Appendix 4: Using Underwriting Data

This appendix describes Butsic's procedure for computing risk adjusted discount rates. The following relationship is used for the computation.

$$C = P(1+i)^{-u} - E(1+i)^{-w} - L(1+i_A)^{-t}$$

Where:

- C is the cash flow on a policy and can be thought of as the present value of the profits, both underwriting and investment income, on the policy,
- P is the policy premium,
- E is expenses and dividends on the policy,
- L is the losses and adjustment expenses,
- $\underline{u}$ is the average duration of the premium, or the average lag between the inception of the policy and the collection of premium,
- w is the average duration of the expenses,
- t is the average duration of the liabilities.
- i is the risk free rate of return
- i_A is the risk adjusted rate of return

This formula says that the present value cash flow or present value profit on a group of policies is equal to the present value of the premium minus the present value of the components of expenses minus the present value of losses. Premiums and expenses are discounted at the risk free rate. Each item is discounted for a time period equal to its duration, or the time difference between inception of the policy or accident period and expiration of all cash flows associated with the item. Losses are discounted at the risk-adjusted rate. Underwriting data in ratio form, i.e., expense ratios, loss ratios, etc. can be plugged into the formula. When that is done, P enters the formula as 1, since the ratios are to premium.

In ratio form this formula would be:

$$c = 1(1+i)^{-u} - e(1+i)^{-w} - l(1+i_A)^{-t}$$

- c is the ratio of present value profit to premium
- e is the expense ratio, including dividends to policyholder
- l is the loss ratio

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Using as a starting point the rate of return on surplus, where the surplus supporting a group of policies is assumed to be eV_m, or the leverage ratio times the average discounted reserve, Butsic (Bustic, 1988) derived the following simplified expression for the risk adjustment:

$$Z = e(R - i) = (1 + i)C/V_m \ ,$$

where:
Z is the risk adjustment to the interest rate or the percentage amount to be subtracted from the risk free rate = $e(R - i)$
C and i are as defined above
V_m is the average discounted reserve for the period

V_m is generally taken as the average of the discounted unpaid liabilities at the beginning of the accident or policy period (typically 100% of the policy losses) and the discounted unpaid liabilities at the end of the period. In general, this would be equal to 100% plus the percentage of losses unpaid at the end of the period (one year if annual data is used) divided by 2. The discount rate is the risk-adjusted rate. If V_m is computed as a ratio to premium, then published loss ratios are discounted and used in the denominator.

To complete the calculation, the quantity c, or the ratio of discounted profit to premium should be multiplied by $(1 + i)$ and divided by v_m (V_m in ratio form). To derive initial estimates of the risk adjustment, it is necessary to start with a guess as to the value of the risk adjustment to the discount rate in order to obtain a value for discounted liabilities.

The following is an example of the computation of the risk adjustment using this method. It is necessary to start with a guess for the risk adjustment and then perform the calculation iteratively until it converges on a solution. This example is based on data in Butsic's (1988) paper.

Parameter assumptions	
Interest Rate R_f	0.0972
Fraction of losses OS after 1 year	0.591
Initial Risk Adjustment	0.044

Variable	Nominal Value	Duration	Discounted Value
1 Loss&LAE	0.767	2.300	0.681
2 Premium	1.000	0.250	0.977
3 UW Expense	0.268	0.250	0.262
4 Pol Dividends	0.016	2.250	0.013
5 Average Liabilities	0.610	1.800	0.556

	Calculation	
6	Premium-Expenses Discounted (2) - (3) - (4)	0.702
7	Premiums-Expenses-Losses Disc (6)-(1)	0.021
8	C*(1+I) (7)*(1+I)	0.024
9	Z=C*(1+I)/V_m (8)/(5)	0.042

An additive risk load

An additive or dollar risk load can be computed from the same data. The formula for the computation of a risk load is:

$$c = p(1+i)^{-u} - e(1+i)^{-w} - l(1+i)^{-t}$$

$$rl = c / l(1+i)^{-t}$$

Where *rl* is the additive risk load and *i* is the risk free interest rate.
An example is shown below:

Parameter assumptions	
Interest Rate Rf	0.0972

	Variable	Nominal Value	Duration	Discounted Value
1	Loss&LAE	0.767	2.300	0.620
2	Premium	1.000	0.250	0.977
3	UW Expense	0.268	0.250	0.262
4	Pol Dividends	0.016	2.250	0.013

	Calculation	
5	Premium-Expenses Discounted (2) - (3) - (4)	0.702
6	C =Premiums-Expenses-Losses Disc (5)-(1)	0.083
7	C/PV(Losses) (6)/(1)	0.133