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A COMPARISON OF PROPERTY/CASUALTY INSURANCE FINANCIAL PRICING MODELS

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Abstract

A number of property/casualty insurance pricing models that attempt to integrate underwriting and investment performance considerations have been proposed, developed, and/or applied. Generally, empirical tests of these models have involved examining how well the models fit historical data at an industry level. This paper demonstrates how to apply a variety of property/casualty insurance financial pricing techniques to a single hypothetical, but representative, company. Both company and economic parameters are varied in order to examine the sensitivity of indicated underwriting profit margins from these techniques to different company situations and economic environments, and to highlight the differences between the techniques at a practical level. This paper *also serves as a practical guide for applying these models in order to encourage more widespread use of these approaches.*

1. INTRODUCTION

The determination of a "fair," or competitive, rate of return for property/casualty insurance underwriting operations has been the subject of increasing scrutiny over the last several decades among both academics and insurance practitioners. The five percent target underwriting profit margin promulgated by the National Convention of Insurance Commissioners in 1921 represented the first of many techniques that have been considered, and in some cases employed, to determine a fair rate of return. Although that first approach had little, if any, statistical or financial foundation, subsequent methods have attempted to determine insurance prices more rigorously, and with due consideration given to relevant insurance, economic, and financial market characteristics. An appropriate determination of fair insurance prices is important because capital will be attracted to—and retained by—the insurance industry only if its rates of return are comparable to those in other industries that are perceived to have similar levels of risk.

A variety of financial pricing models has now been proposed for property/casualty insurance, including the Target Total Rate of Return approach, the Capital Asset Pricing Model, several Discounted Cash Flow approaches, the Option Pricing Model, and the Arbitrage Pricing Model. In general, these models have been applied individually and without clearly showing how the necessary parameters can be determined from insurance financial statements. Several important studies do provide a degree of comparison among the different models. Myers and Cohn [25] compare the discounted cash flow model and the insurance CAPM, including sensitivity analysis of the various parameters. Cummins [9] provides a comparison of the discounted cash flow model and internal rate of return approach and illustrates the results of each method on one set of data. Doherty and Garven [15] contrast the insurance CAPM and the option pricing models over a range of values for each parameter. However, there has been no systematic comparison of all the financial pricing models or any documentation explaining how the relevant parameters should be determined for a particular insurer. This paper addresses those needs, first generating a financial statement for a hypothetical, but representative, insurer, and then applying each pricing model to this insurer to determine the appropriate premium level and underwriting profit margin. Finally, the models are examined over a range of parameter values that occur across insurers and over time to demonstrate which parameters need to be measured most accurately, and which models are most impacted by changes in different variables. This analysis illustrates potential strengths and weaknesses of each technique. By comparing the indications of fair underwriting profit margins under each of these pricing methods, their differences will be highlighted. This will allow both company management and regulators to better gauge the potential impact on prices of adopting one or another technique in various business environments.

The insurance pricing techniques applied to our representative insurance company include:

- target underwriting profit margin model,
- target total rate of return model,
- insurance capital asset pricing model,
- discounted cash flow (Myers–Cohn) model,
- internal rate of return model,
- option pricing model,
- arbitrage pricing model.

The company to which these techniques are applied is a fictitious entity, but quite representative of companies actively involved in the property/casualty insurance industry. As many of the techniques examined are best applied in a single line of business framework, we have chosen to model a company that writes only private passenger automobile (PPA) insurance. Representative financial values and ratios, as well as payout patterns, were selected based on an examination of both aggregate industry and individual company values. Values for other economic and insurance industry variables are derived from appropriate sources as described in Section 3 of the paper. The considerations involved in obtaining each of the parameters used in the models are shown, in order to illustrate how a company could use each technique.

2. REVIEW OF THE ALTERNATIVE PRICING MODELS

In 1921, the National Convention of Insurance Commissioners, by an overwhelming margin, approved the Majority report of the Committee on Fire Insurance. For two years, the Committee had been considering the issue of what was a reasonable underwriting profit margin. The report's conclusions included the following items:

- "Underwriting profit (or loss) is arrived at by deducting from earned premiums, all incurred losses and incurred expenses."
- "A reasonable underwriting profit is 5 percent, plus 3 percent for conflagrations … "

(See National Convention of Insurance Commissioners [28] and National Association of Insurance Commissioners [26] for more details.) A minority report recommended that investment income also be considered in determining a reasonable profit provision, but this recommendation was defeated (see Webb [35]). Thus, the position of the insurance regulatory community at that time was that only underwriting, and not investment, operations were relevant to the determination of a reasonable property/casualty profit level. Furthermore, the specific profit level recommended, five percent, was established apparently without meaningful statistical support.

Subsequent studies and reports began to question the appropriateness of ignoring investment income. This concern intensified in the 1960s and 1970s, as interest rates, and their volatility, increased. The National Association of Insurance Commissioners (NAIC) in 1970 [27] said that, "In determining profits, it is submitted that income from all sources should be considered." The NAIC, however, while criticizing the 1921 formula, did not recommend an alternative until its 1984 Investment Income Task Force Report, which recommended that the total rate of return on net worth should be used to measure insurance profitability.

In the meantime, actuaries started to develop (and sometimes use) several alternative pricing techniques that attempt to address both underwriting and investment considerations in pricing property/casualty insurance policies. Initially, something of a dichotomy existed among the techniques proposed: some concentrated on the underwriting side of the insurance process, with little consideration given to meaningful analysis of the investment process; others focused primarily on the investment side, without adequate understanding of the unique aspects of the insurance underwriting process. Recent research has attempted to give appropriate consideration to *both* aspects of the property/casualty insurance business.

This paper examines seven different pricing models, and applies each to a fictitious but representative insurance company. Each of the seven techniques is described below; additional details regarding specific calculations for each of the financial models are included in the Appendix. Variables used in the following formulas include:

P = premium

UPM = underwriting profit margin

L = losses and loss adjustment expenses

E = other expenses

S = equity (or adjusted statutory surplus)

IA = invested assets

IR = investment return

r_e = return on equity

r_f = risk-free rate of return

r_m = market rate of return

β_e = beta of the insurance company's stock

β_u = beta of the insurance underwriting process

k = funds-generating coefficient

t_i = tax rate on investment income

t_u = tax rate on underwriting income.

A. Target Underwriting Profit Margin Model

The Target Underwriting Profit Margin (Target UPM) Model determines an appropriate premium for a property/casualty insurance policy based upon a pre-selected underwriting profit margin. Thus, no consideration is given to the investment earnings produced by the insurance policy due to either the allocation of surplus in support of the policy or the delay between receipt of the premium and payment of the losses and expenses. The premium is determined strictly as a function of the expected losses, expenses, and the target underwriting profit margin as a percentage of premium. Historically, the target UPMs used have typically been 2.5 percent for workers compensation, and 5 percent for all other lines in most jurisdictions. However, in 1986, Florida adopted rule 4ER86-1 that established a formal procedure for including investment income in the ratemaking process by adjusting the target UPM downward to reflect the additional investment income attained in long-tailed lines over short-tailed lines.

For our representative PPA insurance company, the pricing and profit equations are:

$$UPM = 0.05 \tag{2.1}$$

$$P = \frac{L+E}{1-UPM} \qquad \text{or} \qquad UPM = 1 - \frac{L}{P} - \frac{E}{P}. \tag{2.2}$$

While this approach has been used in the property/casualty insurance industry for decades, and is relatively simple to apply, it is clearly the least "financially sophisticated" of the pricing models examined in this study, and in fact—efforts such as Florida's notwithstanding—the Target UPM model is not supported by financial considerations.

B. Target Total Rate of Return Model

A straightforward way of incorporating investment income into the ratemaking calculation is simply to target, rather than merely the underwriting margin, the *combined underwriting and investment* returns of an insurance policy. The total rate of return of a policy is viewed as having two components: investment and underwriting. If two of these three items are known, the third can be derived. Thus, in the Target Total Rate of Return (Target TRR) Model, the underwriting profit margin is determined based on a selected total rate of return and an estimate of the investment income on a policy. This is analogous to the process that has been historically used for the utility industry.

The target total rate of return can be calculated as

$$TRR = \frac{(IA \times IR) + (P \times UPM)}{S}. \tag{2.3}$$

The TRR reflects both investment income (the first term in Equation 2.3) and underwriting income (the second term) as a proportion of equity. Solving Equation 2.3 for the underwriting profit margin yields

$$UPM = \frac{(S \times TRR) - (IA \times IR)}{P}. \tag{2.4}$$

Now we need to specify an appropriate target total rate of return. As with utility regulation, this is the crux of the model. Although any number of methods might be viable, the Capital Asset Pricing Model (CAPM) has typically been used to select the TRR. This approach will be used in this paper. The CAPM formula is:

$$\mathrm{E}[r_e] = r_f + \beta_e(\mathrm{E}[r_m] - r_f), \tag{2.5}$$

where β_e is defined as

$$\beta_e = \frac{\mathrm{Cov}(r_e, r_m)}{\mathrm{Var}(r_m)}.$$

The TRR is then set equal to the expected return on equity, or the cost of equity capital, $\mathrm{E}[r_e]$. Thus, substituting Equation 2.5 into Equation 2.4 yields:

$$\begin{aligned} UPM &= \frac{(S \times [r_f + \beta_e(\mathrm{E}[r_m] - r_f)] - (IA \times IR)}{P} \qquad \text{or} \\ UPM &= \frac{S}{P}\left([r_f + \beta_e(\mathrm{E}[r_m] - r_f)] - \frac{IA \times IR}{S}\right) \end{aligned} \tag{2.6}$$

C. Insurance Capital Asset Pricing Model

The CAPM was first introduced into the finance literature in the mid-1960s by Sharpe [32], Lintner [22], and Mossin [24]. The CAPM, as described in Equation 2.5, expresses expected return on equity as consisting of two components: a risk-free component and a risk premium, which is essentially a reward for taking on risk. The degree of compensation for risk-taking is measured by the equity beta, which quantifies systematic, as opposed to nonsystematic (or diversifiable), risk. Diversifiable risk is not compensated by the market, since it can be eliminated through an appropriate investment diversification strategy.

The CAPM has been applied to insurance by several authors. Among the first were Biger and Kahane [5]. Fairley [16] developed the following underwriting profit margin formula based on the CAPM:

$$UPM = -kr_f + \beta_u(\mathrm{E}[r_m] - r_f). \tag{2.7}$$

Here, the appropriate underwriting profit margin is calculated as the risk premium associated with the systematic risk of the insurance underwriting process, offset by investment income, which is credited at the risk-free rate of return. The funds-generating coefficient reflects the fact that the insurance process produces investable assets generated by premium income prior to payout of expenses and claims. This coefficient is often estimated by a reserves-to-premium ratio. For a steady state insurer, this approach would be correct; if the company has changed premium or exposure volume, however, this calculation would need to be refined.

In addition to Fairley, Hill [18] and Hill and Modigliani [19] have also developed CAPM applications to property/casualty insurance. In particular, Hill and Modigliani have developed a model that considers the impact of taxes and in fact allows for differential tax rates. Letting t_i and t_u be tax rates as defined above, the Hill and Modigliani model can be expressed as:

$$UPM = -kr_f \frac{1-t_i}{1-t_u} + \beta_u(\mathrm{E}[r_m] - r_f) + \frac{S}{P} r_f \frac{t_i}{1-t_u}. \tag{2.8}$$

It is Equation 2.8 that is modeled in this study.

D. *Discounted Cash Flow Model (Myers–Cohn)*

The Discounted Cash Flow (DCF) Model was developed for use in Massachusetts as a counterpart to the CAPM model that had been used there beginning in the 1970s. The model is described by Myers and Cohn [25] and takes the following general form:

$$P = \mathrm{PV}(L + E) + \mathrm{PV}(UWPT) + \mathrm{PV}(IBT), \qquad (2.9)$$

where

PV = present value operator,

UWPT = tax generated on underwriting income, and

IBT = tax generated on income from the investment balance.

One of the keys to using the DCF model is to properly determine a method of discounting each component of the above equation. Those cash flows that are certain should be discounted at the risk-free rate, while risky cash flows must be discounted at an appropriate risk-adjusted rate.

D'Arcy and Garven [14] test the following DCF model, where all cash flows are discounted based on the risk-free rate (which is equivalent to assuming that $\beta_u = 0$ in the CAPM):

$$1 = \mathrm{PV}\left(\frac{E}{P}\right) + \mathrm{PV}\left(\frac{L}{P}\right) + \mathrm{PV}\left(t\left[1 - \frac{E}{P} - \frac{L}{P}\right]\right) + \mathrm{PV}\left(t\left[1 + \frac{S}{P}\right]\frac{L}{P}LPP\right), \qquad (2.10)$$

where *LPP* = the loss payout pattern. This equation is solved for L/P, the loss ratio. Then, the indicated UPM is calculated as $1 - (L/P) - (E/P)$.

In this study, we use this general approach, but refine it to reflect the discounting of risky cash flows at a risk-adjusted rate. Specifically, in order to determine an indicated DCF premium level, we have used Equation 6.2 of D'Arcy and Dyer [13], with the enhancement that different tax rates are allowed on underwriting versus investment operations. The UPM is then determined as $(P - E - L)/P$.

E. Internal Rate of Return Model

Whereas the Myers–Cohn discounted cash flow model described above considers flows between the insurer and the policyholder, the internal rate of return (IRR) model, for example as used by the National Council on Compensation Insurance (NCCI), looks at flows between the investor and the company. In particular, the flows under the IRR model include the commitment of surplus, the release of surplus, the investment income, and the underwriting profit (both of the last two being net of applicable taxes). The discount rate of these flows is solved for, so that the present value of the flows is zero; then, this discount rate is compared to the cost of capital. A financially fair premium is determined by setting the IRR equal to the cost of capital.

In this study, we have used the same approach as Cummins [9]. The cost of capital is determined by the CAPM. Exhibit 6, Part 2 displays the calculation of the IRR model fair premium for the base case.

F. Option Pricing Model

Recently, the option pricing model (OPM) has received increasing attention among both insurance academics and practitioners. The OPM is seen as having a great deal of promise as a property/casualty insurance pricing framework since an insurance policy can, essentially, be viewed as a package of contingent claims. The primary application of the OPM to property/casualty insurance to date is Doherty and Garven [15], who show that the present values of the claims held by the three claimholders to an insurance contract—shareholders, policyholders, and the tax authorities (government)—can be modeled as European call options. In order to actually value these claims, and then determine a competitive UPM and premium, Doherty and Garven assume two alternative valuation frameworks:

- asset returns are normally distributed, and investors exhibit constant absolute risk aversion (CARA) with regard to their preferences;
- asset returns are lognormally distributed, and investors exhibit constant relative risk aversion (CRRA) with respect to their preferences.

Although closed-form solutions are not derived, the premiums and UPMs can be found for both frameworks via a straightforward iteration process. The appropriate formulas in Doherty and Garven relating to these two valuation assumptions are their Equations 19 and 30, respectively. We apply the first of these two models in this study. (See the Appendix for further details.) A spreadsheet was created wherein the difference between the market value of the residual claim of the shareholders (V) and the initial paid-in equity (S) is "backsolved" to zero by varying the premium; the solving value represents the fair premium indication. This premium is net of expenses, which are then added in. The UPM is calculated as $(P^* - L - E)/P^*$, where P^* includes expenses.

G. Arbitrage Pricing Model

The Arbitrage Pricing Theory (APT), developed initially by Ross [31] and extended by Roll and Ross [30] and others, is, like the CAPM, an equilibrium model of security returns. However, the APT makes fewer assumptions than does the CAPM, and it also admits the possibility of more than one "factor" to which security returns are sensitive. The theory behind the APT specifies neither the number of such factors, nor their identity. Unlike the CAPM, the APT does not posit a special, or even necessarily *any*, role for a market return in determining individual security returns.

According to the Arbitrage Pricing Theory, security returns follow the process

$$r_i = \mathrm{E}[r_i] + \sum_{j=1}^{J} \beta_{ij} f_j + \epsilon_i, \tag{2.11}$$

where

r_i = return on the ith security,

β_{ij} = the sensitivity of the return on the ith security to the jth factor, and

f_j = a factor that influences security returns.

Then, the absence of arbitrage requires that the excess return on each security be a linear combination of the betas:

$$\mathrm{E}[r_i] - r_f = \sum_{j=1}^{J} \beta_{ij} \lambda_j, \tag{2.12}$$

where

λ_j = the risk premium corresponding to the factor f_j.

The APT has been applied to insurance by Kraus and Ross [21] and Urrutia [34]. Urrutia derives UPM formulae for an Arbitrage Pricing Model (APM), based on the above theoretical relationships. His differential-tax UPM equation takes the form

$$UPM = -\frac{1-t_i}{1-t_u} r_f k + \sum_{j=1}^{J} \beta_{UPM,j} \lambda_j + r_f \left(\frac{S}{P}\right) \frac{t_i}{1-t_u}, \tag{2.13}$$

where

$$\beta_{UPM,j} = \frac{\mathrm{Cov}(UPM, f_j)}{\mathrm{Var}(f_j)}.$$

Generally, there are two approaches to testing the APT model. The first involves factor analysis, a statistical methodology that determines factors and betas that best "explain" the data (i.e., that minimize the covariance of residual returns). The second involves the pre-specification of variables that are hypothesized to influence returns, as in Chen, Roll, and Ross [7]. This second approach allows for economic intuition in the interpretation of results; it is this approach which we use in this paper. In particular, a number of macroeconomic variables were tested, with the inflation rate and the growth in industrial production being the two variables that appear most significant in explaining historical underwriting profit margins. Multivariate regression analysis is used to determine sensitivities of UPMs to these two variables. Selected parameter values are incorporated into Equation 2.13 to determine fair UPMs.

3. DATA AND METHODOLOGY

In addition to the selection of the pricing techniques and the identification of the appropriate formulas for each, as documented above, the following steps were involved in this study:

- development of the representative statutory company model,
- collection and development of information regarding company and economic variables,
- application of each of the pricing techniques to the representative company, and
- sensitivity tests of the models by varying certain company and economic parameters.

Each of these steps is discussed below.

A. Development of the Representative Statutory Company Model

It was decided, for the sake of simplicity and clarity of presentation, to concentrate on a fictitious but representative property/casualty insurance company that writes only one line of business in one state. Private passenger automobile insurance was selected due to its size and significance in the industry. Other lines can easily be modeled by the same techniques presented here, with appropriate changes in parameters.

The 1994 editions of several A. M. Best publications provided the basis for the development of the representative company model. The most recent statement year reflected in these editions is 1993. To begin, some basic financial values for the largest PPA companies in the industry were accumulated. Exhibit 9 summarizes the asset, liability, surplus, and net written premium values for the main PPA companies within each of the 20 largest PPA groups. The calculated ratios vary considerably, sometimes due to different operating philosophies, sometimes because a company writes a large amount of other business in addition to PPA. These ratios served as the basis for certain company parameter ranges, discussed later, to test model sensitivity.

Pages 2, 3, and 4 of the Property/Casualty Annual Statement are simulated for the representative insurance company in Exhibit 10. These simulated pages were developed from the consolidated data from A. M. Best. First, 0.1% of consolidated industry (all lines) earned premium was taken as the starting point for the fictitious company (Exhibit 10, Part 3). Then, company asset, liability, and surplus values were derived. Total assets for the company were calculated by applying an industry asset/earned premium ratio to the selected company earned premium. Consolidated industry total values for specific asset, liability, and income categories were compared to aggregate figures for companies in which private passenger automobile and homeowners predominate. Comparisons were generally made on the basis of percentages relative to the appropriate major item—e.g., each asset item as a percentage of total assets. These percentages are shown on the first three sheets of Exhibit 10. Generally, the percentages applying to PPA-and-homeowners-predominating companies were used as the basis for our selected company values.

The selected percentages—as well as the resulting asset, liability, and income items—for the representative company are shown on Exhibit 10, Parts 1 through 3.

PPA loss development patterns for the representative company were determined by analyzing the consolidated Schedule P data from A. M. Best, using standard actuarial techniques. PPA liability/medical and physical damage patterns were analyzed separately. The derivation of these patterns is included in Exhibit 11.

B. Collection and Development of Company and Economic Information

Exhibit 1 documents the information required by each of the pricing techniques in order to apply it to property/casualty insurance ratemaking. The initial or "base case" value assumed in this study for each variable, as well as a range of reasonable values, is included in the table. The variables are classified into three categories: Company Variables, Economic Variables, and Government Policy Variables, depending upon whether a particular variable is most influenced by company operating decisions, general economic conditions, or governmental policy decisions.

The Company Variables include equity, investment rate of return, standard deviation of investment returns, equity beta, and the funds-generating coefficient. The rationale for placing these variables in this category is that the company, through operating and investment decisions, determines the premium-to-surplus level, the investment policy (which affects both the investment rate of return and the standard deviation), internal factors which influence the beta of the firm's equity, and—to some extent—the claims payment patterns and philosophy.

The Economic Variables include the risk-free rate, market risk premium, risk adjusted discount rate, underwriting beta (and, analogously, the investment-claims correlation), the standard deviation of market returns, the standard deviation of losses, and annual growth rates and betas for inflation and industrial production. These parameters vary primarily due to effects exogenous to the company.

The final category of factors are considered Government Policy Variables, which include the tax rate, the ratio of the investment tax rate to the total tax rate, and the annual tax discount factor. Obviously, the government has sole control over the basic tax rate. The ratio of investment/total tax rate can be affected by the company, by changing the investment allocation, or by the government, by changing the rules about taxability of various investments. In light of the significant effect of such tax regulations as the Tax Reform Act of 1986, the government is assumed to have the greater influence over this variable. Similarly, although the tax discount factor is currently the 60-month moving average of mid-maturity Treasury issues, which would tend to make this an Economic Variable, the definition and calculation of this parameter could be changed by the government at any time.

The most critical step in applying any of the financial models is determining the values for the variables. No matter how accurate a particular model is felt to be, unless the correct parameters are included, the results will not be useful. Many of the prior applications of financial pricing models have either simply assumed particular values for certain variables or determined the values based on a one-time study of industry results. These approaches do not provide much guidance for someone who wants to apply these techniques to an individual company situation on an ongoing basis. In order to facilitate future applications of these models, the determination of parameter values is shown by basing the calculation, where possible, on the fictitious insurer's annual statement or supplementary financial reports, or on general economic information.

C. Company Variables

- *Equity*: Each of the financial pricing models requires either the premium-to-surplus ratio or a value for surplus itself. Although this may appear straightforward, it is not. The reason for the difficulty is the different definitions and uses of the surplus value. For example:

  — In the Target Total Rate of Return model, surplus relates to the amount of assets that an investor chooses to invest in any insurance operation, as opposed to deploying those assets in another investment.

  — In the Discounted Cash Flow model, surplus relates to the amount of invested funds that generate taxes that need to be covered by the premium.

  — In the Insurance CAPM, Internal Rate of Return, Option Pricing, and Arbitrage Pricing models, the surplus is both the amount of capital invested in the firm in support of writing a particular amount of business and the invested assets earning taxable investment income.

  Although each model terms this value "surplus," each model technically requires a slightly different definition of surplus. For consistency, the same value is used as the starting point for each method. As this parameter is extremely important, care should be taken in selecting the appropriate figure. In this study, the value for surplus is an adjusted statutory surplus value, or equity, that is determined as follows:

  Equity = (Statutory Surplus)
  + (Equity in the Unearned Premium Reserve)
  + (Difference Between Nominal and Risk-Free-Discounted Loss Reserves)
  + (Excess of Statutory Over Statement Reserves)

+ (Difference Between Market Value and Book Value for Bonds)

+ (Non-Admitted Assets)

– (Tax Liability on Equity in Unrealized Capital Gains).

For the fictitious company, equity equals $189,360 (dollar values in thousands). Premium-to-statutory surplus ratios for the top twenty private passenger auto insurers (Exhibit 9) range from 0.67 for ITT Hartford to 2.89 for American Premier Underwriters. This range, combined with the adjustments to statutory surplus, which were held constant, determined the range for equity values to be $122,132 to $399,692, as documented in Exhibit 10, Part 4.

Another way to determine the economic value of the insurer, which could be used for publicly traded firms, is to use market value, calculated by multiplying the number of shares outstanding times the current stock price. Our fictitious company is assumed to have a market value of $220,399, reflecting the average market-to-book ratio for stock property/casualty insurance companies at the end of 1993 of 1.46 multiplied by the statutory surplus. The market value could differ from the equity value for any number of reasons, including additional accounting conventions that cause a divergence between reported and economic value or other assets that are not reflected in an insurer's balance sheet (reputation, market niche, a book of business that will generate profits on renewal). For the models that consider surplus to be the investor's value in an insurer (all the models illustrated in this paper except the discounted cash flow), the market value represents the amount that the investor could obtain for giving up its investment in insurance. The market-to-book value ratio ranged from a low of 0.92 (market value of $138,881) to a high of 2.43 (market value of $366,828) for personal lines insurers. As these values all fall within the range obtained by varying only the premium-to-statutory surplus ratio, no further adjustments were made. However, the size of this range illustrates the importance of the selection of the appropriate equity value.

- *Investment rate of return*: The investment rate of return was calculated by summing net investment income, realized capital gains and unrealized capital gains and dividing the total by the average investable assets during the year. All investment income, realized or unrealized, was used to reflect the full effect of investment earnings. A base case value of eight percent was selected based on this calculation, with a range of plus or minus two percent. In practice, the average returns over a number of years should be taken to avoid distortions that could be caused by short-term fluctuations in investment results.

- *Standard deviation of investment returns*: The base case of 20 percent is the same value as used by Doherty and Garven [15]. This value could be calculated for a particular insurer by obtaining the standard deviation of the company's total investment rate of return (including both realized and unrealized capital gains). Our selected range is plus or minus ten percent around the base case.

- *Equity beta*: The base case is 1.0, which is the overall market beta. With regard specifically to insurance stocks, Hill [18] found equity betas averaging 0.61, and Hill and Modigliani [19] and Fairley [16] found betas of approximately 1.00. Fama and French [17] formed portfolios based on beta, and the portfolio betas ranged from .81 to 1.73. Thus, the selected range for insurance betas was .60 to 1.70, ranging from the value determined by Hill to approximately the 95th percentile based on Fama and French. Note that, rather than separately testing the sensitivity of UPMs to a selected range of internal rate of return values, we have assumed that the IRR values are determined via a CAPM approach and so have embedded UPM sensitivity to internal rates of return in our equity beta range.

- *Funds-generating coefficient*: k is the average length of time that the insurer holds (and invests) premiums before they are used to pay expenses and losses. The coefficient is calculated by multiplying the loss payments in the first year by 0.5, the loss payments in the second year by 1.5, and so forth. These values are then summed and divided by premiums. (Expenses are assumed to be paid when the premium is received, so they do not increase the total time-weighted sum of outgo.) For the base case, the ratio is 1.18. The sensitivity of UPM indications to the coefficient is examined by assuming that the company pays its losses and expenses, on average, one quarter of a year either faster, $k = .93$, or slower, $k = 1.43$, than the base case.

  The calculation of the coefficient for the Option Pricing Model is similar, but as this method calculates premiums net of expenses, the appropriate adjustment is to divide the weighted sum of loss payments by total losses, rather than premiums. This produces a k value of 1.5 for the base case, with a range of 1.18 to 1.81 to correspond with the insurance CAPM adjustment. The Discounted Cash Flow and Internal Rate of Return models both also depend on the loss payment pattern, but as the actual payments are used instead of a weighted average, the adjustment must be made to each payment. The calculation of the funds-generating coefficient for each model and the adjusted loss payout patterns are displayed in Exhibit 10, Part 4.

D. *Economic Variables*

- *Risk-free rate*: As is frequently done, we used the interest rate on U. S. Treasury bills as a proxy for the risk-free rate. As of February, 1996, both three- and six-month Treasury bills had a yield to maturity of approximately five percent. As the appropriate rate for this variable is the current, and not a past, rate, the base case value was set at five percent. This rate has ranged from 2.9 percent to 14.7 percent over the period 1974 to 1993 based on Ibbotson [20] data. The range was set at the twenty-year high and low values.

- *Market risk premium*: This is generally determined as the average excess return in the stock market over an investment in short-term Treasury bills. The time period 1926 through the most recent year is frequently used based on the Ibbotson Associates data series. For 1926 through 1994, the market risk premium is eight percent. Depending on the number of years included in the measurement and the selected years, the market risk premium fluctuates. The selected range is six to ten percent.

- *Risk-adjusted discount rate*: The RADR is used in the discounted cash flow model to discount risky cash flows, primarily losses. The consensus of research on the issue of discounting loss reserves indicates that the appropriate risk-adjusted discount rate is less than the risk-free rate. (See, for example, Butsic [6], Cummins [9], and D'Arcy [12].) This is because the RADR reflects a risky liability to the insurer, and the insurer may be viewed as a risk-averse entity. Conversely, the insurance policy can be viewed as a risky asset for the policyholder, an asset that has a negative beta since it increases in value when the value of the other assets of the policyholder decline due to a loss. Thus, the policyholder would expect to earn a rate of return below the risk-free rate based on either the Capital Asset Pricing Model or the Arbitrage Pricing Model. How *much* less, however, is an unsettled issue. If the CAPM is used to determine the risk-adjusted discount rate, then the differential will be a constant value, regardless of the level of interest rates. For extremely low interest rates, the RADR could even turn out to be negative. Conversely, if the RADR is proportional to the level of interest rates, then the differential will increase as interest rates increase and, regardless of how low interest rates were to fall, would be non-negative as long as interest rates were non-negative. This is an area requiring further research. For this paper, the RADR is assumed to be proportional to the level of interest rates. The risk-adjusted rate is held at 60 percent of the risk-free rate, with the RADR-to-risk-free ratio ranging from zero percent to 100 percent. (The zero percent is consistent with undiscounted loss reserves.) Given the base case risk-free rate assumption of five percent, the base case risk-adjusted discount rate is selected as three percent.

- *Underwriting beta*: There is no generally-accepted theoretical reason why underwriting results should be correlated with market returns, so measuring the value of the underwriting beta and the investment-claims correlation must be based on empirical results. Cummins and Harrington [10] test underwriting betas over the period 1970 to 1981 and find that values appear to range from −.20 to +.20, although the average is not significantly different from zero. D'Arcy and Garven [14] calculate a long-term correlation (1926 through 1985) of 0.0763. The base case is set at zero, with a range of −0.40 to +0.40.

- *Standard deviation of stock market returns*: This variable has historically been 22 percent (Doherty and Garven [15]), which is used here as the base case. The range of 12 to 40 percent was selected judgmentally.

- *Standard deviation of losses*: This should be measured by comparing actual losses with expected losses over a number of years. There is no information about initial expected losses in any financial statement of an insurer, although Schedule P shows loss development after the first accident year has occurred. Doherty and Garven [15] assume a value of 25 percent of losses for this parameter (i.e., a coefficient of variation, or Cov, of 25 percent), and that value is used here as the base case. The range of 12.5 percent of losses to 50 percent was determined judgmentally; these values correspond to assuming a Cov of one-half to twice the base case Cov.

- *CPI change*: Historical values were taken from Ibbotson Associates [20]. Based on recent inflation rates, an annual growth rate of three percent was chosen as the base case value, with a range from zero to six percent.

- *Industrial production growth rate*: Historical values were taken from Federal Reserve data—log-differences of index values were used. Based on recent growth rates, a base case value of two percent was chosen, with a range of zero to four percent.

- *CPI and industrial production betas*: The beta values were determined by running multivariate regressions of annual inflation and industrial production growth rates against the following dependent variable: historical auto UPMs (taken from A. M. Best *Aggregates and Averages*), plus the historical risk-free rate multiplied by the estimated historical funds generating coefficient. (See Urrutia [34], Equation 13.) Examination of the coefficients of the regressions, as well as covariance and variance calculations, over different periods of time led to the selection of base case values and ranges. (See Exhibit 8, Part 2.) While there is some evidence from the regressions for a base case inflation beta closer to 0.70 than to 0.50 based on much of the historical data period from 1948 to 1993, the last ten years of this period indicate an inflation beta much closer to zero. We have chosen a wide and symmetric range around our selected base case of 0.50 to indicate that these estimates require refinement; this could be the subject of future research.

E. Government Policy Variables

- *Tax rate*: The current corporate tax rates range from 15 to 39 percent, based on taxable income level. The base rates are 15, 25, 34 and 35 percent, with surcharges of three and five percent applying to segments of taxable income in order to equalize average and marginal tax rates. The appropriate tax rate is the projected marginal tax rate for the tax year in which the coverage will apply. This necessarily involves a projection. For the fictitious insurer, taxable income is projected to be between $335,000 and $10,000,000 for which a 34 percent marginal tax rate applies. The range is selected to be 28 to 40 percent, reflecting uncertainty over future, potentially retroactive, government tax policy.

- *Investment/total tax ratio*: Investment income is taxed at different rates depending on the source. Interest on federal and corporate bonds is fully taxable. Interest on municipal bonds purchased after August 7, 1986, is taxed at the 15 percent level. Seventy percent of dividends from non-controlled corporations are taxed at the 15 percent level, with the remainder fully taxed. Long-term capital gains are subject to a maximum tax rate of 28 percent. Based on the distribution of investment income earned by the fictitious insurer for the most recent year, the tax rate on investment income is 80 percent of the maximum level. This calculation is illustrated in Exhibit 10, Part 4. This value is allowed to range from 60 to 100 percent.
- *Tax discount factor*: The tax discount factor based on the 60-month moving average of mid-maturity Treasury issues ending February 1996 is 6.55 percent (Massachusetts Workers' Compensation Rate Filing [23]). The value for the period ending October 1994 was 7.03 percent. Thus, the base case value is seven percent. The range of four to ten percent was selected judgmentally.

F. Application of Techniques to the Representative Company

Exhibits 3 through 8 document the application of each of the six financial pricing techniques to the representative company described above and in Exhibit 10. Each exhibit shows, for the respective pricing model, the relevant parameter values and the indicated base case UPM. In addition, the sensitivity of the UPM indications to different premium/equity ratios is shown both numerically and graphically. For three of the models, an additional parameter is also varied in the graphs. For each model, UPM is an inverse function of premium/equity ratio.

G. Sensitivity Tests

For each pricing model, the sensitivity of UPM indications to each relevant parameter is determined by allowing the parameter to vary from the base case to the low and high ends of the reasonable range, keeping all other parameters constant at their base case values. The results of these sensitivity tests are summarized in Exhibit 1. In addition, Exhibit 2 summarizes the sensitivity of UPM indications to simultaneous changes in groups of variables. Relevant variables in each of the three categories (company, economic, and government policy) are varied while keeping the other groups constant; in addition, all relevant variables across all groups are varied simultaneously.

4. EMPIRICAL RESULTS AND ANALYSIS

A. Evaluation of Base Case Results

The six base case underwriting profit margin indications displayed on Exhibit 1 range from −4.9 percent to 1.7 percent. The Insurance CAPM, which produces the lowest value, is known to under-price, as it ignores insurance-specific risk (Ang and Lai [4] and Turner [33]). (This deficiency also applies to the Arbitrage Pricing model, at least in the form presented in this paper.) The next lowest value, the Target Total Rate of Return, does not consider the effect of taxes. The Discounted Cash Flow, Internal Rate of Return and Option Pricing models all cluster between 0.1 percent and 1.7 percent. A good case can be made for selecting the average of these three models, 0.7 percent, as the underwriting profit margin target for this insurer under base case economic and company conditions. Coincidentally, this is the same value that the Target Total Rate of Return model indicates for an equity beta of 1.7, which would produce a target rate of return, before taxes, of 18.6 percent. Note that this underwriting profit margin, and in fact the entire range of base case values, is significantly below the Target UPM model provision of five percent.

B. Sensitivity Analysis

B.1. Single Factor Variation

By examining the effect of changing each parameter over the range of reasonable values, the sensitivity of the pricing models to different conditions, as well as the importance of accurately measuring each variable, can be discerned. Examination of Exhibit 1 suggests that the variables most affecting the results are the equity, or premium-to-surplus ratio (for all except the Insurance CAPM and Arbitrage Pricing models), and the risk-free rate (for all except the Discounted Cash Flow model). For example, over the range of equity values examined, the underwriting profit margin changes by 14.6 percentage points for the Target Total Rate of Return model, 4.9 percentage points for the Discounted Cash Flow model, 12.7 percentage points for the Internal Rate of Return model, and 4.8 percentage points for the Option Pricing model. In each case, the higher the initial equity, the higher the indicated underwriting profit margin; the greater the amount that the insurance company has invested for a given volume of premium, the higher the price needs to be to provide an adequate return on capital. This volatility is a problem for the financial pricing models since it is difficult to measure true equity for a single-line, single-state insurer. The problem is vastly more complicated for a multiline, multistate insurer. The effect of varying the equity is illustrated in Exhibit 1, Part 2, which compares UPM indications across all models under a range of premium/equity ratio assumptions. In the graph, the low premium/equity ratio corresponds with a high equity value.

For an insurer that had the highest amount of equity, the indicated underwriting profit margins are 7.5 percent for the Target Total Rate of Return, −3.2 percent for the Insurance CAPM, 3.8 percent for the Discounted Cash Flow, 11.0 percent for the Internal Rate of Return, 3.2 percent for the Option Pricing model, and −1.2 percent for the Arbitrage Pricing model. Determining the appropriate underwriting profit margin target in this situation is difficult. Even ignoring the Insurance CAPM and Arbitrage Pricing models, the results vary by 7.8 percentage points, with an average of 6.4 percent. Selecting the appropriate value will be a difficult judgment call.

Similarly, the results are highly sensitive to the risk-free rate: across the range of reasonable parameter values, the results vary by 9.1 percentage points for the Target Total Rate of Return model, 11.6 percentage points for the Insurance CAPM and Arbitrage Pricing models, 0.8 percentage points for the Discounted Cash Flow model, 12.6 percentage points for the Internal Rate of Return model, and 11.0 percentage points for the Option Pricing model. Also, the effect of increasing the risk-free rate affects the results differently. For the Target Total Rate of Return and the Internal Rate of Return models, increasing the risk-free rate raises the indicated underwriting profit margin (given the model assumptions previously described). However, the indications for the Insurance CAPM, the Discounted Cash Flow, the Option Pricing, and the Arbitrage Pricing models all decline as the risk-free rate increases. Thus, if the risk-free rate were 14.7 percent, as it was in 1981, then the indicated underwriting profit margins vary from −14.5 percent for the Insurance CAPM model to 11.4 percent for the Internal Rate of Return model. Fortunately, the value of the risk-free rate is easy to obtain. Unfortunately, the models are very sensitive, in opposite directions, to this value.

On the opposite extreme, the results are not very sensitive, for example, to the tax discount factor. Since private passenger auto losses are paid relatively quickly, the effect of the tax discount factor is minimal. This would not be the case for a longer-tailed line. Also, neither the investment rate of return, the market risk premium nor the standard deviation of market returns affect the results of the Option Pricing model. This occurs because the base case underwriting beta is zero. In some respects, the choice of zero as a underwriting beta is not an ideal choice because the sensitivity analysis does not illustrate the effect of changing values that are multiplied by beta. However, all empirical analyses measuring the underwriting beta indicate that it is not significantly different from zero; taken in aggregate, zero is the best a priori estimate of the underwriting beta. (See, for example, Cummins and Harrington [10], Fairley [16], and Hill and Modigliani [19].) The effect of altering more than one variable at a time is discussed next.

B.2. Multiple Factor Variation

A point of critical importance in evaluating the results summarized on Exhibit 1 is that each of the high/low UPM indications reflects a change in only one parameter, while all of the others are kept fixed. In many cases, such a simplistic scenario will understate the potential sensitivity of a model to changes in a parameter. Often, this is because the selection of the base case value of another parameter minimizes the parameter's impact on a model's UPM calculation. For example, changes in the market risk premium do not influence the UPM indications for the Option Pricing Model, according to Exhibit 1—however, this is because we have selected an investment-claim correlation of zero for our base case assumption. This assumption, when incorporated into the option formulae, "zeroes out," or makes irrelevant, the market risk parameter.

Because of examples like this, in order to determine the true magnitude of potential impact of certain parameters on the various models, it is necessary and instructive to vary more than one of the parameters simultaneously. This is in keeping with reality, as it is entirely possible—and even probable—that multiple parameters will change concurrently.

The impact of selected multi-parameter changes on UPM indications for various models is demonstrated in Exhibit 2. The results vary markedly. Changing all the Company Variables from the lowest values to the highest values only changes the indicated underwriting profit margin by 9.1 percentage points for the Option Pricing model, 4.8 percentage points for the Discounted Cash Flow model and 5.1 percentage points for the Insurance CAPM and Arbitrage Pricing models. At the other extreme, the same changes shift the indicated underwriting profit margin by 34.5 percentage points for the Internal Rate of Return model and 46.8 percentage points for the Target Total Rate of Return model. Thus, the impact of different financial positions for companies will differ depending on the model.

The effect of changing the Economic Variables affects all the models significantly. The effect ranges from 12.1 percentage points for the Target Total Rate of Return model to 61.9 percentage points for the Option Pricing model. Changing economic conditions will affect all insurance companies and must be reflected in the parameters selected.

The impact of changing the Government Policy Variables, which all relate to taxation, is not as significant as the Company or Economic Variables. The indicated underwriting profit margins shift by only 2.6 percentage points for the Insurance CAPM and Arbitrage Pricing models, 5.6 percentage points for the Internal Rate of Return model, 5.7 percentage points for the Option Pricing model, and 6.1 percentage points for the Discounted Cash Flow model. They have no effect on the Target Total Rate of Return model, which ignores taxation.

Predictably, based on the above results, the impact of changing *all* variables simultaneously is extremely significant for each model.

5. PRACTICAL GUIDE TO USING FINANCIAL PRICING MODELS

When using a variety of financial pricing models to select an appropriate underwriting profit margin, the different models can be expected to generate different indications. Selecting the appropriate profit margin requires actuarial judgment, including a thorough understanding of the reliability of the inputs used in the models and the strengths and weaknesses of the different techniques. For example, if current interest rates are high, but there is considerable uncertainty about future levels, then the models that are least sensitive to changes in the risk-free interest rate (in this example the discounted cash flow model) should be given greater weight. If the value of the insurer's equity cannot be easily valued, such as in the case of a mutual insurer, then the models that are very sensitive to the initial equity value (in this case the Target Total Rate of Return and the Internal Rate of Return) should be given less weight than they would if the initial equity could be valued more accurately. When writing a line of business that is considered to have little insurance-specific risk (for example, fidelity), then models that ignore this factor, the Insurance CAPM or Arbitrage Pricing Model (as formatted in this study), would be more appropriate than they would be for lines with a high degree of insurance-specific risk (for example, homeowners).

For this study, all of the models are tested based on the same input data. These values were intentionally selected to be representative of the property/casualty insurance industry in general and of current economic conditions. Thus, the different indications result from differences in the basic structure of the individual models, and the effect of these differences for a representative insurance company can be quantified. Specifically, the Target Total Rate of Return model, which ignores taxes, produces an indication three to four percentage points below the level where other models tend to cluster. The Insurance CAPM, which ignores insurance-specific risk, produces a value approximately five percentage points below the cluster level, and the Arbitrage Pricing Model, which in the form used here also ignores insurance-specific risk, produces an indication about three percentage points lower than the cluster level. These differences can provide some guidance about adjustments that can be made to reflect omitted factors. However, the differences will need to be recalculated when applied in situations that differ from the example provided in this study, if, for example, the type of business or the financial parameters were to change.

Knowledge about the assumptions inherent in the models is also important for proper application. For example, the Internal Rate of Return model assumes that any underwriting losses are funded at the inception of the policy. Conversely, underwriting profits would also be reflected at the inception of the policy, reducing the initial surplus allocation for the policy. Although this assumption would have little effect when the indicated underwriting profit margins are approximately zero, it could introduce distortions in the case of sizeable underwriting profits or losses. Also, the Discounted Cash Flow model relies heavily on the appropriate risk-adjusted discount factor, a value that is difficult to measure given the lack of a public and liquid market trading insurance liabilities. Care must be taken, when using this model, that the risk-adjusted discount rate reasonably accounts for the risk involved in writing a particular line of business.

Selecting an appropriate underwriting profit margin is as much of an actuarial art as selecting the appropriate loss reserve level. The financial pricing models described in this study, although none is perfect, can be used to determine this value if properly applied and if used in conjunction with other models. Knowing the likely relationships among the respective indications and the sensitivity of the indications to specific parameters can help direct the user to select reasonable underwriting profit margins.

6. CONCLUSION

A. Implications for Insurance Companies and Regulators

The diverse results for the indicated underwriting profit margin, depending on the pricing model selected and the nature of the company and economic environment, should be convincing evidence that no one pricing model can be relied upon to provide the appropriate underwriting profit margin in all situations. Instead, insurers should apply a number of different pricing methods and evaluate the results in combination to select the target underwriting profit margin. The models discussed in this paper are possible techniques for companies to use. However, each model should be understood and its shortcomings noted in order to apply the techniques most appropriately.

Another conclusion that can be drawn from this research is that insurance is a very complex financial transaction. For people working in the insurance industry, this may seem to be an unusual statement. An insurance company collects premiums and pays losses and expenses: what is so complex about that? However, compared to stocks, bonds and options, for which the financial pricing models were originally developed, insurance is very complex. Owners of stock receive a periodic stream of dividends and the value of the stock when it is sold. Bondholders receive a fixed stream of income and a predetermined principal amount at maturity, subject only to default risk or an early call. Option holders receive at maturity the difference between the price of the underlying asset and the exercise price, if this is greater than zero.

Insurance, on the other hand, involves collecting a stream of premium payments over time in return for the promise to pay losses, if they occur, for which both the amount and timing are unknown. The mathematics for dealing with this degree of uncertainly have not been perfected. This complexity means that techniques that are readily applicable to other financial transactions are not necessarily going to provide reasonable results when applied to insurance. Thus, regulating insurance involves more than importing a financial technique that has been applied in another setting.

B. Future Research

Our goal in this paper has been to compare and contrast different asset pricing models in terms of their indicated fair rates of return for property/casualty insurance policies given various corporate and economic environments. We have focused on those pricing models that have been suggested and (at least somewhat) developed in the literature to date. Several of the models would benefit from more extensive development—perhaps the findings of this paper will help to suggest where such resources are best applied.

To date, the Arbitrage Pricing Model has received relatively little theoretical or empirical attention in terms of insurance applications. We have used a very basic pre-specified factors model for purposes of illustrative model comparisons in this paper. Two macroeconomic variables—inflation and industrial production growth rates—were found to be relatively significant in explaining adjusted (for investment income) UPMs over the 1948–1993 period (the other variables tested were a bond default premium, a bond horizon premium, and a New York Stock Exchange value-weighted stock return series). The positive relationship we found between inflation and adjusted UPM is interesting in light of the finding in Kraus and Ross [21] that the competitive premium should be affected by inflation only in so far as real rates of interest are impacted. This relationship should be analyzed further, perhaps by separately determining the sensitivity of UPMs to expected and unexpected inflation. Other insurance specific variables—e.g., catastrophe losses, leverage—should also be examined for significance. In addition, historical tax rates may well have an impact on historical UPM regressions with the pre-specified variables, and should be incorporated into the process of determining beta coefficients. Also, instead of assuming the relevancy of specific macroeconomic and insurance variables, a factor analysis approach might be worth investigating. We intend to examine these and other issues in a separate paper.

There are several other areas in which additional research might prove fruitful. For example, the Option Pricing Model requires distributional and risk preference assumptions in order to price the contingent claims. It would be instructive to examine the impact on OPM pricing indications of assuming return and loss distributions other than the normal and lognormal distributions. Another area involves surplus allocation: for practical applications of these models, a multiline and/or multistate insurer must be able to appropriately allocate surplus to its various business segments. Finally, additional research into appropriate parameter values for each model is certainly warranted before the models are actually used for insurance pricing purposes.

REFERENCES

[1] A. M. Best Company, *Best's Aggregates and Averages*, New Jersey, 1994 and prior.

[2] A. M. Best Company, *Best's Insurance Reports: Property-Casualty*, New Jersey, 1994.

[3] A. M. Best Company, *Best's Key Rating Guide: Property-Casualty*, New Jersey, 1994.

[4] Ang, James S., and Tsong-Yue Lai, "Insurance Premium Pricing and Ratemaking in Competitive Insurance and Capital Markets," *Journal of Risk and Insurance*, 54, 1987, pp. 767–779.

[5] Biger, Nahum, and Yehuda Kahane, "Risk Considerations in Insurance Ratemaking," *Journal of Risk and Insurance*, 45, 1978, pp. 121–132.

[6] Butsic, Robert P. "Determining the Proper Interest Rate for Loss Reserve Discounting: An Economic Approach," *Evaluating Insurance Company Liabilities*, 1988 Casualty Actuarial Society Discussion Paper Program, pp. 147–188.

[7] Chen, Nai-Fu, Richard Roll, and Stephen A. Ross, "Economic Forces and the Stock Market," *Journal of Business*, 59, 1986, pp. 383–403.

[8] Cummins, J. David, "Risk-Based Premiums for Insurance Guaranty Funds," *Journal of Finance*, 43, 1988, pp. 823–839.

[9] Cummins, J. David, "Multi-Period Discounted Cash Flow Ratemaking Models in Property-Liability Insurance," *Journal of Risk and Insurance*, 57, 1990, pp. 79–109.

[10] Cummins, J. David, and Scott E. Harrington, "Property-Liability Insurance Rate Regulation: Estimation of Underwriting Betas Using Quarterly Profit Data," *Journal of Risk and Insurance*, 52, 1985, pp. 16–43.

[11] Cummins, J. David, and Scott E. Harrington, *Fair Rate of Return in Property-Liability Insurance*, Boston: Kluwer-Nijhoff Publishing Company, 1987.

[12] D'Arcy, Stephen P., "Use of the CAPM to Discount Property-Liability Loss Reserves," *Journal of Risk and Insurance*, 55, 1988, pp. 481–491.

[13] D'Arcy, Stephen P., and Michael A. Dyer, "Ratemaking: A Financial Economics Approach," *PCAS* LXXXIV, 1997, pp. 301–310.

[14] D'Arcy, Stephen P., and James R. Garven, "Property-Liability Insurance Pricing Models: An Empirical Evaluation," *Journal of Risk and Insurance*, 57, 1990, pp. 391–430.

[15] Doherty, Neil A., and James R. Garven, "Price Regulation in Property-Liability Insurance: A Contingent Claims Approach," *Journal of Finance*, 41, 1986, pp. 1031–1050.

[16] Fairley, William B., "Investment Income and Profit Margins in Property-Liability Insurance: Theory and Empirical Results," *Bell Journal of Economics*, 10, 1979, pp. 192–210.

[17] Fama, Eugene F., and Kenneth R. French, "The Cross-Section of Expected Stock Returns," *Journal of Finance*, 47, 1992, pp. 427–465.

[18] Hill, Raymond D., "Profit Regulation in Property Liability Insurance," *Bell Journal of Economics*, 10, 1979, pp. 172–191.

[19] Hill, Raymond D., and Franco Modigliani, "The Massachusetts Model of Profit Regulation in Non-Life Insurance: An Appraisal and Extensions," Chapter 2 in Cummins and Harrington, 1987.

[20] Ibbotson Associates, *Stocks, Bonds, Bills, and Inflation: 1996 Yearbook*, Ibbotson Associates, 1996.

[21] Kraus, Alan, and Stephen A. Ross, "The Determination of Fair Profits for the Property-Liability Insurance Firm," *Journal of Finance*, 37, 1987, pp. 1015–1028.

[22] Lintner, John, "The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets," *Review of Economics and Statistics*, 47, 1965, pp. 13–37.

[23] Massachusetts Workers Compensation Rate Filing, 1996.

[24] Mossin, Jan, "Equilibrium in a Capital Market," *Econometrica*, 34, 1966, pp. 768–783.

[25] Myers, Stewart C., and Richard Cohn, "A Discounted Cash Flow Approach to Property-Liability Insurance Rate Regulation," Chapter 3 in Cummins and Harrington, 1987.

[26] National Association of Insurance Commissioners, Report to the Special Subcommittee of the Fire and Marine Committee by Roy C. McCullough, *Proceedings of the National Association of Insurance Commissioners*, Seventy-Ninth Session, 1948, pp. 74–157.

[27] National Association of Insurance Commissioners, "Measurement of Profitability and Treatment of Investment Income in Property and Liability Insurance," *Proceedings of the National Association of Insurance Commissioners*, 1970 Annual Meeting, IIA, 1970, pp. 719–893.

[28] National Convention of Insurance Commissioners, Report of the Fire Insurance Committee, *Proceedings of the National Convention of Insurance Commissioners*, 1922, pp. 20–23.

[29] Roll, Richard, "A Critique of the Asset Pricing Theory's Tests," *Journal of Financial Economics*, 4, 1977, pp. 129–176

[30] Roll, Richard, and Stephen A. Ross, "An Empirical Investigation of the Arbitrage Pricing Theory," *Journal of Finance*, 35, 1980, pp. 1073–1103.

[31] Ross, Stephen A., "The Arbitrage Theory of Capital Asset Pricing," *Journal of Economic Theory*, 13, 1976, pp. 341–360.

[32] Sharpe, William F., "Capital Asset Prices: A Theory of Market Equilibrium Under Conditions of Risk," *Journal of Finance*, 19, 1964, pp. 425–442.

[33] Turner, Andrew, "Insurance in an Equilibrium Asset-Pricing Model," Chapter 4 in Cummins and Harrington, 1987.

[34] Urrutia, Jorge L., "Determination of Competitive Underwriting Profit Margins Using the Arbitrage Pricing Model," *Journal of Insurance Issues and Practice*, 10, 1987, pp. 61–77.

[35] Webb, Bernard L., "Investment Income in Insurance Ratemaking," *Journal of Insurance Regulation*, 1, 1982, pp. 46–76.

EXHIBIT 1

PART 1

INDICATED UNDERWRITING PROFIT MARGINS BASED ON RANGE OF REASONABLENESS FOR EACH VARIABLE

Variable	Base Case	Range	Target Total Rate of Return		Insurance CAPM		Discounted Cash Flow		Internal Rate of Return		Option Pricing Normal/ CARA		Arbitrage Pricing	
Base Case Results			−3.6%		−4.9%		0.1%		1.7%		0.2%		−2.9%	
			Low	High	Low	High	Low	High	Low	High	Low	High	Low	High
Equity	189,360	122,132 to 399,692	−7.1%	7.5%	−5.5%	−3.2%	−1.1%	3.8%	−1.7%	11.0%	−1.6%	3.2%	−3.5%	−1.2%
Investment Rate of Return	8.0%	6.0% to 10.0%	−0.2%	−7.0%					5.5%	−2.5%	0.2%	0.2%		
SD of Investment Returns	20.0%	10.0% to 30.0%									−1.2%	0.2%		
Equity Beta (also for IRR)	1.00	0.60 to 1.70	−6.0%	0.7%					−2.8%	7.8%				
Funds Generating Coefficient	1.18	0.93 to 1.43			−3.5%	−6.3%	0.3%	0.0%	1.5%	1.9%	1.1%	−0.8%	−1.5%	−4.3%
Risk-free Rate	5.0%	2.9% to 14.7%	−5.2%	3.9%	−2.9%	−14.5%	0.1%	−0.7%	−1.2%	11.4%	2.2%	−8.8%	−0.9%	−12.5%
Market Risk Premium	8.0%	6.0% to 10.0%	−5.1%	−2.0%	−4.9%	−4.9%			−1.0%	4.1%	0.2%	0.2%		
Risk-Adj/Risk-Free Rate Ratio	60.0%	0.0% to 100.0%					4.4%	−2.6%	6.2%	−7.3%				
Underwriting Beta & Inv-Cl Cor	0.00	−0.40 to +0.40			−8.1%	−1.7%					6.2%	−7.3%		
SD Market Returns	22.0%	12.0% to 40.0%									0.2%	0.2%		
SD of Losses	48,401	24,201 to 96,802									−0.3%	1.1%		
CPI Change	3.0%	0.0% to 6.0%											−4.4%	−1.4%
CPI Beta	0.50	0.00 to 1.00											−4.4%	−1.4%
Industrial Production Growth	2.0%	0.0% to 4.0%											−3.4%	−2.4%
Industrial Production Beta	0.25	0.00 to 0.50											−3.4%	−2.4%
Tax Rate (Total)	34.0%	28.0% to 40.0%			−5.2%	−4.6%	−0.8%	1.4%	0.5%	3.0%	−1.1%	1.6%	−3.2%	−2.6%
Investment/Total Tax Ratio	80.0%	60.0% to 100.0%			−5.9%	−3.9%	−1.5%	2.0%	0.2%	3.2%	−1.3%	1.7%	−3.9%	−1.9%
Tax Discount Factor	7.0%	4.0% to 10.0%					0.1%	0.2%	1.6%	1.8%				

Note: Each UPM indication represents a change in the relevant variable, with all other variables kept constant at base case values.

EXHIBIT 1

PART 2

SUMMARY OF MODEL INDICATIONS
SENSITIVITY OF UNDERWRITING PROFIT MARGINS TO PREMIUM/EQUITY RATIOS
(All other variables held constant at base case values)

	Premium / Equity Ratio									
Model	0.50	0.70	0.90	1.10	1.30	1.50	1.70	1.90	2.10	2.30
Target UPM	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.050
Target Total Rate of Return	0.124	0.050	0.009	−0.017	−0.036	−0.049	−0.059	−0.067	−0.074	−0.079
Insurance CAPM	−0.024	−0.036	−0.042	−0.046	−0.049	−0.051	−0.053	−0.054	−0.055	−0.056
Discounted Cash Flow	0.061	0.034	0.019	0.009	0.002	−0.002	−0.006	−0.009	−0.011	−0.013
Internal Rate of Return	0.193	0.113	0.069	0.041	0.021	0.007	−0.004	−0.012	−0.019	−0.025
Option Pricing—Normal/CARA	0.048	0.028	0.017	0.009	0.003	−0.003	−0.008	−0.013	−0.018	−0.023
Arbitrage Pricing	−0.004	−0.016	−0.022	−0.026	−0.029	−0.031	−0.033	−0.034	−0.035	−0.036

EXHIBIT 1

PART 2—PAGE 2

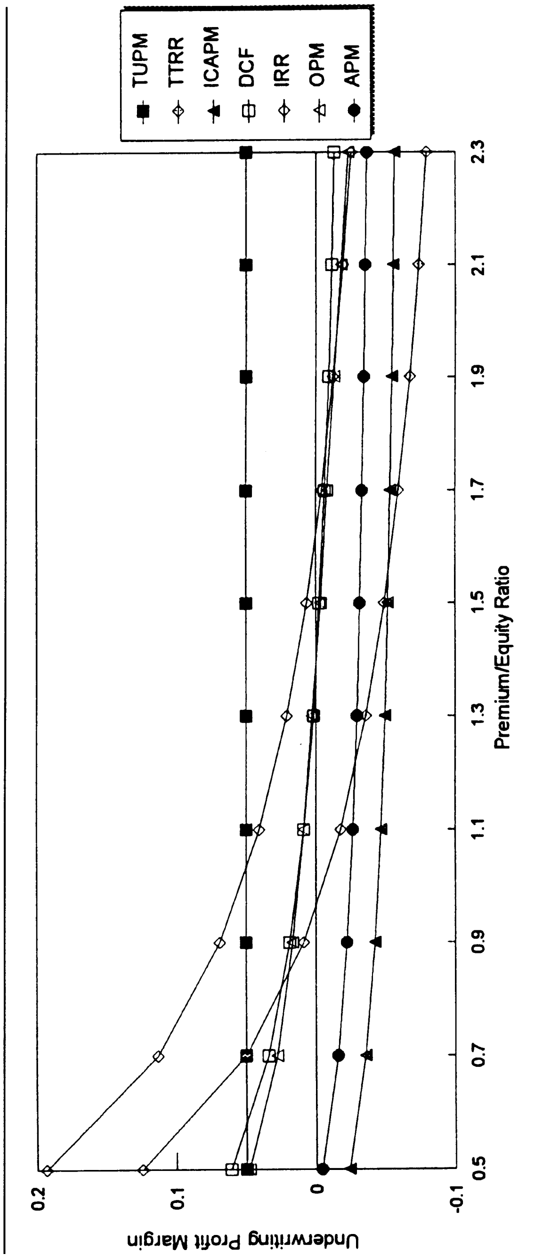

EXHIBIT 2

INDICATED UNDERWRITING PROFIT MARGINS BASED ON COMBINED RANGE EFFECTS OF GROUPS OF VARIABLES

Variable	Target Total Rate of Return		Insurance CAPM		Discounted Cash Flow		Internal Rate of Return		Option Pricing Normal/CARA		Arbitrage Pricing	
Base Case Results	−3.6%		−4.9%		0.1%		1.7%		0.2%		−2.9%	
	Low	High	Low	High	Low	High	Low	High	Low	High	Low	High
Company Variables—Can be Influenced by the Company	−21.4%	25.4%	−6.9%	−1.8%	−1.4%	3.4%	−7.1%	27.4%	−2.9%	6.2%	−4.9%	0.2%
Economic Variables—Determined by Economic Conditions	−6.7%	5.4%	−18.5%	−0.5%	−7.9%	11.2%	−4.2%	12.9%	−26.4%	35.5%	−14.5%	5.1%
Government Policy Variables—Tax-Related			−5.9%	−3.3%	−2.0%	4.1%	−0.7%	4.9%	−2.2%	3.5%	−3.9%	−1.3%
All Variables Combined	−23.1%	46.6%	−26.6%	4.4%	−19.5%	31.9%	−12.1%	43.2%	−47.0%	40.4%	−22.6%	8.4%

Note: Each UPM indication represents a change in each of the relevant groups of variables, with all variables in the other groups kept constant at base case values.

Company Variables:	Equity Investment Rate of Return SD of Investment Returns Equity Beta (also for IRR) Funds Generating Coefficient	Economic Variables:	Risk-free Rate Market Risk Premium Risk-Adj./Risk-Free Rate Ratio Underwriting Beta & Inv-Cl Cor SD Market Returns SD of Losses CPI Change CPI Beta Industrial Production Growth Industrial Production Beta
Gov't. Policy Variables:	Investment/Total Tax Ratio Investment/Total Tax Ratio Tax Discount Factor		

EXHIBIT 3

Target Total Rate of Return Model
Sensitivity of Underwriting Profit Margins to Premium/Equity Ratios and Equity Betas

Risk-free rate	5.0%		Base Case UPM:		−0.036					
Expected market return	13.0%									
Investment rate of return	8.0%									
Invested assets	417,338									
Equity	189,360									
UPMs:										
Premium/equity ratio:	0.50	0.70	0.90	1.10	1.30	1.50	1.70	1.90	2.10	2.30
Equity beta = 1.70	0.236	0.130	0.071	0.033	0.007	−0.012	−0.026	−0.038	−0.047	−0.055
Equity beta = 1.40	0.188	0.096	0.044	0.012	−0.011	−0.028	−0.040	−0.050	−0.058	−0.065
Equity beta = 1.20	0.156	0.073	0.027	−0.003	−0.023	−0.038	−0.050	−0.059	−0.066	−0.072
Equity beta = 1.00	0.124	0.050	0.009	−0.017	−0.036	−0.049	−0.059	−0.067	−0.074	−0.079
Equity beta = 0.80	0.092	0.027	−0.009	−0.032	−0.048	−0.060	−0.069	−0.076	−0.081	−0.086
Equity beta = 0.60	0.060	0.004	−0.027	−0.047	−0.060	−0.070	−0.078	−0.084	−0.089	−0.093

EXHIBIT 3

PART 2

TARGET TOTAL RATE OF RETURN MODEL

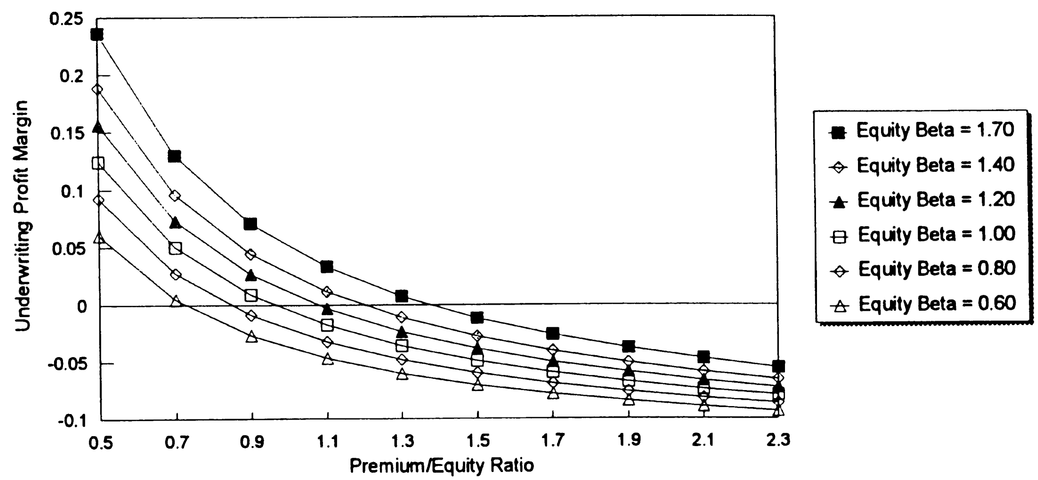

EXHIBIT 4

INSURANCE CAPITAL ASSET PRICING MODEL
SENSITIVITY OF UNDERWRITING PROFIT MARGINS TO PREMIUM/EQUITY RATIOS AND UNDERWRITING BETAS

Risk-free rate	5.0%			Base Case UPM:	−0.049					
Expected market return	13.0%									
Funds generating coeff.	1.18									
Tax rate on investment inc.	27.2%									
Tax rate on U/W income	34.0%									
UPMs:										
Premium/equity ratio:	0.50	0.70	0.90	1.10	1.30	1.50	1.70	1.90	2.10	2.30
U/W beta = 0.40	0.008	−0.004	−0.010	−0.014	−0.017	−0.019	−0.021	−0.022	−0.023	−0.024
U/W beta = 0.20	−0.008	−0.020	−0.026	−0.030	−0.033	−0.035	−0.037	−0.038	−0.039	−0.040
U/W beta = 0	−0.024	−0.036	−0.042	−0.046	−0.049	−0.051	−0.053	−0.054	−0.055	−0.056
U/W beta = −0.20	−0.040	−0.052	−0.058	−0.062	−0.065	−0.067	−0.069	−0.070	−0.071	−0.072
U/W beta = −0.40	−0.056	−0.068	−0.074	−0.078	−0.081	−0.083	−0.085	−0.086	−0.087	−0.088

EXHIBIT 4

PART 2

INSURANCE CAPITAL ASSET PRICING MODEL

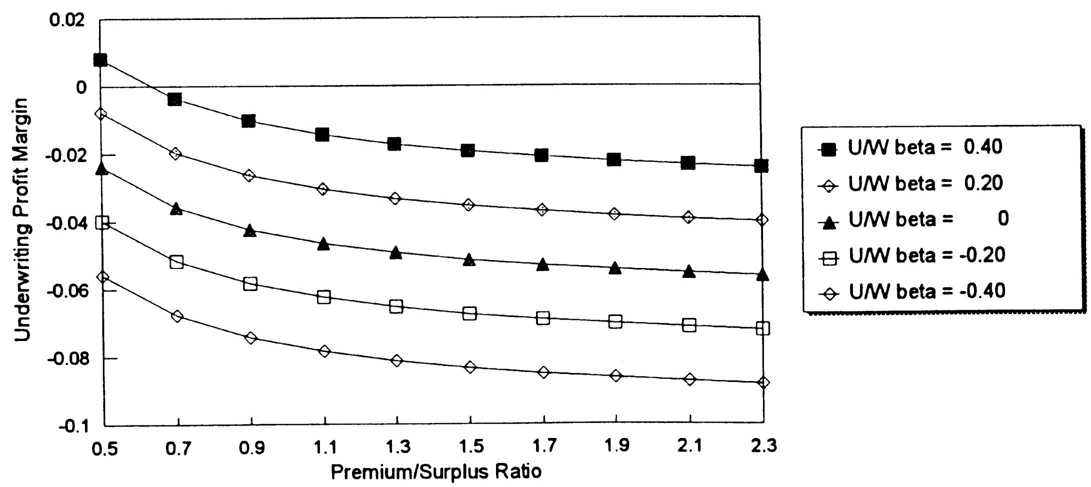

EXHIBIT 5

DISCOUNTED CASH FLOW MODEL
SENSITIVITY OF UNDERWRITING PROFIT MARGINS TO PREMIUM/EQUITY RATIOS AND RISK-ADJUSTED DISCOUNT RATES

Risk-free rate	5.0%	Indicated Premium:		253,040						
Risk-adj. discount rate	3.0%	Premium/Equity Ratio:		1.34						
Tax discount rate	7.0%	Base Case UPM:		0.001						
Tax rate on U/W income	34.0%									
Tax rate on investment inc.	27.2%									
Funds generating coeff.	1.18									
Expected losses	193,605									
Expenses	59,062									
Equity	189,360									
UPMs:										
Premium/equity ratio:	0.50	0.70	0.90	1.10	1.30	1.50	1.70	1.90	2.10	2.30
RADR = 0%	0.102	0.076	0.062	0.053	0.046	0.042	0.038	0.035	0.033	0.031
RADR = 3%	0.061	0.034	0.019	0.009	0.002	−0.002	−0.006	−0.009	−0.011	−0.013
RADR = 6%	0.021	−0.007	−0.023	−0.033	−0.039	−0.045	−0.048	−0.051	−0.054	−0.056

EXHIBIT 5

PART 2

DISCOUNTED CASH FLOW MODEL

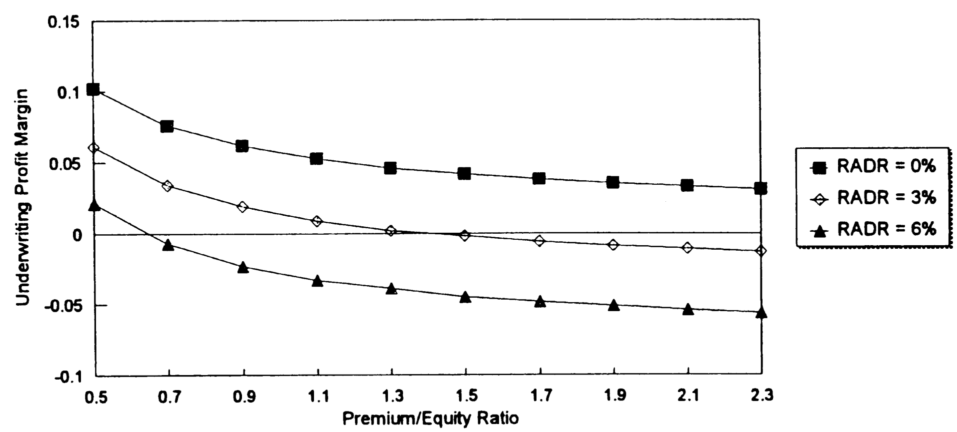

EXHIBIT 6

PART 1

INTERNAL RATE OF RETURN MODEL
(Based on Cummins)
SENSITIVITY OF UNDERWRITING PROFIT MARGIN TO PREMIUM/EQUITY RATIO

Tax Discount Rate:	0.07									
Investment Return Rate:	0.08		Base Case UPM:		0.017					
Tax Rate:	0.34									
Investment Tax Rate:	0.27									
Equity:	189,360									
Expected Losses:	193,605									
Equity/Reserves:	0.98									
IRR:	0.1300									
Premium/Equity Ratio:	0.50	0.70	0.90	1.10	1.30	1.50	1.70	1.90	2.10	2.30
UPM:	0.193	0.113	0.069	0.041	0.021	0.007	−0.004	−0.012	−0.019	−0.025

EXHIBIT 6

Part 1—Page 2

Internal Rate of Return Model

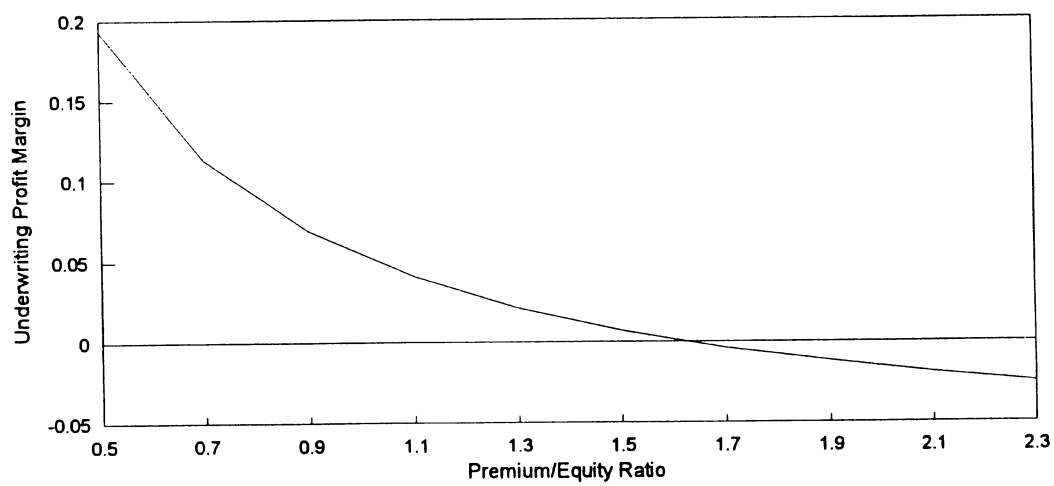

EXHIBIT 6

PART 2

INTERNAL RATE OF RETURN MODEL
(Based on Cummins)

CALCULATION OF DISCOUNTED NET CASH FLOWS FOR BASE CASE

QTR	Prems. (1)	Exps. (2)	Expected Loss (3)	Fed Tax Flow (4)	Total U/W Flow (5)	Accum U/W Acct (6)	Loss Reserve (7)	Supt. Surplus (8)	Avg. Surplus (9)	Invest. on Surp (10)	Income on U/W (11)	Surplus Flow (12)	Net Cash Flow (13)	IRR Discount Factor (14)	Disc. NCF (15)
					(2,866)			189,360				(186,494)	(186,494)	1.0000	(186,494)
1	257,009	(59,062)	(6,389)	(1,119)	190,439	92,354	42,012	183,111	186,236	3,725	1,847	6,249	10,305	0.9848	10,149
2	0	0	(19,361)	(1,119)	(20,480)	177,333	71,053	164,175	173,643	3,473	3,547	18,936	24,046	0.9552	22,969
3	0	0	(32,138)	(1,119)	(33,258)	150,464	87,316	132,741	148,458	2,969	3,009	31,434	35,786	0.9265	33,154
4	0	0	(44,916)	(1,119)	(46,036)	110,818	90,801	88,810	110,776	2,216	2,216	43,932	47,158	0.8986	42,375
5	0	0	(11,665)	351	(11,314)	82,143	79,136	77,401	83,105	1,662	1,643	11,409	13,815	0.8715	12,040
6	0	0	(11,665)	351	(11,314)	70,829	67,471	65,992	71,696	1,434	1,417	11,409	13,484	0.8453	11,398
7	0	0	(11,665)	351	(11,314)	59,515	55,807	54,583	60,287	1,206	1,190	11,409	13,153	0.8199	10,784
8	0	0	(11,665)	351	(11,314)	48,200	44,142	43,174	48,879	978	964	11,409	12,822	0.7952	10,196
12	0	0	(20,329)	701	(19,628)	32,729	23,813	23,291	33,233	2,659	2,618	19,883	23,724	0.7367	17,478
16	0	0	(10,842)	377	(10,465)	17,683	12,972	12,687	17,989	1,439	1,415	10,604	12,682	0.6520	8,268
20	0	0	(5,808)	204	(5,604)	9,649	7,163	7,006	9,847	788	772	5,681	6,816	0.5770	3,933
24	0	0	(2,904)	114	(2,790)	5,452	4,259	4,166	5,586	447	436	2,840	3,483	0.5106	1,779
28	0	0	(1,742)	67	(1,676)	3,220	2,517	2,462	3,314	265	258	1,704	2,085	0.4518	942

EXHIBIT 6

PART 2—PAGE 2

QTR	Prems. (1)	Exps. (2)	Expected Loss (3)	Fed Tax Flow (4)	Total U/W Flow (5)	Accum U/W Acct (6)	Loss Reserve (7)	Supt. Surplus (8)	Avg. Surplus (9)	Invest. on Surp (10)	Income on U/W (11)	Surplus Flow (12)	Net Cash Flow (13)	IRR Discount Factor (14)	Disc. NCF (15)
32	0	0	(968)	39	(929)	1,917	1,549	1,515	1,988	159	153	947	1,174	0.3999	470
36	0	0	(387)	26	(362)	1,272	1,162	1,136	1,326	106	102	379	530	0.3539	188
40	0	0	(194)	20	(173)	1,005	968	947	1,041	83	80	189	309	0.3132	97
44	0	0	(194)	17	(176)	830	774	757	852	68	66	189	287	0.2771	80
48	0	0	(194)	14	(180)	652	581	568	663	53	52	189	266	0.2452	65
52	0	0	(194)	10	(183)	470	387	379	473	38	38	189	244	0.2170	53
56	0	0	(194)	6	(187)	285	194	189	284	23	23	189	223	0.1921	43
60	0	0	(194)	2	(191)	96	0	0	95	8	8	189	200	0.1700	34
64	0	0	0	0	0	0	0	0	0	0	0	0	0	0.1504	0
68	0	0	0	0	0	0	0	0	0	0	0	0	0	0.1331	0
72	0	0	0	0	0	0	0	0	0	0	0	0	0	0.1178	0
76	0	0	0	0	0	0	0	0	0	0	0	0	0	0.1042	0
80	0	0	0	0	0	0	0	0	0	0	0	0	0	0.0923	0
84	0	0	0	0	0	0	0	0	0	0	0	0	0	0.0816	0
88	0	0	0	0	0	0	0	0	0	0	0	0	0	0.0722	0
92	0	0	0	0	0	0	0	0	0	0	0	0	0	0.0639	0
96	0	0	0	0	0	0	0	0	0	0	0	0	0	0.0566	0
100	0	0	0	0	0	0	0	0	0	0	0	0	0	0.0501	0
TOT	257,009	(59,062)	(193,605)	(1,476)	2,866					23,797	21,854	2,866	222,594		0

EXHIBIT 7

OPTION PRICING MODEL—NORMAL/CONSTANT ABSOLUTE RISK AVERSION (Based on Doherty–Garven) SENSITIVITY OF UNDERWRITING PROFIT MARGIN TO PREMIUM/EQUITY RATIOS

Initial Equity S(0)	189,360	Premium P(0):	194,060
Fund Generating Coefficient k	1.50		
Std. Dev. of Investment Returns s(i)	0.20	Equity V(0):	189,360
Expected Claims Costs E[L]	193,605		
Std. Dev. of Claims Costs s(L)	48,401	V(0)–S(0):	0.000
Inv.-Claims Correlation r(iL)	0.00		
Risk-free Interest Rate r(f)	0.05		
Statutory Tax Rate t	0.34		
Tax-Adjustment Parameter h	0.80		
Beta of Inv. Portfolio B(i)	0.38	Base Case UPM:	0.002
Expected Return on Market E[r(M)]	0.13		
Std. Dev. of Market Return s(M)	0.22		

UPMs:

Premium/Equity Ratio:	0.50	0.70	0.90	1.10	1.30	1.50	1.70	1.90	2.10	2.30
r(iL) = −0.40	0.114	0.095	0.083	0.074	0.067	0.060	0.053	0.046	0.040	0.034
r(iL) = −0.20	0.083	0.063	0.051	0.043	0.036	0.030	0.024	0.018	0.012	0.007
r(iL) = 0.00	0.048	0.028	0.017	0.009	0.003	−0.003	−0.008	−0.013	−0.018	−0.023
r(iL) = 0.20	0.011	−0.010	−0.021	−0.028	−0.034	−0.038	−0.043	−0.047	−0.051	−0.055
r(iL) = 0.40	−0.030	−0.051	−0.062	−0.069	−0.074	−0.078	−0.082	−0.085	−0.088	−0.091

EXHIBIT 7

PART 2

OPTION PRICING MODEL

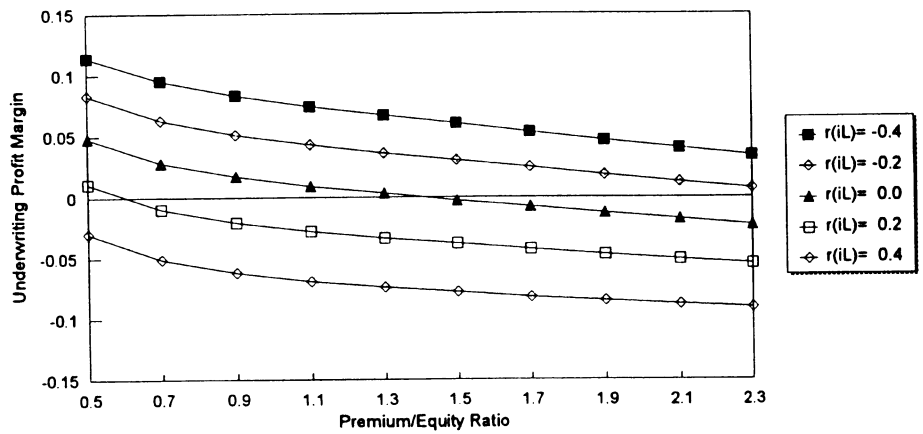

EXHIBIT 8

PART 1

ARBITRAGE PRICING MODEL
SENSITIVITY OF UNDERWRITING PROFIT MARGIN TO PREMIUM/EQUITY RATIO

Risk-Free Rate		5.0%	Base Case UPM:		−0.029					
Funds-Generating Coefficient		1.18								
Investment Tax Rate		27.2%								
Underwriting Tax Rate		34.0%								
Inflation Rate		3.0%								
Inflation Beta		0.500								
Industrial Prod. Growth Rate		2.0%								
Industrial Production Beta		0.250								
Premium/Equity Ratio:	0.5	0.7	0.9	1.1	1.3	1.5	1.7	1.9	2.1	2.3
UPM:	−0.004	−0.016	−0.022	−0.026	−0.029	−0.031	−0.033	−0.034	−0.035	−0.036

EXHIBIT 8

PART 1—PAGE 2

ARBITRAGE PRICING MODEL

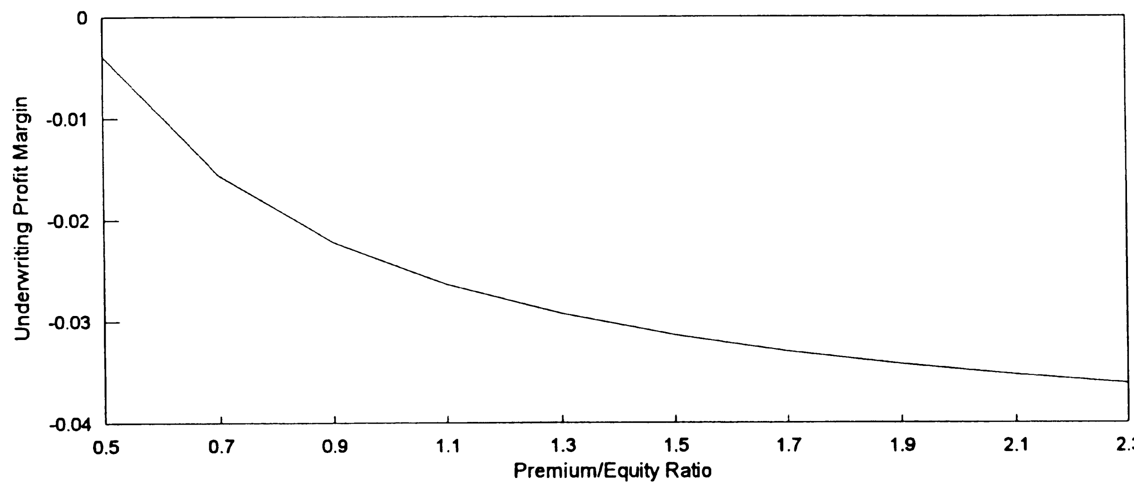

EXHIBIT 8

PART 2

ARBITRAGE PRICING MODEL
MULTIVARIATE REGRESSIONS OF PRE-SPECIFIED MACROECONOMIC FACTORS

This exhibit reports the results of multivariate regressions of pre-specified macroeconomic variables against the following dependent variable:

(Historical UPM + {(Historical Risk-Free Rate) × (Estimated Historical Funds-Generating Coefficient)}.

The regressions are performed with a specified constant of zero; a negative R-squared value indicates that the zero constant assumption is not appropriate. The time series for each of the five independent variables is taken from Ibbotson Associates [20] data, with the exception of the Change in Industrial Production, the source time series is A.M. Best's Aggregates & Averages data [1].

	CPI Change	Industrial Production Change	Bond Default Premium	Bond Horizon Premium	Value-Weighted NYSE	R-Squared
			1948–1993			
Coefficient	0.534	0.247	−0.414	−0.072	0.094	−0.000
Std. Error of Coeff.	0.152	0.117	0.263	0.082	0.039	
			1954–1993			
Coefficient	0.579	0.239	−0.283	−0.081	0.074	0.222
Std. Error of Coeff.	0.136	0.115	0.241	0.073	0.036	

EXHIBIT 8

PART 2—PAGE 2

	CPI Change	Industrial Production Change	Bond Default Premium	Bond Horizon Premium	Value-Weighted NYSE	R-Squared
			1964–1993			
Coefficient	0.715	0.236	−0.004	−0.045	−0.011	0.414
Std. Error of Coeff.	0.134	0.131	0.245	0.076	0.047	
			1974–1993			
Coefficient	0.673	0.258	−0.115	−0.090	−0.010	0.561
Std. Error of Coeff.	0.176	0.175	0.337	0.094	0.065	
			1948–1993			
Coefficient	0.683	0.242				−0.152
Std. Error of Coeff.	0.142	0.120				
			1964–1993			
Coefficient	0.710	0.245				0.395
Std. Error of Coeff.	0.109	0.125				
			1974–1993			
Coefficient	0.679	0.262				0.492
Std. Error of Coeff.	0.119	0.170				
			1984–1993			
Coefficient	0.191	0.231				0.116
Std. Error of Coeff.	0.176	0.175				

EXHIBIT 9

TOP 20 PRIVATE PASSENGER AUTO COMPANY FINANCIALS
For 1993 ($000)

Rank	Company Group	Assets	Liabilities	Surplus	NWP	Premium/ Surplus Ratio	Liabs./ Assets Ratio	Surplus/ Assets Ratio
1	State Farm	47,536,978	26,267,245	21,269,733	22,225,584	1.045	0.553	0.447
2	Allstate	27,698,530	20,553,474	7,145,056	15,801,617	2.212	0.742	0.258
3	Farmers	8,143,111	6,249,549	1,893,562	4,673,952	2.468	0.767	0.233
4	Nationwide	11,452,314	8,129,370	3,322,944	4,457,259	1.341	0.710	0.290
5	USAA	7,486,440	3,975,518	3,510,922	3,018,013	0.860	0.531	0.469
6	GEICO	3,307,701	2,477,676	830,025	1,882,237	2.268	0.749	0.251
7	Liberty Mutual	16,982,390	14,312,302	2,670,088	4,801,346	1.798	0.843	0.157
8	Prudential	2,690,053	2,045,524	644,529	1,519,302	2.357	0.760	0.240
9	Progressive	1,292,781	983,733	309,048	826,594	2.675	0.761	0.239
10	American Family	3,746,445	2,448,558	1,297,887	2,287,309	1.762	0.654	0.346
11	ITT Hartford	8,845,345	5,531,122	3,314,223	2,220,948	0.670	0.625	0.375
12	California State Auto Association	3,323,187	2,103,424	1,219,763	1,324,836	1.086	0.633	0.367
13	Travelers	10,368,761	8,122,916	2,245,845	2,238,146	0.997	0.783	0.217
14	Metropolitan	1,383,632	924,312	459,320	1,028,209	2.239	0.668	0.332
15	SAFECO	2,433,701	1,637,715	795,986	1,060,087	1.332	0.673	0.327
16	Aetna	2,809,288	2,138,362	670,926	1,097,364	1.636	0.761	0.239
17	20th Century	1,534,861	970,861	564,000	971,515	1.723	0.633	0.367
18	Erie	3,356,003	1,979,801	1,376,202	1,422,558	1.034	0.590	0.410
19	American Premier Underwriters	507,604	355,337	152,267	440,265	2.891	0.700	0.300
20	Allmerica	1,549,400	1,026,387	523,013	740,946	1.417	0.662	0.338
	TOTALS	166,448,525	112,233,186	54,215,339	74,038,087	1.366	0.674	0.326
	Average					1.690	0.690	0.310
	Standard Deviation					0.638	0.080	0.080
	Minimum					0.670	0.531	0.157
	Maximum					2.891	0.843	0.469
Per *1993 Aggregates and Averages* (1994) [1], PPA and HO Predominating:						1.601	0.681	0.319

Note: Per *A. M. Best Key Rating Guide* [3]; values for major PPA company within group.

EXHIBIT 10

PART 1

ASSETS
For 1993 ($000)

		Consolidated Industry		PPA and HO Predominating		Fictitious Company	
		Dollar Values	Proportion of Total Assets	Dollar Values	Proportion of Total Assets	Selected Proportion of Total Assets	Dollar Values
1	Bonds	417,776,558	0.622	135,480,072	0.627	0.627	296,679
2	Stocks:						
	2.1 Preferred Stocks	11,795,552	0.018	3,009,190	0.014	0.014	6,624
	2.2 Common Stocks	91,591,043	0.136	39,304,017	0.182	0.182	86,117
3	Mortgage Loans on Real Estate	4,472,774	0.007	272,741	0.001	0.001	473
4	Real Estate:						
	4.1 Properties Occupied by the Company	6,553,591	0.010	4,379,503	0.020	0.020	9,463
	4.2 Other Properties	2,152,292	0.003	451,660	0.002	0.002	946
5	Collateral Loans	76,435	0.000	21,608	0.000	0.000	0
6.1	Cash on Hand and on Deposit	5,189,814	0.008	1,859,610	0.009	0.009	4,259
6.2	Short-Term Investments	31,699,346	0.047	4,350,201	0.020	0.020	9,463
7	Other Invested Assets	7,321,187	0.011	1,506,706	0.007	0.007	3,312
8	Aggregate Write-Ins for Invested Assets	1,205,308	0.002			0.000	0
	8a Subtotals, Cash and Invested Assets	579,833,900	0.863	190,635,308	0.882	0.882	417,338
9	Agents' Balances or Uncollected Premiums:			14,482,101	0.067		
	9.1 In Course of Collection	14,930,076	0.022			0.020	9,463
	9.2 Booked but Deferred and Not Yet Due	27,724,017	0.041			0.038	17,981
	9.3 Accrued Retrospective Premiums	6,973,364	0.010			0.009	4,259

EXHIBIT 10

PART 1—PAGE 2

		Consolidated Industry		PPA and HO Predominating		Fictitious Company	
		Dollar Values	Proportion of Total Assets	Dollar Values	Proportion of Total Assets	Selected Proportion of Total Assets	Dollar Values
10	Funds Held By or Deposited With Reinsured Cos.	3,264,649	0.005	585,707	0.003	0.003	1,420
11	Bills Receivable, Taken for Premiums	1,065,926	0.002			0.000	0
12	Reinsurance Recoverable on LLAE Payments	10,579,195	0.016	1,200,334	0.006	0.006	2,839
13	Federal Income Tax Recoverable	1,959,091	0.003	1,027,140	0.005	0.005	2,366
14	Electronic Data Processing Equipment	2,083,622	0.003	1,347,599	0.006	0.006	2,839
15	Interest, Dividends and Real Estate Income D&A	7,926,145	0.012	2,761,278	0.013	0.013	6,151
16	Receivable from Parent, Subsidiaries, and Affils.	4,528,444	0.007	938,849	0.004	0.004	1,893
17	Equities & Deposits in Pools and Assocs.	2,491,321	0.004	730,527	0.003	0.003	1,420
18	A/R Relating to Uninsured A&H Plans	39,332	0.000			0.000	28
20	Aggregate Write-Ins for Other Than Invested Assets	8,139,150	0.012	2,485,871	0.011	0.011	5,205
21	TOTALS	671,538,233	1.000	216,194,714	1.000	1.000	473,172
	Selected Asset/Earned Premium Ratio:	2.008					

Note: Page 2, Annual Statement; per *Best's Aggregates & Averages—Property-Casualty* (1994) [1].

EXHIBIT 10

PART 2

LIABILITIES
For 1993 ($000)

		Consolidated Industry		PPA and HO Predominating		Fictitious Company	
		Dollar Values	Proportion of Total Liab + Surp	Dollar Values	Proportion of Total Liab + Surp	Selected Proportion of Total Liab + Surp	Dollar Values
1	Losses	280,522,065	0.418	73,866,581	0.342	0.342	161,825
1A	Reinsurance Payable on Paid Loss and LAE	1,820,414	0.003	407,265	0.002	0.002	946
2	Loss Adjustment Expenses	55,793,999	0.083	16,262,840	0.075	0.075	35,488
3	Contingent Commissions	1,942,406	0.003			0.003	1,420
4	Other Expenses	4,755,596	0.007	3,558,428	0.016	0.008	3,785
5	Taxes, Licenses, and Fees	2,647,299	0.004			0.005	2,366
6	Federal and Foreign Income Taxes	2,294,893	0.003	512,170	0.002	0.002	946
7	Borrowed Money	1,487,664	0.002	111,281	0.001	0.001	473
8	Interest	39,130	0.000	(286)	−0.000	0.000	0
9	Unearned Premiums	93,128,418	0.139	38,704,524	0.179	0.179	84,698
10	Dividends Declared and Unpaid:						
	(a) Stockholders	302,780	0.000	22,805	0.000	0.000	0
	(b) Policyholders	1,524,529	0.002	370,398	0.002	0.002	946
11	Funds Held by Company Under Reinsurance	6,941,513	0.010	1,333,242	0.006	0.006	2,839
12	Amounts Withheld or Retained	4,004,529	0.006	557,001	0.003	0.003	1,420
13	Provision for Reinsurance	3,258,238	0.005			0.000	0
14	Excess of Statutory Reserves	856,114	0.001			0.000	0
15	Net Foreign Exchange Adjustments	577,139	0.001	178,579	0.001	0.001	473

EXHIBIT 10

PART 2—PAGE 2

		Consolidated Industry		PPA and HO Predominating		Fictitious Company	
		Dollar Values	Proportion of Total Liab + Surp	Dollar Values	Proportion of Total Liab + Surp	Selected Proportion of Total Liab + Surp	Dollar Values
16	Drafts Outstanding	3,887,256	0.006	2,058,181	0.010	0.010	4,732
17	Payable to Parent, Subsidiaries, and Affiliates	3,486,585	0.005	881,546	0.004	0.004	1,893
18	Payable for Securities	2,010,972	0.003	396,826	0.002	0.002	946
19	Liability re Uninsured A&H Plans	22,112	0.000	170	0.000	0.000	0
20	Aggregates Write-Ins for Liabilities	17,959,263	0.027	7,916,048	0.037	0.036	17,034
21	Total Liabilities	489,262,914	0.729	147,137,599	0.681	0.681	322,230
22	Aggregate Write-Ins for Special Surplus Funds	15,356,274	0.023			0.000	0
23A	Common Capital Stock	6,539,022	0.010			0.000	0
23B	Preferred Capital Stock	736,974	0.001			0.000	0
23C	Aggregate Write-Ins for Other Than Special	2,347,697	0.003			0.000	0
24A	Gross Paid In and Contributed Surplus	73,893,613	0.110	29,817,617	0.138	0.137	64,825
24B	Unassigned Funds (Surplus)	83,803,704	0.125	39,239,497	0.182	0.182	86,117
24C	Less Treasury Stock, at Cost:						
	(1) Shares Common	378,920	0.001			0.000	0
	(2) Shares Preferred	23,045	0.000			0.000	16
25	Surplus as Regards Policyholders	182,275,319	0.271	69,057,114	0.319	0.319	150,958
26	TOTALS	671,538,233	1.000	216,194,713	1.000	1.000	473,172

Note: Page 3, Annual Statement; per *Best's Aggregates & Averages—Property-Casualty* (1994) [1].

EXHIBIT 10

PART 3

UNDERWRITING AND INVESTMENT EXHIBIT
For 1993 ($000)

		Consolidated Industry		PPA and HO Predominating		Fictitious Company	
	STATEMENT OF INCOME	Dollar Values	Proportion of Earned Premium	Dollar Values	Proportion of Earned Premium	Selected Proportion of Earned Premium	Dollar Values
	UNDERWRITING INCOME						
1	Premiums Earned	235,653,339	1.000	107,640,213	1.000	1.000	235,643
	DEDUCTIONS						
2	Losses Incurred	157,173,380	0.667	72,216,288	0.671	0.671	158,117
3	Loss Expenses Incurred	30,267,387	0.128	12,848,736	0.119	0.119	28,042
4	Other Underwriting Expenses Incurred	63,243,439	0.268	25,919,417	0.241	0.241	56,790
5	Aggregate Write-Ins for Underwriting Deductions	49,155	0.000	(560,465)	−0.005	−0.005	(1,178)
6	Total Underwriting Deductions	250,733,361	1.064	110,423,976	1.026	1.026	241,770
7	Net Underwriting Gain or (Loss)	(15,090,022)	−0.064	(2,783,763)	−0.026	−0.026	(6,127)
	INVESTMENT INCOME						
8	Net Investment Income Earned	32,645,415	0.139			0.078	18,380
9	Net Realized Capital Gains or (Losses)	9,817,573	0.042			0.023	5,420
9A	Net Investment Gain or (Loss)	42,462,988	0.180	10,883,286	0.101	0.101	23,800

EXHIBIT 10

PART 3—PAGE 2

	STATEMENT OF INCOME	Consolidated Industry Dollar Values	Consolidated Industry Proportion of Earned Premium	PPA and HO Predominating Dollar Values	PPA and HO Predominating Proportion of Earned Premium	Fictitious Company Selected Proportion of Earned Premium	Fictitious Company Dollar Values
	OTHER INCOME						
10	Net Balance Charge-Off Gain or (Loss)	(446,819)	−0.002			0.000	0
11	Finance and Service Charges	1,047,268	0.004			0.000	0
12	Aggregate Write-Ins for Miscellaneous Items	(895,740)	−0.004			0.000	0
13	Total Other Income	(295,291)	−0.001			0.000	0
14	Net Income Before Dividends and Income Taxes	27,077,675	0.115	8,099,523	0.075	0.075	17,673
14A	Dividends to Policyholders	2,709,126	0.011	1,124,630	0.010	0.010	2,356
14B	Net Income After Dividends, Before Income Taxes	24,368,549	0.103	6,974,893	0.065	0.065	15,317
15	Federal and Foreign Income Taxes Incurred	5,053,041	0.021	1,781,519	0.017	0.017	4,006
16	Net Income	19,315,508	0.082	5,193,374	0.048	0.048	11,311
	Selected Proportion of Company Earned Premium To That of Entire P/C Industry:	0.001					

Note: Page 4, Annual Statement; per *Best's Aggregates & Averages—Property-Casualty* (1994) [1].

EXHIBIT 10

PART 4

SUPPLEMENTAL FINANCIAL STATEMENT INFORMATION

Calculation of Adjusted Surplus	Base Case	Low	High
Statutory Surplus	150,958	83,730	361,290
Equity in the Unearned Premium Reserve	20,412	20,412	20,412
Nominal–Discounted Loss Reserves	8,289	8,289	8,289
Excess of Statutory Reserves	0	0	0
Market Value–Book Value of Bonds	13,928	13,928	13,928
Non-Admitted Assets	946	946	946
Tax Liability on Unrealized Capital Gains	(5,173)	(5,173)	(5,173)
Total Adjustments	38,402	38,402	38,402
Adjusted Surplus	189,360	122,132	399,692
Range: Premium to Surplus Ratio		2.891	0.670
Statutory Surplus		83,730	361,290

Funds-Generating Coefficient	
Prior Year's Losses	186,159
Time-Weighted Paid Losses	277,003
Prior Year's Earned Premium	235,643
Funds-Generating Coefficient	1.18

Investment Income Tax Rate	
Net Investment Income (Company)	18,380
Realized Capital Gaines (Company)	5,420
% Bonds Exempt from US Tax	0.292
% Dividends	0.086
Exclusion (70% of Dividends)	0.060
Tax Rate on Net Investment Income	0.273
Total Tax Rate on Investment Income	0.275
Ratio to Total Tax Rate	0.808

Investment Rate of Return	1992		1993
Total Invested Assets (PPA/HO)	177,103		193,397
Average Invested Assets		185,250	
Net Investment Income		10,883	
Realized Capital Gains		2,638	
Unrealized Capital Gains		1,786	
Total Investment Income		15,307	
Investment Rate of Return		0.083	

Payout Patterns

Year	Base	Fast	Slow
1	0.531	0.691	0.366
2	0.241	0.151	0.331
3	0.105	0.073	0.147
4	0.056	0.039	0.071
5	0.030	0.020	0.038
6	0.015	0.011	0.020
7	0.009	0.006	0.011
8	0.005	0.003	0.006

Payout Patterns

Year	Base	Fast	Slow
9	0.002	0.001	0.003
10	0.001	0.001	0.001
11	0.001	0.001	0.001
12	0.001	0.001	0.001
13	0.001	0.001	0.001
14	0.001	0.001	0.001
15	0.001	0.000	0.001

EXHIBIT 11

PART 1

PRIVATE PASSENGER AUTO LIABILITY/MEDICAL CONSOLIDATED INDUSTRY TOTALS

	Cumulative Net Paid Losses and ALAE ($000)									
Accident Year	12	24	36	48	60	72	84	96	108	120
1984	7,107,267	13,730,807	16,904,897	18,676,836	19,654,989	20,147,446	20,396,113	20,542,284	20,618,789	20,661,498
1985	7,797,650	15,395,558	19,096,053	21,216,278	22,337,968	22,903,322	23,207,201	23,358,240	23,433,206	
1986	8,680,599	17,211,774	21,458,533	23,840,126	25,120,488	25,765,753	26,075,742	26,243,078		
1987	9,674,040	19,363,480	24,098,471	26,757,800	28,171,290	28,854,812	29,247,314			
1988	10,902,007	21,694,958	26,909,462	29,735,250	31,204,226	31,949,941				
1989	12,016,958	24,023,854	29,718,129	32,761,578	34,382,265					
1990	13,250,396	25,970,550	32,000,373	35,193,034						
1991	13,305,353	25,822,406	31,810,715							
1992	14,372,292	27,676,795								
1993	15,656,520									

Note: Per *Best's Aggregates & Averages—Property-Casualty* (1994) [1].

EXHIBIT 11

PART 1—PAGE 2

Accident Year	12–24	24–36	36–48	48–60	60–72	72–84	84–96	96–108	108–120	120–Ult.	
1984	1.932	1.231	1.105	1.052	1.025	1.012	1.007	1.004	1.002		
1985	1.974	1.240	1.111	1.053	1.025	1.013	1.007	1.003			
1986	1.983	1.247	1.111	1.054	1.026	1.012	1.006				
1987	2.002	1.245	1.110	1.053	1.024	1.014					
1988	1.990	1.240	1.105	1.049	1.024						
1989	1.999	1.237	1.102	1.049							
1990	1.960	1.232	1.100								
1991	1.941	1.232									
1992	1.926										
1993											
Average	1.967	1.238	1.106	1.052	1.025	1.013	1.007	1.003	1.002		
Wtd. Avg.	1.966	1.238	1.106	1.052	1.025	1.013	1.007	1.003	1.002		
3-Yr Wt Avg.	1.942	1.234	1.102	1.050	1.025	1.013	1.007	1.003	1.002		
Selected	1.942	1.234	1.102	1.050	1.025	1.013	1.007	1.003	1.002	1.008	
To-Ultimate	2.937	1.513	1.226	1.112	1.059	1.034	1.020	1.014	1.010	1.008	
% of Ult.	34.1%	66.1%	81.6%	89.9%	94.4%	96.8%	98.0%	98.7%	99.0%	99.2%	
Increm. %	34.1%	32.1%	15.4%	8.3%	4.5%	2.3%	1.3%	0.7%	0.3%	0.2%	0.8%
Avg. Pmt. Date	0.50	1.50	2.50	3.50	4.50	5.50	6.50	7.50	8.50	9.50	12.50
Avg. Time Until Pmt.	1.94										

EXHIBIT 11

PART 2

PRIVATE PASSENGER AUTO LIABILITY/MEDICAL CONSOLIDATED INDUSTRY TOTALS

	Incurred Losses and ALAE—Including Bulk and IBNR ($000)									
Accident Year	12	24	36	48	60	72	84	96	108	120
1984	19,881,411	20,458,822	20,723,492	20,788,321	20,793,344	20,796,050	20,803,403	20,818,151	20,819,122	20,826,114
1985	22,640,558	23,331,210	23,605,717	23,648,833	23,681,530	23,667,361	23,678,227	23,669,417	23,662,652	
1986	26,510,270	26,736,641	26,739,048	26,687,523	26,650,450	26,632,126	26,577,414	26,555,719		
1987	30,132,232	30,073,743	29,980,050	29,935,567	29,905,974	29,846,067	29,814,502			
1988	33,939,925	33,682,143	33,462,947	33,316,467	33,103,685	32,999,670				
1989	37,238,902	37,047,111	36,872,244	36,540,543	36,377,267					
1990	40,941,704	40,335,239	39,609,692	39,257,602						
1991	41,803,021	40,529,610	39,643,933							
1992	44,930,921	43,143,619								
1993	47,594,265									

Note: Per *Best's Aggregates & Averages—Property-Casualty* (1994) [1].

EXHIBIT 11

PART 2—PAGE 2

Accident Year	12–24	24–36	36–48	48–60	60–72	72–84	84–96	96–108	108–120	120–Ult.	
1984	1.029	1.013	1.003	1.000	1.000	1.000	1.001	1.000	1.000		
1985	1.031	1.012	1.002	1.001	0.999	1.000	1.000	1.000			
1986	1.009	1.000	0.998	0.999	0.999	0.998	0.999				
1987	0.998	0.997	0.999	0.999	0.998	0.999					
1988	0.992	0.993	0.996	0.994	0.997						
1989	0.995	0.995	0.991	0.996							
1990	0.985	0.982	0.991								
1991	0.970	0.978									
1992	0.960										
1993											
Average	0.996	0.996	0.997	0.998	0.999	0.999	1.000	1.000	1.000		
Wtd. Avg.	0.991	0.994	0.996	0.998	0.999	0.999	1.000	1.000	1.000		
3-Yr Wt Avg.	0.971	0.985	0.992	0.996	0.998	0.999	1.000	1.000	1.000		
Selected	0.971	0.985	0.992	0.996	0.998	0.999	1.000	1.000	1.000	1.000	
To-Ultimate	0.943	0.971	0.985	0.993	0.997	0.999	1.000	1.000	1.000	1.000	
% of Ult.	106.1%	103.0%	101.5%	100.7%	100.3%	100.1%	100.0%	100.0%	100.0%	100.0%	
Increm. %	106.1%	−3.0%	−1.6%	−0.8%	−0.4%	−0.2%	−0.1%	−0.0%	−0.0%	0.0%	0.0%

EXHIBIT 11

PART 3

PRIVATE PASSENGER AUTO LIABILITY/MEDICAL CONSOLIDATED INDUSTRY TOTALS

Accident Year	Case Incurred Losses and ALAE ($000)									
	12	24	36	48	60	72	84	96	108	120
1984	14,971,043	18,740,436	19,858,672	20,351,846	20,561,437	20,657,368	20,711,038	20,743,177	20,758,601	20,777,998
1985	16,658,056	21,135,709	22,525,437	23,165,482	23,402,845	23,472,725	23,543,804	23,562,106	23,578,804	
1986	18,955,064	23,818,148	25,431,280	26,051,927	26,252,369	26,376,438	26,411,278	26,435,323		
1987	21,538,487	26,777,218	28,445,740	29,103,087	29,444,813	29,539,908	29,628,106			
1988	24,123,339	29,860,663	31,489,589	32,288,202	32,544,480	32,658,638				
1989	26,546,705	32,652,373	34,673,136	35,444,420	35,762,026					
1990	28,779,228	35,233,227	37,193,945	37,956,021						
1991	29,037,439	35,149,766	36,938,537							
1992	31,393,255	37,390,576								
1993	33,994,407									

Note: Per *Best's Aggregates & Averages—Property-Casualty* (1994) [1].

EXHIBIT 11

PART 3—PAGE 2

Accident Year	12–24	24–36	36–48	48–60	60–72	72–84	84–96	96–108	108–120	120–Ult.	
1984	1.242	1.060	1.025	1.010	1.005	1.003	1.002	1.001	1.001		
1985	1.269	1.066	1.028	1.010	1.003	1.003	1.001	1.001			
1986	1.257	1.068	1.024	1.008	1.005	1.001	1.001				
1987	1.243	1.062	1.023	1.012	1.003	1.003					
1988	1.238	1.055	1.025	1.008	1.004						
1989	1.230	1.062	1.022	1.009							
1990	1.224	1.056	1.020								
1991	1.210	1.051									
1992	1.191										
1993											
Average	1.235	1.060	1.024	1.009	1.004	1.002	1.001	1.001	1.001		
Wtd. Avg.	1.230	1.059	1.024	1.009	1.004	1.002	1.001	1.001	1.001		
3-Yr Wt Avg.	1.208	1.056	1.023	1.009	1.004	1.002	1.001	1.001	1.001		
Selected	1.208	1.056	1.023	1.009	1.004	1.002	1.001	1.001	1.001	1.002	
To-Ultimate	1.332	1.102	1.044	1.021	1.011	1.007	1.005	1.004	1.003	1.002	
% of Ult.	75.1%	90.7%	95.8%	98.0%	98.9%	99.3%	99.5%	99.6%	99.7%	99.8%	
Increm. %	75.1%	15.6%	5.1%	2.2%	0.9%	0.4%	0.2%	0.1%	0.1%	0.1%	0.2%

EXHIBIT 11

PART 4

PRIVATE PASSENGER AUTO LIABILITY/MEDICAL CONSOLIDATED INDUSTRY TOTALS

Earned Premiums ($000)

Accident Year	Earned Premiums
1984	23,929,028
1985	26,545,102
1986	31,282,435
1987	35,721,716
1988	39,659,210
1989	42,942,782
1990	46,792,329
1991	49,990,387
1992	54,155,823
1993	57,846,167

Ratio of Paid Loss and ALAE to Earned Premiums

Accident Year	12–24	24–36	36–48	48–60	60–72	72–84	84–96	96–108	108–120	120–Ult.
1984	0.297	0.574	0.706	0.781	0.821	0.842	0.852	0.858	0.862	0.863
1985	0.294	0.580	0.719	0.799	0.842	0.863	0.874	0.880	0.883	
1986	0.277	0.550	0.686	0.762	0.803	0.824	0.834	0.839		
1987	0.271	0.542	0.675	0.749	0.789	0.808	0.819			
1988	0.275	0.547	0.679	0.750	0.787	0.806				
1989	0.280	0.559	0.692	0.763	0.801					
1990	0.283	0.555	0.684	0.752						
1991	0.266	0.517	0.636							
1992	0.265	0.511								
1993	0.271									

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Accident Year	Ratio of Case Incurred Loss and ALAE to Earned Premiums									
	12–24	24–36	36–48	48–60	60–72	72–84	84–96	96–108	108–120	120–Ult.
1984	0.626	0.783	0.830	0.851	0.859	0.863	0.86	0.867	0.868	0.868
1985	0.628	0.796	0.849	0.873	0.882	0.884	0.887	0.888	0.888	
1986	0.606	0.761	0.813	0.833	0.839	0.843	0.844	0.845		
1987	0.603	0.750	0.796	0.815	0.824	0.827	0.829			
1988	0.608	0.753	0.794	0.814	0.821	0.823				
1989	0.618	0.760	0.807	0.825	0.833					
1990	0.615	0.753	0.795	0.811						
1991	0.581	0.703	0.739							
1992	0.580	0.690								
1993	0.588									

Note: Per *Best's Aggregates & Averages—Property-Casualty* (1994) [1].

EXHIBIT 11

PART 5

AUTO PHYSICAL DAMAGE
CONSOLIDATED INDUSTRY TOTALS

Cumulative Net Paid Losses and ALAE ($000)

Accident Year	12	24	12–24
1989		21,805,677	
1990	19,625,334	21,450,232	1.093
1991	19,037,412	20,637,002	1.084
1992	19,457,475	21,105,812	1.085
1993	20,529,226		

Cumulative Case Incurred Losses and ALAE ($000)

Accident Year	12	24	Ult. Est.	12–24	24–Ult.
1989		21,898,506	22,058,959		1.007
1990	21,415,626	21,618,021	21,741,587	1.009	1.006
1991	20,716,914	20,721,080	20,800,355	1.000	1.004
1992	21,075,920	21,148,591	21,236,170	1.003	1.004
1993	22,075,313				

Ratios of Loss and ALAE to Earned Premiums

Accident	Earned	Paid Loss and ALAE		Case Incurred Loss and ALAE		
Year	Premiums	12	24	12	24	Ult. Est.
1989	34,179,748		0.638		0.641	0.645
1990	35,015,802	0.560	0.613	0.612	0.617	0.621
1991	35,721,624	0.533	0.578	0.580	0.580	0.582
1992	36,737,466	0.530	0.575	0.574	0.576	0.578
1993	37,864,502	0.542		0.583		

Note: Per *Best's Aggregates & Averages—Property-Casualty* (1994) [1].

APPENDIX

In this appendix, we show examples of specific calculations for each of the six financial pricing models examined in this paper. It is hoped that these examples will provide further insight into the models, as well as encourage actuaries to implement some of these techniques themselves.

Target Total Rate of Return

Underwriting profit margins resulting from the Target Total Rate of Return model are shown in Exhibit 3. These UPMs are generated directly from Equation 2.6 in the text, assuming the parameters given on the exhibit. The Target TRR method equates the sum of an insurance company's underwriting and investment returns with a target total rate of return; this target, in our paper, is based on the Capital Asset Pricing Model. Thus, calculations of UPMs according to the Target TRR model require assumptions regarding values for the following parameters: the risk-free interest rate, the expected return on the equity market (which is equal to the risk-free rate plus an equity "risk premium"), the equity beta of the insurer, the company's invested assets and the rate of return on those investments, and the company's equity and its premium-to-equity ratio. Assumptions regarding five of these seven variables are documented at the top of Exhibit 3; alternative assumptions regarding the premium-to-equity ratio and the equity beta are shown below those, and on the graph on the exhibit.

As an example, the 0.236 UPM that is shown on the exhibit (assuming an equity beta of 1.70 and a premium-to-equity ratio of 0.50) is derived from Equation 2.6 as follows:

$$UPM = \left(\frac{1}{0.50}\right)\left([0.05 + 1.70(0.13 - 0.05)] - \frac{417{,}338 \times 0.08}{189{,}360 \times \dfrac{1.30}{0.50}}\right)$$

$$= 0.236,$$

where the (1.30/0.50) factor is an adjustment to equity to bring the premium-to-equity ratio to the assumed value of 0.50. (It is assumed that, for the base case corresponding to an equity value of 189,360, the premium-to-equity ratio is 1.30; thus, to test the sensitivity of the model to a different leverage and a different premium-to-equity ratio, the *equity* is adjusted in the UPM calculation—the premium level is held constant. Assuming that an equity value of 189,360 corresponds to a premium-to-equity ratio of 1.30, a ratio of 0.50 implies an equity of 492,336.)

Insurance Capital Asset Pricing Model

Underwriting profit margins resulting from the Insurance CAPM are shown in Exhibit 4. These UPMs are generated directly from Equation 2.8 in the text, assuming the parameters given on the exhibit. We use a differential tax version of the Insurance CAPM, and show the sensitivity of UPMs to changes in the underwriting beta and the premium-to-equity ratio on the exhibit and in the graph.

As an example, the 0.008 UPM on Exhibit 4, under underwriting beta and premium-to-equity ratio assumptions of 0.40 and 0.50, respectively, is calculated as follows:

$$UPM = -1.18 \times 0.05 \frac{1 - 0.272}{1 - 0.340} + 0.40(0.13 - 0.05) + \frac{1}{0.50} 0.05 \frac{0.272}{1 - 0.340} = 0.008.$$

Discounted Cash Flow Model

Underwriting profit margins resulting from the DCF Model are shown in Exhibit 5. These UPMs are based on the concepts underlying Equation 2.9 in the text. In this framework, the present value of the premiums is set equal to the present value of all the cash flows emanating from the policy, including expenses, losses (and LAE), taxes on underwriting and taxes on investment income. The specific formula used (with one adjustment—see below) is as follows:

$$
\begin{aligned}
P\sum_{i=0}^{N}\frac{a_i}{(1+r_f)^i}
&= L\sum_{i=0}^{N}\frac{b_i}{(1+r_L)^i} + E\sum_{i=-M}^{N}\frac{c_i}{(1+r_f)^i} \\
&\quad + \frac{\left(P - E\sum_{i=-M}^{N}\frac{c_i}{(1+r_f)^i}\right)t}{1+r_f} \\
&\quad - Lt\left(\frac{\sum_{i=1}^{N}\frac{b_i}{(1+r_T)^{i-1}}}{1+r_L} + \sum_{j=2}^{N}\frac{\sum_{i=j}^{N}\frac{r_T b_i}{(1+r_T)^{i-j+1}}}{(1+r_L)^j}\right) \\
&\quad + r_f t\left(\sum_{j=1}^{N}\left[\frac{S\left(\sum_{i=j}^{N} b_i\right) + P - E - L\sum_{i=0}^{j-1} b_i}{(1+r_f)^j}\right]\right),
\end{aligned}
$$

where

a_i = fraction of premium received in time period i,

b_i = fraction of losses paid in time period i,

c_i = fraction of expenses paid in time period i,

S = owners' equity in insurer,

P = premiums,

L = losses and loss adjustment expenses,

E = underwriting expenses,

t = tax rate,

r_T = discount rate required for tax purposes,

r_f = risk free rate,

r_L = risk adjusted rate for losses,

M = number of time periods before policy effective date that the first prepaid expenses are paid, and

N = number of time periods after policy effective date that the last loss payment is made.

The above DCF formula—with an adjustment to allow for differential tax rates between underwriting and investment income—is implemented via a spreadsheet program in which future annual expected loss, expense, and tax cash flows are discounted to determine a fair premium. In order to solve this equation, we need to know the rates at which premium income is received and expenses, losses (including LAE) and taxes are paid, and the discount rates to use to calculate the present values. The cash flows are discounted at different rates. Premiums, expenses and investment income (which is assumed to be earned based on the risk-free rate) are discounted based on the risk-free rate. Losses (and LAE) and taxes on underwriting income based on losses, are discounted at a risk-adjusted rate. As indicated on Exhibit 5, for the base case, the risk-free interest rate is 5%, the risk-adjusted discount rate is 3% and the tax discount rate is 7%.

For the base case, it is assumed that the premium is received and expenses paid entirely when the policy is written. Losses (and LAE) are assumed to be paid in the middle of each year, with the loss payout pattern shown in Table 1 (based on Best's *Aggregates and Averages*, 1994). The discounted values of the loss payments are determined by dividing the percent of losses paid by $(1.03)^{(\text{Year}-0.5)}$.

Taxes are assumed to be paid at the end of each year. Taxes on underwriting income are determined based on the difference between premiums, and expenses and losses, with the loss reserves discounted based on the provisions of the Tax Reform Act of 1986. Thus, the incurred losses for tax purposes are the

TABLE 1

Year	% of Losses Paid	Discounted Value
1	0.531	0.523210
2	0.241	0.230548
3	0.105	0.097521
4	0.056	0.050496
5	0.030	0.026264
6	0.015	0.012749
7	0.009	0.007427
8	0.005	0.004006
9	0.002	0.001556
10	0.001	0.000765
11	0.001	0.000733
12	0.001	0.000712
13	0.001	0.000691
14	0.001	0.000671
15	0.001	0.000651
Total	1.000	0.957989

paid losses each year plus the ending reserves (discounted at the mandated seven percent rate) minus the beginning loss reserves (discounted at seven percent). Since losses are paid out over a 15-year period in this example, then the taxes are also paid out over the same 15-year period.

The calculation of taxes based on underwriting income is determined by a spreadsheet, which is available from the authors, that runs over the entire 15-year period. For the fifteenth (last) year, the incurred losses are the paid losses of 193.605 (.001 times the incurred losses of $193,605) minus the beginning reserve (discounted for half a year based on the mandated seven percent discount rate) of 187.165, or 6.44. This incurred loss is multiplied by the tax rate applicable to underwriting, 34 percent. This negative tax payment is then discounted back for 15 years, based on the risk-adjusted rate of three percent. Similar calculations are performed for every other year to determine the effect of the underwriting tax on losses. In addition, for the first year, the tax rate is multiplied by the difference between the premiums and expenses (both assumed to be paid at the inception of the policy). This tax payment is discounted back for one year (taxes are paid at the end of the year) at the risk-free rate of five percent.

The calculation of taxes based on investments is also determined by a spreadsheet that runs over the 15-year period during which losses are paid. Investment income is earned on the surplus allocated to the policy, which is released in proportion with loss payments, and the difference between the premiums received and expenses and cumulative losses paid out. Investment income is assumed to be earned based on the risk-free rate and discounted at the risk-free rate. For the first year, the entire surplus plus the premiums less expenses is invested. For the second year, 46.9 percent of the inital surplus (since 53.1 percent of the losses have been paid in the first year) plus the premiums less expenses and less 53.1 percent of the losses are invested. This pattern continues until all the losses are paid after 15 years.

For the base case, Equation (2.9) can be broken down as follows:

$\text{PV}(P) = P$.

$\text{PV}(L) = 193{,}605 \times (.957989)$.

$\text{PV}(E) = 59{,}062$.

$\text{PV}(UWPT) = \text{PV}(\text{tax on premiums}) - \text{PV}(\text{tax on expenses}) -$ PV (tax on losses).

$\text{PV}(\text{tax on premiums}) = 0.34 \times (1/1.05) \times P = 0.323810P$.

$\text{PV}(\text{tax on expenses}) = 0.34 \times (59{,}062) \times (1/1.05) = 19{,}125$.

$\text{PV}(\text{tax on losses}) = 0.34 \times (193{,}605) \times (.968011) = 63{,}720$.

$\text{PV}(IBT)$ = PV(taxes on investment income from surplus) + PV(taxes on investment income from premiums minus expenses minus paid losses).

PV(taxes on investment income from surplus) = $0.05 \times (0.272) \times (189{,}360) \times (1.887415)/(1.05)$.

PV(taxes on investment income from premiums minus expenses minus paid losses) = $0.05 \times (.272) \times (P - 59{,}062 -$ losses paid to date$)/(1.05)$.

Solving for P yields 253,040. The underwriting profit margin associated with this premium is:

$$UPM = \frac{253{,}040 - 193{,}605 - 59{,}062}{253{,}040} = 0.001.$$

Internal Rate of Return Model

Underwriting profit margins resulting from the IRR model are shown in Exhibit 6, Part 1, along with the relevant parameter assumptions. The spreadsheet underlying the UPM calculations is shown in Exhibit 6, Part 2—the numbers on the sheet represent base case calculations, with an internal rate of return of 13% (equal to the expected market return of 5% risk-free plus an 8% market risk premium). The leftmost column indicates the timing of the cashflows; quarterly for the first two years and then annual. This spreadsheet can accomodate 25 years of payouts, but for the base case example all losses are settled within 15 years (60 quarters). The next column, labeled (1), is the premium income, which is calculated by the backsolver routine. Column 2 shows the expenses, an outgo, which are given and assumed to be all paid in the first quarter. Column 3 shows the loss payments. Column 4 shows the federal tax cash flow, calculated as $t(P - E -$ discounted losses). The taxes are calculated on an annual basis and, for the first two years, spread evenly over the four quarters.

Column 5 is the cash flow from underwriting, which is the sum of Columns 1 through 4. This cash flow totals $2,866. The insurer is assumed, under this model, to receive this sum as soon as the policy is written. (This model was originally developed for workers compensation, which operated at an underwriting loss. The insurer was assumed to have to fund this underwriting loss at the inception of the policy. The same timing is assumed in this situation.) Column 6 is the accumulated value of the underwriting account. For the first quarter, the value is half of Column 5 plus the initial underwriting flow. For subsequent rows, the value is the average of Column 5 and Column 5 lagged one period plus Column 6 lagged one period.

Column 7 is the total required loss reserve, which is the total losses (193,605) minus paid to date. For the first year, losses are assumed to be incurred evenly over the year. Column 8 is the remaining surplus. The initial surplus allocated to the line is 189,360. Surplus is released as losses are paid. Column 9 is the average surplus, the average of Column 8 and Column 8 lagged one period. Column 10 is investment income on surplus; for the first eight quarters it is Column 9 times two percent; after that it is Column 9 times eight percent. Column 11 is the investment income on underwriting, which is two (or eight) percent of Column 6.

Column 12 shows the cash flows from surplus. For the top row, it is the sum of Columns 5 and 8. For each remaining row, it is Column 8 lagged by one period minus Column 8. Column 13, the net cash flow to capital providers, is the sum of the surplus cash flow (Column 12) and the after-tax investment returns (0.728 times the sum of Columns 10 and 11). Column 14 is the discount factor at the IRR rate; in this example, 13 percent annually. For the first quarter it is $(1/1.13)^{.125}$ (since the payment occurs midway through the first quarter). For the last row it is $(1/1.13)^{24.5}$. Column 15 is Column 13 times Column 14.

UPMs for non-base case parameter assumptions are derived by changing the parameter values, and then "backsolving" the Part 2 spreadsheet by changing the fair premium in Column 1 until the total discounted net cash flow in Column 15 is zero. (This can typically be done in spreadsheet programs by using a "backsolver" function.) As for the DCF model, the indicated

UPMs are then calculated based on the relationships between the premiums and expected losses and expenses. For example, the base case UPM of 0.017 on Exhibit 6, Part 1, is calculated as follows:

$$UPM = \frac{255{,}680 - 193{,}605 - 57{,}733}{255{,}680} = 0.017.$$

Option Pricing Model

Underwriting profit margins resulting from the OPM are shown in Exhibit 7. These UPMs are generated from a spreadsheet modeled after Equation 19 in Doherty and Garven [15]:

$$V_e = R_f^{-1}\left(\mathrm{E}^*(X)\mathrm{N}\left[\frac{\mathrm{E}^*(X)}{\sigma_x}\right] - \tau\mathrm{E}^*(W)\mathrm{N}\left[\frac{\mathrm{E}^*(W)}{\sigma_w}\right] + \sigma_x n\left[\frac{\mathrm{E}^*(X)}{\sigma_x}\right] - \tau\sigma_w \mathrm{n}\left[\frac{\mathrm{E}^*(W)}{\sigma_w}\right]\right),$$

where

V_e = the market value of the residual claim of the shareholders,

$R_f = 1 + r_f$,

E^* = the certainty-equivalent expectation operator,

τ = the corporate tax rate,

σ = standard deviation,

X = random variable representing the insurer's pre-tax end-of-period value after paying claims costs × (initial equity + premium income + investment income − claims costs),

W = random variable representing the insurer's taxable income (from both underwriting and investments), N[ ] = standard normal distribution value, and

n[ ] = standard normal density value.

This equation can be solved iteratively to determine the fair premium (in the spreadsheet, by using backsolver), since the premium level (net of expenses) is embedded in the X and W random variables above. The "fair" premium is defined where the market value of the residual claim of the shareholders is equal to the initial equity. For the base case, the following values apply:

$$V_e = 189{,}360.$$

$$R_f = 1.05.$$

$$\lambda = [\mathrm{E}(r_M) - r_f]/s^2(M) = 1.594388.$$

$$\mathrm{E}^*[L] = \mathrm{E}[L] - [\lambda/\beta(i)] \times [\rho(iL) \times s(i) \times s(L)] = 193{,}605.$$

$$\begin{aligned}\mathrm{E}^*[X] &= S(0) + [(S(0) + (k \times P(0))) \times r_f] + P(0) - \mathrm{E}^*[L] \\ &= 189{,}360 + [(189{,}360 + 1.5P(0))0.05] + P(0) - 193{,}605 \\ &= 213{,}837.\end{aligned}$$

$$\begin{aligned}s(X) &= \{[(S(0) + (k \times P(0)))^2 \times s^2(i)] + s^2(L) \\ &\quad - [2 \times (S(0) + (k \times P(0))) \times \rho(iL) \times s(i) \times s(L)]\}^{0.5} \\ &= 107{,}592.\end{aligned}$$

$$\begin{aligned}\mathrm{E}^*[W] &= [h \times (S(0) + (k \times P(0))) \times r_f] + P(0) - \mathrm{E}^*[L] \\ &= [0.8 \times (189{,}360 + 1.5P(0)) \times 0.05] + P(0) - 193{,}605 \\ &= 19{,}673.\end{aligned}$$

$$\begin{aligned}s(W) &= \{[(S(0) + (k \times P(0)))^2 \times s^2(i) \times h^2] + s^2(L) \\ &\quad - [2 \times (S(0) + (k \times P(0))) \times h \times \rho(iL) \times s(i) \times s(L)]\}^{0.5} \\ &= 90{,}840,\end{aligned}$$

where

$$M = \text{the market},$$
$$i = \text{investment returns, and}$$
$$L = \text{claims costs.}$$

The system is then solved for $P(0)$, which turns out to be 194,060. To check that this value is correct, substitute the above values into the equation:

$$\begin{aligned}
189{,}360 = (1/1.05)&\\
&\times\{(213{,}837)\times \mathrm{N}[213{,}837/107{,}592]\\
&\quad - 0.34\times(19{,}673)\times \mathrm{N}[19{,}673/90{,}840] + 107{,}592\\
&\quad \times \mathrm{n}[213{,}837/107{,}592] - 0.34\times(90{,}840)\\
&\quad \times \mathrm{n}[19{,}673/90{,}840]\}\\
= 0.952381 &\times \{[(213{,}837)\times(0.976566)]\\
&\quad - [0.34\times(19{,}673)\times(0.585726)]\\
&\quad + [(107{,}592)\times(0.55354)]\\
&\quad - [0.34\times(90{,}840)\times(.389696)]\}\\
= 189{,}360.&
\end{aligned}$$

Finally, expenses are added in to the premium, and the UPM is calculated in the usual way—for example, for the base case on Exhibit 7:

$$UPM = \frac{(194{,}060 + 59{,}062) - 193{,}605 - 59{,}062}{(194{,}060 + 59{,}062)} = 0.002.$$

Arbitrage Pricing Model

Underwriting profit margins resulting from the APT are shown in Exhibit 8, Part 1. These UPMs are generated directly from Equation 2.13 in the text, assuming the parameters given on the exhibit. We use a differential tax version of the Insurance APT and show the sensitivity of UPMs to changes in the premium-to-equity ratio on the exhibit and in the graph. Part 2 of Exhibit 8 shows the results of regressions of five different macroeconomic variables against historical underwriting profit margins. Based on these regressions, it was decided to use inflation and the growth in industrial production as the explanatory factors in the Arbitrage Pricing formula.

As an example, the -0.029 base case UPM on Part 1 is calculated as follows:

$$UPM = -\frac{1-0.272}{1-0.340} \times 0.05 \times 1.18 + (0.50 \times 0.03)$$
$$+ (0.25 \times 0.02) + 0.05\left(\frac{1}{1.3}\right)\left(\frac{0.272}{1-0.340}\right) = -0.029.$$