CAS Task Force on Fair Value Liabilities
White Paper on Fair Valuing Property/Casualty Insurance Liabilities

Section K - Appendices

Appendix 7: Direct Estimation of Market Values

Below we illustrate how to estimate the risk adjustment to the interest rate for a single firm, based on empirical data.

Assume that the market value of assets is 1400 and the book (undiscounted) value of the liabilities is 1000. Both of these values are available from the insurer's published financial statements. Also, assume that using the Ronn-Verma method (see the discussion in the Credit Risk Appendix), the estimated market value of the firm's equity is 500 and that the value of the expected default (the credit risk adjustment) is 10. The market value of the equity adjusted to exclude default is 510.

The discounted risk adjusted liabilities equals the market value of the assets minus the market value of the equity or 900 = 1400 – 500. The implied market value of the liabilities adjusted for default equals the market value of the assets minus the market value of the equity adjusted for default, or **890** = 1400 – 510.

Assume that the risk-free interest rate applicable to valuing the insurer's expected liability payments is 6% and that the liability payment pattern is 10% per year for 10 years (paid at the end of each year). The present value of the liabilities at the risk-free rate is 730. Thus, the risk margin, expressed in dollars is **160** = 890 – 730. Alternatively, the interest rate that gives a present value of 890 using the above payment pattern is 2.18%. This value implies a risk adjustment of **3.82%**.

The following discussion provides an example of the Ronn and Verma method.

Let A be the market value of assets, L the market value of liabilities and s the volatility of the asset/liability ratio. The formula for the owners' equity, where there is a possibility of default, is the call option with expiration in one year:

(1) $$E = A \cdot N(d) - L \cdot N(d - s),$$

where $d = \ln(A/L)/s + s/2$ and $N(d)$ is the standard normal distribution evaluated at d.

Notice that equity value with no default is simply $E_n = A - L$. For an insurer with stochastic assets and liabilities, s_E, the volatility of the equity, is related to the asset/liability volatility by

(2) $$s_E = N(d)As/E.$$

Equations (1) and (2) are solved simultaneously to get E and s .

The expected default value equals $E - E_n$, or the derived market value of the equity minus the equity value with no default.

The method is easily demonstrated with a numerical example. Assume that A =130, L = 100 and s_E = 0.5. Solving the simultaneous equations gives E = 40.057 and s = 0.117. Therefore, the value of the expected default is

0.057 = 40.057 – 40.000.

For an insurer, the market value of assets is readily determined from the published balance sheet. Discounting the reserves at a risk-free rate can approximate the market value of liabilities. The equity volatility can be estimated by analyzing the insurer's stock price over a recent time frame, as done by Allen, Cummins and Phillips.