ON APPROXIMATIONS IN LIMITED FLUCTUATION CREDIBILITY THEORY

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Abstract

Different approximation methods to find a full credibility level in limited fluctuation credibility are studied, and it is concluded that, in most cases, there is no significant difference between the various results. Since Venter [9] presented an opposite conclusion, it is emphasized that his approach to the problem is different and that the formula he derives should be used only in his given context.

ACKNOWLEDGEMENT

The author is very grateful to Professor François Dufresne for many improvements and enhancements to this paper and to various anonymous referees for their valuable comments on previous versions of the paper.

1. INTRODUCTION

"Limited fluctuation" credibility is the oldest branch of credibility theory, the other branch being "greatest accuracy" credibility. Also sometimes called American credibility, limited fluctuation credibility originates from the beginning of this century with Mowbray's paper "How Extensive a Payroll Exposure is Necessary to Give a Dependable Pure Premium?" [7]. The title is self-explanatory: Mowbray was interested in finding a level of payroll in workers compensation insurance for which the pure premium of a given risk would be considered fully credible.

The theory has not evolved much since then. The answer to Mowbray's question—which is Mowbray's answer, as a matter of fact—has remained basically the same (see Section 2). With the emergence of risk theory methods, though, the original problem has been formulated in a more general way, and new techniques have been used to find the full credibility level. This paper will first investigate if more powerful and sophisticated approximation methods are more worthwhile than the straightforward normal approximation. Then, because our conclusion will differ from that of Venter [9], the paper will show that Venter's full credibility requirement systematically exceeds that given by the normal approximation.

2. THE MODEL

Let

S = random variable of the total claim amount of a risk over a given period of time (usually 1 year);

X_j = random variable of the amount of the jth claim;

N = random variable of the claim count of the risk over the given period.

Then,

$$S = X_1 + X_2 + \cdots + X_N,$$

where $X_1, X_2, \ldots, X_n$ are independent, identically distributed (i.i.d.) random variables mutually independent of N.[1] This is the classical collective model of risk theory. Most of the situations usually encountered in limited fluctuation credibility can be described by an application of this model. It is also well known (see Gerber [3]) that

$$\mathrm{E}[S] = \mathrm{E}[N]\mathrm{E}[X_j],$$

$$\mathrm{Var}[S] = \mathrm{E}[N]\mathrm{Var}[X_j] + \mathrm{Var}[N]\mathrm{E}[X_j]^2.$$

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[1] In reality, the losses may be only conditionally independent given some parameters, such as the inflation rate, to which the losses will all be exposed jointly.

The fundamental problem of limited fluctuation credibility, only slightly adapted from Mowbray's original idea, is: What are the parameters of the distribution of S such that the Equation

$$\Pr[(1-k)\mathrm{E}[S] \leq S \leq (1+k)\mathrm{E}[S]] \geq p \tag{2.1}$$

is verified? Using distribution functions, Equation 2.1 can also be written

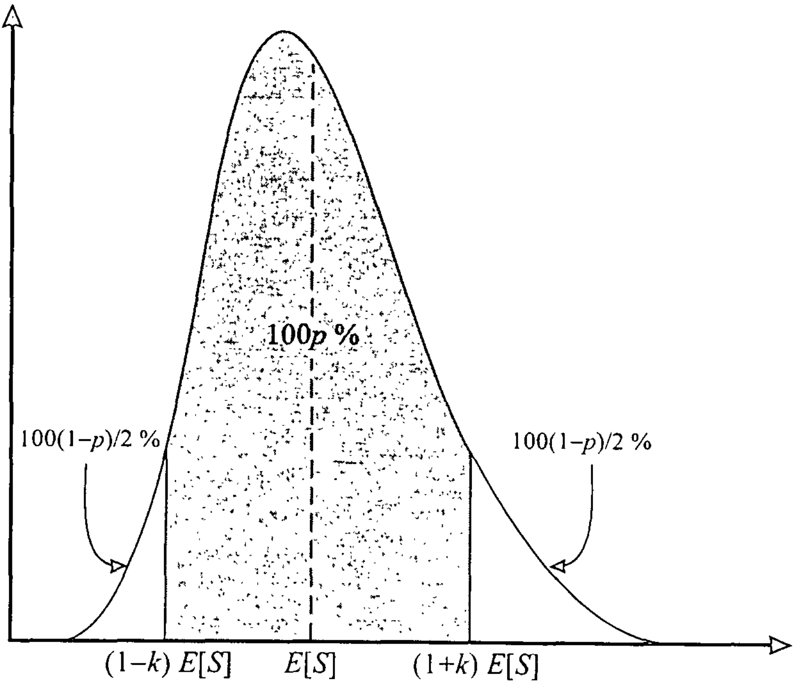
$$F_S((1+k)\mathrm{E}[S]) - F_S((1-k)\mathrm{E}[S]) \geq p. \tag{2.2}$$

This requires that with probability $100p\%$, the total claim amount of a risk stays within $100k\%$ of its expected value (see Figure 1). When a risk meets these requirements, we say that it deserves a full credibility of order (k,p). That is, the risk is charged a pure premium based solely on its own experience. After m periods of time, that premium would simply be the empirical mean $\overline{S} = (S_1 + S_2 + \cdots + S_m)/m$, where each S_i $(i = 1,2,\ldots,m)$ is distributed as S.

In a usual limited fluctuation credibility situation, the parameter k will be quite small, e.g., 5–10%, while the parameter p will be large, often above 90%. Equation 2.1 thus requires the distribution of S to be relatively concentrated around its expected value. Since S is a (random) sum of i.i.d. random variables, one way to achieve such a kind of distribution is to sum a "large" number of those random variables—provided their second moment is finite. The distribution of the sum will then tend towards a normal distribution more relatively concentrated around its mean (that is, the ratio of the standard deviation to the expected value decreases) as the number of terms in the sum increases. Accordingly, the natural way to verify Equation 2.1 is to base the criteria for full credibility on the expected number of claims. (Note that the severity still enters the calculation through the X_js, as it should.) The level of full credibility will then usually be expressed in terms of the expected value of N, which could

FIGURE 1

MOWBRAY'S TWO-SIDED FULL CREDIBILITY CRITERION REQUIRES $100p\%$ OF THE PROBABILITY OF S TO BE CONCENTRATED WITHIN $100k\%$ OF ITS EXPECTED VALUE



represent, for example, the number of claims, the number of employees, or the total payroll. Besides, it is intuitively preferable to base the criterion on some kind of exposure base rather than on the individual amount of the claims.

At this point, most of the theory of limited fluctuation credibility has been covered. What follows are the calculations needed to satisfy Equation 2.1. However, these calculations are more relevant to general risk theory than to credibility theory.

Before going further, we define the skewness of a random variable X as

$$\gamma_1(X) = \mathrm{E}\left[\left(\frac{X - \mathrm{E}[X]}{\sqrt{\mathrm{Var}[X]}}\right)^3\right]. \tag{2.3}$$

Obviously, a symmetrical random variable has $\gamma_1(X) = 0$.

3. THE COMPOUND POISSON CASE

The compound Poisson is a distribution frequently used for S. It is said that the distribution of S is compound Poisson of parameters λ and G when the random variable N follows a Poisson distribution of parameter λ and the random variables X_j $(j = 1,\ldots,n)$ have distribution function G. Let $P_k = \mathrm{E}[X_j^k]$. Then (see, e.g., Gerber [3]):

$$\mathrm{E}[S] = \lambda P_1, \tag{3.1}$$

$$\mathrm{Var}[S] = \lambda P_2, \tag{3.2}$$

$$\gamma_1(S) = \frac{P_3}{\sqrt{\lambda} P_2^{3/2}}. \tag{3.3}$$

Equation 3.1 says that the expected value of the total claim amount is simply the product of the expected values of the number of claims and the amount per claim. In Equation 3.2, we see that the variance of the total claim amount is given by the second moment (P_2) of the claim amount times the expected number (λ) of claims. Finally, Equation 3.3 shows that the skewness of S decreases as the expected number of claims increases.

Further on, we will refer to this model simply as the "compound Poisson case."

4. THREE APPROXIMATION METHODS

In theory, the exact solution of the limited fluctuation credibility problem would be obtained by calculating the exact distribution of S with the convolution formula (see, for example, Gerber

[3]). However, since the expected number of claims is usually quite large, that calculation would represent too long and laborious a task and would, in general, first require transforming the continuous distribution into a discrete one (a procedure known as "discretization"). In fact, nobody ever really intended to calculate the convolutions to solve the problem under study, approximations instead always being used. We present here three common approximation methods that could be used to estimate the distribution of S and then find the parameters that satisfy Equation 2.1.

The first approximation method we present is the most widely used in limited fluctuation credibility: the classic normal approximation. In general, the distribution of S is not symmetrical, even if that of X_j is. However, the limited fluctuation criteria will require the number of claims to be large, thus yielding an almost symmetrical distribution for S. By the version of the Central Limit Theorem applicable to random sums (Feller [2], p. 258), it is reasonable to approximate the distribution of $(S - \mathrm{E}[S])/\sqrt{\mathrm{Var}[S]}$ by a standard normal distribution. Equation 2.1 may then be rewritten

$$\begin{aligned}\Pr\left[-\frac{k\mathrm{E}[S]}{\sqrt{\mathrm{Var}[S]}} \leq \frac{S-\mathrm{E}[S]}{\sqrt{\mathrm{Var}[S]}} \leq \frac{k\mathrm{E}[S]}{\sqrt{\mathrm{Var}[S]}}\right] &\approx \Phi\left(\frac{k\mathrm{E}[S]}{\sqrt{\mathrm{Var}[S]}}\right) - \Phi\left(-\frac{k\mathrm{E}[S]}{\sqrt{\mathrm{Var}[S]}}\right) \\ &= 2\Phi\left(\frac{k\mathrm{E}[S]}{\sqrt{\mathrm{Var}[S]}}\right) - 1 \geq p. \end{aligned} \tag{4.1}$$

Thus,

$$\mathrm{E}[S] \geq \left(\frac{z_{1-\varepsilon/2}}{k}\right)\sqrt{\mathrm{Var}[S]}, \tag{4.2}$$

where $\varepsilon = 1 - p$ and z_α is the αth percentile of a standard normal distribution. In the compound Poisson case, one finds (see, for example, Perryman [8])

$$\lambda \geq \left(\frac{z_{1-\varepsilon/2}}{k}\right)^2 \left(\frac{P_2}{P_1^2}\right). \tag{4.3}$$

The first ratio represents the normality assumption, while the second accounts for the variability of the claim amounts. Indeed, the full credibility level increases with the square of the coefficient of variation of the random variable X_j. The choice $k = 5\%$, $p = 90\%$, and X_j degenerated at 1 (that is, taking value one with probability one) leads to the famous λ value of 1,082.

The popularity of this approximation, aside from its good precision in the limited fluctuation context when the expected value of N is large, comes from the fact that $F(x) = 1 - F(-x)$. This greatly simplifies the calculations, as it may be seen in Equation 4.1. However, even at the price of heavier calculations, one might be interested to take into account the skewness of S by using more refined approximations.

Two approximations that take the skewness of S into account and are generally considered precise and relatively simple to use will be studied here: the normal power II approximation (using the first three moments; simply called normal power hereafter) and the Esscher approximation. The general formula of the normal power approximation as found in Beard et al. [1] is:

Let

$$y = \frac{x - \mathrm{E}[S]}{\sqrt{\mathrm{Var}[S]}} \qquad \text{and} \qquad y_0 = -\sqrt{7/4},$$

then

$$F_S(x) \approx \begin{cases} \Phi\left(-\dfrac{3}{\gamma_1(S)} + \sqrt{1 + \dfrac{9}{\gamma_1^2(S)} + \dfrac{6}{\gamma_1(S)}y}\right), & y \geq 1 \\ \Phi\left(y - \dfrac{\gamma_1(S)}{6}(y^2 - 1) + \dfrac{\gamma_1^2(S)}{36}(4y^3 - 7y)\delta(y_0 - y)\right), & y < 1, \end{cases} \tag{4.4}$$

where $\delta(y) = 0$ if $y = 0$ and 1 otherwise. Note that for $y = 1$, both formulae produce $\Phi(1)$.

To use the Esscher approximation, the moment generating function (m.g.f.) of S must exist (preferably in a known form, to simplify the calculations). If the distribution of S is compound Poisson with parameters λ and G, then the Esscher approximation for the distribution function of S is

$$1 - F_S(x) \approx e^{\lambda[m(h)-1]-hx}\left[E_0(u) - \frac{m'''(h)}{6\lambda^{1/2}(m''(h))^{3/2}}E_3(u)\right]. \tag{4.5}$$

where $m(\cdot)$ is the m.g.f. of G,h the solution of $\lambda m'(h) = x$, and $u = h\sqrt{\lambda m''(h)}$. The functions $E_k(\cdot)$ $(k = 0,1,2,\ldots)$ are the Esscher functions:

$$\begin{aligned} E_0(u) &= e^{u^2/2}[1 - \Phi(u)] \\ E_3(u) &= \frac{1-u^2}{\sqrt{2\pi}} + u^3 E_0(u). \end{aligned} \tag{4.6}$$

A more complete description of the Esscher approximation may be found in Gerber [3].

Quite obviously, it is not possible to simplify Equation 2.1 in a form like Equation 4.1 when using the normal power or Esscher approximations. The search for parameters such that Equation 2.1 is satisfied must then be made iteratively. For example, if S is compound Poisson, one must find the smallest value of λ such that $F_S((1+k)\mathrm{E}[S];\lambda) - F_S((1-k)\mathrm{E}[S];\lambda) \geq p$. If the probability obtained with a particular value of λ is smaller than p, then the value of λ must be raised—and vice-versa—until convergence to a unique minimal value is achieved. Note that if the distribution of S remains right-skewed once the full credibility level has been reached, then there will be more probability mass in the right tail than in the left one.

Now, the question is: Are these more complicated and time consuming approximations better (more precise) than the usual normal approximation, still in the context of limited fluctuation credibility? To study this, we made some tests where the distribution of S was held compound Poisson and the distribution of the individual claim amount changed. The parameters of the latter were chosen such that its expected value remained constant at 5,000, but its variance, and especially its skewness, varied. The idea was to make X_j very skewed and then check if the values of λ given by the three approximations would be significantly different, and what would be the resulting skewness of S. Gamma and lognormal distributions were used for X_j, but as the m.g.f. of the latter does not exist, the Esscher approximation was not calculated. The inversion of characteristic functions (ICF) method has also been used to cross-check the results in the gamma cases. This numerical method is used to calculate distribution functions, and its precision is as high as the user desires (see, for example, the Heckman-Meyers algorithm in [4]). It then appeared that the normal power and Esscher approximations can be considered as almost exact in the present application. Table 1 summarizes the results.

From the results of Table 1, we must conclude that it is not necessary to complicate the estimation of the full credibility level by using more sophisticated approximation methods. Indeed, the differences between the various methods are minor—often less than 0.5%. These results and the conclusion drawn from them should not be very surprising since, as stated in Section 2, it is a requirement of the limited fluctuation problem that most of the probability mass be concentrated around the expected value of S. Thus, for k and p constant, the more X_j is skewed, the more the number of claims has to be large to make S a "concentrated" distribution. Intuitively, such a distribution can not be very skewed, thus leading to a good normal approximation. Besides, a quick look at the last column of Table 1 shows that whatever the skewness of X_j, the value of λ will be sufficiently large to result in a quite symmetrical distribution for S.

There remains a peculiar case to be discussed in Table 1: the first lognormal case, where the difference between the normal

TABLE 1

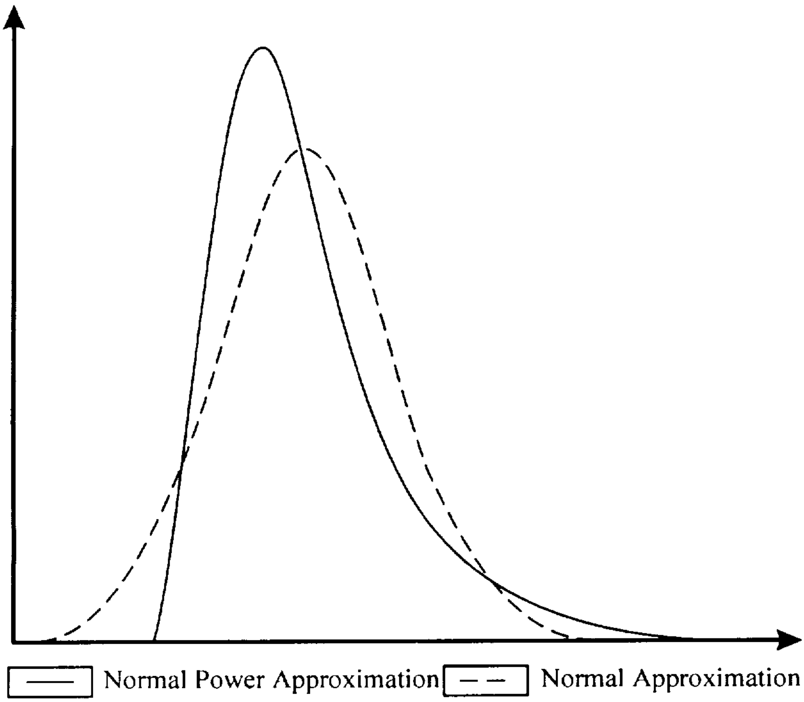
FULL CREDIBILITY LEVELS OBTAINED WITH THREE DIFFERENT APPROXIMATION METHODS IN COMPOUND POISSON CASES

Distribution of X_j	$\gamma_1(X_j)$	k	p	Value of $E[N] = \lambda$: Normal Approximation	Normal Power	Esscher	Largest Difference (%)	$\gamma_1(S)$
Gamma								
$\alpha = 0.01$	20.00	0.050	0.90	109,323	109,258	109,234	0.08	0.06
$\alpha = 0.05$	8.94	0.050	0.95	32,269	32,256	32,257	0.04	0.05
$\alpha = 0.20$	4.47	0.100	0.90	1,624	1,621	1,620	0.23	0.11
$\alpha = 1.10$	1.91	0.025	0.90	8,266	8,264	8,264	0.03	0.02
$\alpha = 5.00$	0.89	0.100	0.95	461	461	461	0.06	0.06
Lognormal								
$\sigma^2 = \ln 50$	364.00	0.050	0.90	54,121	49,232	—	9.03	1.52
$\sigma^2 = 2.00$	23.73	0.050	0.95	11,354	11,301	—	0.47	0.19
$\sigma^2 = 1.50$	12.09	0.100	0.90	1,213	1,203	—	0.77	0.27
$\sigma^2 = 0.75$	4.35	0.025	0.90	9,166	9,163	—	0.03	0.03
$\sigma^2 = 0.65$	3.75	0.100	0.95	736	735	—	0.14	0.10

and the normal power approximations reaches 9%. Clearly, the skewness of 1.52 for S is not insignificant in that case; we would not find that there is precisely a probability of 0.05 both above and below 5% of the mean (in fact, we get 0.089 above and 0.011 below). The normal power estimation of the full credibility level is thus slightly more precise than the normal approximation in that case. The interesting point, though, is that the normal approximation is the higher, or more conservative, of the two. We can thus also conclude from Table 1 that taking the skewness of S into account does not yield higher full credibility levels. In fact, the normal approximation is, in all cases studied, the most conservative one. This can be explained by, for the same expected number of terms in the sum, the normal approximation imputing more probability mass in the left tail than the normal power gains with heavier right tail (see Figure 2).

FIGURE 2

FOR THE SAME EXPECTED NUMBER OF TERMS IN THE SUM, THE NORMAL APPROXIMATION IMPUTES MORE PROBABILITY MASS IN THE LEFT TAIL THAN THE NORMAL POWER GAINS WITH ITS HEAVIER RIGHT TAIL



The above conclusions also mean that the skewness of X_j is not a big issue on the level of full credibility. There is still another interesting way to see that point with the normal power approximation in the compound Poisson case. Since the normal power approximation is only calculated at the points $(1 \pm k)\mathrm{E}[S]$, it is easily seen from Equation 4.4 and Equations 3.1 to 3.3 that all the information one needs about the distribution of X_j to calculate the approximation are the ratios $r_1 \equiv P_1/P_2^{1/2}$ and $r_2 \equiv P_3/P_2^{3/2}$. Assuming that $X_j > 0$, it is easily shown with Jensen's inequality that $r_1 \in [0,1]$ and $r_2 \geq 1$. The left end of the interval for r_1 is not interesting, though, since it represents a zero expected value. The right end represents a zero variance and is thus the frequently used—rightly or wrongly—degenerated case. The ratio r_1 is also the only one needed to calculate the normal approximation and, as such, fully determines in that case the full credibility level—given k and p, of course. Entering in the calculation of $\gamma_1(S)$, the ratio r_2 thus brings the skewness of X_j into the normal power approximation.

Table 2 presents full credibility levels of order (0.05,0.90) for various combinations of the above ratios. For illustration purposes, we have included the $\lambda = 1{,}082$ level, obtained with the combination $r_1 = r_2 = 1$. It should be noted that the entries in the upper left and lower right corners of the table are most unlikely. For the most common distributions (e.g., gamma, lognormal, Pareto), a small r_1 comes with a large r_2, and vice versa. Then, in the really interesting area of the table, we clearly see that the effect of a rather small variation in the value of the ratio r_1 is much more important than a large variation in the value of the ratio r_2. This could also be interpreted as r_1 determining most of the final value of the full credibility level, while r_2 causes only a small, and in most cases negligible, correction to that value.

5. A WORD OF CAUTION

The book *Foundations of Casualty Actuarial Science* published by the Casualty Actuarial Society presents, as the title suggests, different subjects central to casualty insurance practice. The chapter on Credibility Theory—Chapter 7—was written by Gary G. Venter [9]. In the section on limited fluctuation credibility, it is demonstrated by an example (Example 3.1) that the normal power approximation gives a much different estimation of the full credibility level than the one obtained with the nor-

TABLE 2

FULL CREDIBILITY LEVELS OF ORDER (0.05,0.90) IN THE COMPOUND POISSON CASE CALCULATED WITH THE NORMAL POWER APPROXIMATION

	$r_2 = P_3/P_2^{3/2}$		
$r_1 = P_1/P_2^{1/2}$	1	10	300
0.1	108,222	108,210	102,458
0.2	27,055	27,044	24,377
0.3	12,025	12,013	11,172
0.4	6,764	6,753	6,947
0.5	4,329	4,318	4,857
0.6	3,006	2,995	3,652
0.7	2,208	2,198	2,884
0.8	1,691	1,681	2,359
0.9	1,336	1,326	1,981
1.0	1,082	—	—

mal approximation. Naturally, the former is considered the better. This contradiction with the results of the previous section is due to the fact that Venter is not considering exactly the same limited fluctuation problem as above; therefore both normal power approximations can not be directly compared.

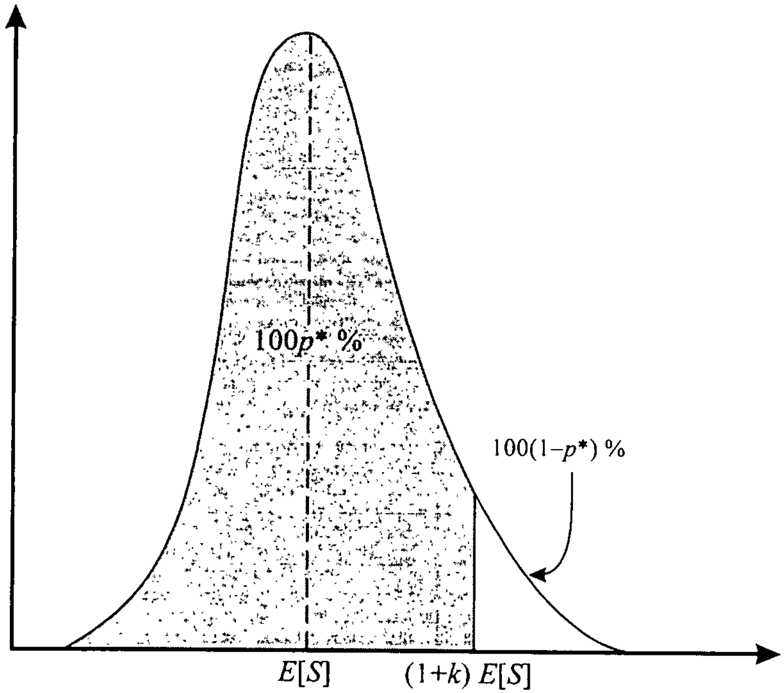
As said before, the normal approximation leads to simple formulae because

$$F_S((1+k)\mathrm{E}[S]) \approx \Phi\left(\frac{k\mathrm{E}[S]}{\sqrt{\mathrm{Var}[S]}}\right) = 1 - \Phi\left(-\frac{k\mathrm{E}[S]}{\sqrt{\mathrm{Var}[S]}}\right)$$
$$\approx 1 - F_S((1-k)\mathrm{E}[S]). \qquad (5.1)$$

Those equalities are not found in the normal power approximation. A simple look at Equation 4.4 is enough to be convinced that $F_S((1+k)\mathrm{E}[S]) \neq 1 - F_S((1-k)\mathrm{E}[S])$. Now, Mr. Venter's approach to the problem is slightly different, as he introduces a simplifying hypothesis right at the beginning. Instead of considering Equation 2.1, he considers a one-sided requirement for full

FIGURE 3

VENTER'S ONE-SIDED FULL CREDIBILITY CRITERION REQUIRES $100p^*\%$ OF THE PROBABILITY OF S TO BE UNDER $100(1+k)\%$ OF ITS EXPECTED VALUE



credibility, namely:

$$\Pr[S \leq (1+k)\mathrm{E}[S]] \geq (1+p)/2 \equiv p^*. \tag{5.2}$$

Thus, instead of requiring that there is a probability of p that S does not deviate from its expected value by more than $100k\%$, it is only required that there is a probability of p^* that S does not exceed its expected value by more than $100k\%$. Therefore, while Equation 2.1 looks at both left and right tails of S, Equation 5.2 looks only at the right tail (see Figure 3).

This different definition of the problem has no effect on the normal approximation since the distribution of S is in any case approximated by a symmetrical distribution. However, when considering the normal power approximation, the necessary iterative calculation can be avoided by applying the simplification assumed in Equation 5.2 to derive a formula of the same form as Equation 4.2. Indeed, Venter [9] defines

$$\begin{aligned} m_2 &= \frac{\mathrm{Var}[N]}{\mathrm{E}[N]} + CV(X)^2 \\ m_3 &= \gamma_1(X)CV(X)^3 + 3\frac{\mathrm{Var}[N]}{\mathrm{E}[N]}CV(X)^2 \\ &\quad + \frac{\mathrm{E}[(N-\mathrm{E}[N])^3]}{\mathrm{E}[N]}. \end{aligned} \tag{5.3}$$

(where $CV(X)$ is the coefficient of variation of the random variable X) and then the following condition is obtained:[2]

$$\mathrm{E}[N] \geq \frac{1}{4k^2}\left[z_{1-\varepsilon/2}\sqrt{m_2} + \sqrt{z_{1-\varepsilon/2}^2 m_2 + \frac{2}{3}\frac{m_3}{m_2}k(z_{1-\varepsilon/2}^2 - 1)}\right]^2. \tag{5.4}$$

In Example 3.1 of [9], S is a compound Poisson distribution. The distribution of the individual claim amount is lognormal with expected value 5,000 and coefficient of variation equal to 7, which amounts to parameter σ^2 equal to ln 50. The full credibility level is defined by $p = 0.90$ ($p^* = 0.95$) and $k = 0.05$. The normal approximation for λ is then correctly given as 54,120. As can be seen in Table 1, the "usual" two-sided normal power approximation would in that case be 53,927, while the result obtained with Equation 5.4 is 80,030. Full credibility levels have also been calculated with Venter's formula for every other case of Table 1. They are compared with previous results in Table 3. In the last column of Table 3 are also displayed the "true" values

[2]There is a misprint in *Foundations of Casualty Actuarial Science*: the square root sign in Equation 3.6 should be longer and end just before the rightmost parenthesis.

TABLE 3

COMPARISON OF FULL CREDIBILITY RESULTS OBTAINED WITH THE NORMAL AND NORMAL POWER APPROXIMATIONS AND WITH VENTER'S FORMULA

Distribution of X_j	k	p	Value of $E[N] = \lambda$: Normal Approximation	Normal Power	Venter's Formula	p for Venter's Results
Gamma						
$\alpha = 0.01$	0.050	0.99	109,323	109,258	111,598	0.9036
$\alpha = 0.05$	0.050	0.95	32,269	32,256	33,042	0.9527
$\alpha = 0.20$	0.100	0.90	1,624	1,621	1,686	0.9065
$\alpha = 1.10$	0.025	0.90	8,266	8,264	8,330	0.9013
$\alpha = 5.00$	0.100	0.95	461	461	474	0.9532
Lognormal						
$\sigma^2 = \ln 50$	0.050	0.90	54,121	49,232	80,029	0.9500
$\sigma^2 = 2.00$	0.050	0.95	11,354	11,301	12,367	0.9596
$\sigma^2 = 1.50$	0.100	0.90	1,213	1,203	1,325	0.9157
$\sigma^2 = 0.75$	0.025	0.90	9,166	9,163	9,268	0.9020
$\sigma^2 = 0.65$	0.100	0.95	736	735	770	0.9552

of p induced by Venter's results and calculated with the normal power approximation.

Venter's one-sided full credibility levels are consistently higher than the two-sided ones calculated with both the normal and normal power approximations. Since the distributions of S are usually positively skewed in the fields where limited fluctuation credibility is applied, this is indeed a direct consequence of the formulation of the problem in the form of Equation 5.2 coupled with the use of the normal power approximation to take the third moment of S into account.

The rationale of the author for adopting a one-sided criterion is not very clear. It is first suggested in [9] that, for most distributions of interest, Mowbray's two-sided criteria will be satisfied if the one-sided is. This can be verified in Table 3. But the main idea was probably to use the normal power approximation to obtain more refined—more accurate—full credibility levels. The task is then facilitated by the one-sided criterion as it leads to the easy to use, closed-form credibility formula, Equation 5.4. The problem with this formula is that it sometimes unnecessarily overstates the full credibility levels, a fact it appears Venter was aware of, as he summarizes Dale Nelson (*PCAS*, 1969):

> ... although the *NP* [normal power approximation] gives useful approximations of the higher percentiles, it may overstate the volume needed for full credibility relative to given standards.

Once the desired degree of conservatism has been fixed through the parameters k and p, there exists a "true" full credibility level satisfying Equation 2.1. We said earlier that our normal power approximation almost gives the true levels[3] and that the normal approximation is sufficiently close to these levels. Now, the normal approximation levels satisfying Equation 5.2 will be the same and as such should be satisfactory. Equation 5.4 may thus be simpler than our application of the normal power approximation, but as it yields higher results than the even simpler normal approximation, its usefulness becomes questionable.

Finally, it is not clearly stated in [9] that Equation 5.4 yields higher—and sometimes much higher, as Table 3 shows—full credibility levels as a solution to a problem defined in the form of Equation 5.2. This could lead to the perception that using the third moment in any full credibility level estimation will necessarily increase these levels. We have concluded earlier that this is not the case. An eventual user of Equation 5.4 should thus be aware of its implications and ensure it is used in conjunction with the one-sided definition of the limited fluctuation credibility problem.

[3] At least in the gamma cases.

6. CONCLUSION

The conclusion of the first part of this paper is drawn from the tests summarized in Table 1. Although giving very accurate results, more sophisticated approximation methods like normal power or Esscher are not worth the added complexity and calculation time as compared to the normal approximation to estimate full credibility levels. It has been shown that when staying with Mowbray's original definition of the limited fluctuation problem, the differences between the various approximations are hardly significant. In more peculiar situations, the normal approximation yields the more conservative result, so we stay on the safe side.

While not necessary in limited fluctuation credibility, we nonetheless emphasize that the normal power and Esscher approximations remain very useful tools in general risk theory because of their good estimation of the percentiles of an aggregate claim distribution.

The paper then discussed the apparently different conclusions put forward by Venter [9]. We mainly argue that it should be stated more clearly in [9] that the definition of the limited fluctuation problem differs from Mowbray's traditional one. Moreover, the formula based on the normal power approximation used in the paper and the conclusions drawn with it pertain only to the problem studied and should not be carried over to general limited fluctuation credibility.

The reader should note that when the limited fluctuation problem is treated as in this paper (that is, with Mowbray's definition), it is not possible to derive a simple, explicit formula for the expected value of N (which usually gives the full credibility level in limited fluctuation credibility) while using the normal power approximation.

Mayerson et al. [6] also obtained significantly higher (from 3% to 10%) full credibility levels when using the third moment of the distribution of S, but this is also due to their conservative approach to the problem. When worked out with the normal power approximation, the compound Poisson examples of Mayerson et al. lead to full credibility levels almost equal to the normal approximation. The most important idea of that paper, though, was that the full credibility level should be based on the pure premium (namely the distribution of S) rather than on only the number of claims (the distribution of N). This should still be stressed today.

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