

# November 2000

## Course 2

Society of Actuaries/Casualty Actuarial Society

1. Two consumers, Jack and Jill, each consume only two normal goods, X and Y .
- The price of X is 5 per unit, and the price of Y is 1 per unit. Although Jack and Jill have identical preferences, at their optimal levels of consumption Jill consumes twice as much X as Jack.

Given this information, which of the following statements must be true?

- (A) Jill's income must be greater than Jack's.
- (B) Jill would be better off by purchasing less X .
- (C) Jack would be better off by purchasing more X .
- (D) Jill's marginal value of X in terms of Y must be greater than Jack's.
- (E) Jill's marginal value of Y in terms of X must be greater than Jack's.

2. The following table shows the annual effective interest rates being credited by an investment account, by calendar year of investment. The investment year method is applicable for the first 3 years, after which a portfolio rate is used:

Calendar Year of Investment	Investment Year Rates			Calendar Year of Portfolio Rate	Portfolio Rate
	$i_1$	$i_2$	$i_3$		
1990	10%	10%	$t\%$	1993	8%
1991	12%	5%	10%	1994	$(t-1)\%$
1992	8%	$(t-2)\%$	12%	1995	6%
1993	9%	11%	6%	1996	9%
1994	7%	7%	10%	1997	10%

An investment of 100 is made at the beginning of years 1990, 1991, and 1992 . The total amount of interest credited by the fund during the year 1993 is equal to 28.40 .

Calculate  $t$  .

- (A) 7.00
- (B) 7.25
- (C) 7.50
- (D) 7.75
- (E) 8.00

3. A company is financed by 1000 shares of stock with a current market value of 100 per share. The company decides to issue 50 5-year bonds with a par value of 100 and an annual coupon rate of 8% and use the proceeds to pay a cash dividend to the company's shareholders. The bonds sell at a market value that provides an annual effective yield of 10% .

Assuming that Modigliani-Miller Proposition I holds, what is the market value per share of the company's stock immediately after the dividend payment?

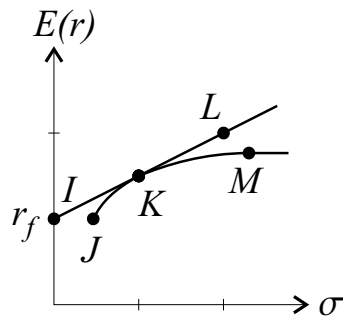
- (A) 95.0
- (B) 95.4
- (C) 100.0
- (D) 104.6
- (E) 105.0

4. An economy is experiencing a declining level of real GDP and a falling price level. The government is running a budget deficit.

Determine an appropriate action for the central bank to take if its short-term goal is to stabilize both output and prices.

- (A) Increase the discount rate.
- (B) Increase reserve requirements on banks.
- (C) Decrease the rate of income taxation.
- (D) Decrease the level of government spending.
- (E) Conduct an open market purchase of government bonds.

5. You are given the following efficient frontier JKM:



Assume you can borrow and lend at the risk-free rate,  $r_f$ .

According to modern portfolio theory, which of the following portfolios would a rational investor hold?

- (A) IJK
- (B) IKL
- (C) IKM
- (D) JKM
- (E) KLM

6. Patrons at an amusement park are all identical and enjoy an uncrowded day at the park more than a crowded day.

The value of a day at the park for each patron is  $100,000 - N^2$  where  $N$  is the total number of park patrons.

If the park management's goal is to operate the park so that total patron value is maximized, what is the optimal number of patrons?

- (A) 0
- (B) 46
- (C) 183
- (D) 224
- (E) 316

7. Which of the following statements about Arbitrage Pricing Theory (APT) is FALSE?

- (A) The CAPM yields an equation equivalent to the one-factor APT with the factor being the stock market index.
- (B) The factors in the APT must be yield curve slope, level of interest rates, level of exchange rates, real GDP, and inflation.
- (C) A portfolio with no exposure to any APT risk factors should earn the risk-free rate on average.
- (D) The APT can be used to estimate the cost of equity capital for a firm.
- (E) All of (A), (B), (C), or (D) are true.



8. An actuary is trying to determine the initial purchase price of a certain physical asset.

The actuary has been able to determine the following:

- (i) Asset is 6 years old.
- (ii) Asset is depreciated using the sinking fund method, with an annual effective rate of 9% .
- (iii) Book value of the asset after 6 years is 55,216.36 .
- (iv) The loss of interest on the original purchase price is at an annual effective rate of 9% .
- (v) Annual cost of the asset is 11,749.22 .
- (vi) Annual maintenance cost of the asset is 3000 .

What was the original purchase price of the asset?

- (A) 72,172
- (B) 107,922
- (C) 112,666
- (D) 121,040
- (E) 143,610

9. Victor invests 300 into a bank account at the beginning of each year for 20 years. The account pays out interest at the end of every year at an annual effective interest rate of  $i\%$ . The interest is reinvested at an annual effective rate of  $\left(\frac{i}{2}\right)\%$ .

The yield rate on the entire investment over the 20 year period is 8% annual effective.

Determine  $i$ .

- (A) 9%
- (B) 10%
- (C) 11%
- (D) 12%
- (E) 13%

10. You are given the following information:

<u>Prices Of:</u>	<u>1997</u>	<u>1998</u>	<u>1999</u>	<u>2000</u>
Ground Beef (per pound)	2.00	2.50	3.00	3.50
Oil (per barrel)	5.00	6.25	7.50	7.70

Calculate the change in the relative price of oil from 1997 to 2000 .

- (A) -12%
- (B) 0%
- (C) +14%
- (D) +54%
- (E) +75%

11. In the 1980s, a certain country was a net borrower on international financial markets. Suppose that the preference of this country's citizens had shifted during this time from domestically produced automobiles to imported automobiles. Assume no change in fiscal policy.

In a simple Keynesian model, determine the effect of this shift in preference.

- (A) A decrease in net exports, with no change in aggregate saving or investment.
- (B) A decrease in net exports, thus decreasing the gap between aggregate saving and investment.
- (C) A decrease in net exports, thus increasing the gap between aggregate saving and investment.
- (D) An increase in net exports, thus increasing the gap between aggregate saving and investment.
- (E) An increase in net exports, thus decreasing the gap between aggregate saving and investment.

12. Kevin takes out a 10-year loan of  $L$ , which he repays by the amortization method at an annual effective interest rate of  $i$ . Kevin makes payments of 1000 at the end of each year.

The total amount of interest repaid during the life of the loan is also equal to  $L$ .

Calculate the amount of interest repaid during the first year of the loan.

- (A) 725
- (B) 750
- (C) 755
- (D) 760
- (E) 765

**13.** Which of the following are true?

- I. According to the “pecking order theory,” firms prefer internal financing to external financing.
- II. An increased likelihood of bankruptcy tends to raise the cost of capital for a firm.
- III. The more senior the claim in a reorganization, the more likely the claimant is to prefer liquidation.

- (A) II only
- (B) I and II only
- (C) I and III only
- (D) II and III only
- (E) I, II, and III

14. Assume that a consumer's Engel curve for a particular product is described by the formula:

$$Y = \begin{cases} \frac{2}{3}X - 2 & \text{for } X \geq 3 \\ 0 & \text{otherwise,} \end{cases}$$

where  $X$  is income and  $Y$  is quantity consumed.

Calculate the income elasticity of demand assuming a change in income from 6 to 9 .

- (A) 0.33
- (B) 0.50
- (C) 0.67
- (D) 1.50
- (E) 2.00

15. You are given the following information:

Long-term debt outstanding:	200,000
Long-term debt is risk free and financed at an interest rate of:	8.0%
Number of shares of common stock:	50,000
Price per share:	16.00
Book value per share:	12.00
Stock's beta:	1.10

The expected market return is 12.0% .

What is the company's before-tax cost of capital?

- (A) 11.0%
- (B) 11.2%
- (C) 11.5%
- (D) 11.9%
- (E) 12.4%



**16.** What is the term for the tendency of an insured individual to take more risks than an uninsured individual?

- (A) Adverse selection
- (B) Moral hazard
- (C) Signaling equilibria
- (D) Principal-agent problem
- (E) Rational expectations

17. An investor wishes to purchase a stock today, sell it at the end of three years, and has the following estimates of dividends paid at the end of each year:

Year	Annual Dividend
1	1.10
2	1.20
3	1.35

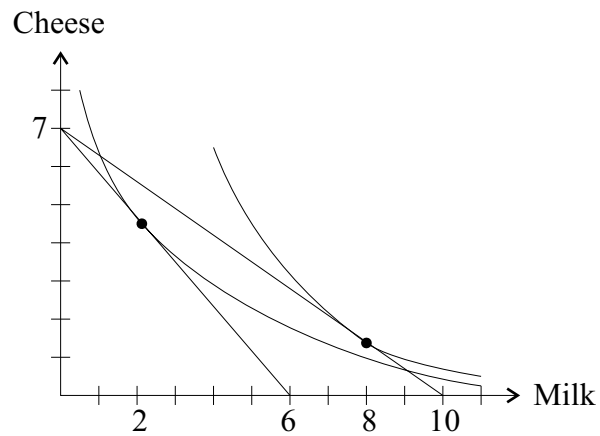
The investor estimates the dividend yield to be 4% at the end of the 3<sup>rd</sup> year.

She anticipates selling immediately following receipt of the 3<sup>rd</sup> dividend.

What would she be willing to pay for the stock today if she has a 14% required rate of return?

- (A) 25.58
- (B) 27.37
- (C) 29.16
- (D) 31.45
- (E) 33.75

18. You are given the following indifference curves that describe a consumer's preferences for milk and cheese. Assume this consumer has a budget constraint of 30 .



What quantity of milk will this consumer demand at a price of 3.3, assuming a linear demand curve?

- (A) 2.9
- (B) 5.0
- (C) 6.4
- (D) 7.1
- (E) 9.1

19. The variance of a company's continuously compounded stock price is 25% per year.  
The nominal risk-free rate payable quarterly is currently 8% .

If the company's stock is currently 100, determine the value of a three-month  
European put with a strike price of 108 .

- (A) 7.56
- (B) 9.03
- (C) 10.50
- (D) 11.98
- (E) 13.45

20. Sandy purchases a perpetuity-immediate that makes annual payments. The first payment is 100, and each payment thereafter increases by 10 .

Danny purchases a perpetuity-due which makes annual payments of 180 .

Using the same annual effective interest rate,  $i > 0$ , the present value of both perpetuities are equal.

Calculate  $i$  .

- (A) 9.2%
- (B) 9.7%
- (C) 10.2%
- (D) 10.7%
- (E) 11.2%

- 21.** The citizens of a country spend 80% of their income and save 20% of their income.

If saving and spending habits do not change, how much total spending is generated if government expenditures increase by 40 ?

- (A) 32
- (B) 40
- (C) 50
- (D) 200
- (E) 250

22. Jerry will make deposits of 450 at the end of each quarter for 10 years.

At the end of 15 years, Jerry will use the fund to make annual payments of  $Y$  at the beginning of each year for 4 years, after which the fund is exhausted.

The annual effective rate of interest is 7% .

Determine  $Y$  .

- (A) 9573
- (B) 9673
- (C) 9773
- (D) 9873
- (E) 9973

23. Company X needs to decide whether or not to develop and market a pill to cure a certain disease. Once developed, then the pill needs to be approved before it can be sold. If the pill is approved, the company will immediately incur marketing costs of 1000 . The probability that the pill will be a success is 75% .

If the pill succeeds, the company will receive a net cash flow perpetuity of 500 at the end of each year. If the pill fails, the company will receive a net cash flow of 500 at the end of year 1, declining by 100 each year for the following 4 years, and no cash flow thereafter.

Using an annual effective rate of 8%, calculate the probability of approval so that the expected net present value of the project is 1400 .

- (A) 35%
- (B) 40%
- (C) 45%
- (D) 50%
- (E) 55%



24. Bill and Jane each sell a different stock short for a price of 1000 . For both investors, the margin requirement is 50%, and interest on the margin is credited at an annual effective rate of 6% .

Bill buys back his stock one year later at a price of  $P$  . At the end of the year, the stock paid a dividend of  $X$  .

Jane also buys back her stock after one year, at a price of  $(P - 25)$  . At the end of the year, her stock paid a dividend of  $2X$  .

Both investors earned an annual effective yield of 21% on their short sales.

Calculate  $P$  .

- (A) 800
- (B) 825
- (C) 850
- (D) 875
- (E) 900

**25.** Which of the following statements about options are true?

- I. Put/call parity implies that puts and calls should trade at the same price, when the stock price equals the exercise price.
- II. Early exercise of an American call option only makes sense when there is positive cash flow prior to maturity on the underlying asset.
- III. As price volatility increases, call option prices rise, and put option prices fall.

- (A) I only
- (B) II only
- (C) III only
- (D) I and II only
- (E) I and III only

26. In a competitive and constant cost industry, the current long-run equilibrium has three identical firms each producing a quantity,  $Q$ , of 7 pencils at a price,  $P$ , of 2 per pencil.

Each firm has a fixed cost of 10 and marginal cost,  $MC$ , given by:

$Q$	$MC$
7	2
8	4
9	7
10	10
11	11
12	13
13	15
14	17
15	19

The market demand schedule for pencils shifts from  $Q$  to  $Q'$  as follows:

$P$	$Q$	$Q'$
2	21	45
4	18	42
7	15	39
10	12	36
11	9	33
13	6	30

What is the new price of pencils in the short run?

- (A) 2
- (B) 7
- (C) 10
- (D) 11
- (E) 13

27. An investor deposits 50 in an investment account on January 1 . The following summarizes the activity in the account during the year:

<u>Date</u>	<u>Value Immediately Before Deposit</u>	<u>Deposit</u>
March 15	40	20
June 1	80	80
October 1	175	75

On June 30, the value of the account is 157.50 . On December 31, the value of the account is  $X$  . Using the time-weighted method, the equivalent annual effective yield during the first 6 months is equal to the (time-weighted) annual effective yield during the entire 1-year period.

Calculate  $X$  .

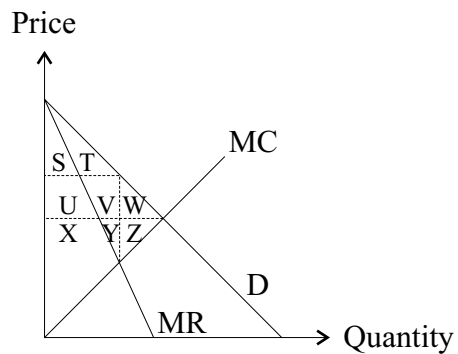
- (A) 234.75
- (B) 235.50
- (C) 236.25
- (D) 237.00
- (E) 237.75

28. You are given that one-year U.S. bonds yield 7.5%, one-year Japanese bonds yield 4.9%, and the current nominal exchange rate is 118 yen per dollar.

Assuming interest rate parity, what is the expected exchange rate one year from now?

- (A) 103
- (B) 115
- (C) 121
- (D) 124
- (E) 133

29. The demand, marginal cost, and marginal revenue curves for a monopolist charging an optimal two-part tariff (a per unit price plus an entry fee) are shown on the graph below. Assume all of the customers are identical and that there are no fixed costs.



What will be the monopolist's profits under the optimal pricing strategy?

- (A)  $S + T$
- (B)  $S + T + U + V + W$
- (C)  $S + T + U + V + W + X + Y + Z$
- (D)  $U + V + X + Y$
- (E)  $X + Y + Z$

30. A 1000 par value 20-year bond with annual coupons and redeemable at maturity at 1050 is purchased for  $P$  to yield an annual effective rate of 8.25% .

The first coupon is 75 . Each subsequent coupon is 3% greater than the preceding coupon.

Determine  $P$  .

- (A) 985
- (B) 1000
- (C) 1050
- (D) 1075
- (E) 1115

**31.** You are given the following information:

Gross Private Domestic Investment	1000
Net Foreign Investment	120
Personal Savings	410
Corporate Retained Earnings	130
Consumption of Fixed Capital (Depreciation)	800
Net Exports	-100
Government Savings	-220

Calculate National Savings.

- (A) 320
- (B) 900
- (C) 990
- (D) 1000
- (E) 1120



- 32.** A stock has a current value of 100 . In each of the next six-month periods, the stock price could rise by 25% or fall by 25% . The risk-free interest rate is 6% per year.

What is the price of a one-year European call on this stock with an exercise price of 90 ?

- (A) 18.5
- (B) 19.0
- (C) 19.6
- (D) 21.3
- (E) 22.5

33. A new device is invented which reduces air pollution when attached to a vehicle's exhaust system. The supply and demand curves for the device are:

Supply:  $P = 0.0003 Q$

Demand:  $P = 20 - 0.0002 Q$ ,

where  $P$  is Price and  $Q$  is Quantity.

To encourage the use of the device, the state offers a price subsidy sufficient to increase the quantity demanded by 25% .

Calculate the cost to the state of this subsidy.

- (A) 25,000
- (B) 100,000
- (C) 150,000
- (D) 200,000
- (E) 250,000

34. An investor took out a loan of 150,000 at 8% compounded quarterly, to be repaid over 10 years with quarterly payments of 5483.36 at the end of each quarter. After 12 payments, the interest rate dropped to 6% compounded quarterly. The new quarterly payment dropped to 5134.62 .

After 20 payments in total, the interest rate on the loan increased to 7% compounded quarterly. The investor decided to make an additional payment of  $X$  at the time of his 20th payment. After the additional payment was made, the new quarterly payment was calculated to be 4265.73, payable for five more years.

Determine  $X$  .

- (A) 11,047
- (B) 13,369
- (C) 16,691
- (D) 20,152
- (E) 23,614

**35.** Which of the following statements is NOT consistent with the “Rational Expectations” hypothesis?

- (A) Actual unemployment rates will not vary from the natural rate for a long period of time.
- (B) Since inflation is strongly correlated with monetary growth, individuals will make unbiased predictions of the inflation rate, as statistics on monetary growth are widely available.
- (C) Monetary policy has no long-run influence on real income.
- (D) A change in the expected inflation rate will generate a similar change in the expected real interest rate.
- (E) There is no long-run trade-off between unemployment and inflation.

36. Which of the following is a reason why the Net Present Value (NPV) technique of evaluating financial projects is preferred over the Internal Rate of Return (IRR) technique?
- (A) IRR does not consider the timing of cash flows, but NPV does.
  - (B) IRR assumes that cash flows are re-invested at the required rate of return and NPV does not have this inherent assumption.
  - (C) IRR may not correctly distinguish between mutually exclusive projects, but NPV does.
  - (D) The IRR on an investment is the return that results in a zero NPV when it is used as the discount rate.
  - (E) None of (A), (B), (C), or (D).

37. Suppose that capital is growing at 3.0% per year and labor hours worked is growing at 1.0% per year. Also, suppose that labor's share of total income is 60% and capital's share is 40% .

If real output growth is 5.0% per year, what is the growth in total factor productivity?

- (A) 1.0%
- (B) 1.8%
- (C) 2.0%
- (D) 2.8%
- (E) 3.2%

38. Chuck needs to purchase an item in 10 years. The item costs 200 today, but its price inflates 4% per year.

To finance the purchase, Chuck deposits 20 into an account at the beginning of each year for 6 years. He deposits an additional  $X$  at the beginning of years 4, 5, and 6 to meet his goal.

The annual effective interest rate is 10% .

Calculate  $X$  .

- (A) 7.4
- (B) 7.9
- (C) 8.4
- (D) 8.9
- (E) 9.4

**39.** Assume a world in which there are two countries and two consumer products.

Country X has a comparative advantage in Product I and Country Y has a comparative advantage in Product II. Further assume that there are no gains from trade that arise from economies of scale.

Which of the following statements is true?

- (A) Although Country X has a comparative disadvantage in the production of Product II, it may still have an absolute advantage over Country Y in the production of Product II.
- (B) By specializing in their respective products of comparative advantage and trading with each other, both countries can increase both their production possibilities and their consumption possibilities.
- (C) By specializing in their respective products of comparative advantage and trading with each other, both countries ensure that the well-being of all their consumers and producers will improve.
- (D) If both countries specialize in and export their respective product of comparative advantage, the sum of the welfare gain of consumers in Country X and Country Y and of producers of the product of comparative advantage in Country X and Country Y will equal the welfare loss suffered by producers of the products of comparative disadvantage in the two countries.
- (E) Countries X and Y would gain most from trade with each other if they specialized in their products of comparative advantage and, at the same time, adopted an import tariff for the product in which they do not have a comparative advantage.



40. Among a company's assets and accounting records, an actuary finds a 15-year bond that was purchased at a premium. From the records, the actuary has determined the following:

- (i) The bond pays semi-annual interest.
- (ii) The amount for amortization of the premium in the 2<sup>nd</sup> coupon payment was 977.19 .
- (iii) The amount for amortization of the premium in the 4<sup>th</sup> coupon payment was 1046.79 .

What is the value of the premium?

- (A) 17,365
- (B) 24,784
- (C) 26,549
- (D) 48,739
- (E) 50,445

**41.** Which of the following statements about non-competitive markets is true?

- (A) When each member of a cartel acts in its own self-interest, the result for the group will be Pareto optimal.
- (B) A regulation requiring a product to meet minimum quality standards may not benefit producers, but it will benefit consumers.
- (C) An oligopoly is more likely to form in a market with low entry and exit costs than in one with high entry and exit costs.
- (D) Government regulation can promote the success of a cartel.
- (E) Products are more likely to be differentiated in a large competitive market than in a market dominated by a few monopolistic competitors.

42. An economy has a fractional reserve banking system. The reserve requirement,  $r$ , in this economy is less than one. People in this economy hold some money in the form of cash in their pockets and some as demand deposits at banks. Banks hold no excess reserves.

Determine the final impact of an increase of  $B$  in the monetary base.

- (A) The level of required reserves will increase by  $Br$ , and the money supply will increase by  $B$ .
- (B) The level of required reserves will increase by  $B$ , and the money supply will increase by  $B/r$ .
- (C) The level of required reserves will increase, and the money supply will increase by more than  $B$  but by less than  $B/r$ .
- (D) The level of required reserves will be unaffected, and the money supply will increase by more than  $B$  but by less than  $B/r$ .
- (E) The level of required reserves will be unaffected, and the money supply will increase by  $B$ .

43. A company has a debt-to-equity ratio of 0.4 . Its common stock is currently selling for 23 . Its next dividend is expected to be 1.20 and the expected long-term growth rate for dividends is 4% . Its bonds currently yield 6%, and it has a marginal tax rate of 35% .

What is the weighted average cost of capital for the company?

- (A) 7.09%
- (B) 7.20%
- (C) 7.70%
- (D) 8.30%
- (E) 9.22%

44. Joe can purchase one of two annuities:

Annuity 1: A 10-year decreasing annuity-immediate, with annual payments of 10, 9, 8,  $\dots$ , 1.

Annuity 2: A perpetuity-immediate with annual payments. The perpetuity pays 1 in year 1, 2 in year 2, 3 in year 3,  $\dots$ , and 11 in year 11. After year 11, the payments remain constant at 11.

At an annual effective interest rate of  $i$ , the present value of Annuity 2 is twice the present value of Annuity 1.

Calculate the value of Annuity 1.

- (A) 36.4
- (B) 37.4
- (C) 38.4
- (D) 39.4
- (E) 40.4

45. A machine has been purchased by a company for 110,000 . The asset will be depreciated over 20 years using the sum-of-digits method. You have been asked by senior management to set up the tax savings of 17,500 at an annual tax rate of 35% and an annual interest rate of 8% .

Calculate the salvage value.

- (A) 8,148
- (B) 16,667
- (C) 20,604
- (D) 20,756
- (E) 27,500

46. An analysis of 72 monthly rates of return on a company's common stock indicates a beta of 1.75 and an alpha of 0.005 per month. One month later, the market is up by 1.0% and the stock is up by 2.0% .

What is the abnormal rate of return?

- (A) -0.250%
- (B) 0.125%
- (C) 0.250%
- (D) 0.500%
- (E) 1.000%

47. In a competitive industry, market demand is given by  $P = 1200 - 15Q$  and market supply is given by  $P = 10 + 2Q$ , where  $P$  is Price and  $Q$  is Quantity.

Firm X in this industry has a total variable cost curve given by  $TVC = Q^3 - 5Q^2 + 125Q$ .

It incurs fixed costs of 150 whether or not it produces.

What is the output of Firm X ?

- (A) 0
- (B) 2
- (C) 5
- (D) 8
- (E) 10



48. A 12-year loan of 8000 is to be repaid with payments to the lender of 800 at the end of each year and deposits of  $X$  at the end of each year into a sinking fund.

Interest on the loan is charged at an 8% annual effective rate. The sinking fund annual effective interest rate is 4% .

Calculate  $X$  .

- (A) 298
- (B) 330
- (C) 361
- (D) 385
- (E) 411

49. A company is considering a project that costs 88,200 at time 0 and expects a revenue stream of 11,400 at the end of each year forever. The company plans to finance the project by issuing debt of 45,000 with the remaining 43,200 coming from the issuance of common stock. This debt to equity is the same as the market value of the company's debt to equity ratio.

You are given the following information about the project:

- (i) Issue costs equal 4% of the gross proceeds on any new issuance of stock.
- (ii) The company must pay an 8% coupon at the end of each year on its debt.
- (iii) The company's weighted average cost of capital = 12% .
- (iv) The company's marginal tax rate = 22.5% .

In order to determine the adjusted present value of the project, the company assumes that its debt will be adjusted each year to keep it at a constant fraction of the future project value.

Calculate the adjusted present value.

- (A) 11,750
- (B) 12,000
- (C) 12,500
- (D) 12,950
- (E) 13,200

50. Assume Bob's weekly income in 1997 was 12.00 and he spent it all on peanut butter and jelly. Peanut butter cost 1.00 per jar and jelly cost 1.50 per jar. In 1998 Bob's weekly income increased to 15.00 while the cost of peanut butter went up to 3.00 per jar and the cost of jelly dropped to 1.00 per jar. At those prices, he spent his 15.00 on 3 jars of peanut butter and 6 jars of jelly.

In which year did Bob consume more peanut butter, in which year did he consume more jelly, and in which year was he happier?

	<u>More Peanut Butter</u>	<u>More Jelly</u>	<u>Happier</u>
(A)	1997	1998	1997
(B)	1998	1997	1997
(C)	1997	1998	1998
(D)	1998	1997	1998
(E)	1998	1998	1998

51. An investor deposits 1000 on January 1 of year  $x$  and deposits another 1000 on January 1 of year  $x + 2$  into a fund that matures on January 1 of year  $x + 4$ . The interest rate on the fund differs every year and is equal to the annual effective rate of growth of the gross domestic product (GDP) during the 4<sup>th</sup> quarter of the previous year.

The following are the relevant GDP values for the past 4 years:

Year	Quarter	
	III	IV
$x - 1$	800.0	808.0
$x$	850.0	858.5
$x + 1$	900.0	918.0
$x + 2$	930.0	948.6

What is the internal rate of return earned by the investor over the 4 year period?

- (A) 1.66%
- (B) 5.10%
- (C) 6.15%
- (D) 6.60%
- (E) 6.78%

52. Company M is a monopolist with a cost function given by  $C(Y) = 0.5 + 2Y$ , where  $Y$  is output per year. The demand for its product is given by  $P(Y) = 8 - 4Y$ . Assume that all of the company's economic rents will remain constant forever and will accrue to its shareholders who expect an 8% rate of return. The government decides to permanently limit the price that the company may charge to  $p_{\max} = 3$ .

Calculate the change in the market value of Company M.

- (A) -21.9
- (B) -12.5
- (C) -8.0
- (D) 0.0
- (E) +18.8

53. At time 0,  $K$  is deposited into Fund X, which accumulates at a force of interest  $\delta_t = 0.006t^2$ . At time  $m$ ,  $2K$  is deposited into Fund Y, which accumulates at an annual effective interest rate of 10%.

At time  $n$ , where  $n > m$ , the accumulated value of each fund is  $4K$ .

Determine  $m$ .

- (A) 1.6
- (B) 2.4
- (C) 3.8
- (D) 5.0
- (E) 6.2

54. You are given the following information about a company:

	Balance Sheet as of:	
	Dec. 31, 1998	Dec. 31, 1999
Cash & Securities	10	15
Receivables	40	45
Inventory	50	45
Property, Plant & Equipment	60	70
Total Assets	160	175
Current Liabilities	70	75
Long-term Debt	40	45
Shareholders' Equity	50	55
Total Liabilities & Shareholders' Equity	160	175

Which of the following increased from 1998 to 1999?

- I. Net working capital
- II. Quick ratio
- III. Debt to equity ratio

- (A) I only
- (B) III only
- (C) I and II only
- (D) I and III only
- (E) II and III only

55. Iggy borrows  $X$  for 10 years at an annual effective rate of 6% . If he pays the principal and accumulated interest in one lump sum at the end of 10 years, he would pay 356.54 more in interest than if he repaid the loan with 10 level payments at the end of each year.

Calculate  $X$  .

- (A) 800
- (B) 825
- (C) 850
- (D) 875
- (E) 900



**Course 2**  
**November 2000**  
**Answer Key**

1	A	31	E
2	D*	32	D
3	B	33	E
4	E	34	C
5	B	35	D
6	C	36	C
7	B	37	E
8	A	38	D
9	B	39	A
10	A	40	D
11	C	41	D
12	C	42	C
13	E	43	C
14	E	44	D
15	C	45	E
16	B	46	A
17	*	47	C
18	D	48	B
19	E	49	B
20	C	50	A
21	D	51	E
22	D	52	B
23	A	53	A
24	E	54	E
25	B	55	B
26	D		
27	C		
28	B		
29	C		
30	E		

\*Scored correct for all candidates.

1. A  
Since both Jack and Jill face the same price, their marginal value of X in terms of Y must be equal, and since they are both at optimal levels of consumption, altering their bundles would make them worse off. Therefore, Jill must simply have more income than Jack.
2. D  
Interest from:  
 $1990 \text{ investment} = 100(1.1)(1.1)(1 + t)(0.08) = 9.68(1 + t)$   
 $1991 \text{ investment} = 100(1.12)(1.05)(0.1) = 11.76$   
 $1992 \text{ investment} = 100(1.08)(t - 0.02) = 108(t - 0.2)$   
 $\text{Total interest} = 9.68 + 9.68t + 11.76 + 108t - 2.16 = 28.40$   
 $117.68t = 9.12$   
 $t = 7.75\%$ .  
 Note: This material was not on the November, 2000 Course 2 syllabus.
3. B  
Under MM I, market value of firm remains unchanged so market value of equity is reduced by the market value of the newly issued debt.  
 Market value of bonds  
 $= 400/1.1 + 400/(1.1)^2 + 400/(1.1)^3 + 400/(1.1)^4 + 5400/(1.1)^5 = 4621$ .  
 Market value of stock reduced by  $4621/1000 = 4.62$  per share.  
 Revised market value of stock = 95.38.
4. E  
The central bank needs to increase the money supply and lower interest rates. This is accomplished by an open market purchase of government bonds.
5. B  
CAPM states that a rational investor will hold a combination of risk free assets and a market portfolio of risky assets. Portfolios J and M are efficient risky portfolios, but will never be held by a rational investor if borrowing and lending at the risk free rate is possible because portfolios with higher expected returns are available for the same level of risk. Hold portfolios IKL
6. C  
Need to maximize social gain  
 $\text{Social gain} = Nf(N) = N(100,000 - N^2)$  which is maximized when  $d/dN = 0$   
 $d/dN = 100,000 - 3N^2 = 0$   
 $N = 183$ .
7. B  
The Arbitrage Pricing Theory is not constrained to a specific set of factors.

8. A  
 From (iii)  $\Rightarrow B_6 = A - (\text{SF deposit}) s_{\overline{6}|0.09}$   
 $\Rightarrow$  (i)  $55,216.36 = A - (\text{SF deposit}) (7.5233346)$   
 From (iv)  $\Rightarrow H = Ai + \underbrace{\frac{A-S}{s_{\overline{n}|j}}}_{\text{This is the SF deposit}} + M$   
 $\Rightarrow 11,749.22 = A(0.09) + \text{SF deposit} + 3000 \Rightarrow \text{SF deposit} = 8,749.22 - A(0.09)$  (ii)  
 Substitute this into (i):  $55,216.36 = A - (8749.22 - A(0.09))(7.5233346) \Rightarrow A = 72,172$ .
9. B  
 $300 \ddot{s}_{\overline{20}|8\%} = 20(300) + 300i (Is)_{\overline{20}|i/2}$   
 $14826.88 = 6000 + 300i \frac{(s_{\overline{21}|i/2} - 21)}{i/2} = 6000 + 600 s_{\overline{21}|i/2} - 12,600$   
 $s_{\overline{21}|i/2} = 35.711467$   
 $i/2 = 0.05$   
 $i = 0.10$
10. A  
 $\left(\frac{7.7}{3.5}\right) / \left(\frac{5}{2}\right) = -12\%$
11. C  
 The country's consumers are buying more imported goods, so net exports must decrease (net exports = exports – imports); in the simple Keynesian model, this produces a decrease in aggregate saving minus investment. Since the country is a net borrower, aggregate saving is already less than investment, so a decrease in aggregate saving minus investment means the negative gap between the two increases.
12. C  
 $L = 1000 \cdot a_{\overline{10}|i}$  (loan)  
 $= 1000 (10 - a_{\overline{10}|i})$  (amount of interest repaid)  
 $1000 \cdot a_{\overline{10}|i} = 10,000 - 1000a_{\overline{10}|i}$   
 $a_{\overline{10}|i} = 5 \Rightarrow i = 15.1\%$   
 Interest in year 1  $= 1000 (1 - V_{0.151}^{10}) = 754.95$ .

13. E  
 I True, if internal funds (i.e. retained earnings) available, there's no need to seek external financing.  
 II True, an increased likelihood of bankruptcy increases the risk of the investment thus increasing the required return for a potential investor.  
 III True, under a liquidation, the senior claimants are likely to receive full recovery of their claims. Under a reorganization, full recovery will be dependent on the future financial success of the reorganized company.
14. E  

$$\frac{\text{percentage change in Y}}{\text{percentage change in X}} = \frac{100\%}{50\%} = 2.00 .$$
15. C  
 CAPM states:  
 Return on stock  $= r_f + \beta_i \times (E[r_m] - r_f)$   
 $= 8\% + 1.1 \times (12\% - 8\%)$   
 $= 12.4\%$   
 Debt = 200,000  
 Equity = 50,000 x 16 = 800,000  
 So debt = 20% and equity = 80%  
 Cost of capital  $= D/V \times r_{\text{debt}} + E/V \times r_{\text{equity}}$   
 $= [20\% \times 8\%] + [80\% \times 12.4\%]$   
 $= 11.5\%$
16. B  
 By definition.
17. Question was discarded.
18. D  
 At P = 5, Q = 2 . At P = 3, Q = 8 . By linear interpolation: at P = 3.3, Q = 7.1 .
19. E  
 We compute:  
 $\sigma = (0.25)^{0.5} = 0.5$   
 $(t)^{0.5} = (0.25)^{0.5} = 0.5$   
 $PV(EX) = 108/1.02$   
 $D1 = \ln(100/(108/1.02))/(1/4) + 1/8 = -0.10363$   
 $D2 = D1 - (1/4) = -0.35363$   
 $N(D1) = 0.45873$   
 $N(D2) = 0.36181$   
 Value of Put =  $N(D1) \times 100 - N(D2) \times 108/1.02 + 108/1.02 - 100 = 13.45$

20.

C

$$180/d = 100/i + 10/i^2$$

$$\frac{180(1+i)}{d} = \frac{100}{i} + \frac{10}{i^2}$$

$$180 + 180i = 100 + 10/i$$

$$180i^2 + 80i - 10 = 0$$

$$i = \frac{-80 \pm \sqrt{80^2 - 4(180)(-10)}}{2(180)} = 10.17\%$$

21.

D

$$\text{Total spending} = 40 \left( \frac{1}{1-0.8} \right) = 200$$

22.

D

$$4(450) \left( \frac{(1.07)^{10} - 1}{0.07^{(4)}} \right) = 1800 \left( \frac{1.96715 - 1}{0.068234} \right) = 25,513.23$$

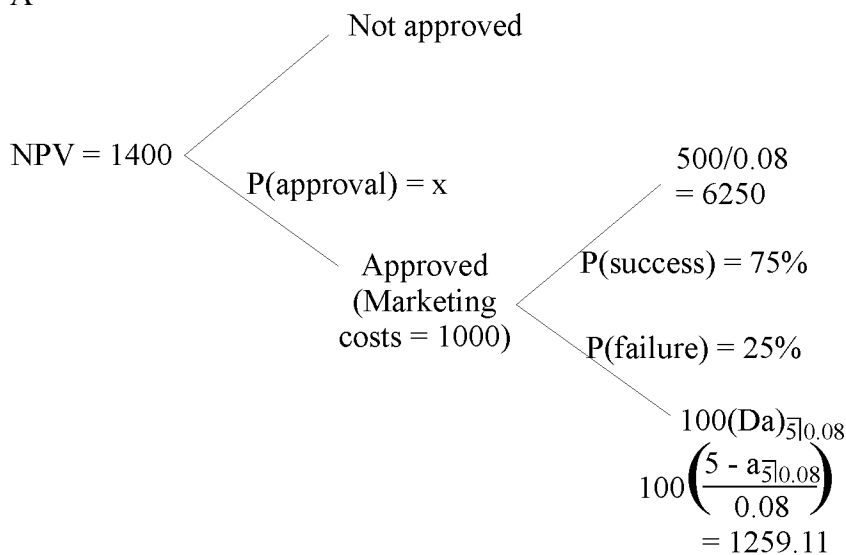
$$25,513.23 \times (1.07)^5 = 35,783.63$$

$$(1 - v^4)/d = \frac{1 - 0.7629}{0.065421} = 3.62422$$

$$\frac{35783.63}{3.62422} = 9873.47$$

23.

A



$$1400 = x[-1000 + 0.75(6250) + 0.25(1259.11)]$$

$$1400 = x[4002.28]$$

$$x = 35\%$$

24.

E

Bill

$1000 - P$

$1000(0.5)0.6 = 30$

$X$

Jane

$1000 - (P - 25) - \text{Profit on sale}$

$30$  – interest on margin

$2X$  – dividend

$$\frac{1000 - P + 30 - X}{500} = \frac{1025 - P + 30 - 2X}{500}$$

$$1030 - X = 1055 - 2X \Rightarrow X = 25$$

$$\frac{1000 - P + 30 - (X = 25)}{500} = \frac{1005 - P}{500} = 0.21$$

$$\Downarrow$$

$$P = 900$$

25.

B

I False, true when stock price equals present value of exercise price.

II True, in the absence of dividends, an American call option is always worth more alive than dead because you earn interest on the exercise money while keeping your option open.

III False, as price volatility rises, both call and put option prices rise.

26.

D

The answer is the price at which short run quantity demanded equals short run supply.  
For one firm, the short run supply curve is equal to the marginal cost curve:

$\frac{P}{}$	$\frac{Q}{}$
2	7
4	8
7	9
10	10
11	11
13	12

In the short run, the total market supply is the sum of the three identical firms:

$\frac{P}{}$	$\frac{Q}{}$
2	21
4	24
7	27
10	30
11	33
13	36

The price P at which market supply equals market demand is 11 .

27.

C

6-month yield =  $(40/50)(80/60)(157.50/160) - 1 = 5\%$  .

Annual equivalent =  $(1.05)^2 - 1 = 10.25\%$  .

1-year yield =  $(40/50)(80/60)(175/160)(X/250) - 1 = 0.1025$

$X = 1.1025/0.004667 = 236.25$  .

28.

B

Using either the exact formula:  $(1 + i) = (1 + i^f) \frac{e_{nom}}{e^{exp}}$

or the approximation formula:  $i \approx i^f - \frac{\Delta e^{exp}}{e_{nom}}$

$$118 \left( \frac{1.049}{1.075} \right) = 115.1 .$$

29.

C

The optimal two-part tariff extracts all consumer surplus, which is  $S + T + U + V + W$  .  
Producer surplus is  $X + Y + Z$  .

30. E  

$$P = 75 [v + 1.03v^2 + 1.03^2 v^3 + \dots + 1.03^{19} v^{20}] + 1050v^{20}$$

$$= \frac{75}{1.0825} \left[ 1 + \frac{1.03}{1.0825} + \left( \frac{1.03}{1.0825} \right)^2 + \dots + \left( \frac{1.03}{1.0825} \right)^{19} \right] + \frac{1050}{1.0825^{20}}$$

$$= \frac{75}{1.0825} \left[ \frac{1 - \left( \frac{1.03}{1.0825} \right)^{20}}{1 - \frac{1.03}{1.0825}} \right] + 215 = 900 + 215 = 1115 .$$
31. E  
 The answer can be obtained in two ways:  
 $1000 + 120 = 1120$   
 $410 + 130 + 800 - 220 = 1120$
32. D  
 At 6 months, share price could be 75 or 125 . At 12 months, stock price could be 56.25, 93.75 or 156.25 .  
 Expected return = (prob. of increase \* 25%) + (1.0 – prob. of increase) \* (–25%) = 2.96%  
 Probability of increase = 0.559 .  
 Prob. of 156.25 =  $0.559^2 = 0.312$  .  
 Prob. of 93.75 =  $2[0.559 * (1 - 0.559)] = 0.247 * 2 = 0.494$  .  
 Prob. of 56.25 =  $(1 - 0.559)^2 = 0.194$  .  
 Value of call before discounting =  $(156.25 - 90) * 0.312 + (93.75 - 90) * 0.494 = 22.52$  .  
 Value of call after discounting =  $22.52/1.06 = 21.25$  .
33. E  
 Original equilibrium quantity Q:  $0.0003 Q = 20 - 0.0002 Q \quad Q = 40,000$   
 Original equilibrium price P:  $P = 0.0003 (40,000) = 20 - 0.0002 (40,000) \quad P = 12$   
 Subsidy increases Q by 25.0% therefore  $Q' = 50,000$   
 Price charged by suppliers when  $Q' = 50,000$ :  $P' = 0.0003 Q' \quad P' = 15$   
 Price consumers are willing to pay when  $Q' = 50,000$ :  $P'' = 20 - 0.0002 Q' \quad P'' = 10$   
 Subsidy =  $P' - P'' = 5$   
 Cost to taxpayers = subsidy times  $Q' = 5 (50,000) = 250,000$
34. C  
 The key to the question is the ability to determine the outstanding balance after 5 years by modifying the retrospective method.  

$$B_{20}^P = 150,000 (1.02)^{12} (1.015)^8 - [5483.36 S_{\overline{12}|0.02} (1.015)^8 + 5134.62 S_{\overline{8}|0.015}]$$

$$= 214,299.75 - 126,145.42 = 88,154.33$$

$$B_{20}^R = 4265.73 a_{\overline{20}|0.0175} = 71,463.27$$

$$\therefore \text{Lump sum} = B_{20}^P - B_{20}^R = 16,691 .$$



35. D  
The relationship is with the NOMINAL interest rate.
36. C  
IRR does not consider the dollar value of the project and assumes that cash flows can be re-invested at the IRR . For this reason a project with a lower IRR may be preferable to a project with a higher IRR if the two are mutually exclusive; the same is not true for NPV.
37. E  
 $5.0\% - 0.6(1.0\%) - 0.4(3.0\%) = 3.2\%$  .
38. D  

$$(20\ddot{s}_{\overline{6}|} + X\ddot{s}_{\overline{3}|})(1.10^4) = 200(1.04)^{10}$$

$$(169.74 + X(3.641)) = 202.20535$$

$$3.641X = 32.46$$

$$X = 8.92$$
39. A  
Absolute advantage has no bearing on comparative advantage.
40. D  
 Given  $n = 30$   
 $P_2 = 977.19$   
 $P_4 = 1046.79$   
 Key point:  $\frac{P_{t+k}}{P_t} = (1+i)^k \Rightarrow \frac{P_4}{P_2} = \frac{1046.79}{977.19} = 1.0712246 = (1+i)^2$   
 $\Rightarrow i = 0.0349998 = 0.035$  .  
 Key point: Sum of amortization of premium column =  $P_1 \cdot s_{\overline{n}|i}$   
 where  $P_1 = P_2(1.035)^{-1} = 944.14 \rightarrow = 944.14 s_{\overline{30}|0.035} = 48.739.03$  .
41. D  
Government regulation can sometimes inadvertently serve to lessen competition and promote a cartel.
42. C  
 $M = (1/k)[Q - (1 - k)C - RF] + T$  where  $C$  = currency and  $k$  is the reserve requirement.  
 If  $C$  increases by an amount  $B$ , then  $M$  increases by  $[(1 - k)/k]B = (B/k) - B$  . Since  $k < 1$ , the answer is C .

43.

C

We need to calculate the expected rate of return for the common stock, find the weights of stocks and bonds in the capital mix, and adjust the return on bonds for the tax shield.

$$r_E = D_1/P_0 + g$$

$$r_E = 1.20/23 + 0.04 = 9.22\%$$

$$D/E = 0.4 \text{ . Thus } D = 0.4E \text{ and } V = D + E = 1.4E$$

$$D/V = 0.4E/1.4E = 0.2857 \text{ and } E/V = 0.7143$$

$$r^* = 0.06*(1 - 0.35)*0.2857 + 0.0922*0.7143 = 0.01114 + 0.06586 = 7.70\%$$

44.

D

$$\text{PV of Annuity 1: } (Da)_{\overline{10}|} = \frac{10 - a_{\overline{10}|}}{i}$$

$$\text{PV of Annuity 2: } (Ia)_{\overline{11}|} + V^{11} \cdot \frac{11}{i} = \frac{\ddot{a}_{\overline{11}|} - 11V^{11}}{i} + \frac{11V^{11}}{i} = \frac{\ddot{a}_{\overline{11}|}}{i}$$

$$\text{Annuity 2} = 2(\text{Annuity 1})$$

$$\frac{\ddot{a}_{\overline{11}|}}{i} = \frac{2(10 - a_{\overline{10}|})}{i}; \quad \ddot{a}_{\overline{11}|} = 1 + a_{\overline{10}|} = 20 - 2a_{\overline{10}|}$$

$$a_{\overline{10}|} = 6.33 \Rightarrow i = 0.093$$

$$(Da)_{\overline{10}|} = \frac{10 - a_{\overline{10}|}}{i} = \frac{10 - 6.33}{0.093} = 39.45 \text{ .}$$

45.

E

$$A = 110,000$$

$$n = 20$$

$$D_t = \frac{n-t+1}{5n}(A-S) = \frac{20-t+1}{210}(110,000 - S)$$

$$\text{Given } 50,000 = \text{p.v. of the } D_t\text{'s} = D_1(1.08)^{-1} + D_2(1.08)^{-2} + \dots + D_{20}(1.08)^{-20}$$

$$= \frac{110,000 - S}{210} \left[ \underbrace{20(1.08)^{-1} + 19(1.08)^{-2} + \dots + (1.08)^{-20}}_{\text{decreasing annuity}} \right]$$

$$= \frac{110,000 - S}{210} \left[ \frac{20 - a_{\overline{20}|0.08}}{0.08} \right] = \frac{110,000 - S}{210} [127.27316] \Rightarrow S = 27,500.28 \text{ .}$$

46.

A

$$2.0 - [0.5 + 1.75(1.0)] = -0.25 \text{ .}$$

47.

C

Intersection of Supply/Demand at:

$$1200 - 15Q = 10 + 2Q$$

$$1190 = 17Q, Q = 70, P = 150$$

Firm X faces industry price = 150

Will supply at Q s.t.  $P = MC$

$$\text{Firm X } MC = 3Q^2 - 10Q + 125 = 150$$

$$3Q^2 - 10Q - 25 = 0$$

$$\frac{10 \pm \sqrt{100 + 300}}{6} = \frac{10 \pm 20}{6} = 5, -10/6$$

$$Q = 5$$

Note at  $Q = 5$ , Firm X profit is negative but  $P > AVC$  so would still produce.

48.

B

$$8000 (1.08)^{12} = 20,145$$

$$800s_{\overline{12}|8\%} = 15,182$$

$$xs_{\overline{12}|4\%} = 4964$$

$$x = \frac{4964}{15.0258} = 330.$$

49.

B

$$APV = -(\text{project cost}) - (\text{equity issue cost}) + (\text{project PV}) + (\text{PV of tax shields})$$

$$\text{Project cost} = 88,200$$

$$\text{Equity issue cost} = 43,200/0.96 - 43,200 = 1800$$

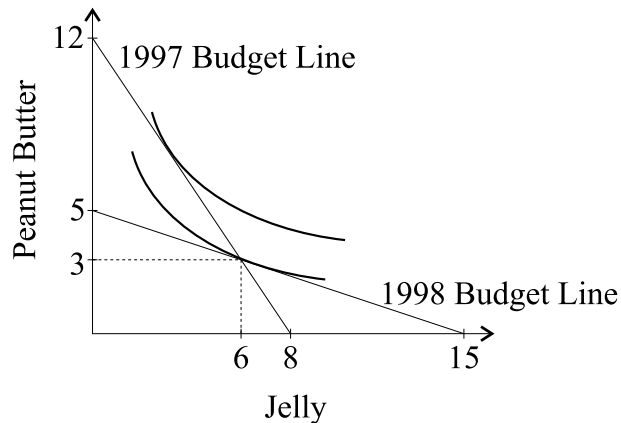
$$\text{Project PV} = 11,400/0.12 = 95,000$$

$$\text{PV tax shield} = 45,000 (0.08/0.12) (0.225) \cdot (1.12/1.08) = 7000$$

$$\therefore APV = -88,200 - 1800 + 95,000 + 7000 = 12,000.$$

50.

A



51. E

For year	Quarterly growth rate = j	$i^{(4)}$	i = equivalent annual rate
x	$808/800 - 1 = 0.01$	0.04	$(1.01)^4 - 1 = 0.040604$
x + 1	$858.5/850 - 1 = 0.01$	0.04	$(1.01)^4 - 1 = 0.040604$
x + 2	$918/900 - 1 = 0.02$	0.08	$(1.02)^4 - 1 = 0.0824322$
x + 3	$948.6/930 - 1 = 0.02$	0.08	$(1.02)^4 - 1 = 0.0824322$

Accumulated value on January 1,  $x + 4 = 1000(1.040604)^2 (1.0824322)^2 + 1000(1.0824322)^2$

= 2440.40

Let  $i = i^{(1)}$  = equivalent annual rate of return. Thus,  $1000(1 + i)^4 + 1000(1 + i)^2$

= 2440.40  $\Rightarrow 1000x^2 + 1000x - 2440.40 = 0 \leftarrow$  let  $x = (1 + i)^2$

$$\Rightarrow x = \frac{-1000 + \sqrt{1000^2 + 4(1000)(2440.40)}}{2(1000)} = \frac{2280.4878}{2000} = 1.1402439$$

$$\Rightarrow i = \sqrt{1.1402439} - 1 = 0.067822 = 6.78\%$$

52. B

A monopolist sets price so that  $MR = MC$

$$R = Y * [8 - 4Y] \quad MR = 8 - 8Y$$

$$MC = 2$$

$$8 - 8Y = 2 \quad Y = 0.75$$

$$P = 5 \quad R = 3.75 \quad C = 2$$

$$\text{Profit} = 1.75$$

$$\text{Market Value} = 1.75 / .08 = 21.875$$

Price capped at 3

$$P = 3 = 8 - 4Y \quad Y = 1.25$$

$$R = 3.75 \quad C = 3$$

$$\text{Profit} = 0.75$$

$$\text{Market Value} = .75 / .08 = 9.375$$

$$\text{Change in MV} = 21.875 - 9.375 = \text{decrease of } 12.5$$

53.

A

$$2k(1.1)^{n-m} = 4k$$

$$1.1^{n-m} = 2$$

$$n - m = 7.27$$

$$ke^{\int_0^n 0.006t^2 dt} = 4k$$

$$e^{0.002 \int_0^n 3t^2 dt} = 4$$

$$e^{0.002t^3} \Big|_0^n = 4$$

$$e^{0.002n^3} = 4$$

$$0.002n^3 = \ln 4 = 1.38629$$

$$n^3 = 693.47$$

$$n = 8.85$$

$$m = 1.58$$

54.

E

Net working capital = current assets – current liabilities

$$\text{In 1998, } (10 + 40 + 50) - 70 = 30$$

$$\text{In 1999, } (15 + 45 + 45) - 75 = 30$$

So no change

Quick ratio = [cash & securities + receivables] / current liabilities

$$\text{In 1998, } [10 + 40]/70 = 0.71$$

$$\text{In 1999, } [15 + 45]/75 = 0.80$$

So increase

Debt to equity ratio = Long term debt / equity

$$\text{In 1998, } 40/50 = 0.80$$

$$\text{In 1999, } 45/55 = 0.82$$

So increase

55.

B

$$x(1.06)^{10} - x - 356.54 = 10 \left( \frac{x}{a_{\overline{10}|0.06}} \right) - x$$

$$1.7908477x - 356.54 = 1.3586796x$$

$$x = 825$$