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LEVELS OF DETERMINISM IN WORKERS COMPENSATION REINSURANCE COMMUTATIONS

GARY BLUMSOHN

Abstract

When commuting workers compensation reinsurance claims, the standard method is to project the future value of the claims using stated assumptions for future medical usage, medical inflation, cost-of-living adjustments, and investment income. The actuary selects a best estimate for each variable, and assumes this deterministic number will be realized in the future. To account for the date of death being stochastic, a mortality table is used to model the future lifetime.

By assuming deterministic values for future medical usage, medical inflation, cost-of-living adjustments, and investment income, the calculation ignores the possibilities of higher or lower values. It is shown that these do not generally balance out, and that the standard method produces biased results. In low reinsurance layers, the *commutation amount is overstated, and in high layers it is understated. By removing deterministic assumptions from the calculation, bias is removed from the results. The paper gives a detailed, realistic, example to illustrate this.*

It is impossible to eliminate all determinism, but it is often appropriate to judgmentally adjust the answers to account for this. In discussing this, the paper draws parallels to the work of economists on "genuine uncertainty."

The implications of the paper reach beyond the narrow realm of workers compensation reinsurance commutations. The most obvious implications are for workers compensation reserving, but the essential message applies to pricing and reserving of any excess insurance and reinsurance: deterministic assumptions often lead to biased results.

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1. INTRODUCTION

Excess reinsurance for workers compensation generally pays out over many decades. While workers compensation claims are usually reported to the insurer soon after the accident, and the insurer may soon report them to the reinsurer, the loss payments are slow, being made over the lifetime of the injured worker or even the lifetime of uninjured dependents. Consequently, even for reinsurance with a relatively modest retention, it can take many years to breach the retention, and many more years to exhaust a layer. Gary Venter [17] has estimated that it takes, on average, over 30 years to pay half the ultimate claim amount.

At some point after an excess reinsurance treaty ends, but before the losses have been fully paid, it is common to commute either the reinsurance treaty or the individual reinsured claims. The commutation entails having the reinsurer pay the ceding company a flat amount, in exchange for canceling future liabilities. This saves costs for both parties, since the expense of reporting on claims to the reinsurer and the cost of paying these claims are eliminated. It allows the parties to shut their reinsurance files and spend their time on more profitable activities.

The actuarial techniques for evaluating workers compensation commutations differ from the techniques generally used in commutations of other lines of business. With workers compensation (and in some other cases, like unlimited medical benefits for no-fault auto) the population of claims is generally known at the time of the commutation—there is very little lag in claims being reported to the primary company. Also, the amount of the payments does not depend on some future court verdict. The payments are based on a fixed annual indemnity amount, subject, in some states, to an annual cost-of-living adjustment (COLA), and on the actual medical payments incurred by the claimant. In the case of permanent-total disability cases, these payments often continue for the rest of the claimant's life. Since the losses are so closely tied to the claimant's life span, it is natural to use the mortality techniques more generally associated with life actuaries than with their property/casualty brethren.

While the actuarial techniques in these calculations are by now well-accepted, this paper will argue that the results are systematically biased and can be improved upon. The life-table techniques generally assume that mortality is stochastic, but that other variables (amount of medical care, inflation rates, investment yields) are deterministic. These deterministic variables can be stripped away, much as earlier actuaries stripped away the assumption of deterministic mortality. By doing this, we improve the accuracy of our calculations and eliminate some biases.

Though this paper will express the issues in terms of commutations, the issues are similar when doing excess workers compensation case reserving using life-table methods. And, as will be noted later, the same issues find their way into most actuarial work.

2. LIFE-TABLE TECHNIQUES

Method 1: Totally Deterministic Calculation

The simplest method for performing the calculation is to assume the claimant will live to his life expectancy and then calculate the present value of the future stream of payments for this time. This method, though simple and appealing, is wrong. As actuaries are well aware, and as will be discussed in detail later, assuming a deterministic life-span leads to systematically incorrect results.

Method 2: Stochastic Date Of Death

The actuarial literature contains several papers that discuss the calculation of reserves for long-term workers compensation cases, and the calculation of a commutation value only differs in minor respects from the calculation of a reserve.[1] Actuaries

[1]The classic paper is Ronald Ferguson's *Actuarial Note on Workmen's Compensation Loss Reserves* [8], which applied life-table methods to excess indemnity reserves. He did not address the issue of the medical portion of the reserve. Richard Snader [15] applied similar methods to long-term medical claims. A recent valuable addition to the literature is by Lee Steeneck [16], who uses an analysis very close to "Method 2" discussed in this paper. Another approach is given in Venter and Gillam [18].

and, to a lesser extent, the wider insurance community, generally accept that the right way to reserve these claims is through the life-table techniques routinely used by life actuaries. Life-table techniques take into account the probabilities of the claimant dying either earlier or later than his life expectancy, rather than assuming he lives to his life expectancy and then dies.

Using a life table to make the number of payments stochastic, rather than deterministic, is a crucial advance in the accuracy of the calculation. A life-table approach allows for the possibility that a claimant may live to age 95, and hence pierce reinsurance layers that would not have been pierced if he had died at his life expectancy. In other words, if the claimant lives to his life expectancy of, say, 75, a retention of $5 million may not be breached. But if he lives another 10 years, to 85, the total payments in the additional 10 years of life may be enough to breach the $5 million retention, and if he lives to 95, it may breach a $10 million retention. The probabilities of living to these ages, and thus breaching higher layers, must be reflected in the commutation price.

Put another way, there will be a positive commutation amount in layers that we do not expect to get hit. The commutation is effectively a purchase of reinsurance by the reinsurer, covering the possibility of the claimant breaching the retention. There need not be a guarantee that the retention will be breached in order for the expected losses in the layer to be positive.

Assumptions

In doing the commutation calculation, the actuary needs to make a number of assumptions:[2]

[2] In practice, some reinsurance contracts have commutation clauses in which the parties have negotiated some of the parameters at the time the contract is drawn up. For example, the clause may specify what mortality table to use and what interest rate to use in discounting the future payments.

- An appropriate *mortality table* must be selected.
- For workers compensation, the indemnity amount is generally known, but it may be subject to *cost-of-living adjustments*, which depend usually on movements in the average weekly wage in the state.
- The amount of medical expenses must be estimated for each year in the future. This is usually done in two steps: first, estimate the future *annual medical expense* in today's dollars, and, second, estimate future *medical price inflation*, to convert today's dollars into tomorrow's dollars.
- The *rate at which to discount* future dollar payments to present value.

Once assumptions have been chosen, the calculations can be performed, and the parties can agree on an amount for settlement.[3]

3. LEVELS OF DETERMINISM

The life-table method ignores fluctuations in other key variables. Just as it is wrong to assume a claimant's life-span is fixed, so it is wrong to assume that medical usage and inflation

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[3]This paper will not address the crucial impact of income tax. In the calculations, one must account for taxes without the commutation, compared to taxes with the commutation.

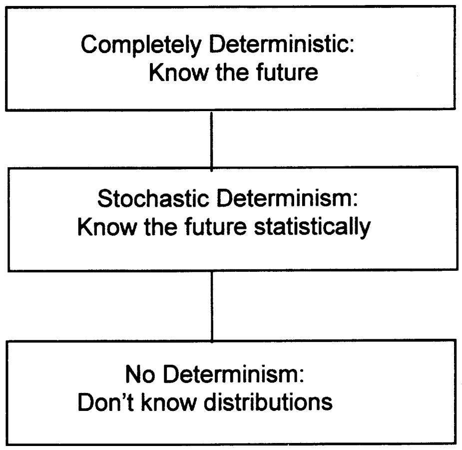
i) If the claim is not commuted, the reinsurer carries a reserve on its books. For tax purposes, this reserve is discounted by the IRS discount factors, and the unwinding of the discount is counted into the incurred losses of the company each year. On the other hand, the investment income earned on the reserve is taxable.
ii) If the claim is commuted, the reinsurer takes down the reserves and puts up a paid loss. If the reserve exceeds the paid loss (as it frequently does, because statutory accounting demands undiscounted, or tabularly-discounted, reserves) the reinsurer's profit rises by the difference between the reserve and the paid loss. This profit is taxable.

The ceding company has the reverse entries on its books.

The tax benefits or costs are as important as any other cash flows, but they are beyond the scope of this paper. For a detailed discussion of the tax effects, see Connor and Olsen [5].

FIGURE 1

LEVELS OF DETERMINISM



are fixed. Assuming a deterministic life-span leads to inaccurate calculations. Likewise, assuming deterministic medical care and inflation will lead to inaccurate calculations. A deterministic life span implies that high layers of reinsurance will not be hit, when they do, in fact, have a chance of getting hit if the claimant lives long enough. Likewise, deterministic medical care and deterministic inflation understate the costs to the highest reinsurance layers.

Actuarial calculations can contain varying levels of determinism, and this can be represented as shown in Figure 1.

At the "completely deterministic" level, our calculations assume we know what the future will bring. This is the viewpoint of typical loss development work: we look at the historical loss development patterns, select patterns to represent the future, and develop the losses to ultimate. We assume that the selected patterns represent loss emergence in the future, and we make no allowance for deviations from the selected patterns. In many uses, this approach is perfectly reasonable. In others, and particularly in dealing with excess reinsurance, it can generate misleading results.

The next stopping point on the continuum of determinism is what I call "stochastic determinism." Here we do not assume that we know what the future will be, but we assume that we know the statistical distributions of the relevant variables. For example, Ferguson [8] pointed out that we do not know when a workers compensation claimant will die, but we have mortality tables that tell us the probability of dying at any given age. Using these probabilities, Ferguson showed, generates a more accurate answer to the required reserve for an excess workers compensation claim.

Note, though, that the typical actuarial approach to workers compensation cases does not have all variables stochastic. For example, the rate of medical inflation, cost-of-living adjustments, investment yields, and the annual real amount of medical expenses are assumed to be fixed. The typical approach (to be labeled "Method 2" later in this paper) is partway between complete determinism and stochastic determinism. The calculations in Section 4 of this paper will shift the approach further towards stochastic determinism.

At the end of the continuum is "no determinism," which is where we assume that we do not know even the distributions that underlie what will happen in the future. We can imagine various scenarios occurring in the future, but we cannot assess the probabilities. We know, for example, that doctors might find a way to surgically fix the damage to a quadriplegic, and thus get him back to work and end his workers compensation claim. But we do not know the probability of this happening. This is obviously the hardest level to deal with from an actuarial standpoint.

We will return later to a more detailed discussion of these various levels of determinism. At this point it is sufficient to notice that Ferguson's paper stripped away some determinism from the workers compensation calculation by making mortality stochastic. To add even greater accuracy, we need to strip away more determinism.

4. A COMPREHENSIVE EXAMPLE

This section gives a realistic example of how one can remove more determinism from the model. The calculations are significantly more complex than the standard life-table method. However, using computers, the problems are not insurmountable, and the results are significantly less biased.

The Data

Suppose we are commuting the following claim:[4]

- Joe Soap has been permanently and totally disabled since 1993. On January 1, 1998, the effective date of the commutation, he will turn 35 years old.
- Through 12/31/97, the primary company has paid out \$300,000 in medical expenses and \$70,000 in indemnity payments.[5] This is an unusually large claim, but by no means unheard of. A smaller claim would not affect any of the conclusions.
- In 1997, Mr. Soap received indemnity payments at the rate of \$20,000 per year, but these are subject to a cost-of-living adjustment that is effective on January 1 of each year, based on

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[4]A similar example was used in a previous version of this paper (Blumsohn [1]). Some items have been updated to incorporate more recent data. Substantive changes from the previous version will be noted.

[5]For simplicity, the example ignores ALAE, which is usually covered by reinsurance and should be included in the calculations. ALAE is usually relatively small in workers compensation, and including it would not change any of the principles discussed in this paper.

the increase in the state average weekly wage over the previous year.

- The best estimate of his future medical expenses is $70,000 per year, in 1997 dollars. These will increase with medical inflation.
- We assume that Joe's mortality follows that for the overall male population, as shown in the 1990 US census (Exhibit 1). Based on this mortality table, his life expectancy is 39.6 years.[6]
- We project future inflation of 4.11% per year.[7] For convenience, we assume that changes in the state average weekly

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[6]Depending on the claimant's condition, one may use impaired-mortality tables. Note that, contrary to the usual intuition, workers compensation lifetime-pension cases do not, overall, appear to have higher mortality rates than the general population. Gillam [9] shows that at some ages, the mortality of workers compensation claimants is even lower than the general population. Gillam's technique weights each claimant equally. That may not be the optimal approach, since some claims are bigger than others. In particular, many of the really big claims are for people who are extremely badly injured and require, say, 24-hour attendant care. One might speculate that a dollar-weighted average of mortality could be found to be significantly worse than the general population.

By using the 1990 census table, we ignore mortality improvements: as medical care improves, mortality rates have historically dropped. By ignoring mortality improvements, we are implicitly assuming Joe Soap has impaired mortality.

[7]The 4.11% rate is the average of Consumer Price Index changes from 1935 to 1997, using data from the US Bureau of Labor Statistics. Using this average is a matter of convenience, rather than a matter of believing that it is a good predictor of future inflation. The data, though not a predictor of future inflation, give an idea of long-term inflationary movements.

The earlier version of this paper (Blumsohn [1]) used 4.2%, based on data from 1935 to 1995. Steeneck ([16], p. 252), when faced with projecting indemnity inflation into the indefinite future, selects 4.0% as his annual rate.

The author admits to cringing at the spurious accuracy implied in publishing an inflation average to two decimal places. Past inflation is a poor way of predicting future inflation, and there's no scientific way to project inflation decades into the future to even the nearest whole percent, never mind two decimals. We are reminded of Gauss's comment that "Lack of mathematical culture is revealed nowhere so conspicuously as in meaningless precision in numerical computations." (Quoted in Coddington [4, p. 160].) However, the problem is that we are trying to contrast various methods of doing the computations, and this requires keeping the assumptions and arithmetic in the methods as consistent as possible, to avoid obscuring the main message by implicitly switching assumptions. The only way to transparently do this was to use more decimal places than are meaningful.

wage follow the overall price inflation in the economy. (Generally, wages actually rise faster than prices over the long run because of productivity improvements.)

- Our best guess of future medical inflation is 5.25% per year.[8] Exhibit 2 shows historical changes in the CPI and medical CPI.

- Selecting an appropriate discount rate is somewhat tricky. The future cash flows are highly uncertain, and the uncertainty arises from two principal places:

  i) *Mortality*: We do not know how long the claimant will live. However, if the insurer and reinsurer both write reasonably large books of workers compensation, the mortality risks of the individual claimants will be diversified away.

  ii) *Inflation*: Wage inflation affects cost-of-living adjustments and medical inflation affects medical payments. This risk cannot be diversified by writing a large book because all claimants are subject to the highs and lows of inflation together.

In setting its investment strategy, the insurer would be wise to hedge against inflation by buying investments that rise when inflation is high—for example common stocks. (See Feldblum [7].) This strategy is particularly appealing for excess workers compensation, where the payouts are extremely slow, so the year-to-year volatility of stock prices are less of a concern.

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[8] As with CPI changes, this average is based on changes in the medical component of the CPI from 1935 to 1997. The earlier version of this paper used data from 1935 to 1995 and had average medical inflation of 5.36% per year. As with the CPI, we are using this number for illustrative purposes, rather than as a considered prediction of future medical inflation. Steeneck [16, p. 252] projects annual medical inflation of 5.5%.

Starting in 1997, another inflation hedge was introduced in the market, namely, inflation-indexed Treasuries. Like other Treasuries, they are considered “risk-free” in the sense of not having default risk, and, unlike other Treasuries, they hedge against inflation as well.[9]

For discounting, we will use inflation-indexed Treasuries. At January 1, 1998, these had a real yield (above inflation) of about 3.75%. In general, discounting should be based on a rate below the investment yield, with the risk adjustment accounting for the riskiness in the flows being discounted (Butsic [3]). I will assume that a reasonable risk adjustment for excess workers compensation is 2.5 percentage points. In other words, we will discount at a real yield of 1.25% (= 3.75% − 2.5%).

As noted above, inflation is assumed to be 4.11% per year. Discounting at a real yield of 1.25% thus entails adding 1.25% to the assumed inflation of 4.11%, to get a discount rate of 5.36% per annum.[10]

- The primary insurer has purchased reinsurance in a number of layers, as shown in Table 1.

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[9] The hedge for excess layers of workers compensation is imperfect because:

i) They are indexed to the CPI, whereas the workers compensation risk is based on changes to the state average weekly wage (for COLAs) and the medical component of the CPI. The CPI is only a proxy for these.

ii) Excess reinsurance layers suffer a leveraged effect from inflation. For example, suppose a reinsurer covers a layer of $1 million excess of $1 million, and there’s a $1.1 million claim, with no inflation. In that case, the reinsurer will pay $100,000. If there’s 10% inflation, raising the claim to $1.21 million, the reinsurer’s portion more than doubles, to $210,000. (Of course, if the claim without inflation were, say, $3 million, inflating it to $3.3 million would have no effect on the reinsurance layer. This does not affect the general point that excess layers are typically more sensitive to inflation than are ground-up layers.)

[10] The earlier version of the paper assumed the risk-adjusted discount rate was exactly equal to the inflation rate.

TABLE 1

REINSURANCE LAYERS

Layer 1	$130,000 excess of $370,000
Layer 2	$500,000 excess of $500,000
Layer 3	$1 million excess of $1 million
Layer 4	$3 million excess of $2 million
Layer 5	$5 million excess of $5 million
Layer 6	$5 million excess of $10 million
Layer 7	$5 million excess of $15 million
Layer 8	$10 million excess of $20 million
Layer 9	$10 million excess of $30 million
Layer 10	$10 million excess of $40 million
Layer 11	$10 million excess of $50 million
Layer 12	$10 million excess of $60 million
Layer 13	$10 million excess of $70 million
Layer 14	$10 million excess of $80 million
Layer 15	$10 million excess of $90 million
Layer 16	Unlimited excess of $100 million

The first layer is somewhat artificial: since $370,000 has already been paid by the end of 1997, the layer will pay from the first dollar in 1998. This allows us to look at the value of all future payments. Also, the top layer is somewhat unusual. Reinsurers do not usually sell unlimited layers. However, it will be instructive to see the value of reinsurance on the unlimited top layer.

Method 1: Totally Deterministic Calculation

Though actuaries would not use a totally deterministic method (i.e., one that assumes Joe lives exactly to his life expectancy and then dies) it is interesting to see what result this produces. Exhibit 3 shows this calculation, and Table 2 summarizes the results.

Total payments are $11.2 million, exhausting the lowest five layers and part of the sixth. The lack of payments in higher layers

TABLE 2

RESULTS OF COMMUTATION CALCULATIONS USING METHOD 1

Layer (in $000's)	Nominal Payments (in $000's)	Present Value of Payments (in $000's)
130 xs 370	130	125
500 xs 500	500	413
1,000 xs 1,000	1,000	612
3,000 xs 2,000	3,000	1,092
5,000 xs 5,000	5,000	970
5,000 xs 10,000	1,606	217
Higher Layers	0	0
Total, All Layers	**11,236**	**3,430**

implies these layers will not be breached, and no commutation payment is needed.

This method ignores the chances of dying before or after one's life expectancy. We correct this by using a life-table approach, following Ferguson [8].

Method 2: Stochastic Date of Death

In Method 2, a mortality table models Joe's life span, as shown in Exhibit 4. Table 3 compares the commutation amounts from Methods 1 and 2.

Comparison of Method 2 Versus Method 1

Several points are worth noting:

- Using Method 2, twelve layers have non-zero commutation amounts, compared to only six layers in Method 1. This is because Method 2 recognizes that people can live beyond their life expectancies. If the person lives to the outer reaches of the mortality table, say to 110, many more layers will be breached. The highest layer reached is $10 million excess of $60 million,

TABLE 3

COMPARISON OF RESULTS OF COMMUTATION CALCULATIONS FOR METHODS 1 AND 2

Layer (in $000's)	Expected Nominal Payments (in $000's)		Expected Present-Value Payments (in $000's)	
	Method 1	Method 2[11]	Method 1	Method 2
130 xs 370	130.0	129.7	124.9	124.6
500 xs 500	500.0	494.9	413.2	409.1
1,000 xs 1,000	1,000.0	970.4	611.7	594.1
3,000 xs 2,000	3,000.0	2,725.1	1,092.4	998.6
5,000 xs 5,000	5,000.0	3,703.0	970.4	729.8
5,000 xs 10,000	1,605.9	2,574.7	217.1	311.2
5,000 xs 15,000	0.0	1,607.4	0.0	139.8
10,000 xs 20,000	0.0	1,359.7	0.0	86.5
10,000 xs 30,000	0.0	293.0	0.0	13.2
10,000 xs 40,000	0.0	39.2	0.0	1.4
10,000 xs 50,000	0.0	3.1	0.0	0.1
10,000 xs 60,000	0.0	0.1	0.0	0.0
Higher layers	0.0	0.0	0.0	0.0
Total, All Layers	**11,235.9**	**13,900.4**	**3,429.8**	**3,408.3**

compared to only the $5 million excess of $10 million layer using Method 1.[12]

- For all layers combined, which translates to the value of all future amounts payable to the claimant, the nominal total from Method 1 ($11.2 million) is considerably lower than the nominal total from Method 2 ($13.9 million). However, the present value from Method 1 ($3.43 million) is about the same as the

[11] "Nominal" payments for Method 2 are discounted for mortality, but not for the time-value of money.

[12] Exhibit 4 in fact shows that the maximum possible loss for Method 2 is $74 million, which is one layer higher than is reflected in the text. The tiny probability of this happening means that the expected losses in the layers above $70 million are below $1,000, and thus do not show up on Table 3. In other words, the numbers are different, even though rounding makes them look the same.

present value from Method 2 ($3.41 million). How can we explain this?

i) *Nominal total from Method 2 considerably greater than Method 1* The easiest way to explain the relation between the nominal totals is by analogy to a more familiar idea involving annuities, namely, that the present value of a life annuity is less than the present value of an annuity certain for the person's life expectancy. (Bowers [2], pp. 149–150 (example 5.13) and p. 158 (exercise 5.45).) In other words, the expected cost of paying someone $1 per year for life is less than the cost of paying $1 per year for a guaranteed period equal to the person's life expectancy. The intuition is that if you pay for the person's actual lifetime, there's a chance of living beyond the life expectancy, and those payments are discounted at a higher rate than the earlier payments. By contrast, the annuity certain ignores the possibility of these higher discounts.

   How does this relate to the nominal payments from Method 2 being much greater than Method 1? In our situation, we have inflation affecting the payments in two ways: the indemnity amounts are increased by the annual cost-of-living increase, and the medical amounts are increased by the annual medical inflation. If the claimant lives to, say, 95 years old, there will be many years of inflation increasing the annual payments beyond the inflation contemplated in Method 1, which halts at the life expectancy. Thus, without inflation, the nominal amounts from Methods 1 and 2 would be identical; with inflation, the nominal amount from Method 1 will be lower than that for Method 2.

ii) *Present value of Method 2 almost the same as Method 1* Without inflation, the payments would be the same each year. Then, as noted above, the present value of Method 1 (an annuity certain for the life expectancy) would exceed the present value for Method 2 (a life annuity). When there is inflation, things are more complicated. The issue is whether the effect of the additional inflation beyond the life expectancy outweighs the effect of the additional discounting. Depending on the rates used for inflation and discounting, the present value of Method 2 could be either higher or lower than the present value of Method 1. Though the total present values for Methods 1 and 2 are close, the amounts in particular layers differ considerably.

- On the layers that are pierced by Method 1, the commutation value from Method 2 is lower than the value from Method 1. For example, on the $500,000 excess $500,000 layer, the value under Method 1 is $413,200, while under Method 2 it's $409,100. This is because Method 1 assumes the amounts are paid for certain, and discounts only for the time-value of money. By contrast, Method 2 recognizes that the claimant may die early, so the amounts may not be paid. Of course, in the layers not pierced in Method 1, the commutation value for Method 2 is always higher.

- We can make no general statement about whether a commutation calculated using Method 1 will produce a total amount, for all layers combined, that is greater than or less than the total for Method 2. For example, if the primary company buys reinsurance on only very low layers, Method 1 will tend to be higher. If it buys reinsurance only on high layers, Method 2 will tend to be higher.

Determinism and Risk

Once a claim has been commuted, the cedent takes the risk of future losses. If the claimant lives to a ripe old age, the primary company will suffer a loss—it would have been better off not to have commuted. That's not a problem: insurance is about taking risks. The commutation calculation measured the mortality risk, and included it in the commutation price. Though the primary company may not be happy to have to pay higher than expected losses, the mortality risk has been priced into the commutation amount.

But there are other risks faced by the ceding company that have not been priced into the commutation amount. Medical inflation is one such example.

The assumed rate of medical inflation is often a contentious issue in commutation negotiations. The parties may argue over whether we should use the average for the past decade (currently about 6%), a longer term average (also about 6% if we average back to World War II), or an econometrician's projection for medical inflation for the next decade. In many cases we are projecting inflation for 70 years or more, so we cannot expect our numbers to be perfect. But often the parties find a number on which they can agree—let us assume it is 5.25%, and let us assume this number is, indeed, the future long-term average medical inflation rate. If the parties use Method 2 with 5.25% medical inflation, and agree on the amount, the ceding company appears to have been compensated for future inflation.

But the ceding company has not, in fact, been compensated for future inflation. It has been compensated for a fixed 5.25% future inflation. It faces the risk that 2 or 3 years hence, there will be high medical inflation, say 20% or 25% per year, for 3 or 4 years, after which medical inflation will drop back to its long-term average. This period of abnormally high medical inflation will quickly erode the retention, which is in nominal dollars, and breach the excess layers much more quickly than the commutation calculation assumes.

There is, similarly, a chance that medical inflation for the next few years will be lower than the long term average, and high medical inflation may not occur for another 60 years. Over the course of the 70 years, one might expect things to even out. So, the skeptic may ask, why should we care? If, on average, it evens out, and if a company does a large number of commutations

TABLE 4

MEDICAL AMOUNT PAYABLE EACH YEAR

Year	Scenario 1: 5% inflation each year	Scenario 2: 20% inflation in year 1; 0% in all other years	Scenario 3: 20% inflation in year 4; 0% in all other years
0	100.00	100.00	100.00
1	105.00	120.00	100.00
2	110.25	120.00	100.00
3	115.76	120.00	100.00
4	121.55	120.00	120.00
Total	**552.56**	**580.00**	**520.00**

over a large number of years, the overall result will be about right.

The problem is that it will not be "about right," as things do not average out in the long run. Just as Method 1 gave biased results, so Method 2, by assuming certain inputs are deterministic, gives biased results.

The Effects of Variable Inflation

To see why things do not average out, let us examine the effects of variable inflation more closely. Consider, on Table 4, an average inflation rate of 5% per year in each of 3 scenarios, and assume the pre-inflation amount payable per year is $100.

Inflation early on (scenario 2) raises the nominal dollar amounts in all future years, causing the total nominal amount to be higher. If there is reinsurance on these payments, the reinsurance retention would be breached earlier, and perhaps a layer will be breached that would not otherwise have been breached. The average inflation over the three scenarios is the same, but

Scenario 2 results in more dollars of medical expenses, and Scenario 3 results in fewer dollars of medical expenses.[13]

For a given average inflation rate, the path of inflation over the life of the claim will affect the future payments: high inflation early on will result in higher amounts; low inflation early on will result in lower amounts. While the total amount over all layers of reinsurance may roughly average out to be the same when present-valued, the amounts within the various layers will differ significantly.

If there is high inflation early on, the reinsurance retention will be breached earlier than expected. There is thus a greater chance that the claimant will still be alive to receive the payment. This greater possibility of payment directly affects the commutation calculation.

The standard commutation calculation fails to include certain risks, and thus neglects to price them. Method 2 assumes mortality is stochastic, but that medical inflation is deterministic. It also assumes wage inflation (and hence cost-of-living adjustments, in states that have them), investment income, and the annual medical usage of the claimant are deterministic. Analogous to Method 1 overstating the lower layers and understating the higher layers, Method 2 will generally bias the commutation amount upwards for lower layers and downwards for higher layers. ("Higher" and "lower" is relative to the size of an individual claim.) Making each of these factors stochastic removes some of the bias in the calculation.

Method 3: Stochastic economic factors and medical costs

Method 3 incorporates several additional random variables into the calculation:

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[13] Lee Steeneck pointed out that it might be more appropriate to use a geometric mean of inflation in this example, rather than an arithmetic mean. Doing so would somewhat complicate the example, without changing the point being made.

- Inflation is not constant over time. It fluctuates, with the year-to-year rates correlated. [A note on terminology: by "inflation," with no modifier, we mean inflation relating to the overall economy, most popularly measured by the CPI. When referring specifically to price rises for medical care, we will refer to "medical inflation."]
- Medical inflation, while roughly tracking the ups and downs of general inflation, will not be the same as inflation, or even some constant difference from inflation.
- Investment yields fluctuate from year to year, but, like inflation, years are correlated.
- The annual medical payment to the claimant will not be a constant real amount each year. As the claimant's health changes, this amount will change. The claimant may take a turn for the worse, and require $200,000 of hospitalization one year; or he may have a stable period where his medical expense is a lot lower than projected.

Each of these variables needs to be modeled. The specific ways they have been modeled here is not the only way it could be done. The details of the example are less important than the general point being made, namely, that additional fluctuations need to be taken into account.

Inflation

Inflation was modeled using an autoregressive process of the following form:

$$\begin{aligned}
&\text{Inflation rate}_{\text{Year } t} \\
&\quad = \text{Long-term average inflation rate} \\
&\qquad + \alpha[\text{Inflation rate}_{\text{Year } (t-1)} \\
&\qquad\qquad - \text{Long-term average inflation rate}] \\
&\qquad + \text{error}_{\text{Year } t}
\end{aligned}$$

Daykin, et al. [6, pp. 218–225], discusses this model, and a number of other inflation models that may better fit the data. In the interests of simplicity, this model was chosen. The model starts with a known inflation rate for 1997, and simulates a series of future paths of inflation.

Using least-squares fitting of inflation data from the Bureau of Labor Statistics from 1935–1995, the following parameters were obtained:

$$\text{Long-term average inflation} = 4.11\% \text{ per year}$$

$$\alpha = 0.511.$$

The error term was modeled using a lognormal distribution. Since the error can be positive or negative, but a lognormal is only defined for positive variables, I shifted the lognormal. The best fit was obtained from a shifted lognormal with parameters $\mu = -2.76$ and $\sigma = 0.501$. To ensure a zero mean for the error term, the lognormal was shifted by the mean of this distribution, or about 0.0718. Exhibit 5 shows the derivation of these parameters.

This inflation variable was used to model the cost-of-living adjustment to the indemnity payments. COLAs are usually tied to changes in the state average weekly wage, and wage inflation was assumed to be the same as overall price inflation—a convenient simplification, not necessarily correct. Since most COLAs are capped, the COLA was assumed to not exceed 5% in any year. It was also assumed that if inflation is negative, the indemnity amount would not drop. Since COLAs are lagged a year, it was assumed that the COLA in 1998 is based on 1997 inflation, etc.

Medical Inflation

Medical inflation may be higher or lower than inflation, but they are linked: if the inflation rate were 20% for a sustained period, one would not expect medical inflation to remain at 2%.

The selected model of medical inflation is tied to the overall inflation rate, but with a degree of error allowed. The model is:

$$\begin{aligned}
&\text{Medical Inflation}_{\text{Year } t} \\
&\quad = \text{Inflation}_{\text{Year } t} \\
&\qquad + \beta[\text{Medical inflation}_{\text{Year } (t-1)} - \text{Inflation}_{\text{Year } (t-1)}] \\
&\qquad + [\text{long-term average medical inflation} \\
&\qquad\quad - \text{long-term average inflation}] \\
&\qquad + \text{error}_{\text{Year } t}
\end{aligned}$$

The error term is assumed to be normally distributed, with a mean of zero.[14]

The longest available data series was used to get these parameters. The Bureau of Labor Statistics has medical CPI numbers back to 1935. From 1935 to 1997, average medical inflation was 1.14 percentage points higher than average inflation. This is what was used for the third term of the above expression. We are assuming the long-term trend will continue, although, there is of course no guarantee of this.

The fitted β was 0.38, and the error term was normally distributed with a mean of 0 and a standard deviation of 0.027. Exhibit 6 shows the derivation.

Investment Yields

As noted above, the firm is assumed to invest in inflation-indexed Treasuries, to hedge the inflation risk.[15] These currently have a real yield of about 3.75%. For discounting purposes, a

[14]The inflation model had a lognormal error term, but the medical inflation model has a normal error term. The author had a strong feeling that the error for inflation was skewed, whereas it is less obvious, both from the data and intuitively, that the difference between overall inflation and medical inflation (which largely drives the medical inflation model) is skewed.

[15]It is beyond the scope of the paper to address the question of whether discounting should be based on the firm's (either the reinsurer's or reinsured's) actual investments, or whether it should be based on market discount rates.

2.5 percentage point risk adjustment was made to the rate, thus discounting at 1.25 percentage points above the inflation rate.

For example, if the annual CPI in a particular year is 5.3%, as generated by the autoregressive model discussed above, the discounting for that year would be at 6.55%.

Even if inflation is negative, one would not expect interest rates to drop below some threshold (e.g., 2.5%), so the risk-adjusted discount rate was assumed to not go below zero, i.e., the rate for discounting was set at the greater of the inflation rate plus 1.25% and zero.

Medical Services Used By Claimant

Medical usage will fluctuate from year to year, but we would expect the services from year to year to be correlated. For example, if a claimant has surgery this year, the costs of post-operative treatment may keep the costs higher than average in the next year. One can model this process using an autoregressive model, similar to the one for inflation:

$$\begin{aligned}\text{Medical amount}_{\text{Year } t} &= \text{Long-term average medical amount} \\ &\quad + \gamma[\text{Medical amount}_{\text{Year } (t-1)} \\ &\qquad - \text{long-term average medical amount}] \\ &\quad + \text{error}_{\text{Year } t}\end{aligned}$$

The long-term average medical amount for this case is, by assumption, \$70,000. Empirically, there does not appear to be a very strong link between last year's medical amount and this year's, so $\gamma = 0.05$ was used. The error term was modeled using a lognormal distribution with $\mu = 10.80089$ and $\sigma = 0.75$. The mean of this lognormal is \$65,000, so the distribution was shifted by 65,000 to ensure the error term has a mean of zero.

TABLE 5

COMPARISON OF RESULTS FROM METHODS 1, 2, AND 3

Layer (in $000's)	Expected Nominal Payments (in $000's)			Expected Present-Value Payments (in $000's)		
	Method 1	Method 2	Method 3	Method 1	Method 2	Method 3
130 xs 370	130	130	130	125	125	125
500 xs 500	500	495	494	413	409	415
1,000 xs 1,000	1,000	970	969	612	594	609
3,000 xs 2,000	3,000	2,725	2,705	1,092	999	1,031
5,000 xs 5,000	5,000	3,703	3,643	970	730	766
5,000 xs 10,000	1,606	2,575	2,591	217	311	344
5,000 xs 15,000	0	1,607	1,788	0	140	175
10,000 xs 20,000	0	1,360	2,093	0	87	152
10,000 xs 30,000	0	293	1,047	0	13	55
10,000 xs 40,000	0	39	558	0	1	23
10,000 xs 50,000	0	3	316	0	0	11
10,000 xs 60,000	0	0	188	0	0	6
10,000 xs 70,000	0	0	117	0	0	3
10,000 xs 80,000	0	0	75	0	0	2
10,000 xs 90,000	0	0	49	0	0	1
Unlimited xs $100MM	0	0	120	0	0	2
Total, All Layers	**11,236**	**13,900**	**16,881**	**3,430**	**3,408**	**3,719**

Running the Model

Each of these parameters was then put into a simulation model. By simulating inflation, medical inflation, and the annual medical amount, one gets a set of input parameters for each simulation. These parameters are then run through the same model as is used in Method 2. The difference is that each time it is run through with different parameters, so that instead of getting a single present value of the future payments, we get a distribution. (Exhibit 7 shows a single simulation from this distribution.)

The means of these distributions, for each layer, are shown on Table 5, compared with the results for Methods 1 and 2.

It is worth noting a few things regarding these results:

- Unlike Methods 1 and 2, Method 3 hits all the reinsurance layers. A less deterministic approach recognizes that higher layers are exposed to loss. Thus, layers that might otherwise have been thought to have no possibility of a loss, are shown to have some commutation value.

- The total nominal value of Method 3 is higher than the nominal value of Method 2 (and Method 2 is higher than Method 1, as discussed earlier).

  This is largely explained by the treatment of inflation. The medical and indemnity amounts paid in some future period depend on the products of (1 + inflation rate) for all prior periods. For example, the amount paid in period 3 depends on what inflation was in periods 1 and 2. The inflation rates are not independent from period to period: the autocorrelation model ensures that they are positively correlated. With positive correlation, the expected value of the product is greater than the product of the expected values, making the overall nominal payments for Method 3 higher than the payments in Method 2.[16]

- The overall present value factor for Method 2 is 25% (= 3,408 ÷ 13,900), but the present value factor for Method 3 is only 22% (= 3,719 ÷ 16,881). In other words, Method 3 has, on average, a steeper discount applied to it.

  This is partly because the year-to-year discount factors (like the inflation factors) are correlated, implying a higher average discount. Also, high medical inflation is correlated with high discount factors, so the higher nominal payments caused by high inflation are more heavily discounted.

- The relationship between the present values of Methods 2 and 3 is complex, largely because the assumptions are not con-

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[16] $E(XY) = E(X)E(Y) + \text{cov}(X,Y)$. Thus, if X and Y are positively correlated, the expected value of the product exceeds the product of the expected values.

sistent between the two methods. Yes, we tried to make them consistent, but the differences in the assumptions become clear once we examine them more carefully.

Consider the indemnity cost-of-living adjustments. In Method 2 we used 4.11% for the cost-of-living adjustment. In Method 3, inflation varies stochastically, with a mean of 4.11%. But our cost-of-living adjustment rules were that it couldn't be above 5%, or below 0%. In Method 3, the average inflation rate is 4.11%, but the average cost-of-living adjustment is about 2.9% because it is sometimes capped. A similar, though smaller, discrepancy occurs in the discount rate, due to assuming that the discount rate cannot be negative.

In general, the relationship between the present values of Methods 2 and 3 will depend on the particular assumptions, and how they interact with the various caps and correlations.

- The present value factor for Method 3 losses declines sharply in the higher layers. For example, for the $5 million excess of $5 million layer, the present value is $766,000, compared to the nominal value of $3,643,000. This translates to a present value factor of 21%. By contrast, in the $10 million excess of $90 million layer, the present value factor is only 2%.
- In the lowest layers, the nominal value of Method 1 is higher than Method 2, and Method 2 is higher than Method 3.[17] This

[17] On the earlier table, the nominal values for Methods 2 and 3 look the same in the low layers, but the numbers in the table are rounded. If the complete numbers had been shown, the nominal values in the low layers would be systematically less (though admittedly by a small amount) for Method 3 than for Method 2:

	Nominal Value (in $000's)	
Layer	Method 2	Method 3
1	129.74	129.69
2	494.88	494.44
3	970.39	968.63
4	2,725.08	2,704.59

is because Method 1 implies these layers will be hit for certain, whereas Methods 2 and 3 recognize that the claimant could die before the layer is penetrated. In addition, Method 3 recognizes that there could be years of unusually low claim amounts, so that it may take longer than expected to breach the retention. This reduces the commutation amount in two ways:

i) The longer it is until the retention is breached, the greater the chance of the claimant dying before breaching the retention.

ii) The longer it is until the retention is breached, the steeper the effect of discounting.

In higher layers, which have a lower probability of being penetrated, this situation reverses itself: Method 3 gives higher results than Method 2. The upper layers are most vulnerable to a period of sustained high inflation or high claim levels. Methods 1 and 2 assume inflation and claim levels are fixed, so they do not contemplate any chance of sustained high inflation or claim levels.

- For the lower layers, where the chances are good that the claimant will live long enough to breach them, Method 2 gives similar results to Method 3. But as the layers get higher, the Method 2 number gets lower and lower as a percentage of Method 3, as shown in Table 6.

5. ARE THERE FURTHER LEVELS OF DETERMINISM?

We have shown that the commutation calculation is significantly affected by making a variety of variables nondeterministic. Have we now stripped away all determinism? Put another way, is Method 3 “the perfect” commutation calculation, or is there further determinism that remains?

There is, indeed, further determinism. This paper has shown how we can strip away determinism in the levels of inflation,

TABLE 6

METHOD 2 RESULT AS PERCENTAGE OF METHOD 3 RESULT

Layer	Nominal	Present Value
1	100%	100%
2	100%	99%
3	100%	98%
4	101%	97%
5	102%	95%
6	99%	90%
7	90%	80%
8	65%	38%
9	28%	24%
10	7%	4%
11	1%	0%
Higher Layers	0%	0%

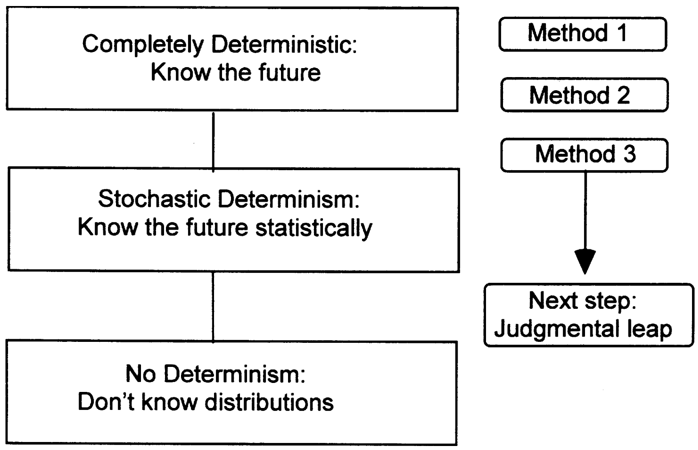
medical utilization, etc. But to measure the paths for these variables, we have relied on statistical measures on past data. Clearly, the historical data may not be valid predictors of the future. For example, the paper assumes that the best predictor of medical inflation is the last 60 years of medical CPI information. One can plausibly argue that what drove medical inflation in the 1930s and 1940s was completely different from what drove it in the 1970s and 1980s, and different from what will drive it in the second half of the twenty-first century. It is quite possible that the drivers of inflation will change periodically over the course of the claimant's lifetime. We have assumed that we know what the future path of medical inflation will be, at the level of a statistical model. But the parameters of the model are deterministic, and so is the structure of the model.

This same issue applies to other variables. For example, advances in medical care could affect the medical utilization for the claimant's condition—and perhaps render the assumed mortality table inappropriate.

In other words, the parameters of our stochastic models could shift, or the model structure itself could change. Method 3 is

FIGURE 2

METHODS 1, 2, AND 3 IN PERSPECTIVE



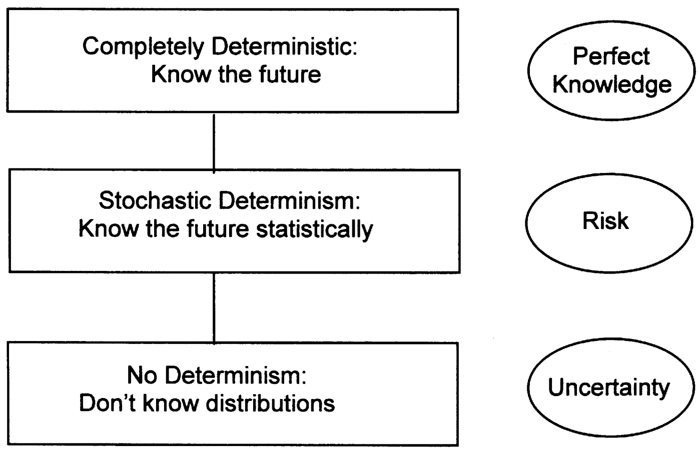
closer to being "stochastically deterministic" than Method 2 is, but it still contains determinism.

The problem is that this next level of determinism is not easily subject to measurement, and hence is not amenable to quantification by the usual actuarial methods. But not being able to quantify does not mean we can simply ignore something.

To put things in perspective, we return now to the graphic introduced at the start of the paper. As Figure 2 makes clear, Method 1 is completely deterministic, Method 2 is somewhat less deterministic, and Method 3 is even less deterministic. But, note carefully that Method 3 is not completely at the level of stochastic determinism, though it is close. There are still various items in Method 3 that are deterministic—for example, mortality rates are assumed to be given. Also, we assume that the parameters of our inflation and interest-rate generators are constant, whereas we could make those parameters themselves stochastic. There are doubts as to whether there is much use in adding these further

FIGURE 3

THE ECONOMIC PERSPECTIVE ON LEVELS OF DETERMINISM



stochastic elements, but the simple point is that Method 3 is not at the level of pure stochastic determinism.

The arrow on Figure 2 shows where we likely need to go after Method 3. The next step requires jumping over the level of pure stochastic determinism and going directly to those items that we cannot measure. Before discussing this, it will be useful to take a brief tour of how economists have viewed some of these issues.

6. THE ECONOMICS OF UNCERTAINTY

The earlier graphic is useful for showing how the ideas in this paper relate to how economists think about risk and uncertainty. Figure 3 repeats the earlier graphic, but now adds some ovals on the right that relate the actuarial ideas to the way economists think about uncertainty.

Many familiar economic models, notably that of perfect competition, assume that people have perfect knowledge. This corresponds with the one end of our continuum: in a completely deterministic calculation, the actuary proceeds as if he or she knows exactly what the future will be.

Moving away from perfect knowledge, economists distinguish between "risk" and "uncertainty."[18] Risk includes things that can be measured statistically, and uncertainty includes things that cannot be measured, but which might occur. For example, if one bets on a fair coin coming up heads, one is facing a risk. But if one bets on the chance of intelligent life being found on an as-yet-undiscovered planet, one faces uncertainty—we have no way of measuring the associated probabilities.

Furthermore, there are events for which we not only do not know the probabilities, but we don't even realize that the event can happen. For example, no actuary pricing liability insurance in the 1930s could even have imagined the wave of asbestos litigation that would hit those policies decades later. This lack of knowledge is sometimes referred to as "sheer ignorance" or "genuine uncertainty."

The economist's idea of risk corresponds to what we called "stochastic determinism": the future is known statistically. And the economist's notion of uncertainty corresponds to what we have called "no determinism."

In practice, most mainstream economics incorporates risk but ignores uncertainty. It is rare to find an economist who deals seriously with uncertainty. And this is, perhaps, for the same reason that one finds so little discussion of this in the actuarial literature—namely, that it is very difficult to include genuine uncertainty in "rigorous" work. Dealing with uncertainty is difficult, and cannot be made numerically precise. Nevertheless, we need to acknowledge that uncertainty is inherent in what we are doing, and that we are fooling ourselves if we believe that our results are perfectly accurate. This applies to both economists and actuaries.

[18]The classic reference on risk and uncertainty is Knight [11]. For more recent discussions of the economics of uncertainty, see O'Driscoll and Rizzo [13] and Kirzner [10].

A focus on uncertainty is mainly found outside the mainstream of economics, and is closely associated with the "Austrian" school of economic thought.[19] Their emphasis is on the role of *sheer ignorance* in the economy:

> For the Austrian approach, imperfect information is seen as involving an element which cannot be fitted at all into neoclassical models, that of "sheer" (i.e., unknown) ignorance…[S]heer ignorance differs from imperfect information in that the discovery which reduces sheer ignorance is necessarily accompanied by the element of *surprise*—one had not hitherto realized one's ignorance. (Kirzner, [10, p. 62])

For the Austrians, uncertainty is an inescapable part of human decision-making. We cannot avoid uncertainty, and the fact that it is difficult for economists to quantify and precisely model is not a reason to ignore it.

7. RISK AND UNCERTAINTY IN INSURANCE

Most insurance problems consist of a mixture of risk and uncertainty. Insurers are good at dealing with risk. By measuring the probabilities of loss and pooling risk, we can work to eliminate risk and make losses more stable in the aggregate. It is far more difficult to deal with uncertainty.[20]

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[19]The "Austrian" school's roots were with Carl Menger at the University of Vienna in the late nineteenth century. Perhaps the best-known Austrian in contemporary times has been Friedrich Hayek, who won the Nobel Prize for Economics in 1974. Today, the main concentrations of "Austrians" are at American universities, most notably New York University and George Mason University. For an introduction to Austrian thought, see Kirzner [10].

Uncertainty is also a concern of some other non-mainstream schools, especially the Post-Keynesians. Some economists, notably George Shackle [14] and Ludwig Lachmann [12] are considered by some to straddle the divide between the Austrians and the Post-Keynesians. For a discussion of Shackle's views on uncertainty, see Coddington [4].

[20]Readers may be tempted to equate the term "risk" with "process risk," and "uncertainty" with "parameter risk." It is advisable to avoid this temptation. Risk, in the sense used by economists, includes both process risk and parameter risk, at least when parameter risk is narrowly defined as the risk of misestimating a parameter due to having a too-small

In this paper, we have been measuring risk: we have only dealt with those things that can be measured. (Insofar as they cannot be modeled well, there are elements of uncertainty.) The next level of determinism consists of uncertainty.

While we cannot easily measure the effect of uncertainty, we can make some qualitative statements about its effects on commutations. Just as removing earlier layers of determinism increased the commutation amount in the higher layers, so removing yet another layer of determinism will increase the commutation amount in higher layers, and higher layers that would not otherwise have been pierced will have some commutation value.

Consider, for example, the inflation model postulated in the example in this paper. There is a real, but very small, chance that the model will generate years where inflation will run above, say, 100% a year as the result of a random blip in the model. In reality, if hyperinflation at that level occurs, it will be more likely to be a result of a structural change in the economy rather than a random event. Since this type of structural change was not included in the data used to fit the model, it is not contemplated in the resulting commutation amount.

Put another way, a completely deterministic model assumes the future will be like the past. Our inflation model, while not completely deterministic, assumes that *fluctuations in* future inflation will be like the past. While this may be more realistic than a completely deterministic model, it is not necessarily true.

All the other variables in the commutation are subject to similar uncertainty: mortality rates might plummet as cures are found for cancer and heart disease; or mortality rates might soar, as a

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sample size. Narrowly defined in this way, parameter risk can be diversified away, just like any other risk.

In popular usage, parameter risk has acquired a more elastic meaning, to include such things as having an incorrectly structured model. (Uncertainty about the structure of the model is sometimes separated from parameter risk, and called "specification risk" or "model risk.") This is much closer to the economist's notion of uncertainty, and is impossible to quantify. Models that quantify parameter risk almost always have a narrower notion of parameter risk in mind, and so it is confusing to equate uncertainty with parameter risk. Furthermore, uncertainty has connotations of the underlying structure of the economy changing over time, and this is not contemplated by parameter risk.

new virus kills half the population. The annual medical usage might drop, if a cure is found for the claimant's ailment, which was previously thought to be permanent. Or the cost of medical care might soar as a new drug is discovered that greatly improves the claimant's quality of life, at twice the cost. What if the government takes over the entire health-care system, and insurers are no longer responsible for medical costs?

We can dream up many different situations that will change what insurers owe to claimants. We can put probabilities on none of these, and we also know that there are many possibilities that we may not even think of, until they actually happen.

In commutations, it is common to ignore this uncertainty, and to commute some of the very high layers without payment. This is unwarranted. Commuting reinsurance is really a matter of pricing future possibilities, and reinsurers do not give away free layers, even if they have only a remote chance of being hit. For example, suppose I want to buy workers compensation reinsurance for a layer of $1 million excess of $800 million. (To avoid catastrophe issues, let us assume the reinsurance is per claim, not per occurrence.) There has never been a workers compensation claim that large, or even remotely close to it. Yet, would a reinsurer be willing to give the layer away free, even assuming they have no costs to service the contract? Of course they won't. Reinsurers recognize the remote possibility of having to pay on this contract, and they need to charge for that risk. The risk is remote, but remote does not mean non-existent. The chance of the layer being hit is not measurable, but not measurable does not mean zero.

8. THE DILEMMA OF THE "AUSTRIAN" ACTUARY

The dilemma of an actuary who recognizes ubiquitous uncertainty described by the Austrian economists is illustrated by a supposed comment of Lord Kelvin that "If you cannot measure, your knowledge is meager and unsatisfactory."[21]

[21] Coddington [4, p. 160] notes that there is no record of Kelvin ever having said exactly this, but it is inscribed on the facade of the Social Science Research Building at the University of Chicago.

As actuaries, we are paid to advise people on the numbers. In the case of a commutation, we are paid to decide whether a particular commutation offer is reasonable. If we are presented with a commutation offer, we can recommend that it be accepted or rejected. But saying "I don't know because the future is uncertain and I can't measure that" won't help. The dilemma of the "Austrian" actuary is that he recognizes that his knowledge *is* "meager and unsatisfactory," but he has to make a recommendation nevertheless.

One way of handling the dilemma is to take the advice of Frank Knight, who commented that the meaning for social scientists of Kelvin's remark is that "If you cannot measure, measure anyhow."[22] But "measuring anyhow" just leads to ignoring things that cannot be measured. If you have no reason to believe that these unmeasurables will bias your answers one way or another, that doesn't matter. But in many cases, especially when dealing with excess reinsurance, the unmeasurables will frequently bias the answers.

We must recognize that we will have to judgmentally adjust our answers for the unmeasurables. Judgmental adjustments are often uncomfortable, because they are hard to justify when attacked by others. But we have no choice other than to make our best judgments and explain the uncertainty of what we are doing.

9. POSSIBLE WAYS TO "MEASURE" THE UNMEASURABLE

When making judgmental adjustments, we are not completely without guidance. For a workers compensation commutation, here are some ways to check one's judgments:

Check 1: How much difference does the uncertainty make?

The first issue is to check the level of uncertainty, and the effects it can have. In the Joe Soap example discussed at length

[22]Quoted in Coddington [4, p. 160].

above, the different reinsurance layers have very different levels of uncertainty. One would expect that the lowest two or three layers will be breached fairly quickly, if the claimant survives. Even fairly dramatic changes in inflation and mortality rates will have relatively little impact on the numbers. The real impact of uncertainty is on the upper layers, where decades of compounded inflation, investment yields, changes in medical practice, and the claimant's condition come together to make the results of the calculations very fuzzy.

In the lower layers, Method 2 gives reasonable results. For medium layers, Method 3, unadjusted, may be reasonable. For higher layers, Method 3 results may need to be judgmentally increased, with the higher the layer, the higher the increase.

Check 2: What would it take to breach the layer?

For high layers, one can ask what it would take to breach the layer. For example, if it would take sustained medical inflation of 25% per annum to breach the layer, one would probably feel that this possibility is remote. But if it would take medical inflation of 10% per annum, which is considerably more likely, it should get a bigger charge. One can do similar reasonability checks for other parameters.

Check 3: What does the market charge?

We can get useful information from finding out what the market charges. To get useful information from market prices, we do not need to assume that the market price is exactly at its equilibrium level. The market price, as some consensus of supply and demand, provides a reality check.

There are, of course, no large, liquid markets for workers compensation commutations, but that doesn't mean there is no available information. A commutation is nothing more than reinsurance pricing, albeit for accidents that have already happened a number of years ago. It is quite reasonable to look at the reinsurance market for help.

For example, we generally find that the higher the layer being covered, the higher the risk load for the layer. [This higher risk load might be expressed in different ways—for example, a lower discounted loss ratio, or a "capacity charge" for layers that are seen to have a remote chance of being breached—but, in essence, these are all just risk loads.] With a commutation, we can look at the market structure of risk loads by layer, and use those to develop commutation risk loads for corresponding layers.

10. OTHER LINES OF BUSINESS; PRICING AND RESERVING, TOO

The issues discussed in this paper apply more broadly than just to workers compensation commutations. A commutation calculation for a general liability treaty would usually develop the expected losses to ultimate, and commute based on the discounted value of those losses. But this ignores risks that are transferred back to the ceding company in the commutation. For example, a general liability treaty being commuted in 1978 would have relieved the reinsurer for liability for environmental claims that were generated by the Superfund law, which passed a couple of years later. It was unknown, at the time of the commutation, that the cedent was giving up coverage for this risk, but it was not unknown that the cedent was taking the risk of some such change in the future. Just as a company selling general liability reinsurance will not give away remote layers free of charge, so the commutation should not be free for these layers either.

And it is not just commutations that are affected by determinism. It applies to regular pricing and reserving work as well. The clearest example would be the reserving of workers compensation reinsurance, where the methods used in this paper can be directly applied. But for pricing and reserving of any excess insurance or reinsurance, it is important to keep in mind the problems of determinism. If we simply assume the future will turn out to be what was expected, or that the future will follow the patterns of the past, we are bound to be led astray. The scary part of writing insurance is the uncertainty of what the future will bring. The uncertainty cannot be quantified, but we must not stick our heads in the sand and assume that if something cannot be quantified, it doesn't exist.

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EXHIBIT 1

1990 US LIFE TABLE (MALES)

Age	$l(x)$	Life Expectancy	Age	$l(x)$	Life Expectancy	Age	$l(x)$	Life Expectancy
0	100,000.0	71.8	37	94,585.0	37.8	74	54,249.0	9.9
1	98,969.0	71.6	38	94,316.0	36.9	75	51,519.0	9.4
2	98,894.0	70.6	39	94,038.0	36.0	76	48,704.0	8.9
3	98,840.0	69.7	40	93,753.0	35.1	77	45,816.0	8.4
4	98,799.0	68.7	41	93,460.0	34.2	78	42,867.0	7.9
5	98,765.0	67.7	42	93,157.0	33.3	79	39,872.0	7.5
6	98,735.0	66.8	43	92,840.0	32.4	80	36,848.0	7.1
7	98,707.0	65.8	44	92,505.0	31.6	81	33,811.0	6.7
8	98,680.0	64.8	45	92,147.0	30.7	82	30,782.0	6.3
9	98,657.0	63.8	46	91,764.0	29.8	83	27,782.0	5.9
10	98,638.0	62.8	47	91,352.0	28.9	84	24,834.0	5.5
11	98,623.0	61.8	48	90,908.0	28.1	85	21,962.0	5.2
12	98,608.0	60.8	49	90,429.0	27.2	86	19,216.8	4.9
13	98,586.0	59.9	50	89,912.0	26.4	87	16,607.4	4.5
14	98,547.0	58.9	51	89,352.0	25.5	88	14,157.7	4.2
15	98,485.0	57.9	52	88,745.0	24.7	89	11,889.0	3.9
16	98,397.0	57.0	53	88,084.0	23.9	90	9,819.5	3.7
17	98,285.0	56.0	54	87,363.0	23.1	91	7,962.6	3.4
18	98,154.0	55.1	55	86,576.0	22.3	92	6,326.9	3.2
19	98,011.0	54.2	56	85,719.0	21.5	93	4,915.0	2.9
20	97,863.0	53.3	57	84,788.0	20.7	94	3,723.5	2.7
21	97,710.0	52.3	58	83,777.0	20.0	95	2,743.0	2.5
22	97,551.0	51.4	59	82,678.0	19.2	96	1,958.3	2.3
23	97,388.0	50.5	60	81,485.0	18.5	97	1,349.7	2.1
24	97,221.0	49.6	61	80,194.0	17.8	98	894.0	1.9
25	97,052.0	48.7	62	78,803.0	17.1	99	566.2	1.8
26	96,881.0	47.8	63	77,314.0	16.4	100	340.6	1.6
27	96,707.0	46.9	64	75,729.0	15.8	101	193.2	1.5
28	96,530.0	45.9	65	74,051.0	15.1	102	102.4	1.3
29	96,348.0	45.0	66	72,280.0	14.5	103	50.1	1.2
30	96,159.0	44.1	67	70,414.0	13.8	104	22.3	1.1
31	95,962.0	43.2	68	68,445.0	13.2	105	8.9	1.0
32	95,758.0	42.3	69	66,364.0	12.6	106	3.1	0.9
33	95,545.0	41.4	70	64,164.0	12.0	107	0.9	0.8
34	95,322.0	40.5	71	61,847.0	11.5	108	0.2	0.7
35	95,089.0	39.6	72	59,419.0	10.9	109	0.0	0.5
36	94,843.0	38.7	73	56,885.0	10.4	110	0.0	

Source: Vital Statistics of the United States, 1990 [US Department of Health and Human Services, 1994].
Note that the published tables extend only to age 85; beyond 85, the numbers are extrapolations.

EXHIBIT 2

INFLATION: CONSUMER PRICE INDEX AND MEDICAL CONSUMER PRICE INDEX

	Index at December		Annual Inflation			Index at December		Annual Inflation	
Year	CPI	Medical CPI	CPI	Medical CPI	Year	CPI	Medical CPI	CPI	Medical CPI
1935	13.8	10.2			1967	33.9	28.9	3.0%	6.3%
1936	14.0	10.2	1.4%	0.0%	1968	35.5	30.7	4.7%	6.2%
1937	14.4	10.3	2.9%	1.0%	1969	37.7	32.6	6.2%	6.2%
1938	14.0	10.3	− 2.8%	0.0%	1970	39.8	35.0	5.6%	7.4%
1939	14.0	10.4	0.0%	1.0%	1971	41.1	36.6	3.3%	4.6%
1940	14.1	10.4	0.7%	0.0%	1972	42.5	37.8	3.4%	3.3%
1941	15.5	10.5	9.9%	1.0%	1973	46.2	39.8	8.7%	5.3%
1942	16.9	10.9	9.0%	3.8%	1974	51.9	44.8	12.3%	12.6%
1943	17.4	11.4	3.0%	4.6%	1975	55.5	49.2	6.9%	9.8%
1944	17.8	11.7	2.3%	2.6%	1976	58.2	54.1	4.9%	10.0%
1945	18.2	12.0	2.2%	2.6%	1977	62.1	58.9	6.7%	8.9%
1946	21.5	13.0	18.1%	8.3%	1978	67.7	64.1	9.0%	8.8%
1947	23.4	13.9	8.8%	6.9%	1979	76.7	70.6	13.3%	10.1%
1948	24.1	14.7	3.0%	5.8%	1980	86.3	77.6	12.5%	9.9%
1949	23.6	14.9	− 2.1%	1.4%	1981	94.0	87.3	8.9%	12.5%
1950	25.0	15.4	5.9%	3.4%	1982	97.6	96.9	3.8%	11.0%
1951	26.5	16.3	6.0%	5.8%	1983	101.3	103.1	3.8%	6.4%
1952	26.7	17.0	0.8%	4.3%	1984	105.3	109.4	3.9%	6.1%
1953	26.9	17.6	0.7%	3.5%	1985	109.3	116.8	3.8%	6.8%
1954	26.7	18.0	−0.7%	2.3%	1986	110.5	125.8	1.1%	7.7%
1955	26.8	18.6	0.4%	3.3%	1987	115.4	133.1	4.4%	5.8%
1956	27.6	19.2	3.0%	3.2%	1988	120.5	142.3	4.4%	6.9%
1957	28.4	20.1	2.9%	4.7%	1989	126.1	154.4	4.6%	8.5%
1958	28.9	21.0	1.8%	4.5%	1990	133.8	169.2	6.1%	9.6%
1959	29.4	21.8	1.7%	3.8%	1991	137.9	182.6	3.1%	7.9%
1960	29.8	22.5	1.4%	3.2%	1992	141.9	194.7	2.9%	6.6%
1961	30.0	23.2	0.7%	3.1%	1993	145.8	205.2	2.7%	5.4%
1962	30.4	23.7	1.3%	2.2%	1994	149.7	215.3	2.7%	4.9%
1963	30.9	24.3	1.6%	2.5%	1995	153.5	223.8	2.5%	3.9%
1964	31.2	24.8	1.0%	2.1%	1996	158.6	230.6	3.3%	3.0%
1965	31.8	25.5	1.9%	2.8%	1997	161.3	237.1	1.7%	2.8%
1966	32.9	27.2	3.5%	6.7%					
							Average	**4.11%**	**5.25%**

Source: US Department of Labor, Bureau of Labor Statistics.

EXHIBIT 3

PART 1—PAGE 1

COMPLETELY DETERMINISTIC COMMUTATION CALCULATION

Parameters:

(A)	Evaluation Date:	1/1/98
(B)	Age at evaluation date:	35
(C)	Annual indemnity payment	20,000
(D)	Annual medical payment: (at mid-1997 price levels)	70,000
(E)	Indemnity paid to date	70,000
(F)	Medical paid to date	300,000
(G)	Life expectancy:	39.6
(H)	Cost-of-Living Adjustment:	4.11%
(I)	Medical Inflation Rate:	5.25%
(J)	Annual Discount Rate:	5.36%

Year	(1) Cost of Living Adjustment	(2) Indemnity Payment	(3) Medical Inflation	(4) Medical Payment	(5) Total Payment (2) + (4)	(6) Cumulative Total Payment Cumulative of (5)
1997 and prior		70,000		300,000	370,000	370,000
1998	4.11%	20,822	5.25%	73,675	94,497	464,497
1999	4.11%	21,678	5.25%	77,543	99,221	563,718
2000	4.11%	22,569	5.25%	81,614	104,183	667,900
2001	4.11%	23,496	5.25%	85,899	109,395	777,295
2002	4.11%	24,462	5.25%	90,408	114,870	892,166
2003	4.11%	25,467	5.25%	95,155	120,622	1,012,788
2004	4.11%	26,514	5.25%	100,150	126,665	1,139,452
2005	4.11%	27,604	5.25%	105,408	133,012	1,272,465
2006	4.11%	28,738	5.25%	110,942	139,681	1,412,145
2007	4.11%	29,920	5.25%	116,767	146,686	1,558,831
2008	4.11%	31,149	5.25%	122,897	154,046	1,712,878
2009	4.11%	32,429	5.25%	129,349	161,778	1,874,656
2010	4.11%	33,762	5.25%	136,140	169,902	2,044,558
2011	4.11%	35,150	5.25%	143,287	178,437	2,222,995
2012	4.11%	36,595	5.25%	150,810	187,404	2,410,400
2013	4.11%	38,099	5.25%	158,727	196,826	2,607,226
2014	4.11%	39,664	5.25%	167,061	206,725	2,813,951
2015	4.11%	41,295	5.25%	175,831	217,126	3,031,077

EXHIBIT 3

PART 1—PAGE 2

Year	(1) Cost of Living Adjustment	(2) Indemnity Payment	(3) Medical Inflation	(4) Medical Payment	(5) Total Payment (2) + (4)	(6) Cumulative Total Payment Cumulative of (5)
2016	4.11%	42,992	5.25%	185,062	228,054	3,259,131
2017	4.11%	44,759	5.25%	194,778	239,537	3,498,668
2018	4.11%	46,598	5.25%	205,004	251,602	3,750,270
2019	4.11%	48,514	5.25%	215,767	264,280	4,014,551
2020	4.11%	50,508	5.25%	227,094	277,602	4,292,152
2021	4.11%	52,583	5.25%	239,017	291,600	4,583,753
2022	4.11%	54,745	5.25%	251,565	306,310	4,890,063
2023	4.11%	56,995	5.25%	264,772	321,767	5,211,830
2024	4.11%	59,337	5.25%	278,673	338,010	5,549,840
2025	4.11%	61,776	5.25%	293,303	355,079	5,904,919
2026	4.11%	64,315	5.25%	308,702	373,017	6,277,935
2027	4.11%	66,958	5.25%	324,909	391,867	6,669,802
2028	4.11%	69,710	5.25%	341,966	411,676	7,081,478
2029	4.11%	72,575	5.25%	359,920	432,495	7,513,973
2030	4.11%	75,558	5.25%	378,815	454,373	7,968,346
2031	4.11%	78,663	5.25%	398,703	477,367	8,445,713
2032	4.11%	81,897	5.25%	419,635	501,532	8,947,245
2033	4.11%	85,263	5.25%	441,666	526,928	9,474,173
2034	4.11%	88,767	5.25%	464,853	553,620	10,027,793
2035	4.11%	92,415	5.25%	489,258	581,673	10,609,466
2036	4.11%	96,213	5.25%	514,944	611,158	11,220,624
2037	4.11%	60,101	5.25%	325,187	385,288	11,605,912
Total		2,060,654		9,545,258		

Future payments = 11,605,912 – 370,000 = 11,235,912

EXHIBIT 3

PART 2—PAGE 1

	(6)	(7)	(8)	(9)	(10)	(11)	(12)
	Cumulative Total Payment	Incremental Payments By Layer					
Year	Cumulative of (5)	$500,000 xs $370,000	$500,000 xs $500,000	$1 million xs $1 million	$3 million xs $2 million	$5 million xs $5 million	$5 million xs $10 million
1997 and prior	370,000	0	0	0	0	0	0
1998	464,497	94,497	0	0	0	0	0
1999	563,718	35,503	63,718	0	0	0	0
2000	667,900	0	104,183	0	0	0	0
2001	777,295	0	109,395	0	0	0	0
2002	892,166	0	114,870	0	0	0	0
2003	1,012,788	0	107,834	12,788	0	0	0
2004	1,139,452	0	0	126,665	0	0	0
2005	1,272,465	0	0	133,012	0	0	0
2006	1,412,145	0	0	139,681	0	0	0
2007	1,558,831	0	0	146,686	0	0	0
2008	1,712,878	0	0	154,046	0	0	0
2009	1,874,656	0	0	161,778	0	0	0
2010	2,044,558	0	0	125,344	44,558	0	0
2011	2,222,995	0	0	0	178,437	0	0
2012	2,410,400	0	0	0	187,404	0	0
2013	2,607,226	0	0	0	196,826	0	0
2014	2,813,951	0	0	0	206,725	0	0
2015	3,031,077	0	0	0	217,126	0	0
2016	3,259,131	0	0	0	228,054	0	0

2017	3,498,668	0	0	0	239,537	0	0
2018	3,750,270	0	0	0	251,602	0	0
2019	4,014,551	0	0	0	264,280	0	0
2020	4,292,152	0	0	0	277,602	0	0
2021	4,583,753	0	0	0	291,600	0	0
2022	4,890,063	0	0	0	306,310	0	0
2023	5,211,830	0	0	0	109,937	211,830	0
2024	5,549,840	0	0	0	0	338,010	0
2025	5,904,919	0	0	0	0	355,079	0
2026	6,277,935	0	0	0	0	373,017	0
2027	6,669,802	0	0	0	0	391,867	0
2028	7,081,478	0	0	0	0	411,676	0
2029	7,513,973	0	0	0	0	432,495	0
2030	7,968,346	0	0	0	0	454,373	0
2031	8,445,713	0	0	0	0	477,367	0
2032	8,947,245	0	0	0	0	501,532	0
2033	9,474,173	0	0	0	0	526,928	0
2034	10,027,793	0	0	0	0	525,827	27,793
2035	10,609,466	0	0	0	0	0	581,673
2036	11,220,624	0	0	0	0	0	611,158
2037	11,605,912	0	0	0	0	0	385,288
Total		130,000	500,000	1,000,000	3,000,000	5,000,000	1,605,912

EXHIBIT 3

PART 2—PAGE 2

	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
		Discounted Value by Layer						
Year	Present Value Factor	\$500,000 xs \$370,000	\$500,000 xs \$500,000	\$1 million xs \$1 million	\$3 million xs \$2 million	\$5 million xs \$5 million	\$5 million xs \$10 million	All Layers Combined
1997 and prior								
1998	0.9742	92,062	0	0	0	0	0	92,062
1999	0.9247	32,829	58,918	0	0	0	0	91,746
2000	0.8776	0	91,434	0	0	0	0	91,434
2001	0.8330	0	91,124	0	0	0	0	91,124
2002	0.7906	0	90,817	0	0	0	0	90,817
2003	0.7504	0	80,917	9,596	0	0	0	90,513
2004	0.7122	0	0	90,212	0	0	0	90,212
2005	0.6760	0	0	89,913	0	0	0	89,913
2006	0.6416	0	0	89,617	0	0	0	89,617
2007	0.6089	0	0	89,324	0	0	0	89,324
2008	0.5780	0	0	89,034	0	0	0	89,034
2009	0.5486	0	0	88,746	0	0	0	88,746
2010	0.5207	0	0	65,261	23,200	0	0	88,461
2011	0.4942	0	0	0	88,178	0	0	88,178
2012	0.4690	0	0	0	87,898	0	0	87,898
2013	0.4452	0	0	0	87,621	0	0	87,621
2014	0.4225	0	0	0	87,346	0	0	87,346
2015	0.4010	0	0	0	87,073	0	0	87,073
2016	0.3806	0	0	0	86,803	0	0	86,803
2017	0.3613	0	0	0	86,536	0	0	86,536

2018	0.3429	0	0	0	86,270	0	0	86,270
2019	0.3254	0	0	0	86,007	0	0	86,007
2020	0.3089	0	0	0	85,747	0	0	85,747
2021	0.2932	0	0	0	85,488	0	0	85,488
2022	0.2783	0	0	0	85,232	0	0	85,232
2023	0.2641	0	0	0	29,034	55,944	0	84,979
2024	0.2507	0	0	0	0	84,727	0	84,727
2025	0.2379	0	0	0	0	84,478	0	84,478
2026	0.2258	0	0	0	0	84,230	0	84,230
2027	0.2143	0	0	0	0	83,985	0	83,985
2028	0.2034	0	0	0	0	83,742	0	83,742
2029	0.1931	0	0	0	0	83,502	0	83,502
2030	0.1832	0	0	0	0	83,263	0	83,263
2031	0.1739	0	0	0	0	83,026	0	83,026
2032	0.1651	0	0	0	0	82,791	0	82,791
2033	0.1567	0	0	0	0	82,559	0	82,559
2034	0.1487	0	0	0	0	78,195	4,133	82,328
2035	0.1411	0	0	0	0	0	82,099	82,099
2036	0.1340	0	0	0	0	0	81,872	81,872
2037	0.1271	0	0	0	0	0	48,988	48,988
Total		124,890	413,209	611,704	1,092,436	970,442	217,093	3,429,774

EXHIBIT 4

PART 1—PAGE 1

METHOD 2: STOCHASTIC MORTALITY (OTHER INPUTS DETERMINISTIC)

Parameters:

(A)	Evaluation Date:	1/1/98
(B)	Current Age:	35
(C)	Annual Indemnity Payment	20,000
(D)	Annual Medical Payment (at mid-1997 price levels)	70,000
(E)	Indemnity Paid to Date	70,000
(F)	Medical Paid to Date:	300,000
(G)	Cost-of-Living Adjustment	4.11%
(H)	Medical Inflation Rate:	5.25%
(I)	Annual Discount Rate:	5.36%

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Year	Cost of Living Adjustment	Indemnity Payment	Medical Inflation	Medical Payment	Total Payment (2) + (4)	Cumulative Total Payment Cum. of (5)	Probability of claimant living to mid-year	Present Value Factor	Discount for mortality & investment income (7) × (8)
1997 and prior		70,000		300,000	370,000	370,000			
1998	4.11%	20,822	5.25%	73,675	94,497	464,497	0.999	0.974	0.973
1999	4.11%	21,678	5.25%	77,543	99,221	563,718	0.996	0.925	0.921
2000	4.11%	22,569	5.25%	81,614	104,183	667,900	0.993	0.878	0.872
2001	4.11%	23,496	5.25%	85,899	109,395	777,295	0.990	0.833	0.825
2002	4.11%	24,462	5.25%	90,408	114,870	892,166	0.987	0.791	0.781
2003	4.11%	25,467	5.25%	95,155	120,622	1,012,788	0.984	0.750	0.739
2004	4.11%	26,514	5.25%	100,150	126,665	1,139,452	0.981	0.712	0.699

2005	4.11%	27,604	5.25%	105,408	133,012	1,272,465	0.978	0.676	0.661
2006	4.11%	28,738	5.25%	110,942	139,681	1,412,145	0.975	0.642	0.625
2007	4.11%	29,920	5.25%	116,767	146,686	1,558,831	0.971	0.609	0.591
2008	4.11%	31,149	5.25%	122,897	154,046	1,712,878	0.967	0.578	0.559
2009	4.11%	32,429	5.25%	129,349	161,778	1,874,656	0.963	0.549	0.528
2010	4.11%	33,762	5.25%	136,140	169,902	2,044,558	0.958	0.521	0.499
2011	4.11%	35,150	5.25%	143,287	178,437	2,222,995	0.954	0.494	0.471
2012	4.11%	36,595	5.25%	150,810	187,404	2,410,400	0.948	0.469	0.445
2013	4.11%	38,099	5.25%	158,727	196,826	2,607,226	0.943	0.445	0.420
2014	4.11%	39,664	5.25%	167,061	206,725	2,813,951	0.936	0.423	0.396
2015	4.11%	41,295	5.25%	175,831	217,126	3,031,077	0.930	0.401	0.373
2016	4.11%	42,992	5.25%	185,062	228,054	3,259,131	0.923	0.381	0.351
2017	4.11%	44,759	5.25%	194,778	239,537	3,498,668	0.915	0.361	0.330
2018	4.11%	46,598	5.25%	205,004	251,602	3,750,270	0.906	0.343	0.311
2019	4.11%	48,514	5.25%	215,767	264,280	4,014,551	0.897	0.325	0.292
2020	4.11%	50,508	5.25%	227,094	277,602	4,292,152	0.886	0.309	0.274
2021	4.11%	52,583	5.25%	239,017	291,600	4,583,753	0.875	0.293	0.257
2022	4.11%	54,745	5.25%	251,565	306,310	4,890,063	0.863	0.278	0.240
2023	4.11%	56,995	5.25%	264,772	321,767	5,211,830	0.850	0.264	0.225
2024	4.11%	59,337	5.25%	278,673	338,010	5,549,840	0.836	0.251	0.210
2025	4.11%	61,776	5.25%	293,303	355,079	5,904,919	0.821	0.238	0.195
2026	4.11%	64,315	5.25%	308,702	373,017	6,277,935	0.805	0.226	0.182
2027	4.11%	66,958	5.25%	324,909	391,867	6,669,802	0.788	0.214	0.169
2028	4.11%	69,710	5.25%	341,966	411,676	7,081,478	0.769	0.203	0.157
2029	4.11%	72,575	5.25%	359,920	432,495	7,513,973	0.750	0.193	0.145
2030	4.11%	75,558	5.25%	378,815	454,373	7,968,346	0.730	0.183	0.134
2031	4.11%	78,663	5.25%	398,703	477,367	8,445,713	0.709	0.174	0.123
2032	4.11%	81,897	5.25%	419,635	501,532	8,947,245	0.686	0.165	0.113
2033	4.11%	85,263	5.25%	441,666	526,928	9,474,173	0.663	0.157	0.104
2034	4.11%	88,767	5.25%	464,853	553,620	10,027,793	0.638	0.149	0.095

EXHIBIT 4

PART 1—PAGE 2

METHOD 2: STOCHASTIC MORTALITY (OTHER INPUTS DETERMINISTIC)

Year	(1) Cost of Living Adjustment	(2) Indemnity Payment	(3) Medical Inflation	(4) Medical Payment	(5) Total Payment (2) + (4)	(6) Cumulative Total Payment Cum. of (5)	(7) Probability of claimant living to mid-year	(8) Present Value Factor	(9) Discount for mortality & investment income (7) × (8)
2035	4.11%	92,415	5.25%	489,258	581,673	10,609,466	0.612	0.141	0.086
2036	4.11%	96,213	5.25%	514,944	611,158	11,220,624	0.584	0.134	0.078
2037	4.11%	100,168	5.25%	541,979	642,146	11,862,770	0.556	0.127	0.072
2038	4.11%	104,285	5.25%	570,433	674,717	12,537,487	0.527	0.121	0.064
2039	4.11%	108,571	5.25%	600,380	708,951	13,246,438	0.497	0.115	0.057
2040	4.11%	113,033	5.25%	631,900	744,933	13,991,372	0.466	0.109	0.051
2041	4.11%	117,679	5.25%	665,075	782,754	14,774,125	0.435	0.103	0.045
2042	4.11%	122,515	5.25%	699,991	822,507	15,596,632	0.403	0.098	0.040
2043	4.11%	127,551	5.25%	736,741	864,292	16,460,924	0.372	0.093	0.035
2044	4.11%	132,793	5.25%	775,420	908,213	17,369,136	0.340	0.088	0.030
2045	4.11%	138,251	5.25%	816,129	954,380	18,323,517	0.308	0.084	0.026
2046	4.11%	143,933	5.25%	858,976	1,002,909	19,326,426	0.277	0.079	0.022
2047	4.11%	149,848	5.25%	904,073	1,053,921	20,380,347	0.246	0.075	0.019
2048	4.11%	156,007	5.25%	951,536	1,107,544	21,487,890	0.217	0.072	0.016
2049	4.11%	162,419	5.25%	1,001,492	1,163,911	22,651,801	0.188	0.068	0.013
2050	4.11%	169,095	5.25%	1,054,070	1,223,165	23,874,966	0.162	0.065	0.010
2051	4.11%	176,044	5.25%	1,109,409	1,285,453	25,160,420	0.137	0.061	0.008
2052	4.11%	183,280	5.25%	1,167,653	1,350,933	26,511,352	0.114	0.058	0.007

2053	4.11%	190,813	5.25%	1,228,955	1,419,767	27,931,120	0.094	0.055	0.005
2054	4.11%	198,655	5.25%	1,293,475	1,492,130	29,423,250	0.075	0.052	0.004
2055	4.11%	206,820	5.25%	1,361,382	1,568,202	30,991,452	0.059	0.050	0.003
2056	4.11%	215,320	5.25%	1,432,855	1,648,175	32,639,626	0.045	0.047	0.002
2057	4.11%	224,170	5.25%	1,508,080	1,732,249	34,371,876	0.034	0.045	0.002
2058	4.11%	233,383	5.25%	1,587,254	1,820,637	36,192,513	0.025	0.042	0.001
2059	4.11%	242,975	5.25%	1,670,585	1,913,560	38,106,073	0.017	0.040	0.001
2060	4.11%	252,961	5.25%	1,758,290	2,011,252	40,117,324	0.012	0.038	0.001
2061	4.11%	263,358	5.25%	1,850,601	2,113,959	42,231,283	0.008	0.036	0.000
2062	4.11%	274,182	5.25%	1,947,757	2,221,939	44,453,222	0.005	0.035	0.000
2063	4.11%	285,451	5.25%	2,050,015	2,335,465	46,788,688	0.003	0.033	0.000
2064	4.11%	297,183	5.25%	2,157,640	2,454,823	49,243,511	0.002	0.031	0.000
2065	4.11%	309,397	5.25%	2,270,916	2,580,314	51,823,825	0.001	0.029	0.000
2066	4.11%	322,113	5.25%	2,390,140	2,712,253	54,536,078	0.0004	0.028	0.000
2067	4.11%	335,352	5.25%	2,515,622	2,850,974	57,387,052	0.0002	0.027	0.000
2068	4.11%	349,135	5.25%	2,647,692	2,996,827	60,383,879	0.0001	0.025	0.000
2069	4.11%	363,485	5.25%	2,786,696	3,150,181	63,534,059	0.00002	0.024	0.000
2070	4.11%	378,424	5.25%	2,932,997	3,311,421	66,845,481	0.00001	0.023	0.000
2071	4.11%	393,977	5.25%	3,086,980	3,480,957	70,326,438	0.000001	0.022	0.000
2072	4.11%	410,170	5.25%	3,249,046	3,659,216	73,985,653	0.0000002	0.020	0.000

EXHIBIT 4

PART 2—PAGE 1

	(10)	(11)	(12)	(13)	(14)	(15)
	Incremental Payments by Layer					
Year	$130,000 xs $370,000	$500,000 xs $500,000	$1 million xs $1 million	$3 million xs $2 million	$5 million xs $5 million	$5 million xs $10 million
1997 and prior						
1998	94,497	0	0	0	0	0
1999	35,503	63,718	0	0	0	0
2000	0	104,183	0	0	0	0
2001	0	109,395	0	0	0	0
2002	0	114,870	0	0	0	0
2003	0	107,834	12,788	0	0	0
2004	0	0	126,665	0	0	0
2005	0	0	133,012	0	0	0
2006	0	0	139,681	0	0	0
2007	0	0	146,686	0	0	0
2008	0	0	154,046	0	0	0
2009	0	0	161,778	0	0	0
2010	0	0	125,344	44,558	0	0
2011	0	0	0	178,437	0	0
2012	0	0	0	187,404	0	0
2013	0	0	0	196,826	0	0
2014	0	0	0	206,725	0	0
2015	0	0	0	217,126	0	0
2016	0	0	0	228,054	0	0
2017	0	0	0	239,537	0	0
2018	0	0	0	251,602	0	0
2019	0	0	0	264,280	0	0
2020	0	0	0	277,602	0	0
2021	0	0	0	291,600	0	0
2022	0	0	0	306,310	0	0
2023	0	0	0	109,937	211,830	0
2024	0	0	0	0	338,010	0
2025	0	0	0	0	355,079	0
2026	0	0	0	0	373,017	0
2027	0	0	0	0	391,867	0
2028	0	0	0	0	411,676	0
2029	0	0	0	0	432,495	0
2030	0	0	0	0	454,373	0
2031	0	0	0	0	477,367	0
2032	0	0	0	0	501,532	0
2033	0	0	0	0	526,928	0
2034	0	0	0	0	525,827	27,793
2035	0	0	0	0	0	581,673
2036	0	0	0	0	0	611,158
2037	0	0	0	0	0	642,146
2038	0	0	0	0	0	674,717
2039	0	0	0	0	0	708,951
2040	0	0	0	0	0	744,933
2041	0	0	0	0	0	782,754
2042	0	0	0	0	0	225,875
2043	0	0	0	0	0	0
2044	0	0	0	0	0	0

(16)	(17)	(18)	(19)	(20)	(21)	(22)
Incremental Payments by Layer						
$5 million xs $15 million	$10 million xs $20 million	$10 million xs $30 million	$10 million xs $40 million	$10 million xs $50 million	$10 million xs $60 million	$10 million xs $70 million
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
596,632	0	0	0	0	0	0
864,292	0	0	0	0	0	0
908,213	0	0	0	0	0	0

EXHIBIT 4

PART 2—PAGE 2

	(10)	(11)	(12)	(13)	(14)	(15)
	Incremental Payments by Layer					
Year	$130,000 xs $370,000	$500,000 xs $500,000	$1 million xs $1 million	$3 million xs $2 million	$5 million xs $5 million	$5 million xs $10 million
2045	0	0	0	0	0	0
2046	0	0	0	0	0	0
2047	0	0	0	0	0	0
2048	0	0	0	0	0	0
2049	0	0	0	0	0	0
2050	0	0	0	0	0	0
2051	0	0	0	0	0	0
2052	0	0	0	0	0	0
2053	0	0	0	0	0	0
2054	0	0	0	0	0	0
2055	0	0	0	0	0	0
2056	0	0	0	0	0	0
2057	0	0	0	0	0	0
2058	0	0	0	0	0	0
2059	0	0	0	0	0	0
2060	0	0	0	0	0	0
2061	0	0	0	0	0	0
2062	0	0	0	0	0	0
2063	0	0	0	0	0	0
2064	0	0	0	0	0	0
2065	0	0	0	0	0	0
2066	0	0	0	0	0	0
2067	0	0	0	0	0	0
2068	0	0	0	0	0	0
2069	0	0	0	0	0	0
2070	0	0	0	0	0	0
2071	0	0	0	0	0	0
2072	0	0	0	0	0	0
	130,000	500,000	1,000,000	3,000,000	5,000,000	5,000,000

(16)	(17)	(18)	(19)	(20)	(21)	(22)
Incremental Payments by Layer						
$5 million xs $15 million	$10 million xs $20 million	$10 million xs $30 million	$10 million xs $40 million	$10 million xs $50 million	$10 million xs $60 million	$10 million xs $70 million
954,380	0	0	0	0	0	0
1,002,909	0	0	0	0	0	0
673,574	380,347	0	0	0	0	0
0	1,107,544	0	0	0	0	0
0	1,163,911	0	0	0	0	0
0	1,223,165	0	0	0	0	0
0	1,285,453	0	0	0	0	0
0	1,350,933	0	0	0	0	0
0	1,419,767	0	0	0	0	0
0	1,492,130	0	0	0	0	0
0	576,750	991,452	0	0	0	0
0	0	1,648,175	0	0	0	0
0	0	1,732,249	0	0	0	0
0	0	1,820,637	0	0	0	0
0	0	1,913,560	0	0	0	0
0	0	1,893,927	117,324	0	0	0
0	0	0	2,113,959	0	0	0
0	0	0	2,221,939	0	0	0
0	0	0	2,335,465	0	0	0
0	0	0	2,454,823	0	0	0
0	0	0	756,489	1,823,825	0	0
0	0	0	0	2,712,253	0	0
0	0	0	0	2,850,974	0	0
0	0	0	0	2,612,948	383,879	0
0	0	0	0	0	3,150,181	0
0	0	0	0	0	3,311,421	0
0	0	0	0	0	3,154,519	326,438
0	0	0	0	0	0	3,659,216
5,000,000	10,000,000	10,000,000	10,000,000	10,000,000	10,000,000	3,985,653

EXHIBIT 4

PART 2—PAGE 3

	(23)	(24)	(25)	(26)	(27)	(28)
	Commutation Value by Layer, Discounted for Both Mortality and Investment Income Columns are derived by multiplying the corresponding column from Exhibit 4, pages 3 and 4, by Column 9, from pages 1 and 2. For example, Column 23 = Column 10 × Column 9					
Year	$500,000 xs $0	$500,000 xs $500,000	$1 million xs $1 million	$3 million xs $2 million	$5 million xs $5 million	$5 million xs $10 million
1997 and prior						
1998	91,943	0	0	0	0	0
1999	32,699	58,685	0	0	0	0
2000	0	90,820	0	0	0	0
2001	0	90,250	0	0	0	0
2002	0	89,677	0	0	0	0
2003	0	79,655	9,446	0	0	0
2004	0	0	88,522	0	0	0
2005	0	0	87,936	0	0	0
2006	0	0	87,340	0	0	0
2007	0	0	86,729	0	0	0
2008	0	0	86,100	0	0	0
2009	0	0	85,451	0	0	0
2010	0	0	62,544	22,234	0	0
2011	0	0	0	84,079	0	0
2012	0	0	0	83,352	0	0
2013	0	0	0	82,593	0	0
2014	0	0	0	81,797	0	0
2015	0	0	0	80,962	0	0
2016	0	0	0	80,080	0	0
2017	0	0	0	79,147	0	0
2018	0	0	0	78,158	0	0
2019	0	0	0	77,111	0	0
2020	0	0	0	76,002	0	0
2021	0	0	0	74,825	0	0
2022	0	0	0	73,573	0	0
2023	0	0	0	24,684	47,561	0
2024	0	0	0	0	70,835	0
2025	0	0	0	0	69,348	0
2026	0	0	0	0	67,783	0
2027	0	0	0	0	66,145	0
2028	0	0	0	0	64,435	0
2029	0	0	0	0	62,653	0
2030	0	0	0	0	60,795	0
2031	0	0	0	0	58,854	0
2032	0	0	0	0	56,823	0
2033	0	0	0	0	54,703	0
2034	0	0	0	0	49,860	2,635
2035	0	0	0	0	0	50,208
2036	0	0	0	0	0	47,844
2037	0	0	0	0	0	45,408
2038	0	0	0	0	0	42,910
2039	0	0	0	0	0	40,359
2040	0	0	0	0	0	37,764
2041	0	0	0	0	0	35,138
2042	0	0	0	0	0	8,924

(29)	(30)	(31)	(32)	(33)	(34)	(35)
\$5 million xs \$15 million	\$10 million xs \$20 million	\$10 million xs \$30 million	\$10 million xs \$40 million	\$10 million xs \$50 million	\$10 million xs \$60 million	\$10 million xs \$70 million
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
23,571	0	0	0	0	0	0

EXHIBIT 4

PART 2—PAGE 4

Year	(23) $500,000 xs $0	(24) $500,000 xs $500,000	(25) $1 million xs $1 million	(26) $3 million xs $2 million	(27) $5 million xs $5 million	(28) $5 million xs $10 million
	Commutation Value by Layer, Discounted for Both Mortality and Investment Income. Columns are derived by multiplying the corresponding column from Exhibit 4, pages 3 and 4, by Column 9, from pages 1 and 2. For example, Column 23 = Column 10 × Column 9					
2043	0	0	0	0	0	0
2044	0	0	0	0	0	0
2045	0	0	0	0	0	0
2046	0	0	0	0	0	0
2047	0	0	0	0	0	0
2048	0	0	0	0	0	0
2049	0	0	0	0	0	0
2050	0	0	0	0	0	0
2051	0	0	0	0	0	0
2052	0	0	0	0	0	0
2053	0	0	0	0	0	0
2054	0	0	0	0	0	0
2055	0	0	0	0	0	0
2056	0	0	0	0	0	0
2057	0	0	0	0	0	0
2058	0	0	0	0	0	0
2059	0	0	0	0	0	0
2060	0	0	0	0	0	0
2061	0	0	0	0	0	0
2062	0	0	0	0	0	0
2063	0	0	0	0	0	0
2064	0	0	0	0	0	0
2065	0	0	0	0	0	0
2066	0	0	0	0	0	0
2067	0	0	0	0	0	0
2068	0	0	0	0	0	0
2069	0	0	0	0	0	0
2070	0	0	0	0	0	0
2071	0	0	0	0	0	0
2072	0	0	0	0	0	0
	124,642	409,088	594,069	998,595	729,794	311,190

Overall Total = 3,408,316

(29)	(30)	(31)	(32)	(33)	(34)	(35)
$5 million xs $15 million	$10 million xs $20 million	$10 million xs $30 million	$10 million xs $40 million	$10 million xs $50 million	$10 million xs $60 million	$10 million xs $70 million
29,848	0	0	0	0	0	0
27,214	0	0	0	0	0	0
24,609	0	0	0	0	0	0
22,052	0	0	0	0	0	0
12,502	7,060	0	0	0	0	0
0	17,169	0	0	0	0	0
0	14,898	0	0	0	0	0
0	12,762	0	0	0	0	0
0	10,777	0	0	0	0	0
0	8,959	0	0	0	0	0
0	7,320	0	0	0	0	0
0	5,868	0	0	0	0	0
0	1,694	2,911	0	0	0	0
0	0	3,530	0	0	0	0
0	0	2,636	0	0	0	0
0	0	1,912	0	0	0	0
0	0	1,342	0	0	0	0
0	0	855	53	0	0	0
0	0	0	589	0	0	0
0	0	0	365	0	0	0
0	0	0	214	0	0	0
0	0	0	118	0	0	0
0	0	0	18	43	0	0
0	0	0	0	29	0	0
0	0	0	0	12	0	0
0	0	0	0	4	1	0
0	0	0	0	0	2	0.00
0	0	0	0	0	0	0.00
0	0	0	0	0	0	0.01
0	0	0	0	0	0	0.01
139,796	86,507	13,185	1,358	88	3	0.02

EXHIBIT 5

FITTING OF AUTO-REGRESSIVE MODEL FOR CPI

Model: Inflation rate = average inflation + α(last year's inflation – average inflation) + error term where error term is represented by a shifted lognormal

$$\alpha = 0.511$$

α is chosen to minimize the sum of the squared errors in Col. 5

Year	(1) CPI at December	(2) Annual % Increase in CPI	(3) Least-Squares Fit of Inflation Model*	(4) Error**	(5) Squared Error***	(6) Error + 0.07	(7) log(error + 0.07)
1935	13.8						
1936	14.0	1.4%					
1937	14.4	2.9%	2.8%	0.00105	0.00000	0.07105	(2.64431)
1938	14.0	−2.8%	3.5%	(0.06249)	0.00390	0.00751	(4.89101)
1939	14.0	0.0%	0.6%	(0.00593)	0.00004	0.06407	(2.74771)
1940	14.1	0.7%	2.0%	(0.01297)	0.00017	0.05703	(2.86421)
1941	15.5	9.9%	2.4%	0.07553	0.00570	0.14553	(1.92739)
1942	16.9	9.0%	7.1%	0.01949	0.00038	0.08949	(2.41361)
1943	17.4	3.0%	6.6%	(0.03666)	0.00134	0.03334	(3.40113)
1944	17.8	2.3%	3.5%	(0.01224)	0.00015	0.05776	(2.85142)
1945	18.2	2.2%	3.2%	(0.00938)	0.00009	0.06062	(2.80321)
1946	21.5	18.1%	3.2%	0.14973	0.02242	0.21973	(1.51537)
1947	23.4	8.8%	11.3%	(0.02436)	0.00059	0.04564	(3.08692)
1948	24.1	3.0%	6.5%	(0.03534)	0.00125	0.03466	(3.36215)
1949	23.6	−2.1%	3.5%	(0.05614)	0.00315	0.01386	(4.27884)
1950	25.0	5.9%	1.0%	0.04980	0.00248	0.11980	(2.12189)
1951	26.5	6.0%	5.0%	0.00958	0.00009	0.07958	(2.53094)
1952	26.7	0.8%	5.1%	(0.04321)	0.00187	0.02679	(3.61990)

1953	26.9	0.7%	2.4%	(0.01648)	0.00027	0.05352	(2.92768)
1954	26.7	−0.7%	2.4%	(0.03138)	0.00098	0.03862	(3.25387)
1955	26.8	0.4%	1.6%	(0.01257)	0.00016	0.05743	(2.85721)
1956	27.6	3.0%	2.2%	0.00782	0.00006	0.07782	(2.55331)
1957	28.4	2.9%	3.5%	(0.00638)	0.00004	0.06362	(2.75477)
1958	28.9	1.8%	3.5%	(0.01731)	0.00030	0.05269	(2.94341)
1959	29.4	1.7%	2.9%	(0.01181)	0.00014	0.05819	(2.84398)
1960	29.8	1.4%	2.9%	(0.01535)	0.00024	0.05465	(2.90674)
1961	30.0	0.7%	2.7%	(0.02035)	0.00041	0.04965	(3.00281)
1962	30.4	1.3%	2.4%	(0.01021)	0.00010	0.05979	(2.81690)
1963	30.9	1.6%	2.7%	(0.01048)	0.00011	0.05952	(2.82140)
1964	31.2	1.0%	2.9%	(0.01881)	0.00035	0.05119	(2.97215)
1965	31.8	1.9%	2.5%	(0.00584)	0.00003	0.06416	(2.74642)
1966	32.9	3.5%	3.0%	0.00465	0.00002	0.07465	(2.59489)
1967	33.9	3.0%	3.8%	(0.00739)	0.00005	0.06261	(2.77080)
1968	35.5	4.7%	3.6%	0.01156	0.00013	0.08156	(2.50644)
1969	37.7	6.2%	4.4%	0.01775	0.00032	0.08775	(2.43327)
1970	39.8	5.6%	5.2%	0.00393	0.00002	0.07393	(2.60458)
1971	41.1	3.3%	4.9%	(0.01590)	0.00025	0.05410	(2.91699)
1972	42.5	3.4%	3.7%	(0.00274)	0.00001	0.06726	(2.69912)
1973	46.2	8.7%	3.8%	0.04955	0.00245	0.11955	(2.12406)
1974	51.9	12.3%	6.5%	0.05879	0.00346	0.12879	(2.04954)
1975	55.5	6.9%	8.3%	(0.01377)	0.00019	0.05623	(2.87830)
1976	58.2	4.9%	5.6%	(0.00690)	0.00005	0.06310	(2.76297)
1977	62.1	6.7%	4.5%	0.02205	0.00049	0.09205	(2.38546)
1978	67.7	9.0%	5.4%	0.03583	0.00128	0.10583	(2.24588)
1979	76.7	13.3%	6.6%	0.06676	0.00446	0.13676	(1.98950)
1980	86.3	12.5%	8.8%	0.03714	0.00138	0.10714	(2.23358)
1981	94.0	8.9%	8.4%	0.00518	0.00003	0.07518	(2.58791)
1982	97.6	3.8%	6.6%	(0.02739)	0.00075	0.04261	(3.15569)
1983	101.3	3.8%	4.0%	(0.00177)	0.00000	0.06823	(2.68482)
1984	105.3	3.9%	3.9%	0.00001	0.00000	0.07001	(2.65914)
1985	109.3	3.8%	4.0%	(0.00230)	0.00001	0.06770	(2.69263)
1986	110.5	1.1%	4.0%	(0.02854)	0.00081	0.04146	(3.18299)

EXHIBIT 5

(Continued)

Year	(1) CPI at December	(2) Annual % Increase in CPI	(3) Least-Squares Fit of Inflation Model*	(4) Error**	(5) Squared Error***	(6) Error + 0.07	(7) log(error + 0.07)
1987	115.4	4.4%	2.6%	0.01862	0.00035	0.08862	(2.42338)
1988	120.5	4.4%	4.3%	0.00143	0.00000	0.07143	(2.63905)
1989	126.1	4.6%	4.3%	0.00378	0.00001	0.07378	(2.60660)
1990	133.8	6.1%	4.4%	0.01721	0.00030	0.08721	(2.43943)
1991	137.9	3.1%	5.1%	(0.02066)	0.00043	0.04934	(3.00906)
1992	141.9	2.9%	3.6%	(0.00676)	0.00005	0.06324	(2.76082)
1993	145.8	2.7%	3.5%	(0.00745)	0.00006	0.06255	(2.77173)
1994	149.7	2.7%	3.4%	(0.00740)	0.00005	0.06260	(2.77105)
1995	153.5	2.5%	3.4%	(0.00839)	0.00007	0.06161	(2.78699)
1996	158.6	3.3%	3.3%	0.00014	0.00000	0.07014	(2.65720)
1997	161.3	1.7%	3.7%	(0.02006)	0.00040	0.04994	(2.99696)
Average		4.11%		0.00023	0.00106	0.07023	(2.76199)
Std. Dev.				0.03284		0.03284	0.50068

*Column 3 is calculated as: [Avg. of Col. 2] + α[Value of Col. 3 for previous yr – Avg. of Col. 2].
**Column 4 is calculated as: {Col. 2 – Col. 3}.
***Column 5 is calculated as: $\{\text{Col. } 4\}^2$.

Shifted lognormal to model the error term is calculated by fitting a lognormal to Col. 6, the error term, plus a shift of 0.07, which ensures that all the error terms are positive. The lognormal is fitted using the method of moments to the underlying normal distribution (rather than directly to the lognormal), yielding:

$$\mu = -2.7620$$

$$\sigma = 0.5007$$

EXHIBIT 6

Fitting of Model for Medical Inflation

Model: Medical inflation$_t$ = inflation$_t$ + β(Medical inflation$_{t-1}$ − Inflation$_{t-1}$) + (Average medical inflation − average inflation) + error$_t$

$$\beta = 0.380$$

β is chosen to minimize the sum of the squared errors in column 6

Year	(1) Medical CPI at December	(2) Annual % Increase in Medical CPI	(3) Annual % Increase in Overall CPI	(4) Least-Squares Fit of Medical Inflation Model*	(5) Error**	(6) Squared Error***
1935	10.2					
1936	10.2	0.0%	1.4%			
1937	10.3	1.0%	2.9%	3.4%	−2.46%	0.00061
1938	10.3	0.0%	−2.8%	−2.4%	2.35%	0.00055
1939	10.4	1.0%	0.0%	2.2%	−1.22%	0.00015
1940	10.4	0.0%	0.7%	2.2%	−2.22%	0.00049
1941	10.5	1.0%	9.9%	10.8%	−9.83%	0.00967
1942	10.9	3.8%	9.0%	6.8%	−2.95%	0.00087
1943	11.4	4.6%	3.0%	2.1%	2.48%	0.00061
1944	11.7	2.6%	2.3%	4.1%	−1.42%	0.00020
1945	12.0	2.6%	2.2%	3.5%	−0.95%	0.00009
1946	13.0	8.3%	18.1%	19.4%	−11.06%	0.01223
1947	13.9	6.9%	8.8%	6.2%	0.68%	0.00005
1948	14.7	5.8%	3.0%	3.4%	2.35%	0.00055
1949	14.9	1.4%	−2.1%	0.1%	1.25%	0.00016
1950	15.4	3.4%	5.9%	8.4%	−5.02%	0.00252
1951	16.3	5.8%	6.0%	6.2%	−0.31%	0.00001
1952	17.0	4.3%	0.8%	1.8%	2.46%	0.00061

EXHIBIT 6

(Continued)

Year	(1) Medical CPI at December	(2) Annual % Increase in Medical CPI	(3) Annual % Increase in Overall CPI	(4) Least-Squares Fit of Medical Inflation Model*	(5) Error**	(6) Squared Error***
1953	17.6	3.5%	0.7%	3.2%	0.30%	0.00001
1954	18.0	2.3%	−0.7%	1.5%	0.82%	0.00007
1955	18.6	3.3%	0.4%	2.7%	0.67%	0.00005
1956	19.2	3.2%	3.0%	5.2%	−2.02%	0.00041
1957	20.1	4.7%	2.9%	4.1%	0.56%	0.00003
1958	21.0	4.5%	1.8%	3.6%	0.90%	0.00008
1959	21.8	3.8%	1.7%	3.9%	−0.09%	0.00000
1960	22.5	3.2%	1.4%	3.3%	−0.08%	0.00000
1961	23.2	3.1%	0.7%	2.5%	0.60%	0.00004
1962	23.7	2.2%	1.3%	3.4%	−1.24%	0.00015
1963	24.3	2.5%	1.6%	3.1%	−0.56%	0.00003
1964	24.8	2.1%	1.0%	2.4%	−0.39%	0.00002
1965	25.5	2.8%	1.9%	3.5%	−0.65%	0.00004
1966	27.2	6.7%	3.5%	4.9%	1.73%	0.00030
1967	28.9	6.3%	3.0%	5.4%	0.85%	0.00007
1968	30.7	6.2%	4.7%	7.1%	−0.85%	0.00007
1969	32.6	6.2%	6.2%	7.9%	−1.72%	0.00030
1970	35.0	7.4%	5.6%	6.7%	0.66%	0.00004
1971	36.6	4.6%	3.3%	5.1%	−0.51%	0.00003
1972	37.8	3.3%	3.4%	5.0%	−1.76%	0.00031
1973	39.8	5.3%	8.7%	9.8%	−4.50%	0.00203
1974	44.8	12.6%	12.3%	12.2%	0.39%	0.00001

1975	49.2	9.8%	6.9%	8.2%	1.66%	0.00028
1976	54.1	10.0%	4.9%	7.1%	2.86%	0.00082
1977	58.9	8.9%	6.7%	9.8%	−0.91%	0.00008
1978	64.1	8.8%	9.0%	11.0%	−2.15%	0.00046
1979	70.6	10.1%	13.3%	14.4%	−4.22%	0.00178
1980	77.6	9.9%	12.5%	12.5%	−2.54%	0.00064
1981	87.3	12.5%	8.9%	9.1%	3.43%	0.00118
1982	96.9	11.0%	3.8%	6.3%	4.67%	0.00218
1983	103.1	6.4%	3.8%	7.7%	−1.26%	0.00016
1984	109.4	6.1%	3.9%	6.1%	0.03%	0.00000
1985	116.8	6.8%	3.8%	5.8%	1.00%	0.00010
1986	125.8	7.7%	1.1%	3.4%	4.34%	0.00188
1987	133.1	5.8%	4.4%	8.1%	−2.28%	0.00052
1988	142.3	6.9%	4.4%	6.1%	0.83%	0.00007
1989	154.4	8.5%	4.6%	6.7%	1.77%	0.00031
1990	169.2	9.6%	6.1%	8.7%	0.87%	0.00008
1991	182.6	7.9%	3.1%	5.5%	2.39%	0.00057
1992	194.7	6.6%	2.9%	5.9%	0.74%	0.00005
1993	205.2	5.4%	2.7%	5.3%	0.09%	0.00000
1994	215.3	4.9%	2.7%	4.8%	0.10%	0.00000
1995	223.8	3.9%	2.5%	4.5%	−0.58%	0.00003
1996	230.6	3.0%	3.3%	5.0%	−1.96%	0.00038
1997	237.1	2.8%	1.7%	2.7%	0.09%	0.00000
Average		5.25%	4.1%		−0.39%	0.00074
					2.71% = Std. Dev. of errors	0.04505 = Sum of square errors

Average difference between medical inflation and inflation (i.e., avg. of Col. 2 − avg. of Col. 3) = 1.14%.
*Column 4 is calculated as Col. 3 for previous year + β[Col. 2 for previous year − Col. 3 for previous year] + [Avg. of Col. 2 − Avg. of Col. 3].
**Column 5 = Column 2 − Column 4.
***Column 6 = {Column 5}^{2}.
β is fitted to minimize the sum of column 6.

EXHIBIT 7

PART 1—PAGE 1

ONE SIMULATION FROM METHOD 3
STOCHASTIC MORTALITY, INFLATION, MEDICAL INFLATION, AND INVESTMENT YIELDS

Parameters:

(A)	Evaluation Date:	1/1/98
(B)	Current Age:	35
(C)	Annual Indemnity Payment	20,000
(D)	Annual Medical Payment (at mid-1997 price levels)	Varies
(E)	Indemnity Paid to Date	70,000
(F)	Medical Paid to Date:	300,000
(G)	Cost-of-Living Adjustment	Varies
(H)	Medical Inflation Rate:	Varies
(I)	Annual Discount Rate:	Varies

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Year	Cost of Living Adjustment	Indemnity Payment	Medical Inflation	Medical Payment	Total Payment (2) + (4)	Cumulative Total Payment Cum. of (5)	Probability of claimant Living to Mid-year	Present Value Factor	Discount for Mortality & Investment Income (7) × (8)
1997 and prior		70,000		300,000	370,000	370,000			
1998	4.2%	20,843	4.22%	153,624	174,467	544,467	0.999	0.9744	0.9732
1999	4.1%	21,691	6.02%	46,557	68,248	612,716	0.996	0.9336	0.9299
2000	2.2%	22,167	2.74%	154,490	176,657	789,373	0.993	0.9168	0.9107

2001	0.0%	22,167	11.60%	26,456	48,624	837,996	0.990	0.8804	0.8719
2002	5.0%	23,275	9.33%	104,053	127,329	965,325	0.987	0.8175	0.8073
2003	5.0%	24,439	2.17%	28,936	53,375	1,018,700	0.984	0.7862	0.7739
2004	0.0%	24,439	−0.47%	95,726	120,166	1,138,866	0.981	0.7807	0.7661
2005	0.0%	24,439	−0.48%	275,599	300,038	1,438,904	0.978	0.7723	0.7554
2006	0.4%	24,540	5.79%	78,292	102,832	1,541,736	0.975	0.7660	0.7465
2007	0.0%	24,540	10.62%	135,876	160,416	1,702,151	0.971	0.7461	0.7244
2008	4.2%	25,559	7.01%	91,516	117,075	1,819,227	0.967	0.7179	0.6943
2009	1.2%	25,870	3.07%	153,420	179,289	1,998,516	0.963	0.6926	0.6669
2010	3.6%	26,805	6.35%	281,942	308,747	2,307,263	0.958	0.6681	0.6403
2011	1.2%	27,133	3.92%	47,016	74,150	2,381,413	0.954	0.6497	0.6195
2012	2.0%	27,663	4.97%	95,496	123,159	2,504,572	0.948	0.6292	0.5966
2013	2.1%	28,236	2.39%	86,667	114,903	2,619,476	0.943	0.6146	0.5794
2014	0.2%	28,284	1.54%	82,222	110,506	2,729,982	0.936	0.6012	0.5630
2015	1.8%	28,797	−2.26%	74,619	103,416	2,833,398	0.930	0.5810	0.5402
2016	2.6%	29,554	1.48%	130,737	160,291	2,993,689	0.923	0.5581	0.5149
2017	3.1%	30,462	3.73%	595,604	626,065	3,619,754	0.915	0.5365	0.4907
2018	2.5%	31,220	2.09%	280,560	311,780	3,931,534	0.906	0.5259	0.4765
2019	0.0%	31,220	−0.36%	73,362	104,582	4,036,116	0.897	0.5173	0.4638
2020	1.8%	31,780	12.41%	241,678	273,458	4,309,574	0.886	0.4811	0.4264
2021	5.0%	33,369	19.56%	121,747	155,116	4,464,690	0.875	0.4206	0.3682
2022	5.0%	35,038	15.70%	78,638	113,675	4,578,366	0.863	0.3574	0.3085
2023	5.0%	36,790	2.77%	152,619	189,408	4,767,774	0.850	0.3136	0.2666
2024	5.0%	38,629	1.02%	134,724	173,353	4,941,127	0.836	0.2955	0.2470
2025	1.8%	39,310	−2.82%	104,389	143,699	5,084,826	0.821	0.2899	0.2380
2026	0.0%	39,310	−2.30%	73,213	112,523	5,197,350	0.805	0.2887	0.2323
2027	0.0%	39,310	2.10%	332,860	372,170	5,569,519	0.788	0.2878	0.2267
2028	0.0%	39,310	2.59%	296,832	336,143	5,905,662	0.769	0.2853	0.2195

EXHIBIT 7

PART 1—PAGE 2

ONE SIMULATION FROM METHOD 3
STOCHASTIC MORTALITY, INFLATION, MEDICAL INFLATION, AND INVESTMENT YIELDS

Year	(1) Cost of Living Adjustment	(2) Indemnity Payment	(3) Medical Inflation	(4) Medical Payment	(5) Total Payment (2) + (4)	(6) Cumulative Total Payment Cum. of (5)	(7) Probability of claimant Living to Mid-year	(8) Present Value Factor	(9) Discount for Mortality & Investment Income (7) × (8)
2029	0.0%	39,310	−3.51%	84,297	123,607	6,029,269	0.750	0.2835	0.2127
2030	0.0%	39,310	5.22%	487,497	526,807	6,556,076	0.730	0.2790	0.2037
2031	2.0%	40,103	3.22%	153,476	193,580	6,749,656	0.709	0.2736	0.1939
2032	0.0%	40,103	12.65%	233,858	273,961	7,023,617	0.686	0.2639	0.1811
2033	5.0%	42,109	10.66%	139,118	181,227	7,204,844	0.663	0.2461	0.1631
2034	5.0%	44,214	6.31%	253,736	297,950	7,502,795	0.638	0.2329	0.1485
2035	2.3%	45,228	6.52%	119,357	164,585	7,667,380	0.612	0.2243	0.1372
2036	2.9%	46,533	20.05%	120,464	166,996	7,834,377	0.584	0.2030	0.1186
2037	5.0%	48,859	10.89%	529,686	578,545	8,412,922	0.556	0.1799	0.1001
2038	5.0%	51,302	8.96%	970,521	1,021,823	9,434,745	0.527	0.1679	0.0885
2039	4.5%	53,620	2.71%	284,077	337,697	9,772,442	0.497	0.1614	0.0802
2040	1.0%	54,155	8.04%	293,634	347,789	10,120,231	0.466	0.1511	0.0705
2041	5.0%	56,863	11.12%	694,986	751,848	10,872,079	0.435	0.1349	0.0587
2042	5.0%	59,706	9.13%	481,244	540,950	11,413,029	0.403	0.1224	0.0494
2043	5.0%	62,691	5.21%	1,250,236	1,312,927	12,725,956	0.372	0.1153	0.0428

2044	2.8%	64,466	7.24%	668,178	732,644	13,458,599	0.340	0.1111	0.0377
2045	2.2%	65,885	13.38%	490,791	556,676	14,015,275	0.308	0.1042	0.0321
2046	5.0%	69,179	5.43%	967,270	1,036,449	15,051,724	0.277	0.0976	0.0270
2047	2.6%	70,949	5.70%	2,033,827	2,104,775	17,156,500	0.246	0.0934	0.0230
2048	3.9%	73,682	7.81%	637,227	710,908	17,867,408	0.217	0.0895	0.0194
2049	2.3%	75,366	0.57%	1,858,383	1,933,749	19,801,158	0.188	0.0871	0.0164
2050	0.7%	75,896	3.92%	1,383,577	1,459,473	21,260,631	0.162	0.0849	0.0137
2051	2.1%	77,460	10.91%	923,621	1,001,081	22,261,712	0.137	0.0816	0.0112
2052	3.6%	80,212	4.92%	1,120,646	1,200,858	23,462,570	0.114	0.0791	0.0090
2053	0.4%	80,529	1.24%	958,304	1,038,833	24,501,403	0.094	0.0780	0.0073
2054	0.0%	80,529	3.62%	1,195,236	1,275,765	25,777,168	0.075	0.0759	0.0057
2055	3.1%	83,039	−1.81%	1,381,415	1,464,454	27,241,622	0.059	0.0740	0.0044
2056	0.0%	83,039	7.28%	1,128,922	1,211,961	28,453,583	0.045	0.0730	0.0033
2057	0.4%	83,359	6.51%	2,148,141	2,231,500	30,685,083	0.034	0.0719	0.0024
2058	0.4%	83,668	12.41%	1,726,074	1,809,742	32,494,825	0.025	0.0684	0.0017
2059	5.0%	87,851	7.14%	1,323,207	1,411,058	33,905,883	0.017	0.0639	0.0011
2060	4.0%	91,399	5.37%	1,560,112	1,651,512	35,557,395	0.012	0.0614	0.0007
2061	1.6%	92,839	9.29%	999,046	1,091,885	36,649,280	0.008	0.0603	0.0005
2062	0.0%	92,839	7.37%	541,530	634,369	37,283,649	0.005	0.0587	0.0003
2063	3.5%	96,131	9.95%	1,645,362	1,741,493	39,025,142	0.003	0.0549	0.0002
2064	5.0%	100,937	4.86%	1,090,053	1,190,990	40,216,132	0.002	0.0513	0.0001
2065	3.9%	104,845	4.80%	2,549,822	2,654,666	42,870,799	0.001	0.0487	0.0000
2066	4.3%	109,377	11.04%	1,219,660	1,329,038	44,199,836	0.0004	0.0465	0.0000
2067	2.7%	112,317	8.81%	3,720,340	3,832,657	48,032,494	0.0002	0.0451	0.0000
2068	1.2%	113,665	−2.52%	1,892,894	2,006,560	50,039,053	0.0001	0.0445	0.0000
2069	0.0%	113,665	0.52%	1,166,240	1,279,905	51,318,959	0.00002	0.0443	0.0000
2070	0.0%	113,665	−0.14%	1,513,593	1,627,258	52,946,217	0.00001	0.0442	0.0000
2071	0.0%	113,665	3.76%	11,045,559	11,159,225	64,105,442	0.000001	0.0434	0.0000
2072	2.2%	116,181	0.46%	5,311,459	5,427,640	69,533,082	0.0000002	0.0424	0.0000

EXHIBIT 7

PART 2—PAGE 1

	(10)	(11)	(12)	(13)	(14)	(15)
	Incremental Payments by Layer					
Year	$130,000 xs $370,000	$500,000 xs $500,000	$1 million xs $1 million	$3 million xs $2 million	$5 million xs $5 million	$5 million xs $10 million
1997 and prior						
1998	130,000	44,467	0	0	0	0
1999	0	68,248	0	0	0	0
2000	0	176,657	0	0	0	0
2001	0	48,624	0	0	0	0
2002	0	127,329	0	0	0	0
2003	0	34,675	18,700	0	0	0
2004	0	0	120,166	0	0	0
2005	0	0	300,038	0	0	0
2006	0	0	102,832	0	0	0
2007	0	0	160,416	0	0	0
2008	0	0	117,075	0	0	0
2009	0	0	179,289	0	0	0
2010	0	0	1,484	307,263	0	0
2011	0	0	0	74,150	0	0
2012	0	0	0	123,159	0	0
2013	0	0	0	114,903	0	0
2014	0	0	0	110,506	0	0
2015	0	0	0	103,416	0	0
2016	0	0	0	160,291	0	0
2017	0	0	0	626,065	0	0
2018	0	0	0	311,780	0	0
2019	0	0	0	104,582	0	0
2020	0	0	0	273,458	0	0
2021	0	0	0	155,116	0	0
2022	0	0	0	113,675	0	0
2023	0	0	0	189,408	0	0
2024	0	0	0	173,353	0	0
2025	0	0	0	58,873	84,826	0
2026	0	0	0	0	112,523	0
2027	0	0	0	0	372,170	0
2028	0	0	0	0	336,143	0
2029	0	0	0	0	123,607	0
2030	0	0	0	0	526,807	0
2031	0	0	0	0	193,580	0
2032	0	0	0	0	273,961	0
2033	0	0	0	0	181,227	0
2034	0	0	0	0	297,950	0
2035	0	0	0	0	164,585	0
2036	0	0	0	0	166,996	0
2037	0	0	0	0	578,545	0
2038	0	0	0	0	1,021,823	0
2039	0	0	0	0	337,697	0
2040	0	0	0	0	227,558	120,231
2041	0	0	0	0	0	751,848
2042	0	0	0	0	0	540,950
2043	0	0	0	0	0	1,312,927

(16)	(17)	(18)	(19)	(20)	(21)
Incremental Payments by Layer					
$5 million xs $15 million	$10 million xs $20 million	$10 million xs $30 million	$10 million xs $40 million	$10 million xs $50 million	$10 million xs $60 million
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
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0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0

EXHIBIT 7

PART 2—PAGE 2

	(10)	(11)	(12)	(13)	(14)	(15)
	Incremental Payments by Layer					
Year	$130,000 xs $370,000	$500,000 xs $500,000	$1 million xs $1 million	$3 million xs $2 million	$5 million xs $5 million	$5 million xs $10 million
2044	0	0	0	0	0	732,644
2045	0	0	0	0	0	556,676
2046	0	0	0	0	0	984,725
2047	0	0	0	0	0	0
2048	0	0	0	0	0	0
2049	0	0	0	0	0	0
2050	0	0	0	0	0	0
2051	0	0	0	0	0	0
2052	0	0	0	0	0	0
2053	0	0	0	0	0	0
2054	0	0	0	0	0	0
2055	0	0	0	0	0	0
2056	0	0	0	0	0	0
2057	0	0	0	0	0	0
2058	0	0	0	0	0	0
2059	0	0	0	0	0	0
2060	0	0	0	0	0	0
2061	0	0	0	0	0	0
2062	0	0	0	0	0	0
2063	0	0	0	0	0	0
2064	0	0	0	0	0	0
2065	0	0	0	0	0	0
2066	0	0	0	0	0	0
2067	0	0	0	0	0	0
2068	0	0	0	0	0	0
2069	0	0	0	0	0	0
2070	0	0	0	0	0	0
2071	0	0	0	0	0	0
2072	0	0	0	0	0	0
	130,000	500,000	1,000,000	3,000,000	5,000,000	5,000,000

(16)	(17)	(18)	(19)	(20)	(21)
Incremental Payments by Layer					
$5 million xs $15 million	$10 million xs $20 million	$10 million xs $30 million	$10 million xs $40 million	$10 million xs $50 million	$10 million xs $60 million
0	0	0	0	0	0
0	0	0	0	0	0
51,724	0	0	0	0	0
2,104,775	0	0	0	0	0
710,908	0	0	0	0	0
1,933,749	0	0	0	0	0
198,842	1,260,631	0	0	0	0
0	1,001,081	0	0	0	0
0	1,200,858	0	0	0	0
0	1,038,833	0	0	0	0
0	1,275,765	0	0	0	0
0	1,464,454	0	0	0	0
0	1,211,961	0	0	0	0
0	1,546,417	685,083	0	0	0
0	0	1,809,742	0	0	0
0	0	1,411,058	0	0	0
0	0	1,651,512	0	0	0
0	0	1,091,885	0	0	0
0	0	634,369	0	0	0
0	0	1,741,493	0	0	0
0	0	974,858	216,132	0	0
0	0	0	2,654,666	0	0
0	0	0	1,329,038	0	0
0	0	0	3,832,657	0	0
0	0	0	1,967,506	39,053	0
0	0	0	0	1,279,905	0
0	0	0	0	1,627,258	0
0	0	0	0	7,053,783	4,105,442
0	0	0	0	0	5,427,640
5,000,000	10,000,000	10,000,000	10,000,000	10,000,000	9,533,082

EXHIBIT 7

Part 3—Page 1

	(22)	(23)	(24)	(25)	(26)	(27)
	Commutation Value by Layer, Discounted for Both Mortality and Investment Income Columns are derived by multiplying the corresponding column from Exhibit 4, pages 3 and 4, by Column 9, from pages 1 and 2. For example, Column 24 = Column 10 × Column 9					
Year	\$500,000 xs \$0	\$500,000 xs \$500,000	\$1 million xs \$1 million	\$3 million xs \$2 million	\$5 million xs \$5 million	\$5 million xs \$10 million
1997 and prior						
1998	126,511	43,274	0	0	0	0
1999	0	63,463	0	0	0	0
2000	0	160,878	0	0	0	0
2001	0	42,396	0	0	0	0
2002	0	102,789	0	0	0	0
2003	0	26,835	14,472	0	0	0
2004	0	0	92,054	0	0	0
2005	0	0	226,638	0	0	0
2006	0	0	76,768	0	0	0
2007	0	0	116,211	0	0	0
2008	0	0	81,284	0	0	0
2009	0	0	119,565	0	0	0
2010	0	0	950	196,744	0	0
2011	0	0	0	45,935	0	0
2012	0	0	0	73,479	0	0
2013	0	0	0	66,570	0	0
2014	0	0	0	62,213	0	0
2015	0	0	0	55,868	0	0
2016	0	0	0	82,533	0	0
2017	0	0	0	307,205	0	0
2018	0	0	0	148,548	0	0
2019	0	0	0	48,501	0	0
2020	0	0	0	116,604	0	0
2021	0	0	0	57,106	0	0
2022	0	0	0	35,074	0	0
2023	0	0	0	50,503	0	0
2024	0	0	0	42,825	0	0
2025	0	0	0	14,011	20,187	0
2026	0	0	0	0	26,140	0
2027	0	0	0	0	84,370	0
2028	0	0	0	0	73,781	0
2029	0	0	0	0	26,296	0
2030	0	0	0	0	107,320	0
2031	0	0	0	0	37,540	0
2032	0	0	0	0	49,623	0
2033	0	0	0	0	29,549	0
2034	0	0	0	0	44,253	0
2035	0	0	0	0	22,578	0
2036	0	0	0	0	19,813	0
2037	0	0	0	0	57,891	0
2038	0	0	0	0	90,393	0
2039	0	0	0	0	27,092	0
2040	0	0	0	0	16,034	8,472
2041	0	0	0	0	0	44,133
2042	0	0	0	0	0	26,704

(28)	(29)	(30)	(31)	(32)	(33)
$5 million xs $15 million	$10 million xs $20 million	$10 million xs $30 million	$10 million xs $40 million	$10 million xs $50 million	$10 million xs $60 million
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
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0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
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0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0

EXHIBIT 7

PART 3—PAGE 2

	(22)	(23)	(24)	(25)	(26)	(27)
	Commutation Value by Layer, Discounted for Both Mortality and Investment Income Columns are derived by multiplying the corresponding column from Exhibit 4, pages 3 and 4, by Column 9, from pages 1 and 2. For example, Column 24 = Column 10 × Column 9					
Year	\$500,000 xs \$0	\$500,000 xs \$500,000	\$1 million xs \$1 million	\$3 million xs \$2 million	\$5 million xs \$5 million	\$5 million xs \$10 million
2043	0	0	0	0	0	56,253
2044	0	0	0	0	0	27,654
2045	0	0	0	0	0	17,864
2046	0	0	0	0	0	26,577
2047	0	0	0	0	0	0
2048	0	0	0	0	0	0
2049	0	0	0	0	0	0
2050	0	0	0	0	0	0
2051	0	0	0	0	0	0
2052	0	0	0	0	0	0
2053	0	0	0	0	0	0
2054	0	0	0	0	0	0
2055	0	0	0	0	0	0
2056	0	0	0	0	0	0
2057	0	0	0	0	0	0
2058	0	0	0	0	0	0
2059	0	0	0	0	0	0
2060	0	0	0	0	0	0
2061	0	0	0	0	0	0
2062	0	0	0	0	0	0
2063	0	0	0	0	0	0
2064	0	0	0	0	0	0
2065	0	0	0	0	0	0
2066	0	0	0	0	0	0
2067	0	0	0	0	0	0
2068	0	0	0	0	0	0
2069	0	0	0	0	0	0
2070	0	0	0	0	0	0
2071	0	0	0	0	0	0
2072	0	0	0	0	0	0
	126,511	439,635	727,941	1,403,719	732,859	207,656
	Overall Total = 3,813,435					

(28)	(29)	(30)	(31)	(32)	(33)
$5 million xs $15 million	$10 million xs $20 million	$10 million xs $30 million	$10 million xs $40 million	$10 million xs $50 million	$10 million xs $60 million
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
1,396	0	0	0	0	0
48,369	0	0	0	0	0
13,781	0	0	0	0	0
31,742	0	0	0	0	0
2,731	17,315	0	0	0	0
0	11,188	0	0	0	0
0	10,837	0	0	0	0
0	7,573	0	0	0	0
0	7,274	0	0	0	0
0	6,402	0	0	0	0
0	4,021	0	0	0	0
0	3,779	1,674	0	0	0
0	0	3,058	0	0	0
0	0	1,568	0	0	0
0	0	1,196	0	0	0
0	0	506	0	0	0
0	0	177	0	0	0
0	0	269	0	0	0
0	0	78	17	0	0
0	0	0	104	0	0
0	0	0	24	0	0
0	0	0	28	0	0
0	0	0	5	0.11	0
0	0	0	0	1.18	0
0	0	0	0	0.41	0
0	0	0	0	0.38	0.22
0	0	0	0	0.00	0.04
98,019	68,389	8,526	178	2.07	0.26
	Overall Total = 3,813,435				