of the Society of Actuaries; the tables on pages 16 through 20 are copyright material of the Casualty Actuarial Society.

We are furnishing a set of tables for statistical tests as well as a set of distribution functions for Exam MAS-I. We do not have a single authoritative textbook for Statistics. The format of the tables may vary from one textbook to the next. The nomenclature used to describe the distribution functions may vary from one textbook to the next. To avoid confusion on the part of the candidates we will use the tables and distribution functions definitions that follow when writing exam questions for Exam MAS-I.

Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC)

AIC and BIC are two common measures of relative model fit often used as criteria to select among competing models. These topics are covered in a number of the syllabus materials. Although the conceptual treatment are the same in each, the actual AIC and BIC formulas vary by source.

Candidates are expected to understand AIC and BIC as presented in all of the texts. However if an exam question requires a calculation of AIC and/or BIC, unless an alternative formula is explicitly given, the following will be used:

$$\text{AIC} = -2 * l(\hat{\pi}; y) + 2 * p$$

$$\text{BIC} = -2 * l(\hat{\pi}; y) + \ln(n) * p$$

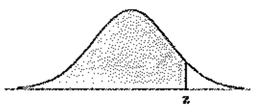
where:

$l(\hat{\pi}; y)$ is the log-likelihood of the observed data, Y, under the model of interest and the maximum likelihood estimates of the parameters

p is the number of parameters in the model

n is the number of observations in the modeled dataset

Tables of the Normal Distribution



Probability Content from -∞ to Z

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

Values of z for selected values of Pr(Z<z)

z	0.842	1.036	1.282	1.645	1.960	2.326	2.576
Pr(Z<z)	0.800	0.850	0.900	0.950	0.975	0.990	0.995

Illustrative Life Table: Basic Functions and Single Benefit Premiums at $i = 0.06$

x	l_x	$1000q_x$	$\ddot{a}_x$	$1000A_x$	$1000(^2A_x)$	$1000\,{}_5E_x$	$1000\,{}_{10}E_x$	$1000\,{}_{20}E_x$	x
0	10,000,000	20.42	16.8010	49.00	25.92	728.54	541.95	299.89	0
5	9,749,503	0.98	17.0379	35.59	8.45	743.89	553.48	305.90	5
10	9,705,588	0.85	16.9119	42.72	9.37	744.04	553.34	305.24	10
15	9,663,731	0.91	16.7384	52.55	11.33	743.71	552.69	303.96	15
20	9,617,802	1.03	16.5133	65.28	14.30	743.16	551.64	301.93	20
21	9,607,896	1.06	16.4611	68.24	15.06	743.01	551.36	301.40	21
22	9,597,695	1.10	16.4061	71.35	15.87	742.86	551.06	300.82	22
23	9,587,169	1.13	16.3484	74.62	16.76	742.68	550.73	300.19	23
24	9,576,288	1.18	16.2878	78.05	17.71	742.49	550.36	299.49	24
25	9,565,017	1.22	16.2242	81.65	18.75	742.29	549.97	298.73	25
26	9,553,319	1.27	16.1574	85.43	19.87	742.06	549.53	297.90	26
27	9,541,153	1.33	16.0873	89.40	21.07	741.81	549.05	297.00	27
28	9,528,475	1.39	16.0139	93.56	22.38	741.54	548.53	296.01	28
29	9,515,235	1.46	15.9368	97.92	23.79	741.24	547.96	294.92	29
30	9,501,381	1.53	15.8561	102.48	25.31	740.91	547.33	293.74	30
31	9,486,854	1.61	15.7716	107.27	26.95	740.55	546.65	292.45	31
32	9,471,591	1.70	15.6831	112.28	28.72	740.16	545.90	291.04	32
33	9,455,522	1.79	15.5906	117.51	30.63	739.72	545.07	289.50	33
34	9,438,571	1.90	15.4938	122.99	32.68	739.25	544.17	287.82	34
35	9,420,657	2.01	15.3926	128.72	34.88	738.73	543.18	286.00	35
36	9,401,688	2.14	15.2870	134.70	37.26	738.16	542.11	284.00	36
37	9,381,566	2.28	15.1767	140.94	39.81	737.54	540.92	281.84	37
38	9,360,184	2.43	15.0616	147.46	42.55	736.86	539.63	279.48	38
39	9,337,427	2.60	14.9416	154.25	45.48	736.11	538.22	276.92	39
40	9,313,166	2.78	14.8166	161.32	48.63	735.29	536.67	274.14	40
41	9,287,264	2.98	14.6864	168.69	52.01	734.40	534.99	271.12	41
42	9,259,571	3.20	14.5510	176.36	55.62	733.42	533.14	267.85	42
43	9,229,925	3.44	14.4102	184.33	59.48	732.34	531.12	264.31	43
44	9,198,149	3.71	14.2639	192.61	63.61	731.17	528.92	260.48	44
45	9,164,051	4.00	14.1121	201.20	68.02	729.88	526.52	256.34	45
46	9,127,426	4.31	13.9546	210.12	72.72	728.47	523.89	251.88	46
47	9,088,049	4.66	13.7914	219.36	77.73	726.93	521.03	247.08	47
48	9,045,679	5.04	13.6224	228.92	83.06	725.24	517.91	241.93	48
49	9,000,057	5.46	13.4475	238.82	88.73	723.39	514.51	236.39	49
50	8,950,901	5.92	13.2668	249.05	94.76	721.37	510.81	230.47	50
51	8,897,913	6.42	13.0803	259.61	101.15	719.17	506.78	224.15	51
52	8,840,770	6.97	12.8879	270.50	107.92	716.76	502.40	217.42	52
53	8,779,128	7.58	12.6896	281.72	115.09	714.12	497.64	210.27	53
54	8,712,621	8.24	12.4856	293.27	122.67	711.24	492.47	202.70	54
55	8,640,861	8.96	12.2758	305.14	130.67	708.10	486.86	194.72	55
56	8,563,435	9.75	12.0604	317.33	139.11	704.67	480.79	186.32	56
57	8,479,908	10.62	11.8395	329.84	147.99	700.93	474.22	177.53	57
58	8,389,826	11.58	11.6133	342.65	157.33	696.85	467.12	168.37	58
59	8,292,713	12.62	11.3818	355.75	167.13	692.41	459.46	158.87	59
60	8,188,074	13.76	11.1454	369.13	177.41	687.56	451.20	149.06	60
61	8,075,403	15.01	10.9041	382.79	188.17	682.29	442.31	139.00	61
62	7,954,179	16.38	10.6584	396.70	199.41	676.56	432.77	128.75	62
63	7,823,879	17.88	10.4084	410.85	211.13	670.33	422.54	118.38	63
64	7,683,979	19.52	10.1544	425.22	223.34	663.56	411.61	107.97	64
65	7,533,964	21.32	9.8969	439.80	236.03	656.23	399.94	97.60	65

Illustrative Life Table: Basic Functions and Single Benefit Premiums at $i = 0.06$

x	l_x	$1000q_x$	$\ddot{a}_x$	$1000A_x$	$1000(^2A_x)$	$1000\,{}_5E_x$	$1000\,{}_{10}E_x$	$1000\,{}_{20}E_x$	x
66	7,373,338	23.29	9.6362	454.56	249.20	648.27	387.53	87.37	66
67	7,201,635	25.44	9.3726	469.47	262.83	639.66	374.36	77.38	67
68	7,018,432	27.79	9.1066	484.53	276.92	630.35	360.44	67.74	68
69	6,823,367	30.37	8.8387	499.70	291.46	620.30	345.77	58.54	69
70	6,616,155	33.18	8.5693	514.95	306.42	609.46	330.37	49.88	70
71	6,396,609	36.26	8.2988	530.26	321.78	597.79	314.27	41.86	71
72	6,164,663	39.62	8.0278	545.60	337.54	585.25	297.51	34.53	72
73	5,920,394	43.30	7.7568	560.93	353.64	571.81	280.17	27.96	73
74	5,664,051	47.31	7.4864	576.24	370.08	557.43	262.31	22.19	74
75	5,396,081	51.69	7.2170	591.49	386.81	542.07	244.03	17.22	75
76	5,117,152	56.47	6.9493	606.65	403.80	525.71	225.46	13.04	76
77	4,828,182	61.68	6.6836	621.68	421.02	508.35	206.71	9.61	77
78	4,530,360	67.37	6.4207	636.56	438.42	489.97	187.94	6.88	78
79	4,225,163	73.56	6.1610	651.26	455.95	470.57	169.31	4.77	79
80	3,914,365	80.30	5.9050	665.75	473.59	450.19	151.00	3.19	80
81	3,600,038	87.64	5.6533	680.00	491.27	428.86	133.19	2.05	81
82	3,284,542	95.61	5.4063	693.98	508.96	406.62	116.06	1.27	82
83	2,970,496	104.28	5.1645	707.67	526.60	383.57	99.81	0.75	83
84	2,660,734	113.69	4.9282	721.04	544.15	359.79	84.59	0.42	84
85	2,358,246	123.89	4.6980	734.07	561.57	335.40	70.56	0.22	85
86	2,066,090	134.94	4.4742	746.74	578.80	310.56	57.83	0.11	86
87	1,787,299	146.89	4.2571	759.03	595.79	285.44	46.50	0.05	87
88	1,524,758	159.81	4.0470	770.92	612.51	260.21	36.61	0.02	88
89	1,281,083	173.75	3.8442	782.41	628.92	235.11	28.17	0.01	89
90	1,058,491	188.77	3.6488	793.46	644.96	210.36	21.13	0.00	90
91	858,676	204.93	3.4611	804.09	660.61	186.21	15.41	0.00	91
92	682,707	222.27	3.2812	814.27	675.83	162.90	10.91	0.00	92
93	530,959	240.86	3.1091	824.01	690.59	140.69	7.47	0.00	93
94	403,072	260.73	2.9450	833.30	704.86	119.79	4.93	0.00	94
95	297,981	281.91	2.7888	842.14	718.61	100.43	3.13	0.00	95
96	213,977	304.45	2.6406	850.53	731.83	82.78	1.90	0.00	96
97	148,832	328.34	2.5002	858.48	744.50	66.97	1.10	0.00	97
98	99,965	353.60	2.3676	865.99	756.60	53.09	0.60	0.00	98
99	64,617	380.20	2.2426	873.06	768.13	41.14	0.31	0.00	99
100	40,049	408.12	2.1252	879.70	779.08	31.12	0.15	0.00	100
101	23,705	437.28	2.0152	885.93	789.44	22.91	0.07	0.00	101
102	13,339	467.61	1.9123	891.76	799.21	16.37	0.03	0.00	102
103	7,101	498.99	1.8164	897.19	808.41	11.33	0.01	0.00	103
104	3,558	531.28	1.7273	902.23	817.02	7.56	0.00	0.00	104
105	1,668	564.29	1.6447	906.90	825.06	4.86	0.00	0.00	105
106	727	597.83	1.5685	911.22	832.53	2.99	0.00	0.00	106
107	292	631.64	1.4984	915.19	839.46	1.76	0.00	0.00	107
108	108	665.45	1.4341	918.82	845.84	0.98	0.00	0.00	108
109	36	698.97	1.3755	922.14	851.69	0.52	0.00	0.00	109
110	11	731.87	1.3223	925.15	857.04	0.26	0.00	0.00	110

Interest Functions

Interest Functions at $i = 0.06$

m	$i^{(m)}$	$d^{(m)}$	$i/i^{(m)}$	$d/d^{(m)}$	$\alpha(m)$	$\beta(m)$
1	0.06000	0.05660	1.00000	1.00000	1.00000	0.00000
2	0.05913	0.05743	1.01478	0.98564	1.00021	0.25739
4	0.05870	0.05785	1.02223	0.97852	1.00027	0.38424
12	0.05841	0.05813	1.02721	0.97378	1.00028	0.46812
∞	0.05827	0.05827	1.02971	0.97142	1.00028	0.50985

Special Note: Unless specified, the force of interest is constant in each question .

Excerpts from the Appendices to *Loss Models: From Data to Decisions, 2nd edition*

April 21, 2005

Appendix A

An Inventory of Continuous Distributions

A.1 Introduction

The incomplete gamma function is given by

$$\Gamma(\alpha; x) = \frac{1}{\Gamma(\alpha)} \int_0^x t^{\alpha-1} e^{-t}\, dt, \quad \alpha > 0,\ x > 0$$

$$\text{with } \Gamma(\alpha) = \int_0^\infty t^{\alpha-1} e^{-t}\, dt, \quad \alpha > 0.$$

Also, define

$$G(\alpha; x) = \int_x^\infty t^{\alpha-1} e^{-t}\, dt, \quad x > 0.$$

Integration by parts produces the relationship

$$G(\alpha; x) = -\frac{x^\alpha e^{-x}}{\alpha} + \frac{1}{\alpha} G(\alpha + 1; x)$$

For negative α, this can repeated until the first argument is postive, say at $\alpha + k$. Then the incomplete gamma function can be evaluated from

$$G(\alpha + k; x) = \Gamma(\alpha + k)[1 - \Gamma(\alpha + k; x)].$$

The incomplete beta function is given by

$$\beta(a, b; x) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} \int_0^x t^{a-1}(1-t)^{b-1}\, dt, \quad a > 0,\ b > 0,\ 0 < x < 1.$$

A.2 Transformed beta family

A.2.3 Three-parameter distributions

A.2.3.1 Generalized Pareto (beta of the second kind)—α, θ, τ

$$
\begin{aligned}
f(x) &= \frac{\Gamma(\alpha+\tau)}{\Gamma(\alpha)\Gamma(\tau)} \frac{\theta^\alpha x^{\tau-1}}{(x+\theta)^{\alpha+\tau}} \qquad F(x) = \beta(\tau, \alpha; u), \quad u = \frac{x}{x+\theta} \\
\mathrm{E}[X^k] &= \frac{\theta^k \Gamma(\tau+k)\Gamma(\alpha-k)}{\Gamma(\alpha)\Gamma(\tau)}, \quad -\tau < k < \alpha \\
\mathrm{E}[X^k] &= \frac{\theta^k \tau(\tau+1)\cdots(\tau+k-1)}{(\alpha-1)\cdots(\alpha-k)}, \quad \text{if } k \text{ is an integer} \\
\mathrm{E}[(X \wedge x)^k] &= \frac{\theta^k \Gamma(\tau+k)\Gamma(\alpha-k)}{\Gamma(\alpha)\Gamma(\tau)} \beta(\tau+k, \alpha-k; u) + x^k[1-F(x)], \quad k > -\tau \\
\text{mode} &= \theta \frac{\tau-1}{\alpha+1}, \quad \tau > 1, \text{ else } 0
\end{aligned}
$$

A.2.3.2 Burr (Burr Type XII, Singh-Maddala)—α, θ, γ

$$
\begin{aligned}
f(x) &= \frac{\alpha\gamma(x/\theta)^\gamma}{x[1+(x/\theta)^\gamma]^{\alpha+1}} \qquad F(x) = 1 - u^\alpha, \quad u = \frac{1}{1+(x/\theta)^\gamma} \\
\mathrm{E}[X^k] &= \frac{\theta^k \Gamma(1+k/\gamma)\Gamma(\alpha-k/\gamma)}{\Gamma(\alpha)}, \quad -\gamma < k < \alpha\gamma \\
\mathrm{E}[(X \wedge x)^k] &= \frac{\theta^k \Gamma(1+k/\gamma)\Gamma(\alpha-k/\gamma)}{\Gamma(\alpha)} \beta(1+k/\gamma, \alpha-k/\gamma; 1-u) + x^k u^\alpha, \quad k > -\gamma \\
\text{mode} &= \theta \left(\frac{\gamma-1}{\alpha\gamma+1} \right)^{1/\gamma}, \quad \gamma > 1, \text{ else } 0
\end{aligned}
$$

A.2.3.3 Inverse Burr (Dagum)—τ, θ, γ

$$
\begin{aligned}
f(x) &= \frac{\tau\gamma(x/\theta)^{\gamma\tau}}{x[1+(x/\theta)^\gamma]^{\tau+1}} \qquad F(x) = u^\tau, \quad u = \frac{(x/\theta)^\gamma}{1+(x/\theta)^\gamma} \\
\mathrm{E}[X^k] &= \frac{\theta^k \Gamma(\tau+k/\gamma)\Gamma(1-k/\gamma)}{\Gamma(\tau)}, \quad -\tau\gamma < k < \gamma \\
\mathrm{E}[(X \wedge x)^k] &= \frac{\theta^k \Gamma(\tau+k/\gamma)\Gamma(1-k/\gamma)}{\Gamma(\tau)} \beta(\tau+k/\gamma, 1-k/\gamma; u) + x^k[1-u^\tau], \quad k > -\tau\gamma \\
\text{mode} &= \theta \left(\frac{\tau\gamma-1}{\gamma+1} \right)^{1/\gamma}, \quad \tau\gamma > 1, \text{ else } 0
\end{aligned}
$$

A.2.4 Two-parameter distributions

A.2.4.1 Pareto—α, θ

$$
\begin{aligned}
f(x) &= \frac{\alpha\theta^\alpha}{(x+\theta)^{\alpha+1}} \qquad F(x) = 1 - \left(\frac{\theta}{x+\theta}\right)^\alpha \\
\mathrm{E}[X^k] &= \frac{\theta^k\Gamma(k+1)\Gamma(\alpha-k)}{\Gamma(\alpha)}, \quad -1 < k < \alpha \\
\mathrm{E}[X^k] &= \frac{\theta^k k!}{(\alpha-1)\cdots(\alpha-k)}, \quad \text{if } k \text{ is an integer} \\
\mathrm{E}[X \wedge x] &= \frac{\theta}{\alpha-1}\left[1 - \left(\frac{\theta}{x+\theta}\right)^{\alpha-1}\right], \quad \alpha \neq 1 \\
\mathrm{E}[X \wedge x] &= -\theta \ln\left(\frac{\theta}{x+\theta}\right), \quad \alpha = 1 \\
\mathrm{E}[(X \wedge x)^k] &= \frac{\theta^k\Gamma(k+1)\Gamma(\alpha-k)}{\Gamma(\alpha)}\beta[k+1, \alpha-k; x/(x+\theta)] + x^k\left(\frac{\theta}{x+\theta}\right)^\alpha, \quad \text{all } k \\
\text{mode} &= 0
\end{aligned}
$$

A.2.4.2 Inverse Pareto—τ, θ

$$
\begin{aligned}
f(x) &= \frac{\tau\theta x^{\tau-1}}{(x+\theta)^{\tau+1}} \qquad F(x) = \left(\frac{x}{x+\theta}\right)^\tau \\
\mathrm{E}[X^k] &= \frac{\theta^k\Gamma(\tau+k)\Gamma(1-k)}{\Gamma(\tau)}, \quad -\tau < k < 1 \\
\mathrm{E}[X^k] &= \frac{\theta^k(-k)!}{(\tau-1)\cdots(\tau+k)}, \quad \text{if } k \text{ is a negative integer} \\
\mathrm{E}[(X \wedge x)^k] &= \theta^k\tau\int_0^{x/(x+\theta)} y^{\tau+k-1}(1-y)^{-k}dy + x^k\left[1 - \left(\frac{x}{x+\theta}\right)^\tau\right], \quad k > -\tau \\
\text{mode} &= \theta\frac{\tau-1}{2}, \quad \tau > 1, \text{ else } 0
\end{aligned}
$$

A.2.4.3 Loglogistic (Fisk)—γ, θ

$$
\begin{aligned}
f(x) &= \frac{\gamma(x/\theta)^\gamma}{x[1+(x/\theta)^\gamma]^2} \qquad F(x) = u, \quad u = \frac{(x/\theta)^\gamma}{1+(x/\theta)^\gamma} \\
\mathrm{E}[X^k] &= \theta^k\Gamma(1+k/\gamma)\Gamma(1-k/\gamma), \quad -\gamma < k < \gamma \\
\mathrm{E}[(X \wedge x)^k] &= \theta^k\Gamma(1+k/\gamma)\Gamma(1-k/\gamma)\beta(1+k/\gamma, 1-k/\gamma; u) + x^k(1-u), \quad k > -\gamma \\
\text{mode} &= \theta\left(\frac{\gamma-1}{\gamma+1}\right)^{1/\gamma}, \quad \gamma > 1, \text{ else } 0
\end{aligned}
$$

A.2.4.4 Paralogistic—α, θ

This is a Burr distribution with $\gamma = \alpha$.

$$
\begin{aligned}
f(x) &= \frac{\alpha^2 (x/\theta)^\alpha}{x[1+(x/\theta)^\alpha]^{\alpha+1}} \qquad F(x) = 1 - u^\alpha, \quad u = \frac{1}{1+(x/\theta)^\alpha} \\
\mathrm{E}[X^k] &= \frac{\theta^k \Gamma(1+k/\alpha)\Gamma(\alpha - k/\alpha)}{\Gamma(\alpha)}, \quad -\alpha < k < \alpha^2 \\
\mathrm{E}[(X \wedge x)^k] &= \frac{\theta^k \Gamma(1+k/\alpha)\Gamma(\alpha - k/\alpha)}{\Gamma(\alpha)} \beta(1+k/\alpha, \alpha - k/\alpha; 1-u) + x^k u^\alpha, \quad k > -\alpha \\
\text{mode} &= \theta \left(\frac{\alpha - 1}{\alpha^2 + 1} \right)^{1/\alpha}, \quad \alpha > 1, \text{ else } 0
\end{aligned}
$$

A.2.4.5 Inverse paralogistic—τ, θ

This is an inverse Burr distribution with $\gamma = \tau$.

$$
\begin{aligned}
f(x) &= \frac{\tau^2 (x/\theta)^{\tau^2}}{x[1+(x/\theta)^\tau]^{\tau+1}} \qquad F(x) = u^\tau, \quad u = \frac{(x/\theta)^\tau}{1+(x/\theta)^\tau} \\
\mathrm{E}[X^k] &= \frac{\theta^k \Gamma(\tau + k/\tau)\Gamma(1 - k/\tau)}{\Gamma(\tau)}, \quad -\tau^2 < k < \tau \\
\mathrm{E}[(X \wedge x)^k] &= \frac{\theta^k \Gamma(\tau + k/\tau)\Gamma(1 - k/\tau)}{\Gamma(\tau)} \beta(\tau + k/\tau, 1 - k/\tau; u) + x^k [1 - u^\tau], \quad k > -\tau^2 \\
\text{mode} &= \theta \left(\tau - 1 \right)^{1/\tau}, \quad \tau > 1, \text{ else } 0
\end{aligned}
$$

A.3 Transformed gamma family

A.3.2 Two-parameter distributions

A.3.2.1 Gamma—α, θ

$$
\begin{aligned}
f(x) &= \frac{(x/\theta)^\alpha e^{-x/\theta}}{x\Gamma(\alpha)} \qquad F(x) = \Gamma(\alpha; x/\theta) \\
M(t) &= (1 - \theta t)^{-\alpha}, \quad t < 1/\theta \qquad \mathrm{E}[X^k] = \frac{\theta^k \Gamma(\alpha + k)}{\Gamma(\alpha)}, \quad k > -\alpha \\
\mathrm{E}[X^k] &= \theta^k (\alpha + k - 1) \cdots \alpha, \quad \text{if } k \text{ is an integer}
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{E}[(X \wedge x)^k] &= \frac{\theta^k \Gamma(\alpha + k)}{\Gamma(\alpha)} \Gamma(\alpha + k; x/\theta) + x^k [1 - \Gamma(\alpha; x/\theta)], \quad k > -\alpha \\
&= \alpha(\alpha + 1) \cdots (\alpha + k - 1) \theta^k \Gamma(\alpha + k; x/\theta) + x^k [1 - \Gamma(\alpha; x/\theta)], \quad k \text{ an integer} \\
\text{mode} &= \theta(\alpha - 1), \quad \alpha > 1, \text{ else } 0
\end{aligned}
$$

A.3.2.2 Inverse gamma (Vinci)—α, θ

$$
\begin{aligned}
f(x) &= \frac{(\theta/x)^\alpha e^{-\theta/x}}{x\Gamma(\alpha)} \qquad F(x) = 1 - \Gamma(\alpha;\theta/x) \\
\mathrm{E}[X^k] &= \frac{\theta^k\Gamma(\alpha-k)}{\Gamma(\alpha)}, \quad k<\alpha \qquad \mathrm{E}[X^k] = \frac{\theta^k}{(\alpha-1)\cdots(\alpha-k)}, \quad \text{if } k \text{ is an integer} \\
\mathrm{E}[(X\wedge x)^k] &= \frac{\theta^k\Gamma(\alpha-k)}{\Gamma(\alpha)}[1-\Gamma(\alpha-k;\theta/x)] + x^k\Gamma(\alpha;\theta/x) \\
&= \frac{\theta^k\Gamma(\alpha-k)}{\Gamma(\alpha)}G(\alpha-k;\theta/x) + x^k\Gamma(\alpha;\theta/x), \text{ all } k \\
\text{mode} &= \theta/(\alpha+1)
\end{aligned}
$$

A.3.2.3 Weibull—θ, τ

$$
\begin{aligned}
f(x) &= \frac{\tau(x/\theta)^\tau e^{-(x/\theta)^\tau}}{x} \qquad F(x) = 1 - e^{-(x/\theta)^\tau} \\
\mathrm{E}[X^k] &= \theta^k\Gamma(1+k/\tau), \quad k>-\tau \\
\mathrm{E}[(X\wedge x)^k] &= \theta^k\Gamma(1+k/\tau)\Gamma[1+k/\tau;(x/\theta)^\tau] + x^k e^{-(x/\theta)^\tau}, \quad k>-\tau \\
\text{mode} &= \theta\left(\frac{\tau-1}{\tau}\right)^{1/\tau}, \quad \tau>1, \text{ else } 0
\end{aligned}
$$

A.3.2.4 Inverse Weibull (log Gompertz)—θ, τ

$$
\begin{aligned}
f(x) &= \frac{\tau(\theta/x)^\tau e^{-(\theta/x)^\tau}}{x} \qquad F(x) = e^{-(\theta/x)^\tau} \\
\mathrm{E}[X^k] &= \theta^k\Gamma(1-k/\tau), \quad k<\tau \\
\mathrm{E}[(X\wedge x)^k] &= \theta^k\Gamma(1-k/\tau)\{1-\Gamma[1-k/\tau;(\theta/x)^\tau]\} + x^k\left[1-e^{-(\theta/x)^\tau}\right], \quad \text{all } k \\
&= \theta^k\Gamma(1-k/\tau)G[1-k/\tau;(\theta/x)^\tau] + x^k\left[1-e^{-(\theta/x)^\tau}\right] \\
\text{mode} &= \theta\left(\frac{\tau}{\tau+1}\right)^{1/\tau}
\end{aligned}
$$

A.3.3 One-parameter distributions

A.3.3.1 Exponential—θ

$$
\begin{aligned}
f(x) &= \frac{e^{-x/\theta}}{\theta} \qquad F(x) = 1 - e^{-x/\theta} \\
M(t) &= (1-\theta t)^{-1} \qquad \mathrm{E}[X^k] = \theta^k\Gamma(k+1), \quad k>-1 \\
\mathrm{E}[X^k] &= \theta^k k!, \quad \text{if } k \text{ is an integer} \\
\mathrm{E}[X\wedge x] &= \theta(1-e^{-x/\theta}) \\
\mathrm{E}[(X\wedge x)^k] &= \theta^k\Gamma(k+1)\Gamma(k+1;x/\theta) + x^k e^{-x/\theta}, \quad k>-1 \\
&= \theta^k k!\Gamma(k+1;x/\theta) + x^k e^{-x/\theta}, \quad k \text{ an integer} \\
\text{mode} &= 0
\end{aligned}
$$

A.3.3.2 Inverse exponential—θ

$$\begin{aligned}
f(x) &= \frac{\theta e^{-\theta/x}}{x^2} \qquad\qquad F(x) = e^{-\theta/x} \\
\mathrm{E}[X^k] &= \theta^k \Gamma(1-k), \quad k<1 \\
\mathrm{E}[(X \wedge x)^k] &= \theta^k G(1-k;\theta/x) + x^k(1-e^{-\theta/x}), \quad \text{all } k \\
\text{mode} &= \theta/2
\end{aligned}$$

A.4 Other distributions

A.4.1.1 Lognormal—μ,σ (μ can be negative)

$$\begin{aligned}
f(x) &= \frac{1}{x\sigma\sqrt{2\pi}}\exp(-z^2/2) = \phi(z)/(\sigma x), \quad z = \frac{\ln x - \mu}{\sigma} \qquad\qquad F(x) = \Phi(z) \\
\mathrm{E}[X^k] &= \exp(k\mu + k^2\sigma^2/2) \\
\mathrm{E}[(X \wedge x)^k] &= \exp(k\mu + k^2\sigma^2/2)\Phi\left(\frac{\ln x - \mu - k\sigma^2}{\sigma}\right) + x^k[1-F(x)] \\
\text{mode} &= \exp(\mu - \sigma^2)
\end{aligned}$$

A.4.1.2 Inverse Gaussian—μ, θ

$$\begin{aligned}
f(x) &= \left(\frac{\theta}{2\pi x^3}\right)^{1/2} \exp\left(-\frac{\theta z^2}{2x}\right), \quad z = \frac{x-\mu}{\mu} \\
F(x) &= \Phi\left[z\left(\frac{\theta}{x}\right)^{1/2}\right] + \exp\left(\frac{2\theta}{\mu}\right)\Phi\left[-y\left(\frac{\theta}{x}\right)^{1/2}\right], \quad y = \frac{x+\mu}{\mu} \\
M(t) &= \exp\left[\frac{\theta}{\mu}\left(1-\sqrt{1-\frac{2t\mu^2}{\theta}}\right)\right], \quad t < \frac{\theta}{2\mu^2}, \qquad\qquad \mathrm{E}[X] = \mu, \quad \mathrm{Var}[X] = \mu^3/\theta \\
\mathrm{E}[X \wedge x] &= x - \mu z \Phi\left[z\left(\frac{\theta}{x}\right)^{1/2}\right] - \mu y \exp\left(\frac{2\theta}{\mu}\right)\Phi\left[-y\left(\frac{\theta}{x}\right)^{1/2}\right]
\end{aligned}$$

A.4.1.3 log-t—r, μ, σ (μ *can be negative*)

Let Y have a t distribution with r degrees of freedom. Then $X = \exp(\sigma Y + \mu)$ has the log-t distribution. Positive moments do not exist for this distribution. Just as the t distribution has a heavier tail than the normal distribution, this distribution has a heavier tail than the lognormal distribution.

$$\begin{aligned}
f(x) &= \frac{\Gamma\left(\frac{r+1}{2}\right)}{x\sigma\sqrt{\pi r}\,\Gamma\left(\frac{r}{2}\right)\left[1+\frac{1}{r}\left(\frac{\ln x - \mu}{\sigma}\right)^2\right]^{(r+1)/2}}, \\
F(x) &= F_r\left(\frac{\ln x - \mu}{\sigma}\right) \text{ with } F_r(t) \text{ the cdf of a } t \text{ distribution with } r \text{ d.f.,}
\end{aligned}$$

$$F(x) = \begin{cases} \frac{1}{2}\beta\left[\frac{r}{2}, \frac{1}{2}; \frac{r}{r + \left(\frac{\ln x - \mu}{\sigma}\right)^2}\right], & 0 < x \le e^{\mu}, \\ 1 - \frac{1}{2}\beta\left[\frac{r}{2}, \frac{1}{2}; \frac{r}{r + \left(\frac{\ln x - \mu}{\sigma}\right)^2}\right], & x \ge e^{\mu}. \end{cases}$$

A.4.1.4 Single-parameter Pareto—α, θ

$$\begin{aligned} f(x) &= \frac{\alpha\theta^{\alpha}}{x^{\alpha+1}}, \quad x > \theta & F(x) &= 1 - (\theta/x)^{\alpha}, \quad x > \theta \\ \mathrm{E}[X^k] &= \frac{\alpha\theta^k}{\alpha - k}, \quad k < \alpha & \mathrm{E}[(X \wedge x)^k] &= \frac{\alpha\theta^k}{\alpha - k} - \frac{k\theta^{\alpha}}{(\alpha - k)x^{\alpha-k}} \\ \text{mode} &= \theta \end{aligned}$$

Note: Although there appears to be two parameters, only α is a true parameter. The value of θ must be set in advance.

A.5 Distributions with finite support

For these two distributions, the scale parameter θ is assumed known.

A.5.1.1 Generalized beta—a, b, θ, τ

$$\begin{aligned} f(x) &= \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} u^a (1-u)^{b-1} \frac{\tau}{x}, \quad 0 < x < \theta, \quad u = (x/\theta)^{\tau} \\ F(x) &= \beta(a, b; u) \\ \mathrm{E}[X^k] &= \frac{\theta^k \Gamma(a+b)\Gamma(a+k/\tau)}{\Gamma(a)\Gamma(a+b+k/\tau)}, \quad k > -a\tau \\ \mathrm{E}[(X \wedge x)^k] &= \frac{\theta^k \Gamma(a+b)\Gamma(a+k/\tau)}{\Gamma(a)\Gamma(a+b+k/\tau)} \beta(a+k/\tau, b; u) + x^k[1 - \beta(a, b; u)] \end{aligned}$$

A.5.1.2 beta—a, b, θ

$$\begin{aligned} f(x) &= \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} u^a (1-u)^{b-1} \frac{1}{x}, \quad 0 < x < \theta, \quad u = x/\theta \\ F(x) &= \beta(a, b; u) \\ \mathrm{E}[X^k] &= \frac{\theta^k \Gamma(a+b)\Gamma(a+k)}{\Gamma(a)\Gamma(a+b+k)}, \quad k > -a \\ \mathrm{E}[X^k] &= \frac{\theta^k a(a+1)\cdots(a+k-1)}{(a+b)(a+b+1)\cdots(a+b+k-1)}, \quad \text{if } k \text{ is an integer} \\ \mathrm{E}[(X \wedge x)^k] &= \frac{\theta^k a(a+1)\cdots(a+k-1)}{(a+b)(a+b+1)\cdots(a+b+k-1)} \beta(a+k, b; u) \\ &\quad + x^k[1 - \beta(a, b; u)] \end{aligned}$$

Appendix B

An Inventory of Discrete Distributions

B.2 The $(a, b, 0)$ class

B.2.1.1 Poisson—λ

$$\begin{aligned} p_0 &= e^{-\lambda}, \quad a = 0, \quad b = \lambda \qquad\qquad p_k = \frac{e^{-\lambda}\lambda^k}{k!} \\ \mathrm{E}[N] &= \lambda, \quad \mathrm{Var}[N] = \lambda \qquad\qquad P(z) = e^{\lambda(z-1)} \end{aligned}$$

B.2.1.2 Geometric—β

$$\begin{aligned} p_0 &= \frac{1}{1+\beta}, \quad a = \frac{\beta}{1+\beta}, \quad b = 0 \qquad\qquad p_k = \frac{\beta^k}{(1+\beta)^{k+1}} \\ \mathrm{E}[N] &= \beta, \quad \mathrm{Var}[N] = \beta(1+\beta) \qquad\qquad P(z) = [1-\beta(z-1)]^{-1}. \end{aligned}$$

This is a special case of the negative binomial with $r = 1$.

B.2.1.3 Binomial—$q, m, (0 < q < 1,\ m$ an integer$)$

$$\begin{aligned} p_0 &= (1-q)^m, \quad a = -\frac{q}{1-q}, \quad b = \frac{(m+1)q}{1-q} \\ p_k &= \binom{m}{k} q^k (1-q)^{m-k}, \quad k = 0, 1, \ldots, m \\ \mathrm{E}[N] &= mq, \quad \mathrm{Var}[N] = mq(1-q) \qquad\qquad P(z) = [1+q(z-1)]^m. \end{aligned}$$

B.2.1.4 Negative binomial—β, r

$$\begin{aligned} p_0 &= (1+\beta)^{-r}, \quad a = \frac{\beta}{1+\beta}, \quad b = \frac{(r-1)\beta}{1+\beta} \\ p_k &= \frac{r(r+1)\cdots(r+k-1)\beta^k}{k!(1+\beta)^{r+k}} \\ \mathrm{E}[N] &= r\beta, \quad \mathrm{Var}[N] = r\beta(1+\beta) \qquad\qquad P(z) = [1-\beta(z-1)]^{-r}. \end{aligned}$$

Tail areas (two-sided) for t-distributions

df	0.20	0.10	0.05	0.02	0.01
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
30	1.310	1.697	2.042	2.457	2.750
35	1.306	1.690	2.030	2.438	2.724
40	1.303	1.684	2.021	2.423	2.704
45	1.301	1.679	2.014	2.412	2.690
50	1.299	1.676	2.009	2.403	2.678
55	1.297	1.673	2.004	2.396	2.668
60	1.296	1.671	2.000	2.390	2.660
70	1.294	1.667	1.994	2.381	2.648
80	1.292	1.664	1.990	2.374	2.639
90	1.291	1.662	1.987	2.368	2.632
100	1.290	1.660	1.984	2.364	2.626
120	1.289	1.658	1.980	2.358	2.617
400	1.284	1.649	1.966	2.336	2.588
Inf	1.282	1.645	1.960	2.326	2.576

For use on the CAS exams

Selected Upper-tail areas for F-distributions

	Numerator df	1	2	3	4	5	6	7	8	9	10	11	12	14	16	20	24	30	40	50	75	100	200	500	Inf		
Denominator df	Upper-tail																										
1	0.20	9.472	12	13.064	13.644	14.008	14.258	14.439	14.577	14.685	14.772	14.844	14.904	14.998	15.07	15.171	15.238	15.306	15.374	15.415	15.47	15.497	15.539	15.563	15.58	0.20	1
1	0.10	39.86	49.5	53.59	55.83	57.24	58.2	58.91	59.44	59.86	60.19	60.47	60.71	61.07	61.35	61.74	62	62.26	62.53	62.69	62.9	63.01	63.17	63.26	63.33	0.10	1
1	0.05	161.4	199.5	215.7	224.6	230.2	234	236.8	238.9	240.5	241.9	243	243.9	245.4	246.5	248	249.1	250.1	251.1	251.8	252.6	253	253.7	254.1	254.3	0.05	1
1	0.02	1013	1249	1351	1406	1441	1464	1482	1495	1505	1514	1521	1526	1535	1542	1552	1558	1565	1571	1575	1581	1583	1587	1590	1591	0.02	1
1	0.01	4052	4999	5403	5625	5764	5859	5928	5981	6022	6056	6083	6106	6143	6170	6209	6235	6261	6287	6303	6324	6334	6350	6360	6366	0.01	1
2	0.20	3.556	4	4.156	4.236	4.284	4.317	4.34	4.358	4.371	4.382	4.391	4.399	4.41	4.419	4.432	4.44	4.448	4.456	4.461	4.468	4.471	4.476	4.479	4.481	0.20	2
2	0.10	8.526	9	9.162	9.243	9.293	9.326	9.349	9.367	9.381	9.392	9.401	9.408	9.42	9.429	9.441	9.45	9.458	9.466	9.471	9.478	9.481	9.486	9.489	9.491	0.10	2
2	0.05	18.51	19	19.16	19.25	19.3	19.33	19.35	19.37	19.38	19.4	19.4	19.41	19.42	19.43	19.45	19.45	19.46	19.47	19.48	19.48	19.49	19.49	19.49	19.5	0.05	2
2	0.02	48.51	49	49.17	49.25	49.3	49.33	49.36	49.37	49.39	49.4	49.41	49.42	49.43	49.44	49.45	49.46	49.46	49.47	49.48	49.48	49.49	49.49	49.5	49.5	0.02	2
2	0.01	98.5	99	99.17	99.25	99.3	99.33	99.36	99.37	99.39	99.4	99.41	99.42	99.43	99.44	99.45	99.46	99.47	99.47	99.48	99.49	99.49	99.49	99.5	99.5	0.01	2
3	0.20	2.682	2.886	2.936	2.956	2.965	2.971	2.974	2.976	2.978	2.979	2.98	2.981	2.982	2.982	2.983	2.983	2.984	2.984	2.984	2.984	2.984	2.985	2.985	2.985	0.20	3
3	0.10	5.538	5.462	5.391	5.343	5.309	5.285	5.266	5.252	5.24	5.23	5.222	5.216	5.205	5.196	5.184	5.176	5.168	5.16	5.155	5.148	5.144	5.139	5.136	5.134	0.10	3
3	0.05	10.128	9.552	9.277	9.117	9.013	8.941	8.887	8.845	8.812	8.786	8.763	8.745	8.715	8.692	8.66	8.639	8.617	8.594	8.581	8.563	8.554	8.54	8.532	8.526	0.05	3
3	0.02	20.62	18.86	18.11	17.69	17.43	17.25	17.11	17.01	16.93	16.86	16.81	16.76	16.69	16.63	16.55	16.5	16.45	16.39	16.36	16.32	16.3	16.26	16.24	16.23	0.02	3
3	0.01	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.35	27.23	27.13	27.05	26.92	26.83	26.69	26.6	26.5	26.41	26.35	26.28	26.24	26.18	26.15	26.13	0.01	3
4	0.20	2.351	2.472	2.485	2.483	2.478	2.473	2.469	2.465	2.462	2.46	2.457	2.455	2.452	2.449	2.445	2.442	2.439	2.436	2.434	2.432	2.43	2.428	2.427	2.426	0.20	4
4	0.10	4.545	4.325	4.191	4.107	4.051	4.01	3.979	3.955	3.936	3.92	3.907	3.896	3.878	3.864	3.844	3.831	3.817	3.804	3.795	3.784	3.778	3.769	3.764	3.761	0.10	4
4	0.05	7.709	6.944	6.591	6.388	6.256	6.163	6.094	6.041	5.999	5.964	5.936	5.912	5.873	5.844	5.803	5.774	5.746	5.717	5.699	5.676	5.664	5.646	5.635	5.628	0.05	4
4	0.02	14.04	12.142	11.344	10.899	10.616	10.419	10.274	10.162	10.074	10.003	9.944	9.894	9.815	9.755	9.67	9.612	9.554	9.495	9.46	9.412	9.388	9.352	9.33	9.315	0.02	4
4	0.01	21.2	18	16.69	15.98	15.52	15.21	14.98	14.8	14.66	14.55	14.45	14.37	14.25	14.15	14.02	13.93	13.84	13.75	13.69	13.61	13.58	13.52	13.49	13.46	0.01	4
5	0.20	2.178	2.259	2.253	2.24	2.228	2.217	2.209	2.202	2.196	2.191	2.187	2.184	2.178	2.173	2.166	2.161	2.156	2.151	2.148	2.144	2.141	2.138	2.136	2.134	0.20	5
5	0.10	4.06	3.78	3.619	3.52	3.453	3.405	3.368	3.339	3.316	3.297	3.282	3.268	3.247	3.23	3.207	3.191	3.174	3.157	3.147	3.133	3.126	3.116	3.109	3.105	0.10	5
5	0.05	6.608	5.786	5.409	5.192	5.05	4.95	4.876	4.818	4.772	4.735	4.704	4.678	4.636	4.604	4.558	4.527	4.496	4.464	4.444	4.418	4.405	4.385	4.373	4.365	0.05	5
5	0.02	11.323	9.454	8.67	8.233	7.953	7.758	7.614	7.503	7.415	7.344	7.285	7.235	7.156	7.095	7.009	6.951	6.893	6.833	6.797	6.749	6.724	6.687	6.665	6.65	0.02	5
5	0.01	16.258	13.274	12.06	11.392	10.967	10.672	10.456	10.289	10.158	10.051	9.963	9.888	9.77	9.68	9.553	9.466	9.379	9.291	9.238	9.166	9.13	9.075	9.042	9.02	0.01	5
6	0.20	2.073	2.13	2.113	2.092	2.076	2.062	2.051	2.042	2.034	2.028	2.022	2.018	2.01	2.004	1.995	1.989	1.982	1.976	1.972	1.966	1.963	1.959	1.956	1.954	0.20	6
6	0.10	3.776	3.463	3.289	3.181	3.108	3.055	3.014	2.983	2.958	2.937	2.92	2.905	2.881	2.863	2.836	2.818	2.8	2.781	2.77	2.754	2.746	2.734	2.727	2.722	0.10	6
6	0.05	5.987	5.143	4.757	4.534	4.387	4.284	4.207	4.147	4.099	4.06	4.027	4	3.956	3.922	3.874	3.841	3.808	3.774	3.754	3.726	3.712	3.69	3.678	3.669	0.05	6
6	0.02	9.876	8.052	7.287	6.859	6.585	6.393	6.251	6.141	6.055	5.984	5.925	5.876	5.797	5.737	5.651	5.593	5.534	5.474	5.438	5.389	5.364	5.327	5.304	5.289	0.02	6
6	0.01	13.745	10.925	9.78	9.148	8.746	8.466	8.26	8.102	7.976	7.874	7.79	7.718	7.605	7.519	7.396	7.313	7.229	7.143	7.091	7.022	6.987	6.934	6.902	6.88	0.01	6
7	0.20	2.002	2.043	2.019	1.994	1.974	1.957	1.945	1.934	1.925	1.918	1.911	1.906	1.897	1.89	1.879	1.872	1.865	1.857	1.852	1.845	1.842	1.837	1.833	1.831	0.20	7
7	0.10	3.589	3.257	3.074	2.961	2.883	2.827	2.785	2.752	2.725	2.703	2.684	2.668	2.643	2.623	2.595	2.575	2.555	2.535	2.523	2.506	2.497	2.484	2.476	2.471	0.10	7
7	0.05	5.591	4.737	4.347	4.12	3.972	3.866	3.787	3.726	3.677	3.637	3.603	3.575	3.529	3.494	3.445	3.41	3.376	3.34	3.319	3.29	3.275	3.252	3.239	3.23	0.05	7
7	0.02	8.988	7.203	6.454	6.035	5.765	5.576	5.435	5.327	5.241	5.171	5.113	5.064	4.985	4.925	4.839	4.781	4.722	4.662	4.625	4.576	4.551	4.513	4.49	4.475	0.02	7
7	0.01	12.246	9.547	8.451	7.847	7.46	7.191	6.993	6.84	6.719	6.62	6.538	6.469	6.359	6.275	6.155	6.074	5.992	5.908	5.858	5.789	5.755	5.702	5.671	5.65	0.01	7
8	0.20	1.951	1.981	1.951	1.923	1.9	1.883	1.868	1.856	1.847	1.838	1.831	1.825	1.815	1.807	1.796	1.787	1.779	1.77	1.765	1.757	1.753	1.748	1.744	1.742	0.20	8
8	0.10	3.458	3.113	2.924	2.806	2.726	2.668	2.624	2.589	2.561	2.538	2.519	2.502	2.475	2.455	2.425	2.404	2.383	2.361	2.348	2.33	2.321	2.307	2.298	2.293	0.10	8
8	0.05	5.318	4.459	4.066	3.838	3.687	3.581	3.5	3.438	3.388	3.347	3.313	3.284	3.237	3.202	3.15	3.115	3.079	3.043	3.02	2.99	2.975	2.951	2.937	2.928	0.05	8
8	0.02	8.389	6.637	5.901	5.489	5.223	5.036	4.897	4.79	4.705	4.635	4.577	4.528	4.449	4.389	4.304	4.245	4.186	4.125	4.088	4.038	4.013	3.975	3.952	3.936	0.02	8
8	0.01	11.259	8.649	7.591	7.006	6.632	6.371	6.178	6.029	5.911	5.814	5.734	5.667	5.559	5.477	5.359	5.279	5.198	5.116	5.065	4.998	4.963	4.911	4.88	4.859	0.01	8
9	0.20	1.913	1.935	1.901	1.87	1.846	1.826	1.811	1.798	1.787	1.778	1.771	1.764	1.753	1.745	1.732	1.723	1.714	1.704	1.698	1.69	1.686	1.68	1.676	1.673	0.20	9
9	0.10	3.36	3.006	2.813	2.693	2.611	2.551	2.505	2.469	2.44	2.416	2.396	2.379	2.351	2.329	2.298	2.277	2.255	2.232	2.218	2.199	2.189	2.174	2.165	2.159	0.10	9
9	0.05	5.117	4.256	3.863	3.633	3.482	3.374	3.293	3.23	3.179	3.137	3.102	3.073	3.025	2.989	2.936	2.9	2.864	2.826	2.803	2.771	2.756	2.731	2.717	2.707	0.05	9
9	0.02	7.961	6.234	5.51	5.103	4.84	4.654	4.517	4.41	4.325	4.256	4.198	4.149	4.071	4.011	3.925	3.866	3.806	3.745	3.708	3.658	3.632	3.593	3.57	3.554	0.02	9
9	0.01	10.561	8.022	6.992	6.422	6.057	5.802	5.613	5.467	5.351	5.257	5.178	5.111	5.005	4.924	4.808	4.729	4.649	4.567	4.517	4.449	4.415	4.363	4.332	4.311	0.01	9

	Numerator df	1	2	3	4	5	6	7	8	9	10	11	12	14	16	20	24	30	40	50	75	100	200	500	Inf		
Denominator df	Upper-tail																										
10	0.20	1.883	1.899	1.861	1.829	1.803	1.782	1.766	1.752	1.741	1.732	1.723	1.716	1.705	1.696	1.682	1.673	1.663	1.653	1.646	1.637	1.633	1.626	1.621	1.618	0.20	10
10	0.10	3.285	2.924	2.728	2.605	2.522	2.461	2.414	2.377	2.347	2.323	2.302	2.284	2.255	2.233	2.201	2.178	2.155	2.132	2.117	2.097	2.087	2.071	2.062	2.055	0.10	10
10	0.05	4.965	4.103	3.708	3.478	3.326	3.217	3.135	3.072	3.02	2.978	2.943	2.913	2.865	2.828	2.774	2.737	2.7	2.661	2.637	2.605	2.588	2.563	2.548	2.538	0.05	10
10	0.02	7.638	5.934	5.218	4.816	4.555	4.371	4.235	4.129	4.044	3.975	3.917	3.868	3.79	3.73	3.644	3.585	3.525	3.463	3.425	3.374	3.348	3.309	3.285	3.269	0.02	10
10	0.01	10.044	7.559	6.552	5.994	5.636	5.386	5.2	5.057	4.942	4.849	4.772	4.706	4.601	4.52	4.405	4.327	4.247	4.165	4.115	4.048	4.014	3.962	3.93	3.909	0.01	10
11	0.20	1.859	1.87	1.83	1.796	1.768	1.747	1.73	1.716	1.704	1.694	1.685	1.678	1.666	1.656	1.642	1.632	1.622	1.611	1.604	1.594	1.589	1.582	1.577	1.574	0.20	11
11	0.10	3.225	2.86	2.66	2.536	2.451	2.389	2.342	2.304	2.274	2.248	2.227	2.209	2.179	2.156	2.123	2.1	2.076	2.052	2.036	2.016	2.005	1.989	1.979	1.972	0.10	11
11	0.05	4.844	3.982	3.587	3.357	3.204	3.095	3.012	2.948	2.896	2.854	2.818	2.788	2.739	2.701	2.646	2.609	2.57	2.531	2.507	2.473	2.457	2.431	2.415	2.404	0.05	11
11	0.02	7.388	5.701	4.993	4.594	4.336	4.153	4.017	3.912	3.828	3.758	3.701	3.652	3.573	3.513	3.427	3.367	3.307	3.245	3.207	3.155	3.129	3.089	3.065	3.048	0.02	11
11	0.01	9.646	7.206	6.217	5.668	5.316	5.069	4.886	4.744	4.632	4.539	4.462	4.397	4.293	4.213	4.099	4.021	3.941	3.86	3.81	3.742	3.708	3.656	3.624	3.602	0.01	11
12	0.20	1.839	1.846	1.804	1.768	1.74	1.718	1.7	1.686	1.673	1.663	1.654	1.646	1.634	1.624	1.609	1.598	1.587	1.576	1.568	1.558	1.553	1.545	1.54	1.537	0.20	12
12	0.10	3.177	2.807	2.606	2.48	2.394	2.331	2.283	2.245	2.214	2.188	2.166	2.147	2.117	2.094	2.06	2.036	2.011	1.986	1.97	1.949	1.938	1.921	1.911	1.904	0.10	12
12	0.05	4.747	3.885	3.49	3.259	3.106	2.996	2.913	2.849	2.796	2.753	2.717	2.687	2.637	2.599	2.544	2.505	2.466	2.426	2.401	2.367	2.35	2.323	2.307	2.296	0.05	12
12	0.02	7.188	5.516	4.814	4.419	4.162	3.98	3.845	3.74	3.656	3.587	3.529	3.48	3.402	3.341	3.254	3.195	3.134	3.071	3.033	2.981	2.954	2.913	2.889	2.872	0.02	12
12	0.01	9.33	6.927	5.953	5.412	5.064	4.821	4.64	4.499	4.388	4.296	4.22	4.155	4.052	3.972	3.858	3.78	3.701	3.619	3.569	3.501	3.467	3.414	3.382	3.361	0.01	12
13	0.20	1.823	1.826	1.783	1.746	1.717	1.694	1.676	1.661	1.648	1.637	1.628	1.62	1.607	1.596	1.581	1.57	1.558	1.546	1.539	1.528	1.523	1.514	1.509	1.506	0.20	13
13	0.10	3.136	2.763	2.56	2.434	2.347	2.283	2.234	2.195	2.164	2.138	2.116	2.097	2.066	2.042	2.007	1.983	1.958	1.931	1.915	1.893	1.882	1.864	1.853	1.846	0.10	13
13	0.05	4.667	3.806	3.411	3.179	3.025	2.915	2.832	2.767	2.714	2.671	2.635	2.604	2.554	2.515	2.459	2.42	2.38	2.339	2.314	2.279	2.261	2.234	2.218	2.206	0.05	13
13	0.02	7.024	5.366	4.669	4.276	4.02	3.84	3.705	3.6	3.516	3.447	3.39	3.341	3.262	3.201	3.114	3.054	2.993	2.93	2.891	2.838	2.811	2.77	2.745	2.728	0.02	13
13	0.01	9.074	6.701	5.739	5.205	4.862	4.62	4.441	4.302	4.191	4.1	4.025	3.96	3.857	3.778	3.665	3.587	3.507	3.425	3.375	3.307	3.272	3.219	3.187	3.165	0.01	13
14	0.20	1.809	1.809	1.765	1.727	1.697	1.674	1.655	1.639	1.626	1.615	1.606	1.598	1.584	1.573	1.557	1.546	1.534	1.521	1.513	1.502	1.497	1.488	1.482	1.479	0.20	14
14	0.10	3.102	2.726	2.522	2.395	2.307	2.243	2.193	2.154	2.122	2.095	2.073	2.054	2.022	1.998	1.962	1.938	1.912	1.885	1.869	1.846	1.834	1.816	1.805	1.797	0.10	14
14	0.05	4.6	3.739	3.344	3.112	2.958	2.848	2.764	2.699	2.646	2.602	2.565	2.534	2.484	2.445	2.388	2.349	2.308	2.266	2.241	2.205	2.187	2.159	2.142	2.131	0.05	14
14	0.02	6.888	5.241	4.549	4.158	3.904	3.724	3.589	3.485	3.401	3.332	3.274	3.225	3.146	3.086	2.998	2.938	2.876	2.812	2.773	2.72	2.692	2.651	2.625	2.608	0.02	14
14	0.01	8.862	6.515	5.564	5.035	4.695	4.456	4.278	4.14	4.03	3.939	3.864	3.8	3.698	3.619	3.505	3.427	3.348	3.266	3.215	3.147	3.112	3.059	3.026	3.004	0.01	14
15	0.20	1.797	1.795	1.749	1.71	1.68	1.656	1.637	1.621	1.608	1.596	1.587	1.578	1.564	1.553	1.537	1.525	1.513	1.5	1.491	1.48	1.474	1.465	1.459	1.455	0.20	15
15	0.10	3.073	2.695	2.49	2.361	2.273	2.208	2.158	2.119	2.086	2.059	2.037	2.017	1.985	1.961	1.924	1.899	1.873	1.845	1.828	1.805	1.793	1.774	1.763	1.755	0.10	15
15	0.05	4.543	3.682	3.287	3.056	2.901	2.79	2.707	2.641	2.588	2.544	2.507	2.475	2.424	2.385	2.328	2.288	2.247	2.204	2.178	2.142	2.123	2.095	2.078	2.066	0.05	15
15	0.02	6.773	5.135	4.447	4.058	3.805	3.626	3.492	3.387	3.303	3.235	3.177	3.128	3.049	2.988	2.9	2.84	2.777	2.713	2.674	2.62	2.592	2.55	2.524	2.506	0.02	15
15	0.01	8.683	6.359	5.417	4.893	4.556	4.318	4.142	4.004	3.895	3.805	3.73	3.666	3.564	3.485	3.372	3.294	3.214	3.132	3.081	3.012	2.977	2.923	2.891	2.868	0.01	15
16	0.20	1.787	1.783	1.736	1.696	1.665	1.641	1.621	1.605	1.591	1.58	1.57	1.561	1.547	1.536	1.519	1.507	1.494	1.481	1.472	1.46	1.454	1.445	1.439	1.435	0.20	16
16	0.10	3.048	2.668	2.462	2.333	2.244	2.178	2.128	2.088	2.055	2.028	2.005	1.985	1.953	1.928	1.891	1.866	1.839	1.811	1.793	1.769	1.757	1.738	1.726	1.718	0.10	16
16	0.05	4.494	3.634	3.239	3.007	2.852	2.741	2.657	2.591	2.538	2.494	2.456	2.425	2.373	2.333	2.276	2.235	2.194	2.151	2.124	2.087	2.068	2.039	2.022	2.01	0.05	16
16	0.02	6.674	5.046	4.361	3.974	3.721	3.543	3.409	3.304	3.221	3.152	3.094	3.045	2.966	2.905	2.817	2.756	2.693	2.628	2.589	2.534	2.506	2.463	2.437	2.419	0.02	16
16	0.01	8.531	6.226	5.292	4.773	4.437	4.202	4.026	3.89	3.78	3.691	3.616	3.553	3.451	3.372	3.259	3.181	3.101	3.018	2.967	2.898	2.863	2.808	2.775	2.753	0.01	16
17	0.20	1.778	1.772	1.724	1.684	1.652	1.628	1.608	1.591	1.577	1.566	1.555	1.547	1.532	1.52	1.503	1.491	1.478	1.464	1.455	1.443	1.437	1.427	1.421	1.416	0.20	17
17	0.10	3.026	2.645	2.437	2.308	2.218	2.152	2.102	2.061	2.028	2.001	1.978	1.958	1.925	1.9	1.862	1.836	1.809	1.781	1.763	1.738	1.726	1.706	1.694	1.686	0.10	17
17	0.05	4.451	3.592	3.197	2.965	2.81	2.699	2.614	2.548	2.494	2.45	2.413	2.381	2.329	2.289	2.23	2.19	2.148	2.104	2.077	2.04	2.02	1.991	1.973	1.96	0.05	17
17	0.02	6.589	4.968	4.286	3.901	3.649	3.471	3.337	3.233	3.149	3.08	3.023	2.973	2.894	2.833	2.745	2.683	2.62	2.555	2.515	2.46	2.431	2.388	2.361	2.343	0.02	17
17	0.01	8.4	6.112	5.185	4.669	4.336	4.102	3.927	3.791	3.682	3.593	3.519	3.455	3.353	3.275	3.162	3.084	3.003	2.92	2.869	2.8	2.764	2.709	2.676	2.653	0.01	17
18	0.20	1.77	1.762	1.713	1.673	1.641	1.616	1.596	1.579	1.565	1.553	1.543	1.534	1.519	1.507	1.489	1.477	1.463	1.449	1.44	1.428	1.421	1.411	1.404	1.4	0.20	18
18	0.10	3.007	2.624	2.416	2.286	2.196	2.13	2.079	2.038	2.005	1.977	1.954	1.933	1.9	1.875	1.837	1.81	1.783	1.754	1.736	1.711	1.698	1.678	1.665	1.657	0.10	18
18	0.05	4.414	3.555	3.16	2.928	2.773	2.661	2.577	2.51	2.456	2.412	2.374	2.342	2.29	2.25	2.191	2.15	2.107	2.063	2.035	1.998	1.978	1.948	1.929	1.917	0.05	18
18	0.02	6.515	4.9	4.221	3.837	3.586	3.408	3.275	3.171	3.087	3.018	2.96	2.911	2.832	2.77	2.682	2.62	2.557	2.491	2.45	2.395	2.366	2.322	2.295	2.277	0.02	18
18	0.01	8.285	6.013	5.092	4.579	4.248	4.015	3.841	3.705	3.597	3.508	3.434	3.371	3.269	3.19	3.077	2.999	2.919	2.835	2.784	2.714	2.678	2.623	2.589	2.566	0.01	18
19	0.20	1.763	1.754	1.704	1.663	1.631	1.605	1.585	1.568	1.554	1.542	1.531	1.522	1.507	1.495	1.477	1.464	1.45	1.436	1.427	1.414	1.407	1.396	1.39	1.385	0.20	19
19	0.10	2.99	2.606	2.397	2.266	2.176	2.109	2.058	2.017	1.984	1.956	1.932	1.912	1.878	1.852	1.814	1.787	1.759	1.73	1.711	1.686	1.673	1.652	1.639	1.631	0.10	19
19	0.05	4.381	3.522	3.127	2.895	2.74	2.628	2.544	2.477	2.423	2.378	2.34	2.308	2.256	2.215	2.155	2.114	2.071	2.026	1.999	1.96	1.94	1.91	1.891	1.878	0.05	19
19	0.02	6.449	4.84	4.164	3.781	3.531	3.353	3.22	3.116	3.032	2.963	2.906	2.856	2.777	2.715	2.626	2.564	2.501	2.434	2.394	2.337	2.308	2.264	2.236	2.218	0.02	19
19	0.01	8.185	5.926	5.01	4.5	4.171	3.939	3.765	3.631	3.523	3.434	3.36	3.297	3.195	3.116	3.003	2.925	2.844	2.761	2.709	2.639	2.602	2.547	2.512	2.489	0.01	19

Denominator df	Upper-tail	1	2	3	4	5	6	7	8	9	10	11	12	14	16	20	24	30	40	50	75	100	200	500	Inf		
20	0.20	1.757	1.746	1.696	1.654	1.622	1.596	1.575	1.558	1.544	1.531	1.521	1.512	1.496	1.484	1.466	1.452	1.439	1.424	1.414	1.401	1.394	1.383	1.377	1.372	0.20	20
20	0.10	2.975	2.589	2.38	2.249	2.158	2.091	2.04	1.999	1.965	1.937	1.913	1.892	1.859	1.833	1.794	1.767	1.738	1.708	1.69	1.664	1.65	1.629	1.616	1.607	0.10	20
20	0.05	4.351	3.493	3.098	2.866	2.711	2.599	2.514	2.447	2.393	2.348	2.31	2.278	2.225	2.184	2.124	2.082	2.039	1.994	1.966	1.927	1.907	1.875	1.856	1.843	0.05	20
20	0.02	6.391	4.788	4.113	3.731	3.482	3.304	3.171	3.067	2.984	2.915	2.857	2.808	2.728	2.666	2.577	2.515	2.451	2.384	2.343	2.286	2.257	2.212	2.184	2.165	0.02	20
20	0.01	8.096	5.849	4.938	4.431	4.103	3.871	3.699	3.564	3.457	3.368	3.294	3.231	3.13	3.051	2.938	2.859	2.778	2.695	2.643	2.572	2.535	2.479	2.445	2.421	0.01	20
21	0.20	1.751	1.739	1.688	1.646	1.614	1.588	1.567	1.549	1.535	1.522	1.511	1.502	1.487	1.474	1.455	1.442	1.428	1.413	1.403	1.39	1.383	1.371	1.364	1.36	0.20	21
21	0.10	2.961	2.575	2.365	2.233	2.142	2.075	2.023	1.982	1.948	1.92	1.896	1.875	1.841	1.815	1.776	1.748	1.719	1.689	1.67	1.644	1.63	1.608	1.595	1.586	0.10	21
21	0.05	4.325	3.467	3.072	2.84	2.685	2.573	2.488	2.42	2.366	2.321	2.283	2.25	2.197	2.156	2.096	2.054	2.01	1.965	1.936	1.897	1.876	1.845	1.825	1.812	0.05	21
21	0.02	6.339	4.74	4.068	3.687	3.438	3.261	3.128	3.024	2.94	2.872	2.814	2.764	2.685	2.623	2.533	2.471	2.406	2.339	2.298	2.24	2.211	2.165	2.137	2.118	0.02	21
21	0.01	8.017	5.78	4.874	4.369	4.042	3.812	3.64	3.506	3.398	3.31	3.236	3.173	3.072	2.993	2.88	2.801	2.72	2.636	2.584	2.512	2.475	2.419	2.384	2.36	0.01	21
22	0.20	1.746	1.733	1.682	1.639	1.606	1.58	1.559	1.541	1.526	1.514	1.503	1.494	1.478	1.465	1.446	1.433	1.418	1.403	1.393	1.379	1.372	1.361	1.353	1.349	0.20	22
22	0.10	2.949	2.561	2.351	2.219	2.128	2.06	2.008	1.967	1.933	1.904	1.88	1.859	1.825	1.798	1.759	1.731	1.702	1.671	1.652	1.625	1.611	1.59	1.576	1.567	0.10	22
22	0.05	4.301	3.443	3.049	2.817	2.661	2.549	2.464	2.397	2.342	2.297	2.259	2.226	2.173	2.131	2.071	2.028	1.984	1.938	1.909	1.869	1.849	1.817	1.797	1.783	0.05	22
22	0.02	6.292	4.698	4.028	3.647	3.399	3.222	3.089	2.985	2.902	2.833	2.775	2.725	2.646	2.584	2.494	2.431	2.366	2.299	2.257	2.199	2.169	2.123	2.095	2.075	0.02	22
22	0.01	7.945	5.719	4.817	4.313	3.988	3.758	3.587	3.453	3.346	3.258	3.184	3.121	3.019	2.941	2.827	2.749	2.667	2.583	2.531	2.459	2.422	2.365	2.329	2.305	0.01	22
23	0.20	1.741	1.728	1.676	1.633	1.599	1.573	1.552	1.534	1.519	1.506	1.495	1.486	1.47	1.457	1.438	1.424	1.41	1.394	1.384	1.37	1.362	1.351	1.343	1.338	0.20	23
23	0.10	2.937	2.549	2.339	2.207	2.115	2.047	1.995	1.953	1.919	1.89	1.866	1.845	1.811	1.784	1.744	1.716	1.686	1.655	1.636	1.609	1.594	1.572	1.558	1.549	0.10	23
23	0.05	4.279	3.422	3.028	2.796	2.64	2.528	2.442	2.375	2.32	2.275	2.236	2.204	2.15	2.109	2.048	2.005	1.961	1.914	1.885	1.844	1.823	1.791	1.771	1.757	0.05	23
23	0.02	6.249	4.66	3.991	3.611	3.363	3.187	3.054	2.95	2.867	2.798	2.74	2.69	2.61	2.548	2.458	2.395	2.33	2.262	2.22	2.162	2.132	2.085	2.056	2.037	0.02	23
23	0.01	7.881	5.664	4.765	4.264	3.939	3.71	3.539	3.406	3.299	3.211	3.137	3.074	2.973	2.894	2.781	2.702	2.62	2.535	2.483	2.411	2.373	2.316	2.28	2.256	0.01	23
24	0.20	1.737	1.722	1.67	1.627	1.593	1.567	1.545	1.527	1.512	1.499	1.488	1.479	1.463	1.45	1.43	1.416	1.401	1.385	1.375	1.361	1.353	1.341	1.334	1.329	0.20	24
24	0.10	2.927	2.538	2.327	2.195	2.103	2.035	1.983	1.941	1.906	1.877	1.853	1.832	1.797	1.77	1.73	1.702	1.672	1.641	1.621	1.593	1.579	1.556	1.542	1.533	0.10	24
24	0.05	4.26	3.403	3.009	2.776	2.621	2.508	2.423	2.355	2.3	2.255	2.216	2.183	2.13	2.088	2.027	1.984	1.939	1.892	1.863	1.822	1.8	1.768	1.747	1.733	0.05	24
24	0.02	6.211	4.625	3.958	3.579	3.331	3.155	3.022	2.919	2.835	2.766	2.708	2.658	2.578	2.516	2.426	2.363	2.297	2.229	2.187	2.128	2.097	2.05	2.021	2.001	0.02	24
24	0.01	7.823	5.614	4.718	4.218	3.895	3.667	3.496	3.363	3.256	3.168	3.094	3.032	2.93	2.852	2.738	2.659	2.577	2.492	2.44	2.367	2.329	2.271	2.235	2.211	0.01	24
25	0.20	1.733	1.718	1.665	1.622	1.588	1.561	1.539	1.521	1.506	1.493	1.482	1.472	1.456	1.443	1.423	1.409	1.394	1.378	1.367	1.353	1.345	1.333	1.325	1.32	0.20	25
25	0.10	2.918	2.528	2.317	2.184	2.092	2.024	1.971	1.929	1.895	1.866	1.841	1.82	1.785	1.758	1.718	1.689	1.659	1.627	1.607	1.579	1.565	1.542	1.527	1.518	0.10	25
25	0.05	4.242	3.385	2.991	2.759	2.603	2.49	2.405	2.337	2.282	2.236	2.198	2.165	2.111	2.069	2.007	1.964	1.919	1.872	1.842	1.801	1.779	1.746	1.725	1.711	0.05	25
25	0.02	6.176	4.593	3.928	3.549	3.302	3.126	2.993	2.89	2.806	2.737	2.679	2.629	2.549	2.487	2.396	2.333	2.267	2.199	2.156	2.097	2.066	2.018	1.989	1.969	0.02	25
25	0.01	7.77	5.568	4.675	4.177	3.855	3.627	3.457	3.324	3.217	3.129	3.056	2.993	2.892	2.813	2.699	2.62	2.538	2.453	2.4	2.327	2.289	2.23	2.194	2.169	0.01	25
26	0.20	1.729	1.713	1.66	1.617	1.583	1.556	1.534	1.516	1.5	1.487	1.476	1.466	1.45	1.437	1.417	1.402	1.387	1.371	1.36	1.345	1.337	1.325	1.317	1.312	0.20	26
26	0.10	2.909	2.519	2.307	2.174	2.082	2.014	1.961	1.919	1.884	1.855	1.83	1.809	1.774	1.747	1.706	1.677	1.647	1.615	1.594	1.566	1.551	1.528	1.514	1.504	0.10	26
26	0.05	4.225	3.369	2.975	2.743	2.587	2.474	2.388	2.321	2.265	2.22	2.181	2.148	2.094	2.052	1.99	1.946	1.901	1.853	1.823	1.782	1.76	1.726	1.705	1.691	0.05	26
26	0.02	6.144	4.564	3.9	3.522	3.275	3.099	2.967	2.863	2.78	2.711	2.652	2.603	2.523	2.46	2.369	2.306	2.24	2.171	2.128	2.068	2.037	1.989	1.959	1.939	0.02	26
26	0.01	7.721	5.526	4.637	4.14	3.818	3.591	3.421	3.288	3.182	3.094	3.021	2.958	2.857	2.778	2.664	2.585	2.503	2.417	2.364	2.29	2.252	2.193	2.156	2.131	0.01	26

Lower-tail areas for Chi-square distributions

df	0.005	0.010	0.025	0.050	0.950	0.975	0.990	0.995
1	0.00	0.00	0.00	0.00	3.84	5.02	6.63	7.88
2	0.01	0.02	0.05	0.10	5.99	7.38	9.21	10.60
3	0.07	0.11	0.22	0.35	7.81	9.35	11.34	12.84
4	0.21	0.30	0.48	0.71	9.49	11.14	13.28	14.86
5	0.41	0.55	0.83	1.15	11.07	12.83	15.09	16.75
6	0.68	0.87	1.24	1.64	12.59	14.45	16.81	18.55
7	0.99	1.24	1.69	2.17	14.07	16.01	18.48	20.28
8	1.34	1.65	2.18	2.73	15.51	17.53	20.09	21.95
9	1.73	2.09	2.70	3.33	16.92	19.02	21.67	23.59
10	2.16	2.56	3.25	3.94	18.31	20.48	23.21	25.19
11	2.60	3.05	3.82	4.57	19.68	21.92	24.72	26.76
12	3.07	3.57	4.40	5.23	21.03	23.34	26.22	28.30
13	3.57	4.11	5.01	5.89	22.36	24.74	27.69	29.82
14	4.07	4.66	5.63	6.57	23.68	26.12	29.14	31.32
15	4.60	5.23	6.26	7.26	25.00	27.49	30.58	32.80
16	5.14	5.81	6.91	7.96	26.30	28.85	32.00	34.27
17	5.70	6.41	7.56	8.67	27.59	30.19	33.41	35.72
18	6.26	7.01	8.23	9.39	28.87	31.53	34.81	37.16
19	6.84	7.63	8.91	10.12	30.14	32.85	36.19	38.58
20	7.43	8.26	9.59	10.85	31.41	34.17	37.57	40.00
21	8.03	8.90	10.28	11.59	32.67	35.48	38.93	41.40
22	8.64	9.54	10.98	12.34	33.92	36.78	40.29	42.80
23	9.26	10.20	11.69	13.09	35.17	38.08	41.64	44.18
24	9.89	10.86	12.40	13.85	36.42	39.36	42.98	45.56
25	10.52	11.52	13.12	14.61	37.65	40.65	44.31	46.93
26	11.16	12.20	13.84	15.38	38.89	41.92	45.64	48.29
27	11.81	12.88	14.57	16.15	40.11	43.19	46.96	49.64
28	12.46	13.56	15.31	16.93	41.34	44.46	48.28	50.99
29	13.12	14.26	16.05	17.71	42.56	45.72	49.59	52.34
30	13.79	14.95	16.79	18.49	43.77	46.98	50.89	53.67
31	14.46	15.66	17.54	19.28	44.99	48.23	52.19	55.00
32	15.13	16.36	18.29	20.07	46.19	49.48	53.49	56.33
33	15.82	17.07	19.05	20.87	47.40	50.73	54.78	57.65
34	16.50	17.79	19.81	21.66	48.60	51.97	56.06	58.96
35	17.19	18.51	20.57	22.47	49.80	53.20	57.34	60.27
36	17.89	19.23	21.34	23.27	51.00	54.44	58.62	61.58
37	18.59	19.96	22.11	24.07	52.19	55.67	59.89	62.88
38	19.29	20.69	22.88	24.88	53.38	56.90	61.16	64.18
39	20.00	21.43	23.65	25.70	54.57	58.12	62.43	65.48
40	20.71	22.16	24.43	26.51	55.76	59.34	63.69	66.77
41	21.42	22.91	25.21	27.33	56.94	60.56	64.95	68.05
42	22.14	23.65	26.00	28.14	58.12	61.78	66.21	69.34
43	22.86	24.40	26.79	28.96	59.30	62.99	67.46	70.62
44	23.58	25.15	27.57	29.79	60.48	64.20	68.71	71.89
45	24.31	25.90	28.37	30.61	61.66	65.41	69.96	73.17
46	25.04	26.66	29.16	31.44	62.83	66.62	71.20	74.44
47	25.77	27.42	29.96	32.27	64.00	67.82	72.44	75.70
48	26.51	28.18	30.75	33.10	65.17	69.02	73.68	76.97
49	27.25	28.94	31.55	33.93	66.34	70.22	74.92	78.23
50	27.99	29.71	32.36	34.76	67.50	71.42	76.15	79.49

For use on the CAS exams