A BUYER'S GUIDE FOR OPTIONS ON A CATASTROPHE INDEX

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Abstract

In the wake of recent catastrophes, a new way of transferring insurance risk was born. In December 1992, the Chicago Board of Trade began trading contracts on an index sensitive to insurer catastrophe experience. Such indices provide an insurer a means to transfer a portion of its catastrophe risk to the capital markets by buying future and option contracts.

The cost of using these contracts to transfer catastrophe risk is compared to the cost of raising sufficient capital to retain the risk and the cost of conventional reinsurance. We derive equations that give the optimal participation in the future and option contracts, and in reinsurance. The cost of using these contracts can be compared to the cost of the capital that they replace.

1. INTRODUCTION

In the wake of recent catastrophes, a new way of transferring insurance risk was born. In December 1992, the Chicago Board of Trade began trading contracts on an index sensitive to insurer catastrophe experience.

These contracts gave insurers an additional financial strategy for handling catastrophe risk. Two other common strategies are:

1. buying reinsurance; and
2. raising sufficient capital to maintain solvency while retaining the risk.

Another innovation that has gained popularity in the wake of recent catastrophes is the use of catastrophe models in insurance ratemaking and underwriting. These models combine meteorological and geological science with engineering damageability studies and insurance exposure information to estimate potential losses for an insurance portfolio.

The purpose of this paper is to show how to use catastrophe models to estimate the costs and benefits of contracts on a catastrophe index relative to other means of managing the catastrophe risk.

2. MOTIVATION FOR TRADING CONTRACTS ON A CATASTROPHE INDEX

Risk of loss is usually transferred from one with insufficient capital to absorb a loss to one(s) who can absorb it. The size of an insurance catastrophe, which at its worst is measured in billions, is small compared to the money invested in the capital markets, which is measured in trillions. There are insurers with a demand for risk transfer, and there are investors who can meet this demand. However, one needs to find a contract that meets the institutional needs of the investors and the insurers.

Investors are ill-equipped to deal with counterparty risk, i.e., the risk that the insurer knows something about the transfer that will be to the investors disadvantage. One way to reduce this risk is to base the contract on the combined results of several insurers, i.e., a catastrophe index.

Trading contracts on an index introduces additional risk for the insurer in that the money it recovers from a catastrophe index contract may differ substantially from its own catastrophe losses. In investment language, this is referred to as basis risk.[1]

[1] For a more complete explanation of basis risk, see Hull [5, p. 32].

The insurer would like its losses to be highly correlated with the index, as is the case for reinsurance,[2] so that its basis risk is small.

The investor seeks to maximize profit while adding the least amount of risk to its total investment portfolio. Usually the returns on available investments tend to be positively correlated over time. For example, the returns on stocks tend to be correlated with the general economy. If the value of the index is uncorrelated with the seller's other investments, the investor will take on less risk by selling contracts on the index than he would if he took on an otherwise equivalent investment on the stock market.

Both the insurer and the investor want their risk to be quantified. As this paper will illustrate, both risks can be quantified with the use of a catastrophe model and a tabulation of the underlying exposures.

3. A STATISTICAL DESCRIPTION OF THE CONTRACTS

This paper will focus on catastrophe index contracts as they are traded on an exchange such as the Chicago Board of Trade. The form of the contracts that are traded is explained below. The scale of the index is arbitrary. In this paper we set the scale so that the expected value of the index at expiration is $1.00.

A **call option contract** gives the buyer the right to buy the index at an agreed upon price at a specified date. The agreed upon price is called the strike price.

As an example, suppose an investor sells a one year option contract with a strike price of $1.00 for a premium of $0.20 to an insurer. If there are no catastrophes during the year and the value of the index is zero on December 31, the insurer would

[2]The coefficient of correlation between losses and reinsurance recoveries will be 1.00 for quota share reinsurance agreements. If there is a reinsurance limit, the coefficient of correlation will be less than 1.00.

not want to buy the index for $1.00, so it would not exercise its option. The investor would keep the $0.20. However, if the index is valued at $3.00 on December 31, the insurer would buy the index for $1.00 and the investor would lose $1.80.

A **call option spread** is a package of two option contracts where one buys an option at one strike price and simultaneously sells another option at a higher strike price. The difference between the two strike prices is called the covered layer of the spread.

To continue our example, suppose the investor sells a call option spread to an insurer for the $1.00 to $2.00 layer for a net premium of $0.10. This means that the insurer is buying insurance on the index for the $1.00 to $2.00 layer for $0.10.

In terms of the transaction details, this means that the investor sells the insurer an option with a strike price of $1.00 for a premium of $0.20, and insurer sells the investor an option with a strike price of $2.00 for a premium of $0.10. If the final value of the index is zero, neither party exercises its option and the investor keeps its $0.10. If the final value of the index is $3.00, the investor exercises its option to buy the index from the insurer for $2.00 and the insurer exercises its option to buy the index from the investor for $1.00. The net effect is that the investor gives the insurer (and loses) $0.90. This is the most that the investor can lose on this contract.

If the final value of the index is $1.50, the insurer exercises its option and the investor does not. The investor pays the insurer $0.50 and ends up losing $0.40.

The purpose of the call option spread is to limit the liability of the seller, in much the same way that reinsurers limit their liability on catastrophe reinsurance contracts. If an insurer wants the full coverage, it can buy a series of call option spreads from different sellers, with the cost of the coverage being the sum of the premiums for the call option spreads.

When an insurer buys these contracts, it reduces the overall variability of its financial results and, at least in principle, it will need less capital to support its business.

We illustrate these points with a statistical argument. Let:

- X be a random variable for the insurer's losses prior to buying contracts on a catastrophe index;
- Y be a random variable for the final contract value;
- ρ be the coefficient of correlation between X and Y; and
- σ_Z be the standard deviation of any random variable, Z.

If an insurer buys n contracts on the index, the random variable for its net loss is $X - nY$, and a quantification of its risk is given by:

$$\sigma_{X-nY} = \sqrt{\sigma_X^2 - 2n\rho\sigma_X\sigma_Y + n^2\sigma_Y^2}. \tag{3.1}$$

Note that the insurer will reduce its risk if $2\rho\sigma_X > n\sigma_Y$. There may be motivation to buy an options contract if ρ is positive and n is not too large. Exactly how many contracts will be bought depends upon the price. More on this below.

Let:

- U be a random variable for the investor's gain on its current portfolio;
- V be a random variable for the investor's gain on a prospective investment; and
- ν be the coefficient of correlation between U and V.

Further suppose that $\sigma_V^2 = n^2\sigma_Y^2$ and that U and Y are uncorrelated.

If the investor buys the prospective investment, a quantification of its risk is given by:

$$\sigma_{U+V} = \sqrt{\sigma_U^2 + 2\nu\sigma_U\sigma_V + \sigma_V^2}. \tag{3.2}$$

If an investor sells n contracts on the index, the random variable for its net return is $U + nY$, and the equivalent quantification of its risk is given by:

$$\sigma_{U+nY} = \sqrt{\sigma_U^2 + n^2\sigma_Y^2}. \tag{3.3}$$

Since $\sigma_V^2 = n^2\sigma_Y^2$, the investor will face less risk by selling the catastrophe contracts when $\nu > 0$. Thus the investor should have a preference for selling the catastrophe contracts.[3] Again, it depends upon the price.

4. THE COST OF CAPITAL

The ultimate reason an insurer would want to purchase contracts on a catastrophe index is to reduce its cost of doing business. One of the key costs of the insurance business is the cost of capital. In this paper, we assume that the amount of capital needed for an insurer to adequately support the risks it writes is given by:

$$C = T\sigma_X. \tag{4.1}$$

Our choice of Equation 4.1 deserves some discussion since there is no universal agreement on a capitalization formula. For example, the NAIC risk based capital formula might be one possible alternative, but it does not recognize the catastrophe risk. Another alternative is the "expected policyholder deficit," which is the expected payment by the policyholders (or guaranty fund) in case the insurer goes insolvent (see AAA Report [1]). This formula is sensitive only to the tail of the loss distribution.

We offer the following two arguments in favor of Equation 4.1. First, we feel that most insurers are worried about losing even a small portion of their capital. Equation 4.1 is sensitive to the entire range of losses. Second, the mathematics needed to im-

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[3]This is often called the "zero beta" argument. This is in reference to the Capital Asset Pricing Model. See Chapter 8 of Brealy and Myers [3].

plement this formula are relatively simple. However, many of the ideas in this paper can be implemented with other capitalization formulas.

Continuing, if the insurer buys n contracts on the catastrophe index, the needed capital becomes:

$$C(n) \equiv T\sigma_{X-nY} = T\sqrt{\sigma_X^2 - 2n\rho\sigma_X\sigma_Y + n^2\sigma_Y^2}. \qquad (4.2)$$

To obtain the reduction of capital indicated by the difference between Equations 4.1 and 4.2, the insurer must buy n contracts at a price determined by the market forces of supply and demand. Let P be equal to the price of a single contract less the expected return on the contract, i.e., the net cost of the contract. Then nP is the net cost of the contracts being substituted for capital.

Let K denote the rate of return the insurer pays to secure the needed capital. K will depend on the riskiness of the insurer's enterprise and the cost of competing investments.

When the insurer buys n contracts, its cost of capital plus its capital substitute is:

$$R(n) \equiv KT\sqrt{\sigma_X^2 - 2n\rho\sigma_X\sigma_Y + n^2\sigma_Y^2} + nP. \qquad (4.3)$$

To minimize its cost of providing insurance, the insurer will choose the value of n that minimizes $R(n)$. To determine this n, we find:

$$R'(n) = \frac{KT(n\sigma_Y^2 - \rho\sigma_X\sigma_Y)}{\sqrt{\sigma_X^2 - 2\rho n\sigma_X\sigma_Y + n^2\sigma_Y^2}} + P. \qquad (4.4)$$

Setting $R'(n) = 0$, and then solving for n yields:[4]

$$n = \frac{\rho\sigma_X}{\sigma_Y} - \frac{\sigma_X}{\sigma_Y}\sqrt{\frac{P^2(1-\rho^2)}{K^2T^2\sigma_Y^2 - P^2}}. \qquad (4.5)$$

[4]The details of this derivation are provided in the Appendix.

Here we see that the number of contracts needed to minimize the cost of providing insurance decreases:

1. as the price of the contracts, quantified by P, increases;
2. as ρ decreases, i.e. as the basis risk, quantified by ρ, increases;
3. as the cost of capital, quantified by K and T, decreases; and
4. as the scale of the index, quantified by σ_Y, increases.

If you set $P = 0$, Equation 4.5 reduces to a familiar expression for the "optimal hedge postion" otherwise known as the "hedge ratio".[5]

The quantities P and K depend upon market conditions. K also depends on the overall risk of the insurer. T depends upon the risk aversion of the insurer. To obtain the quantities σ_X, σ_Y and ρ you need a catastrophe model. It is to this we now turn.

5. AN ILLUSTRATIVE CATASTROPHE MODEL

The following information can be provided by a catastrophe model:

1. h—the natural event causing the catastrophe, numbered from 1 to s;
2. p_h—the probability of event h;
3. i—the location, e.g. county or ZIP code, numbered from 1 to m;
4. E_i—the number of exposure units at location i for all the insurers in the index, appropriately scaled so that the expected value of the index at expiration is \$1.00;

[5] See, for example, Hentschel and Smith [4]. There are several articles that may be of interest in this volume of the *JRI*, which is titled *Symposium on Financial Risk Management in Insurance Firms.*

5. e_i—the number of exposure units for the insurer at location i; and

6. L_{ih}—the damage caused to a unit of exposure at location i by event h.

For the examples in this paper, we will assume only one class of property. In practice one should add another subscript to allow for different classes each with different L_{ih}s.

The assembling of this information is a formidable task, and those who have done so regard the results of their efforts as proprietary. In this paper we use an illustrative catastrophe model published by Glenn Meyers [6]. Meyers' model has the following properties.

1. The covered area consists of a state with 50 counties. The east coast is exposed to the ocean and therefore to hurricanes.

2. Hurricanes travel only from east to west. They come in various strengths and affect either five or ten counties.

3. For the inland counties, the damage per exposure unit is 70% of the damage per unit in the county immediately to the east.

Table 1 provides a schematic map of the state along with the index exposures, E_i.

Tables 2A and 2B provide the probability, p_h, of each event h, and the loss per unit of exposure, L_{ih}, by each landfall county for each event. L_{ih} decreases by 70% as each event moves inland by one county. The index loss for each event h is given by:

$$\text{Index}(h) = \frac{\sum_{i=1}^{50} p_h e_i L_{ih}}{\text{Average Annual Hurricane Loss}}. \qquad (5.1)$$

TABLE 1

INDEX EXPOSURES BY COUNTY

i	E_i	i	E_i	i	E_i	i	E_i	i	E_i	Ocean
1	0.010	2	0.030	3	0.030	4	0.010	5	0.010	
6	0.010	7	0.030	8	0.030	9	0.010	10	0.010	
11	0.010	12	0.010	13	0.010	14	0.010	15	0.010	
16	0.010	17	0.010	18	0.010	19	0.010	20	0.010	
21	0.010	22	0.010	23	0.010	24	0.090	25	0.090	
26	0.010	27	0.010	28	0.010	29	0.010	30	0.010	
31	0.010	32	0.010	33	0.010	34	0.010	35	0.010	
36	0.050	37	0.010	38	0.050	39	0.050	40	0.010	
41	0.050	42	0.010	43	0.050	44	0.050	45	0.010	
46	0.010	47	0.030	48	0.010	49	0.010	50	0.010	

In this example we assume that only one hurricane can happen in a given year. To allow for multiple hurricanes in a year, one could create synthetic "events" by randomly selecting hurricanes that can happen in a single year, and simulating a very large version of Table 2.

The probability of a hurricane happening is 0.5000.

We also give the probability distribution of the final index values in Table 2. We consider this information to be valuable to potential investors who want to estimate the risk they are taking. This probability distribution is also shown graphically in Figure 1.

6. CALCULATING σ_X, σ_Y, AND ρ

Given the information from the previous section, we calculate:

$$\sigma_Y = \sqrt{\sum_{h=1}^{s}\left(\sum_{i=1}^{m} E_i L_{ih}\right)^2 p_h - \left(\sum_{h=1}^{s}\sum_{i=1}^{m} E_i L_{ih} p_h\right)^2}. \qquad (6.1)$$

It is possible for a large multiline insurer to have the same catastrophe exposure as a small monoline property insurer. The capital

TABLE 2A

SMALL HURRICANES

h	i, at Landfall	L_{ih}	p_h	Index Loss for h
1	5	41.46	0.016181	0.4601
2	5	82.91	0.012945	0.9201
3	5	124.37	0.004854	1.3802
4	10	41.46	0.016181	0.4601
5	10	82.91	0.012945	0.9201
6	10	124.37	0.004854	1.3802
7	15	41.46	0.016181	0.2874
8	15	82.91	0.012945	0.5748
9	15	124.37	0.004854	0.8622
10	20	41.46	0.016181	0.2874
11	20	82.91	0.012945	0.5748
12	20	124.37	0.004854	0.8622
13	25	41.46	0.016181	1.6969
14	25	82.91	0.012945	3.3938
15	25	124.37	0.004854	5.0907
16	30	41.46	0.016181	0.2874
17	30	82.91	0.012945	0.5748
18	30	124.37	0.004854	0.8622
19	35	41.46	0.016181	0.2874
20	35	82.91	0.012945	0.5748
21	35	124.37	0.004854	0.8622
22	40	41.46	0.016181	0.8803
23	40	82.91	0.012945	1.7605
24	40	124.37	0.004854	2.6408
25	45	41.46	0.016181	0.8803
26	45	82.91	0.012945	1.7605
27	45	124.37	0.004854	2.6408
28	50	41.46	0.016181	0.3585
29	50	82.91	0.012945	0.7170
30	50	124.37	0.004854	1.0755

carried by each insurer depends on its entire book of business and should be taken into account when calculating the coefficient of correlation of its losses with the catastrophe index. To do this let:

$$X = X_1 + X_2 \tag{6.2}$$

TABLE 2B

LARGE HURRICANES

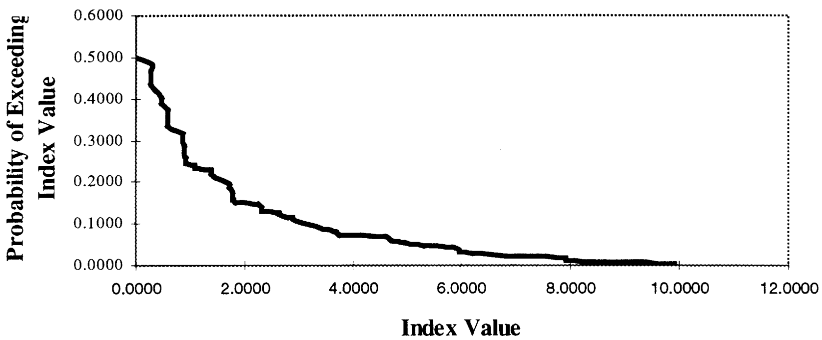
h	i, at 1st Landfall	i, at 2nd Landfall	L_{ih} at 1st and 2nd Landfall	p_h	Index Loss for h
31	5	10	124.37	0.004854	2.7604
32	5	10	165.82	0.006472	3.6806
33	5	10	207.28	0.003236	4.6007
34	10	15	124.37	0.004854	2.2424
35	10	15	165.82	0.006472	2.9899
36	10	15	207.28	0.003236	3.7374
37	15	20	124.37	0.004854	1.7244
38	15	20	165.82	0.006472	2.2992
39	15	20	207.28	0.003236	2.8740
40	20	25	124.37	0.004854	5.9530
41	20	25	165.82	0.006472	7.9373
42	20	25	207.28	0.003236	9.9216
43	25	30	124.37	0.004854	5.9530
44	25	30	165.82	0.006472	7.9373
45	25	30	207.28	0.003236	9.9216
46	30	35	124.37	0.004854	1.7244
47	30	35	165.82	0.006472	2.2992
48	30	35	207.28	0.003236	2.8740
49	35	40	124.37	0.004854	3.5030
50	35	40	165.82	0.006472	4.6707
51	35	40	207.28	0.003236	5.8384
52	40	45	124.37	0.004854	5.2816
53	40	45	165.82	0.006472	7.0422
54	40	45	207.28	0.003236	8.8027
55	45	50	124.37	0.004854	3.7163
56	45	50	165.82	0.006472	4.9551
57	45	50	207.28	0.003236	6.1939
58	5		124.37	0.004854	1.3802
59	5		165.82	0.006472	1.8403
60	5		207.28	0.003236	2.3003
61	50		124.37	0.004854	1.0755
62	50		165.82	0.006472	1.4340
63	50		207.28	0.003236	1.7925

where:

- X_1 represents the catastrophe losses that are estimated with a catastrophe model; and

FIGURE 1

INDEX LOSS EXCEEDING PROBABILITY



- X_2 represents the other insurer losses, which are assumed to be uncorrelated with X_1.

Then:

$$\sigma_{X_1} = \sqrt{\sum_{h=1}^{s}\left(\sum_{i}^{m} e_i L_{ih}\right)^2 - \left(\sum_{h=1}^{s}\sum_{i=1}^{m} e_i L_{ih} p_h\right)^2}. \qquad (6.3)$$

σ_{X_2} must be obtained from an analysis of the insurer's other business.

Let ρ_k be the coefficient of correlation of X_k with the index. We assume $\rho_2 = 0$.

Then:

$$\rho_1 = \frac{\displaystyle\sum_{h=1}^{s}\left(\sum_{i=1}^{m}(e_i L_{ij})\right)\left(\sum_{i=1}^{m}(E_i L_{ih})\right) p_h - \left(\sum_{h=1}^{s}\sum_{i=1}^{m} e_i L_{ih} p_h\right)\cdot\left(\sum_{h=1}^{s}\sum_{i=1}^{m} E_i L_{ih} p_h\right)}{\sigma_{X_1}\sigma_Y} \qquad (6.4)$$

and

$$\rho = \frac{\rho_1 \sigma_{X_1} \sigma_Y + \rho_2 \sigma_{X_2} \sigma_Y}{\sigma_{X_1+X_2} \sigma_Y} = \frac{\rho_1 \sigma_{X_1}}{\sqrt{\sigma_{X_1}^2 + \sigma_{X_2}^2}}. \tag{6.5}$$

7. EXAMPLES USING THE ILLUSTRATIVE MODEL

The examples given in this section will be based on an option with a zero strike price contract as described in Section 2. We chose this contract because it offers the insurer the maximum amount of protection and can be replicated by a series of the more popular call option spreads.

Using Table 1 as a reference, we create six sample insurers. Each insurer's book of business has a different geographical distribution.

1. All County Insurance Company has exposure in all counties in proportion to the industry as charted in Table 1.
2. Uni-County Insurance Company has the same exposure in all counties.
3. Northern Counties Insurance Company has exposure in counties 1–25 in proportion to the industry as charted in Table 1. It has no exposures in counties 26–50.
4. Big County Insurance Company has all its exposure in county 25.
5. Southern Counties Insurance Company has exposure in counties 26–50 in proportion to the industry as charted in Table 1. It has no exposures in counties 1–25.
6. Small County Insurance Company has all its exposure in county 1.

To facilitate comparisons among the six insurers, we have scaled the exposure of each so that σ_{X_i} is the same for each insurer.

TABLE 3
INSURER PARAMETERS

Parameter	Value
K	0.20
T	10
σ_{X_1}	30,000,000
σ_{X_2}	40,000,000
σ_Y	1.819

TABLE 4
INSURER PARAMETERS

Insurer #	Expected Loss	ρ_1	ρ
1	16,496,571	1.000	0.600
2	19,404,690	0.867	0.520
3	11,246,179	0.743	0.446
4	6,942,082	0.693	0.416
5	11,255,277	0.609	0.365
6	6,942,082	0.147	0.088

Table 2 lists the parameters, both selected and calculated from the model, common to each insurer.

The parameters in Table 3 are sufficient to describe the cost of providing coverage without buying any contracts on the catastrophe index. The needed insurer capital is:

$$C(0) = T\sigma_X = 10\sqrt{30{,}000{,}000^2 + 40{,}000{,}000^2} = 500{,}000{,}000.$$

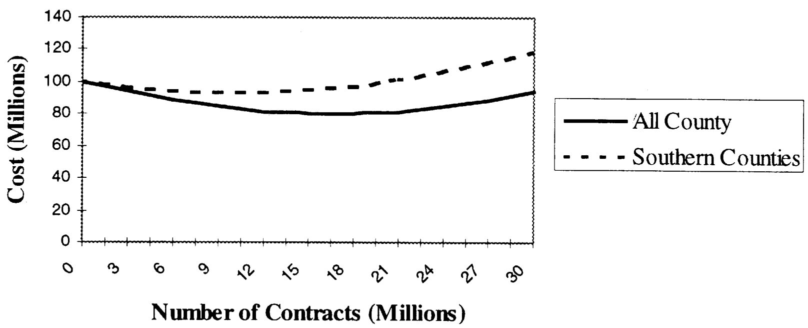
The cost of providing this capital is:

$$R(0) = KC(0) = 100{,}000{,}000.$$

We now introduce option contracts on the catastrophe index. Table 4 gives the expected loss for each insurer resulting from scaling the exposure, along with ρ_1 and ρ calculated from the illustrative model using Equations 6.4 and 6.5.

FIGURE 2

COST OF CAPITAL+NET COST OF CONTRACTS



As discussed in Section 4, the insurer wants to choose n so as to minimize its cost of capital, $KC(n)$, plus the net cost of the n contracts, nP. Figure 2 shows the cost for selected insurers as a function of n for $P = 0$.

As Figure 2 illustrates, there is an optimal number, n, of contracts that will minimize the cost of writing insurance subject to catastrophes. The number n can be calculated using Equation 4.5. Tables 5 and 6 show the ns calculated from Equation 4.5 for each of the insurers in our example. The cost of insuring is then given by Equation 4.3 for these ns.

Table 5 is sorted in order of P to illustrate the effect of the contract price. As the price increases, the optimal number of contracts decreases and the cost of insuring increases.

Table 6 is sorted in order of insurer to illustrate the effect of the insurer's correlation with the catastrophe index. As the correlation increases, the optimal number of contracts increases, and the cost of insuring decreases.

Without the catastrophe contracts, All County must raise an additional \$20,000,000 in capital. This provides a yardstick for measuring the efficiency of the contracts. For example, if $P = 0.6$, the cost of insuring catastrophes for All County is only an additional \$8,801,889 if it buys the optimal number of contracts. All County reduces its cost of insuring its catastrophe exposure by 56%. At the same time, Big County Insurance's additional cost of insuring its catastrophe exposure is reduced by only 17%.

It is possible for n to be negative. This simply indicates that if the price of the contract is sufficiently high, it is better to be a seller than a buyer of the catastrophe contracts.

8. CONTRACTS ON A CATASTROPHE INDEX VS. REINSURANCE

The examples given show that contracts on a catastrophe index can reduce the cost of providing insurance, even if the correlation between the insurer's catastrophe losses are not highly correlated with the index. However, it is possible that conventional reinsurance may be an even lower cost of providing insurance. In this section we show how to investigate this possibility.

Reinsurance can be viewed as an option contract on a catastrophe index, with the index being the insurer's own experience. We take this view here. Properly interpreted, Equations 4.3 and 4.5 provide the means of finding out how much reinsurance to buy, and the expected benefit of buying it.

We will use the examples in the preceding section to show that reinsurance can give a lower cost of providing insurance.

A quota-share reinsurance contract corresponds to the option contract with $\rho_1 = 1$. We find a net cost of reinsurance, denoted by P_R, that provides the same cost of insurance as the corresponding contract on the catastrophe index. If reinsurance can be obtained for a lower net cost, we conclude that insurance can be provided at a lower cost.

TABLE 5

THE EFFECT OF THE CONTRACT PRICE

Insurer #	Number of Contracts	Cost of Insuring	P
1	16,496,571	80,000,000	0.0
1	15,285,243	83,178,275	0.2
1	14,062,815	86,113,360	0.4
1	12,817,677	88,801,889	0.6
1	11,537,127	91,238,074	0.8
2	14,306,818	85,394,944	0.0
2	13,013,800	88,127,104	0.2
2	11,708,935	90,599,676	0.4
2	10,379,829	92,809,065	0.6
2	9,012,923	94,749,092	0.8
3	12,264,212	89,500,107	0.0
3	10,909,035	91,817,535	0.2
3	9,541,442	93,862,895	0.4
3	8,148,442	95,632,421	0.6
3	6,715,825	97,119,635	0.8
4	11,428,496	90,951,642	0.0
4	10,051,340	93,099,730	0.2
4	8,661,567	94,971,339	0.4
4	7,245,975	96,562,639	0.6
4	5,790,124	97,867,049	0.8
5	10,048,063	93,082,705	0.0
5	8,638,639	94,951,482	0.2
5	7,216,303	96,537,301	0.4
5	5,767,543	97,836,244	0.6
5	4,277,580	98,841,576	0.8
6	2,425,986	99,609,960	0.0
6	917,729	99,944,446	0.2
6	– 604,346	99,976,132	0.4
6	– 2,154,698	99,700,825	0.6
6	– 3,749,142	99,111,318	0.8

The P_Rs were calculated by trial and error as follows:

1. Select a P_R.
2. Find n_R using Equation 4.5.

TABLE 6

THE EFFECT OF INSURER CORRELATION WITH THE INDEX

Insurer #	Number of Contracts	Cost of Insuring	P
1	16,496,571	80,000,000	0.0
2	14,306,818	85,394,944	0.0
3	12,264,212	89,500,107	0.0
4	11,428,496	90,951,642	0.0
5	10,048,063	93,082,705	0.0
6	2,425,986	99,609,960	0.0
1	15,285,243	83,178,275	0.2
2	13,013,800	88,127,104	0.2
3	10,909,035	91,817,535	0.2
4	10,051,340	93,099,730	0.2
5	8,638,639	94,951,482	0.2
6	917,729	99,944,446	0.2
1	14,062,815	86,113,360	0.4
2	11,708,935	90,599,676	0.4
3	9,541,442	93,862,895	0.4
4	8,661,567	94,971,339	0.4
5	7,216,303	96,537,301	0.4
6	−604,346	99,976,132	0.4
1	12,817,677	88,801,889	0.6
2	10,379,829	92,809,065	0.6
3	8,148,442	95,632,421	0.6
4	7,245,975	96,562,639	0.6
5	5,767,543	97,836,244	0.6
6	−2,154,698	99,700,825	0.6
1	11,537,127	91,238,074	0.8
2	9,012,923	94,749,092	0.8
3	6,715,825	97,119,635	0.8
4	5,790,124	97,867,049	0.8
5	4,277,580	98,841,576	0.8
6	−3,749,142	99,111,318	0.8

3. Find the cost of insurance using Equation 4.3 with $P = P_R$ and $n = n_R$.
4. If the cost of insurance is not equal to the target cost, try another P_R.

TABLE 7

OPTIONS VS. REINSURANCE

Insurer #	Cost of Insuring	P	P_R
1	88,801,889	0.6000	0.6000
2	92,809,065	0.6000	0.7820
3	95,632,421	0.6000	2.0073
4	96,562,639	0.6000	5.6528
5	97,836,244	0.6000	2.0165
6	99,700,825	0.6000	8.1631

We use the option contract from Table 6 with $P = 0.6$. The P_Rs that provide the same cost of providing insurance are given in Table 7.

For Insurer 1, All County Insurance Company, there is no difference because its losses correlate perfectly with the index losses. If the net cost for reinsurance to Insurer 2, Uni-County Insurance Company, is between 0.6000 and 0.7820, reinsurance is less expensive. There is more leeway for reinsurance for the regional insurers, Insurers 3 and 5, and considerably more leeway for reinsurance with the single-county insurers, Insurers 4 and 6.

9. SUMMARY

The cost of capital and its substitutes is determined by a variety of market conditions that are beyond the control of the insurer. To efficiently use its capital, the insurer has to constantly analyze the opportunities that are presented to it. This paper shows how a catastrophe model can be used to evaluate the costs and benefits of alternative catastrophe risk management tools for insurers. The alternatives include:

1. raising sufficient capital to contain the catastrophe risk;
2. buying options on a catastrophe index; and
3. buying reinsurance.

These alternatives are quantified by the cost of providing insurance, which depends upon:

1. the price of the contracts and/or reinsurance, as quantified by P and P_R;
2. the basis risk, as quantified by ρ; and
3. the cost of capital, as quantified by K, T and σ_X.

The quantities P and K depend upon market conditions, and T depends upon the risk aversion of the insurer. The quantities σ_X, σ_Y, and ρ are obtained from the catastrophe model.

With these quantities one can calculate the optimal number of contracts (or the optimal amount of reinsurance) to buy with Equation 4.5 and then quantify the cost of providing insurance with Equation 4.3. The cost of the various alternatives can be compared to provide the best insurance value.

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APPENDIX

DERIVATION OF EQUATION 4.5

We seek to solve the equation:

$$\frac{K \cdot T \cdot (n\sigma_Y^2 - \rho\sigma_X\sigma_Y)}{\sqrt{\sigma_X^2 - 2n\rho\sigma_X\sigma_Y + n^2\sigma_Y^2}} + P = 0.$$

Moving the P and the denominator to the other side of the equation, and squaring yields:

$$\begin{aligned} P^2(\sigma_X^2 - 2n\rho\sigma_X\sigma_Y + n^2\sigma_Y^2) &= K^2T^2(n\sigma_Y^2 - \rho\sigma_X\sigma_Y)^2 \\ &= K^2T^2(n^2\sigma_Y^2 - 2n\rho\sigma_X\sigma_Y^3 + \rho^2\sigma_X^2\sigma_Y^2) \end{aligned}$$

The above equation can be put into the form: $an^2 + bn + c = 0$ with

$$\begin{aligned} a &= \sigma_Y^2(K^2T^2\sigma_Y^2 - P^2); \\ b &= -2\rho\sigma_X\sigma_Y(K^2T^2\sigma_Y^2 - P^2); \qquad \text{and} \\ c &= \sigma_X^2(K^2T^2\sigma_Y^2\rho^2 - P^2). \end{aligned}$$

The solution for n is of the form

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

with:

$$\frac{-b}{2a} = \frac{2\rho\sigma_X\sigma_Y(K^2T^2\sigma_Y^2 - P^2)}{2\sigma_Y^2(K^2T^2\sigma_Y^2 - P^2)} = \frac{\rho\sigma_X}{\sigma_Y}; \qquad \text{and}$$

$$\begin{aligned} \frac{b^2 - 4ac}{4a^2} &= \frac{4\sigma_X^2\sigma_Y^2\rho^2(K^2T^2\sigma_Y^2 - P^2)^2 - 4\sigma_Y^2(K^2T^2\sigma_Y^2 - P^2)\sigma_X^2(K^2T^2\sigma_Y^2\rho^2 - P^2)}{4\sigma_Y^4(K^2T^2\sigma_Y^2 - P^2)^2} \\ &= \frac{\sigma_X^2\rho^2(K^2T^2\sigma_Y^2 - P^2) - \sigma_X^2(K^2T^2\sigma_Y^2\rho^2 - P^2)}{\sigma_Y^2(K^2T^2\sigma_Y^2 - P^2)} \\ &= \frac{\sigma_X^2}{\sigma_Y^2} \cdot \frac{P^2(1 - \rho^2)}{K^2T^2\sigma_Y^2 - P^2}. \end{aligned}$$

Then:

$$n = \frac{-b - \sqrt{b^2 - 4ac}}{2a} = \frac{\rho\sigma_X}{\sigma_Y} - \frac{\sigma_X}{\sigma_Y}\sqrt{\frac{P^2(1-\rho^2)}{K^2T^2\sigma_Y^2 - P^2}}.$$

Squaring the equation in the first step introduces an extraneous root. The solution with the positive square root is the extraneous root since it indicates one should buy more contracts when $P > 0$ than when $P = 0$.