

CAS Task Force on Fair Value Liabilities
White Paper on Fair Valuing Property/Casualty Insurance Liabilities

Section K - Appendices

Appendix 3: Single Period RAD model

All balance sheet values are at fair value.

The discussion of the income tax liability is the same as in Appendix 2.

Here, there is no iteration needed, since the risk adjustment is derived directly from the equations relating the variables to each other. Butsic (2000) derives this result.

The formula is

$$z = c \left[\frac{R - r_f}{1 - t} \right] + (r_A - r_f) \left[1 + c \frac{1 + r_f}{1 + r_f(1 - t)} \right],$$

where the variables are:

- z risk adjustment to the risk-free rate
- c capital as a ratio to the fair value of the liability
- R required rate of return on capital (ROE)
- r_A expected return on assets (includes bond yields net of expected default)
- r_f risk-free rate
- t income tax rate

Although the risk adjustment can be calculated directly from the above formula, we have provided Exhibit A3, which shows that the risk adjustment in fact produces the required ROE and internal rate of return. The format of Exhibit A3 is similar to that of Exhibit A2. However, only a single time period is needed.

Note that exhibits A2 and A3 give slightly different results for the risk adjustment. This is because capital is needed for both asset and liability risk. In a multiple period model, the relationship between the assets and loss reserve fair value is not strictly proportional. This creates a small discrepancy.

CAS Task Force on Fair Value Liabilities
White Paper on Fair Valuing Property/Casualty Insurance Liabilities

Section K - Appendices

Exhibit A3

Calculation of Risk Adjustment Using Single Period ROE Model

Fixed Inputs		
1	Risk-free rate	0.060
2	Expected investment return	0.080
3	Income tax rate	0.350
4	Equity beta	0.800
5	market risk premium	0.090
6	Capital/reserve	0.500
7	Loss & LAE	1000.00
8		
Calculated values		
10	Required ROE	0.1320
11	Risk-adjusted yield	0.0348
12	After-tax risk-free rate	0.0390
13		
14	Premium	981.38
15	Risk adjustment	0.02518
16		
Balance sheet, at fair value		
17		Time
18		0 1
19	<i>Assets</i>	
20	Investments, before dividend	976.12 546.96
21	Investments, after dividend	1459.30 0.00
22		
23	<i>Liabilities</i>	
24	Loss & LAE	966.35 0.00
25	Income tax liability	15.02 0.00
26	Capital, before dividend	0.00 546.96
27	Capital after div (required amount)	483.18 0.00
28		
Income		
30	Underwriting income	15.02 -33.65
31	Investment income	
32	<u>Net income, pretax</u>	<u>15.02 83.10</u>
33	Inv income, capital (risk-adjusted)	28.99
34		
Insurance Cash Flows		
36	Premium	981.38 0.00
37	Loss & LAE	0.00 -1000.00
38	Income tax	-5.26 -29.08
39		
40	Income tax, capital (risk-adjusted)	10.15
41		
42	Capital flow (dividend)	483.18 -546.96
43		
44	ROE	13.20%
45		
46	Internal rate of return	13.20%

CAS Task Force on Fair Value Liabilities
White Paper on Fair Valuing Property/Casualty Insurance Liabilities
Section K - Appendices

Notes to Exhibit A3

Rows (Note that “R1” denotes Row 1, “R2” denotes Row 2, etc.):

1. Rate for portfolio of U. S. Treasury securities having same expected cash flows as the losses.
2. Expected return for the insurer’s investment portfolio. Note that the yield on a bond is not an expected return. The yield must be adjusted to eliminate expected default. Municipal bond yields are adjusted to reflect the implied return as if they were fully taxable.
3. Statutory income tax rate on taxable income.
4. Estimates can be obtained from Value Line, Yahoo Finance or other services.
5. Estimates are commonly available in rate filings (e.g., Massachusetts).
6. All-lines value can be estimated by adjusting historical industry reserve values to present value and adding back the after-tax discount to GAAP equity. See Butsic (1999) for an example. For individual lines, a capital allocation method can be used, such as Myers and Read (1999).
7. An arbitrary round number used to illustrate the method.
10. $R1 + (R4 \times R5)$.
11. $R1 - R15$
12. $(1 - R3) \times R1$
14. $R24 + R25$ (at time 0).
15. $R6 \times (R10 - R1) / (1 - R3) - (R2 - R1) \times [1 + R6 \times (1 + R1) / (1+R12)]$.
20. $R21$ (Prior Year) + $R36 + R37 + R38$.
21. $R20 + R42$.
24. Present value of $R7$ using interest rate $R11$.
25. Present value of $R40$ using interest rate $R12$. Result is divided by $(1 - R3)$.
26. Time 0: 0; Time 1: $R20 - R24 - R25$.
27. $R6 \times R24$.
30. Time 0: $R36 - R24$. Time 1: $- R11 \times R24$ (Prior Year).
31. $(R21, \text{Prior Year}) \times R2$.
32. $R30 + R31$.
33. $(R27, \text{Prior Year}) \times R1$.
36. $R14$.
37. Time 0: 0. Time 1: $- R7$.
38. $- R3 \times R32$.
40. $R3 \times R33$.
42. $R27 - R26$
44. $(R26, \text{Time 1}) / (R27, \text{Time 0}) - 1$.
46. Internal rate of return on Row 42 cash flows.