Volume LXXXVI, Part 2 **No. 165**

PROCEEDINGS
November 14, 15, 16, 17, 1999

RESIDUAL MARKET PRICING

RICHARD B. AMUNDSON

Abstract

Residual market plans often review their rates based on the experience of the plans themselves. The typical result is an indication for a large increase, which the regulator then judgmentally reduces. To the extent that equilibrium exists between voluntary and residual markets, it results from ignoring the indications. Plans' experience can call for rate decreases as well as increases, especially with no allowance for profit. Indications for decreases are politically harder to ignore and could destroy the voluntary market if followed.

Break-even residual market pricing, if truly followed, has unpredictable consequences on prices and market shares for the residual and the voluntary markets. This paper proposes an alternative to break-even pricing. With input from all concerned, a state should first

establish specific goals for the residual market plan in terms of market share, burden on insureds in the voluntary market, and maximum surcharge for insureds in the plan. Regulators can then set plan prices at a consistent level above voluntary prices to meet the established goals.

1. INTRODUCTION

1.1. A Paradox

In 1983, the State of Minnesota merged its departments of insurance, banking, and securities into a single Department of Commerce. The first commissioner of the newly created Department was determined to keep consumer prices down wherever possible. Among the duties of the Department was to review the rates of the assigned risk plan (ARP). During the seven years ending with 1989, despite many requests for rate increases, the Department allowed only a single increase in the state's private passenger automobile assigned risk plan. At the beginning of that period, ARP judged its rates to be adequate; at the end, ARP calculated a needed increase of 10.3%, with the one increase in the interim being 14.8%. That implies an average annual needed increase of 3.4% during those seven years ($1.034 = (1.103 \times 1.148)^{1/7}$). Annual increases of 3.4% were modest at the time, so the commissioner's strategy of holding down ARP rates appeared to be successful.

A change of commissioners in 1989 brought a new philosophy, one that permitted ARP rates to rise. Between 1989 and 1994, ARP took increases of 12.0%, 20.4%, 19.5%, and 13.8%; and there was still an indication of 33.7% at the end of that period. That implies an average annual needed increase of 17.3% during those five years. ARP was smaller at the end of the period, but the goal that it be self-supporting was as far away as ever. Loss ratios stayed high as rates went up, and the drivers that remained insured with ARP had little to celebrate. External economic indices did nothing to explain the sudden shift from annual cost increases of 3.4% to increases of 17.3%. The only obvious change was the Department's change in attitude toward change itself: the culprit appeared to be the strategy of letting ARP follow its own indicated rate increases.

1.2. An Actuarial Explanation

None of this was hard to explain. In the beginning, insurers rejected only the very most unwanted drivers—the worst of the worst. They were happy to write a borderline driver for $1,000. But when inflation pushed the voluntary market price for that driver's policy up to $1,100, ARP, whose rates had not budged, might write the driver at $1,050. These borderline drivers moving into ARP were the best of the worst, and they improved the quality of ARP's book of business as it grew. Exactly the opposite occurred when ARP shrank. When ARP's prices began increasing faster than those of the voluntary market, ARP's insureds began moving to the voluntary market to get better prices. The voluntary market was interested only in the best of ARP's business, of course; and, when ARP lost its best customers, its loss ratio began to climb.

After years of increases, when things were back to the original balance between voluntary and assigned risk, the indications for ARP were as high as ever. The actuary at the Department wrote a memo explaining why this was and what one might have to do in the future to keep everything in balance. To continue following indications blindly seemed sure to lead to the disappearance of ARP—not a bad idea in the eyes of some, but not politically viable in this case. The presence of a contingency factor in the analysis posed a problem; it added to the price of each policy, not unlike a profit margin, even though this was non-profit business. ARP rates tended to rise mercilessly; and the contingency factor only exacerbated the tendency, pushing rates for the dwindling number of policyholders to truly unaffordable levels. It seemed a good idea to get rid of the contingency factor.

1.3. A Second Paradox

The Department also regulates the workers compensation assigned risk plan. In 1995, something surprising began to occur: this ARP, whose rates were already low, needed rate *decreases*. Whether this was just random noise or a true reflection of the risks in ARP, it seemed unwise for the rates to get too close to the voluntary market rates. The voluntary market charges for the same coverages as ARP but, in addition, charges for profit because of the risk of writing business. The ARP analysis had no charge for risk even though, of course, the ARP business is just as risky as the voluntary business. This gave ARP a rate advantage—it could pick up market share and constantly improve its book, and the voluntary market could eventually disappear. The Department actuary reasoned that one might prevent that disaster by including a contingency factor in the analysis to keep rates from falling too low.

All this was strangely familiar. The same actuary (who happens to be the author of this paper) had argued, not so long before, against a contingency factor in the case of auto assigned risk. What was wrong? What was the truth?

1.4. The Scales Fall From Our Eyes

The truth is all of the above. Both of these scenarios can happen, even though they are complete opposites. A residual market that bases its prices on its own experience has no certainty of reaching an acceptable equilibrium, as this paper will demonstrate. To achieve the goals normally desired for an assigned risk plan, the state should base the plan's rates on voluntary market rates and not on the plan's own experience.

2. A MODEL OF RESIDUAL MARKET PRICING

2.1. Some Assumptions

We will look at residual market plans that set prices to break even based on their own experience. Of course, with break-even pricing, a plan may still realize profits or losses. The plan design may or may not give the profit to insurers, but it will virtually always give insurers the loss. The examples in this paper assume that insurers get the profit as well as the loss. The conclusions of the paper are still valid if insurers do not get the profit, but the examples are a bit more complex.

We will ignore self-insurance. Assume that all employers, drivers, etc., must buy insurance and that they have two options: an insurance company in the voluntary market or ARP, our surrogate for all residual markets. Assume further that within each classification there is a continuum of expected losses per exposure: there are insureds with very few losses expected for each exposure unit, there are others with very high expected losses, and there is everything in between.

Let us look at a simplified financial model that illustrates some important relationships between the residual and voluntary markets. First suppose there is no ARP. Now imagine an insurer that needs a \$100 investment in surplus to take on \$200 of expected loss at the end of the coming year and that there are no expenses. Further suppose that one can earn 5% risk-free on invested assets and that, given the uncertainty in the expected losses, the insurer needs a 15% return on the venture. Thus, if it collects \$200 in premium up front and invests it along with the \$100 of surplus, it will earn \$15 during the year. Then if losses materialize as expected, the insurer will pay out \$200 at year-end and will keep the original \$100 plus \$15 of investment income—the expected return is exactly what the insurer needs.

From the extreme where a for-profit, voluntary market collects all the premium, let us go to the opposite extreme where the non-profit ARP collects all the premium and pays the entire \$200 of loss. The voluntary insurer now has no premium, but it has continued responsibility for potential bottom line losses of ARP. Even with no premium, the insurer still needs the entire \$100 in surplus that it needed when it was the one collecting premium and paying claims. That \$100 was to protect against insolvency, and all the risks that it protected against still exist. Not only do they still exist, but they are all on the back of the insurer. ARP carries no surplus and assesses the insurer for any losses at the end of the day, whether they arise from excessive claims or from investments or from anything else.

Remember, moreover, that one can get a return of \$5 with no risk. An insurer might want to add some risk in exchange for an increased return. In the extreme case where the insurer has no premium, though, if the insurer did not share in ARP's profit, it would be taking on risk in exchange for a *decreased* return. The insurer will be interested in assuming ARP's risk only if it gets the full profit that it would have gotten in the absence of ARP. In order to realize the full profit, ARP must charge the full \$200 of premium. Thus, *no matter what market share ARP has*, the system still needs the full \$100 of surplus and the full \$200 of premium.

The preceding argument assumed that private insurers are at risk for residual market losses, so one might be tempted to assume that the result does not hold in the absence of private insurance. By eliminating private insurance, might premiums be reduced? No. The argument did not rely on the private status of the insurers; the risk remains whether or not private investors are bearing it. The risk takers, whether taxpayers or policyholders, will put up the surplus and reap the rewards explicitly or implicitly.

Let us turn our attention away from the extremes and consider the more usual case. Typically, ARP will have part of the market and insurers will have the rest. Consider a single premium group: all insureds of like size in a single class. Suppose ARP charges a premium of R for a member of this group. ARP may vary its rate somewhat due to merit rating; but, unlike the voluntary market, it does not do any underwriting, so it will not charge the variety of rates typical of the voluntary market. Assume that ARP charges the same rate to all insureds in the group. The voluntary market by contrast, through the forces of underwriting and competition, charges a rate proportional to expected losses. This will result from a combination of schedule rating, experience rating, retrospective rating, and underwriting by companies with differing rates and differing niches. Remember that there is a whole spectrum of expected losses. For the moment, assume that the underwriting cost is negligible; it will not change the result to assume it is significant, but it clutters the argument. Let the market price be ax, where x is the expected loss. In order to attract any business the market must charge less than ARP.

2.2. A Natural Limit: Assigned Risk Must Charge Strictly More Than Market Average

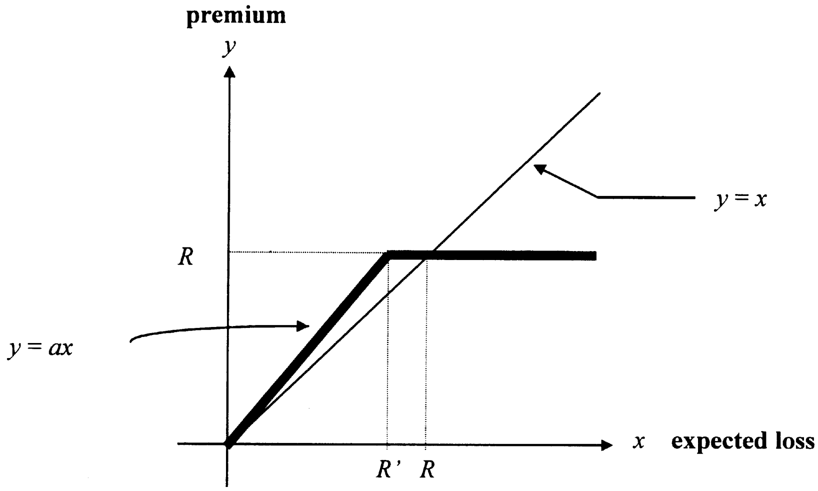
The graph in Figure 1 illustrates the market in equilibrium. The x-axis represents expected losses; the y-axis, premium. We continue to ignore expenses and to assume that investment income alone will generate appropriate profit for insurers. In an unfettered market, the line $y = x$ represents the appropriate relationship between premium and loss. With ARP charging a premium of R and the voluntary market charging ax, the bold curve represents actual prices charged. If the expected loss is greater than R', where $R' = R/a$, ARP will write the risk. If the expected loss is less than R', the voluntary market will write the risk.

Insureds whose expected losses are less than R pay more than they would in a completely free market, while insureds whose expected losses are greater than R pay less.

We will show that if $R \leq L$, where L is the average expected loss, there is no solution to the pricing problem of insurers. That is, there is no premium they can charge that would attract customers and would give them enough to pay claims and adequately reward them for the risks they would be taking.

FIGURE 1

THE MARKET IN EQUILIBRIUM



If $R > L$, there is a solution but it is not necessarily stable. If R increases or decreases depending on ARP's own experience, ARP will most likely not be in equilibrium: it will grow or shrink depending on the distribution of expected losses.

For the case $R \leq L$, it is almost self-evident that insurers can not compete. If there are n insureds, the total premium needed is nL. If ARP has m insureds, its premium will be mR. The voluntary market must then collect a total of $nL - mR$ from the remaining $n - m$ policyholders. If $R \leq L$, then $(nL - mR)/(n - m) \geq (nR - mR)/(n - m) = R$; that is, the voluntary market would have to charge on average *at least* as much as ARP.

Given that there is a continuum of expected losses, one can prove the stronger result that the insurers' pricing problem is solvable if and only if R is strictly greater than L. Furthermore the solution, when it exists, is unique.

This is easy to visualize with the help of Figure 1. The bold curve on the graph represents the prices of the combined voluntary and residual markets. The line $y = x$ represents prices in the absence of a residual market. The voluntary market seeks a value of a for which the overall average price of the bold curve is exactly the same as for the line $y = x$. For small values of a, the entire bold curve will be below the line $y = x$. As a increases, the bold curve approaches the horizontal line $y = R$. The average price will increase from 0 to R as a increases from 0 to ∞, but the average will never quite reach R for any finite value of a. Thus if $R \leq L$, the average price generated by the bold curve can never be as great as L, the average generated by the line $y = x$. If $R > L$, there must be some point at which, as a approaches ∞, the average price represented by the bold curve equals L. (Appendix A provides a more complete proof of this result.)

What this has demonstrated so far is that, however ARP sets its rates, it should not simply gear them to the average risk. They must be higher; otherwise the voluntary market will deconstruct. The danger that ARP will gear its rates to the average risk increases as ARP's market share increases. Because the argument above applies to a single class, the danger is not limited to the case where average ARP rates are higher than the overall market rates—ARP can take over the market segment by segment. If ARP sets its rates for the average risk and, in addition, includes no allowance for profit, the voluntary market has no choice but to abandon the segment in question.

3. THE ELUSIVE SEARCH FOR EQUILIBRIUM

3.1. The Rate Review

Let us suppose that $R > L$ and that the market has spent some time in equilibrium in the sense that the relative prices and market shares of ARP and the voluntary market have remained stable. Now the time has arrived for ARP to review its rates. What happens? Look back to the graph in Figure 1. ARP has been overcharging insureds with expected losses between R' and R and undercharging those with expected losses greater than R. The net effect is an undercharge, which the voluntary market makes up by overcharging all its insureds.

Because ARP has been undercharging, shouldn't its experience indicate that it needs an increase? Not necessarily. ARP has been undercharging when one considers the need for profit, but ARP does not include a profit margin in its rate analysis. It is possible that ARP has charged enough to pay claims and that its analysis on a non-profit basis will show a need for a rate decrease. This is not the normal course of events with residual market plans, but it is possible, especially for individual segments of the market. Whether ARP's analysis will show the need for an increase or for a decrease is a function of the distribution of expected losses. One can construct distributions that go both ways, as the examples in Tables 1 through 4 (discussed later in this paper) will illustrate.

If ARP uses a market-level profit margin in its analysis, it will generally see the need for an increase. Residual market plans often do include a "contingency" allowance, which serves somewhat the same purpose and does increase the probability that the analysis will indicate the need for a rate increase. For just the right distribution, just the right value of R, and just the right contingency factor, equilibrium may occur; but it will be precarious.

The tendency is rather for continual indications for rate increases, or continual indications for decreases. In the first case, if ARP follows the indications, it will eventually price itself out of existence; in the second case, it is the voluntary market that will disappear if ARP follows the indications. The more common scenario is the first; and equilibrium usually occurs only because ARP ignores the indications: ARP takes lesser increases at the insistence of the regulator. Because this is an inherently unpredictable road to equilibrium, it opens the door to many problems.

The more serious scenario, and fortunately the more rare so far, is the one in which ARP sees a need for a decrease. It is more serious because if ARP follows its indications under this scenario, the voluntary market may well disappear. As in the case where increases are indicated, the only sure way to remain in equilibrium is to ignore the indications; but that is not easy in the face of political pressures to lower rates. Let us look at some simple, finite examples that show the two possibilities.

3.2. Assigned Risk Plans That Follow Their Own Experience May Grow

First, continuing with our earlier assumptions, imagine a distribution of expected losses with ten equally likely possible outcomes: the integers ranging from 20 to 29. The voluntary market with its diversity of players and underwriting capabilities distinguishes among policies with different expectations and charges accordingly, while ARP takes all comers at the same price. The voluntary market sets its prices for a break-even underwriting return, getting its profit from investment income. ARP prices at a 5% discount in order to break even *after* investment income (i.e., ARP is non-profit). Table 1 summarizes this situation.

X is the random variable representing a policy's expected losses, with its ten possible outcomes (in column 1) each having a probability of 0.10 (column 2). The data in columns 3, 4, 5 and 6 assume that ARP writes all risks with expected losses greater than the value of x in column 1. If ARP writes all the risks with expected losses greater than 20, for example, it will have to charge 23.81 per risk in order to break even (column 4, first row). With investment income, it will have $25.00 = 23.81 \times 1.05$ to pay claims (25.00 is the average value of expected losses for policies whose expected losses are greater than 20).

The first entry in the third column, 30.71, is what the voluntary market would have to charge for a risk with expected losses of 20, given that ARP writes everything with greater expected losses. The voluntary market must collect not only the

TABLE 1
AN EXPANDING ARP WITH LIMITED EQUILIBRIUM

(1)	(2)	(3)	(4)	(5)	(6)
		If ARP writes all risks with expected losses greater than x			
x	$P[X = x]$	voluntary market rate for x	ARP rate (for all)	voluntary market rate for $x+1$	1: ARP gains -1: ARP loses 0: equilibrium
20	0.10	30.71	23.81	32.25	1
21	0.10	25.98	24.29	27.21	1
22	0.10	25.03	24.76	26.16	1
23	0.10	25.02	25.24	26.11	0
24	0.10	25.40	25.71	26.46	0
25	0.10	25.97	26.19	27.01	0
26	0.10	26.65	26.67	27.67	0
27	0.10	27.39	27.14	28.40	1
28	0.10	28.18	27.62	29.19	1
29	0.10	29.00			

24.5 = average expected loss = $E[X]$

Column (4): ARP rate = $A(x) = E[X \mid X > x]/1.05$
Column (3): vol mkt rate for $x = V(x) = ax$,
where $a = (E[X] - A(x) \cdot P[X > x])/(E[X \mid X \leq x] \cdot P[X \leq x])$
Column (5): vol mkt rate for $x + 1 = V(x) \cdot (x+1)/x = a \cdot (x+1)$

20 needed to pay the claims and provide for the profit for the risks that it writes, but it must also collect enough to provide for the profit on all the risks that ARP writes, since it (and not ARP) is taking on the risk. The combined premium that the voluntary market and ARP collect would then be, on average, 24.5 ($0.1 \times 30.71 + 0.9 \times 23.81$). The overall expected loss is 24.5 and exactly what is needed to keep the voluntary market in the game. That forces the voluntary market to charge more than ARP ($30.71 > 23.81$), so the voluntary market would lose the risks with expected losses of 20 to ARP in this situation. The 1 in the sixth column of the first row is a flag to indicate that ARP would capture this risk, too, once it had all the larger risks.

We assume that the voluntary market uniformly loads its expected ARP assessment by applying the multiplier, a to the rate that it would otherwise charge. The voluntary market would then charge 32.25 (column 5) for a risk with expected losses of 21, again given that ARP writes everything with expected losses greater than 20. If the voluntary market rate in column 5 were less than the ARP rate in column 4, then ARP would lose the risks with expected losses of 21 to the voluntary market; in that case, the flag in column 6 would be set to -1. A zero in column 6 indicates equilibrium, and occurs when the voluntary market rate for x is less than the ARP rate, which is in turn less than the voluntary market rate for $x+1$ (i.e., column 3 < column 4 < column 5).

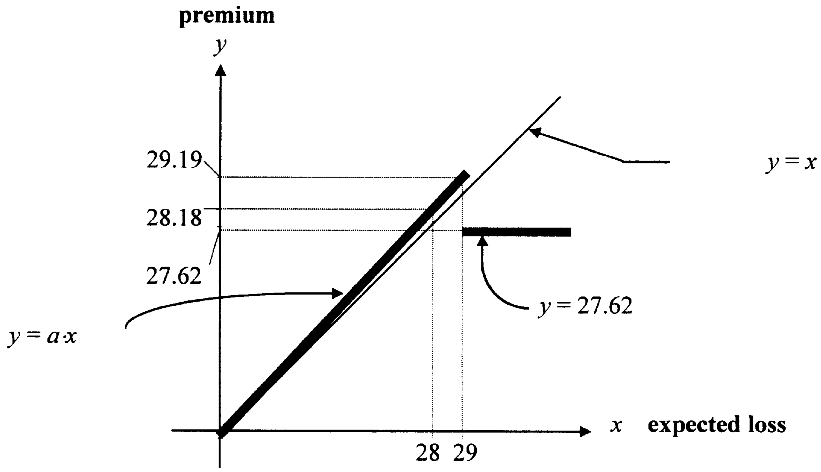
Each row represents a distinct rating scenario: the columns of voluntary market rates for x and $x+1$ are not lists of rates all of which would be available at the same time. For example, the table contains two voluntary market rates for risks with expected losses of 21: 32.25 in row 1, column 5, and 25.98 in row 2, column 3. 32.25 is the voluntary market rate if ARP writes everything greater than 20, while 25.98 is the voluntary market rate if ARP writes everything greater than 21. The full schedule of voluntary market rates is not displayed for every ARP rate; the table displays only the two rates (in columns 3 and 5), which lie at the boundary of ARP's book of business for the row in question. To know if ARP will grow or shrink or remain in equilibrium, we need only look at the boundary.

For each row of Table 1, one could construct a graph similar to that in Figure 1. Figure 2, for example, corresponds to the row $x = 28$ of Table 1. As in Figure 1, the bold line segment through the origin represents the premium that the voluntary market charges, while the bold horizontal segment represents ARP's premium. The premiums represented by the bold line segments generate an average premium of $L = E[X]$, just as in the case of Figure 1. The obvious difference is that the graph in Figure 2 is discontinuous.

For Figure 1, we required the two segments to join at (R', R); and we varied R' (by varying a) to obtain an adequate total premium, without regard for the adequacy of ARP by itself. We

FIGURE 2

THE MARKET IN DISEQUILIBRIUM



showed that, for $R > L$, there is always an R' that solves this problem.

For Figure 2, we fix the left end point of the horizontal ARP segment at 29 on the x-axis and allow the segment to move up or down until ARP's premium balances its own discounted expected losses. The voluntary market segment then pivots at the origin to attain the desired total premium. The discontinuity in the graph represents a state of disequilibrium between ARP and the voluntary market. ARP is momentarily in balance but the system is not: ARP sets its rates for one group of insureds, but the rates themselves will cause that group to change.

If ARP starts out writing only risks with expected losses greater than 28, it will charge 27.62 (29.00/1.05). Because the voluntary market must then charge 28.18 for a risk with expected losses = 28, ARP, with its lower price, will take over this level as well. ARP's price (based on its own new experience for the

TABLE 2
A VANISHING VOLUNTARY MARKET

(1)	(2)	(3)	(4)	(5)	(6)
		If ARP writes all risks with expected losses greater than x			
x	$P[X = x]$	voluntary market rate for x	ARP rate (for all)	voluntary market rate for $x + 1$	1: ARP gains −1: ARP loses 0: equilibrium
20	0.0028	429.57	23.35	451.05	1
21	0.0095	115.79	23.38	121.30	1
22	0.0316	47.96	23.46	50.14	1
23	0.1053	29.86	23.65	31.16	1
24	0.3508	25.23	24.21	26.28	1
25	0.3508	25.23	25.14	26.24	1
26	0.1053	26.06	26.04	27.07	1
27	0.0316	27.02	26.89	28.02	1
28	0.0095	28.01	27.62	29.00	1
29	0.0028	29.00			

24.5 = average expected loss

risks with expected losses of 28 and 29) will drop to 27.14 (row $x = 27$ of Table 1). The voluntary market then needs to charge 27.39 for a risk with expected losses of 27, but that still exceeds ARP's rate, so ARP will capture the risks with expected losses of 27 too. Now, based on the experience of risks with expected losses of 27, 28 and 29, ARP will again lower its rate, this time to 26.67 (row $x = 26$ of Table 1). This time however, because the voluntary market will need only 26.65 for risks with expected losses of 26, it will keep risks with that level of expectation or better; and the market will be in equilibrium.

There is nothing robust or inevitable about this equilibrium. Table 2 presents the same scenario as Table 1, except that the probabilities have changed. The overall expected loss is still 24.5, but the distribution is more concentrated. In this case, if ARP starts with risks whose expected losses are greater than 28 and bases its future rates on its own experience, it will capture the entire market before reaching equilibrium. ARP will undercut the voluntary market at the high-priced end of the voluntary market's book, causing the high-priced business to move to ARP. This will improve ARP's experience, and ARP will lower its price. The voluntary market will have a higher risk load, which will increase the voluntary market's price. After the price adjustments, ARP will undercut the voluntary market at the next level. With the distribution shown in Table 2, the cycle will continue until ARP has all the business.

One needs to take care with the conclusions that one draws from these examples. It is true that as a distribution becomes more dispersed ARP is less likely to take over, but not all uniform distributions result in a balanced equilibrium between ARP and the voluntary market. Since one can construct examples where nearly anything happens, the only firm conclusion that one can draw is that the evolution of ARP is sensitive to the distribution of expected losses among insureds. There is no mathematical certainty of equilibrium or even of the direction that the evolution will take.

3.3. Assigned Risk Plans That Follow Their Own Experience May Shrink

Let us look at some examples where ARP's experience will lead to a rate increase. The distribution of the random variable X in Table 3 is essentially a shifted, truncated Poisson. (Think of X as defined by $X = \min(1 + Y, 10)$, where Y has a Poisson distribution with $\lambda = 2.74$. We concentrate the probabilities of the tail at 10 simply to make a readable table.) Now we see negative flags in column 6, meaning that ARP will be increasing rates and losing business to the voluntary market if it follows its own indications—even with non-profit pricing. If it starts out writing everything with expected losses greater than 2, it will have a beginning rate of 4.17. The voluntary market will undercut it with a rate of 4.13 for risks with expected losses of 3. ARP's market share will drop, ARP's rate will increase, and the voluntary market will then beat ARP's price for risks with expected losses of

TABLE 3
A SHRINKING ARP WITH EQUILIBRIUM ONLY AT TWO EXTREMES

(1)	(2)	(3)	(4)	(5)	(6)
		If ARP writes all risks with expected losses greater than x			
x	$P[X = x]$	voluntary market rate for x	ARP rate (for all)	voluntary market rate for $x + 1$	1: ARP gains −1: ARP loses 0: equilibrium
1	0.0646	3.71	3.74	7.42	0
2	0.1769	2.76	4.17	4.13	−1
3	0.2424	3.32	4.79	4.43	−1
4	0.2214	4.16	5.52	5.20	−1
5	0.1516	5.08	6.32	6.10	−1
6	0.0831	6.04	7.16	7.05	−1
7	0.0379	7.02	8.02	8.02	−1
8	0.0149	8.01	8.85	9.01	0
9	0.0051	9.00	9.52	10.00	0
10	0.0021	10.00			

3.74 = average expected loss

4. The cycle will continue until the market reaches equilibrium, with ARP writing only risks with expected losses of 9 and 10 at a rate of 8.85.

This is an interesting example not just because it illustrates that ARP's experience can cause it to lose, as well as gain, market share; it also illustrates that equilibrium, even within a single distribution, can occur at extremely different points. ARP and the voluntary market can be in equilibrium if ARP writes all risks with expected losses larger than 1 at a rate of 3.74, or if ARP writes all risks with expected losses larger than 8 with a rate of 8.85. In the first case ARP will have a market share of 93.6%; in the second, 1.7% (see Table 3A of Appendix B for calculation of market shares). ARP and the voluntary market will not be in equilibrium anywhere in-between these two extremes.

A market share of 1.7% for ARP is certainly not extreme, but there is no guarantee that ARP will stop at 1.7%. Look at one

TABLE 4
A VANISHING ARP

(1)	(2)	(3)	(4)	(5)	(6)
		If ARP writes all risks with expected losses greater than x			
x	$P[X = x]$	voluntary market rate for x	ARP rate (for all)	voluntary market rate for $x+1$	1: ARP gains -1: ARP loses 0: equilibrium
1	0.5400	1.12	2.71	2.23	−1
2	0.2484	2.07	3.66	3.11	−1
3	0.1143	3.05	4.61	4.07	−1
4	0.0526	4.03	5.56	5.04	−1
5	0.0242	5.02	6.49	6.02	−1
6	0.0111	6.01	7.40	7.01	−1
7	0.0051	7.01	8.26	8.01	−1
8	0.0024	8.00	9.01	9.00	−1
9	0.0011	9.00	9.52	10.00	0
10	0.0009	10.00			

1.85 = average expected loss

last example: Table 4 shows a truncated geometric distribution. For x less than 10, $P[X = x] = 0.54 \times 0.46^{x-1}$; the balance of the distribution is concentrated at $x = 10$. In this case, there is no equilibrium for the voluntary market at the small end of the market; ARP has either all of the market or nearly none of it. Equilibrium can occur with ARP writing risks with expected losses of 10, at a rate of 9.52, and a market share of 0.5% (Table 4A, Appendix B). Even this equilibrium occurs only because the distribution is truncated; if it were not truncated, equilibrium would not occur until ARP's market share was less than 0.01% and its rate nearly 17, more than 9 times the average market rate (Table 4B, Appendix B). By tweaking the parameters a little, one can push this equilibrium market share to any extreme.

The above examples assume that the voluntary market operates freely. If regulatory constraint becomes too severe, none of these examples will bear much resemblance to the real behavior of the market. They are still relevant though—just as the force of gravity is relevant to an engineer—because they show the natural forces at work against the barriers of regulation.

4. HOW TO SET THE RATES

4.1. An Alternative to Break-Even Pricing

One might be tempted to argue that because the above examples are filled with instances of equilibrium, it is reasonable for assigned risk plans to base their prices on their own experience. Unfortunately, the equilibrium is capricious—one never knows where or whether it will occur. Equilibrium, moreover, desirable as it is, is not an end in itself. Society will probably not accept an equilibrium that leaves insurers a tiny fraction of the market, or that charges assigned risk plan members ten times the voluntary market rates. In any case, ARP's pricing strategy should be consistent with public goals. The public may accept letting some residual markets price themselves out of existence and may be well served by so doing. In those cases break-even pricing with a contingency factor may work well, provided ARP really follows the indications. Where the consensus is in favor of keeping and controlling the residual market, however, the break-even approach is not a good one.

So how should ARP set its rates? If one starts with the assumptions that there should (or in any case *will*) be an assigned risk plan, that it should not be overly burdensome on the insureds in the voluntary market and that it should not have wild swings in market share, there is a reasonable solution to the rate problem. The solution is to base ARP's rates on total industry experience, but set at a level consistently higher than that which a typical insurer would need to charge in the voluntary market. One can start with industrywide pure premiums, for example, and load them with an expense and profit factor which is 25% above that of the industry average (or whatever percentage seems reasonable in line with studies of the market and the philosophy of a given state). The market will seek its own equilibrium; in the typical case ARP will lose money, but the burden on voluntary insureds will not be excessive. At the same time ARP's rates will be high, but not intolerably high. Thus a start-up employer who truly has a contribution to make to society, for example, will have a chance.

4.2. Setting Specific Goals

Words such as *reasonable* and *excessive* are rather vague; one must define them in order to use them in actually setting rates. Their definitions may vary from state to state and from line to line, and probably with the passage of time as well. They will come through compromise and consensus—there is no optimal solution that everyone will accept. The key is to have specific goals and to structure the pricing to accomplish those goals.

The voluntary market attempts to identify true costs underlying whatever it is insuring; and, by varying its prices according to those costs, it steers production of goods and services toward those that are most efficient. This feature of insurance is very beneficial to society. A state should choose a goal for residual market share that guarantees the continuation of a large voluntary market so as to give society the benefit of an efficient economy, with the ideal being a totally voluntary market.

On the other hand, rightly or wrongly, the government has constrained the operation of the insurance market for many decades. Workers compensation statutes are a prime example: despite the benefits of the statutes, they raise a high hurdle for many small employers. Residual market plans often enable such employers to enter and compete in the marketplace, something that could occur naturally in the absence of the workers compensation statutes. One could view residual markets as intervention needed because states interfered with the natural flow of the marketplace when they first created laws such as the workers compensation statutes. Residual markets will almost surely continue to have their adherents and, if their prices are unaffordable for virtually everyone, consumers will revolt and probably revolt successfully.

So in determining the parameters of the pricing problem, one has two somewhat conflicting goals: the bigger the voluntary market the better, and residual market rates should not be unaffordable for all. A third guiding factor is consideration for the voluntary market insureds—the expected assessment of residual market losses on these innocent bystanders should not be punitive. A fourth guiding factor is the *status quo*. Too abrupt a change can be harmful—partly because it might unleash unexpected and uncontrollable consequences, and partly because it would be in some sense a change of the rules under which many people have been operating in good faith.

Reasonable goals for a residual market plan might be a market share of under 1%, a rate of under 150% of the voluntary market, an expected assessment on the voluntary market of under 0.5%, and (during the catch-up period if one is needed) annual price adjustments of under 10% relative to the voluntary market. This paper is not trying to suggest the exact parameters to use; it is merely suggesting a way to approach them.

Of course, the voluntary market does not charge a single rate that one can use as a basis for the ARP rates. In the above example where ARP rates are under 150% of the voluntary market, what is "voluntary market?" A reasonable starting place is to use statewide pure premiums loaded with average industry expenses and profits. In place of statewide pure premiums one might also use the pure premiums or rates generated by a large ratemaking bureau operating in the state, provided that the bureau's members represent a significant enough market share.

It will be helpful to look not only at the *average* voluntary market rates, but also at the *spread* of rates. In particular, some companies specialize in non-standard business and provide a valuable service to the marketplace. Before arbitrarily selecting an upper bound of say 150% of average, it will be helpful to know where the rates of the non-standard writers fall relative to the overall average. A state could do its citizens a disservice if it sets a limit that cuts out the non-standard carriers.

Finally, although this paper suggests abandoning break-even pricing, ARP's own experience still has an important role in ARP pricing. In order to measure the expected assessment on the voluntary market, ARP still must analyze its own experience. If ARP's experience indicates excessive future assessments, ARP will need to adjust its rates within the constraints of the other goals. The state may even need to change the goals if all the goals are already at the limit of their constraints. In addition, an analysis of ARP's experience can be helpful to the voluntary market in identifying opportunities to depopulate ARP.

4.3. Using the Goals to Set Prices

With a set of specific residual market goals in hand, a state does not need to fight the unpredictability of break-even pricing. It can take the more stable path of setting residual market prices as a direct multiple of voluntary market prices, and it can measure its success directly from its goals.

Suppose that a state sets ARP rates by looking at ARP's own experience, judgmentally modifying the indication (essentially ignoring it), and finally ending up with rates that currently average 105% of voluntary rates. Now consider the following alternative. Having first set specific goals for ARP, the state gathers all the data it needs to monitor the goals. What are the market shares of the residual and voluntary markets? What are the average rates of voluntary writers (paying separate attention to companies specializing in non-standard business)? What are the average expense ratios? What are the underlying loss costs? Then the state measures its goals against the data. Are all the goals met? If so, the state leaves the prices at 105% of voluntary (as measured by loss costs and average expense ratios) and the job is done.

Probably, though, 105% of voluntary will not achieve the goals. So the state increases the rates to 110% or 115% of voluntary, depending on the "catch-up" parameter. Next year it looks again at the experience and market data. Gradually the state adjusts the ARP-to-voluntary ratio until it meets its goals—not break-even goals with all their unpredictability, but goals based directly on society's specific expectations of ARP.

Once the state finds the multiplier that meets its goals, it sets future rates using the same multiplier. As long as the goals are met, ARP's own experience will have no effect on ARP's rates. For example, if the goals call for a market share of under 1% and a burden on the voluntary market of under 0.5%, ARP could consistently lose 50 cents or more on each dollar of premium provided its market share remains sufficiently small. Its market share will remain sufficiently small as long as the multiplier is sufficiently large. By the same token, a fortuitous ARP profit will have no effect on the rates either; ARP's insureds will be rewarded for good experience not by ARP rate decreases but rather by movement into the voluntary market.

The advantage of this market-based pricing approach is not necessarily to reduce the overall losses of the residual market, but rather to enable more conscious control over the residual market. Rather than having an official ratemaking procedure (break-even pricing) that is not actually followed and that could lead to totally unacceptable results if it were followed, states would articulate their true goals and consciously manage them. Some residual markets might very well shrink as a result and would probably produce fewer losses, but that is not a necessary consequence of moving to market-based pricing. What will happen will depend on the goals of the individual states. In any case, one can not measure the true cost of a residual market by its bottom-line losses alone. Voluntary market insureds bear the risk charge for the residual market even when the residual market is profitable, and all of society pays for the loss of diversity when a residual market gets too big.

5. FINAL THOUGHTS

The original impetus for this paper sprang from real-life observation of the outcomes that this simple model predicts; the predictions are not merely theoretical. Of course, the worst examples of residual market problems arise not from using break-even pricing, but rather from suppressing rates and ignoring the effects. What appears to be an easy solution to that problem—namely basing residual market rates directly on residual market experience—is in general not a solution at all.

This paper demonstrates that under break-even residual market pricing, regardless of the goal that one sets for residual market share, one can find a loss distribution that leads to a market share very different from the goal. The paper does not look at empirical loss distributions to predict how specific residual markets would behave under them. That is an interesting area for additional research, but the paper's thesis is that such research is not essential if there is an approach to residual market pricing whose success is independent of loss distribution. It turns out that there is such an approach; namely, to base residual market prices on total market experience, at a level consistently above that of voluntary market prices. That approach not only solves the market-share problem, but it also enables focusing on and achieving all of the other goals of the residual market to the extent that the goals are achievable.

APPENDIX A

PROOF OF EXISTENCE AND UNIQUENESS OF SOLUTION TO PRICING PROBLEM

The insurers' pricing problem—to solve for a in Equation (A.1) below—has a solution if and only if $L < R$, where L is the average expected loss and R is the average ARP premium. The solution, when it exists, is unique.

Proof Let F be the distribution function of the expected losses. As a distribution function, F is right-continuous. Assume furthermore that $F(0) = 0$. To allow $F(0) > 0$ would be to assume that for some insureds not even the *possibility* of a loss exists; F, remember, is the distribution of *expected* losses, not of actual losses. We have:

$$L = \int_0^\infty x\,dF = \int_0^{R/a} ax\,dF + \int_{R/a}^\infty R\,dF. \tag{A.1}$$

Equation (A.1) merely says that the expected losses are equal to the premium of the voluntary market plus the premium of ARP. The insurers' pricing problem is to solve for a. Set

$$g(a) = L - \int_0^{R/a} ax\,dF - \int_{R/a}^\infty R\,dF. \tag{A.2}$$

Solving equation (A.1) for a is equivalent to finding a zero of the function g defined by equation (A.2). g is a continuous, monotonically decreasing function on the interval $(0,\infty)$, so it has at most one zero. If it ever changes sign, it has exactly one zero

$$g(1) = \int_0^\infty x\,dF - \int_0^R x\,dF - \int_R^\infty R\,dF = \int_R^\infty (x-R)\,dF > 0.$$

Thus $g(a)$ is positive for $a \le 1$. Now look at $g(a)$ as a increases. For $0 \le x \le R/a$, $ax \le R$, so

$$\int_0^{R/a} ax\,dF \le \int_0^{R/a} R\,dF = R(F(R/a) - F(0)).$$

Since F is right-continuous, $\lim_{a\to\infty} R(F(R/a) - F(0)) = 0$, so also

$$\lim_{a\to\infty} \int_0^{R/a} ax\, dF = 0. \tag{A.3}$$

Because $F(0) = 0$ and again because F is right-continuous,

$$\lim_{a\to\infty} \int_{R/a}^{\infty} R\, dF = R. \tag{A.4}$$

Finally, combining equations (A.2), (A.3), and (A.4) we have

$$\lim_{a\to\infty} g(a) = L - R,$$

which is negative if and only if $L < R$. Thus if $L \geq R$, there is no a for which $g(a) = 0$, and equation (A.1) has no solution. If $L < R$, there is a unique solution.

If we removed the requirement that there exist insureds with arbitrarily large expected losses, our conclusion would not change. For values of R greater than the largest expected loss, the solution would be $a = 1$ and all the business would be in the voluntary market. If we removed the requirement that there exist insureds with arbitrarily small expected losses, there might be some degenerate solutions. In that case, g would no longer be monotonically decreasing on the entire interval $(0,\infty)$, but only on $(0,R/b)$, where b is the smallest possible expected loss—more precisely, $b = \inf\{x : F(x) > 0\}$. For all $a > R/b$, we'd have $g(a) = L - R$, so that for $R = L$ there would be infinitely many solutions of the equation $g(a) = 0$. These solutions are rather trivial; they are simply all multipliers, a, large enough to charge the tiniest risk more than R, so that ARP writes all of the business.

APPENDIX B

ARP MARKET SHARE CALCULATIONS

This appendix contains Tables 3A, 4A and 4B; these tables extend Tables 3 and 4 to show calculations of ARP market shares. In addition, Table 4B extends the truncation point of the geometric distribution from 10 to 20 to show a more extreme example of diminishing ARP market share. The data in the first six columns of Tables 3A and 4A come directly from the corresponding Tables 3 and 4 of the paper. The reader will find explanations of the additional columns (columns 7 through 11) in the tables themselves.

TABLE 3A

MARKET SHARE CALCULATIONS FOR SHRINKING ARP

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
		If ARP writes all risks with expected losses greater than x								
x	$P[X = x]$	voluntary market rate for x	ARP rate (for all)	voluntary market rate for $x+1$	1: ARP gains −1: ARP losses 0: equilibrium	cost for expected loss $= x$ $(1)\times(2)$	cost for expected loss $> x$	ARP premium (8)/1.05	voluntary market premium $(7)_{tot}-(9)$	ARP market share $(9)/(7)_{tot}$
1	0.0646	3.71	3.74	7.42	0	0.0646	3.6747	3.4997	0.2396	93.6%
2	0.1769	2.76	4.17	4.13	−1	0.3538	3.3208	3.1627	0.5766	84.6%
3	0.2424	3.32	4.79	4.43	−1	0.7272	2.5937	2.4702	1.2691	66.1%
4	0.2214	4.16	5.52	5.20	−1	0.8855	1.7082	1.6268	2.1124	43.5%
5	0.1516	5.08	6.32	6.10	−1	0.7582	0.9500	0.9047	2.8345	24.2%
6	0.0831	6.04	7.16	7.05	−1	0.4986	0.4514	0.4299	3.3094	11.5%
7	0.0379	7.02	8.02	8.02	−1	0.2656	0.1857	0.1769	3.5624	4.7%
8	0.0149	8.01	8.85	9.01	0	0.1188	0.0669	0.0637	3.6756	1.7%
9	0.0051	9.00	9.52	10.00	0	0.0458	0.0211	0.0201	3.7192	0.5%
10	0.0021	10.00				0.0211			3.7393	0.0%
Total	1.0000					3.7393				

Column (8): row x = sum of Column (7) from row $x+1$ through row 10

TABLE 4A

MARKET SHARE CALCULATIONS FOR VANISHING ARP

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
		If ARP writes all risks with expected losses greater than x								
x	$P[X = x]$	voluntary market rate for x	ARP rate (for all)	voluntary market rate for $x + 1$	1: ARP gains −1: ARP losses 0: equilibrium	cost for expected loss = x (1) × (2)	cost for expected loss > x	ARP premium (8)/1.05	voluntary market premium $(7)_{tot} - (9)$	ARP market share $(9)/(7)_{tot}$
1	0.5400	1.12	2.71	2.23	−1	0.5400	1.3111	1.2486	0.6024	67.4%
2	0.2484	2.07	3.66	3.11	−1	0.4968	0.8143	0.7755	1.0756	41.9%
3	0.1143	3.05	4.61	4.07	−1	0.3428	0.4715	0.4490	1.4020	24.3%
4	0.0526	4.03	5.56	5.04	−1	0.2102	0.2612	0.2488	1.6023	13.4%
5	0.0242	5.02	6.49	6.02	−1	0.1209	0.1403	0.1337	1.7174	7.2%
6	0.0111	6.01	7.40	7.01	−1	0.0667	0.0736	0.0701	1.7810	3.8%
7	0.0051	7.01	8.26	8.01	−1	0.0358	0.0378	0.0360	1.8151	1.9%
8	0.0024	8.00	9.01	9.00	−1	0.0188	0.0190	0.0181	1.8330	1.0%
9	0.0011	9.00	9.52	10.00	0	0.0097	0.0092	0.0088	1.8423	0.5%
10	0.0009	10.00				0.0092			1.8511	0.0%
Total	1.0000					1.8511				

Column (8): row x = sum of Column (7) from row $x + 1$ through row 10

TABLE 4B

EXTENDED MARKET SHARE CALCULATIONS FOR VANISHING ARP

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
		If ARP writes all risks with expected losses greater than x								
x	$P[X = x]$	voluntary market rate for x	ARP rate (for all)	voluntary market rate for $x+1$	1: ARP gains −1: ARP losses 0: equilibrium	cost for expected loss = x (1) × (2)	cost for expected loss > x	ARP premium (8)/1.05	voluntary market premium $(7)_{tot} - (9)$	ARP market share $(9)/(7)_{tot}$
1	0.5400	1.12	2.72	2.23	−1	0.5400	1.3119	1.2486	0.6025	67.47%
2	0.2484	2.07	3.67	3.11	−1	0.4968	0.8151	0.7755	1.0756	41.92%
3	0.1143	3.05	4.62	4.07	−1	0.3428	0.4723	0.4490	1.4021	24.29%
4	0.0526	4.03	5.57	5.04	−1	0.2102	0.2620	0.2488	1.6023	13.47%
5	0.0242	5.02	6.53	6.02	−1	0.1209	0.1411	0.1337	1.7174	7.26%
6	0.0111	6.01	7.48	7.01	−1	0.0667	0.0744	0.0701	1.7810	3.83%
7	0.0051	7.01	8.43	8.01	−1	0.0358	0.0386	0.0360	1.8151	1.98%
8	0.0024	8.00	9.38	9.00	−1	0.0188	0.0198	0.0181	1.8330	1.02%
9	0.0011	9.00	10.33	10.00	−1	0.0097	0.0100	0.0088	1.8423	0.51%
10	0.0005	10.00	11.29	11.00	−1	0.0050	0.0050	0.0048	1.8471	0.26%
11	0.0002	11.00	12.24	12.00	−1	0.0025	0.0025	0.0024	1.8495	0.13%
12	0.0001	12.00	13.19	13.00	−1	0.0013	0.0012	0.0012	1.8507	0.06%
13	0.0000	13.00	14.14	14.00	−1	0.0006	0.0006	0.0006	1.8513	0.03%
14	0.0000	14.00	15.08	15.00	−1	0.0003	0.0003	0.0003	1.8516	0.02%
15	0.0000	15.00	16.01	16.00	−1	0.0002	0.0001	0.0001	1.8517	0.01%
16	0.0000	16.00	16.92	17.00	0	0.0001	0.0001	0.0001	1.8518	0.00%
17	0.0000	17.00	17.78	18.00	0	0.0000	0.0000	0.0000	1.8518	0.00%
18	0.0000	18.00	18.53	19.00	0	0.0000	0.0000	0.0000	1.8518	0.00%
19	0.0000	19.00	19.05	20.00	0	0.0000	0.0000	0.0000	1.8518	0.00%
20	0.0000	20.00				0.0000		0.0000	1.8519	0.00%
Total	1.0000					1.8519				

Column (8): row x = sum of Column (7) from row $x+1$ through row 20