CAS Task Force on Fair Value Liabilities
White Paper on Fair Valuing Property/Casualty Insurance Liabilities

Section D - Methods of Estimating Risk Adjustments

Section introduction and scope

The previous sections discussed general conceptual issues relative to fair value accounting and the principle alternatives to fair value for insurance liabilities. No detail was given as to how the fair value would actually be calculated. This section takes the next step, discussing specific methods that may be used in calculating the fair value of insurance liabilities.

Risk adjustments

Fair value estimates reflect expected cash flows, the time value of money and an adjustment for risk. This section focuses on the last of these components, the risk adjustment. The methods discussed here assume that expected cash flows and risk-free discount rates are already available. For the purpose of all subsequent discussion the starting point for the discount rate before risk adjustment is the risk-free rate.

Risk to the insurer

All the methods discussed here focus on the riskiness of the insured liabilities to the insurer, not the risk that the insurer will default on the liabilities. This latter risk, called credit risk, is very controversial as to its role in estimating the fair value of liabilities. As such, it is being addressed separately, in Section H. Therefore, while some of the methods discussed below may implicitly reflect this credit risk, quantifying that risk is not the intent of this section.

Risk to loss (and loss expense) liabilities

The risk adjustments discussed here generally apply to two major liability categories on the balance sheet: 1) liabilities already incurred (for example, loss reserves) and 2) liabilities not yet incurred for policies already written. The latter liabilities are called the unearned premium (or "unexpired policy") liabilities Although all the other methods we describe for liabilities already incurred could be used for, the unearned premium liabilities, we provide a separate discussion at the end of this section on methods for computing their risk margins.

Other balance sheet insurance items, such as contingent commissions and deductible recoverable amounts may also be subject to a risk adjustment in estimating their fair value. The risk adjustment for these items is not addressed in this section, although some of the methods discussed here may also be feasible for estimating their fair value.

This section begins with a conceptual discussion of risk margins, including a discussion of diversifiable versus nondiversifiable risk. Next, the methods listed below are presented. These presentations are meant to give the reader a brief conceptual overview of the methods (a more involved discussion is included in the appendices). At the end of this section, a chart comparing the listed methods is provided.

Section D - Methods of Estimating Risk Adjustments

(Note: Neither the inclusion of a method, exclusion of a method, nor the order of the methods listed is meant to imply any preference or priority by the task force. Methods were listed if members of the task force felt it deserved consideration, whether or not consensus was achieved.)

1) Capital Asset Pricing Model (CAPM) based methods, where the liability beta is calculated from insurers' asset and equity betas.

2) Internal Rate of Return (IRR) method, where the risk adjustment is derived from cash flow and rate of return on equity (ROE) estimates.

3) Single Period Risk-Adjusted Discount method, where the calendar year ROE is used to find a risk adjusted interest rate.

4) Methods that use historical underwriting results to derive a risk adjustment.

5) Methods using probability distributions of aggregate losses.

6) Determining fair value estimates from reinsurance transactions.

7) Direct estimation of liability market values based on share prices of property-liability insurance companies.

8) Transformed distribution methods, where the probability distribution of liability outcomes is altered to produce a higher expected value.

9) Naive methods using rules of thumb.

10) Other methods.

Conceptual overview - risk margins

The IASC (paragraph 243) and FASB (Concept statement 5 paragraphs 62 – 71) documents require the use of a risk margin when measuring the fair value of an uncertain liabilities (such as an insurer's liabilities) by discounting the expected liability cash flows. The finance and actuarial literature generally support this approach. (Butsic, Cummins, D'Arcy, and Myers-Cohn.)

The economic rationale for a risk margin is that a third party would not accept compensation for a transfer of liabilities if such payment reflected only the present value of the cash flows at a risk-free interest rate. The acquiring entity would get an expected risk-free return while bearing

risk. A market exchange of the liability would therefore require a premium or risk margin over and above the present value of the liability discounted at the risk-free rate.

In this section we discuss various possible feasible methods for estimating a risk margin. All of these methods have been used for estimating risk margins, either for direct application to balance sheet liabilities or in ratemaking. Financial theory indicates that the same principles for estimating the risk margin in pricing would also apply to a fair valuation of outstanding liabilities. For certain kinds of short tail liabilities, such as claim liabilities associated with catastrophes, the risk margins for pricing may be much larger than the risk margins for liabilities, however. This is because, once a catastrophe has occurred the uncertainty regarding future payments may be relatively modest, compared to the quite large level of uncertainty before the event has occurred.

There are two major paradigms used to compute risk loads that are represented in this paper: the finance perspective and the actuarial perspective. These two paradigms differ in their treatment of diversifiable versus nondiversifiable risk. In the context of liability fair value, diversifiable risk is defined as risk that can be reduced, per unit of liability volume, as more volume is added. For example, if two statistically independent risks are combined, their joint risk will be reduced due to the tendency of bad outcomes from one being offset by good outcomes in the other. In contrast, nondiversifiable (or systematic) risk is defined as risk that cannot be reduced, per unit of liability volume, as more volume is added. Here, bad or good outcomes in one risk are matched with the same result in the other.

The amount of diversification depends on the correlation between the units being added. This

$$s(x+y) = \sqrt{(s_x^2 + s_y^2 + 2 r s_x s_y}$$

effect is evident in the square root rule for summing standard deviations:
Where ρ is the correlation between x and y, σ_x is the standard deviation of x and σ_y is the standard deviation of y.

Adding more units to a portfolio may or may not reduce its risk. If the correlation between the units is one, then there is no reduction in risk per unit volume from adding more of the units. In this case the standard deviation of the sum will equal the sum of the standard deviations, and when this is normalized by dividing by the mean of the portfolio, the risk per unit is unchanged. In investing, for instance, adding more shares of a given company's stock to one's portfolio will not reduce the portfolio's risk, since the shares added will be perfectly correlated with the shares the investor already owns.

If the correlation between the units is less than one, then there is a reduction in risk per unit volume from adding the units. Thus, if an investor adds to the portfolio shares of a company not already in it, the risk should decline since the correlation of the new stock with stocks in the

portfolio should be less than one. If the correlation is negative then there can be a significant reduction in risk.

An example of diversifiable risk from insurance is the random occurrence of losses — where the fortuitous amount of one claim does not influence the amount of another claim. An example of nondiversifiable risk from insurance is medical inflation, where a change in the cost of medical care will simultaneously effect the value of general liability and workers compensation reserves. Another example is parameter risk, where the mean (or other parameter) of a loss distribution is unknown. Here the uncertainty in the mean affects all losses included in the distribution.

The finance perspective:
The classical finance perspective, as reflected in such methods as CAPM and internal rate of return, posits that an investor is compensated only for that risk that is not diversifiable. Diversifiable risk is not rewarded in the financial markets, because an investor can eliminate this risk by holding a diversified portfolio of securities. The finance perspective quantifies nondiversifiable risk, which is also called systematic risk, by measuring the correlation of a security's return with the market's return. From the finance perspective, if an investor owns a sufficiently diversified portfolio of securities, the only portion of the securities' return that cannot be diversified away is due to its co-movement with the market. Thus, much of the finance literature tends to treat systematic risk and covariance with the stock market as synonymous, and ignores other possible approaches to defining and quantifying diversifiable risk.[18] For determining risk loads in insurance, this may translate into measuring the correlation between insurance companies' returns from underwriting and market returns on its shareholder's equity.

The actuarial perspective:
In determining risk loads, what has come to be known as the actuarial perspective, in general, looks at the contribution of a policy to the total risk of the enterprise. (Risk loads based on aggregate probability distribution reflect the actuarial perspective.) The contribution to total risk will have a component that is diversifiable (process risk) and a component that is nondiversifiable (parameter risk). For many lines, especially in large insurers, the component due to process risk will be small, however, due to the law of large numbers. The actuarial perspective views the nondiversifiable or parameter risk component as that portion of total uncertainty due to the enterprise's inability to accurately measure its true liability and expense costs. While parameter risk may sound analogous to systematic risk, as both are viewed by their users are nondiversifiable, they are different concepts. Systematic risk is measured by calculating correlations with market returns. Parameter risk, where quantified, is measured through the use of Bayesian statistics.

[18] Certain approaches, such as Arbitrage Pricing Theory allow factors other than beta to be used in the quantification of risk. Except for some very recent research work, these approaches have not influenced the finance-based methods used to compute risk loads in property and casualty insurance.

Section D - Methods of Estimating Risk Adjustments

The characterization of the finance approach as quantifying only nondiversifiable risk and the actuarial approach as including both diversifiable and nondiversifiable risk is an oversimplification. Stulz[19] points out that in the real world, total risk often matters, and costs incurred by companies to control total risk are rewarded in the financial markets and the failure to do so may be punished. For some kinds of insurance, such as catastrophe insurance, it could be difficult to find a market unless some kinds of "diversifiable" risk were rewarded. Property catastrophe risk is diversifiable in a perfect market, but the mechanisms for doing so are so costly that in practice it is only partially diversifiable. As in the case of formally nondiversifiable risk, the whole industry is in the same boat, so the market treats the risk as systematic and policyholders in catastrophe-exposed areas pay a risk premium for insurance coverage. If an efficient means of diversification were to arise, then that situation would change.

While the actuarial based methods often explicitly incorporate process (diversifiable) and parameter (nondiversifiable) risk components into the risk load formulas, some of the finance-based methods, such as internal rate of return, may implicitly incorporate this risk as part of the total return on equity required by an insurance company.

The discussion surrounding diversifiable versus nondiversifiable risk is still evolving. The reader should be aware that differing views exist as to whether only diversifiable, or both diversifiable and nondiversifiable risk should be included in risk adjustments. The reader should also be aware that there are also very different approaches to measuring the nondiversifiable component.

[19] Stulz, Rene, "Whats wrong with modern capital budgeting?", Address to the Eastern Finance Association, April, 1999

Section D - Methods of Estimating Risk Adjustments

Method 1. The CAPM Approach

(Note: references to specific authors mentioned below and in the discussion of subsequent methods can be found in the Appendix.)

CAPM is the method used in Massachusetts rate filings in the Automobile and Workers Compensation lines. Myers and Cohn developed the underlying theory.

The method equates the present value of the premium to be charged on a policy to the present value of the losses plus the present value of the underwriting profits tax plus the present value of the tax on invested surplus and premium.

$$PV(P) = PV(L) + PV(UWPT) + PV(IT),$$

where P = Premium, net of underwriting expenses
L = losses plus loss adjustment expenses
$UWPT$ = underwriting profits tax
IT = tax on investments

Losses are discounted at a risk-adjusted rate. The premium portion of underwriting profits is discounted at a risk-free rate and the liability portion is discounted at a risk-adjusted rate. Investment tax is discounted at the risk-free rate. The risk-adjusted rate used in the calculations is derived from CAPM.

$$r_L = r_f + b_L(r_m - r_f)$$

where r_L = risk-adjusted rate
r_f = one period risk-free rate
b_L = Cov(r_L,r_m)/Var(r_m) = the liability or underwriting beta
r_m = expected rate of return on market portfolio

b_L, the underwriting beta, is a measure of the covariance between the underwriting profits for a line of business and the stock market. It represents the systematic risk to the insurer for writing the policy. Note that β_L is usually considered to be negative. Otherwise insurance companies would incur exposure to risk for a reward equal to or less than the risk-free rate, an illogical conclusion.

Although the Myers-Cohn approach is typically applied in ratemaking to compute risk adjusted premiums for new policies, the risk-adjusted discount rate from the calculation can be used to discount outstanding reserve liabilities as well.

Section D - Methods of Estimating Risk Adjustments

There are at least three approaches to computing b_L. The first method is broadly similar to the direct estimation technique (Method 7 of this section). Here, a time series of publicly traded insurer data is analyzed. A beta of equities is determined from insurance company stock prices. A beta of assets is determined from a weighted average of insurance company asset betas. The liability beta is determined by subtracting the asset and equity betas, weighted by their respective leverage values. The risk margin, as a reduction of the risk-free rate, equals the liability beta times the market risk premium. This is the method used in Massachusetts.

The second method uses accounting data to measure the covariance between insurance underwriting returns and the market.[20] A third CAPM-based approach measures beta for a line of business by quantifying the covariance of that line's underwriting return with the return for all property and casualty lines.[21]

A numerical illustration of the method is shown in the Appendix.

Advantages

- The method has actually been done. In Massachusetts it is the standard method used in the workers compensation and personal auto, with risk margins being positive and stable. Note that this has only been applied to lines that are relatively homogeneous, and where public data is generally available.
- The method is objective and the analysis is reproducible.
- The method has been in use for over a decade and has been reviewed by many economists.

Disadvantages

- Several stages of estimation can produce measurement errors.
  a) Some insurers in the data are also life insurers; carving them out requires estimating the equity beta of the life operation.
  b) The liabilities may be under- or overstated in the financial statements.
  c) Mutual insurers, nonpublic companies, self insurers and captives are not included in the analysis, introducing a potential bias
- Intangible assets like franchise value could distort the results. Another similar problem is that the present value of income taxes is embedded in the liability value and cannot be easily separated from it.
- Measurement errors on the beta for assets have a leveraged effect on the measurement of underwriting betas.
- It relies on the CAPM model, which may not accurately predict returns for insurance firms, as discussed below.

[20] Kozik, Thomas, "Underwriting Betas-The Shadows of Ghosts," Proceedings of the Casualty Actuarial Society (PCAS) LXXXI, 1994, pp. 303-329.
[21] Feldblum, Shalom, "Risk Load for Insurers", PCAS LXXVII, 1990, pp. 160- 195

Section D - Methods of Estimating Risk Adjustments

The CAPM beta has come under considerable criticism recently in the finance literature. CAPM only recognizes nondiversifiable risk, assuming an efficient, friction-free market. The magnitude of transaction costs to diversify an insurance portfolio violates the friction-free assumption, casting doubt as to the applicability of CAPM to valuing insurance liabilities.

Fama and French have shown that factors other than beta contribute significantly to the explanation of company stock returns.[22] Their work has caused a great deal of discussion in the finance community about the use of CAPM and beta for estimating equity returns and computing cost of capital. Alternatives to CAPM that look CAPM-like but incorporate factors other than beta into the determination of the risk premium have attempted to address some of the deficiencies of the CAPM model. For instance, Fama and French have presented a method for deriving costs of equity that uses two additional factors as well as beta.[23] Some of the models that appear to be generalizations of CAPM and use factors other than beta are better known as examples of the Arbitrage Pricing Model. An introduction to this more general approach is provided by D'Arcy and Doherty.[24]

Members of the actuarial community (as opposed to members of the finance community) have also criticized CAPM approaches. Much of the criticism focuses on the unreliability of estimates of underwriting betas as opposed to estimates of equity betas examined by Fama and French. Kozik[25] notes that a number of authors have measured the underwriting beta to be zero or negative (i.e., no risk load necessary on insurance). He provides a detailed discussion of the flaws in current methods of measurements of the underwriting beta, which can cause such results to be obtained.

Note that much of the underlying theory of CAPM is widely used and accepted, although the actual mechanisms for measurement have been criticized. Some of the criticisms of CAPM have been addressed in extensions of CAPM such as contained in the Automobile Insurance Bureau's Massachusetts Rate Filing (1998). Extending CAPM to address some of its limitations is currently an area of active research.

It should be noted that many of the limitations of the CAPM approach may apply to other methods presented in this paper, whenever those methods use CAPM to determine a rate of return.

---

[22] Fama, Eugene and French, Kenneth, "The Cross Section of Expected Stock Returns" Journal of Finance, Vol 47, 1992, pp. 427-465

[23] Fama, Eugene and French, Kenneth, "The Cross Section of Expected Stock Returns" Journal of Finance, Vol 47, 1992, pp. 427-465

[24] D'Arcy, S. P., and Doherty, N. A., "The Financial Theory of Pricing Property-Liability Insurance Contracts," Huebner Foundation, 1988

[25] Kozik, Thomas, "Underwriting Betas-The Shadows of Ghosts," Proceedings of the Casualty Actuarial Society (PCAS) LXXXI, 1994, pp. 303-329.

Section D - Methods of Estimating Risk Adjustments

The Pricing-Based Methods (Methods 2 and 3)

Under this general category of methods, the fair premium for a group of polices (which could be those of a line of business or an entire company) is first determined. In this calculation, the value of all nonliability premium components (such as commissions and general expenses) is excluded from the fair premium calculation. The resulting premium amount, by definition, is the fair value of the liability (losses and loss adjustment expenses). Since the liability fair value and its expected payments are known, the implicit risk-adjusted interest rate at which the payments are discounted can be readily found. Subtracting this value from the risk-free rate gives an estimate of the risk adjustment to the risk-free rate. Note that this approach can be used to compute a dollar-value risk load (to apply to liabilities discounted at the risk-free rate) rather than an adjustment to the discount rate.

This method can be applied to any prospective pricing model that uses expected cash flows. The most prevalent cash flow approaches are the internal rate of return (IRR) and the risk-adjusted discount (RAD) models.

It should be noted that the standard pricing-based methods give a risk margin that is a composite of the risk characteristics of liabilities already incurred and the unexpired policy liability. As the time since policy issuance increases, there may be a significant information gain in a book of liabilities (e.g., the insurer knows more about claims once they are reported) This effect is most pronounced for property insurance with significant catastrophe potential. To separately measure the risk margins in the reserve and unexpired policy portions of the insurer's liabilities, the pricing methods can be modified. For example, in the IRR model, the capital requirement and/or the required ROE may be different per unit of liability for the two liability types.

Method 2 - The IRR method

The IRR method is used by the NCCI in workers compensation rate filings.[26] It does not directly produce a risk margin, but it can easily be adapted to do so. The underlying theory is standard capital budgeting.[27]

Under the IRR method, a cohort of policies, written at the same time, is modeled over time until all claim payments are made. At each stage (usually quarterly or annually) the cash flows (premiums, losses, expenses, income taxes and investment returns) and balance sheet values are estimated. Capital is added based on capital allocation rules, frequently as a fixed proportion to liabilities. The application of these capital allocation rules results in an initial amount of capital,

[26] Cummins, J. David, "Multi-Period Discounted Cash Flow Ratemaking Models in Property-Liability Insurance," Journal of Risk and Insurance, March 1990, Volume 57:1, pp. 79-109.
[27] Brealy, Richard A. and Stuart C. Myers, 1996, "Principles of Corporate Finance (5th Edition)", McGraw-Hill, New York

then a subsequent capital flow, based on the amount of additional or withdrawal of capital necessary to maintain the capital allocation assumption at each point in the policy flows.

When the internal rate of return on the capital contributions and withdrawals equals the required rate of return on the capital (equity), then the fair premium is obtained.

The inputs to the IRR method are the capital allocation rules (e.g., the required amount of equity per unit of liability), the expected payments pattern of the policy flows, the investment return on cash flows, the income tax rate and the required return on equity. Note that the expenses and the premium cash flows need not be included in this calculation, since we are only trying to value the liability itself.

The required ROE can be determined using a variety of approaches. A simple approach often used by insurance companies is to select a rate of return based on examining actual historical rates of return on equity for insurance companies. Roth advocates this approach.[28] Another approach is to use CAPM to estimate the industry-average insurer equity beta and then to derive the appropriate ROE, given beta. An alternative way to estimate the required ROE is to use the dividend growth model, which has been documented in rate filings. Still another approach might use the "hurdle rate" for an insurer that is derived from its experience raising capital.

The required capital could be based on the company's internal capital allocation rules. Absent this, industry-wide "rules of thumb" or rating agency dictated norms might be used. Note that the capital typically used in this calculation is "required" or "target" capital, not actual capital. Care must be taken where the capital allocation assumption is dependent on the required ROE assumption.

An additional complication arises where fair value rules require the use of "market assumptions" wherever possible, over individual company assumptions. This could imply that the capital allocation rules that drive the market price (if one can be said to exist) should be used instead of the company's own internal capital assumptions.

The investment return under a fair value paradigm typically is the set of currently available market yields for investments. This may be complicated by investment in tax-exempt investments, especially where the company has significant tax advantages or disadvantages relative to the market. Many users of IRR models make the simplifying assumption that all investments are made in taxable securities.

A numerical illustration of the method is shown in the Appendix.

[28] Roth, R., "Analysis of Surplus and Rates of Return Using Leverage Ratios,", 1992 Casualty Actuarial Society Discussion Paper Program - Insurer Financial Solvency, Volume 1, pp 439-464

Advantages

- The IRR is commonly used to price insurance products. The extension to calculate risk margins is straightforward and will produce positive and stable risk margins.
- The method is conceptually simple and easy to explain.
- The method is objective and the analysis is reproducible.
- The method will work at the individual insurer level.

Disadvantages

- All of the methods for determining the required return on equity have problems and they can produce different answers:
  a) A required ROE based on historical returns depends on the historical period chosen.
  b) A required ROE based on CAPM is subject to the limitations and criticisms that apply to CAPM (see Method # 1 above).
  c) The dividend growth method requires some subjective estimation — it will not work for companies with erratic or no dividends.
  d) Internal management "hurdle" rates, based on a company's experience in raising capital, are very subjective and may not be consistent with the market value approach under fair value.
- The number of steps required makes this a fairly indirect method.
- Estimating the present value of income taxes requires a modification to the method.
- A required capital estimate is needed. There is no agreed upon method for doing this, and no consensus as to whether it should be the company's or the industry's capital allocation or requirement.

Method 3 - The Single-Period RAD (Risk-Adjusted Discount) method

This method shares some features of the above IRR method. It is based on the risk-adjusted discount method.[29,30] Here the relationship between the required ROE, the expected investment return, the income tax rate and the capital ratio is used to find the implied risk-adjusted interest rate. Like the above IRR method, the balance sheet values are fair value quantities. It is simpler than the IRR model since the risk adjustment is derived directly from a formula (shown in the Appendix), rather than by an iterative process.

The inputs to the single-period RAD method are the required amount of equity, the investment return on cash flows, the risk-free rate, the effective income tax rate and the required return on

[29] Butsic, Robert, "Determining the Proper Discount Rate for Loss Reserve Discounting: An Economic Approach," 1988 Casualty Actuarial Society Discussion Paper Program - Evaluating Insurance Company Liabilities, pp. 147-188.
[30] D'Arcy, Stephen P., 1988, "Use of the CAPM to Discount Property-Liability Loss Reserves", Journal of Risk and Insurance, September 1988, Volume 55:3, pp. 481-490.

equity. The required ROE can be determined using one of the methods described above for the IRR approach. The required capital and the investment return are estimated using historical industry data, or from one of the alternative methods described above for the IRR approach. Note that the required capital needs to be consistent with the fair value of the liabilities. For example, if the fair value of reserves were less than a non-fair value such as ultimate undiscounted liabilities, the required capital would go up.

The simplicity of this method arises from the assumption that the risk adjustment (as a reduction to the risk-free rate) is uniform over time. Thus, evaluating an insurance contract over a single period will be sufficient to determine the risk adjustment. To illustrate the method, we assume the following:

- capital is 50% of liability fair value,
- required ROE is 13%,
- expected investment return (EIR) is 7%,
- risk-free rate (RFR) is 6%,
- income tax rate is zero, and
- fair value for the liability is $100 at time zero.

The formula for the risk adjustment is:

risk adjustment = capital ratio x (ROE – EIR) + RFR – EIR

= 0.02 = 0.5 x (0.13 – 0.07) + 0.06 – 0.07

The formula for the resulting risk-adjusted interest rate is:

risk-adjusted interest rate = RFR - risk adjustment

= 0.04 = 0.06 - 0.02

To see that this works, note that the beginning assets are the fair premium for the liability of $100 plus the required capital of $50. This amount grows to $160.50 (i.e., $150 x 1.07) at the end of the year. The expected amount of liability grows at the risk-adjusted rate of 4% to $104. Subtracting this amount from assets gives $56.50, which represents the required 13% return (56.5 / 50 = 1.13) .

The income tax rate, however, is not zero, so the formula for the risk adjustment (see the Appendix) is somewhat more complicated than shown here. The Appendix provides the complete formula and also gives a numerical illustration of the method.

Advantages

- The method is very simple and transparent. It is easy to explain and to demonstrate with a spreadsheet.
- The method is reliable, robust and will produce positive and stable risk margins.
- Inputs are presently available from published sources. For example, many rate filings with state insurance departments have estimates for required ROE and capital leverage.

Disadvantages

- The method will only produce an industry-average or company-average risk adjustment (to the risk-free rate). It would be difficult to apply the method to produce specific lines of business risk adjustments.
- This method has the same disadvantages relative to the selected ROE as the IRR method.
- This method has the same disadvantages relative to the selected "required capital" as the IRR method.

Section D - Methods of Estimating Risk Adjustments

Method 4 - Methods Based on Underwriting Data

A pragmatic approach to developing liability risk adjustments is to use published underwriting data. Over a sufficiently long period of time companies are assumed to earn enough in profit on the policies they write to be adequately compensated for the risk they bear. This method assumes that the historical returns indicate the true market perception of the fair profit for bearing insurance risk. The historic profit or risk load can then be related to the risk adjustment required for discounted liabilities.

Typically, risk adjustments based on underwriting data use information published in insurance companies' annual statements. To obtain stable results by line of business applicable to a typical company, data aggregated to industry level by sources such as A. M. Best can be used.

The published literature on risk adjustments using underwriting data primarily focuses on estimating a risk adjustment to the factor used to discount liabilities. Alternative methods for computing risk-adjusted discount rates use a CAPM approach to compute the risk adjustment.

Although we focus on using underwriting data to compute risk-adjusted discount rates, the same data can be used to derive an additive risk load instead.[31] Risk adjustments incorporated through the discount rate are discussed first, followed by discussion of risk adjustment via an additive risk load.

Using Underwriting Data to Adjust the Discount Rate

Butsic introduced the concept of using risk adjusted discount rates to discount insurance liabilities.[32] He argued that a liability whose value is certain should be discounted at a risk free rate. The appropriate risk free rate to use for the certain liabilities is the spot rate for maturities equal to the duration of the liabilities. If certain liabilities are discounted at the risk free rate, then uncertain liabilities should be discounted at a rate below the risk free rate. The formula for the risk-adjusted rate is:

[31] There are several different ways to make a risk adjustment. One way is through an additive risk load to the otherwise calculated present value estimate (based on risk-free discount rates). A second is by discounting the expected cash flows using a risk-adjusted discount rate. A third is by adjusting the individual expected cash flow amounts for each time period, replacing each uncertain amount with the certainty equivalent amount (i.e. the fixed amount for which the market would be indifferent between it and the uncertain amount being estimated.) A fourth is by adjusting the timing of the estimated cash flows (sometimes used when timing risk is thought to dominate amount risk).

[32] Butsic, Robert, "Determining the Proper Discount Rate for Loss Reserve Discounting: An Economic Approach," 1988 Casualty Actuarial Society Discussion Paper Program - Evaluating Insurance Company Liabilities, pp. 147-188.

Section D - Methods of Estimating Risk Adjustments

$$i_L = i - e\,(R - i),$$

where i_L = the risk-adjusted discount rate for liabilities,
i = the risk free rate for duration equal to the duration of the liabilities,
e = a leverage factor, equal to surplus divided by the present value of liabilities,
$(R - i)$ = the market risk premium, i.e., the excess of the market's return over the risk-free rate. The market return is usually measured as the return on a stock market index such as the S&P 500 or the return for all NYSE stocks, but other interpretations are possible.

The above term "$e\,(R - i)$" represents the adjustment to the risk free rate for the riskiness of the liabilities.

There is an analogy between this formula and that for a company's cost of equity based on the CAPM.

$$i_E = i + b_c\,(R - i)$$

where i_E = the cost of equity for a company,
i = the risk-free rate,
b_c = the company's beta, based on the covariance between the return on the company's stock and the market's return,

The specific procedure for computing the adjustment is described in detail in the Appendix.

Note that the method's results can be very sensitive to the historical time period used as the source of the underwriting data. For example, the selection of an historical period that includes a major market disruption, such as a workers' compensation crisis, major catastrophe, or mass tort eruption, can produce drastically different indications than a time period that excluded this major disruption. Thus, it is necessary to consider how long a time period is required to obtain stable and reasonable results and whether the method is unstable over time. The longer the historical period used for computing the risk adjustment, the more stable the results will be, but the less likely they are to reflect current trends in the underwriting cycle or business environment. The shorter the historical period used, the more likely it is that the adjustment will reflect the current environment, but at a cost of being more unstable and more susceptible to infrequent random events such as catastrophes (or the short-term absence of the long-term catastrophe or large loss risk).

An additional effect that must be considered is the effect of taxes. As shown by Myers and Cohn[33]

[33] Myers, S and Cohn, R, "A Discounted Cash Flow Approach to Property-Liability Rate Regulation," Fair Rate of Return in Property-Liability Insurance, Cummins, J.D., Harrington S.A., Eds, Kluwer-Nijhoff Publishing, 1987, pp. 55-78

and Butsic[34], taxes increase the premium needed to obtain a target rate of return and therefore decrease the effective risk-adjusted discount rate. This effect is embedded in the data used to derive the risk-adjusted discount rate. It might be desirable to segregate this effect from the pure risk adjustment. A procedure for doing this is discussed in the Appendix.

Advantages

- The approach produces an adjustment to the discount rate without requiring the computation of a liability beta. As discussed above in the CAPM method for estimating a risk adjustment, the liability beta is one of the more controversial features of the CAPM approach.
- The approach does not require the computation of a leverage ratio
- The approach is relatively easy to implement. Spreadsheets can be placed on a web site containing a sample calculation
- The data required, such as Bests Aggregates and Averages, is relatively inexpensive and readily available
- A paper presenting the approach has been included in the syllabus of the Casualty Actuarial Society for over 10 years. A description of this technique is, therefore, readily accessible to actuaries (or anyone else who accesses the CAS web site.)
- This method can easily be applied to individual lines where annual statement data is available.

Disadvantages

- Results can be very different depending on the historical time period used. This committee's research indicates that changing the time period used for the calculation in one instance changed the all-lines risk adjustment from 4.5% to 1.0%. The committee believes that the results for recent historical periods reflect certain well-known market disruptions such as the impact of the recognition of asbestos and environmental liabilities. Also, the industry has been in a protracted soft market, which has depressed underwriting profitability in the recent historical data.
- Results for a single line can be unstable. Some lines are unprofitable for extended periods of time and this method may not produce a positive risk load. Useful data for lines with very long tails (or without industry data available) may be a problem. Examples of such include medical malpractice-occurrence and directors & officers (D&O, for which industry accident year data may not be available).
- Pricing adequacy may vary by line based upon individual line characteristics such as regulatory environment, market conditions, geography, etc. An impact of this is cross subsidization of lines where some lines are undercharges at the expenses of other lines. Thus the results for a single line, even over relatively long time periods can be misleading. (Our research showed that at least one regulated line had a negative risk adjustment using this approach for 30 years.)

[34] Butsic, Robert P., 2000, Treatment of Income Taxes in Present Value Models of Property-Liability Insurance, Unpublished Working Paper.

- Results will be affected by "smoothing" in published financial numbers.
- The method requires accident year data to do the computation correctly, or else it is susceptible to distortion from events with long-term latency issues, such as mass torts or construction defect.
- Results using individual company data may be too volatile, hence, the method has usually been applied mostly to industry data.

Computing Additive Risk Loads Instead of Risk Adjustments to the Discount Rate

Since the procedures described here focuses on computing a risk adjustment to the discount rate, the procedure to compute an additive, dollar-value risk load must convert the risk-adjusted rate into a risk load (as a ratio to the liability value). However, it is possible to compute the risk load directly using the same data for computing a risk adjustment to the discount rate. This approach might be preferred for a short tail line.

One approach to computing an additive risk load is simply to calculate the ratio of the profit on the policies at the beginning of the period to the average discounted losses, where losses are discounted at a risk-free rate rather than a risky rate. Thus, the risk load (expressed as a percentage of the present value losses) is equal to the present value of the premiums minus the present value of expenses minus the present value of the losses (plus loss adjustment expenses) divided by the present value of the losses. All quantities are discounted at the risk-free rate.

Unlike the adjustment to the discount rate, this risk load would not be meaningful unless computed by line, since the duration of the liabilities varies by line. An example of this computation is shown in the Appendix.

Section D - Methods of Estimating Risk Adjustments

Method 5 - Actuarial Distribution-Based Risk Loads[35]

The evolution of this approach relative to pricing is given first, followed by the extension to the valuation of liabilities.

Pricing context

Probability-based actuarial risk loads are among the oldest procedures developed by actuaries for estimating the risk adjustment to losses. These approaches continue to develop, even as other approaches, which largely evolved from other disciplines (such as economics and finance), continue to add to the tools used for deriving risk loads. Distribution based loads arose in the context of insurance pricing to fill the perceived need to apportion the targeted underwriting profit to different classes of business according to their actual riskiness, as described mathematically by the probability distribution of the loss.

The first approaches to the problem focused on the volatility of the individual loss, characterized mainly by the severity distribution. In 1970, Hans Bühlmann set forth three possible principles that might be applied to the problem:

- The Standard Deviation Principle: Risk Load = λ SD[Loss],
- The Variance Principle: Risk Load = λ Var[Loss],
- The Utility Principle: U(Equity) = E[U(Equity + Premium - Loss)].

Actuarial distribution-based risk loads often invoke collective risk theory to explain the derivation of the risk load. Collective risk theory provides a model of the insurance loss generating process that can be used to derive aggregate probability distributions. The theory also allows derivation of the distribution parameters such as standard deviations or variances, which are used in the risk load formulas. Recent developments in collective risk theory have given rise to an additional principle used to derive risk loads:

- The expected policyholder deficit (EPD[36]) principle: Risk Load = λ Surplus Requirement.

Surplus is determined based on the expected policyholder deficit, which is derived from the

[35] This exposition draws heavily on Glenn Meyers' September 18, 1998 presentation to Casualty Actuaries of New England (CANE).

[36] The "expected policyholder deficit" is the total expected level of uncompensated losses over the total expected level of all losses, for a given level of assets (reserves plus surplus) supporting a risk. For example, assume 99% of the time losses are only $1, 1% of the time they are $100, and the total level of assets supporting this risk is $90. Then expected uncompensated losses are $0.10. Total expected losses are $1.99. The expected policyholder deficit is 0.10/1.99, or around 5%. For further discussion of this concept, see " Solvency Measurement for Property-Liability Risk-Based Capital Applications" By Robert P. Butsic, published in the 1992 CAS discussion paper program titled "Insurer Financial Solvency".

aggregate probability distribution of either losses or surplus (assets minus losses). This principle is very similar to the tail-value-at-risk principle proposed by Meyers.[37]

Each of the above principles contains an arbitrary coefficient λ, constant across classes of business (and concealed in the utility function), that can be adjusted to yield the desired overall underwriting profit or rate of return on surplus. In much of the literature the time element is not addressed explicitly. It is straightforward, however to apply the risk load to discounted liabilities.

The first two of the principles were applied in the practical context of increased limits ratemaking at the Insurance Services Office (ISO) in the late seventies and early eighties.

During the eighties, regulatory pressures brought the Capital Asset Pricing Model (CAPM) into the debate regarding how to incorporate risk into insurance prices. CAPM is founded on certain axioms that are violated in the context of insurance pricing (e.g., no default, frictionless markets), but this intrusion of modern financial theory stimulated much thought as to how the risk load formalism can address enterprise-wide and market-wide issues that had been neglected in the earlier formulations. The concept of systematic risk, already familiar to actuaries as parameter risk, was incorporated into practical treatments intended for actual insurance pricing.

The Competitive Market Equilibrium approach to risk load incorporates parameter uncertainty and other mechanisms, which generate correlations among distinct insurance contracts (e.g., the catastrophe mechanism, which can affect many contracts, in different lines of insurance, in a single event).[38] This scheme attempts to integrate capital market theory and collective risk theory in the development of risk loads for insurance pricing. The procedure requires all parties to agree that more variance is worse and less is better. (Note that the CAPM disagrees. It treats variance not related to the market as not valued by the market and not a concern, as it can be diversified away. It assumes no transaction cost to do so.)

The answer given by this scheme gives a contract risk loading proportional to the change in the variance of the insurer's bottom line caused by the addition of that one contract to the insurer's portfolio. This raised an interesting parallel with work being done at about the same time on reinsurance pricing based on marginal surplus requirements.[39] The Competitive Market Equilibrium result can be re-expressed in terms of the marginal surplus (risk capital) required to support the additional business, and thus linked to the cost of risk capital. More recent work using probability distributions has referenced the expected policyholder deficit concept, rather than standard deviation, variance or probability of ruin to motivate the computation of marginal

[37] Meyers, Glenn, "The Cost of Financing Insurance", paper presented to the NAIC's Insurance Securitization Working Group at the March 2000 NAIC quarterly meeting.
[38] Meyers, Glenn G., "The Competitive Market Equilibrium Risk Load Formula for Increased Limits Ratemaking," Proceedings of the Casualty Actuarial Society (PCAS), LXXVIII, 1991
[39] Kreps, Rodney E., "Reinsurer Risk Loads from Marginal Surplus Requirements," Proceedings of the Casualty Actuarial Society (PCAS), LXXVII, 1990, p. 196

surplus requirement and, therefore, of risk load.[40, 41]

Extension to Loss and Expense Reserves

The above methods apply prospectively to situations where the losses have not yet taken place and only rating information is available. For risk-adjusted valuation of insurance liabilities, such methods would apply to the Unearned Premium Reserve (UPR) and Incurred But Not Reported Reserves (IBNR). As long as one has some kind of runoff schedule giving estimates of number and type of claims not yet reported, one can apply these methods to estimate the variability of unreported claims.

Estimating the variability of reported claims is a different problem because of the information available to the insurance company about actual reported claims. Meyers has addressed the problem in the context of reserving for workers' compensation pensions, using a parametric model for the mortality table and calculating the variance of conditional future payments.[42] Hayne has used the collective risk model with information about claim counts and severities as the claim cohort ages and assumptions as to distributions and correlation structures to estimate the distribution of outstanding losses.[43] Heckman has applied distribution and regression techniques to estimating the expected ultimate value of claims already reported and of IBNR claims.[44] For the two latter methods, the conditional loss distribution provides the information needed to calculate risk loads for the reserves.

There are some unsolved problems associated with approaches based on probability distributions. Research is in progress to develop methods for measuring correlations of lines or segments of the business with other segments, but there is no generally accepted approach for incorporating correlations into the measure of risk. This is believed to be important, as these correlations may make a significant contribution to, and in some cases may reduce overall risk. In addition, some of the risk load procedures such as those based on standard deviation and variance approaches are not value additive. That is, the risk load of the sum is not equal to the sum of the risk loads.

Advantages

- Actuaries have used the approaches for a long time to compute risk loads.

[40] Meyers, Glenn, "The Cost of Financing Insurance", paper presented to the NAIC's Insurance Securitization Working Group at the March 2000 NAIC quarterly meeting.
[41] Philbrick, Stephen W., "Accounting for Risk Margins," Casualty Actuarial Society Forum, Spring 1994, Volume 1, pp. 1-87.
[42] Meyers, Glenn G., "Risk Theoretic Issues in Loss Reserving: The Case of Workers Compensation Pension Reserves," Proceedings of the Casualty Actuarial Society (PCAS), LXXVI, 1989, p. 171
[43] Hayne, Roger M., "Application of Collective Risk Theory to Estimate Variability in Loss Reserves," Proceedings of the Casualty Actuarial Society (PCAS), LXXVI, 1989, p. 77-110
[44] Heckman, Philip, "Seriatim, Claim Valuation from Detailed Process Models," paper presented at Casualty Loss Reserve Seminar, 1999.

- This is an area of active research with many worked out examples of how the method can be applied.
- The method is intuitive: risk load is related to actual risk for a body of liabilities.
- The data required to compute the risk loads is readily available within many insurance companies and many actuaries are qualified to perform the computation.
- Many reserving actuaries are familiar with using aggregate loss probabilities to establish confidence intervals around their reserve estimates.
- This method can be used with company-specific data.
- This method can be used by line to reflect unique line of business risks.

Disadvantages

- The approaches have often been criticized as being inconsistent with modern financial theory, as classically formulated, relative to compensation for diversifiable risk. For example, the risk loads often fail to satisfy the *one-price rule*, whereby two insurers offering identical insurance coverage would charge the same price.
- Sometimes the weight given to process risk relative to parameter risk in determining the risk load can appear to be too large. Many researchers and practitioners believe that risk loads apply only to nondiversifiable (parameter or systematic) risk not to unique (or process) risk. It should be noted that it is not universally accepted that only diversifiable risk matters when computing risk loads.[45,46]
- The risk loads may not satisfy value additivity. As a result, two companies with identical lines but a different mix can have different risk margins (see discussion below).
- A large number of methods for doing these calculations exist, yielding a variety of results. There is little guidance regarding which of the available methods is appropriate for a given set of circumstances.
- Certain parameters are not only subjective, but there is little guidance on how to calibrate them. For instance, only the more recent papers discuss a conceptual framework for selecting λ.
- Parameters are often determined in a subjective manner and may therefore be inaccurate.
- Actuaries are still struggling with measuring the correlations between lines of business. This may be a significant source of risk to companies.

Note that the lack of value additivity is not universally accepted as a disadvantage. For example, some believe there is much less risk in a $1 million (undiscounted) share of a large company's auto liability reserves than in the entire $1 million in undiscounted auto liability reserves for a small regional insurer. Thus, the former may be worth more than the latter (i.e., valued with a smaller risk margin).

[45] Cornell, Bradford, "Risk. Duration and Capital Budgeting: New Evidence on Some Old Questions", Journal of Business, 1999 vol 72, pp 183-200.
[46] Stulz, Rene, "Whats wrong with modern capital budgeting?", Address to the Eastern Finance Association, April, 1999

Section D - Methods of Estimating Risk Adjustments

Section D - Methods of Estimating Risk Adjustments

Method 6 - Using the reinsurance market to estimate the fair value of liabilities

The reinsurance market offers the most direct approach to estimating the fair value of an insurance company's liabilities. Blocks of liabilities are often sold either on a retrospective basis, in transactions such as loss portfolio transfers, or on a prospective basis in more commonly purchased excess of loss treaties. The price structures associated with these contracts provide another glimpse of the implicit risk load required to record the liabilities at their fair value.

Reinsurance prices may require some adjustment before they could be used to estimate the fair value of liabilities. For example, market prices offered by some reinsurers reflect an embedded option value equal to the value of their default on their liabilities. Such market prices would have to be adjusted upward to remove this default value. Another example is portfolio transfers that include customer lists or renewal rights. The effect of these lists or rights on the total price would have to be isolated and removed before the portfolio transfer price could be used for a fair value estimate.

There are numerous practical issues that need to be addressed before the method can be implemented in practice. For example, how would a ceding company measure the risk loading in the reinsurer's price structure? How could the analysis of a particular treaty structured to reinsure a portion of the company's liability be generalized to estimate the fair value of all its liabilities? Possible approaches are:

- Reinsurance Surveys: On a regular basis, leading companies can be surveyed to evaluate the risk loading implicit in their reinsurance structure. The survey can be structured to discriminate between various lines of insurance and sizes of ceding companies. The implicit risk loading can then be published and employed by all companies with a particular set of attributes (size, type of business, balance sheet leverage, etc.). Note that this is a controversial suggestion. (*Asking companies to share loss information is one thing. Asking them to share pricing information is something else entirely. First, the pricing "assumption" may not be as objective an item as a loss amount. It may be a gut call that varies by sale. Second, there are many more antitrust issues in sharing pricing information than in sharing loss information.*)

Conceptually, this would operate similarly to the PCS Catastrophe Options currently offered by the Chicago Board of Trade. These options are priced based on an index, which is constructed in the following way:

> *"A survey of companies, agents, and adjusters is one part of the estimating process. PCS conducts confidential surveys of at least 70% of the market based on premium-written market share. PCS then develops a composite of individual loss and claim estimates reported by these sources. Using both actual and projected claim figures, PCS extrapolates to a total industry estimate by comparing this information to market share data."* [47]

- Extrapolating from a company's own reinsurance program: Companies that submit their reinsurance programs to bid will receive reinsurance market price information from a number of providers. At a minimum, even the information contained in one well-documented bid may be sufficient to compare the reinsurer's price to the ceding company's best estimate of the ceded liabilities discounted at the risk-free rate. In practice, a number of adjustments to this risk load may be appropriate. For example, if the only reinsurance purchased is high layer excess, then the risk loading will be commensurate with the increased risk associated with that layer. Publicly available increased limits tables (e.g., ISO) might be suitable in some cases to evaluate the relative risk at each layer of coverage. An insurer's policy limits profile can then be employed to evaluate the weighted total limits of their liability portfolio and the resulting risk load.

Advantages

- The reinsurance market is the closest structure to a liquid market for insurance liabilities;
- Most insurers have access to the reinsurance market and can therefore gain information regarding their unique risk profile;
- Similar to catastrophe options, once the survey results are published, it would be relatively straightforward to estimate fair value

Disadvantages

- Results can be sensitive to capacity changes in the reinsurance market. As such, the values at any point in time may not represent future values. In fact, in highly competitive market cycles, a negative risk load could be obtained for some coverages.
- Unstable reinsurance prices also make it difficult to update estimates for each reporting period. If the information required for the fair value estimate could not be obtained quickly enough, all estimates would have to be recalculated each reporting period.
- The credit risk of the reinsurer's default on its obligation is embedded in the price. For reinsurance, this can be material, and would have to be removed, but the isolation of this item from the total price (and other risks) may be problematic.
- This approach would also raise difficulties in updating the values, as it would require

[47] Chicago Board of Trade web site: PCS Catastrophe Insurance Options – Frequently Asked Questions

regular surveys or continual shopping of ceded business to reset the risk charges.[48]

- Some reinsurance quotes are not transparent, so that the implied risk loading may be difficult to ascertain. Often, the insurer and reinsurer would each have different estimates of the expected loss and other components of price.
- The users of this method will only sample the reinsurance market. I.e., they will not be using the entire market for estimation. This could introduce bias.
- Reinsurance markets focus much more on prospective exposures rather than past exposures, partly due to current accounting treatment of most retroactive reinsurance contracts. As such, there are fewer market prices potentially available (and a much smaller market) for reinsurance of existing claim liabilities.
- Reinsurance prices embed antiselection bias. The price of reinsurance for the portion of an insurer's portfolio ceded may be higher than the price if all risks were ceded.

[48] Note that continual updates would be required under fair value accounting. This is because fair value accounting is meant to be an idealized market value, i.e., an actual market value if a sufficiently active market exists, or an estimate of what a fair market value would be otherwise. As such, a fair value estimate would have to be updated as often as an active market value would be updated. In general, market values in an active market change constantly.

Section D - Methods of Estimating Risk Adjustments

Method 7 - Direct estimation of market values

This is the method of Allen, Cummins and Phillips.[49] In this approach, a time series of publicly traded insurer data is analyzed. The output of the analysis is an estimate of the market value of each insurer's liabilities for each year of the history. The market value of liabilities is derived by subtracting the market value of the equity from the market value of total assets. The market value of equity is calculated by extending the method of Ronn and Verma to avoid the problem of including intangible asset values in the equity measurement.[50] Here, the equity value is determined so that the measured volatility of the insurer's stock price and of its asset values are consistent. This method is described in the section on measurement of credit risk The market value of assets is estimated from the separate asset categories, most of which are publicly traded.

The market value of liabilities thus obtained contains an embedded option value equal to the value of default on the liabilities. This value of the default can be separately determined by the of Ronn-Verma method.

Adding back the default value gives the market value of the liability as if there were no credit risk. Next, the nominal (undiscounted) value of the liability is compared to the no-default market value to determine the implied interest rate at which the nominal value is discounted to get the market value. This calculation requires an estimation of the payment pattern of the liabilities (also used in the above-average payment duration). The risk margin, as a reduction to the risk-free rate, is the difference between the risk-free rate and the implied rate underlying the market value.

A numerical illustration of the method is shown in the Appendix.

Advantages

- The method is theoretically sound. It produces a risk load consistent with modern financial theory without requiring the calculation of a beta.
- The method is objective and the analysis is reproducible
- The method is a type of direct measurement of liabilities that may be desirable by the accounting profession. However, the measurement is direct for the industry, but not for a particular company

Disadvantages

- There are difficulties with the estimation of parameters:
  a) Some insurers in the data are also life insurers, or involved in multiple lines not relevant to a particular company at issue; carving them out requires estimating the

[49] Allen, Franklin, J. David Cummins and Richard D. Phillips, 1998, "Financial Pricing of Insurance in a Multiple Line Insurance Company", Journal of Risk and Insurance, 1998, volume 65, pp. 597-636.
[50] Ronn, Ehun I., and Avinash K. Verma, 1986, Pricing Risk-Adjusted Deposit Insurance: An Option-Based Model, Journal of Finance, 41(4): 871-895.

market equity value of these other operations.

b) Some companies are members of financial conglomerates, or general conglomerates (e.g., General Electric).

c) Not all insurers are publicly traded. These include foreign companies, privately held companies and mutuals or reciprocals.

- The liabilities may be under- or overstated in the financial statements. Therefore, the market value may reflect an adjustment to the book value, based on market perceptive of this bias. Any perceived change in this bias may make prior history unusable.
- Measurement problems make it difficult to provide a stable estimate for individual line of business risk margins. It is also difficult to get a reliable estimate for an individual firm.
- Most actuaries don't have any experience with this method. It has not yet been used in practice.

Section D - Methods of Estimating Risk Adjustments

Method 8 - Distribution Transform Method

A number of authors have proposed risk-loading procedures based on transforming the aggregate loss probability distribution.[51] The risk-loaded losses are computed from the mean of the transformed distribution. A simple example of such a transform is the scale transform:

$$x \rightarrow kx$$

where x = the aggregate losses
$k > 1$

As a simple, but unrealistic example (because insurance losses tend to have positive skewness), x is a normal variable, that is, if aggregate losses follow a normal distribution and k is 1.1, then the loss distribution's expected mean is shifted upwards by 10%. Thus, a company purchasing the liabilities would require 10% above the present value of the liabilities (at a risk-free rate), in order to be adequately compensated for the riskiness of the liabilities. If one is using this distribution to compute primary losses for an exposure where the limits applied to losses in the aggregate, the expected mean would be increased by less than 10%, but losses excess of the primary limit will be increased by more than 10%.

In the more recent literature on the transform method the power transform is used.[52] (Other transforms such as the Esscher transform also appear in the literature). This approach raises the survival or tail probability to a power.

$$S^*(x) = S(x)^r$$

where $S(x)$ = the original survival distribution, $1-F(x)$, or 1 minus the cumulative probability distribution);
$S^*(x)$ = the transformed survival probability.

If r is between 0 and one, the tail probabilities will increase and the transformed distribution will have a higher mean than the original distribution.

The choice of the transformation parameter r is guided by the uncertainty of the business being

[51] Venter, Gary G., 1991, Premium Implications of Reinsurance Without Arbitrage, ASTIN Bulletin, 21 No. 2: 223-232. Also,
Wang, Shaun, 1998, Implementation of the PH-Transform in Ratemaking, [Presented at the Fall, 1998 meeting of the Casualty Actuarial Society]. Also,
Butsic, Robert P, 1999, Capital Allocation for Property Liability Insurers: A Catastrophe Reinsurance Application. Casualty Actuarial Society Forum, Fall 1999.

[52] Wang, Shaun, 1998, Implementation of the PH-Transform in Ratemaking, [Presented at the Fall, 1998 meeting of the Casualty Actuarial Society]. Also,
Venter, Gary G., 1998, (Discusssion of) Implementation of the PH-Transform in Ratemaking, [by Shaun Wang; presented at the Fall, 1998 meeting of the Casualty Actuarial Society]

priced. The greater the uncertainty, the lower *r* will be. In practice, this may mean that one calibrates the parameter by selecting a transformation that approximates current market premiums for a given class of exposures. Wang suggests that using a distribution transformation to derive risk loads is the equivalent of including a provision for parameter risk, but not process risk, into the formula for risk loads. Thus, one might select *r* based on subjective probabilities about the parameter uncertainty of the business.

Wang (1998) has suggested that one could apply this approach in two ways.[53] The first applies a transform separately to the frequency and severity distributions used to price policies. The second transforms the probability distribution of aggregate losses (i.e., the convolution of the frequency and severity distributions). However, Venter suggests that one could obtain inconsistent results when applying a transform to aggregate losses, and prefers working with the frequency and severity distributions.[54]

Option pricing theory and the distribution transform method are related. The parameters of the probability distributions used in the option pricing formulas typically reflect "risk neutral" probabilities, rather than real probabilities. Thus, for example, the parameters used to price interest rate options are generally derived from current actual prices of bonds of different maturities, or from the current yield curve, rather than from empirical time series data of the various interest rates. One could view the "risk neutral" probabilities as a transformation of the distribution for the underlying asset values.

Advantages

- The method produces a risk load consistent with modern financial theory without requiring the calculation of a beta. Risk loads are value additive. (Note again that there is not universal agreement among actuaries that risk loads should be value additive.) The approach is similar to that used in pricing options.
- The method is conceptually straightforward to understand and explain. Once r or a similar parameter has been selected, it can be reused subsequently.
- This approach is currently used in reinsurance pricing.
- It is theoretically viable for estimating risk loads by layer. Many of the other methods do not address layers or deductibles.
- It is an area of active research for those investigating risk load methodologies.

Disadvantages

- It is not in common use for producing prices or risk loads on primary business. Currently its primary use is in producing risk load for layers.

[53] Wang, Shaun, 1998, Implementation of the PH-Transform in Ratemaking, [Presented at the Fall, 1998 meeting of the Casualty Actuarial Society].
[54] Venter, Gary G., 1998, (Discusssion of) Implementation of the PH-Transform in Ratemaking, [by Shaun Wang; presented at the Fall, 1998 meeting of the Casualty Actuarial Society]

- As currently applied, in order to calibrate the parameters, it often requires knowledge of the risk loads on primary business.
- Because it is a new approach, actuaries are not as familiar with it as with some of the others presented in this paper.
- The parameters may be selected based on the analyst's experience with a particular line of business. This introduces an element of subjectivity, where different analysts may choose different values for the parameter.
- It is not clear which transform choice to use. Many of the transformation methods are chosen for their mathematical tractability, and are not supported with empirical evidence.

Section D - Methods of Estimating Risk Adjustments

Method 9 - The Rule-of-Thumb Method

The methods presented so far require that the person computing the risk-adjusted present value of liabilities do original analytical work. In some situations there may not be adequate data or other resources to develop the risk adjustment from scratch. In such situations it might be appropriate to use a rule of thumb that provides a "quick and dirty" way to derive a risk adjustment. Such methods would be relatively easy to apply but would produce broadly reasonable results. Examples of rules of thumb would be:

- Compute a risk adjusted discount rate by subtracting 3% from the risk-free rate.
- The risk load should be 10% of the present value of General Liability liabilities and 5% of the present value of Homeowners liabilities.

The numbers in the examples above are for illustrative purposes only. A separate body of actuaries and other experts could determine actual guideline values. This group would review existing research and perform additional studies where necessary. Quite likely, it would consolidate results from using one or more of the other methods in this document.

Advantages

- For the individual company, it would be simpler to apply than any of the other alternatives. It would reduce the work effort for actuaries and others, who would not have to separately develop risk adjustments.
- This approach may lead to industry standard risk adjustments being used, thus creating comparability from company to company.
- It may reduce the likelihood that a risk adjustment methodology can be used to manipulate a company's financial statements.

Disadvantages

- Fair values produced using this approach may be less accurate because the unique risk factors for a company may not be reflected.
- It precludes actuaries from applying methods that reflect new developments for determining risk adjustments.
- An industry body may be required to perform research to parameterize the risk adjustments. This may create antitrust issues. It is not clear that the industry body would be sufficiently authoritative for its research to be used in financial valuations.

Section D - Methods of Estimating Risk Adjustments

Method 10 - Alternative Methods

This paper has presented a number of possible approaches to estimating the fair value of insurance liabilities. Most of these approaches are rooted in analytical methods documented in the actuarial literature. However, research continues into how to determine risk adjustments. Not all current developments are covered in this paper and undoubtedly others will be published. A company may wish to use alternative approaches not presented in this paper. In such cases, there are a number of points one should consider:

- Once selected, the approaches should be used consistently. Changing approaches from year to year may result in inappropriate income statements.
- If the method is changed, it should be documented adequately.
- The risk margin should be positive.

Section D - Methods of Estimating Risk Adjustments

Converting a risk adjusted discount to an additive risk load

A number of the methods presented in this paper produce an adjustment to the risk-free discount rate. Risk adjusted present values of liabilities are then derived by discounted the liabilities using the risk-adjusted rate. An approach to deriving a dollar-value risk load is to work from the risk-adjusted discount rates. This approach might be used if one wanted to discount losses at the risk-free rate and apply the risk load to the losses directly. The procedure begins by discounting the liabilities at the risk-adjusted and the risk-free rate. It then computes the difference between the two discounted quantities. The risk load is this difference divided by the present value of the liabilities, discounted at the risk-free rate. The table below presents an example where this calculation is performed for liabilities of various durations, when the assumed risk-free rate and the risk adjustment remain constant.

Risk Free Rate:	6.0%		
Risk Adjustment	3.0%		
Duration	PV @ Risk-Free Rate	PV @ Risk-Adjusted Rate	Risk Load
1	94.3%	97.1%	2.8%
2	89.0%	94.3%	5.6%
3	84.0%	91.5%	8.3%
4	79.2%	88.8%	10.8%
5	74.7%	86.3%	13.4%
6	70.5%	83.7%	15.8%
7	66.5%	81.3%	18.2%
8	62.7%	78.9%	20.5%
9	59.2%	76.6%	22.8%
10	55.8%	74.4%	25.0%

Section D - Methods of Estimating Risk Adjustments

Unearned Premium (or Unexpired Policy) liability methods

As noted in the background section, a fair value accounting system focuses on the measurement of assets and liabilities, not income. As such, the current recording of unearned premium under U.S. GAAP accounting conventions would be replaced with the fair value of the business written but not yet earned. The methods used to estimate this fair value have much in common with the above methods that estimate the fair value of the liabilities for unpaid losses. However, additional methods may be applicable since it may be easier to discern the market prices underlying earned premium. One can argue that the booked premium represents the "market price" charged by the particular insurer.

One area where such additional methods may be needed is property insurance, particularly where catastrophe exposure exists.

Possible methods to consider include:

- The price at which the business was written, the original entry price. The initial fair value for a policy's liability may be the premium charged (less expenses).

- The price at which the company is currently writing similar business.

- The price at which similar business is currently being written by the market, e.g., a broad average price. It is an indication of the current entry price. (This value may only be available retrospectively shortly after the balance sheet date.)

- The price at which reinsurance is being purchased for this risk, both quota share reinsurance, which prices the entire risk, or excess of loss reinsurance, which should provide a market guide to one of the more volatile components of the risk. This also is an indication of the current exit price.

- An actuarial estimate of the expected value of discounted losses associated with the business written but not yet earned, adjusted for risk. The estimate of the necessary risk adjustment would be based on the above methods for estimating the market value of unpaid losses. In particular, return on equity models, internal rate of return models, and models based on the aggregate probability distribution of losses, can be directly applied to future losses (losses not yet incurred on business written).

Note that the actuarial methods applicable to lines of business that contain a significant catastrophe potential may require modification to consider the seasonality of the exposures.

Section D - Methods of Estimating Risk Adjustments

Summary

A number of methods for computing risk adjustments to discounted liabilities have been presented. These are the approaches that the committee thought were worthy of discussion. Not all would be feasible for the individual company actuary to implement. As fair value becomes established as an accounting procedure, more research and application will be performed, and more methods will become feasible.

Some methods would require an "official" body such as a committee of the American Academy of Actuaries to perform research to establish parameters. Once established, the parameters could thereafter be used at individual companies without further research or analysis being required. This would hold only if one agrees that it is acceptable to ignore risks that are unique to companies, such as those classified under diversifiable risk.

Methods such as those based on CAPM and IRR pricing models should be straightforward to modify for estimating the fair value of liabilities. Actuaries are also well acquainted with methods based on aggregate probability distributions. Actuaries should be able to apply one or more of the methods to a line of business for which they are computing risk-adjusted discounted reserves.

Some methods are more appropriate for some lines of business. For instance, methods based on using risk-adjusted discount rates have been applied to lines of business with longer tails such as Automobile Liability and Workers Compensation. However, they may be inappropriate for short tail volatile lines such as property catastrophe because the risk is not time-dependent. Methods based on applying aggregate probability distributions might be appropriate for such short tail volatile lines. However, their use outside of increased limits and catastrophe pricing has not been well researched.

The direct estimation method is relatively new and has only been applied by academic researchers. Therefore, it could be difficult for practitioners to apply until further study has been done. Using reinsurance pricing to develop a risk load is, in principle, the most consistent with computing market-based estimates of liabilities. However, due to limitations on available data, the extent of the market and a lack of published research on the approach, it might be difficult to apply in practice. There might be special situations where it could be used, such as in evaluating catastrophe liabilities.

In general, risk adjustments based on industry-wide information will be more stable than risk adjustments based entirely on company-specific data. Also, risk adjustments based on individual line of business data will be less stable than risk adjustments established using all-lines data. However, such risk adjustments will fail to incorporate some of the risk components of that are unique to lines of business or to companies.

Section D - Methods of Estimating Risk Adjustments

This summary and discussion provided by the task force of methods available for computing the risk adjusted present value of liabilities demonstrates that actuaries have the theoretical understanding needed to implement fair valuing of insurance liabilities. We have identified a number of models that are available and appropriate for actuaries to use in estimating fair value liabilities. No issues have been identified that are not susceptible of actuarial estimation.

The following table summarizes our findings on the methods of deriving risk adjustments.

Section D - Methods of Estimating Risk Adjustments

Summary of Features of Estimation Methods

Method	Uses Industry Data	Uses Company Specific Data	Has Specific Time Element	Uses Leverage Ratios	Incorporates Systematic Risk	Incorporates Process Risk	Is Value Additive	Commonly Used in Pricing	Commonly Used for Reserve Margins
CAPM	X		X	X	X		X		
Internal Rate of Return	X	X	X	X	X		X	X	
Single Period RAD	X	X	X	X	X		X	X	
Using Under-writing Results	X		X		X	X	X		
Based on Probability Distributions	X	X			X	X			X
Based on Reinsurance	x	x							
Direct Estimation	x		x		x		x		
Distribution Transforms	x			x	x	x			
Naive Methods	x				x		x		