ADJUSTING INDICATED INSURANCE RATES: FUZZY RULES THAT CONSIDER BOTH EXPERIENCE AND AUXILIARY DATA

VIRGINIA R. YOUNG

Abstract

This paper describes how an actuary can use fuzzy logic to adjust insurance rates by considering both claim experience data and supplementary information. This supplementary data may be financial or marketing data or statements that reflect the philosophy of the actuary's company or client. The paper shows how to build and fine-tune a rate-making model by using workers compensation insurance data from an insurance company.

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1. INTRODUCTION

Through the education programs of the Society of Actuaries and the Casualty Actuarial Society, actuaries are equipped with statistical tools to analyze experience data and to determine necessary rate changes for their insurance products. Students are often surprised to learn that those rate changes are frequently not accepted "as is" by company management. Actuaries work with sales, marketing, and underwriting personnel to develop rates that will be competitive and adequate.

Actuaries frequently consider statistical data specific to rates, such as the results of experience studies. In setting premiums, actuaries also consider constraints that supplement experience data. These constraints may reflect company philosophy, such as "We wish to increase our market share *moderately* from year to year." They may also include financial data, such as "Raise the rates if we experience *high* loss ratios or *low* profit margins."

The theory of fuzzy sets provides a natural setting in which to handle such statements. Through fuzzy sets, one can account for vague notions whose boundaries are not clearly defined, such as "*large* amount of business." Fuzzy logic provides a uniform way to handle such factors that influence the indicated rate change (Zadeh [20]). A fuzzy logic system is a type of expert system. An advantage of using a fuzzy logic system is that it provides a systematic way to develop mathematical rules from linguistic ones. This paper describes step-by-step how an actuary can adjust rates by beginning with linguistic rules that consider both experience data and supplementary information.

Fuzzy sets have only recently been applied to problems in actuarial science. DeWit [5] and Lemaire [13] show how to apply fuzzy sets in individual underwriting, and Young [16] indicates how to use fuzzy sets in group health underwriting. Ostaszewski [15] suggests several areas in actuarial science in which fuzzy sets may prove useful. Cummins and Derrig [2] apply a form of fuzzy logic to calculate fuzzy trends in property-liability insurance. Derrig and Ostaszewski [4] employ fuzzy clustering in risk classification and provide an example in automobile insurance. Cummins and Derrig [3] use fuzzy arithmetic in pricing property-liability insurance. In an earlier paper [18], I show how to develop a fuzzy logic model with which actuaries can adjust insurance rates by considering only constraints or information that are ancillary to experience data.

Section 2 introduces fuzzy sets and defines operators corresponding to the linguistic connectors *and* and *or* and the modifier *not*. It also describes a simple fuzzy inference system. References for fuzzy sets include Dubois and Prade [7], Kosko [12], and Zadeh [19]. Some references for fuzzy logic and fuzzy inference are Bellman and Zadeh [1], Driankov et al. [6], Kandel and Langholz [9], Klir and Folger [11], Mamdani [14], Zadeh [20], and Zimmermann [21].

Section 3 describes how to construct and fine-tune a pricing model using fuzzy inference. Section 4 shows how to build a pricing model using workers compensation insurance data from an insurance company. Finally, Section 5 summarizes the paper's key points.

2. FUZZY INFERENCE

Fuzzy sets describe concepts that are vague (Zadeh [19]). The fuzziness of a set arises from the lack of well-defined boundaries. This lack is due to the imprecise nature of language; that is, objects can possess an attribute to various degrees. A fuzzy set corresponding to a given characteristic assigns a value to an object, the degree to which the object possesses the attribute.

Examples of fuzzy sets encountered in insurance pricing are *stable* rates, *large* profits, and *small* amounts of business renewed or written. Indeed, rates can be *stable* to different degrees depending on the relative or absolute changes in the premium rate. Also, profits can be *large* to different degrees depending on the relative or absolute amount of profits.

Fuzzy sets generalize nonfuzzy, or crisp, sets. A crisp set, C, is given by a characteristic function:

$$\chi_C : X \to \{0,1\},$$

in which $\chi_C(x) = 1$ if x is in C; otherwise, $\chi_C(x) = 0$. Fuzzy sets recognize that objects can belong to a given set to different degrees. They essentially expand the notion of set to allow partial membership in a set.

DEFINITION 2.1 *A fuzzy set, A, in a universe of discourse, X, is a function m_A on X that takes values in the unit interval* [0, 1]*:*

$$m_A : X \to [0,1].$$

The function m_A is called the membership function of A, and for any x in X, $m_A(x)$ in [0, 1] *represents the grade of membership of x in A.*

EXAMPLE 2.1 One may define *stable* rates by the following hypothetical fuzzy set:

$$m_{stable}(r) = \begin{cases} 0, & \text{if} \quad r < -0.10, \\ \dfrac{r+0.10}{0.05}, & \text{if} \quad -0.10 \le r < -0.05, \\ 1, & \text{if} \quad -0.05 \le r < 0.05, \\ \dfrac{0.10-r}{0.05}, & \text{if} \quad 0.05 \le r < 0.10, \\ 0, & \text{if} \quad r \ge 0.10, \end{cases}$$

in which r is the relative rate change. For example, the degree to which a rate increase of 8% is *stable* is 0.40. It does *not* mean, however, that one will view an 8% rate increase as stable 40% of the time and unstable the rest of the time. See Figure 1 for the graph of this fuzzy set. The points ± 0.05 and ± 0.10 depend on the line of business. Also, one may want to use a fuzzy set that is not necessarily piecewise linear.

We now define three basic operations on fuzzy sets.

DEFINITION 2.2 *The union, $A \cup B$, of two fuzzy sets, A and B, is given by*

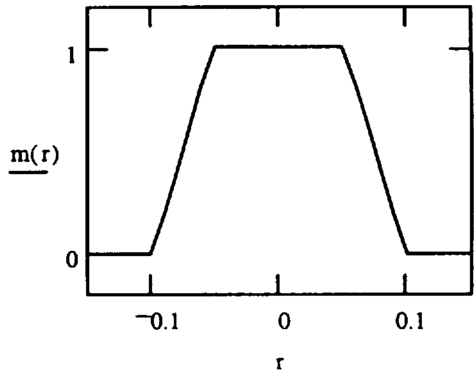
$$m_{A \cup B}(x) \equiv \max[m_A(x), m_B(x)], \qquad x \in X,$$

and the intersection, $A \cap B$, is given by

$$m_{A \cap B}(x) \equiv \min[m_A(x), m_B(x)], \qquad x \in X.$$

FIGURE 1

GRAPH OF FUZZY SET OF STABLE RATES, EXAMPLE 2.1



The complement, $-A$, of fuzzy set A is given by

$$m_{-A}(x) \equiv 1 - m_A(x), \qquad x \in X.$$

The union operation acts as an *or* operator, the intersection operation acts as *and*, and the complement operation acts as *not*. Thus, for example, $m_{A \cap B}(x)$ represents the degree to which x is a member of both A and B. The given definitions are not the only acceptable ones for these operations. Klir and Folger [11] specify axioms that union, intersection, and complement satisfy. Also, Dubois and Prade [7] and Young [16] discuss alternative operators. One in particular is the intersection operator called the algebraic product. The algebraic product of two fuzzy sets A and B is given by

$$m_{AB}(x) = m_A(x) \cdot m_B(x).$$

The algebraic product allows the fuzzy sets to interact in the intersection. That is, both fuzzy sets contribute to the value of the intersection, as opposed to the min operator in which the minimum of the two values determines the value of the intersection.

We will consider this intersection operator in some of the examples below. Unless otherwise stated, however, the intersection operator is the min operator.

EXAMPLE 2.2 Suppose we want to intersect the fuzzy set of *stable* rates from Example 2.1 with the fuzzy set of *low* actual-to-expected ratios given by

$$m_{low}(x) = \begin{cases} 1, & \text{if} \quad x < 0.90, \\ \dfrac{1.10 - x}{0.20}, & \text{if} \quad 0.90 \le x < 1.10, \\ 0, & \text{if} \quad 1.10 \le x, \end{cases}$$

in which x is the ratio of actual claims to expected claims (A/E ratio).[1] We first imbed these fuzzy sets in the product space of pairs $\{(r,x) : r \ge -1.00,\ x \ge 0\}$, as follows

$$m_{stable}(r,x) = m_{stable}(r)$$

$$m_{low}(r,x) = m_{low}(x).$$

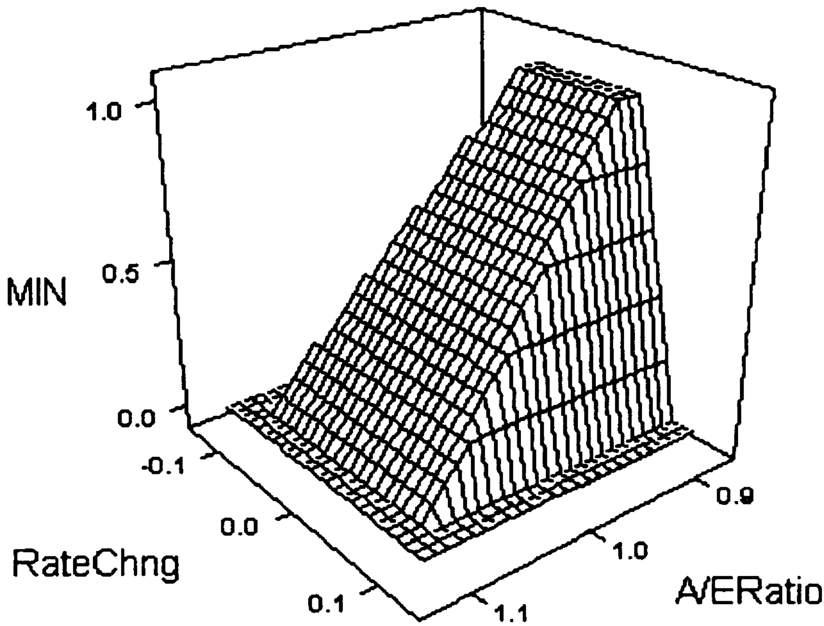
See Figure 2 for the graph of the intersection of these two fuzzy sets using the min operator and Figure 3 for the graph of the intersection of these two fuzzy sets using the algebraic product.

Note that the algebraic product operator allows the two fuzzy sets to interact more than does the min operator. For example, suppose the rate decrease is 6% and the A/E ratio is 0.95. Then, the degree to which the rate change is *stable* is 0.80, and the degree to which the A/E ratio is *low* is 0.75. The degree to which the rate change is stable *and* the A/E ratio is low is min(0.80,0.75) = 0.75 if we use the min operator to intersect the

[1]One type of experience study is called an actual-to-expected study. In this study, one compares the actual (incurred) claims relative to the expected claims built into the premium. If this study is performed before the claims have run out, then one develops the actual claims to an ultimate basis to estimate actual incurred claims. One result of this study is the ratio of actual claims to expected claims, called the *actual-to-expected ratio*, or briefly A/E ratio. A high A/E ratio indicates that the allowance for claims in the premium is too low.

FIGURE 2

INTERSECTION OF STABLE RATES AND LOW ACTUAL-TO-EXPECTED RATIO (USING THE MIN OPERATOR)

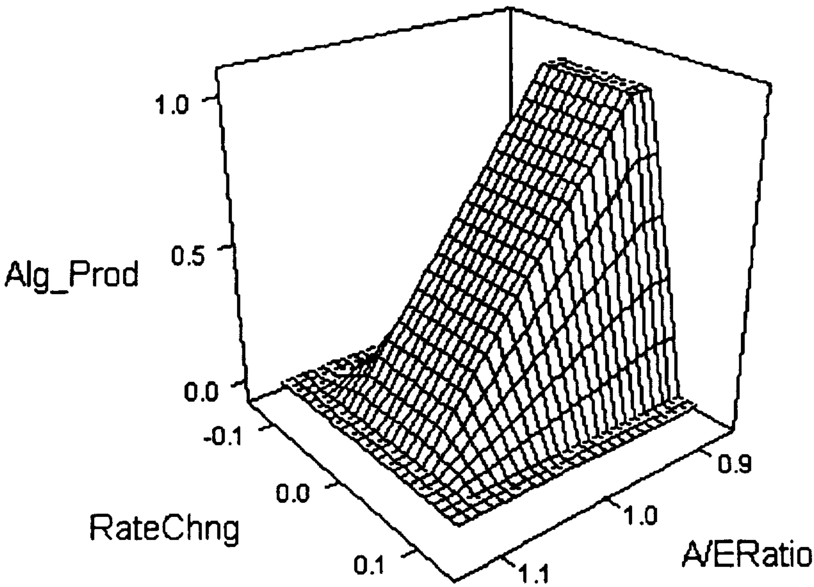


two fuzzy sets, and is (0.80)(0.75) = 0.60 if we use the algebraic product to intersect them.

A few years after fuzzy sets were introduced, Bellman and Zadeh [1] developed the first fuzzy logic model in which goals and constraints were defined as fuzzy sets and their intersection was the fuzzy set of the decision. Cummins and Derrig [2] use the method of Bellman and Zadeh to calculate a trend factor in property-liability insurance. They calculate several possible trends using accepted statistical procedures. For each trend, they determine the degree to which the estimate is *good* by intersecting several fuzzy goals. They suggest that one may choose the trend

FIGURE 3

INTERSECTION OF STABLE RATES AND LOW ACTUAL-TO-EXPECTED RATIO (USING THE ALGEBRAIC PRODUCT)



that has the highest degree of *goodness*. Cummins and Derrig also propose that one may calculate a trend that accounts for all the trends by forming a weighted average of these trends using the membership degrees as weights. It is this latter method that more closely relates to the technique proposed below.

This paper shows how actuaries may incorporate supplementary information in their pricing models, for example, amount of business written or profit earned. Instead of using the method designed by Bellman and Zadeh [1], we follow Zadeh [20] by applying fuzzy inference. In particular, we use a simple form of fuzzy inference proposed by Mamdani [14], who has been a pioneer in applying fuzzy logic in industry. We describe this fuzzy inference after the following example.[2]

EXAMPLE 2.3

(a) If the A/E ratio is *high* and the amount of business is *large*, then raise the rates.

(b) If the A/E ratio is *moderate* and the amount of business is *moderate*, then do not change the rates.

(c) If the A/E ratio is *low* and the amount of business is *small*, then lower the rates.

An actuary can only apply a crisp rate change, not a fuzzy expression such as "*raise* the rates." We therefore set the phrase "raise the rates" equal to the largest rate increase we are willing to administer; similarly, "lower the rates" is replaced by the largest rate decrease we are willing to administer. The reason for doing so will become evident as we proceed below.

In general, our fuzzy system is a collection of n fuzzy rules:

$$\text{If } x \text{ is } A_1, \text{ then } y \text{ is } y_1.$$
$$\text{If } x \text{ is } A_2, \text{ then } y \text{ is } y_2.$$
$$\ldots$$
$$\text{If } x \text{ is } A_n, \text{ then } y \text{ is } y_n.$$

If we are given specific input, or explanatory, data $\tilde{x}$ (possibly multi-dimensional if the A_i are compound hypotheses, as in Example 2.3), then measure the degree to which $\tilde{x}$ satisfies the hypothesis A_i in rule i, $i = 1,\ldots,n$, namely, $m_{A_i}(\tilde{x})$. To calculate

[2]Throughout this paper, by default, assume that if none of the hypotheses is satisfied to a positive degree, then do nothing. In the following example, this would mean "do not change the rates." This convention is consistent with the weighting scheme defined below in Equation 2.1 if one sets 0/0 equal to 0.

the output $\tilde{y}$, form the weighted average

$$\tilde{y} = \frac{\sum_{i=1}^{n} y_i m_{A_i}(\tilde{x})}{\sum_{i=1}^{n} m_{A_i}(\tilde{x})}. \tag{2.1}$$

A fuzzy hypothesis A may be a compound statement, such as "our company has been writing a *great deal* of business and earning a *small* amount of profit." In this case, we intersect the fuzzy sets corresponding to a *great deal* of business and a *small* amount of profit with the min operator, as in Definition 2.2. Alternatively, one may use the algebraic product operator to intersect the fuzzy sets, as in Example 2.2. Also, if a compound hypothesis involves the connector *or* and modifier *not*, then use the max and negative operators, respectively, to combine the individual fuzzy sets. In Section 3, we describe how to obtain a specific output y_i, $i = 1, \ldots, n$, if the conclusion is expressed as a fuzzy statement, such as "raise the rates a *great deal*."

EXAMPLE 2.4 To continue with Example 2.3, suppose that we have determined the following values of y_i that correspond to the conclusions in the fuzzy rules that we state in that example:

(a) If the A/E ratio is *high* and the amount of business is *large*, then raise the rates 15%.

(b) If the A/E ratio is *moderate* and the amount of business is *moderate*, then do not change the rates.

(c) If the A/E ratio is *low* and the amount of business is *small*, then lower the rates 10%.

Again, if none of the hypotheses is satisfied, then do not change the rates. We are given that the actual-to-expected (A/E) ratio is 1.05, and the amount of business is 3.0 (on some appropriate scale). Given fuzzy sets for the components of the hypotheses, the next step is to calculate the degree to which the input satisfies each hypothesis. Evaluate the degree of membership of the A/E ratio, 1.05, in the fuzzy sets for *high*, *moderate*, and *low*. Hypothetically, suppose that the A/E ratio is *high* to degree 0.75, *moderate* to degree 0.25, and *low* to degree 0.0. Similarly, evaluate the degree of membership of the amount of business, 3.0, in the fuzzy sets for *large*, *moderate*, and *small*. Suppose that the amount of business is *large* to degree 0.50, *moderate* to degree 0.50, and *small* to degree 0.0. The hypothesis of the first rule is, thus, satisfied to degree min(0.75,0.50) = 0.50; the second rule, min(0.25,0.50) = 0.25; and the third rule, min(0.0,0.0) = 0.0. Our rate change is, therefore,

$$\tilde{y} = \frac{0.50(0.15) + 0.25(0.00) + 0.0(-0.10)}{0.50 + 0.25 + 0.0} = 0.10,$$

or increase the rates 10%. Compare the expression for $\tilde{y}$ with Equation 2.1. If, instead of the min operator, we were to use the algebraic product operator for intersection, the rate change would be

$$\tilde{y} = \frac{0.375(0.15) + 0.125(0.00) + 0.0(-0.10)}{0.375 + 0.125 + 0.0} = 0.1125.$$

In Examples 2.3 and 2.4, we incorporate experience data, the actual-to-expected ratio, in the hypotheses of our fuzzy rules. One may also include experience data in the conclusion, as in the following example.

EXAMPLE 2.5 The following fuzzy rules may more accurately reflect the philosophy of the company:

(a) If the amount of business is increasing *greatly* and the profit margin is decreasing *greatly*, then raise the rates *more* than indicated by the A/E ratio.

(b) If the amount of business is *stable* and the profit margin is *stable*, then change the rates as indicated by the A/E ratio.

(c) If the amount of business is decreasing *greatly* and the profit margin is increasing *greatly*, then *lower* the rates *more* than indicated by the A/E ratio.

3. BUILDING A FUZZY INFERENCE MODEL

The previous section describes how to obtain a crisp output $\tilde{y}$ given a fuzzy inference model and crisp input $\tilde{x}$. This section explains how to construct and fine-tune a fuzzy logic model. Young [18] presents steps that may be followed to build a fuzzy logic model. They are repeated here so that this work is self-contained. Section 4 shows how to follow these steps in creating and fine-tuning a fuzzy logic model. Because the following procedure formalizes the discussion in Section 2, the casual or first-time reader may wish to skip to Section 4.

1. *Verbally state linguistic rules.* These rules may reflect current or desired company philosophy. They may arise from the business sense of actuaries. They may result from the combined input of several functions in the insurance company.

2. *Create the fuzzy sets corresponding to the hypotheses.* Assume that the linguistic variables used are naturally ordered. For example, the linguistic variable of amount of business is naturally ordered because *large* amounts of business correspond with large numbers that measure the amount of business, and similarly for *small* amounts of business.

   (a) To create the fuzzy sets for the jth dimension of the input, partition the input space $X_j = [x_{j,1}, x_{j,n(j)}]$ into $n(j) - 1$ disjoint subintervals, one fewer than the number, $n(j)$, of fuzzy sets defined on X_j. Write the boundary points of the subintervals:

$$x_{j,1} \leq x_{j,2} \leq \cdots \leq x_{j,n(j)}.$$

Even though the input space X_j may be infinitely long, the example below describes how to determine $x_{j,1}$ and $x_{j,n(j)}$ so that we can effectively limit X_j to the finite interval $[x_{j,1}, x_{j,n(j)}]$.

(b) The graph of the leftmost fuzzy set $A_{j,1}$ is defined to be the line segment joining the points $(x_{j,1}, 1)$ and $(x_{j,2}, 0)$ and 0 elsewhere. The graph of each of the middle $n(j) - 2$ sets $A_{j,k(j)}$ is the triangular fuzzy set that connects the points $(x_{j,k(j)-1}, 0)$, $(x_{j,k(j)}, 1)$, and $(x_{j,k(j)+1}, 0)$ and 0 elsewhere, $k(j) = 2, \ldots, n(j) - 1$. Finally, the $n(j)$-th fuzzy set $A_{j,n(j)}$ is the line segment joining the points $(x_{j,n(j)-1}, 0)$ and $(x_{j,n(j)}, 1)$ and 0 elsewhere. Note that for any input value of x_j, the sum (over $k(j)$) of its membership values in the sets $A_{j,k(j)}$ is 1; thus, we say that the $A_{j,k(j)}$ form a *fuzzy partition* of X_j.

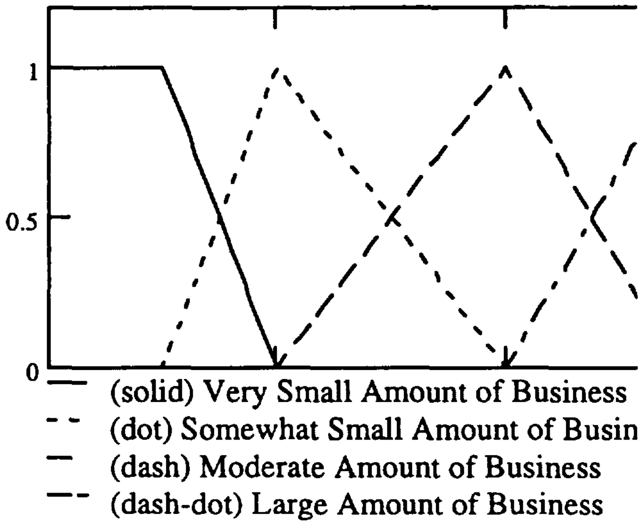
See Figure 4 for an illustration of a partition of the variable of amount of business into four fuzzy sets. Other forms of fuzzy sets may be used to partition a variable, but triangular fuzzy sets are easy to compute and are completely determined by the points in the partition of X_j.

(c) Combine the fuzzy sets that comprise each hypothesis into one fuzzy set using the operators min, max, and negative, corresponding to the linguistic connectors *and* and *or* and modifier *not*, respectively.

3. *Determine the output values $\{y_i\}$ for the conclusions.* Set the output value y_i to the desired output if the hypothesis of rule i is met to degree 1.0.

FIGURE 4

A FUZZY PARTITION OF AMOUNT OF BUSINESS



4. *Fine-tune the fuzzy rules, if applicable.* If learning data is available, either historical data that is still relevant or hypothetical data from experts, then use that data to modify the values $x_{j,k(j)}$ and the values y_i. This is done to optimize any one of a number of objectives. In this work, we minimize a squared-error loss function.

Given data of the form $\{(x_l^*, y_l^*) : l = 1, \ldots, L\}$, pairs of input and output values, either from prior rating periods or from experts' opinions, the model may be fine-tuned using the following simple method: Perturb the parameters $\{x_{j,k(j)}\}$ and $\{y_i\}$ to minimize the squared error

$$\sum_{l=1}^{L} (y_l^* - \tilde{y}(x_l^*))^2,$$

in which $\tilde{y}(x_l^*)$ is the output of the fuzzy logic model, given the input x_l^*. These errors may also be weighted to reflect the relative importance of each ordered pair. In the next section, we minimize such a weighted sum of squared errors:

$$\sum_{l=1}^{L} w_l(y_l^* - \tilde{y}(x_l^*))^2, \tag{3.1}$$

in which w_l, $l = 1,\ldots,L$, is the weight for the pair (x_l^*,y_l^*). The data, $\{(x_l^*,y_l^*) : l = 1,\ldots,L\}$, is called *learning data* because one "trains" the fuzzy logic system to follow the data to the degree measured by Equation 3.1.

The interested reader may wish to explore other methods for fine-tuning a fuzzy logic model. Glorennec [8], Katayama et al. [10], and Driankov et al. [6] describe several methods for adjusting the parameters to fit learning data. Also, Young [17] proposes using a measure of implication derived from fuzzy subsethood to fine-tune fuzzy logic models. This measure of implication measures the degree to which the input implies the output. To fine-tune a given model, therefore, perturb the parameters of the model to maximize this measure of implication.

4. WORKERS COMPENSATION EXAMPLE

Here is an example of building and fine-tuning fuzzy logic models, using workers compensation insurance data from an insurance company for four consecutive rating periods. Call the insurance company Workers Compensation Insurer (WCI). To protect the interests of this insurance company, the data has been masked by linearly transforming it and by relabeling the geographic regions and the dates involved.

There is a distinction between prescriptive and descriptive modeling. The first part of this section briefly explains the decision process that WCI works through every six months, and proposes and builds fuzzy logic systems that model that process. That is, fuzzy models are built based on the expert opinions of the actuaries and other managers at WCI. This is prescriptive modeling, and it corresponds to Steps 1 through 3 in Section 3. The second part of this section fits three fuzzy logic models based on the data that WCI provides, using Step 4 in Section 3. That is, we seek to find fuzzy models that describe what WCI has actually done in the past.

WCI files rates for its workers compensation insurance line in various states. Every six months, WCI determines the adequacy of those filed rates. WCI represents that adequacy by an *indicated target*. For example, an indicated target of +5% in a state means that WCI requires premiums equal to 105% of its filed rates to reach a specified return on surplus. Similarly, an indicated target of −7% means that WCI requires premium equal to 93% of its filed rates.

In the fuzzy models, the indicated target is based on the experience data. WCI calculates it by comparing the filed rates in a state with the sum of the experience loss ratio and expense ratio in that state, among other items. Based on the indicated target and supplementary (financial and marketing) data, WCI then chooses a selected target for each state. (See Section 4.1.1 for more about how WCI selects a target.) Financial data include competitively-driven rate departures with respect to previous selected targets. For example, a rate departure of −1% means that actual premium was 99% of (filed rates)∗(1 + selected target). Marketing data include retention ratios and actual versus planned initial premium.

4.1. *Prescriptive Modeling*

4.1.1 *Verbally state linguistic rules.* To develop linguistic rules for a prescriptive model, the pricing actuaries and product developers at WCI provide information about how an ideal "target selector" would use the data for choosing a target. As a rule of thumb, if the indicated target increases over the previous six months, then the selected target increases, and vice versa. However, this rate change is tempered by how well the region met its previous targets and by how much business is written in the region. For example, if the region had a positive rate departure recently, then WCI might consider increasing the selected target. Also, if the amount of business is low relative to planned, then WCI might consider decreasing the selected target in order to stimulate growth. On the other hand, a large amount of initial business (relative to planned initial business) may not be desirable because of the legal or competitive climate in a given state.

In view of the opinions of the experts at WCI, the following linguistic rules are developed on which to base a prescriptive fuzzy logic model:

(a) If the change in indicated target from time $t-1$ to time t is *positive*, and if the recent rate departure is *positive*, and if the amount of business is *good*, then the change in selected target from time $t-1$ to time t is *positive*.

(b) If the change in indicated target from time $t-1$ to time t is *zero*, and if the recent rate departure is *zero*, and if the amount of business is *moderate*, then the change in selected target from time $t-1$ to time t is *zero*.

(c) If the change in indicated target from time $t-1$ to time t is *negative*, and if the recent rate departure is *negative*, and if the amount of business is *bad*, then the change in selected target from time $t-1$ to time t is *negative*.

Methods for measuring the amount of business include premium, number of accounts, retention ratio, close ratio (percentage of new business written to new business quoted), and premium for new business. This paper measures amount of business by the sum of the retention ratio and the minimum of the ratio of actual initial premium to planned initial premium and the inverse of that ratio, that is, min(actual/planned, planned/actual). This minimum lies between 0.0 and 1.0, and it takes into account that writing a great deal of business (relative to the planned initial premium) is not necessarily a profitable goal. The closer the minimum is to 1.0, the better the region has met its target. If the ratio actual/planned is very small or very large, then min(actual/planned, planned/actual) is close to 0.0. Therefore, a *good* amount of business is measured relative to a maximum of 2.0, after expressing the retention ratio as a decimal.

4.1.2 *Create the fuzzy sets corresponding to the hypotheses and determine the output values for the conclusions.* In the above linguistic rules, each hypothesis is a compound statement that combines three fuzzy sets with the connector *and*. Denote the space of change in indicated target by X_1, the space of rate departures (RD) by X_2, and the space of amount of business by X_3. On each of these spaces, define three fuzzy sets—one for each fuzzy rule.

To get the endpoints of each of these spaces and the intermediate boundary points, work backwards as follows: Determine the maximum and minimum changes in the selected target from time $t-1$ to time t. For example, suppose that the maximum allowable change in selected target is +10%, and the minimum is −10%. Then, determine the changes in indicated target, the rate departures, and the sum of retention ratio and min(A/P, P/A) that would lead to those maximum and minimum changes. Suppose that the selected target would be increased 10% if the indicated target increased by at least 15%, if the rate departure were at least +3%, and if the measure of the amount of business were greater than or equal to 1.8. Also, suppose that the selected target would be decreased by 10% if the indicated target decreased by at least 10%, if the rate departure were at least −5%, and if the measure of the amount of business were less than or equal to 1.0.

Then, the space of change in indicated target is effectively $X_1 = [-10\%, 15\%]$, the space of rate departures is $X_2 = [-5\%, 3\%]$, and the space of amount of business is $X_3 = [1.0, 1.8]$. In this representation, all rate departures less than -5% are identified with -5% because the change in selected target resulting from any rate departure less than -5% is the same as the change in selected target if the rate departure were identically -5%. Observed values outside the ranges selected for other variables are treated similarly.

To get the intermediate points at which no change in selected target occurs, decide what values of change in indicated target, rate departure, and amount of business would lead to no change. Suppose that these values are 0%, 0%, and 1.6, respectively. The defining equations of the fuzzy sets for *positive*, *zero*, and *negative* changes in indicated target (*chind*) are, respectively,

$$m_{positive}(chind) = \max\left[0, \min\left(\frac{chind - 0}{15 - 0}, 1\right)\right]$$

$$m_{zero}(chind) = \max\left[0, \min\left(\frac{15 - chind}{15 - 0}, \frac{chind + 10}{0 + 10}\right)\right]$$

$$m_{negative}(chind) = \max\left[0, \min\left(1, \frac{0 - chind}{0 + 10}\right)\right].$$

Similarly, the defining equations of the fuzzy sets for *positive*, *zero*, and *negative* rate departures are, respectively,

$$m_{positive}(rd) = \max\left[0, \min\left(\frac{rd - 0}{3 - 0}, 1\right)\right]$$

$$m_{zero}(rd) = \max\left[0, \min\left(\frac{3 - rd}{3 - 0}, \frac{rd + 5}{0 + 5}\right)\right]$$

$$m_{negative}(rd) = \max\left[0, \min\left(1, \frac{0 - rd}{0 + 5}\right)\right],$$

and the defining equations of the fuzzy sets for *good*, *moderate*, and *bad* amounts of business are, respectively,

$$m_{good}(bus) = \max\left[0, \min\left(\frac{bus - 1.6}{1.8 - 1.6}, 1\right)\right]$$

$$m_{moderate}(bus) = \max\left[0, \min\left(\frac{1.8 - bus}{1.8 - 1.6}, \frac{bus - 1.0}{1.6 - 1.0}\right)\right]$$

$$m_{bad}(bus) = \max\left[0, \min\left(1, \frac{1.6 - bus}{1.6 - 1.0}\right)\right].$$

Finally, the change in selected target is given by

$$\begin{aligned}&[m_{A_1}(chind, rd, bus) \cdot 10 + m_{A_2}(chind, rd, bus) \cdot 0\\ &\quad + m_{A_3}(chind, rd, bus) \cdot (-10)]\\ &\div [m_{A_1}(chind, rd, bus) + m_{A_2}(chind, rd, bus)\\ &\quad + m_{A_3}(chind, rd, bus)],\end{aligned}$$

(see Equation 2.1) in which

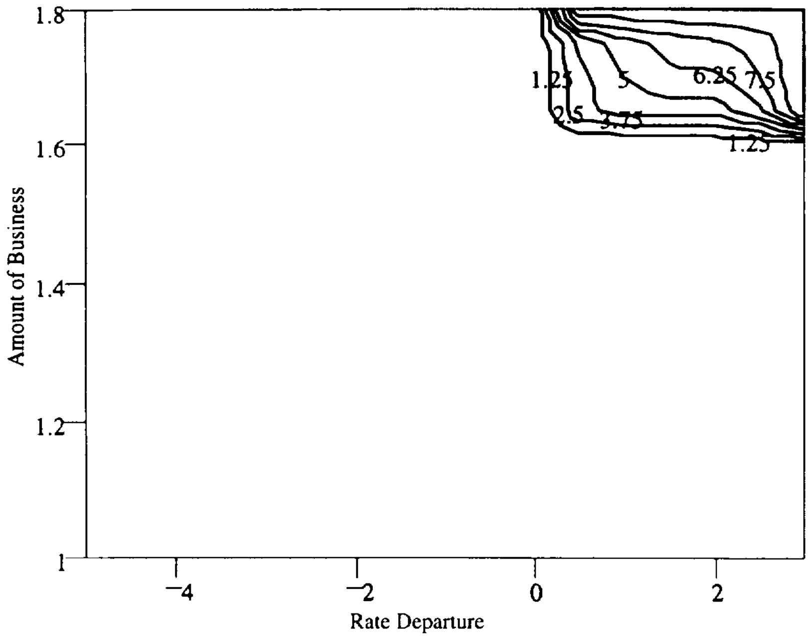
$$\begin{aligned}&m_{A_1}(chind, rd, bus)\\ &\quad = \min[m_{positive}(chind), m_{positive}(rd), m_{good}(bus)]\\ &m_{A_2}(chind, rd, bus)\\ &\quad = \min[m_{zero}(chind), m_{zero}(rd), m_{moderate}(bus)]\\ &m_{A_3}(chind, rd, bus)\\ &\quad = \min[m_{negative}(chind), m_{negative}(rd), m_{bad}(bus)].\end{aligned} \tag{4.1}$$

Figure 5 plots contours of the change in selected target against rate departure and amount of business while fixing the change in indicated target at +10%. Amount of business is along the vertical and rate departure lies along the horizontal. Note that the region for "no change" is relatively large.

If the three variables—change in indicated target, rate departure, and amount of business—may interact when connected by

FIGURE 5

CONTOURS OF CHANGE IN SELECTED TARGET AGAINST AMOUNT OF BUSINESS AND RD (USING AND IN ALL THREE RULES)



and, then consider replacing the min operator with the algebraic product. Also other intersection operators may be used, including those that form weighted averages of the values of the membership functions. There are many ways to formulate the fuzzy rules, but an actuary should, at a minimum, check contour plots to see which formulation coincides with the philosophy or practices of the company. For example, in Figure 5, it should be verified that such a large area of no change is consistent with the company's pricing philosophy when the change in the indicated target is +10%.

TABLE 1

Variable 1	Variable 2	Weighted Correlations
Current Indicated	Current Selected	0.952
Change in Indicated	Change in Selected	0.812
Previous Selected	Current Selected	0.909
Previous RD	Change in Selected	0.281
Current RD	Change in Selected	0.151
Previous Retention	Current Selected	−0.412
Previous Retention	Change in Selected	0.236
Current Retention	Current Selected	−0.429
Current Retention	Change in Selected	−0.035
Actual/Planned Initial	Current Selected	−0.115
Actual/Planned Initial	Change in Selected	−0.149
min(Act/Plan,Plan/Act)	Current Selected	−0.270
min(Act/Plan,Plan/Act)	Change in Selected	0.005

4.2. Descriptive Modeling

Turning to the descriptive portion of fuzzy modeling, fuzzy models are fit to the data that WCI provided. In selecting a target, the actuaries consider the relative amount of business in each state. For this reason, the fuzzy models were fine-tuned by minimizing a weighted sum of squared errors, as in Equation 3.1. The data for each period were weighted according to the premium in each state, after normalizing the weights so that they add to 1.00. Then, each six-month period was weighted equally. That is, a weighted sum of squared errors was calculated for each six months, then those four numbers were added together to get a total sum of squared errors. As a benchmark for the fuzzy models, linear functions were fitted via weighted least squares regression.

Weighted correlations were calculated between variables of interest. See Table 1 for those correlations. The weights used were the same as those used in fine-tuning the fuzzy logic models and in calculating the linear regressions. Note that the correlations between the current indicated and current selected, between

TABLE 2

WEIGHTED CORRELATIONS BETWEEN ERRORS AND REMAINING VARIABLES

	Model (1), Exhibit 1	Model (1), Exhibit 3	Model (2), Exhibit 3
Previous RD	0.025	−0.075	−0.074
Current RD	0.085	0.027	−0.058
Previous Retention	−0.124	−0.059	−0.079
Current Retention	0.160	−0.063	−0.132
Actual/Planned Initial	0.064	−0.007	0.006
min(Actual/Plan,Plan/Actual)	−0.066	0.081	0.027

the change in indicated and change in selected, and between the previous and current selected targets are fairly high.

An actuary may begin by considering simple fuzzy logic models involving one or two explanatory variables that correlate highly with the change in the selected target or the target itself. Starting from the simple models that fit most closely to the data, an actuary may then expand them to include more complicated models with two or more explanatory variables. Those more complicated models may not be substantially more accurate than the simple ones. In this case, the correlations between the errors from the simple models and remaining variables of interest (see Table 2) are fairly small, thus confirming that more complicated models may not add accuracy to the description of WCI's target selecting practices.

In general, models with two rules are nearly as accurate as those with three or more rules. For this reason, only the results of fine-tuning models with two rules (plus the default rule of no action if none of the hypotheses is satisfied) are presented here. Exhibits 1 through 3 display the results obtained using three simple models. To fine-tune these fuzzy logic models, Excel's Solver was used to minimize the weighted sum of squared errors from the four six-month periods, given the starting values, as listed in Exhibits 1 through 3. Solver uses a gradient-descent method (beginning with the starting values) to optimize an objective function subject to constraints. The starting values are the endpoints of the input spaces and the changes in the selected (or the selected target itself) that correspond to the conclusions. Each model described in the exhibits involves only two fuzzy rules, plus the default rule of no action, so only the boundary points of the input spaces and the two output values (either the selected target or changes in the selected) were specified. In the optimization, the left-hand endpoint was constrained to be less than or equal to the right-hand endpoints. In general, the solution obtained by Excel's Solver yields a local minimum. Although the global minimum may not have been reached, the solution may be desirable because, in some sense, it is close to the initial system.

Exhibit 1 considers rules that depend on the value of the change in the indicated target from the previous selection period. To compare with a standard model, the weighted least squares regression line that uses the same explanatory variable, namely, the change in indicated target, is included. The fuzzy model fits only slightly better, as measured by the sum of squared errors, than does the linear regression. In the fuzzy model in Exhibit 1, the starting values of -10.00 and 10.00 for the change in the indicated target from the previous period imply that the space of indicated targets is partitioned into the two fuzzy sets graphed in Figure 6. The set $\{-10.00, 10.00\}$ associated with the change in the selected target means that if the change in the indicated target were -10.00 or less, then the change in the selected would be -10.00. Similarly, if the change in the indicated target were 10.00 or more, then the change in the selected would be 10.00. Figure 7 graphs the change in the selected target as a function of the change in the indicated, before fine-tuning using Excel's Solver.

To minimize the squared-error loss in Equation 3.1, both the endpoints of the interval for the change in the indicated tar-

FIGURE 6

PARTITION OF CHANGE IN THE INDICATED TARGET BEFORE SOLVER SOLUTION

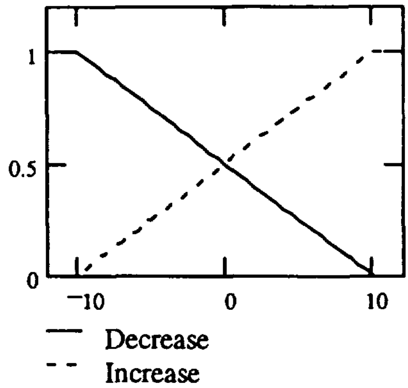


FIGURE 7

CHANGE IN SELECTED TARGET AS A FUNCTION OF CHANGE IN INDICATED TARGET BEFORE SOLVER SOLUTION

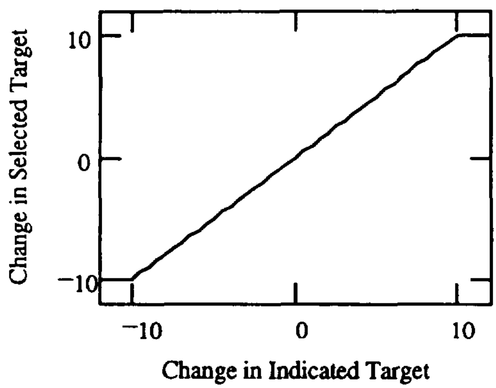
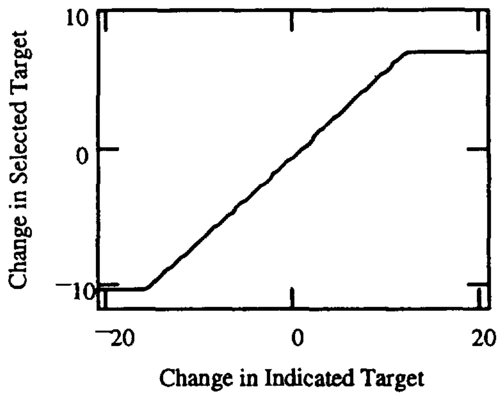


FIGURE 8

CHANGE IN SELECTED TARGET AS A FUNCTION OF CHANGE IN INDICATED TARGET AFTER SOLVER SOLUTION



get, −10.00 and 10.00, and the changes in the selected target, −10.00 and 10.00, are varied. The interval for change in the indicated target becomes the interval [−15.73, 12.34], and the interval for changes in selected target becomes [−10.52, 6.92]. Thus, the maximum decrease in selected target is −10.52 and the maximum increase is 6.92. See Figure 8 for a graph of the change in the selected target as a function of the change in the indicated target, after fine-tuning using Excel's Solver.

The form of the presentation and the results are similar in the following two exhibits. Exhibit 2 expands on the model in Exhibit 1 by considering the most recent and the previous rate departures. Exhibit 3 calculates the selected target itself as a function of the current indicated target and the previous selected target. Two fuzzy models were fitted—one that joins the phrases in the hypotheses with *or* (the max operator), and another that uses *and* (the min operator). The former model provides the better fit of the two models, and it has an average error 0.35% smaller than that of linear regression.

5. SUMMARY AND CONCLUSIONS

This paper demonstrates how to build and fine-tune a fuzzy logic system from linguistic rules to finished model, while distinguishing between the prescriptive phase and the descriptive phase. It emphasizes models that combine experience data with supplementary data. It compares those fuzzy models with linear regressions to judge their performance.

Even though a given fuzzy logic model may fit only slightly better than a standard linear regression model, the main advantage of fuzzy logic is that an actuary can begin with verbal rules and create a mathematical model that follows those rules. Fuzzy logic allows linguistic rules to be handled in a consistent manner; it allows possibly conflicting goals and constraints to be combined. By fine-tuning a model using historical data, an actuary can judge whether his or her company has followed those rules. A model can also be fine-tuned based on information from several (possibly conflicting) experts.

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EXHIBIT 1

CHANGE IN SELECTED TARGET AS A FUNCTION OF THE CHANGE IN INDICATED TARGET

(1) **Fuzzy model:**

(a) If the indicated target decreases from time $t-1$ to time t, then decrease the selected target from time $t-1$ to time t.

(b) If the indicated target increases from time $t-1$ to time t, then increase the selected target from time $t-1$ to time t.

	Starting Values	Solver Solution
Indicated Change	$[-10.00, 10.00]$	$[-15.73, 12.34]$
Selected Change	$\{-10.00, 10.00\}$	$\{-10.52, 6.92\}$

Sum of squared errors 34.98

Average error $\sqrt{34.98/4} = 2.96$

(2) **Linear regression:**

Change in selected = $-0.84 + 0.48 *$(change in indicated)

Sum of squared errors 39.62

Average error $\sqrt{39.62/4} = 3.15$

EXHIBIT 2

CHANGE IN SELECTED TARGET AS A FUNCTION OF THE CHANGE IN INDICATED TARGET AND OF THE RATE DEPARTURE

(1) **Fuzzy model using the most recent rate departure:**

(a) If the indicated target decreases from time $t-1$ to time t and if the recent rate departure (RD_t) is negative, then decrease the selected target from $t-1$ to time t.

(b) If the indicated target increases from time $t-1$ to time t and if the recent rate departure (RD_t) is positive, then increase the selected target from time $t-1$ to time t.

Note: By default, if both hypotheses have zero weight, then do not change the selected target.

	Starting Values	Solver Solution
Indicated Change	$[-10.00, 10.00]$	$[-15.73, 12.34]$
RD_t	$[-10.00, 10.00]$	$[-37.87, 23.20]$
Selected Change	$\{-10.00, 10.00\}$	$\{-11.09, 7.32\}$

Sum of squared errors 32.88

Average error $\sqrt{32.88/4} = 2.87$

(2) **Fuzzy model using the previous rate departure:**

(a) If the indicated target decreases from time $t-1$ to time t and if the previous rate departure (RD_{t-1}) is negative, then decrease the selected target from time $t-1$ to time t.

(b) If the indicated target increases from time $t-1$ to time t and if the previous rate departure (RD_{t-1}) is positive, then increase the selected target from time $t-1$ to time t.

	Starting Values	Solver Solution
Indicated Change	$[-10.00, 10.00]$	$[-13.31, 11.85]$
RD_{t-1}	$[-10.00, 10.00]$	$[-23.06, 12.67]$
Selected Change	$\{-10.00, 10.00\}$	$\{-11.25, 7.09\}$

Sum of squared errors 32.74

Average error $\sqrt{32.74/4} = 2.86$

(3) **Linear regression using the most recent rate departure:**

Change in selected $= -0.75 + 0.48 *$(change in indicated) $+ 0.22 * RD_t$

Sum of squared errors 38.28

Average error $\sqrt{38.28/4} = 3.09$

(4) **Linear regression using the previous rate departure:**

Change in selected $= -0.34 + 0.47 *$(change in indicated) $+ 0.42 * RD_{t-1}$

Sum of squared errors 37.26

Average error $\sqrt{37.26/4} = 3.05$

EXHIBIT 3

SELECTED TARGET AS A FUNCTION OF THE INDICATED TARGET AND OF THE PREVIOUS SELECTED TARGET

(1) **Fuzzy model using *or*:**

(a) If the current indicated target or the previous selected target is low, then the current selected target is low.

(b) If the current indicated target or the previous selected target is high, then the current selected target is high.

	Starting Values	Solver Solution
Indicated_t	[−20.00, 20.00]	[−131.38, 62.67]
Selected_{t-1}	[−20.00, 20.00]	[−143.55, 48.75]
Selected_t	{−20.00, 20.00}	{−124.99, 48.03}

Sum of squared errors 27.09

Average error $\sqrt{27.09/4} = 2.60$

(2) **Fuzzy model using *and*:**

(a) If the current indicated target and the previous selected target are low, then the current selected target is low.

(b) If the current indicated target and the previous selected target are high, then the current selected target is high.

	Starting Values	Solver Solution
Indicated_t	[−20.00, 20.00]	[−20.27, 48.18]
Selected_{t-1}	[−20.00, 20.00]	[−31.18, 36.64]
Selected_t	{−20.00, 20.00}	{−21.55, 28.53}

Sum of squared errors 34.99

Average error $\sqrt{34.99/4} = 2.96$

(3) **Linear regression:**

$\text{Selected}_t = -3.84 + 0.45 * \text{Indicated}_t + 0.38 * \text{Selected}_{t-1}$

Sum of squared errors 34.90

Average error $\sqrt{34.90/4} = 2.95$