CAS Task Force on Fair Value Liabilities
White Paper on Fair Valuing Property/Casualty Insurance Liabilities

Section K - Appendices

Table of Contents

Section K - Appendices

Appendix 1: CAPM Method

This appendix presents an example of computing a risk-adjusted discount rate using CAPM.

In its simplest form, the approach used in Massachusetts assumes that the equity beta for insurance companies is a weighted average of an asset beta and an underwriting beta. The underwriting beta can therefore be backed into from the equity beta and the asset beta.

here
b_e is the equity beta for insurance companies, or alternatively for an individual insurer
b_A is the beta for insurance company assets
b_u is the beta for insurance company underwriting profits
k is the funds generating coefficient, and represents the lag between the receipt of premium and the average payout of losses in a given line
s is a leverage ratio

Since

$$b_e = \frac{\mathrm{Cov}(r_e, r_M)}{\mathrm{Var}(r_M)}$$

or the equity beta is the covariance between the company's stock return and the overall market return divided by the variance of the overall market return. It can be measured by regressing historical P&C insurance company stock returns on a return index such as the S&P 500 Index. Similarly, b_A can be measured by evaluating the mix of investments in insurance company portfolios. The beta for each asset category, such as corporate bonds, stocks, real estate is determined. The overall asset beta is a weighted average of the betas of the individual assets, where the weights are the market values of the assets.

Example:
Assume detailed research using computerized tapes of security returns such as those available from CRISP concluded that β_e for the insurance industry is 1.0 and b_A for the insurance industry is 0.15. By examining company premium and loss cash flow patterns, it has been determined that k is 2. The leverage ratio s is assumed to equal 2. The underwriting Beta is

$$b_u = \frac{b_e - (ks+1)b_A}{s}$$

or $b_U = .5*(1. – (2*2+1).15) = .125$

Once b_U has been determined overall for the P&C industry, an approach to deriving the beta for a particular line is to assume that the only factor affecting the covariance of a given line's losses with the market is the duration of its liabilities:

$$b_L = -k b_U$$

So if the average duration in a given line is 2, its beta is -2*.125= -.25

In order to derive the risk-adjusted rate, the risk free rate and the market risk premium are needed. Assume the current risk free rate is 6% and the market risk premium (i.e., the excess of the market return over the risk free return) is 9%. Then the risk-adjusted rate is:

$$r_L = r_f + b_L(r_m - r_f)$$

or $r_L = .06 - .25 * (.09) = .06 - .0225 = .0375$

An alternative approach to computing the underwriting beta is to regress accounting underwriting returns in a line of business on stock market returns. The method suffers from the weakness that the reported underwriting returns often contain values for the liabilities that have been smoothed over the underwriting cycle, thus depressing their variability.

Appendix 2: IRR Method

All balance sheet values are at fair value. Thus, the liability value at each evaluation date must be calculated using a risk-adjusted interest rate. Since we are trying to find this value, it is an input that is iterated until the IRR equals the desired ROE. (This is easily done using the "Goal Seek" function in an Excel spreadsheet.)

The present value of the income taxes is a liability under a true economic valuation method. However, in the FASB and IASC proposals, it is not included.[1] The basis for this calculation is found in Butsic (Butsic, 2000). To a close approximation, the PV of income taxes equals the present value of the tax on investment income from capital, divided by 1 minus the tax rate. The PV is taken at an after-tax risk-free rate.

Exhibit A2 shows an example of the risk adjustment calculation, using the IRR method, for a liability whose payments extend for three periods.

[1] Note that the present value of income taxes is not the same as the deferred tax liability. For example, the present value of income taxes includes the PV of taxes on *future* underwriting and investment income generated by the policy cash flows.

Exhibit A2
Calculation of Risk Adjustment Using Internal Rate of Return Model

	Fixed Inputs		**Loss & LAE cash flow patterns**	
1	Risk-free rate	0.060		
2	Expected investment return	0.080		Proportion
3	Income tax rate	0.350	Time	of Total
4	Equity beta	0.800	0	0.000
5	market risk premium	0.090	1	0.500
6	Capital/reserve	0.500	2	0.300
7	Loss & LAE	1000.00	3	0.200
8			Total	1.000
9	**Calculated values**			
10	Required ROE	0.1320		
11	Risk-adjusted yield	0.0346		
12	After-tax risk-free rate	0.0390		
13	Premium	968.75		
14				
15	**Iterative Input**			
16	Risk adjustment	0.0254		
17				

		Time			
18	**Balance sheet, at fair value**				
19		**0**	**1**	**2**	**3**
20	*Assets*				
21	Investments, before dividend	960.14	1018.12	469.03	110.55
22	Investments, after dividend	1432.21	725.53	292.97	0.00
23					
24	*Liabilities*				
25	Loss & LAE	944.15	476.81	193.31	0.00
26	Income tax liability	24.60	10.31	3.01	0.00
27	Capital, before dividend	0.00	531.00	272.71	110.55
28	Capital after div (required amount)	472.07	238.41	96.66	0.00
29					
30	**Income**				
31	Underwriting income	24.60	-32.67	-16.50	-6.69
32	Investment income		114.58	58.04	23.44
33	Net income, pretax	24.60	81.91	41.54	16.75
34	Inv income, capital (risk-adjusted)		28.32	14.30	5.80
35					
36	**Insurance Cash Flows**				
37	Premium	968.75	0.00	0.00	0.00
38	Loss & LAE	0.00	-500.00	-300.00	-200.00
39	Income tax	-8.61	-28.67	-14.54	-5.86
40					
41	Income tax, capital (risk-adjusted)		9.91	5.01	2.03
42					
43	**Capital flow (dividend)**	472.07	-292.59	-176.06	-110.55
44					
45	**Internal rate of return**	**13.20%**			

Section K - Appendices

Notes to Exhibit A2

Rows (Note that "R1" denotes Row 1, "R2" denotes Row 2, etc.):

1. Rate for portfolio of U. S. Treasury securities having same expected cash flows as the losses.
2. Expected return for the insurer's investment portfolio. Note that the yield on a bond is not an expected return. The yield must by adjusted to eliminate expected default. Municipal bond yields are adjusted to reflect the implied return as if they were fully taxable.
3. Statutory income tax rate on taxable income.
4. Estimates can be obtained from Value Line, Yahoo Finance or other services.
5. Estimates are commonly available in rate filings (e.g., Massachusetts).
6. All-lines value an be estimated by adjusting historical industry reserve values to present value and adding back the after-tax discount to GAAP equity. See Butsic (1999) for an example. For individual lines, a capital allocation method can be used, such as Myers and Read (1999).
7. An arbitrary round number used to illustrate the method.

10. R1 + (R4 x R5).
11. R1 – R16.
12. (1 – R3) x R1
13. R25 + R26 (at time 0).

16. This value is iterated until the IRR (Row 45) equals R10.

21. R22 (Prior Year) + R37 + R38 + R39.
22. R21 + R43.

25. Present value of negative R38 using interest rate R11.
26. Present value of R41 using interest rate R12. Result is divided by (1 – R3).
27. (R6, capital/reserve) x R25.
28. R27 + R43.

31. Time 0: R37 – R25. Time 1 to 3: – R11 x R25 (Prior Year).
32. (R22, Prior Year) x R2.
33. R31 + R32.
34. (R28, Prior Year) x R1.

37. R13.
38. – R7 x payment pattern in Rows 4 though 7.
39. – R3 x R33.

41. R3 x R34.

43. R28 – R27

45. Internal rate of return on Row 43 cash flows.

Section K - Appendices

Appendix 3: Single Period RAD model

All balance sheet values are at fair value.

The discussion of the income tax liability is the same as in Appendix 2.

Here, there is no iteration needed, since the risk adjustment is derived directly from the equations relating the variables to each other. Butsic (2000) derives this result.

The formula is

$$z = c\left[\frac{R - r_f}{1 - t}\right] + (r_A - r_f)\left[1 + c\frac{1 + r_f}{1 + r_f(1 - t)}\right],$$

where the variables are:

z	risk adjustment to the risk-free rate
c	capital as a ratio to the fair value of the liability
R	required rate of return on capital (ROE)
r_A	expected return on assets (includes bond yields net of expected default)
r_f	risk-free rate
t	income tax rate

Although the risk adjustment can be calculated directly from the above formula, we have provided Exhibit A3, which shows that the risk adjustment in fact produces the required ROE and internal rate of return. The format of Exhibit A3 is similar to that of Exhibit A2. However, only a single time period is needed.

Note that exhibits A2 and A3 give slightly different results for the risk adjustment. This is because capital is needed for both asset and liability risk. In a multiple period model, the relationship between the assets and loss reserve fair value is not strictly proportional. This creates a small discrepancy.

Exhibit A3
Calculation of Risk Adjustment Using Single Period ROE Model

	Fixed Inputs		
1	Risk-free rate	0.060	
2	Expected investment return	0.080	
3	Income tax rate	0.350	
4	Equity beta	0.800	
5	market risk premium	0.090	
6	Capital/reserve	0.500	
7	Loss & LAE	1000.00	
8			
9	**Calculated values**		
10	Required ROE	0.1320	
11	Risk-adjusted yield	0.0348	
12	After-tax risk-free rate	0.0390	
13			
14	Premium	981.38	
15	Risk adjustment	0.02518	
16			
17	**Balance sheet, at fair value**	Time	
18		**0**	**1**
19	*Assets*		
20	Investments, before dividend	976.12	546.96
21	Investments, after dividend	1459.30	0.00
22			
23	*Liabilities*		
24	Loss & LAE	966.35	0.00
25	Income tax liability	15.02	0.00
26	Capital, before dividend	0.00	546.96
27	Capital after div (required amount)	483.18	0.00
28			
29	**Income**		
30	Underwriting income	15.02	-33.65
31	Investment income		116.74
32	Net income, pretax	15.02	83.10
33	Inv income, capital (risk-adjusted)		28.99
34			
35	**Insurance Cash Flows**		
36	Premium	981.38	0.00
37	Loss & LAE	0.00	-1000.00
38	Income tax	-5.26	-29.08
39			
40	Income tax, capital (risk-adjusted)		10.15
41			
42	**Capital flow (dividend)**	483.18	-546.96
43			
44	**ROE**		**13.20%**
45			
46	**Internal rate of return**	**13.20%**	

Section K - Appendices

Notes to Exhibit A3

Rows (Note that “R1” denotes Row 1, “R2” denotes Row 2, etc.):

1. Rate for portfolio of U. S. Treasury securities having same expected cash flows as the losses.
2. Expected return for the insurer’s investment portfolio. Note that the yield on a bond is not an expected return. The yield must by adjusted to eliminate expected default. Municipal bond yields are adjusted to reflect the implied return as if they were fully taxable.
3. Statutory income tax rate on taxable income.
4. Estimates can be obtained from Value Line, Yahoo Finance or other services.
5. Estimates are commonly available in rate filings (e.g., Massachusetts).
6. All-lines value an be estimated by adjusting historical industry reserve values to present value and adding back the after-tax discount to GAAP equity. See Butsic (1999) for an example. For individual lines, a capital allocation method can be used, such as Myers and Read (1999).
7. An arbitrary round number used to illustrate the method.

10. R1 + (R4 x R5).
11. R1 – R15
12. (1 – R3) x R1

14. R24 + R25 (at time 0).
15. R6 x (R10 – R1) / (1 – R3) – (R2 – R1) x [1 + R6 x (1 + R1) / (1+R12)].

20. R21 (Prior Year) + R36 + R37 + R38.
21. R20 + R42.

24. Present value of R7 using interest rate R11.
25. Present value of R40 using interest rate R12. Result is divided by (1 – R3).
26. Time 0: 0; Time 1: R20 – R24 – R25.
27. R6 x R24.

30. Time 0: R36 – R24. Time 1: – R11 x R24 (Prior Year).
31. (R21, Prior Year) x R2.
32. R30 + R31.
33. (R27, Prior Year) x R1.

36. R14.
37. Time 0: 0. Time 1: – R7.
38. – R3 x R32.

40. R3 x R33.

42. R27 – R26

44. (R26, Time 1) / (R27, Time 0) – 1.

46. Internal rate of return on Row 42 cash flows.

Appendix 4: Using Underwriting Data

This appendix describes Butsic's procedure for computing risk adjusted discount rates. The following relationship is used for the computation.

$$C = P(1+i)^{-u} - E(1+i)^{-w} - L(1+i_A)^{-t}$$

Where:

C is the cash flow on a policy and can be thought of as the present value of the profits, both underwriting and investment income, on the policy,
P is the policy premium,
E is expenses and dividends on the policy,
L is the losses and adjustment expenses,
$\underline{u}$ is the average duration of the premium, or the average lag between the inception of the policy and the collection of premium,
w is the average duration of the expenses,
t is the average duration of the liabilities.
i is the risk free rate of return
i_A is the risk adjusted rate of return

This formula says that the present value cash flow or present value profit on a group of policies is equal to the present value of the premium minus the present value of the components of expenses minus the present value of losses. Premiums and expenses are discounted at the risk free rate. Each item is discounted for a time period equal to its duration, or the time difference between inception of the policy or accident period and expiration of all cash flows associated with the item. Losses are discounted at the risk-adjusted rate. Underwriting data in ratio form, i.e., expense ratios, loss ratios, etc. can be plugged into the formula. When that is done, P enters the formula as 1, since the ratios are to premium.

In ratio form this formula would be:

$$c = 1(1+i)^{-u} - e(1+i)^{-w} - l(1+i_A)^{-t}$$

c is the ratio of present value profit to premium
e is the expense ratio, including dividends to policyholder
l is the loss ratio

Section K - Appendices

Using as a starting point the rate of return on surplus, where the surplus supporting a group of policies is assumed to be eV_m, or the leverage ratio times the average discounted reserve, Butsic (Bustic, 1988) derived the following simplified expression for the risk adjustment:

$$Z = e(R - i) = (1 + i)C/V_m \quad ,$$

where:
Z is the risk adjustment to the interest rate or the percentage amount to be subtracted from the risk free rate = $e(R - i)$
C and i are as defined above
V_m is the average discounted reserve for the period

V_m is generally taken as the average of the discounted unpaid liabilities at the beginning of the accident or policy period (typically 100% of the policy losses) and the discounted unpaid liabilities at the end of the period. In general, this would be equal to 100% plus the percentage of losses unpaid at the end of the period (one year if annual data is used) divided by 2. The discount rate is the risk-adjusted rate. If V_m is computed as a ratio to premium, then published loss ratios are discounted and used in the denominator.

To complete the calculation, the quantity c, or the ratio of discounted profit to premium should be multiplied by $(1 + i)$ and divided by v_m (V_m in ratio form). To derive initial estimates of the risk adjustment, it is necessary to start with a guess as to the value of the risk adjustment to the discount rate in order to obtain a value for discounted liabilities.

The following is an example of the computation of the risk adjustment using this method. It is necessary to start with a guess for the risk adjustment and then perform the calculation iteratively until it converges on a solution. This example is based on data in Butsic's (1988) paper.

Parameter assumptions	
Interest Rate R_f	0.0972
Fraction of losses OS after 1 year	0.591
Initial Risk Adjustment	0.044

Variable	Nominal Value	Duration	Discounted Value
1 Loss&LAE	0.767	2.300	0.681
2 Premium	1.000	0.250	0.977
3 UW Expense	0.268	0.250	0.262
4 Pol Dividends	0.016	2.250	0.013
5 Average Liabilities	0.610	1.800	0.556

	Calculation	
6	Premium-Expenses Discounted	
	(2) - (3) - (4)	0.702
7	Premiums-Expenses-Losses Disc	0.021
	(6)-(1)	
8	C*(1+I)	0.024
	(7)*(1+I)	
9	$Z=C^*(1+I)/V_m$	0.042
	(8)/(5)	

An additive risk load

An additive or dollar risk load can be computed from the same data. The formula for the computation of a risk load is:

$$c = p(1+i)^{-u} - e(1+i)^{-w} - l(1+i)^{-t}$$

$$rl = c / l(1+i)^{-t}$$

Where *rl* is the additive risk load and *i* is the risk free interest rate.
An example is shown below:

Parameter assumptions	
Interest Rate Rf	0.0972

	Variable	Nominal Value	Duration	Discounted Value
1	Loss&LAE	0.767	2.300	0.620
2	Premium	1.000	0.250	0.977
3	UW Expense	0.268	0.250	0.262
4	Pol Dividends	0.016	2.250	0.013

	Calculation	
5	Premium-Expenses Discounted	
	(2) - (3) - (4)	0.702
6	C =Premiums-Expenses-Losses Disc	0.083
	(5)-(1)	
7	C/PV(Losses)	0.133
	(6)/(1)	

Section K - Appendices

Appendix 5: The Tax Effect

More recent work by Butsic (Butsic, 2000) has examined the effect of taxes on the risk adjusted discount rates and insurance premium. Butsic argued that, due to double taxation of corporate income, there is a tax effect from stockholder supplied funds. Stockholder funds are the equity supplied by the stockholder to support the policy. In the formulas above, stockholder supplied funds are denoted by E and taken to be the ratio of *e* to the present value of losses $V = L(1+i_A)^{-T}$. For a one period policy an amount *E* is invested at the risk free rate *i*, an amount E_i of income is earned, but because it is taxed at the rate *t*, the after tax income is $E_i(1 - t)$. The reduced investment income on equity will be insufficient to supply the amount needed to achieve the target return. In order for the company to earn its target after tax return, the amount lost to taxes must be included in the premium. However, the underwriting profit on this amount will also be taxed. The amount that must be added to premium to compensate for this tax effect is:

$$\frac{Eit}{(1-t)\left[1+i(1-t)\right]}$$

This is the tax effect for a one period policy if the discount rate for taxes is the same as the discount rate for pricing the policy, i.e., the risk adjusted discount rate. Butsic shows that there is an additional tax effect under the current tax law, where losses are discounted at a higher rate than the risk adjusted rate. There is also a premium collection tax effect, due to lags between the writing and collecting of premium. This is because some premium is taxed before it is collected. Butsic developed an approximation for all of these effects taken together, as well as the multiperiod nature of cash flows into the following adjustment to the risk adjusted discount rate:

$i_A' = i - e(1-t)(r_T - i)$,where
i_A' is the tax and risk adjusted rate,
e is a leverage ratio,
t is the tax rate,
r_T is the pre tax return on equity.

This is the effective rate used to discount losses to derive economic premium. The tax effect acts like an addition to the pure risk adjustment. Since premiums as stated in aggregate industry data already reflect this tax effect, no adjustment is needed for the risk adjusted discount rate used for pricing. However, for discounting liabilities, it may be desirable to segregate the tax adjustment from the pure risk adjustment, since the tax effect really represents a separate tax liability. Using the formula above, as well as the formula for determining the pure risk adjustment to the discount rate the two effects could be segregated. One would need to have an estimate of the total pre tax return on equity.

Appendix 6: Using Aggregate Probability Distributions

This example uses the Collective Risk Model to compute a risk load. It represents only one of the many approaches based on aggregate probability distributions. This is in order to keep the illustration simple.

The approach is based on the following model for risk load:

- Risk Load = λ SD[Loss] or Risk Load =λ Var[Loss],

Therefore, in order or compute a risk load, two quantities are needed: λ and Var[Loss], since SD(Loss) = $\text{Var[Loss]}^{1/2}$. The following algorithm from Meyers (Meyers, 1994) will be used to compute the variance of aggregate losses.

The Model:

1. Assume claim volume has an unconditional Poisson distribution.

2. Assume the Poisson parameter, n (the claim distribution mean), varies from risk to risk.

3. Select a random variable c from a distribution with mean 1 and variance c.

4. Select the claim count, K, at random from a Poisson distribution with mean cn, where the random variable c is multiplied by the random Poisson mean n.

<u>The Variability of Insurer Losses</u>

5. Select occurrence severities, $Z_1, Z_2, .., Z_K$, at random from a distribution with mean m and variance s^2.

6. The total loss is given by:

$$X = \sum_{j=1}^{K} Z_j$$

The expected occurrence count is n (i.e. E[(c n] = E[n] = n). n is used as a measure of exposure.
When there is no parameter uncertainty in the claim count distribution c = 0,

$$\text{Var}[x] = n\,(m^2 + s^2),$$

and variance is a linear function of exposures.

When there is parameter uncertainty:

$$\text{Var}[x] = nu + n^2v,$$

where

$$u = (m^2 + s^2)$$

and

$$v = cm^2$$

nu is the process risk and n^2v is the parameter risk.

For example, assume an insurer writes two lines of business. The expected claim volume for the first line is 10,000 and the expected claim volume for the second line is 20,000. The parameter c for the first line is 0.01 and for the second line is 0.005. Let the severity for line 1 be lognormal with a mean of $10,000 and volatility parameter (the standard deviation of the logs of losses) equal to 1.25 and the severity for line 2 be lognormal with severity of $20,000 and volatility equal to 2. Applying the formula above for the variance of aggregate losses, we find that the variance for line 1 is $1.05x\ 10^{14}$ and the variance of line two is 1.24×10^{15} and the sum of the variances for the two lines is 1.34×10^{15}. The standard deviation is $36,627,257.

One approach to determining the multiplier l would be to select the multiplier ISO uses in its increased limits rate filings. In the increased limits rate filings, l is applied to the variance of losses and is on the order of 10^{-7}.(Meyers, 1998)

In recent actuarial literature, the probability of ruin has been used to determine the multipliers of SD(loss) or Var(Loss). (Kreps 1998, Meyers 1998, Philbrick, 1994). The probability of ruin or expected policyholder deficit is used to compute the amount of surplus required to support the liabilities. To keep the illustration simple, we use the probability of ruin approach. However, the expected policyholder deficit or tail value at risk (which is similar to expected policyholder deficit) approaches better reflect the current literature on computing risk loads. Suppose the company wishes to be 99.9% sure that it has sufficient surplus to pay the liabilities, ignoring investment income, the company will require surplus of 3.1 times the standard deviation of losses, if one assumes that losses are normally distributed.[2] In order to complete the calculation, we need to know the company's required return on equity, r_e. This can be determined by examining historical return data for the P&C insurance industry. Then the required risk margin for one year is r_e x 3.1x 36,627,257. For instance, if r_e is 10% then the risk margin is

[2] If one assumes that aggregate losses are lognormally distributed, then the company needs approximately $e^{(2.33*.06)}*$ the expected losses as surplus, where .06 is the volatility parameter, derived from the mean and variance of the distribution..

11,354,450 or about 2.0% of expected losses. In this example, the parameter lambda is equal to 3.1 r_e. The result computed above could be converted into a risk margin for discounted losses by applying the 2% to losses discounted at the risk free rate. This would require the assumption that the risks of investment income on the assets supporting the losses being less than expected is much less than the risk that losses will be greater than expected. When the assets supporting the liabilities are primarily invested in high quality bonds, this assumption is probably reasonable. (see D'Arcy et. al., 1997)

Philbrick in his paper commissioned by the CAS "Accounting for Risk Margins" had a slightly different approach to determining the risk margin. Philbrick's formula for risk margin, given a total surplus requirement *S*, (i.e. 3.1* standard deviation in this example), a rate of return on equity r_e and a risk free rate *i* is:

$$RM = \frac{(r_e - i)\, x S}{1 + r_e}$$

This is a risk margin for discounted losses not undiscounted losses. . The formula above assumes that some of the required return on surplus is obtained from investing the surplus at the risk free rate. If i = 5%, and r_e = 10% the risk margin in this illustration would be $5,161,113.

In this example, it should be noted that the majority of the standard deviation is due to parameter risk, as process risk for such large claim volumes is minimal. However, only parameter risk for claims volumes has been incorporated. A more complete model would incorporate parameter risk for the severity distribution. This risk parameter has been denoted the "mixing parameter" in the actuarial literature. The algorithm for incorporating this variance into the measure of aggregate loss variance is as follows:

> 1 - 5. Follow steps 1 through 5 above, describing the selection of frequency and severity parameters for a distribution
> 6. Select a random variable *B* from a distribution with mean 1 and variance *b*.
>
> 7. The total loss is given by:

$$X = \sum_{j=1}^{K} Z_j / \mathrm{B}$$

The variance reflecting the mixing parameter is given by:

$$\mathrm{Var}[x] = n(1+b)\left(m^2 + s^2\right) + n^2(b + c + bc)m^2 .$$

Procedures for estimating *b* and *c* are provided by Meyers and Schenker. The procedures use the means and variances of the claim count and the loss distribution to compute *b* and *c*. The parameter *b* can also be viewed as the uncertainty contributed to the total estimate of losses due to uncertainty in the trend and development factors. Methods for measuring the variance due to development are presented by Hayne, Venter and Mack. Regression statistics containing information about the variances of trend factors are published in ISO circulars and can be developed from internal data. To continue our example, we will assume that the *b* parameter for line 1 is 0.02 and for line 2 is 0.05. Then the standard deviation of aggregate losses is $95,663,174. The risk load using Philbrick's formula is $13,479,811 or 2.7% of expected undiscounted losses. The load is intended to be applied to <u>discounted</u> liabilities where liabilities are discounted at the risk free rate. Thus if losses take one year to pay out the risk margin is 2.8% of the present value of liabilities.

The above risk load is consistent with liabilities that expire in one year. When losses take more than one year to pay, Philbrick uses the following formula to derive a risk load.

$$RM = \sum_j \frac{(r_e - i) S_j}{(1 + r_e)^j}$$

This formula can be applied to liabilities of any maturity. Where S_j is the surplus requirement for outstanding liabilities as of year *j*. In the above example if losses pay out evenly over 3 years then the risk margin is $20,693,737or 4.6% of he discounted liabilities. The calculation is shown below.

(1)	(2)	(3)	(4)	(5)
	Surplus			
t	.227*PV(OS Losses)	1/(1+r(e)^t	(3)*(2)	(r(e)-.05)*(4)
0	219,965,641	1.000	219,965,641	10,998,282
1	146,643,760	0.909	133,312,510	6,665,625
2	73,321,880	0.826	60,596,595	3,029,830
				20,693,737

The computation above assumes that the relative variability of the liabilities remains constant as the liabilities mature. As this may not be the case, refinements to the measure of variability by age of liability may be desirable. One approach to modeling the uncertainty in reserves would derive measures of variability from observed loss development variability. This is the approach used by Zenwirth, Mack and Hayne. Another approach, consistent with how risk base capital is computed, would measure historic reserve development for P&C companies for a line of business from Schedule P.

Appendix 7: Direct Estimation of Market Values

Below we illustrate how to estimate the risk adjustment to the interest rate for a single firm, based on empirical data.

Assume that the market value of assets is 1400 and the book (undiscounted) value of the liabilities is 1000. Both of these values are available from the insurer's published financial statements. Also, assume that using the Ronn-Verma method (see the discussion in the Credit Risk Appendix), the estimated market value of the firm's equity is 500 and that the value of the expected default (the credit risk adjustment) is 10. The market value of the equity adjusted to exclude default is 510.

The discounted risk adjusted liabilities equals the market value of the assets minus the market value of the equity or 900 = 1400 – 500. The implied market value of the liabilities adjusted for default equals the market value of the assets minus the market value of the equity adjusted for default, or **890** = 1400 – 510.

Assume that the risk-free interest rate applicable to valuing the insurer's expected liability payments is 6% and that the liability payment pattern is 10% per year for 10 years (paid at the end of each year). The present value of the liabilities at the risk-free rate is 730. Thus, the risk margin, expressed in dollars is **160** = 890 – 730. Alternatively, the interest rate that gives a present value of 890 using the above payment pattern is 2.18%. This value implies a risk adjustment of **3.82%**.

The following discussion provides an example of the Ronn and Verma method.

Let *A* be the market value of assets, L the market value of liabilities and s the volatility of the asset/liability ratio. The formula for the owners' equity, where there is a possibility of default, is the call option with expiration in one year:

(1) $E = A \cdot N(d) - L \cdot N(d - s)$,

where $d = \ln(A/L)/s + s/2$ and $N(d)$ is the standard normal distribution evaluated at *d*.

Notice that equity value with no default is simply $E_n = A - L$. For an insurer with stochastic assets and liabilities, s_E, the volatility of the equity, is related to the asset/liability volatility by

(2) $s_E = N(d) A s / E$.

Section K - Appendices

Equations (1) and (2) are solved simultaneously to get E and s .

The expected default value equals $E - E_n$, or the derived market value of the equity minus the equity value with no default.

The method is easily demonstrated with a numerical example. Assume that A =130, L = 100 and s_E = 0.5. Solving the simultaneous equations gives E = 40.057 and s = 0.117. Therefore, the value of the expected default is

0.057 = 40.057 – 40.000.

For an insurer, the market value of assets is readily determined from the published balance sheet. Discounting the reserves at a risk-free rate can approximate the market value of liabilities. The equity volatility can be estimated by analyzing the insurer’s stock price over a recent time frame, as done by Allen, Cummins and Phillips.

Section K - Appendices

Appendix 8: Distribution Transform Method

Assume expected claim counts for a policy equal 100 and ground up severities follow a Pareto distribution:

$$F(x) = 1 - \left[b/(b+x) \right]^q \text{ for x>0.}$$

Therefore $G(x) = \left[b/(b+x) \right]^q$

$\text{E}[X] = b/(q-1)$

E(aggregate loss) = 100* E[X]

For the transformation *r*, $G(x) = \left[b/(b+x) \right]^{qr}$.

If the market risk premium is 10% then risk loaded premiums equal:

$$100 \frac{b}{q-1} 1.1 = 100 \frac{b}{qr-1}$$

This expression can be solves for r:

$$r = \left[(q-1)/1.1 + 1 \right]/q = (q+0.1)/1.1q.$$

If *q* were 2, *r* would be 0.95.

Expected values for higher layers could be computed by replacing *q* with *qr* in the Pareto distribution and using the Pareto formula for limited expected value to price the excess layers.:

E(X, x) = Limited Expected Value function = $\text{E}[X]\left\{1 - \left[b/(b+x) \right]^{q-1}\right\}$

In the above example, a transformation was applied only to the severity distribution. However, with a little more work, the transformation could be applied to both the frequency and severity distribution.

For instance the formula for the transformed mean of a Poisson distribution with a mean of 100 and transformation parameter *r* is:

$$\sum_j ((e^{200}\Gamma(j) - \Gamma(j,100))/\Gamma(j))^r$$

This formula could be combined with the formula for the transformed severity distribution to produce a risk loaded mean.

CAS Task Force on Fair Value Liabilities
White Paper on Fair Valuing Property/Casualty Insurance Liabilities

Section K - Appendices

Appendix 9: Credit Risk

The Time Horizon Problem

In general, long-tail liabilities are subject to greater default risk than are short-tail liabilities. To see why this is so, assume that an ongoing insurer has a 1% chance of insolvency each year. The insurer has two lines of business: line A has claims that are paid in one year and line B has claims that are paid in five years. The probability of a claim from line A not being paid in full is 1%. Assuming that each year's insolvency potential is independent of the other years, the probability of a claim from line B not being fully paid is $4.9\% = 1-(1-0.01)^5$, or about 5 times as great as for line A.

An insurance firm's owners normally make capital decisions at an approximate annual frequency, so to truly measure the long-term value of the potential default, it is necessary to consider the future capital flows as well as the current level of capital. (However, note that the fair value accounting proposals purposely ignore future transactions that are not based on current contractual obligations.) The complexity due to future capital flows (which are contingent on future company results and market conditions) makes the estimation of credit risk extremely difficult.

To make the credit risk adjustment calculation more tractable, it is customary to assume an annual time horizon and that future insolvencies have the same probability as for the current one-year horizon. For longer-term liabilities, one can further assume that the insolvency probabilities are independent year-to year and then determine the overall expected default by a formula suggested by the above 5-year calculation:

$$D = \frac{D_1}{p}[1-w_1(1-p)-w_2(1-p)^2-\ldots-w_n(1-p)^n] \quad \cong \quad D_1[w_1+2w_2+\ldots+nw_n].$$

Here, D_1 is the fair value of the expected default for the one-year horizon, *p* is the one-year insolvency probability and the weight w_i is the expected proportion of loss paid in year *i* (the weights sum to 1). Using the approximation above, the fair value over an n year time horizon of a company's option to default can be expressed as a function of its one year default value.

It should be noted that the published research relating to bond default rates does not support the assumption that annual default rates over the life of a bond are independent and identically distributed. That is, for many categories of bonds, the default rate during the third and fourth year is higher than the default rate during the first and second years after issuance. If the assumption of independent and identically distributed default rates is inappropriate for bonds, it may be inappropriate for some of the companies issuing bonds (i.e. insurance companies) and therefore the approximations in the above formula would not be appropriate.

A related technical issue that must be addressed in calculating the credit risk adjustment is the length of the *time horizon* over which defaults are recognized. At one extreme, it may be argued the applicable horizon is unlimited. Insurers are obliged to pay claims occurring during the contractual coverage period, no matter how long the reporting and settlement processes take. On the other hand, solvency monitoring and financial reports have a quarterly or annual cycle. Also, it is important to recognize that capital funding and withdrawal decisions are made with an approximate quarterly or annual cycle. An approach that often makes the solution easier to derive is to assume that one may view the time horizon as being a fairly short duration. According to this view, if the company is examined over short increments such as one year, corrective action is applied and insolvency over a longer term is avoided. The task force considers this view to be controversial. The alternative view is that insurance liabilities are often obligations with relatively long time horizons, and these longer horizons need to be considered when evaluating the companies' option to default on its obligations.

In the numerical examples below, we have determined the annual fair value of default. The extension to longer-duration liabilities is straightforward, using the above formula, if one assumes the formula to be appropriate. If one assumes the formula to be inappropriate, many of the methods below can be modified to adjuste for the longer time horizon of insurance liabilities.

Numerical Examples of Credit Risk Adjustment Estimation Methods

1. Implied Option Value: Example

The following (until #2, the DFA example), is a repeat of a few pages ago immediately following Appendix 7,

The following discussion provides an example of the Ronn and Verma method.

Let A be the market value of assets, L the market value of liabilities and s the volatility of the asset/liability ratio. The formula for the owners' equity, where there is a possibility of default, is the call option with expiration in one year:

(1) $$E = A \cdot N(d) - L \cdot N(d - s),$$

where $d = \ln(A/L)/s + s/2$ and $N(d)$ is the standard normal distribution evaluated at d.

Notice that equity value with no default is simply $E_n = A - L$. For an insurer with stochastic assets and liabilities, s_E, the volatility of the equity, is related to the asset/liability volatility by

(2) $s_E = N(d)As\ /E$.

Equations (1) and (2) are solved simultaneously to get E and s .

The expected default value equals $E - E_n$, or the derived market value of the equity minus the equity value with no default.

The method is easily demonstrated with a numerical example. Assume that A =130, L = 100 and s_E = 0.5. Solving the simultaneous equations gives E = 40.057 and s = 0.117. Therefore, the value of the expected default is

0.057 = 40.057 – 40.000.

For an insurer, the market value of assets is readily determined from the published balance sheet. Discounting the reserves at a risk-free rate can approximate the market value of liabilities. The equity volatility can be estimated by analyzing the insurer’s stock price over a recent time frame, as done by Allen, Cummins and Phillips.

2. Dynamic Financial Analysis: Example

An insurer has initial liabilities of $100 million, measured at fair value, but under the assumption that all contractual obligations will be paid. Assume that the DFA model has been run using 10,000 simulations. The time horizon is one year. We examine all observations where the terminal fair value (before default) of liabilities exceeds the market value of the assets. Suppose that there are 22 of them, with a total deficit (liability minus asset value) of $660 million. The average default amount per simulation is $0.066 million.

The expected terminal fair value is then discounted at a risk-adjusted interest rate to get the fair value of the credit risk adjustment. With a 4% risk-adjusted interest rate, for example, the fair value of the default is **$0.063 million** = 0.066/1.04. Thus, the fair value of the liabilities, adjusted for credit risk, is **$99.94 million** ($100 million - $.06 million.

Section K - Appendices

3. Rating Agency Method: Example

This example shows how the table of default ratios might look, if a one-year time horizon approach was used. Alternatively, a matrix of default ratios by rating and lag year could be used, similar to those available from Moody's (e.g., Moody's January 2000 report titled "Historical Default Rates of Corporate Bond Issuers, 1920-1999"). Here the ratings are the current A. M. Best categories. The values in the table below are purely hypothetical.

Rating	Annual Expected Default Ratio (Raw Results)	Annual Expected Default Ratio (Adjusted)
A++	0.000%	0.001%
A+	0.000%	0.004%
A	0.013%	0.010%
A-	0.043%	0.050%
B++	0.122%	0.100%
B+	0.155%	0.150%
B	0.432%	0.300%
B-	0.619%	0.500%
C++	0.653%	0.800%
C+	1.221%	1.000%
C	1.554%	1.500%
C-	2.221%	2.000%
D	4.689%	5.000%
E	13.658%	15.000%

The raw results would be based on historical insolvency data. A simulation model or a closed-form model could be applied to a large sample of companies within each rating group to produce the adjusted results. These results might be further adjusted to ensure that a higher rating had a corresponding lower default expectation.

To show how the above table would be applied, assume that an insurer has initial liabilities of $100 million. These are measured at fair value, but under the assumption that all contractual obligations will be paid. Assume also that the insurer has an A- Best's rating. The expected default is 0.05% of $100 million, or $50,000.

The expected terminal fair value is then discounted at a risk-adjusted interest rate to get the fair value of the credit risk adjustment. With a 4% risk-adjusted interest rate, for example, the fair value of the default is **$48,100** = 50,000/1.04. Thus, the fair value of the liabilities, adjusted for credit risk, is **$99.95 million**.

CAS Task Force on Fair Value Liabilities
White Paper on Fair Valuing Property/Casualty Insurance Liabilities

Section K - Appendices

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