CAS Task Force on Fair Value Liabilities
White Paper on Fair Valuing Property/Casualty Insurance Liabilities

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Appendix 9: Credit Risk

The Time Horizon Problem

In general, long-tail liabilities are subject to greater default risk than are short-tail liabilities. To see why this is so, assume that an ongoing insurer has a 1% chance of insolvency each year. The insurer has two lines of business: line A has claims that are paid in one year and line B has claims that are paid in five years. The probability of a claim from line A not being paid in full is 1%. Assuming that each year's insolvency potential is independent of the other years, the probability of a claim from line B not being fully paid is $4.9\% = 1-(1-0.01)^5$, or about 5 times as great as for line A.

An insurance firm's owners normally make capital decisions at an approximate annual frequency, so to truly measure the long-term value of the potential default, it is necessary to consider the future capital flows as well as the current level of capital. (However, note that the fair value accounting proposals purposely ignore future transactions that are not based on current contractual obligations.) The complexity due to future capital flows (which are contingent on future company results and market conditions) makes the estimation of credit risk extremely difficult.

To make the credit risk adjustment calculation more tractable, it is customary to assume an annual time horizon and that future insolvencies have the same probability as for the current one-year horizon. For longer-term liabilities, one can further assume that the insolvency probabilities are independent year-to year and then determine the overall expected default by a formula suggested by the above 5-year calculation:

$$D = \frac{D_1}{p}[1 - w_1(1-p) - w_2(1-p)^2 - \ldots - w_n(1-p)^n] \quad \cong \quad D_1[w_1 + 2w_2 + \ldots + nw_n].$$

Here, D_1 is the fair value of the expected default for the one-year horizon, *p* is the one-year insolvency probability and the weight w_i is the expected proportion of loss paid in year *i* (the weights sum to 1). Using the approximation above, the fair value over an n year time horizon of a company's option to default can be expressed as a function of its one year default value.

It should be noted that the published research relating to bond default rates does not support the assumption that annual default rates over the life of a bond are independent and identically distributed. That is, for many categories of bonds, the default rate during the third and fourth year is higher than the default rate during the first and second years after issuance. If the assumption of independent and identically distributed default rates is inappropriate for bonds, it may be inappropriate for some of the companies issuing bonds (i.e. insurance companies) and therefore the approximations in the above formula would not be appropriate.

A related technical issue that must be addressed in calculating the credit risk adjustment is the length of the *time horizon* over which defaults are recognized. At one extreme, it may be argued the applicable horizon is unlimited. Insurers are obliged to pay claims occurring during the contractual coverage period, no matter how long the reporting and settlement processes take. On the other hand, solvency monitoring and financial reports have a quarterly or annual cycle. Also, it is important to recognize that capital funding and withdrawal decisions are made with an approximate quarterly or annual cycle. An approach that often makes the solution easier to derive is to assume that one may view the time horizon as being a fairly short duration. According to this view, if the company is examined over short increments such as one year, corrective action is applied and insolvency over a longer term is avoided. The task force considers this view to be controversial. The alternative view is that insurance liabilities are often obligations with relatively long time horizons, and these longer horizons need to be considered when evaluating the companies' option to default on its obligations.

In the numerical examples below, we have determined the annual fair value of default. The extension to longer-duration liabilities is straightforward, using the above formula, if one assumes the formula to be appropriate. If one assumes the formula to be inappropriate, many of the methods below can be modified to adjuste for the longer time horizon of insurance liabilities.

Numerical Examples of Credit Risk Adjustment Estimation Methods

1. Implied Option Value: Example

The following (until #2, the DFA example), is a repeat of a few pages ago immediately following Appendix 7,

The following discussion provides an example of the Ronn and Verma method.

Let *A* be the market value of assets, L the market value of liabilities and s the volatility of the asset/liability ratio. The formula for the owners' equity, where there is a possibility of default, is the call option with expiration in one year:

(1) $E = A \cdot N(d) - L \cdot N(d - s)$,

where $d = \ln(A/L)/s + s/2$ and $N(d)$ is the standard normal distribution evaluated at *d*.

Notice that equity value with no default is simply $E_n = A - L$. For an insurer with stochastic assets and liabilities, s_E, the volatility of the equity, is related to the asset/liability volatility by

(2) $s_E = N(d)As \ / E$.

Equations (1) and (2) are solved simultaneously to get E and s .

The expected default value equals $E - E_n$, or the derived market value of the equity minus the equity value with no default.

The method is easily demonstrated with a numerical example. Assume that A =130, L = 100 and s_E = 0.5. Solving the simultaneous equations gives E = 40.057 and s = 0.117. Therefore, the value of the expected default is

0.057 = 40.057 – 40.000.

For an insurer, the market value of assets is readily determined from the published balance sheet. Discounting the reserves at a risk-free rate can approximate the market value of liabilities. The equity volatility can be estimated by analyzing the insurer's stock price over a recent time frame, as done by Allen, Cummins and Phillips.

2. Dynamic Financial Analysis: Example

An insurer has initial liabilities of $100 million, measured at fair value, but under the assumption that all contractual obligations will be paid. Assume that the DFA model has been run using 10,000 simulations. The time horizon is one year. We examine all observations where the terminal fair value (before default) of liabilities exceeds the market value of the assets. Suppose that there are 22 of them, with a total deficit (liability minus asset value) of $660 million. The average default amount per simulation is $0.066 million.

The expected terminal fair value is then discounted at a risk-adjusted interest rate to get the fair value of the credit risk adjustment. With a 4% risk-adjusted interest rate, for example, the fair value of the default is **$0.063 million** = 0.066/1.04. Thus, the fair value of the liabilities, adjusted for credit risk, is **$99.94 million** ($100 million - $.06 million.

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3. Rating Agency Method: Example

This example shows how the table of default ratios might look, if a one-year time horizon approach was used. Alternatively, a matrix of default ratios by rating and lag year could be used, similar to those available from Moody's (e.g., Moody's January 2000 report titled "Historical Default Rates of Corporate Bond Issuers, 1920-1999"). Here the ratings are the current A. M. Best categories. The values in the table below are purely hypothetical.

Rating	Annual Expected Default Ratio (Raw Results)	Annual Expected Default Ratio (Adjusted)
A++	0.000%	0.001%
A+	0.000%	0.004%
A	0.013%	0.010%
A-	0.043%	0.050%
B++	0.122%	0.100%
B+	0.155%	0.150%
B	0.432%	0.300%
B-	0.619%	0.500%
C++	0.653%	0.800%
C+	1.221%	1.000%
C	1.554%	1.500%
C-	2.221%	2.000%
D	4.689%	5.000%
E	13.658%	15.000%

The raw results would be based on historical insolvency data. A simulation model or a closed-form model could be applied to a large sample of companies within each rating group to produce the adjusted results. These results might be further adjusted to ensure that a higher rating had a corresponding lower default expectation.

To show how the above table would be applied, assume that an insurer has initial liabilities of $100 million. These are measured at fair value, but under the assumption that all contractual obligations will be paid. Assume also that the insurer has an A- Best’s rating. The expected default is 0.05% of $100 million, or $50,000.

The expected terminal fair value is then discounted at a risk-adjusted interest rate to get the fair value of the credit risk adjustment. With a 4% risk-adjusted interest rate, for example, the fair value of the default is **$48,100** = 50,000/1.04. Thus, the fair value of the liabilities, adjusted for credit risk, is **$99.95 million**.