Section K - Appendices

Appendix 8: Distribution Transform Method

Assume expected claim counts for a policy equal 100 and ground up severities follow a Pareto distribution:

$$F(x) = 1 - \left[b / (b + x) \right]^q \text{ for x>0.}$$

Therefore $G(x) = \left[b / (b + x) \right]^q$
$\mathrm{E}[X] = b/(q-1)$
E(aggregate loss) = 100* E[X]

For the transformation *r*, $G(x) = \left[b / (b + x) \right]^{qr}$.

If the market risk premium is 10% then risk loaded premiums equal:

$$100 \frac{b}{q-1} 1.1 = 100 \frac{b}{qr-1}$$

This expression can be solves for r:

$$r = \left[(q - 1)/1.1 + 1 \right] / q = \ (q + 0.1)/1.1q .$$

If *q* were 2, *r* would be 0.95.

Expected values for higher layers could be computed by replacing *q* with *qr* in the Pareto distribution and using the Pareto formula for limited expected value to price the excess layers.:

E(X, x) = Limited Expected Value function = $\mathrm{E}[X] \left\{ 1 - \left[b / (b + x) \right]^{q-1} \right\}$

In the above example, a transformation was applied only to the severity distribution. However, with a little more work, the transformation could be applied to both the frequency and severity distribution.

For instance the formula for the transformed mean of a Poisson distribution with a mean of 100 and transformation parameter *r* is:

$$\sum_{j}((e^{200}\Gamma(j)-\Gamma(j,100))/\Gamma(j))^{r}$$

This formula could be combined with the formula for the transformed severity distribution to produce a risk loaded mean.