CAS Task Force on Fair Value Liabilities
White Paper on Fair Valuing Property/Casualty Insurance Liabilities

Section K - Appendices

Appendix 6: Using Aggregate Probability Distributions

This example uses the Collective Risk Model to compute a risk load. It represents only one of the many approaches based on aggregate probability distributions. This is in order to keep the illustration simple.

The approach is based on the following model for risk load:

- Risk Load = λ SD[Loss] or Risk Load =λ Var[Loss],

Therefore, in order or compute a risk load, two quantities are needed: λ and Var[Loss], since SD(Loss) = $\text{Var[Loss]}^{1/2}$. The following algorithm from Meyers (Meyers, 1994) will be used to compute the variance of aggregate losses.

The Model:

1. Assume claim volume has an unconditional Poisson distribution.

2. Assume the Poisson parameter, *n* (the claim distribution mean), varies from risk to risk.

3. Select a random variable *c* from a distribution with mean 1 and variance c.

4. Select the claim count, *K*, at random from a Poisson distribution with mean *cn*, where the random variable *c* is multiplied by the random Poisson mean *n*.

<u>The Variability of Insurer Losses</u>

5. Select occurrence severities, Z_1, Z_2, .., Z_K, at random from a distribution with mean *m* and variance s^2.

6. The total loss is given by:

$$X = \sum_{j=1}^{K} Z_j$$

The expected occurrence count is *n* (i.e. E[(*c n*] = E[n] = n). *n* is used as a measure of exposure.
When there is no parameter uncertainty in the claim count distribution *c* = 0,

$$\text{Var}[x] = n\,(m^2 + s^2),$$

and variance is a linear function of exposures.

When there is parameter uncertainty:

$$\text{Var}[x] = nu + n^2 v,$$

where

$$u = (m^2 + s^2)$$

and

$$v = cm^2$$

nu is the process risk and n^2v is the parameter risk.

For example, assume an insurer writes two lines of business. The expected claim volume for the first line is 10,000 and the expected claim volume for the second line is 20,000. The parameter c for the first line is 0.01 and for the second line is 0.005. Let the severity for line 1 be lognormal with a mean of $10,000 and volatility parameter (the standard deviation of the logs of losses) equal to 1.25 and the severity for line 2 be lognormal with severity of $20,000 and volatility equal to 2. Applying the formula above for the variance of aggregate losses, we find that the variance for line 1 is $1.05 \text{x}\ 10^{14}$ and the variance of line two is $1.24 \text{ x } 10^{15}$ and the sum of the variances for the two lines is $1.34 \text{ x } 10^{15}$. The standard deviation is $36,627,257.

One approach to determining the multiplier l would be to select the multiplier ISO uses in its increased limits rate filings. In the increased limits rate filings, l is applied to the variance of losses and is on the order of 10^{-7}.(Meyers, 1998)

In recent actuarial literature, the probability of ruin has been used to determine the multipliers of SD(loss) or Var(Loss). (Kreps 1998, Meyers 1998, Philbrick, 1994). The probability of ruin or expected policyholder deficit is used to compute the amount of surplus required to support the liabilities. To keep the illustration simple, we use the probability of ruin approach. However, the expected policyholder deficit or tail value at risk (which is similar to expected policyholder deficit) approaches better reflect the current literature on computing risk loads. Suppose the company wishes to be 99.9% sure that it has sufficient surplus to pay the liabilities, ignoring investment income, the company will require surplus of 3.1 times the standard deviation of losses, if one assumes that losses are normally distributed.[2] In order to complete the calculation, we need to know the company's required return on equity, r_e. This can be determined by examining historical return data for the P&C insurance industry. Then the required risk margin for one year is r_e x 3.1x 36,627,257. For instance, if r_e is 10% then the risk margin is

[2] If one assumes that aggregate losses are lognormally distributed, then the company needs approximately $e^{(2.33*.06)}*$ the expected losses as surplus, where .06 is the volatility parameter, derived from the mean and variance of the distribution..

11,354,450 or about 2.0% of expected losses. In this example, the parameter lambda is equal to 3.1 r_e. The result computed above could be converted into a risk margin for discounted losses by applying the 2% to losses discounted at the risk free rate. This would require the assumption that the risks of investment income on the assets supporting the losses being less than expected is much less than the risk that losses will be greater than expected. When the assets supporting the liabilities are primarily invested in high quality bonds, this assumption is probably reasonable. (see D'Arcy et. al., 1997)

Philbrick in his paper commissioned by the CAS "Accounting for Risk Margins" had a slightly different approach to determining the risk margin. Philbrick's formula for risk margin, given a total surplus requirement *S*, (i.e. 3.1* standard deviation in this example), a rate of return on equity r_e and a risk free rate *i* is:

$$RM = \frac{(r_e - i)\,xS}{1 + r_e}$$

This is a risk margin for discounted losses not undiscounted losses. . The formula above assumes that some of the required return on surplus is obtained from investing the surplus at the risk free rate. If i = 5%, and r_e = 10% the risk margin in this illustration would be $5,161,113.

In this example, it should be noted that the majority of the standard deviation is due to parameter risk, as process risk for such large claim volumes is minimal. However, only parameter risk for claims volumes has been incorporated. A more complete model would incorporate parameter risk for the severity distribution. This risk parameter has been denoted the "mixing parameter" in the actuarial literature. The algorithm for incorporating this variance into the measure of aggregate loss variance is as follows:

1 - 5. Follow steps 1 through 5 above, describing the selection of frequency and severity parameters for a distribution
6. Select a random variable *B* from a distribution with mean 1 and variance *b*.

7. The total loss is given by:

$$X = \sum_{j=1}^{K} Z_j / B$$

The variance reflecting the mixing parameter is given by:

$$\text{Var}[x] = n(1+b)(m^2 + s^2) + n^2(b + c + bc)m^2.$$

Procedures for estimating *b* and *c* are provided by Meyers and Schenker. The procedures use the means and variances of the claim count and the loss distribution to compute *b* and *c*. The parameter *b* can also be viewed as the uncertainty contributed to the total estimate of losses due to uncertainty in the trend and development factors. Methods for measuring the variance due to development are presented by Hayne, Venter and Mack. Regression statistics containing information about the variances of trend factors are published in ISO circulars and can be developed from internal data. To continue our example, we will assume that the *b* parameter for line 1 is 0.02 and for line 2 is 0.05. Then the standard deviation of aggregate losses is $95,663,174. The risk load using Philbrick's formula is $13,479,811 or 2.7% of expected undiscounted losses. The load is intended to be applied to discounted liabilities where liabilities are discounted at the risk free rate. Thus if losses take one year to pay out the risk margin is 2.8% of the present value of liabilities.

The above risk load is consistent with liabilities that expire in one year. When losses take more than one year to pay, Philbrick uses the following formula to derive a risk load.

$$RM = \sum_j \frac{(r_e - i) S_j}{(1 + r_e)^j}$$

This formula can be applied to liabilities of any maturity. Where S_j is the surplus requirement for outstanding liabilities as of year *j*. In the above example if losses pay out evenly over 3 years then the risk margin is $20,693,737or 4.6% of he discounted liabilities. The calculation is shown below.

(1)	(2)	(3)	(4)	(5)
	Surplus			
t	.227*PV(OS Losses)	1/(1+r(e)^t	(3)*(2)	(r(e)-.05)*(4)
0	219,965,641	1.000	219,965,641	10,998,282
1	146,643,760	0.909	133,312,510	6,665,625
2	73,321,880	0.826	60,596,595	3,029,830
				20,693,737

The computation above assumes that the relative variability of the liabilities remains constant as the liabilities mature. As this may not be the case, refinements to the measure of variability by age of liability may be desirable. One approach to modeling the uncertainty in reserves would derive measures of variability from observed loss development variability. This is the approach used by Zenwirth, Mack and Hayne. Another approach, consistent with how risk base capital is computed, would measure historic reserve development for P&C companies for a line of business from Schedule P.