RETROSPECTIVE RATING: 1997 EXCESS LOSS FACTORS

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"Fly me to the moon, and let me swing among the stars ... "
—Bart Howard

Abstract

The NCCI methodology for deriving Excess Loss Factors (ELFs), based largely on research performed in 1986, is documented in "Retrospective Rating: Excess Loss Factors" by William R. Gillam [2]. This paper updates that 1991 paper. The changes in the way ELFs are produced have been significant, if not extensive. The work done to support those changes was extensive.

In the Fall of 1992, after an intense but focused study, NCCI updated the parametric size-of-loss distributions described in Gillam's paper. The associated changes were in production for most 1993 filings.

A much more in-depth review of the ELF model was completed in 1995. In this report, we detail some of the investigations made in that review and the features of the resulting model.

The researchers checked to see that the existing groupings of claim types were optimal, or at least superior to any other obvious groupings. They also determined that the groupings of states by benefit type (escalating, non-escalating, and limited) was not justified.

Loss distributions by claim group have again been updated, this time using a new method to model fifth-to-ultimate loss development, overcoming the lack of individual loss information after that report. (The Workers Compensation Statistical Plan ends at fifth report.)

The risk loadings for parameter risk and contagion were also updated to be more appropriate in an open competition rating environment.

1. MODELING OF LOSSES BY INJURY TYPE

Under the Workers Compensation Statistical Plan (WCSP), an *injury type* code is reported for each claim—corresponding to the carrier's belief at the valuation date as to the ultimate injury type of the claim. The injury types are: Fatal, Permanent Total (PT), Permanent Partial (PP), Temporary Total (TT), Medical-Only, and Contract Medical. For ratemaking, NCCI makes the distinction between a *Major* and *Minor* Permanent Partial claim according to whether its indemnity component is above or below a state-specific *critical value*. This results in seven injury types being coded into NCCI's databases.

As described in the 1991 paper by Gillam, Excess Loss Factors (ELFs) were based on weighted excess ratios for each of three injury groups. In the 1995 study, we tried to determine the ideal grouping of injury types.

Description of NCCI Approach

In the 1986 study, curves were fit to data from each of a sample of states. Combining data for various states prior to curve fitting was not done, apparently due to concern over differences in scale between the states. Consequently, one problem the researchers encountered was the scarcity of data within each state for PT claims. Their solution was to combine PT claims with Major PP claims, yielding the composite injury type *PT/Major*.

Similarly, *TT/Minor* is a combination of TT and Minor PP claims.

In the 1992 study, NCCI developed a procedure for combining multiple states' data. At each report, losses for each state were grouped into three categories: Fatal, PT/Major, and TT/Minor. Next, differences in scale by state were removed through *normalization*; for each claim group, this was done by dividing each claim by the average cost per case for the appropriate state-injury-type combination. Then claim sizes would be calibrated by "entry ratio" to the average cost per case. For example, a Florida claim in the PT/Major category would be divided by the average cost per case for Florida PT/Major claims. For Fatal and PT/Major, claims were combined respective of benefit type (escalating, limited, and non-escalating). This normalization is consistent with the methodology for production of ELFs, wherein excess ratios are calibrated for entry ratios.

Statistical distributions were then fit to the normalized empirical distributions using maximum likelihood.

Since by definition, no Major PP claim can be less than the critical value and it is unlikely that a permanent total claim would be, it made sense to fit a shifted distribution to the normalized fifth report PT/Major claims; that is, all normalized PT/Major claims were reduced by some flat amount prior to curve fitting.

The average state critical value for the claims in the database was roughly a fourth of the average cost per case for PT/Major claims. Consequently, a shift parameter of .25 or 25% of the average cost per case was reasonable. The actual dollar value would of course vary by state and year.

Performance Testing Injury Groups

Exhibit 1 summarizes the testing used to gauge the effectiveness of these three ways of grouping claims: 1) PT and PP modeled separately, 2) PT and Major PP combined, TT and Minor PP combined, 3) a single distribution combining PT and all PP, leaving TT by itself, and 4) a control, the simple use of last year's raw data. The testing attempted to determine which approach best predicted the relative magnitude of the empirical fifth report excess ratio at given loss limits. We tested using the following loss limits: 12,500, 50,000, 250,000, and 500,000.

We first considered the option of separating out PT and consolidating Major and Minor PP claims, then modeling claims according to the normally reported injury type. This would be the common-sense approach. In recent studies, we observed considerable variation in the proportion of PT/Major corresponding to PT. In some states, PT is nearly 40% of the PT/Major loss dollars; whereas, in others it is only 5%. This variation seemed to argue against a model which combined PT and Major PP.

To review the rationale behind the current option, labeled Option 2 in Exhibit 1, it is apparent that the practical distinction between PT and Major PP varies from state to state and year to year. A claim classified as PT in one state might well be considered a PP claim in another. This blurring of PT and Major PP would not be a factor if PT and Major PP were combined prior to curve fitting. The combination of Minor PP with TT is made for ease of computation and has little impact on the final factors.

As a third option for grouping claims, we considered the use of a single ground-up distribution combining PT and all of PP, calling this the "Permanent Claims."

Excess ratio tables were calculated at fifth report for each of the groupings of claims. Each of the three models above were used to calculate ELFs. For fixed loss limits, the relative magnitude of the modeled excess ratios by state should roughly track the empirical ratios. If a model predicts a higher excess ratio in, say, Georgia than in Florida, the empirical fifth report excess ratio for Georgia should be higher than that for Florida. The accuracy of the tracking can be quantified using R^2. The models used as inputs the average cost per case and injury weight values corresponding to the target data.

The model using the current grouping of claims produced the best estimates, as measured by R^2. That is, the current injury groupings did the best job of predicting which states would have high or low empirical excess ratios. It may be that PT average costs per case and injury weights, which are based on relatively small samples, are too volatile, leading to unstable partial excess ratios when PT is modeled separately.

The selected model is based on the injury groupings: Fatal, PT/Major, and TT/Minor.

2. THE GROUPING OF STATES BY BENEFIT TYPE

Description of NCCI Approach

In the 1992 study described above, differences in scale by state were removed by dividing each claim by the average cost per case for the appropriate state/claim group combination. Once the differences in scale were removed for claims of each injury group, the states were combined according to state benefit type. There were five groupings: 1) Escalating Fatal, 2) Non-escalating or Limited Fatal, 3) Escalating or Limited PT, 4) Non-escalating PT, and 5) TT/Minor. States would be in either 1) or 2), 3) or 4), and all states would be in 5).

Performance Testing State Groupings

We have tried to determine whether there exists a systemic relationship between the shape of the distribution (after removing the effects of scale) and the state benefit type. Three injury types were tested: Fatal, PT, and PP. These have by far the most weight in the calculation of excess ratios. Fatal and PT are the ones that could be logically impacted by escalation, non-escalation, or limitation, but we also tested PP for completeness.

We first examined the variance and skewness statistics of the normalized fifth report losses for each of the three injury types—Fatal, PT, and PP.

For Fatal claims, neither the variance (Exhibit 2-A) nor the skewness (Exhibit 2-B) of the normalized loss seem to have any significant relationship to state benefit type. Similarly, no useful relationship could be deduced for PT claims (Exhibits 3-A and 3-B) or PP claims (Exhibit 4-A and 4-B).

Treating benefit type as a categorical variable, we performed ANOVA testing and calculated coefficients of determination (R^2)

for each of the comparisons. The categorical variable *state benefit type* appears to be of little use in predicting fifth report normalized loss skewness or variance for Fatal or PT losses.

Analysis based on the likelihood ratio test further supports this argument. For both Fatal and PT claims, we compared normalized loss distributions for claims in states grouped by benefit type with those for all other states combined. Differences for these groupings were statistically insignificant.

We have decided that states should not be grouped by benefit classification, based on the large variation by state in higher moments of the distribution and on the fact that these are not correlated with benefit type.

As another possible change from the prior procedure, we considered eliminating the countrywide distributions and using distributions for each state. For most states we found the statistical significance of the difference between the state and countrywide normalized size-of-loss distributions for Fatal and PT is questionable—as indicated by the likelihood ratio test (Exhibits 5-A and 5-B). The enhanced credibility and utility of using countrywide distributions, on the other hand, are of clear value.

3. MODELING LOSS DEVELOPMENT

The impact of loss development on individual claims is not uniform since claims obviously have unique development patterns. Some settle for less than originally estimated, some for more. Accordingly, as losses mature, the dispersion among losses increases and so we expect the shape of the size-of-loss distribution at an ultimate report to be very different from that at a fifth report. We would expect the former to be more heavy-tailed than the latter.

For our purposes, it is the shape of the ultimate-report normalized loss distribution that we wish to model for each injury type. Unfortunately, Workers Compensation Statistical Plan data is available only through a fifth report. We are able to account for average claim size development on open serious claims using financial data. What is needed is a procedure which can account for the distortion of the shape of the size-of-loss distribution due to post-fifth-report loss development. In this section, we describe just such a procedure—the Random Development Divisor algorithm.

In the Appendix, we discuss the Black–Scholes model used by stock traders to price securities options, noting the similarity between the mathematics of pricing an option in the financial arena and that of excess of loss pricing in insurance. The Random Development Divisor algorithm described below bears more than coincidental resemblance to the Black–Scholes model.

The Random Development Divisor algorithm was designed to account for the post-fifth-report development in the shape of the severity distribution. The process is to 1) organize the partially developed fifth report loss distribution into a series of uniform distributions derived from empirical grouped data, 2) model loss development using a gamma distributed divisor, whose parameters are determined by matching the moments of the loss development factors for individual claims, and 3) compound the uniform and gamma distributions to derive an ultimate report distribution. The use of the piecewise linear approximation to a continuous distribution is a standard technique.

The basic building blocks of the model are a prior uniform distribution representing open or closed claims in each layer of fifth report loss range, and a corresponding gamma distribution quantifying development for such losses in the layer.

Empirical Fifth Report Severity Distribution

We construct n intervals of the grouped empirical claim distribution F_y for the fifth report size-of-loss random variable

Y. Each interval may contain several claims. These $n+1$ points,

$$(a_1 = 0, F_y(a_1) = 0), (a_2, F_y(a_2)), \ldots, (a_n, F_y(a_n)),$$
$$(a_{n+1}, F_y(a_{n+1}) = 1),$$

divide the probability space of Y into n intervals. Let p_k represent the probability associated with the kth interval:

$$p_k = F_y(a_{k+1}) - F_y(a_k), \qquad k = 1, 2, \ldots, n.$$

p_k is the number of empirical claims in the kth interval divided by the total number of claims.

The following discussion is in terms of a basic building block. However, it should be kept in mind that the complete model would involve an application of the method to each subinterval of fifth report size-of-loss. Compounding the posterior distributions for all layers is a task made easy by the computer.

Gamma Distributed Fifth-to-Ultimate Development Divisors

Let Z denote the random variable representing the reciprocal of the fifth-to-ultimate loss development factor. Our a priori assumption is that such loss development is dependent on the size of the fifth report losses and whether they are open or not. Of course, the proportion of open claims varies by layer, so we were able to model loss development using two gamma distributions, one for open, one for closed.

Modeling development using a divisor rather than a multiplier facilitates the derivation of closed form formulas for the cumulative distribution function and excess ratio functions.

Constructive Model of Ultimate Losses

A heuristic description of the process for generating ultimate losses ($X = Y/Z$) is as follows:

STEP 1 Select one of the n fifth report loss intervals. The probability of selecting a given interval equals the amount of probability in the interval (p_k).

STEP 2 Assume that losses are uniformly distributed within each selected interval. Randomly select a fifth report loss (Y), which may be open or closed, from the uniform distribution chosen in Step 1.

STEP 3 The result of Step 1 determines which gamma distribution will be used to select a loss development divisor (Z). Randomly choose Z from the respective gamma distribution with parameters (α_o, β_o) or (α_c, β_c), where o is open and c is closed.

STEP 4 Divide Y by Z. The result is X (the ultimate report loss).

The Relationship Between the Conditional Distribution Functions of X and Y

$$\begin{aligned} F_x(x \mid z) &= \Pr(X \leq x \mid z) \\ &= \Pr(Y/Z \leq x \mid z) \\ &= \Pr(Y \leq zx) \\ F_x(x \mid z) &= F_y(zx). \end{aligned} \tag{1}$$

Derivation of Distribution Function of X

We treat each y-interval as a separate random variable. Let $u_k(z)$ denote the probability density function (p.d.f.) for Z. Note that there are n such conditional distributions—one for each y-loss interval.

Then, using Equation 1 in

$$F_x(x) = \int_0^\infty F_x(x \mid z) u_k(z)\, dz,$$

we have

$$F_x(x) = \int_0^\infty F_y(zx)u_k(z)\,dz. \tag{2}$$

For each interval $(a_k, a_{k+1}]$, we assume that fifth report losses are uniformly distributed. Then,

$$F_y(y) = \begin{cases} 0 & \text{for} \quad y \le a_k \\ \dfrac{y - a_k}{a_{k+1} - a_k} & \text{for} \quad a_k < y \le a_{k+1} \\ 1 & \text{for} \quad y > a_{k+1} \end{cases}.$$

For (fixed) $x > 0$, $k = 1, 2, \ldots, n$

$$\begin{aligned} &a_k < y \le a_{k+1} \\ \Leftrightarrow\ &a_k < xz \le a_{k+1} \\ \Leftrightarrow\ &\frac{a_k}{x} < z \le \frac{a_{k+1}}{x}. \end{aligned}$$

Thus,

$$F_y(zx) = \begin{cases} 0 & \text{for} \quad z \le a_k/x \\ \dfrac{y - a_k}{a_{k+1} - a_k} & \text{for} \quad a_k/x < z \le a_{k+1}/x \\ 1 & \text{for} \quad z > a_{k+1}/x \end{cases}.$$

Using the above in Equation 2, we can calculate $F_x(x)$:

$$F_x(x) = \int_{a_k/x}^{a_{k+1}/x} \frac{zx - a_k}{a_{k+1} - a_k} u_k(z)\,dz + \int_{a_{k+1}/x}^{\infty} u_k(z)\,dz. \tag{3}$$

The above applies to the kth interval ($k = 1, 2, \ldots, n$) treated in isolation. To calculate $F_x(x)$ over all intervals, we take a probability-weighted average. $F_x(x)$ is the fully developed sample to which we fit the final parametrized distributions leading to the excess ratio table used in production of ELFs.

4. DERIVATION OF EXPECTED EXCESS LOSS FUNCTION FOR EACH INTERVAL

Let x denote the loss limit. Then for a random loss Y/Z, the excess of Y/Z over x is: $(Y/Z - x)$ for $Y/Z > x$, and 0 otherwise.

Thus to calculate the expected excess loss, we need to integrate over the set of (y,z) for which y/z is greater than x.

Let $f(y,z)$ denote the joint probability density function for Y and Z. Since Y and Z are independent,

$$f(y,z) = f(y)f(z).$$

Now, recall that Y is uniformly distributed in $(a_k, a_{k+1}]$ which means that $f(y,z)$ is zero whenever $y < a_k$ or $y > a_{k+1}$. This reduces the area over which we must integrate to a trapezoidal region.

This trapezoidal region consists of the rectangular "AREA A" and the triangular "AREA B" in Figure 1. The expected excess loss can then be calculated as

$$\begin{aligned}\text{Excess}_x = &\int_0^{a_k/x} \int_{a_k}^{a_{k+1}} \left(\frac{y}{z} - x\right) f(y,z)\,dy\,dz \\ &+ \int_{a_k/x}^{a_{k+1}/x} \int_{xz}^{a_{k+1}} \left(\frac{y}{z} - x\right) f(y,z)\,dy\,dz. \qquad (4)\end{aligned}$$

DEFINITION *If Z is gamma distributed with parameters (α, β), then the cumulative distribution function (c.d.f.) of Z is*

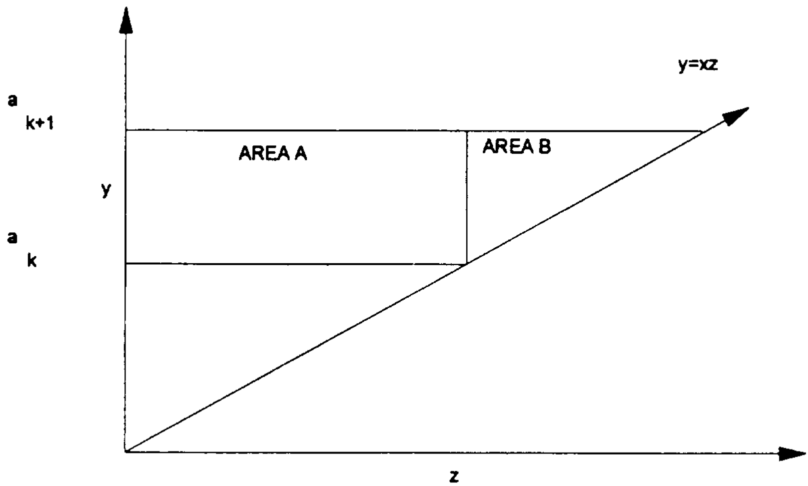
$$F_z(z) = \Gamma(\alpha; \beta z) = \frac{\int_0^{\beta z} u^{\alpha-1} e^{-u}\,du}{\Gamma(\alpha)}. \qquad (5)$$

For this distribution, non-central moments can be calculated as follows:

$$\mathrm{E}[Z^n] = \frac{\Gamma(\alpha + n)}{\beta^n \Gamma(\alpha)}. \qquad (6)$$

FIGURE 1

AREA OVER WHICH EXCESS LOSS INTEGRAL IS EVALUATED



THEOREM 1 *Let the fifth report size-of-loss random variable Y ($Y > 0$) be uniformly distributed on the interval $(a_k, a_{k+1}]$, and let the loss development divisor random variable Z ($z > 0$) be gamma distributed with parameters (α, β). Let Y and Z be independent. Then the ultimate report size-of-loss random variable X, equal to the ratio of Y to Z ($X = Y/Z$), has cumulative distribution function*

$$\begin{aligned} F_x(x) = {} & \frac{\alpha x}{\beta(a_{k+1} - a_k)}\left[\Gamma\left(\alpha + 1; \frac{a_{k+1}\beta}{x}\right) - \Gamma\left(\alpha + 1; \frac{a_k \beta}{x}\right)\right] \\ & - \frac{a_k}{a_{k+1} - a_k}\left[\Gamma\left(\alpha; \frac{a_{k+1}\beta}{x}\right) - \Gamma\left(\alpha; \frac{a_k \beta}{x}\right)\right] \\ & + 1 - \Gamma\left(\alpha; \frac{a_{k+1}\beta}{x}\right). \end{aligned}$$

THEOREM 2 *Let the random variable X be as defined in Theorem 1. Then for any x ($x > 0$) the expected portion of loss in* *excess of x for a randomly selected claim is equal to*

$$\begin{aligned}
\text{Excess}_x &= \frac{\beta(a_k + a_{k+1})}{2(\alpha-1)}\Gamma\left(\alpha-1;\frac{a_k\beta}{x}\right) - x\Gamma\left(\alpha;\frac{a_k\beta}{x}\right)\\
&+ \frac{a_{k+1}^2\beta}{2(\alpha-1)(a_{k+1}-a_k)}\left\{\Gamma\left(\alpha-1;\frac{a_{k+1}\beta}{x}\right) - \Gamma\left(\alpha-1;\frac{a_k\beta}{x}\right)\right\}\\
&+ \frac{\alpha x^2}{2\beta(a_{k+1}-a_k)}\left\{\Gamma\left(\alpha+1;\frac{a_{k+1}\beta}{x}\right) - \Gamma\left(\alpha+1;\frac{a_k\beta}{x}\right)\right\}\\
&- \frac{x a_{k+1}}{a_{k+1}-a_k}\left\{\Gamma\left(\alpha;\frac{a_{k+1}\beta}{x}\right) - \Gamma\left(\alpha;\frac{a_k\beta}{x}\right)\right\}.
\end{aligned}$$

Excess_x *is the numerator of the excess ratio, whose denominator is*

$$\mathrm{E}[X] = \beta\frac{(\alpha_{k+1} + \alpha_k)}{2(\alpha-1)}.$$

Illustration #1: Estimation of Z-parameters

For fifth report losses in the interval (20,000, 30,000) suppose we have observed that the first moment (mean) of the fifth-to-ultimate loss development factor distribution is $m_1 = 1.00$ and the second moment is $m_2 = 1.81$.

Estimate the α and β parameters of random development division that correspond to the observed moments.

Solution:

Using Equation 6, set m_1 equal to $\mathrm{E}[1/Z]$ and m_2 equal to $\mathrm{E}[1/Z^2]$

$$\mathrm{E}[1/Z] = \beta/(\alpha-1) = m_1 = 1.00,$$

and

$$\mathrm{E}[1/Z^2] = \beta^2/\{(\alpha-1)(\alpha-2)\} = m_2 = 1.81.$$

Solving the two equations simultaneously gives $\alpha = 3.2346$ and $\beta = 2.2346$.

Illustration #2: Calculation of $F_x(x)$

Given the scenario in Illustration 1, estimate the probability that a fully developed claim will not exceed $35,000.

Solution:

We apply Theorem 1 with the following parameter values: $a_k = 20{,}000$, $a_{k+1} = 30{,}000$, $\alpha = 3.2346$, $\beta = 2.2346$, and $x = 35{,}000$

$$F_x(35{,}000) = .825178.$$

Illustration #3: Calculation of Excess Ratio

Given the assumptions in Illustration 1, estimate the expected proportion of loss dollars in excess of 35,000.

Solution:

We apply Theorem 2 with the following parameter values: $a_k = 20{,}000$, $a_{k+1} = 30{,}000$, $\alpha = 3.2346$, $\beta = 2.2346$, and $x = 35{,}000$

$$\text{Excess}_x(35{,}000) = 4092.57$$

$$\text{E}[X] = 25{,}000 \times 1.00 = 25{,}000.$$

The ratio is $4097.57/25000 = 0.1639$.

5. THE ISSUE OF RISK LOAD

The Flat Loading

The flat loading, which accounts for parameter risk and anti-selection, was added to the ELF which is a ratio to premium that includes expenses; it was .005, subject to a maximum of half the ELF. In the selected procedure, we have chosen to remove ourselves from the expense arena by instead applying the .005 flat loading to the pure excess ratio and limiting it to half of that.

Prior Load for Contagion

The loss distributions underlying the prior ELF procedure correspond to individual claims by injury type; however, ELFs apply on a per occurrence basis (a single occurrence may contain multiple claims). The adjustment used to account for the per occurrence basis of the coverage was to inflate the average cost per case for each injury type by a factor of 1.1. So, for example, if the average Fatal claim for a given state and hazard group was projected to be \$100,000, an average value of \$110,000 was assumed in the ELF calculations. In other words, the Fatal occurrence size distribution was scaled to an average value of \$110,000—from which the Fatal contribution to the ELF (the partial excess ratio) is calculated. This was done for all claim types.

Selected Contagion Load

As stated in Section 1, removing the differences in scale by state made it possible to combine experience from more than one state. For each injury type, normalized claims had an average size of unity. Parametrized statistical distributions were then fit to the sample distributions by maximum likelihood. The scale parameters of the fitted distributions did not necessarily result in a mean of unity but had to be adjusted once again to normalize the result.

We are sampling from highly skewed distributions for PT/Major and Fatal. Consider the distributions of the sample means. In theory, these distributions approach normality as the sample sizes approach infinity; but is this the case in practice? These sampling distributions of the means at an ultimate report, based on finite state sample sizes, are likely still skewed. This means that in more cases than not, the sample means will be less than the true means.

The empirical cumulative distribution function (c.d.f.) is based on a sample, and a sample contains a largest observed claim.

A theoretical distribution such as a transformed beta would not have a maximum possible claim; some probability (albeit small) would be assigned to claims greater than the largest claim observed in the sample. For this reason, the mean of the fitted distribution (the maximum likelihood estimator) may be greater than that observed in the data (the method of moments estimator).

The choice of an adequate statistical model produces a fitted statistical distribution whose cdf very closely matches that observed for the data, except at very high entry ratios. For example, if 25% of the observed normalized claims are below entry ratio 1.00, we expect the theoretical model's cdf to be very close to .25 at input value 1.00. As stated above, the prior approach was to re-scale the distribution fitted to combined data to a mean of unity. A consequence of this re-scaling is that the cumulative distribution values do not match the empirical. In 1995, we chose not to re-scale the fitted distributions, thereby providing, in effect, a natural contagion load. We are using distributions that closely match the observed empirical distribution values, but assign small probabilities to large unobserved claim values. As in the prior procedure, the small probabilities assigned to the tail of each distribution are determined by the fitting procedure. The difference is that the means of the models are greater than unity. By allowing the means to float, our models more closely match the observed claim distributions and at the same time provide some risk load.

The way the Fatal and PT/Major claim data is fit enhances the impact of the above strategy. The model accounts for these occurrences by fitting a distribution to the claim data censored from above; heuristically, the observed values correspond to single-claim occurrences and the censored portion of the distribution corresponds to multiple claim occurrences. The result is an occurrence size-of-loss distribution, with entry ratios to the average cost per claim. This is described in more detail below.

6. DEVELOPMENT OF PRODUCTION MODEL

The above sections cover the major issues addressed by our research and decisions made on these issues. Following is the application of these decisions in creating a new model.

Construction of Normalized Database

The countrywide claims database comes from Workers Compensation Statistical Plan data. This database contains fifth report claims for NCCI states along with an open/closed claim indicator. Each claim is identified by injury type.

Fifth-to-ultimate development factors (from a separate database used for class ratemaking) by state and injury type were used to develop these open claims. The development factor for open claims was such that the overall development (on open and closed claims) averaged to the loss development factors in our class ratemaking database.

Claims were then normalized (scaled to unity) by state and injury group, retaining the open/closed indicator. At this point, states can be combined, and the distributions can be grouped into n uniform claim size intervals.

In the procedure described thus far, no adjustment has been made for dispersion in the development by claim, other than the application of a flat factor by state to open claims only.

Application of Random Development Divisor (RDD) Algorithm

As in the 1992 study, we assume that only the distributions for Fatal and PT/Major claims change shape beyond a fifth report.

We introduced development uncertainty via the Random Development Divisor (RDD) algorithm. Based officially on judgment, but unofficially on an analysis of confidential data, we used a coefficient of variation (cv) of .9 for open claims and .1 for closed claims. Section 4 explains how we developed the open claim sub-intervals using a cv of .9. A similar procedure was used for the intervals of closed claims. We weighted together all $2n$ resulting distributions to form the sample for the next step.

Curve Fitting

Using maximum likelihood, we fit parametric distributions to the developed sample claim distributions. Actual fifth report data was used without further adjustment to fit the TT/Minor model.

Fit to Fatal Claims

The fatal loss size distribution encompasses two distinct types of claim—those with and those without survivor. Survivor benefits range over a lot of possible values, generally large to larger. Without a survivor, there is still a range of values depending on medical care, but a cluster of smallish values for claims in which medical care is minimal. Looking at the actual data, we concluded this could not be easily modeled by a single parametrized distribution.

A linear mixture of three distributions is used to model Fatal losses. Let R represent the "entry ratio" random variable. $F(r)$ is the cumulative distribution function of R:

$$F(r) = w_1 F_1(r) + w_2 F_2(r) + w_3 F_3(r),$$

where the w's represent the weights given to each of the three pieces.

For $R < 1$, the distribution of R is modeled using a *censored* Weibull distribution. The censoring parameter, c, is 1. This distribution, $F_1(r)$, received the largest weight (w_1) of 0.608.

For $R > 1$, we model, $R - 1$, the *excess* above entry ratio 1, with a transformed beta distribution. Each normalized occurrence in this interval can be thought of as unity plus a transformed beta deviate. This distribution received the next largest weight (w_2).

To eliminate clustering and improve the fit of the model, a small portion of the claims in the interval $(.75, 1)$ were modeled separately using a conditional (truncated and censored) Weibull. This distribution received a weight (w_3). The parameters are the same as those for the Weibull used in the $(0, 1)$ interval, except for the truncation point. Following is a comparison of the composite fitted distribution, $F(r)$, and the sample distribution generated by the RDD model, $F_n^*(r)$.

COMPARISON OF FATAL DISTRIBUTIONS

r	$F_n^*(r)$	$F(r)$
0.10	0.176340	0.176835
0.50	0.413960	0.419246
1.00	0.626340	0.626340
5.00	0.987300	0.986424
10.00	0.997330	0.996420
50.00	0.999980	0.999779

The severity distribution for Fatal has a mean of 1.039.

Fit to PT/Major

Following is a comparison of PT/Major cdfs.

The empirical ultimate report cdf for normalized claims prior to application of the RDD algorithm (but after development of open claims) is $F_n(r)$; after RDD it is $F_n^*(r)$. To account for the per occurrence basis of the coverage, a conditional distribution $F(r \mid r \leq 90)$ was fit via maximum likelihood to $F_n^*(r)$, also censored at 90. The corresponding uncensored distribution $F(r)$ is used to model occurrences.

The RDD algorithm causes most claims to develop downward but at the same time makes the tail of the distribution thicker, as can be noted from a comparison of $F_n(r)$ and $F_n^*(r)$.

The fitted conditional distribution ($F(r) \leq 90$) fits the post-RDD cdf ($F_n^*(r)$) well.

r	$F_n(r)$	$F_n^*(r)$	$F(r \mid r \leq 90)$	$F(r)$
0.10	0.00030	0.00101	0.00123	0.00123
0.50	0.37237	0.41468	0.41221	0.41214
1.00	0.71205	0.74970	0.74759	0.74745
5.00	0.98692	0.98053	0.97958	0.97940
10.00	0.99741	0.99472	0.99350	0.99332
50.00	0.99986	0.99967	0.99970	0.99951

The severity distribution for PT/Major has a mean of 1.066.

Fit to TT/Minor Claims

A Transformed Beta was fit to TT/Minor claims.

r	$F(r)$	$F_n(r)$
1.00	0.69826	0.68660
5.00	0.97017	0.96897
10.00	0.99635	0.99731

In other words, finis.

REFERENCES

[1] Brealey, R. A., and S. C. Myers, *Principles of Corporate Finance* (Fourth Edition), 1991, McGraw-Hill, Manchester, MO.

[2] Gillam, W. R., "Retrospective Rating: Excess Loss Factors," *PCAS* LXXVIII, 1991, pp. 1–40.

[3] Heckman, P. E., and G. G. Meyers, "The Calculation of Aggregate Loss Distributions from Claim Severity and Claim Count Distributions," *PCAS* LXX, 1983, pp. 22–61.

[4] Hogg, R. V., and S. A. Klugman, *Loss Distributions*, 1984, John Wiley & Sons, Somerset, NJ.

EXHIBIT 1

COMPARISON OF INJURY GROUPING SCHEMES

PERFORMANCE TESTING SUMMARY
THEORETICAL STATE EXCESS RATIOS REGRESSED ON EMPIRICAL 5TH REPORT EXCESS RATIOS

35 States

Loss Limit	Option 1 R-squared	Option 2 R-squared	Option 3 R-squared	Option 4 R-squared
$12,500	0.969	0.960	0.970	0.903
$50,000	0.853	0.953	0.843	0.791
$250,000	0.406	0.408	0.281	0.404
$500,000	0.290	0.213	0.157	0.147

NCCI States At Least 5,000 Serious Claims
(17 States)

Loss Limit	Option 1 R-squared	Option 2 R-squared	Option 3 R-squared	Option 4 R-squared
$12,500	0.958	0.952	0.960	0.936
$50,000	0.843	0.965	0.837	0.921
$250,000	0.618	0.696	0.534	0.553
$500,000	0.361	0.371	0.296	0.215

Option 1: PT and PP modeled separately.
Option 2: PT and Major PP together, TT and Minor PP together.
Option 3: PT and all PP modeled together, TT by itself.
(Control) Option 4: Excess Ratio predicted using previous year's observed values.

EXHIBIT 2-A

FATAL VARIANCE

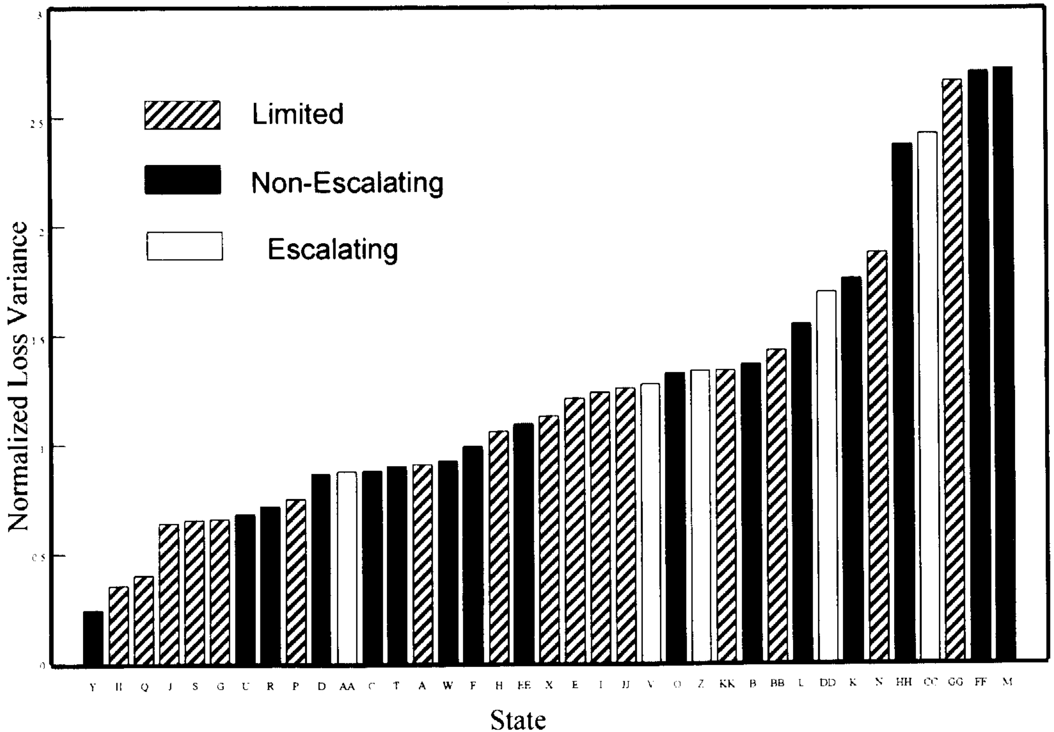


EXHIBIT 2-B

FATAL SKEWNESS

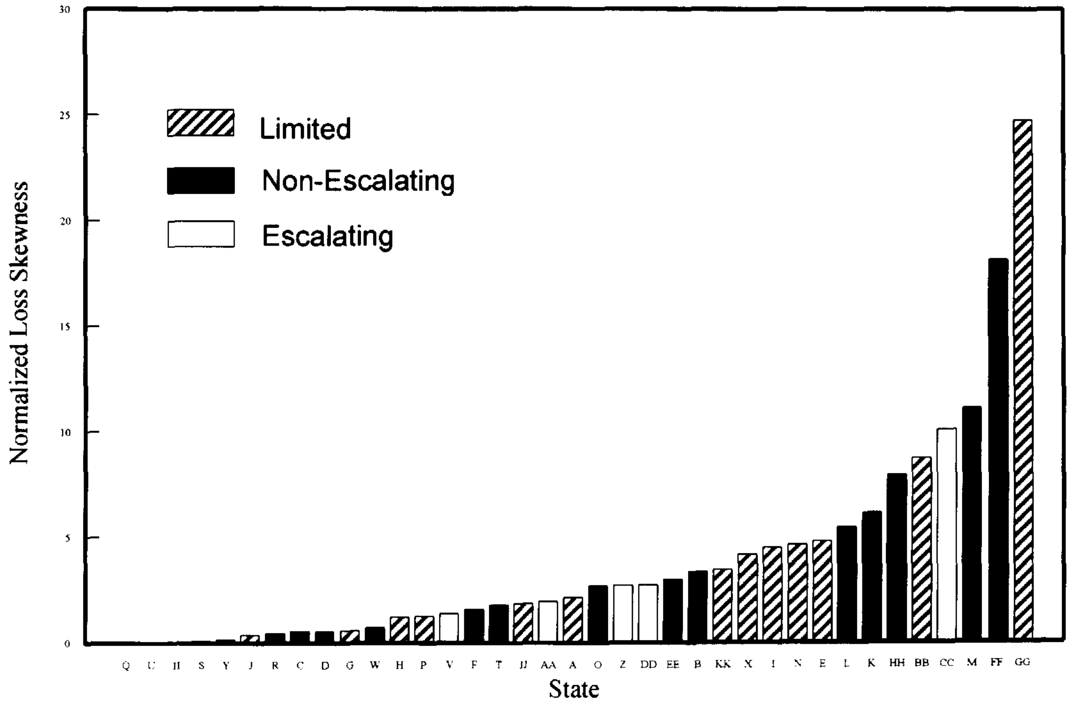


EXHIBIT 3-A

PERMANENT TOTAL VARIANCE

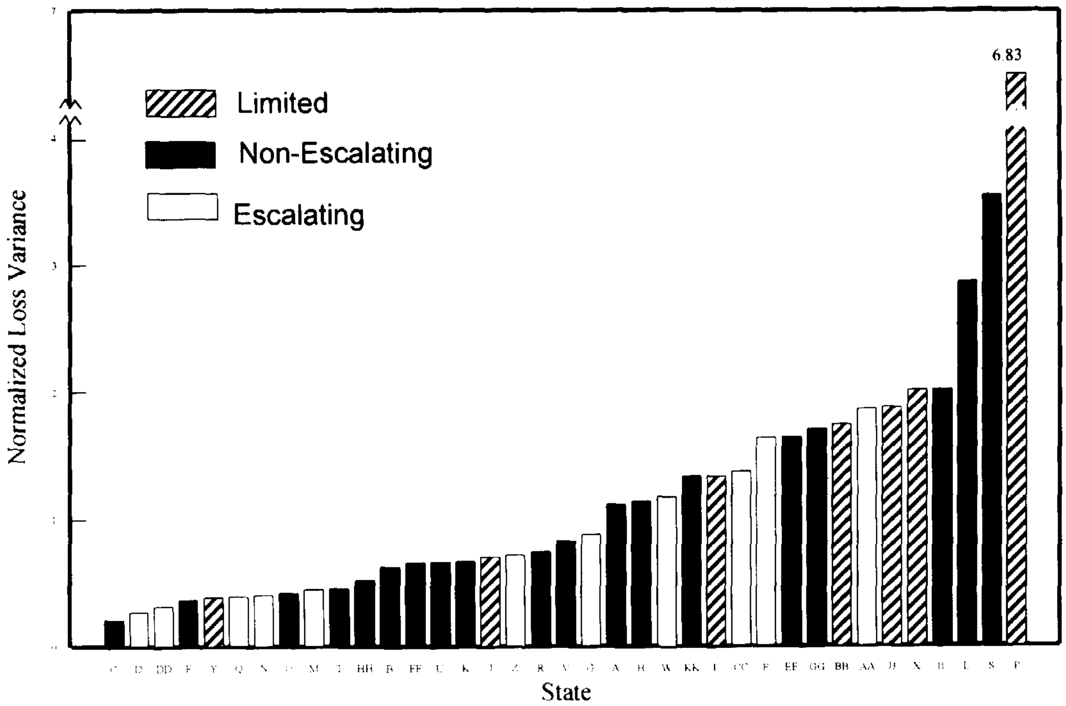


EXHIBIT 3-B

PERMANENT TOTAL SKEWNESS

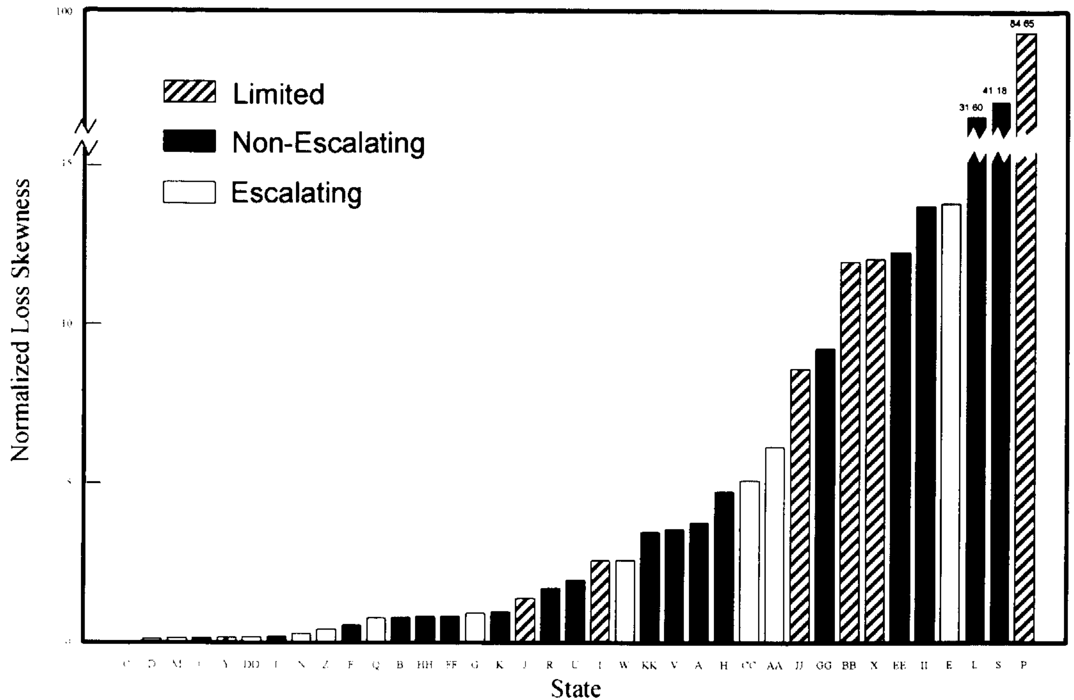


EXHIBIT 4-A

PERMANENT PARTIAL VARIANCE

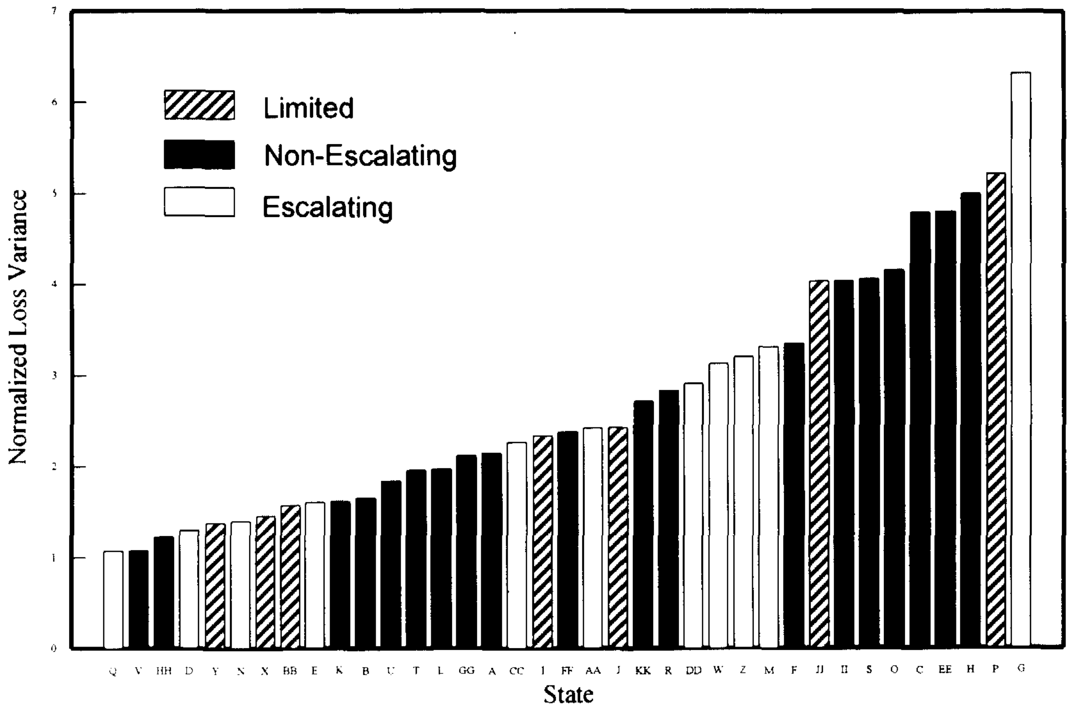


EXHIBIT 4-B

PERMANENT PARTIAL SKEWNESS

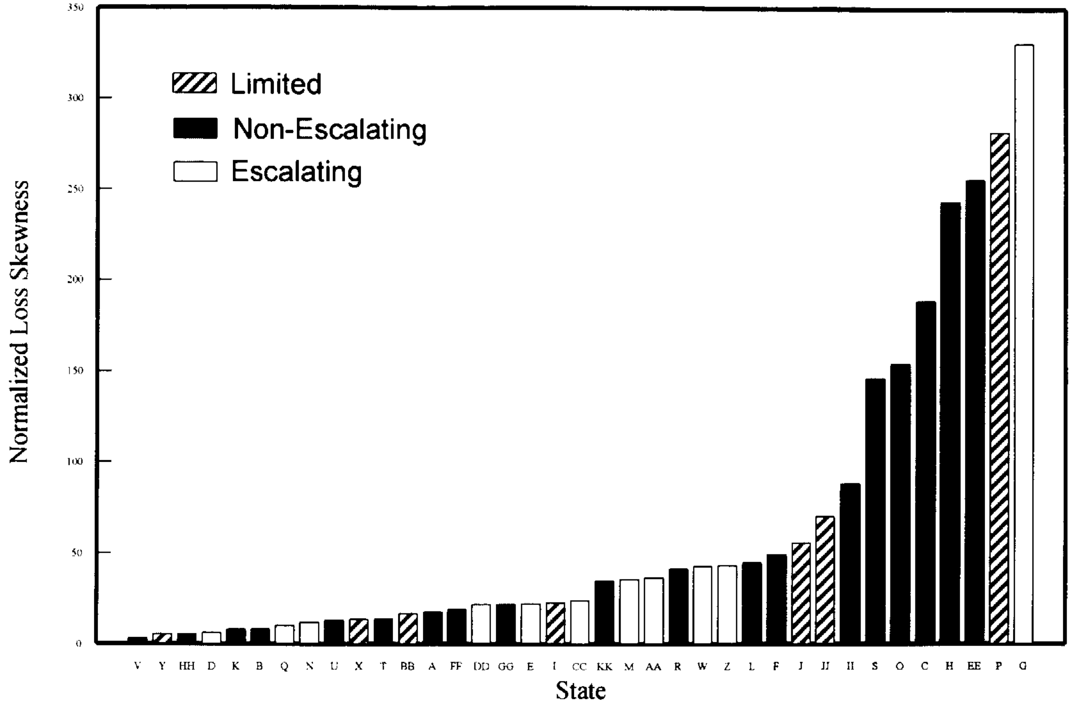


EXHIBIT 5-A

SUMMARY
STATE FATAL CLAIMS DISTRIBUTION COMPARED TO COUNTRYWIDE

State	Claims	Degrees of Freedom	Likelihood Ratio Test Statistic	*p*-value
A	77	3	0.780	0.854
B	36	3	0.435	0.933
C	40	2	1.279	0.528
D	69	2	1.587	0.452
E	187	3	10.567	0.014
F	14	2	0.680	0.712
G	22	2	0.182	0.913
H	168	2	0.767	0.681
I	103	2	4.518	0.104
J	56	2	4.193	0.123
K	51	2	2.188	0.335
L	134	3	5.542	0.136
M	50	3	19.877	0.000
N	15	2	5.653	0.059
O	118	3	0.882	0.830
P	75	3	0.049	0.997
Q	27	1	0.560	0.454
R	28	2	0.382	0.826
S	27	1	0.560	0.454
T	78	2	2.434	0.296
U	79	1	2.968	0.085
V	7	2	2.622	0.270
W	14	2	0.680	0.712
X	84	2	3.253	0.197
Y	4	1	0.451	0.502
Z	10	2	1.825	0.402
AA	66	3	0.208	0.976
BB	61	2	6.313	0.043
CC	59	3	9.003	0.029
DD	13	1	1.860	0.173
EE	132	3	2.510	0.473
FF	47	3	5.757	0.124
GG	131	3	9.280	0.026
HH	10	2	1.825	0.402
II	99	1	4.429	0.035
JJ	91	3	0.389	0.943
KK	78	3	1.392	0.707
All States	2,360	118	117.88	0.486

EXHIBIT 5-B

SUMMARY
STATE PT CLAIMS DISTRIBUTION COMPARED TO COUNTRYWIDE

State	Claims	Degrees of Freedom	Likelihood Ratio Test Statistic	p-value
A	86	3	1.016	0.797
B	29	2	2.076	0.354
C	21	1	1.040	0.308
D	358	2	55.105	0.000
E	738	3	8.843	0.031
F	64	2	0.195	0.907
G	5	1	0.134	0.714
H	144	3	0.133	0.988
I	20	2	2.148	0.342
J	47	2	0.264	0.876
K	98	2	1.887	0.389
L	156	3	7.160	0.067
M	36	1	2.342	0.126
N	29	2	3.464	0.177
O	33	1	3.753	0.053
P	32	3	10.665	0.014
Q	82	2	0.635	0.728
R	13	2	0.619	0.734
S	45	2	3.346	0.188
T	78	1	4.402	0.036
U	109	3	0.511	0.916
V	80	2	2.031	0.362
W	11	2	2.961	0.228
X	56	2	2.422	0.298
Y	14	1	0.693	0.405
Z	5	1	0.134	0.714
AA	75	3	26.425	0.000
BB	61	3	0.246	0.970
CC	48	3	3.016	0.389
DD	7	1	0.043	0.836
EE	130	3	2.049	0.562
FF	32	2	3.041	0.219
GG	102	3	2.864	0.413
HH	28	2	0.765	0.682
II	52	3	1.726	0.631
JJ	51	2	2.197	0.333
KK	52	3	2.117	0.548
All States	3,027	115	162.468	0.002
Excl. D	2,669	113	107.363	0.632

APPENDIX

Black–Scholes Model

Let us think of the future price of a share of stock, S, as the current share price, S_0, times a random "development factor." The development factor random variable is customarily modeled by a lognormal distribution. Then the future price—equal to a lognormally distributed random factor times a constant (the current price) is also lognormally distributed.

Let the indexed random variable S_t represent the unknown future price of a share of stock at time t ($t > 0$). Suppose S_t is lognormally distributed with parameters $(\mu_t, \sigma^2 t)$.

At time zero, we wish to price a call option exercisable at time t, at exercise price d. A call gives the holder the option of buying a share of stock at the exercise price at some future date. Let r_f represent the force of interest at the risk free rate. Then:

$$\begin{aligned}
PV(CALL) &= e^{-r_f t}\int_d^{\infty}(s-d)f(s)ds \\
&= e^{-r_f t}\{\mathrm{E}(S)-\mathrm{E}(S;d)\} \\
&= e^{-r_f t}\left(e^{\mu_t+\sigma^2 t/2} - e^{\mu_t+\sigma^2 t/2}\Phi\left(\frac{\ln(d)-\mu_t-\sigma^2 t}{\sigma\sqrt{t}}\right)\right. \\
&\qquad \left. -d\left\{1-\Phi\left(\frac{\ln(d)-\mu_t}{\sigma\sqrt{t}}\right)\right\}\right) \\
&= e^{-r_f t}\left(e^{\mu_t+\sigma^2 t/2}\left\{1-\Phi\left(\frac{\ln(d)-\mu_t-\sigma^2 t}{\sigma\sqrt{t}}\right)\right\}\right. \\
&\qquad \left. -d\left\{1-\Phi\left(\frac{\ln(d)-\mu_t}{\sigma\sqrt{t}}\right)\right\}\right)
\end{aligned}$$

$$= e^{-r_f t}\left[e^{\mu_t+\sigma^2 t}\Phi\left(\frac{-\ln(d)+\mu_t+\sigma^2 t}{\sigma\sqrt{t}}\right)\right.$$

$$\left.-d\Phi\left(\frac{-\ln(d)+\mu_t}{\sigma\sqrt{t}}\right)\right].$$

The Black–Scholes form for the above is arrived at by equating the mean of the distribution to the current ($t = 0$) share price times the discount factor:

$$e^{\mu_t+\sigma^2 t/2} = S_0 e^{r_f t}.$$

The above is highly suggestive. First, the idea of creating Black–Scholes analogs based on distributions other than the lognormal may come to mind. We could, for example, assume that the stock "development factors" follow the gamma distribution instead of the lognormal; the share price itself would also then be gamma distributed. Not surprisingly, the use of such Black–Scholes analogs is not unknown in the world of finance.

The second item which may come to mind is the resemblance between the mathematics of pricing an option and the reserving of excess of loss coverage. Of major significance is the following: if a fifth report open claim is currently valued below a given retention, it does not follow that the expected contribution of the claim to the excess layer is zero, just as the value of a call for a stock currently priced below the exercise price is not zero.