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SYLLABUS OF BASIC EDUCATION  
2021

Modern Actuarial Statistics-I – Exam MAS-I

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The syllabus for this four-hour exam is defined in the form of learning objectives, knowledge statements, and readings.

**LEARNING OBJECTIVES** set forth, usually in broad terms, what the candidate should be able to do in actual practice. Included in these learning objectives are certain methodologies that may not be possible to perform on an examination, such as calculating the Hat Matrix for a complex Generalized Linear Model, but that the candidate would still be expected to explain conceptually in the context of an examination.

**KNOWLEDGE STATEMENTS** identify some of the key terms, concepts, and methods that are associated with each learning objective. These knowledge statements are not intended to represent an exhaustive list of topics that may be tested, but they are illustrative of the scope of each learning objective.

**READINGS** support the learning objectives. It is intended that the readings, in conjunction with the material on earlier examinations, provide sufficient resources to allow the candidate to perform the learning objectives. Some readings are cited for more than one learning objective. The CAS Syllabus & Examination Committee emphasizes that candidates are expected to use the readings cited in this *Syllabus* as their primary study materials.

Thus, the learning objectives, knowledge statements, and readings complement each other. The learning objectives define the behaviors, the knowledge statements illustrate more fully the intended scope of the learning objectives, and the readings provide the source material to achieve the learning objectives. Learning objectives should not be seen as independent units, but as building blocks for the understanding and integration of important competencies that the candidate will be able to demonstrate.

Note that the range of weights shown should be viewed as a guideline only. There is no intent that they be strictly adhered to on any given examination—the actual weight may fall outside the published range on any particular examination.

The overall section weights should be viewed as having more significance than the weights for the individual learning objectives. Over a number of years of examinations, absent changes, it is likely that the average of the weights for each individual overall section will be in the vicinity of the guideline weight. For the weights of individual learning objectives, such convergence is less likely. On a given examination, in which it is very possible that not every individual learning objective will be tested, there will be more divergence of guideline weights and actual weights. Questions on a given learning objective may be drawn from any of the listed readings, or a combination of the readings. There may be no questions from one or more readings on a particular exam.

After each set of learning objectives, the readings are listed in abbreviated form. Complete text references are provided at the end of this exam syllabus.

Items marked with a bold **OP** (Online Publication) are available at no charge and may be downloaded from the CAS website.

Please check the “*Syllabus Updates*” section of the CAS Web Site for any changes to the *Syllabus*.

A thorough knowledge of calculus and probability is assumed, as is familiarity with discounting cash flows.

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**Materials for Study, 2021 Exam MAS-I**

**Exam MAS-I-1**



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Given the material covered on Section C, we assume that the candidate has knowledge of Linear Algebra concepts at the level commonly assumed as a prerequisite to taking an undergraduate level course in regression analysis. The insurance terminology used in questions for Exam MAS-I will not assume prior knowledge of either reserving or ratemaking actuarial practice, but we may include problems with insurance terms consistent with those stated in the Knowledge Statements for section B.3. The Probability Models section (Section A) covers Stochastic Processes, Markov Chains and Survival Models along with a simplified version of Life Contingencies. Survival models are covered in depth as part of probability modeling in generic terms. Markov Chains provide the means to model how an entity can move through different states. Life Contingencies problems can be viewed as discounted cash flow problems that include the effect of probability of payment and are covered through a Study Note linking the generic survival model concepts to a subset of life actuarial concepts to illustrate how to calculate annuities or single premium insurance amounts.

In general, the material covered under the Statistics section (Section B) covers topics that would be commonly found in a second semester course of a two semester Probability & Statistics sequence at the undergraduate level. Coverage of the topics listed under the Statistics section will vary by college and the candidate may need to supplement that course work with additional reading and problem-solving work from the suggested textbooks listed at the end of Section B.

Extended Linear Models including Generalized Linear Models, a predictive modeling technique commonly used to construct classification plans, are covered in Section C. The ordinary least squares model is covered as one member of the exponential family under the Extended Linear Models section. Many textbooks covering this topic, including the textbook on the syllabus, use statistical software to illustrate the concepts covered in examples, since using a calculator to solve a realistic problem is impractical. While we are not testing a candidate's ability to write R code, some of the examples in the textbooks cited in the Readings require using R code to work the examples. Those candidates that work through the examples or exercises will understand the material better than those who do not.

The Time Series section (Section D) covers an introduction to modeling activity over time like financial results or stock prices using the Auto Regressive Integrated Moving Average (ARIMA) where activity in a given time period may be linked to activity in subsequent time periods. That connection between adjacent time periods violates one of the assumptions behind the Extended Linear Model techniques, but the ARIMA approach incorporates that linkage as an aid in predicting future results. The Time Series section also covers the application of regression models to time series analysis.

A variety of tables will be provided to the candidate with the examination's reference materials. The tables include values for the illustrative life tables, standard normal distribution, abridged inventories of discrete and continuous probability distributions, Chi-square Distribution, t-Distribution, and F-Distribution. Since they will be included with the examination, candidates will not be allowed to bring copies of the tables into the examination room.

**A guessing adjustment will be used in grading this exam.** Details are provided under "Guessing Adjustment" in the "Examination Rules-The Examination" section of the *Syllabus of Basic Education*.



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## A. Probability Models (Stochastic Processes and Survival Models)

Range of weight for Section A: 20-35 percent

Candidates should be able to solve problems using stochastic processes. They should be able to determine the probabilities and distributions associated with these processes. Specifically, candidates should be able to use a Poisson process in these applications. Survival models are simply an extension of the stochastic process probability models where one is estimating the future lifetime of an entity given assumptions on the distribution function used to describe the likelihood of survival. Markov Chains are a useful tool to model movement between states in a given process and underlie modern Bayesian MCMC models. A short section on computer simulation is included, since in some cases a closed form solution to a problem involving random variables is not possible and simulation provides a means to arrive at a solution. The Study Note will re-cast the generic survival model learning objectives to link those concepts to life actuarial symbols to help ensure P&C actuaries can communicate with life actuaries on basic concepts, but we should recognize that many disciplines like engineering or computer science incorporate survival models in their work. Life Contingencies problems can be viewed as discounted cash flow problems that can be set up and solved using Markov Chain concepts or simply viewed as three matrices in a spreadsheet indicating payment amount, likelihood of payment, and discount effect by time period as illustrated by Learning Objective A.7.

LEARNING OBJECTIVES	KNOWLEDGE STATEMENTS
<p>1. Understand and apply the properties of Poisson processes:</p> <ul style="list-style-type: none"> <li>• For increments in the homogeneous case</li> <li>• For interval times in the homogeneous case</li> <li>• For increments in the non-homogeneous case</li> <li>• Resulting from special types of events in the Poisson process</li> <li>• Resulting from sums of independent Poisson processes</li> </ul> <p>Range of weight: 0-5 percent</p>	<ul style="list-style-type: none"> <li>a. Poisson process</li> <li>b. Non-homogeneous Poisson process</li> <li>c. Memoryless property of Exponential and Poisson</li> <li>d. Relationship between Exponential and Gamma</li> <li>e. Relationship between Exponential and Poisson</li> </ul>



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LEARNING OBJECTIVES	KNOWLEDGE STATEMENTS
<p>2. For any Poisson process and the inter-arrival and waiting distributions associated with the Poisson process, calculate:</p> <ul style="list-style-type: none"><li>• Expected values</li><li>• Variances</li><li>• Probabilities</li></ul> <p>Range of weight: 0-5 percent</p>	<ul style="list-style-type: none"><li>a. Probability calculations for Poisson process</li><li>b. Conditional distribution of arrival times</li><li>c. Splitting grouped Poisson rate to subsets of population using probability distribution</li><li>d. Conditional distribution of events by category within a group within a certain time period</li></ul>
<b>READINGS</b>	
<ul style="list-style-type: none"><li>• Daniel</li><li>• Ross, Sections 5.3, 5.4.1</li></ul>	

LEARNING OBJECTIVES	KNOWLEDGE STATEMENTS
<p>3. For a compound Poisson process, calculate moments associated with the value of the process at a given time.</p> <p>Range of weight: 0-5 percent</p>	<ul style="list-style-type: none"><li>a. Compound Poisson process mean and variance</li><li>b. Normal approximation and hypothesis testing</li></ul>
<b>READINGS</b>	
<ul style="list-style-type: none"><li>• Daniel</li><li>• Ross, Sections 5.4.2</li></ul>	

LEARNING OBJECTIVES	KNOWLEDGE STATEMENTS
<p>4. Apply the Poisson process concepts to calculate the hazard function and related survival model concepts.</p> <ul style="list-style-type: none"><li>• Relationship between hazard rate, probability density function and cumulative distribution function</li><li>• Effect of memoryless nature of Poisson distribution on survival time estimation</li></ul> <p>Range of weight: 2-8 percent</p>	<ul style="list-style-type: none"><li>a. Failure time random variables</li><li>b. Cumulative distribution functions</li><li>c. Survival functions</li><li>d. Probability density functions</li><li>e. Hazard functions and relationship to Exponential distribution</li><li>f. Relationships between failure time random variables in the functions above</li><li>g. Greedy algorithms</li></ul>
<b>READINGS</b>	
<ul style="list-style-type: none"><li>• Ross, Section 5.2.1-5.2.4</li></ul>	



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LEARNING OBJECTIVES	KNOWLEDGE STATEMENTS
<p>5. Given the joint distribution of more than one source of failure in a system (or life) and using Poisson Process assumptions:</p> <ul style="list-style-type: none"><li>• Calculate probabilities and moments associated with functions of these random variables' variances.</li><li>• Understand difference between a series system (joint life) and parallel system (last survivor) when calculating expected time to failure or probability of failure by a certain time.</li><li>• Understand the effect of multiple sources of failure (multiple decrement) on expected system time to failure (expected lifetime).</li></ul> <p>Range of weight: 2-8 percent</p>	<ul style="list-style-type: none"><li>a. Joint distribution of failure times</li><li>b. Probabilities and moments</li><li>c. Time until failure of the system (life)</li><li>d. Time until failure of the system (life) from a specific cause</li><li>e. Time until failure of the system (life) for parallel or series systems with multiple components</li><li>f. Paths that lead to parallel or series system failure for systems with multiple components</li><li>g. Relationship between failure time and minimal path and minimal cut sets</li><li>h. Bridge system and defining path to failure</li><li>i. Random graphs and defining path to failure</li><li>j. Effect of multiple sources of failure (multiple decrements) on failure time calculations (competing risk)</li><li>k. Non-uniform probability of component failure (multiple decrement)</li><li>l. Method of inclusion and exclusion as applied to failure time estimates</li><li>m. Expected system lifetime as function of component lifetime and properties of expected lifetime estimates</li><li>n. Linkage between reliability function for a system and future expected lifetime</li></ul>
<b>READINGS</b>	
<ul style="list-style-type: none"><li>• Ross, Sections 9.1-9.6</li></ul>	



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LEARNING OBJECTIVES	KNOWLEDGE STATEMENTS
<p>6. For discrete Markov Chains under both homogeneous and non-homogenous states:</p> <ul style="list-style-type: none"> <li>• Definition of a Markov Chain</li> <li>• Chapman-Kolmogorov Equations for n-step transition calculations</li> <li>• Accessible states</li> <li>• Ergodic Markov Chains and limiting probabilities</li> </ul> <p>Range of weight: 2-8 percent</p>	<ul style="list-style-type: none"> <li>a. Random Walk</li> <li>b. Classification of states and classes of states (absorbing, accessible, transition, irreducible, and recurrent)</li> <li>c. Transition step probabilities</li> <li>d. Stationary probabilities</li> <li>e. Recurrent vs. transient states</li> <li>f. Gamblers ruin problem</li> <li>g. Branching processes</li> <li>h. Homogeneous transition probabilities</li> <li>i. Memoryless property of Markov Chains</li> <li>j. Limiting probabilities</li> </ul>
<p><b>READINGS</b></p>	
<ul style="list-style-type: none"> <li>• Ross, Sections 4.1-4.8</li> </ul>	

LEARNING OBJECTIVES	KNOWLEDGE STATEMENTS
<p>7. Solve Life Contingency problems using a life table in a spreadsheet as the combined result of discount, probability of payment and amount of payment vectors. Understand the linkage between the life table and the corresponding probability models.</p> <ul style="list-style-type: none"> <li>• Calculate annuities for discrete time.</li> <li>• Calculate life insurance single net premiums (or property/casualty pure premiums) for discrete time.</li> <li>• Solve for net level premiums (not including fractional lives).</li> </ul> <p>Range of weight: 2-5 percent</p>	<ul style="list-style-type: none"> <li>a. Discounted cash flow</li> <li>b. Relationship between annuity values and insurance premiums</li> <li>c. Life table linkage to probability models</li> <li>d. Equivalence property</li> </ul>
<p><b>READINGS</b></p>	
<ul style="list-style-type: none"> <li>• Struppeck</li> </ul>	



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<b>LEARNING OBJECTIVES</b>	<b>KNOWLEDGE STATEMENTS</b>
<p>8. The candidate should be familiar with basic computer simulation methods.</p> <ul style="list-style-type: none"><li>• Understand the basic framework of Monte Carlo Simulation.</li><li>• Understand the mechanics of generating uniform random numbers.</li><li>• Generate random numbers from a variety of distributions using the inversion method.</li><li>• Be able to explain when and how to use the Acceptance-Rejection method.</li></ul> <p>Range of weight: 2-5 percent</p>	<ul style="list-style-type: none"><li>a. Random Number Generation</li><li>b. Uniform Random Numbers</li><li>c. Inversion Method</li><li>d. Acceptance-Rejection Method</li></ul>
<b>READINGS</b>	
<ul style="list-style-type: none"><li>• Ross, Sections 11.1, 11.2.1, and 11.2.2</li></ul>	



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## B. Statistics

Range of weight for Section B: 15-30 percent

Candidates should have a thorough understanding of the concepts typically covered in the 2<sup>nd</sup> semester of a two semester undergraduate sequence in Probability and Statistics. The specific topics to be tested are described below. Mastering the concepts listed under Section B is necessary to understand the concepts behind the Generalized Linear Models presented under Section C.

<b>LEARNING OBJECTIVES</b>	<b>KNOWLEDGE STATEMENTS</b>
<p>1. Perform point estimation of statistical parameters using Maximum likelihood estimation (“MLE”).</p> <p>Apply criteria to the estimates such as:</p> <ul style="list-style-type: none"><li>• Consistency</li><li>• Unbiasedness</li><li>• Sufficiency</li><li>• Efficiency</li><li>• Minimum variance</li><li>• Mean square error</li></ul> <p>Calculate parameter estimates using methods other than maximum likelihood.</p>	<ul style="list-style-type: none"><li>a. Equations for MLE of mean, variance from a sample</li><li>b. Estimation of mean and variance based on sample</li><li>c. General equations for MLE of parameters</li><li>d. Recognition of consistency property of estimators and alternative measures of consistency</li><li>e. Application of criteria for measurement when estimating parameters through minimization of variance, mean square error</li><li>f. Definition of statistical bias and recognition of estimators that are unbiased or biased</li><li>g. Application of Rao-Cramer Lower Bound and Efficiency</li><li>h. Relationship between Sufficiency and Minimum Variance</li><li>i. Develop and estimate a sufficient statistic for a distribution</li><li>j. Factorization Criterion for sufficiency</li><li>k. Application of Rao-Cramer Lower Bound and Fisher Information</li><li>l. Application of MVUE for the exponential class of distributions</li><li>m. Linkage between Score Function, Fisher Information and maximum likelihood</li><li>n. Method of Moments</li><li>o. Percentile Matching</li><li>p. Kernel Density Estimation</li><li>q. Maximum Likelihood with Censoring &amp; Truncation</li></ul>





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<b>LEARNING OBJECTIVES</b>	<b>KNOWLEDGE STATEMENTS</b>
<p>Range of weight: 5-15 percent</p> <p>2. Test statistical hypotheses including Type I and Type II errors using:</p> <ul style="list-style-type: none"> <li>• Neyman-Pearson theorem</li> <li>• Likelihood ratio tests</li> <li>• First principles</li> </ul> <p>Apply Neyman-Pearson theorem to construct likelihood ratio equation.</p> <p>Use critical values from a sampling distribution to test means and variances.</p> <p>Range of weight: 5-15 percent</p>	<p>a. Presentation of fundamental inequalities based on general assumptions and normal assumptions</p> <p>b. Definition of Type I and Type II errors</p> <p>c. Significance levels</p> <p>d. One-sided versus two-sided tests</p> <p>e. Estimation of sample sizes under normality to control for Type I and Type II errors</p> <p>f. Determination of critical regions</p> <p>g. Definition and measurement of likelihood ratio tests</p> <p>h. Determining parameters and testing using tabular values</p> <p>i. Recognizing when to apply likelihood ratio tests versus chi-square or other goodness of fit tests</p> <p>j. Apply paired t-test to two samples</p> <p>k. Test for difference in variance under Normal distribution between two samples through application of F-test</p> <p>l. Test of significance of means from two samples under Normal distribution assumption in both large and small sample cases</p> <p>m. Test for significance of difference in proportions between two samples under Binomial distribution assumption in both large and small sample case</p> <p>n. Application of contingency tables to test independence between effects</p> <p>o. Asymptotic relationship between likelihood ratio tests and the Chi-Square distribution</p> <p>p. Application of Neyman-Pearson theorem to Uniformly Most Powerful hypothesis tests</p> <p>q. Equivalence between critical regions and confidence intervals</p> <p>r. Kolmogorov –Smirnov test</p>



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LEARNING OBJECTIVES	KNOWLEDGE STATEMENTS
<p>3. For the Exponential, Gamma, Weibull, Pareto, Lognormal, Beta, and mixtures thereof:</p> <ul style="list-style-type: none"> <li>• Identify the applications to Insurance claim modeling in which each distribution is used and reasons why.</li> <li>• Transformation of distributions</li> </ul> <p>Range of weight: 2-5 percent</p>	<ul style="list-style-type: none"> <li>a. Frequency, severity, and aggregate loss</li> <li>b. Common continuous distributions for modeling claim severity</li> <li>c. Mixing distributions</li> <li>d. Tail properties of claim severity</li> <li>e. Effects of coverage modifications including, for example: limits, deductibles, loss elimination ratios and effects of inflation</li> </ul>
<p>4. Calculate Order Statistics of a sample for a given distribution</p> <p>Range of weight: 0-5 percent</p>	<ul style="list-style-type: none"> <li>a. General Form for distribution of nth largest element of a set</li> <li>b. Application to a given distributional form</li> </ul>
<p><b>READINGS</b></p>	
<p>There are many good introductory statistics textbooks that do an excellent job of covering the material on Section B. In the interest of clarity for the candidates though we have selected two. One should note that the Tse textbook is also used as a reference on Exam MAS-II.</p> <ul style="list-style-type: none"> <li>• Hogg, McKean, and Craig</li> <li>• Tse</li> </ul> <p>For a mapping of the sections of these texts to the learning objectives, candidates should refer to the “Knowledge Statement Mapping for Exam MAS-I” document posted on the CAS website under the Syllabus Material section for this exam.</p> <p>For those candidates who would like to work additional problems or see additional examples to illustrate the concepts in Section B, a couple of sources are listed below that are from the Schaum’s Outline series. We are not expanding the range of material covered and only mention these additional sources as a study aid.</p> <ul style="list-style-type: none"> <li>• Schiller, Spiegel, and Srinivasan: Chapters 4-9</li> <li>• Spiegel and Stephens: Chapters 8-12</li> </ul>	



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## C. Extended Linear Models

Range of weight for Section C: 30-50 percent

This section covers the Extended Linear Model Family and treats Ordinary Least Squares as one type of a Linear Model that may be used when the dependent variable follows the Normal distribution and the observations are independent and identically distributed with constant variance. There is a range of models shown in this section with varying degrees of depth of coverage in the readings. The most commonly used form of the Linear Model for insurance modeling work today is the Generalized Linear Model. Generalized Additive Models are an extension of Generalized Linear Models in which the explanatory variables in the linear equation contain functions, which are in turn modeled when running the software. The models presented in this section all assume that the underlying data consists of independent and identically distributed observations from a member of the exponential distribution family. Also, we assume there is a formula describing the behavior of the dependent variable can be described as a linear process of the dependent variables after applying a link function and that the variance is a function of the mean. There are linear models in which those assumptions are violated, but that is a topic for Exam MAS-II. The specific topics to be tested are described below

LEARNING OBJECTIVES	KNOWLEDGE STATEMENTS
<p>1. Understand the assumptions behind different forms of the Extended Linear Model and be able to select the appropriate model from list below:</p> <ul style="list-style-type: none"> <li>• Ordinary Least Squares</li> <li>• Generalized Linear Model</li> <li>• ANOVA</li> <li>• Generalized Additive Models</li> <li>• Local Regression</li> <li>• Lasso</li> <li>• Ridge Regression</li> <li>• Partial Least Squares</li> <li>• Principle Component Analysis (PCA) Regression</li> </ul> <p>Range of weight: 5-15 percent</p>	<ul style="list-style-type: none"> <li>a. Understand the relationship between mean and variance by model family member for the exponential distribution</li> <li>b. Understand how to select the appropriate distribution function for the dependent variable and the implication for the appropriate model form</li> <li>c. Link Functions (Identity, Log, Logit, Power, Inverse)</li> <li>d. Characteristics of Exponential Family (Binomial, Normal, Exponential, Gamma, Poisson, Inverse Gaussian, Negative Binomial, and Tweedie)</li> <li>e. Canonical Forms of link function and effect of non-canonical link function on bias</li> <li>f. Penalized Regression as implemented using the Lasso or Ridge Regression</li> <li>g. Understand concept of models within models for Generalized Additive Models</li> <li>h. Understand dimension reduction using Partial Least Squares or PCA Regression</li> </ul>



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LEARNING OBJECTIVES	KNOWLEDGE STATEMENTS
<p>2. Evaluate models developed using Extended Linear Model approach.</p> <p>Range of weight: 5-15 percent</p>	<ul style="list-style-type: none"><li>a. Raw and studentized Residuals</li><li>b. R-Squared statistic</li><li>c. Cook's Distance and outliers</li><li>d. Influential points</li><li>e. Leverage</li><li>f. Akaike Information Criterion (AIC) and BIC penalized log likelihood measures</li><li>g. Standardized/Studentized Residuals</li><li>h. Deviance, Deviance Residuals and relationship to likelihood</li><li>i. Pearson Residuals vs. Deviance Residuals</li><li>j. Scatter, QQ and Box Plots</li><li>k. Type III Sequential Chi-Square test</li><li>l. T-test and Wald test for significance of regression coefficients</li><li>m. Prediction intervals for response variable</li><li>n. Mean square error and standard error</li><li>o. Calculation and validity of F test to compare two models (under OLS)</li><li>p. Cross Validation</li><li>q. Test vs. Train Error</li><li>r. Bootstrapping to test model validity</li><li>s. Prediction vs. Forecast Error</li><li>t. Overfitting</li><li>u. Bias-Variance Tradeoff</li><li>v. Evaluate collinearity using variance inflation factor<sup>1</sup></li><li>w. Evaluate appropriateness of underlying assumptions including:<ul style="list-style-type: none"><li>• Homoscedasticity</li><li>• Autocorrelation of residuals</li></ul></li></ul>

<sup>1</sup> See Readings for a definition of variance inflation factor (VIF).



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LEARNING OBJECTIVES	KNOWLEDGE STATEMENTS
<p>3. Understand the algorithms behind the numerical solutions for the different forms of the Extended Linear Model family to enable interpretation of output from the statistical software employed in modeling and to make appropriate modeling choices when selecting modeling options.</p> <p>Range of weight: 5-15 percent</p>	<ul style="list-style-type: none"><li>a. Maximum Likelihood and Ordinary Least Squares</li><li>b. Fisher Scoring (iterative weighted least squares as implemented using the Information and Score functions covered in Section B.1)</li><li>c. Quasi-Likelihood and relationship to maximum likelihood</li><li>d. Collinearity (Aliasing) and model stability</li><li>e. Hat matrix</li><li>f. Design matrix</li><li>g. Fitting adjoining, overlapping observations in groups for Local Regression</li><li>h. Supervised vs. Unsupervised learning methods</li><li>i. Modeling functions within functions for Generalized Additive Models</li><li>j. Penalty function in Penalized regression models (Lasso and Ridge Regression)</li><li>k. Partial Least Squares Supervised learning vs. PCA Regression Unsupervised learning</li></ul>



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LEARNING OBJECTIVES	KNOWLEDGE STATEMENTS
<p>4. Understand and be able to select the appropriate model structure for an Extended Linear Model given the behavior of the data set to be modeled.</p> <p>Range of weight: 5-15 percent</p>	<ul style="list-style-type: none"><li>a. Predictor variables</li><li>b. Response variables</li><li>c. Regression through the origin</li><li>d. Transformation of variables</li><li>e. Categorical vs. continuous explanatory variables</li><li>f. Interaction terms</li><li>g. Significance and model comparison statistics</li><li>h. Residuals and model parameter selection</li><li>i. Piecewise Linear and Smoothing Splines</li><li>j. Smoothing parameter for splines</li><li>k. Basis Functions</li><li>l. Knot Selection for Splines</li><li>m. Weighting function for local regression</li><li>n. Selection of functions within functions for Generalized Additive Models</li><li>o. Selection of appropriate tuning factor for Lasso or Ridge Regression</li><li>p. Select either Lasso or Ridge Regression depending on desired effect from penalized regression</li><li>q. Curse of High Dimensionality</li><li>r. Forward or backward or best subset selection</li></ul>



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LEARNING OBJECTIVES	KNOWLEDGE STATEMENTS
<b>READINGS</b>	
<ul style="list-style-type: none"><li>• Dobson and Barnett, Chapters 1-9, excluding 6.3.3, 6.7, 6.8, &amp; 7.9 in the 4<sup>th</sup> edition only</li><li>• Hogg, McKean, and Craig, Sections 4.4 (only the sections related to QQ and Box Plots), 6.3 through 6.5, 7.5, and 9.1 through 9.6</li><li>• James et al., Chapters 1, 2 (excluding Section 2.2.3), 3 (Sections 3.1 through 3.3, and 3.6 labs on Sections 3.1 through 3.3), 4 (Sections 4.1 through 4.4; the Linear Discrimination Analysis portion of Section 4.4 will not be tested), 5, 6, and 7</li><li>• Larsen</li></ul> <p>For a mapping of the sections of these texts to the learning objectives, candidates should refer to the “Knowledge Statement Mapping for Exam MAS-I” document posted on the CAS website under the Syllabus Material section for this exam.</p> <p>The definition of variance inflation factor (VIF), as referenced in Knowledge Statement C.2.v, varies across the syllabus readings. Candidates should familiarize themselves with the widely accepted VIF formula found on p.102 of James et al. and p. 101 (3<sup>rd</sup> edition) or p. 119 (4<sup>th</sup> edition) of Dobson. The VIF formula on p. 101 of James et al. will not be used on the exam.</p> <p>Exam questions from this section may contain parameter tables and diagnostic tables or plots of the type shown in the texts. Candidates should understand how to interpret these tables. Candidates who become familiar with a statistical language capable of generating this type of output, such as R, will have an easier time understanding and applying the concepts covered in the syllabus material. In particular, candidates that work the lab exercises at the end of the chapters in the James, Gareth, et al. textbook will have a better grasp of the material than that obtained by simply reading the textbook. However, for exam questions from this section, candidates will not be asked to write or interpret R code.</p> <p>Candidates are encouraged to seek out examples of GLM problems to enhance their understanding of GLM concepts. Sources for such examples will be posted on the CAS website under the Study Tips, Tools, and Past Pass Marks section for this exam. Candidates will not be tested on concepts that are outside of the scope of the required reading that may appear in those examples. The examples are furnished so that candidates might reinforce concepts covered in the Dobson and Barnett textbook.</p> <p>For those candidates who would like to work additional problems or see additional examples to illustrate the concepts in Section C, a couple of sources are listed below that are from the Schaum’s Outline series. We are not expanding the range of material covered and only mention these additional sources as a study aid.</p> <ul style="list-style-type: none"><li>• Salvatore and Reagle: Chapters 6-9</li><li>• Schiller, Spiegel, and Srinivasan: Chapters 8 and 9</li><li>• Spiegel and Stephens: Chapters 13 and 16</li></ul>	



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## D. Time Series with Constant Variance

Range of weight for Section D: 10-20 percent

This section will cover basic applications of the Auto Regressive Integrated Moving Average time series model. The specific topics to be tested are described below.

LEARNING OBJECTIVES	KNOWLEDGE STATEMENTS
1. Use time series to model trends. <ul style="list-style-type: none"> <li>• Estimation, data analysis, and forecasting</li> <li>• Forecast errors and confidence intervals</li> </ul> Range of weight: 2-8 percent	a. Mean-reverting time series b. Elimination of trends using differencing c. Relationship between seasonality and autocorrelation
2. Model relationships of current and past values of a statistic / metric. <ul style="list-style-type: none"> <li>• Estimation, data analysis, and forecasting</li> <li>• Forecast errors and confidence intervals</li> </ul> Range of weight: 2-8 percent	a. Calculation and use of lag k autocorrelation statistic and cross correlation statistics in determining model structure b. Stationary series c. Autoregressive models of order 1, AR(1) d. Autoregressive integrated moving average models (ARIMA) <ul style="list-style-type: none"> <li>• AR(p) models</li> <li>• Moving average models (MA)</li> <li>• Autoregressive moving average models (ARMA)</li> <li>• ARIMA model vs. ARMA model</li> </ul> e. Invertible time series and relationship between AR and MA models f. Converting between AR and MA models g. Interpretation of auto-correlation function as aid to model selection (AR vs. MA and number of lags to include in model) h. Relationship between time series input and item modeled for AR vs. MA
3. Understand forecasts produced by ARIMA.  Range of weight: 2-5 percent	a. Forecast using ARIMA models b. One step ahead prediction vs. many step ahead projection c. Change in variance in prediction by AR vs. MA model





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LEARNING OBJECTIVES	KNOWLEDGE STATEMENTS
4. Time Series with Regression  Range of weight: 2-5 percent	a. Deterministic vs. Stochastic Trend b. Serial correlation in regression error results c. Correction in regression via Generalized Least Squares d. Transformation of data using natural logarithms for regression modeling e. Forecast error correction under natural logarithm transformation
<b>READINGS</b>	
<ul style="list-style-type: none"><li>Cowpertwait, Chapters 1-5 (excluding Sections 3.3 and 3.4), 6, 7 (Sections 7.1, 7.2 and 7.3)</li></ul> <p>Exam questions from this section may contain snippets of simple R code and illustrative output of the type shown in the text. Candidates should understand the general functionality of the R commands listed in the “Summary of commands used in examples” sections at the end of Chapters 1-5 and 6. Candidates will not be asked to write R code, nor will they be required to interpret complex applications or complete R programs.</p>	



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## Complete Text References for Exam MAS-I

Text references are alphabetized by the citation column.

Citation	Abbreviation	Learning Objective	Source
Cowpertwait, P. and Metcalfe, A., <i>Introductory Time Series with R</i> , Springer, 2009.	Cowpertwait	D1-D4	<b>B</b>
Daniel, J.W., "Poisson processes (and mixture distributions)," CAS Study Note, June 2008.	Daniel	A1-A3	<b>OP</b>
Dobson, A and Barnett, A, <i>An Introduction to Generalized Linear Models</i> , 3 <sup>rd</sup> Edition or 4 <sup>th</sup> Edition, Chapman and Hall/CRC Press, 2008/2018.  <b>Note: Candidates may use either the 3<sup>rd</sup> or 4<sup>th</sup> edition for 2021 exam administrations.</b>	Dobson & Barnett	C1-C4	<b>B</b>
Hogg, R.V.; McKean, J.W.; and Craig, A.T., <i>Introduction to Mathematical Statistics</i> , 7 <sup>th</sup> Edition, Prentice Hall, 2013.	Hogg, McKean, and Craig	B1-B4, C1-C4	<b>B</b>
James, G., et al., <i>An Introduction to Statistical Learning, with Application in R</i> , 1 <sup>st</sup> ed. 2013, Corr. 8 <sup>th</sup> printing, Springer, 2017.  Note: Although page iv of the text identifies it as the corrected 8 <sup>th</sup> printing, the publisher refers to it as the corrected 7 <sup>th</sup> printing. Candidates should use the version found in the Complete Online Text References web page for this exam.	James et al.	C1-C4	<b>OP</b>
Larsen, M., "Generalized Linear Models," CAS Study Note, December 2015, revised June 2016.	Larsen	C1-C4	<b>OP</b>
Ross, S. M., <i>Introduction to Probability Models</i> , 11 <sup>th</sup> Edition or 12 <sup>th</sup> Edition, Academic Press (an imprint of Elsevier, Inc.), 2014/2019.  <b>Note: Candidates may use either the 11th or 12th edition for the 2021 exam administrations.</b>	Ross	A1-A6, A8	<b>B</b>
Struppeck, T., "Life Contingencies," CAS Study Note, October 2014, revised September 2015.	Struppeck	A7	<b>OP</b>
Tse, Y., <i>Nonlife Actuarial Models, Theory Methods and Evaluation</i> , Cambridge University Press, 2009.	Tse	B1-B3	<b>B</b>



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## Additional Study Aids

Citation	Abbreviation	Learning Objective	Source
Salvatore, D. and Reagle, D., <i>Schaum's Outline of Statistics and Econometrics</i> , McGraw-Hill, 2 <sup>nd</sup> Edition, paperback, January 27, 2011, Chapters 6-9.	Salvatore	C1-C4	<b>BO</b>
Schiller, J.; Spiegel, M.; and Srinivasan, R., <i>Schaum's Outlines of Probability and Statistics: 897 Solved Problems + 20 Videos</i> , McGraw-Hill, 4th Edition, Chapters 4-9.	Schiller	B1-B3, C1-C4	<b>BO</b>
Spiegel, M. and Stephens, L., <i>Schaum's Outline of Statistics</i> , McGraw-Hill, 5 <sup>th</sup> Edition, Chapters 8-13 and 16.	Spiegel	B1-B3, C1-C4	<b>BO</b>

## Source Key

<b>B</b>	Book—may be purchased from the publisher or bookstore or borrowed from the CAS Library.
<b>BO</b>	Book (Optional)—may be purchased from the publisher or bookstore.
<b>NEW</b>	Indicates new or updated material.
<b>OP</b>	All text references marked as Online Publications will be available on a web page titled Complete Online Text References.
<b>SK</b>	Material included in the 2021 Study Kit.
<b>SKU</b>	Material included in both the 2021 CAS Study Kit and the 2021 Update to the 2020 Study Kit.

Items printed in **red** indicate an update, clarification, or change.



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