RATEMAKING: A FINANCIAL ECONOMICS APPROACH

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Abstract

Financial pricing models are replacing traditional ratemaking techniques for property-liability insurers. This paper provides an introduction to the target total rate of return approach, the capital asset pricing model, the discounted cash flow technique, and the option pricing model, all in an insurance context. Examples of each method, along with discussions of their advantages and weaknesses, are provided.

ACKNOWLDGEMENTS

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1. INTRODUCTION TO FINANCIAL ECONOMICS

Financial economics deals with the acquisition, issuance, valuation, and investment of securities in capital markets. Much of the early work in financial economics dealt with determining the appropriate value of stocks. Models were developed to predict the value of a stock, which was compared with its actual price. The strategy of buying underpriced stocks and selling overpriced stocks was expected to produce returns above the general market performance. Benjamin Graham and David Dodd were major proponents of this approach [18]. However, valuation of individual stocks proved to be difficult, and to this day no consensus exists among financial economists about what the price of a given stock should be.

In 1952, Harry Markowitz directed the focus away from individual stock picking with his work entitled "Portfolio Selection" [24]. Markowitz calculated the variance, which was used as a measure of risk of returns, and demonstrated the effect on portfolio risk of the addition and subtraction of stocks to and from a group of stocks. He showed that a portfolio of stocks could generate a higher return at a lower level of risk than individual stocks held alone. This concept, known as portfolio diversification, reduced the emphasis on individual stock picking.

However, investors were still interested in the returns of individual stocks. Building upon Markowitz's work, William Sharpe [29] published an article in 1964 that explained the expected return of individual securities in a well-diversified portfolio. In this model, termed the Capital Asset Pricing Model (CAPM), the investor is compensated only for bearing *systematic risk*, which cannot be diversified away by adding more stocks to a portfolio. *Unsystematic risk*, which can be diversified away, is the second component of total risk of a portfolio. Markowitz-like portfolio-diversified investors do not need to be compensated for unsystematic risk. The expected return of a security, thus, is the rate of return on a risk-free asset, plus the security's beta multiplied by the market risk premium:

$$\mathrm{E}(R_i) = R_f + \beta_i[\mathrm{E}(R_m) - R_f], \tag{1.1}$$

where

$$\begin{aligned} \mathrm{E}(R_i) &= \text{expected return for security } i, \\ R_f &= \text{risk-free rate}, \\ \mathrm{E}(R_m) - R_f &= \text{market risk premium, and} \\ \beta_i &= \text{beta of security } i \\ &= \mathrm{Cov}(R_i, R_m)/\mathrm{Var}(R_m). \end{aligned}$$

The market risk premium is the amount by which a portfolio of stocks diversified against all unsystematic risk is expected to exceed the risk-free rate. This value is frequently determined based on historical experience.

Initially, empirical tests of the CAPM tended to support the model. However, some studies, notably Roll [28], have stated that the tests were essentially joint tests of the model and the market proxy. The market portfolio should include such assets as bonds, real estate, collectibles, and even human capital, but valid measures of the total value of these assets on a regular basis are not available. Since empirical tests of the model tend to use a stock market portfolio, which is readily available, as the market portfolio, the model has not been, and actually cannot be, fully tested. More recently, Fama and French [16] report the results of an extensive test of the CAPM on stock market data from 1941 to 1990 and conclude that size and the ratio of book-to-market value are more important than beta in explaining returns. Over the entire period, the relationship between beta and average returns is insignificant. Despite this damaging evidence, the Capital Asset Pricing Model's risk and return relationship is still considered important today and is the foundation of several financial models that have been applied to insurance ratemaking.

Merton Miller, in his 1958 work with Franco Modigliani entitled "The Cost of Capital, Corporation Finance, and the Theory of Investment," laid the groundwork for corporate financial theory [25]. This work examined the impact that the use of debt and dividends had upon the value of the firm. Miller found that the value of the firm is independent of the level of debt and the dividend payout level chosen by the firm. This conclusion, derived from strict assumptions including no taxes, was controversial, but led to the understanding of optimal capital structure and dividend policy used by corporations today.

One other important development in financial economics is the option pricing theory developed by Black and Scholes [4] in the early 1970s. Options are derivative securities, meaning that they derive their value from their relationship with another security. Options give the holder the right to buy, in the case of a *call* option, or to sell, in the case of a *put* option, an asset at a specified price. Options exist on stocks, bonds, futures, commodities, stock indices, and even insurance catastrophe losses. For example, an investor may own a call option on IBM stock to buy IBM for $100 a share. If the price of IBM is greater than $100, the investor can exercise the option to buy IBM at $100 and then sell the stock at the higher price, thereby earning a profit. Many assets and claims exist that can be thought of as options, or contingent claims. For example, stockholders of a corporation can be thought of as holding an option on the company's assets being greater than its liabilities. If the assets are less than the liabilities, stockholders receive nothing; if greater, stockholders receive the entire difference.

Option pricing models such as the Black–Scholes model have been fairly successful at valuing options. The Black–Scholes model was so successful that its model prices were used by option traders as actual market prices in the early 1970s when organized option exchanges were opened.

The option concept along with the use of option pricing models can also be applied to the claims of insurers. The claims of policyholders, stockholders, and tax authorities against the insurer can be thought of as options. These applications will be discussed in detail later.

The contributions of Markowitz, Sharpe, Miller, Modigliani, Black, and Scholes have led to the development of financial models that have been applied to investment and corporate finance. These models have also been applied to ratemaking in the insurance industry. The formulation of these financial models and their application to insurance will be explained in this paper.

Even a cursory review of insurance profitability demonstrates that, at least since the 1970s, the industry has not achieved the target underwriting profit margin of five percent based on the 1921 National Convention of Insurance Commissioners profit formula. This result could be due to an inability to achieve the appropriate rate of return because of unexpected inflation, disasters and other insured catastrophes, social changes that raised costs in an unpredictable manner, or other unforeseen developments. However, the persistency of the shortfall, the fact that insurance markets have remained attractive enough to continue to draw new entrants and investment capital, and the fact that bankruptcies and failures among insurers have not risen to especially unusual levels suggest that a more appropriate explanation is that the model for determining the profit margin is at fault, and insurers have not been trying to obtain a 5 percent underwriting profit margin. The search for an alternative pricing model has yet to be concluded. For a description of the early regulatory decisions repudiating the 5 percent underwriting profit margin and a summary of alternative models, see Derrig [11].

2. TARGET TOTAL RATE OF RETURN MODEL

Early alternative pricing models were proposed by Bailey [2], Ferrari [17], and Cooper [6]. In these models, the total return of an insurer, the sum of underwriting and investment results, was recognized as the key measure of profitability. When investment income increases, as it did in the 1960s due to longer-tailed claim payments and higher interest rates, the underwriting income can be expected to decline, depending on the required total rate of return. An example of this approach is the Target Total Rate of Return Model.

The target total rate of return combines the two sources of income for an insurer: investment income and underwriting income. In this approach, a target total rate of return is set equal to the total return from investments plus the total rate of return from underwriting. Once the investment income is projected, the required underwriting profit margin can be calculated. The formula for the target total rate of return for insurers can be written as:

$$TRR = (IA/S)(IR) + (P/S)(UPM), \qquad (2.1)$$

where

$$TRR = \text{target total rate of return,}$$
$$IA = \text{investable assets,}$$
$$S = \text{owners' equity in the insurer,}$$
$$IR = \text{investment return,}$$
$$P = \text{premium, and}$$
$$UPM = \text{underwriting profit margin.}$$

In Equation 2.1, investment income and underwriting income are expressed as a percentage of equity.

In order to use this technique, the appropriate target total rate of return must be determined. Various procedures could be used to determine the total rate of return, such as an industry average return on equity, an arbitrary target such as 15 percent, a variable value tied to an alternative investment such as 5 percent over long-term Treasury bonds, or some appropriate rate of return for the investor based on the riskiness of the firm. The latter procedure of providing investors with an appropriate rate of return to compensate for the risk that they undertake is used in public utility rate regulation. The Capital Asset Pricing Model, discussed in detail in the next section, is often used in utility rate regulation to determine the appropriate risk-adjusted return that stockholders should expect to receive.

To apply the target total rate of return model, $E(R_e)$ from the CAPM in Equation 1.1 is set equal to the target total rate of return TRR in Equation 2.1. Solving for the underwriting profit margin UPM leads to the following equation:

$$UPM = (S/P)[R_f + \beta_e(E(R_m) - R_f) - (IA/S)(IR)]. \quad (2.2)$$

To use Equation 2.2 for a stock insurer, current company data for the ratios of investable assets to equity and premium are used along with a forecast of the insurer's investment rate of return.

The insurer's beta and the market risk premium can be gathered through historical estimates, and the current one year Treasury bill rate can be used as the risk-free rate in this single period model.

For example, assume the risk-free rate is 7 percent, the insurer's beta is 1.0, the market risk premium is 8 percent, the insurer's ratio of investable assets to equity is 3 to 1, the insurer's investment return is 7 percent, and the ratio of premiums to equity is 2 to 1. The target total rate of return is given by the CAPM in Equation 1.1 as follows:

$$TRR = 7\% + 1.0(8\%) = 15\%.$$

The underwriting profit margin is given by Equation 2.2:

$$UPM = (1/2)[15\% - 2(7\%)] = 0.5\%.$$

The investment return on equity of 14 percent is subtracted from the total rate of return of 15 percent yielding an underwriting return on equity of 1 percent, which translates into an underwriting profit margin of 0.5 percent.

The target underwriting profit margin for an insurer with equity of \$500,000, premiums of \$1,250,000, investable assets of \$2,000,000, investment return of 7.5 percent, and beta of 1.15, when the risk-free rate is 7 percent, and the market risk premium is 9 percent is determined as follows:

$$\begin{aligned} UPM &= (S/P)[R_f + \beta_e(\mathrm{E}(R_m) - R_f) - (IA/S)(IRR)] \\ &= (500{,}000/1{,}250{,}000) \\ &\quad \times [7\% + 1.15(9\%) - (2{,}000{,}000/500{,}000) \times (7.5\%)] \\ &= -5.06\%. \end{aligned}$$

In addition to the difficulty in determining the target total rate of return for this model, measuring the owners' equity in the insurer is another complex issue. This value should represent the current investment in the company, the amount that could be deployed elsewhere if the owners decided not to continue to write insurance. For a stockholder-owned insurer, this can be estimated in total by the market value of the company. However, insurers do not set rates in aggregate, but on a by-line by-state basis. Estimating the owners' equity in, for example, Kansas private passenger automobile, is far more difficult.

Equity is not statutory surplus. If statutory surplus, instead of the insurer's actual equity, is used in the target total rate of return method, the required underwriting profit margin derived from the model will be distorted. The statutory surplus figure is lower than most insurers' actual equity levels, since statutory surplus ignores the time value of money in loss reserves, excludes the value of tangible assets and non-admitted reinsurance, and values bonds and real estate at other than market values. Thus, the target total rate of return calculation based on statutory surplus will generate a lower underwriting profit margin than if the true equity figure were used. If this lower underwriting profit margin were forced upon insurers, they might react by investing in more risky assets to boost their investment rate of return in order to compensate for the lower underwriting profit margin. The reaction of increased risk taking by insurers could lead to an increase in insolvency among insurance companies.

A statutory surplus figure higher than actual equity levels, which could occur in times of increasing interest rates, would indicate a higher than necessary underwriting profit margin. This would cause excessive premiums to be charged to customers.

By itself, the target total rate of return approach lacks any theoretical justification for a proper rate of return. A model well supported by theory will be discussed next.

3. CAPITAL ASSET PRICING MODEL

The CAPM, developed in the 1960s, is one of the most powerful tools of finance and one of the foundations of most current financial theories. The CAPM has been applied to many financial issues: estimating stock returns and prices, determining appropriate corporate capital budgeting rates of return, establishing allowable rates of return for utilities, and pricing insurance. Insurance applications of the CAPM include estimating underwriting profit margins for insurance pricing purposes and determining the appropriate rate for discounting loss reserves.

The CAPM is based on several straightforward investment principles—asset allocation, portfolio return and risk, efficient portfolios, and portfolio diversification—that will be described and explained in this section.

Asset allocation involves dividing capital among broad asset categories. These asset categories include stocks, bonds, real estate, bank deposits, certificates of deposit (CDs), and Treasury bills. Most of the former asset categories can be described as risky assets, meaning they have an uncertain return. Some of the latter categories, such as Treasury bills and, to a lesser extent, bank deposits and CDs, are considered to be risk-free assets, meaning they have a virtually guaranteed return. The following simplified example will describe and illustrate asset allocation.

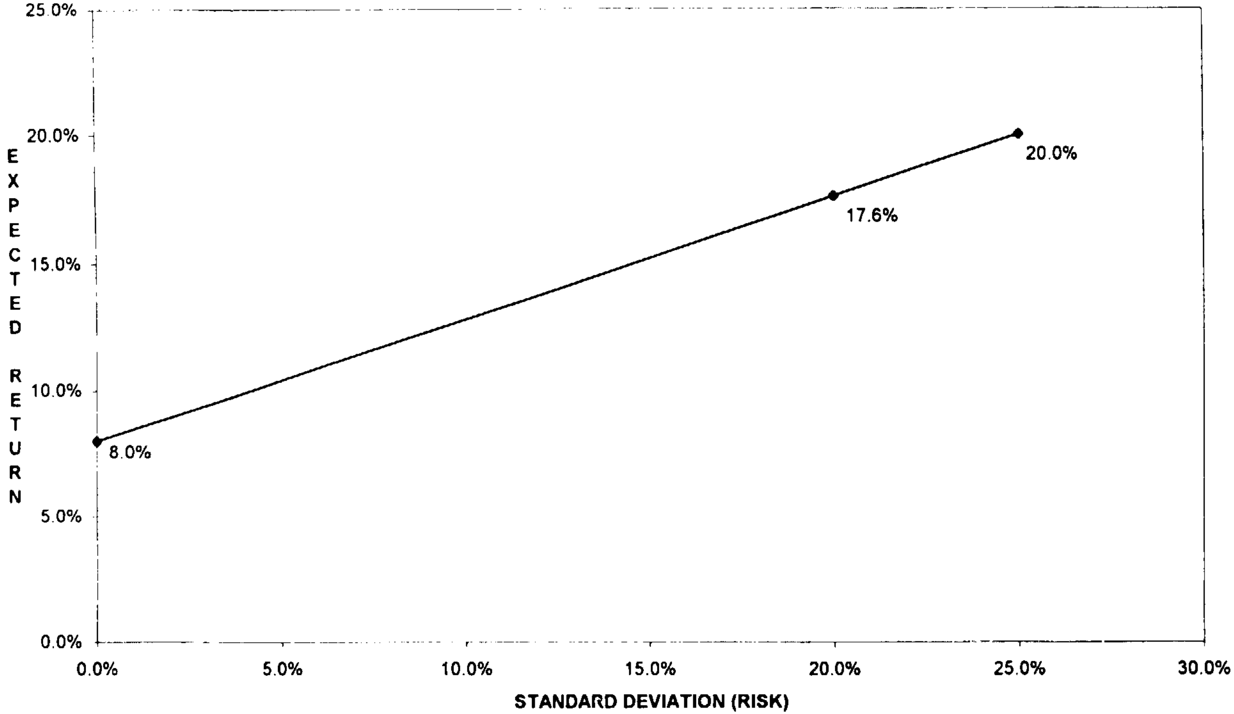
Two Asset Allocation Case

Assume that there are only two assets available to investors, one risk-free asset and one risky asset. The risk-free asset has a rate of return of 8 percent and has no risk. The risky asset has an expected rate of return of 20 percent and a standard deviation of 25 percent. The standard deviation measures the total variability of returns over time and is frequently used as a risk measure. This standard deviation of return will be our risk measure initially.

The two assets' expected return and risk are known, and an investor wants to allocate money between these two assets. Figure 1 illustrates this asset allocation choice. The y-axis intercept

FIGURE 1

TWO ASSET ALLOCATION



represents one possible asset allocation choice: investing all of the money in the risk-free asset, giving the investor a guaranteed return of 8 percent. Another possibility is the endpoint of the line plotted in Figure 1, which represents investing all of the money in the risky asset. This choice has an expected return of 20 percent and a standard deviation of 25 percent. The line between these two points, termed the Capital Allocation Line (CAL), represents the expected return and standard deviation of different combinations of the two assets. The expected return of each portfolio is simply a weighted average of each asset's expected return and is given by the following general formula:

$$\mathrm{E}(R_p) = (1 - W)R_f + W\mathrm{E}(R_k), \tag{3.1}$$

where

$\mathrm{E}(R_p)$ = the expected return of the combination portfolio,

R_f = the risk-free rate,

W = the proportion of money invested in the risky asset, and

$\mathrm{E}(R_k)$ = the expected return of the risky asset.

In this example, with R_f = 8 percent, and $\mathrm{E}(R_k)$ = 20 percent, the formula for the expected combination portfolio return is:

$$\mathrm{E}(R_p) = (1 - W)(8\%) + W(20\%). \tag{3.2}$$

The riskiness of the portfolio is given by the portfolio's standard deviation, which depends on the standard deviation of each asset, the proportion invested in each asset, and the covariance between the two assets' returns. The general formula for a two asset portfolio's standard deviation is:

$$\sigma_p = [(1 - W)^2\sigma_1^2 + 2(1 - W)(W)\mathrm{Cov}(R_1,R_2) + W^2\sigma_2^2]^{(1/2)}, \tag{3.3}$$

where

σ_i = the standard deviation of returns for asset i,

W = the proportion invested in asset 2, and

$\mathrm{Cov}(R_1, R_2)$ = the covariance of returns between asset 1 and asset 2.

The covariance of returns equals the product of the standard deviation of each of the two assets and the correlation coefficient between the two assets. In this example, the first asset is a risk-free asset, meaning it has a standard deviation of zero; therefore, the covariance between the risk-free asset and the risky asset is also zero, which yields the following simple formula for the standard deviation of our example's portfolio:

$$\sigma_p = [W^2 \sigma_k^2]^{1/2} = W\sigma_k. \tag{3.4}$$

Thus, in this example, the portfolio's standard deviation is the proportion of money invested in the risky asset multiplied by the standard deviation of the risky portfolio, which is 25 percent.

For illustrative purposes, consider a sample portfolio to verify a point on the Capital Allocation Line in Figure 1. Assume 20 percent of an investor's money is invested in the risk-free asset and the remaining 80 percent in the risky asset. The expected return for this portfolio would be:

$$\mathrm{E}(R_p) = (0.2)(8\%) + (0.8)(20\%) = 17.6\%.$$

The standard deviation of the portfolio would be:

$$\mathrm{SD}(\mathrm{E}(R_p)) = (0.8)(25\%) = 20\%.$$

This point is shown on the Capital Allocation Line in Figure 1.

To apply this technique, an individual investor would choose the desired level of risk and/or return and would solve for the appropriate proportion to invest in the risky asset from the risk and return Equations 3.2 and 3.4 above. The investor selects the point along the Capital Allocation Line that indicates the expected return and risk level of the investor's choice. For example, if an investor wanted an expected rate of return of 12 percent, he or she could use the expected return formula given in Equation 3.1 to solve for the risky asset proportion that would yield the 12 percent expected return. Solving Equation 3.1, the appropriate risky asset proportion would be:

$$W = [\mathrm{E}(R_p) - R_f]/[\mathrm{E}(R_k) - R_f]. \tag{3.5}$$

In this example Equation 3.5 would be written as the following:

$$W = [12\% - 8\%]/[20\% - 8\%] = 0.33 \text{ or } 33\%.$$

If the investor wanted an expected return of 12 percent, based on Equation 3.5, the investor would have to invest 33 percent of the portfolio in the risky asset and the remaining 67 percent in the risk-free asset. According to Equation 3.4, this portfolio would have a standard deviation of (0.33)(25%) or 8.25 percent.

An investor could also establish the portfolio according to the amount of risk desired. For example, if an investor could tolerate a risk level of only a 10 percent standard deviation in the expected return, he or she could use the portfolio standard deviation equation given in Equation 3.4 to solve for the risky portfolio proportion that would yield the 10 percent combination portfolio standard deviation. Solving Equation 3.4, the appropriate risky proportion would be:

$$W = \sigma_p/\sigma_k. \tag{3.6}$$

From the above example, the appropriate risky asset proportion would be:

$$W = 10\%/25\% = .40 \text{ or } 40\%.$$

Therefore, to achieve the desired risk level of 10 percent, an investor would have to invest 40 percent of the portfolio in the risky asset and 60 percent in the risk-free asset. From Equation 3.2, this would give the investor an expected portfolio return of

(.6)(8%) + (.4)(20%), or 12.8 percent for the desired 10 percent risk level.

What if an investor wanted an expected return greater than the expected return of the risky asset of 20 percent? In this case, the investor would have to invest more than 100 percent in the risky asset by borrowing. For simplicity, financial models often assume that investors can borrow and lend at the same interest rate. Figure 2 illustrates borrowing at the risk-free rate. The extension of the Capital Allocation Line beyond the horizontal line at an expected return of 20 percent represents a negative investment at the risk-free rate (borrowing), giving the investor the necessary funds to invest more than 100 percent in the risky asset. For example, if an investor wanted an expected return of 26 percent, the investor would solve for W, the proportion invested in the risky asset, from Equation 3.5. In this example, Equation 3.5 is written as the following:

$$W = [26\% - 8\%]/[20\% - 8\%] = 1.5 \text{ or } 150\%.$$

To achieve an expected return of 26 percent, the investor would have to invest 150 percent of the value of the portfolio in the risky portfolio and borrow an amount equal to 50 percent of the portfolio at the risk-free rate. This portfolio would have a standard deviation of (1.5)(25%), or 37.5 percent.

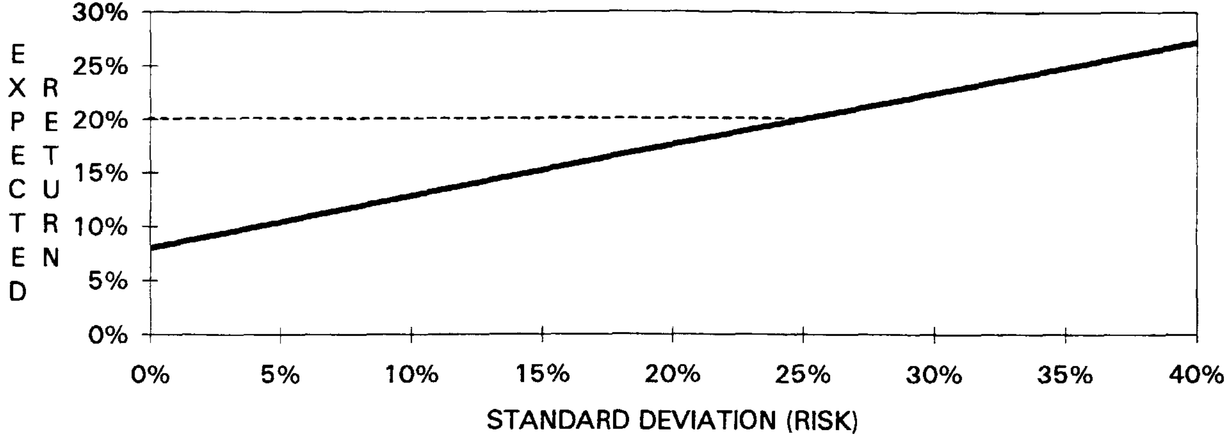
This concludes the discussion of the simple two asset allocation case. Next, the case that is more realistic, with a myriad of assets available for investment, is introduced. It will then be shown that this "entire universe" case can be simplified to a two asset allocation choice, leading to the concept of the Capital Asset Pricing Model.

Multiple Asset Allocation Case

Assume that any or all risky assets in the world are available for investment, all investors know the expected return and standard deviation of each asset and the covariance of returns

FIGURE 2

2 ASSET ALLOCATION WITH BORROWING AT THE RISK-FREE RATE



among different assets, and everyone has the same expectations regarding these returns and standard deviations. All the risk and return information could be used to form millions of different portfolios of these assets, and the expected return and standard deviation of each portfolio could be calculated. The risk and return calculation would be performed using a more general form of Equations 3.1 and 3.3 as given by Equations 3.7 and 3.8:

$$E(R_p) = \Sigma_i(W_i E(R_i)), \qquad \text{and} \tag{3.7}$$

$$\sigma_p^2 = \Sigma_i W_i \sigma_i^2 + \Sigma_j \Sigma_i W_i W_j \sigma_{ij}, \tag{3.8}$$

where

j does not equal i, and

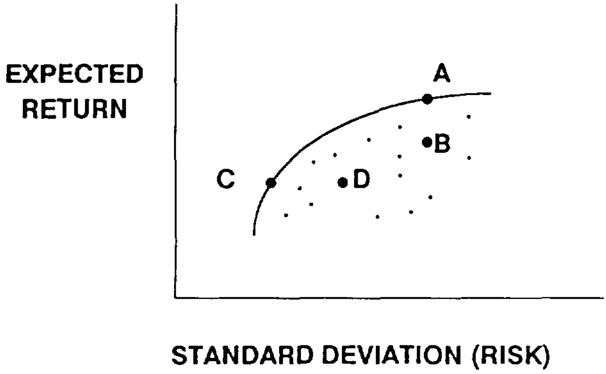
σ_{ij} = covariance between stocks i and j.

A sample of these portfolios is plotted on Figure 3. The next step is to decide which portfolios investors might select. Assume investors are rational risk averse investors. This means these investors prefer to maximize return for the same level of risk and to minimize risk for the same level of return. First, look at portfolios A and B in Figure 3. Notice that both portfolios have the same standard deviation, but portfolio A has a higher expected return. Rational risk averse investors would prefer portfolio A to portfolio B because it has a higher level of expected return for the same level of risk. Portfolio A is said to dominate portfolio B and any other portfolio below portfolio A on the graph with the same level of risk but a lower expected return.

Now, examine portfolios C and D in Figure 3. Both portfolios have the same expected return, but portfolio C has a lower standard deviation than portfolio D. Again, rational risk averse investors would rather invest in portfolio C because it has a lower level of risk for the same level of return when compared to portfolio D. Portfolio C is said to dominate portfolio D and any other portfolio to the right of portfolio C in the graph with the same expected return but a higher standard deviation.

FIGURE 3

EFFICIENT FRONTIER

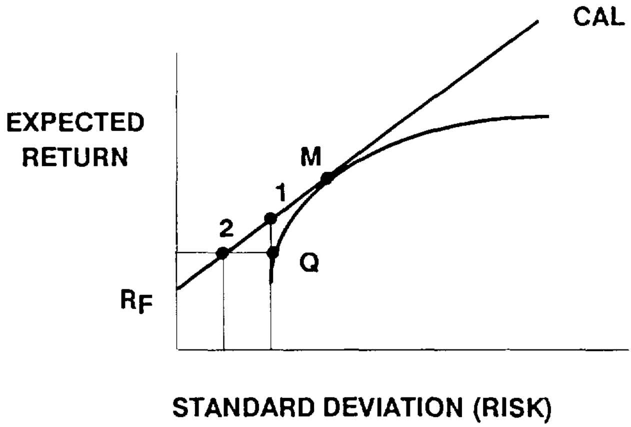


Portfolios A and C are called efficient portfolios because they have the highest level of return for a given level of risk and the lowest level of risk for a given level of return. All the portfolios on the curve in Figure 3 are efficient portfolios and investors would want to invest only in this set of efficient portfolios because their risk and return characteristics dominate all portfolios under the curve. The curve representing the efficient portfolio set is called the efficient frontier.

An investor could select a portfolio on this efficient frontier according to the investor's desired risk and return level. However, there is a better way to choose a portfolio which coincides with our earlier two asset allocation example.

Assume the risk-free asset still exists and a line can be drawn from the risk-free asset to the efficient frontier. The line could intersect the efficient frontier at any point on the curve, but the line from the risk-free rate that is tangent to the curve is the most desirable line from the standpoint of the investor. This line, included in Figure 4, is exactly the same as the capital allocation line discussed earlier in the two asset example. In this case, the

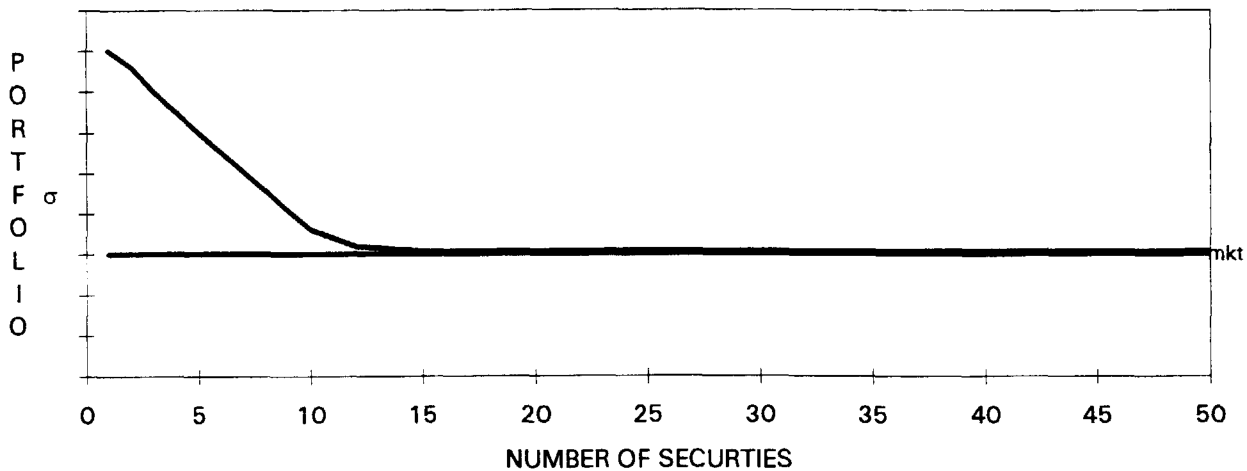
FIGURE 4



risky asset is the one portfolio, M, that is common to both the line and the efficient frontier. Since every investor has the same (homogenous) expectations about asset risk and return, portfolio M in Figure 4 would be the risky portfolio that all investors would want to own. In this case, the investor's portfolio choice is the same as the earlier two asset case of a combination between the risk-free asset and the efficient risky asset portfolio M. The investor's portfolio would be divided between the two assets according to the level of risk and/or return desired. If the investor wanted the level of risk and return that portfolio M offered, then 100 percent of the money would be placed in portfolio M. An investor wanting a level of risk lower than portfolio M would prefer to invest a portion of the total portfolio in the risk-free asset and the rest in portfolio M according to Equation 3.6 instead of investing in a portfolio on the efficient frontier like Q in Figure 4. A portfolio on the CAL has either a higher return for the same level of risk, such as point 1, or a lower level of risk for the same level of return, such as point 2, when compared to portfolio Q. In other words, the CAL is more efficient than the efficient frontier at every point except M; that is why this expanded asset world example collapses to the two asset allocation choice.

Measurement of Expected Returns on Individual Securities

At this point, the investor holds a well-diversified portfolio that consists of the risk-free asset and the optimal risky portfolio M which contains many assets (theoretically all assets available in the world). To define a well-diversified portfolio, we must first discuss the two types of risk of a portfolio. As in insurance, where an insurer can reduce risk by writing more policies, an investor can reduce risk by adding more assets to a portfolio. Figure 5 illustrates this point where the standard deviation, or total risk, of a portfolio is denoted on the y-axis and the number of assets on the x-axis. Generally, by choosing assets at random and adding them to the portfolio, the investor can reduce the overall risk of a portfolio. However, eventually the investor reaches a saturation point where more assets added to the portfolio do not significantly reduce the total risk of the portfolio. At this saturation point, the investor still has risk remaining in the portfolio. This remaining risk has three different names in finance, but all mean the same thing: nondiversifiable, systematic, or market risk. This market risk is risk that cannot be diversified away by adding more assets to a portfolio and is the inherent risk associated with the market portfolio of all risky assets. However, if asset returns were uncorrelated, then this residual risk would disappear in the same way that the law of large numbers applies to insurance. Risk associated with individual assets that can be diversified away is called diversifiable, unsystematic, or company-specific risk. Therefore, total risk is equal to company-specific risk plus market risk. This means that the investor should be concerned with both risks, the total risk, if a portfolio contains only a few different assets, and with just market risk if the portfolio is well-diversified. Returning to the case discussed earlier where the investor holds a portfolio that consists of a combination of the risk-free asset and the "market" portfolio M in Figure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NUMBER OF SECURTIES

4, this investor holds a well-diversified portfolio and should be concerned only with market risk rather than total risk.

How can this market risk be measured? Consider an investor who holds a well-diversified portfolio and is thinking about adding a new asset to the portfolio. The investor should be concerned with how the new asset's returns vary with the market portfolio's returns. The following simple linear regression can be calculated to measure the new asset's market risk:

$$R_{it} = a_i + b_i R_{mt} + e_{it}, \tag{3.9}$$

where

a_i and b_i = the regression intercept and slope coefficient,

R_i = the return of asset i,

R_m = the return of the market portfolio,

e_{it} = the residual error at time t, and

t = time.

The regression line slope coefficient, $b_i = \text{Cov}(R_i, R_m)/\text{Var}(R_m)$, measures the time series variation between the asset's return and the market portfolio's return and can be used as a measure of the asset's market risk. Let's now call b_i, beta or β. The market portfolio has a β of one. If an asset's β is greater than one, it means the asset's return tends to go up more than the market when the market rises and decline more when the market return drops.

Returning to the asset allocation choice between the risk-free asset and the market portfolio M, the formula for the expected return of this portfolio is given by rewriting Equation 3.1 as:

$$\text{E}(R_p) = (1 - W)R_f + (W)\text{E}(R_m). \tag{3.10}$$

Equation 3.10 can be rewritten as

$$\text{E}(R_p) = R_f + W(\text{E}(R_m) - R_f). \tag{3.11}$$

The second half of Equation 3.11 can be called a market risk premium, which is the return an investor expects to receive above the risk-free rate for investing in the market portfolio. If W is 1 in Equation 3.11, the expected portfolio return equals the expected market return, and the portfolio risk equals the standard deviation of the market portfolio. Figure 4 shows this relationship but still uses total risk as a risk measure although the investor should only be interested in market risk, as measured by β, when looking at adding a new asset to a well-diversified portfolio.

β measures an individual asset's sensitivity to movements in the market portfolio, and $E(R_m) - R_f$ is the excess return demanded on the market portfolio, or market risk premium. The excess return demanded on an individual asset added to a well-diversified portfolio should be $\beta[E(R_m) - R_f]$ which is the asset risk premium. From this relationship, the following formula for the expected or required return for an individual asset in a well-diversified portfolio can be developed:

$$E(R_i) - R_f = \beta_i[E(R_m) - R_f], \tag{3.12}$$

which is the formula for the asset risk premium just explained above. Rewriting Equation 3.12 gives a formula for the expected return on asset i,

$$E(R_i) = R_f + \beta_i[E(R_m) - R_f]. \tag{3.13}$$

Equation 3.13, shown previously as Equation 1.1, is known as the Capital Asset Pricing Model, and it is a single period linear relationship between market risk as denoted by β and expected return. The CAPM was developed by Sharpe [29], Lintner [21], and Mossin [26] independently in the 1960s. The assumptions of the model are:

1. Investors are risk averse diversifiers who try to maximize expected return and minimize risk.

2. Investors are price takers, in that they act as if their trades have no effect on asset prices.

3. Investors have homogeneous or identical expectations about asset expected returns and standard deviations.
4. Investors have no transaction costs or taxes.
5. Investors can borrow or invest at the risk-free rate without any limit.
6. Assets are infinitely divisible.

Equation 3.13 forms a line in Figure 6 known as the Security Market Line (SML). The y-axis in Figure 6 is the expected return and the x-axis is beta. The slope of the SML is the market risk premium, $\mathrm{E}(R_m) - R_f$, and the market portfolio has a beta of 1.

In the example in Figure 6, the risk-free rate is 8 percent, and the market risk premium return is 9 percent. This leads to an expected market return, where beta equals 1, of 17 percent, as depicted by the horizontal line at 17 percent. From this graphical relationship, we can find the expected return of any asset as long as we know its beta. For example, assume stock A has a beta of 1.2. From Equation 3.13, stock A's expected return would be:

$$\mathrm{E}(R_A) = 8\% + 1.2[9\%] = 18.8\%.$$

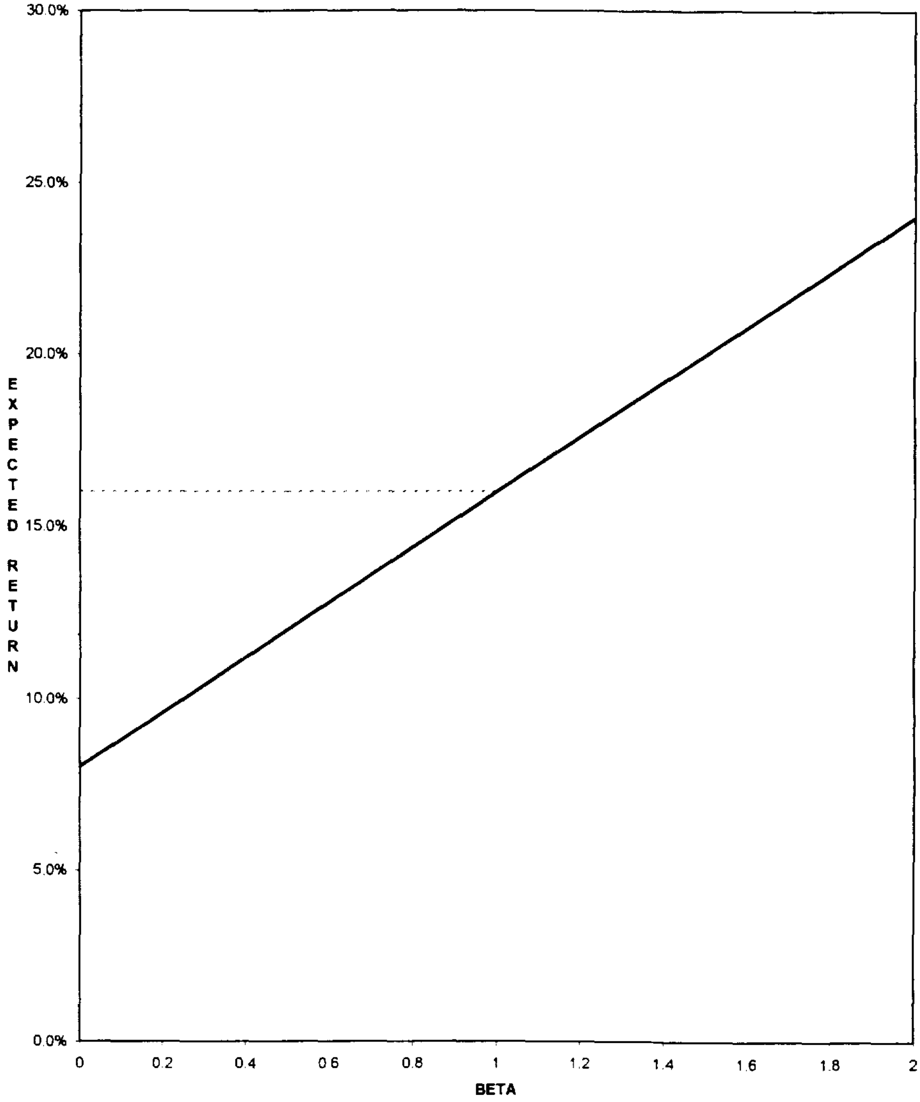
This point is shown on the SML in Figure 6. A stock with a beta of 0.6 would have an expected return of (8% + 0.6[9%]), or 13.4 percent.

An asset with a negative beta is assumed to have returns that move in the opposite direction of the return of the market portfolio. Examples of assets that may have negative betas are gold and gold mining stocks, which tend to have increased returns when the market falls. Continuing with the previous examples, a stock with a beta of −0.4 would have an expected return of (8% + (−0.4)[9%]), or 4.4 percent.

The implication of the CAPM for asset prices is that when an asset price is in equilibrium, its actual expected return equals its expected return as given by the CAPM. If an asset, such as

FIGURE 6

CAPITAL ASSET PRICING MODEL



stock A with a beta of 1.2, has an expected return of 20 percent, higher than its equilibrium expected return of 18.8 percent, then its return should decline to 18.8 percent. In order for stock A's return to go down, its price must go up. Investors, noticing that stock A's return lies above the SML, will put buying pressure on the stock until the price rises to the equilibrium point where its expected return equals 18.8 percent. The opposite would happen if the actual expected return were below the expected return given on the SML.

Some cautions about using the CAPM must be mentioned. The model requires the use of the market risk premium and past market portfolio returns and individual asset returns to arrive at beta estimates for individual assets. This assumes that such relationships are stable, when in fact they are likely to change over time. There has also been much debate in finance literature about what the market portfolio actually is. Most of the debate centers around the fact that the market portfolio in theory consists of all the assets in the world, and its return has never been measured. To use the CAPM in practice, a proxy for the market portfolio is used. Typical market proxies are stock market indices such as the Standard & Poor's 500 stock index, the New York Stock Exchange Composite stock index, the Wilshire 5000 stock index, the Value Line Investment Survey 1700 stock index, the American Stock Exchange Index, or combinations of some of these indices along with bond and real estate indices. Despite the problems with choosing an appropriate market proxy, the CAPM is a model that can easily be applied to many applications and has been applied to insurance.

4. APPLICATION OF THE CAPITAL ASSET PRICING MODEL TO INSURANCE

The Capital Asset Pricing Model as defined in Equation 3.13 has been used to determine insurance underwriting profit margins by Fairley [15], Hill [19], and Hill and Modigliani [20] among others. The basic form of Fairley's CAPM model is given by the following equation:

$$UPM = -kR_f + \beta_u[\mathrm{E}(R_m) - R_f], \tag{4.1}$$

where

k = the funds generating coefficient, and

β_u = the underwriting beta.

In this form of the model, the appropriate underwriting profit margin is equal to the insurer's systematic underwriting risk premium, $\beta_u[\mathrm{E}(R_m) - R_f]$, which is offset by the investment inflow rate of return, $-kR_f$. The underwriting beta is determined by the historical movements of underwriting returns in relation to the market portfolio returns, and can be applied to individual lines of business. The investment inflow rate of return arises because of the time lag between the receipt of premiums by the insurer and the payment of losses and expenses. The funds generating coefficient, k, represents the average time the insurer holds premiums. This model ignores actual insurance company investment performance but assumes insurers will earn the risk-free rate of return. The insurer bears the risk and incurs the gain or loss on any risky investment.

Use of the Fairley CAPM requires an estimate of the underwriting beta and the funds generating coefficient for the company as a whole or for the line of business under consideration. The underwriting beta is frequently estimated by running a simple linear regression of historical underwriting returns against the returns of the market portfolio as described in Equation 3.9. The beta coefficient, the estimate for β_u, from this regression is equal to $\mathrm{Cov}(R_u, R_m)/\mathrm{Var}(R_m)$.

Cummins and Harrington [9] used quarterly underwriting results for insurers to arrive at an empirical β_u estimate that was insignificantly different from zero. Other empirical studies have estimated the beta of insurer liabilities, which is then converted to an underwriting beta. Hill found a liability beta of −0.23 through a regression approach. Fairley used an indirect estimation approach depending on the insurer's financial leverage, the systematic risk of investable assets (the asset beta), and the funds generating coefficient, from which an estimate of −0.21 was found for the liability beta. Fairley then used the relationship of $\beta_u = -k\beta_L$ which yielded a positive underwriting beta of approximately 0.2.

The funds generating coefficient estimate, k, can be given by the insurer's projection of the loss and expense payment pattern expected from the insurer's current exposures. The estimate k would be the weighted average of the length of time expected between the receipt of premium and the payment of losses and expenses among these different exposures. A value of 1 for k would mean an expected time lag of one period between the receipt of premium and payment of losses and expenses, and a value of 0 would mean that losses and expenses are paid as soon as the premiums are received. Fairley found empirical estimates of k for various lines of insurance that ranged from 0.31 and 0.35 for auto property damage and homeowners to 1.60 for both auto bodily injury and workers compensation to a high of 3.74 for medical malpractice.

To illustrate the use of the model, consider the following example. Assume an insurer wants to determine the minimum annual underwriting profit margin to factor into the upcoming year's premiums for homeowners insurance. The company has determined that homeowners coverages consist of three distinct payment pattern groups. Group 1, which represents 30 percent of the homeowners premium, has an expected loss payment pattern of three months or 0.25 years after receipt of premiums. Group 2, which represents 40 percent of the line's premium, has an expected loss payment pattern of 0.5 years after receipt of premiums. Group 3, which represents 30 percent of the line's premium, has an expected payment pattern of 0.75 years after receipt of premiums. The estimate of the funds generating coefficient, k, for the line of business would be:

$$k = .3(0.25) + .4(0.5) + .3(0.75) = 0.5.$$

The insurer has determined the homeowners underwriting beta to be 0.2 based on historical information. The risk-free rate is 6 percent and the market risk premium is 8 percent. The appropriate homeowners insurance underwriting profit margin according to the model in Equation 4.1 would be:

$$UPM = -0.5(6\%) + 0.2[8\%] = -1.4\%.$$

In this example, the underwriting risk premium, $0.2[8\%] = 1.6\%$, is offset by the interest received from the investment of premiums at 6 percent for one-half year to yield a negative underwriting profit margin of 1.4 percent.

Now, assume a lower risk-free rate of 4 percent and a higher underwriting beta of .50. The underwriting profit margin in the above example would be:

$$UPM = -0.5(4\%) + 0.5[8\%] = 2.0\%.$$

In this second example, the lower risk-free rate results in a lower investment rate of return and the higher beta produces a larger underwriting risk premium, yielding a higher indicated underwriting profit margin.

The insurance CAPM described in Equation 4.1 does not include the effects of taxation. The Hill and Modigliani tax version of the insurance CAPM takes into account the corporate taxation of underwriting income and differential tax rates for the assets in an insurer's investment portfolio of tax-exempt bonds, capital gains on stocks and bonds, and corporate dividend income from other non-controlled corporations. The tax version insurance CAPM can be written as the following equation:

$$\begin{aligned} UPM = {} & -kR_f(1 - T_A)/(1 - T) + \beta_u[\mathrm{E}(R_m) - R_f] \\ & + (S/P)R_f[T_A/(1 - T)], \end{aligned} \tag{4.2}$$

where

$$T_A = \text{the tax rate on investment income,}$$

$$T = \text{the tax rate on underwriting income, and}$$

$$S/P = \text{the insurer's equity to premium ratio.}$$

In the above equation, T_A is the weighted average of the different tax rates on the insurer's investment portfolio.

The first term in Equation 4.2 is the after-tax adjusted risk-free return on the insurer's investment portfolio during the time lag between receipt of premiums and payment of losses. The second term is the underwriting risk premium.

To illustrate the use of the tax version insurance CAPM, consider the following example: the risk-free rate = 6%; the market risk premium = 8%; the underwriting beta = 0.2; the funds generating coefficient = 0.5; the corporate tax rate = 35%; the equity to premium ratio = 1.0; and the insurer invests 30 percent of its investment portfolio in tax exempt bonds, 20 percent in corporate dividend income stocks which are taxed at 30 percent of the corporate tax rate, and 50 percent in investments that are taxed as ordinary taxable income. The investment income tax rate is the weighted average of the tax rates of each investment category and is given by the following:

$$T_A = .3(0\%) + .2(.3)(35\%) + .5(35\%) = 19.6\% \text{ or } .196.$$

The first term above is for the tax exempt bonds, the second term for the corporate dividend income which has an effective tax rate of .3(35%) = 10.5%, and the third term for ordinary income. Given the investment income tax rate, the tax version insurance CAPM yields the following underwriting profit margin:

$$\begin{aligned} UPM = {} & -0.5(6\%)(1 - .196)/(1 - .35) + 0.2[8\%] \\ & + 1.0(6\%)(.196/(1 - .35))] = -0.30\%. \end{aligned}$$

Notice this is the same as the first insurance CAPM example but with the addition of taxes; the effect of taxes is to generate a higher underwriting profit margin.

The models that apply the CAPM to insurance have been criticized for ignoring risk unique to insurance that is not systematic with investment risk. Ang and Lai [1] determine that insurance premiums should be based on both systematic insurance risk and systematic investment risk. Turner [30] indicates that the insurance market cannot simply be appended to the stock market. Both studies conclude that CAPM insurance pricing models would underprice insurance. These conclusions are supported in D'Arcy and Garven [10] by the finding that actual underwriting profit margins significantly exceeded the CAPM indications over most of the period from 1926 to 1985. Thus, while it is important to understand the mechanics of both the CAPM and its applications to insurance, this method is not necessarily the appropriate pricing technique.

For example, consider the following situation in which the CAPM is not likely to produce the correct indication. An insurer is pricing earthquake insurance and assumes that the underwriting beta for this coverage is zero, the funds generating coefficient is .4, the risk-free rate is 5 percent, and the market risk premium is 7 percent. The insurer operates at a 2 to 1 premium to equity ratio, has a 35 percent tax rate on underwriting income, and a 15 percent tax rate on investment income. The indicated underwriting profit margin based on the tax version insurance CAPM is:

$$\begin{aligned} UPM &= -.4(5\%)(1-.15)/(1-.35) + 0(7\%) \\ &\quad + (1/2)(5\%)(.15/(1-.35)) \\ &= -2.0\%. \end{aligned}$$

What factors are not reflected in this calculation that would affect the appropriate underwriting profit margin? The CAPM provides a risk premium only for risk that is systematic with market returns, ignoring catastrophe risk. Also, the CAPM ignores bankruptcy costs. Insurers must be concerned with insurance-specific risk and with bankruptcy. Thus, the CAPM indicated underwriting pricing margin is likely to be too low.

5. DISCOUNTED CASH FLOW ANALYSIS

Discounted cash flow analysis is another foundation of most financial theories and models. Discounted cash flow (DCF) analysis converts cash flows from different times to a common point based on the time value of money so that cash inflows and outflows can be more easily compared. Discounted cash flow analysis is used to value bonds, stocks, and corporate investments in capital projects. DCF can also be useful in insurance where differences in timing between receipt of premiums and payment of losses are common.

The typical DCF analysis is a straightforward calculation that finds the present value of expected future cash flows by discounting these cash flows at the appropriate discount rate. The present values are then summed to determine the value of the investment. The basic concept behind the time value of money is that a dollar in the future is worth less than a dollar today. A dollar today can be invested and earn interest so that more than a dollar will be available in the future, or can be used for current consumption, which is assumed to be worth more than a similar amount of consumption in the future.

To illustrate this approach, consider the following example: an insurer sells a one-year policy for a premium of $1,200 that has $200 of expenses paid concurrently with the receipt of premium and an expected loss of $1,050 that is paid at the end of the year, and the insurer can invest the premium less expenses at 7 percent. The insurer would like to know the gross profit from both underwriting and investment on this policy in today's dollars. This problem can be approached in two ways.

First, the end of period gross profit could be determined by finding the future value of the net premium investment and then subtracting the end of period expected loss. The \$1,000 net premium is invested at 7 percent and the future, or end of period, value is given by the following:

$$\begin{aligned} FV &= PV(1 + r) \\ &= \$1{,}000(1 + .07) = \$1{,}070, \end{aligned} \tag{5.1}$$

where

FV = the future value,

PV = the present value (which is the premium in this case), and

r = the interest rate.

The end of period gross profit would be \$1,070 – \$1,050 = \$20, but the insurer wants to know what this is worth in today's dollars. This means the present value of the \$20 end of period gross profit must be found. Assuming the discount rate is equal to the insurer's investment rate of 7 percent, the present value can be found by solving for PV in Equation 5.1:

$$PV = FV/(1 + r). \tag{5.2}$$

The present value of the future profit of \$20 is:

$$PV = \$20/(1 + .07) = \$18.69.$$

A second and more direct approach to find the present value of the policy's gross profit is to subtract the present value of the expected loss from the premium:

$$\begin{aligned} PV \text{ (gross profit)} &= \$1{,}000 - \$1{,}050/(1 + .07) \\ &= \$1{,}000 - \$981.31 = \$18.69. \end{aligned}$$

Again, the present value of the policy's gross profit is found to be \$18.69.

More general versions for the future value and present value equations with a time span of more than one year are given in Equations 5.3 and 5.4:

$$FV_t = PV(1 + r)^t, \quad (5.3)$$

$$PV = FV_t/(1 + r)^t, \quad (5.4)$$

where

FV_t = the future value at time t,

PV = the present value at time 0, and

r = the interest (discount) rate.

To illustrate the use of the above formulae, consider the following examples.

EXAMPLE 1 An investor places \$500 today in an account paying 8 percent annually for three years. How much would the investor have in the account at the end of three years?

$$FV_3 = \$500(1.08)^3 = \$629.86.$$

The investor will have \$629.86 in the account at the end of three years.

EXAMPLE 2 An insurer expects to make a loss payment of \$5,000 five years from now and has a discount rate of 9 percent; the insurer wants to know the present value of this future payment:

$$PV = \$5{,}000/(1.09)^5 = \$3{,}249.66.$$

The present value of the expected \$5,000 payment in five years is \$3,249.66.

DCF analysis can also be used to find the present value of multiple cash flows, as illustrated by examples valuing bonds, corporate investment projects, and stocks.

For example, assume a bond matures in two years and pays annual interest of \$100 per year, with the next payment occurring in one year and the last payment occurring in two years. In addition to the interest payments, the bond has a maturity value of \$1,000 also payable in two years. If the appropriate rate of return on this bond is 8 percent, the value of the bond can be determined according to the following formula:

$$V = \Sigma_t\{CF_t/(1+r)^t\}, \tag{5.5}$$

where

V = the value of the investment,

CF_t = the cash flow at time t, and

r = the discount rate.

For the above bond example, Equation 5.5 is as follows:

$$\begin{aligned} V &= \$100/(1.08) + \$100/(1.08)^2 + \$1000/(1.08)^2 \\ &= \$92.59 + \$85.73 + \$857.34 = \$1{,}035.66. \end{aligned}$$

The present value of the cash flows discounted at 8 percent from the bond is \$1,035.66. A use for this technique is to determine the appropriate price to pay for an investment. If the investor requires an 8 percent return to make the above investment desirable, then the maximum purchase price that would be paid to obtain these cash flows is \$1,035.66. Any higher price would generate a return less than 8 percent.

A variation of the formula in Equation 5.5, which includes a cash flow at time zero, can be used to help corporate managers determine whether to invest in a given project

$$NPV = CF_0 + \Sigma_t\{CF_t/(1+r)^t\}. \tag{5.6}$$

Equation 5.6 used in this corporate capital budgeting environment is called the net present value (*NPV*) of the project. To calculate the *NPV* of an investment, the manager simply needs the estimates of the future cash flows from the investment, the estimated cost of the investment, and the required return or discount rate demanded by the firm on this type of investment. If the *NPV* is positive, the present value of the expected cash inflows is greater than the expected cost of the investment, and the project would be profitable to invest in, assuming the projected cash inflows turned out to be correct. A negative *NPV* means that the estimated costs of the investment exceed the present value of the expected cash inflows, and such a project would be considered unacceptable for investment purposes.

Consider the following example for net present value analysis.

NPV ANALYSIS : $r = 15\%$

Period	Cash Flow
0	−$10,000
1	$4,000
2	$5,000
3	$4,000
4	$2,000
5	$1,000

The *NPV* from Equation 5.6 for this sample would be written as:

$$NPV = -\$10{,}000 + \$4{,}000/(1.15) + \$5{,}000/(1.15)^2$$

$$+ \$4{,}000/(1.15)^3 + \$2{,}000/(1.15)^4 + \$1{,}000/(1.15)^5$$

$$= -\$10{,}000 + \$3{,}478 + \$3{,}781 + \$2{,}630 + \$1{,}144 + \$497$$

$$= \$1{,}530.$$

The project in this example has a positive *NPV* of $1,530, which means it is an acceptable project for investment purposes.

Another application of the discounted cash flow analysis on the same project evaluation is the internal rate of return (*IRR*) method. The *IRR* is simply the discount rate that gives the project a net present value of zero. Equation 5.7 is the formula for the internal rate of return:

$$NPV = CF_0 + \Sigma_t\{CF_t/(1 + IRR)^t\} = 0. \tag{5.7}$$

The *IRR* is found by trial and error or by computer programs that iterate to find the appropriate rate. The decision rule for the *IRR* approach is as follows: if the *IRR* is greater than the required rate of return for the project, then accept the project; if the *IRR* is less than the required rate of return, the project is rejected. The *IRR* for the previous example is 22.63 percent. The *NPV* approach with a discount rate of 22.63 percent is used below to prove the *IRR* result

$$\begin{aligned} NPV = & -10{,}000 + 4{,}000/(1.2263) + 5{,}000/(1.2263)^2 \\ & + 4{,}000/(1.2263)^3 + 2{,}000/(1.2263)^4 + 1{,}000/(1.2263)^5 \\ = & -10{,}000 + 3{,}262 + 3{,}324 + 2{,}169 + 884 + 361 \\ = & \ 0. \end{aligned}$$

For typical projects, the *NPV* and *IRR* will always provide the same decision.

However, problems exist with the *IRR* method. If the cash flows change sign (positive to negative or vice-versa) more than once, then multiple *IRR*s can occur. For example, a project may require an initial investment (negative cash flow), then generate a positive cash flow, and then negative cash flows. In this case, one *IRR* may be negative and another one very high, but usually only one *IRR* appears reasonable. For example, consider a project with the following cash flows:

Period	Cash Flow
0	−$5,000
1	$5,000
2	$4,000
3	−$3,000
4	$2,000
5	−$1,000

This project has a *NPV* at 15 percent of $1,046; however, the project has two *IRR* values: −46.9 percent and 36.4 percent. Since the project has a positive *NPV* at 15 percent, the latter *IRR* of 36.4 percent must be the reasonable value for the internal rate of return.

Stock Valuation

Another application of discounted cash flow analysis is stock valuation. The value of a stock can be thought of as the present value of its future cash flows, similar to the earlier bond valuation example. The relevant cash flows for a stock are its expected future cash dividends. The valuation is a little more difficult for stocks because they have no maturity value. One stock valuation model is the Gordon growth model. It assumes the following present value of expected dividends model:

$$V = \Sigma_t\{D_t/(1 + r_s)^t\}, \tag{5.8}$$

where

D_t = the dividend expected at time t, and

r_s = the required return on stock.

The Gordon model is a specialized version of the model in Equation 5.5 that assumes a constant annual growth rate in dividends, which causes Equation 5.5 to reduce to the following:

$$V = D_0(1 + g)/(r_s - g), \tag{5.9}$$

where

D_0 = the current dividend paid,

g = the constant dividend growth rate, and

r_s = the required return on stock.

The model in Equation 5.9 cannot be used if the growth rate is greater than or equal to the required stock return rate.

To illustrate the use of Gordon's stock valuation model, consider the following example. Stock A currently pays a dividend of \$3 per share and has a required rate of return of 17 percent. Stock A's dividend is expected to grow at a constant rate of 9 percent annually. Equation 5.9 can be used in this case and would be written as follows:

$$V = \$3(1.09)/(.17 - .09) = \$40.875.$$

The value of \$40.875 given by the model for Stock A is called the stock's intrinsic value. An investor could compare the model price to the actual market price for Stock A and decide whether to buy or sell the stock.

A word of caution about using stock valuation models. The model is only as good as the estimates used in it. It is quite difficult to arrive at accurate estimates of future growth rates in dividends.

6. DISCOUNTED CASH FLOW MODELS APPLIED TO INSURANCE

Two basic methods of applying discounted cash flow analysis to insurance have developed. One, termed the Risk Adjusted Discount Technique, analyzes the cash flows from the point of view of the policyholder, and was first applied at the 1982 Massachusetts automobile rate hearings [27]. The other approach, used by the NCCI, is an internal rate of return calculation. Cummins [7] explains and compares these two approaches. Derrig [13] explains how the Risk Adjusted Discount Technique has been used in Massachusetts to set automobile and workers compensation rates and discusses the key issues in selecting parameter values. In retrospect, as cited by Derrig, the factor most responsible for underpricing these coverages has been the underestimation of losses and expenses, rather than the choice of financial model or value. The material presented here does not attempt to duplicate the specific approach in Massachusetts, but does apply the same general technique. Although the calculations seek to determine an appropriate premium rather than an underwriting profit margin, the underwriting profit margin can be calculated in the conventional manner after the premium is determined.

The basic premise of the Risk Adjusted Discount Technique is that, on a risk adjusted basis, the present value of the premium equals the present value of all the cash flows resulting from writing an insurance policy. Specifically, the present value of the premium equals the sum of the present values of the losses, expenses, and taxes on both underwriting and investment income, generated by the contract. For an explanation of the importance of considering taxes post-Tax Reform Act of 1986, see Derrig [12]. The term "risk adjusted" means that the interest rate selected to discount cash flows varies to account for the degree of risk inherent in the cash flow: a risky cash flow will be discounted at a different rate than a certain cash flow.

To illustrate this concept, assume that an insurer is trying to set a premium level for a one year policy. The premiums will be collected when the policy is effective. Expenses on the policy are $20 and will be paid when the policy is written. Losses on the policy are expected to be $80, and will be paid at the end of the year. (Assume, for example, that the average loss will occur halfway through the coverage period and there will be a six month lag in paying the claim.) The insurer will incur taxes on the underwriting profit (or a tax reduction on an underwriting loss) at the 35 percent level. The insurer will earn investment income on the premium less the expenses paid, and on the surplus, or equity, devoted to this policy. The insurer will assign \$50 of equity to support writing this policy. In this case, the insurer pays the same 35 percent tax rate on investment income as on underwriting income. All taxes will be paid at the end of the year. In this first example, risk will be ignored and all cash flows will be discounted at the same interest rate of 7 percent.

The general format of the discounting approach is quantified as follows:

$$PV(P) = PV(L) + PV(E) + PV(TUW) + PV(TII), \qquad (6.1)$$

where

PV = present value operator,

P = premiums,

L = losses and loss adjustment expenses,

E = underwriting expenses,

TUW = taxes on underwriting profit or loss,

TII = taxes on investment income, and

UPM = underwriting profit margin.

For the first example, the calculation becomes:

$$\begin{aligned} P &= \frac{80}{1.07} + 20 + \frac{(P-20-80)(.35)}{1.07} \\ &\quad + \frac{(50+P-20)(.07)(.35)}{1.07} \\ P &= 74.766 + 20 + .327P - 6.542 - 26.168 + .687 + .023P \\ .65P &= 62.743 \\ P &= \$96.53 \\ UPM &= 1 - \frac{80}{96.53} - \frac{20}{96.53} = -3.59\%. \end{aligned}$$

In the first case, the premium is \$96.53 for an underwriting profit margin of negative 3.59 percent. This represents the

TABLE 1

SUMMARY OF NOMINAL AND DISCOUNTED VALUES
EXAMPLE 1

	Nominal Values	Discounted Values
Losses	$ 80.00	$74.77
Expenses	20.00	20.00
Taxes on Underwriting	− 1.21	− 1.14
Taxes on Investments	3.10	2.90
Total	$101.89	$96.53*

*Premium = Sum of the Discounted Value of Losses, Expenses, and Taxes

present value of the losses ($80/1.07), the expenses ($20), the tax reduction on the underwriting loss ($[P - 100][.35]/1.07$), and the tax on the equity and premiums, less expenses, invested at interest for one year ($[50 + P - 20][.07][.35]/1.07$). The nominal and discounted values from Example 1 are shown on Table 1. Note that an underwriting loss occurs and the tax on this underwriting loss is negative, representing a cash inflow or an offset to other taxes. Since investment income is positive, the tax on investment income is also positive, raising the required premium. This calculation demonstrates discounting, and the various cash flows generated by writing an insurance policy. It does not represent risk adjusted discounting, though, which will be introduced in the next example.

Example 2 will recognize that some of the cash flows from the insurance contract are risky. Specifically, losses will vary around the expected value. Since risk is involved, it is not reasonable to discount them at what was, in Example 1, a risk-free rate. However, the premium income is certain once the policy is written and the underwriting expenses can be assumed to be known. Taxes emanating from these certain cash flows can also be assumed to be risk-free. However, a critical problem rests with how to determine an appropriate risk adjusted discount rate. One approach is outlined below.

The insurance company is assuming the risk of guaranteeing to pay losses for the insured. The insurer should not be expected to place its capital at risk without compensation. In Example 1, where all cash flows were discounted at the risk-free rate, the insurer would be better off investing the equity directly in financial markets and not assuming the risk involved in paying claims. Thus, discounting the risky cash flows at an interest rate below the risk-free rate represents a form of compensation to the insurer for placing its capital at risk in the insurance contract.

Conversely, the policyholder in an insurance contract is receiving a guarantee from the insurer to pay claims. The guarantee represents a value to the policyholder. Thus, much in the manner that a life insurance policyholder is willing to accept a guaranteed interest rate below the market interest rate, a property/liability insurance policyholder is willing to accept a lower interest rate on the risky cash flows relating to that insurance policy. Another way to view this issue is on a CAPM basis. The insurance policy represents an asset with a negative beta because it has value when the policyholder's tangible assets are reduced in value. The required return on a negative beta asset is below the risk-free rate. The problem becomes, though, the determination of an appropriate risk adjusted discount rate.

For Example 2 we will sidestep that thorny issue and select a discount rate of 4 percent for the risky loss payment cash flow, but maintain the 7 percent discount rate for the risk-free cash flows. The calculation for Example 2 becomes:

$$P = \frac{80}{1.04} + 20 + \frac{(P-20)(.35)}{1.07} - \frac{80(.35)}{1.04} + \frac{(50+P-20)(.07)(.35)}{1.07}$$

$$P = 76.923 + 20 + .327P - 6.542 - 26.923 + .687 + .023P$$

$$P = 64.145/.65 = \$98.68$$

$$UPM = 1 - \frac{80}{98.68} - \frac{20}{98.68} = -1.34\%.$$

The effect of discounting loss payments at a risk adjusted rate is to increase the appropriate premium level and reduce the underwriting loss. The increase in the value of discounted losses (76.923 versus 74.767) is partially offset by the increased reduction in taxes generated by the losses (26.923 versus 26.168). The higher the tax rate, the less the overall effect of a lower risk adjusted discount rate would be.

Reflecting a more realistic loss payment pattern makes the determination a bit more complex. For Example 3, assume that the losses will still total $80, but half will be paid after one year and the other half after two years. Now we have to address the issue of how long equity should be allocated to a given policy. Conventional insurance accounting deals with premium to surplus ratios as if surplus were necessary only to support writing policies. However, it is not the writing of policies that requires a surplus, but the assumption of the obligation to pay claims. Surplus, or equity, is required in the event that claims exceed the expected values so that the insurer can absorb the excess without defaulting on the commitment to pay claims. Thus, equity should not be released as soon as the premium is written, or even earned, but more realistically should continue to be allocated to a given policy until the obligation to pay claims is extinguished, that is, when all losses are settled. In Example 3, the equity devoted to this policy will be released in proportion to the payment of losses. Thus, the full $50 of equity will be invested for the first year of the policy, but only $25 will be invested during the second year because one-half of the losses have already been settled. Similarly, the full premium, less expenses, is available for investment the first year, but for the second year the premium less expenses and losses paid in the first year is available to invest.

Another complication is the calculation of the taxes on underwriting income. The Tax Reform Act of 1986 requires discounting of loss reserves based on a five year moving average of mid-maturity U.S. government obligations. Insurers use either industry or company loss payment patterns to discount outstanding reserves. For this example, the company pattern will be used. The interest rate required for discounting bears no relationship to rates actually earned by the insurer and, since the required rate is based on a five year moving average, the required rate may not even be available to the insurer. When interest rates have been rising, the required discount rate may be below the current risk-free rate. At other times the required rate will exceed the risk-free rate. Since the mid-maturity rate is based on three to nine year maturities for U.S. government obligations, in normal times this rate will be slightly above the rate for short term U.S. bonds on which the risk-free rate is frequently based. Thus, in this example, the outstanding reserve will be discounted at a rate of 1 percent above the risk-free rate, or at 8 percent. In determining the tax on underwriting income, the incurred losses in the first year are reduced to reflect the discount at the mid-maturity interest rate. In the second year, the incurred losses equal the difference between the paid losses and the initial, discounted, loss reserve. The Tax Reform Act of 1986 also reduces the unearned premium reserve deduction by 20 percent to reflect the timing difference between earning premiums and paying expenses. This adjustment does not affect these examples, as the premium is considered fully earned at the end of the year.

The calculation for Example 3 is:

$$\begin{aligned}
P = {} & \frac{40}{1.04} + \frac{40}{(1.04)^2} + 20 \\
& + \frac{(P-20)(.35)}{1.07} - \frac{(40 + 40/1.08)(.35)}{1.04} \\
& - \frac{(40 - 40/1.08)(.35)}{(1.04)^2} + \frac{(50 + P - 20)(.07)(.35)}{1.07} \\
& + \frac{(50(.5) + P - 20 - 40)(.07)(.35)}{(1.07)^2}
\end{aligned}$$

$$P = 38.462 + 36.982 + 20 + .327P - 6.542 - 25.926 - .959$$

$$+ .687 + .023P - .749 + .021P$$

$$P = 61.955/.629 = \$98.50$$

$$UPM = 1 - \frac{80}{98.50} - \frac{20}{98.50} = -1.52\%.$$

In this case, the delay in claim payments decreases the premium level and increases the underwriting loss. The present value of the loss payments declines, but this decline is partly offset by an increase in taxes on investment income.

The prior examples assumed that the expenses were paid when the premium was received, which is a common assumption in insurance ratemaking. Realistically, however, many expenses are incurred well before the premium is collected. The work involved in setting premium levels is done years before the premium is actually collected. Computer systems, underwriting guidelines, contract language, advertising, and many other aspects of an insurance transaction are developed well before a given policy is written. The expenses associated with training staff are incurred before the work for which they are trained is actually performed. Although some expenses are contemporaneous with the receipt of premium, primarily commissions, premium taxes, underwriting inspection reports, and clerical policy insurance expenses, other expenses are paid before the policy is written. To reflect the prepayment of some expenses, Example 4 is calculated on the basis that \$10 of expenses was paid two years before the premium was collected and \$10 was paid when the policy was written. For simplicity it will be assumed that the insurer is content to earn the risk-free rate on the prepaid expenses, although a higher rate may be more reasonably expected, as some prepaid expenses may not be recovered by future policy writings. This calculation is:

$$P = \frac{40}{1.04} + \frac{40}{(1.04)^2} + 10(1.07)^2 + 10$$

$$+ \frac{(P - 10(1.07^2) - 10)(.35)}{1.07} - \frac{(40 + 40/1.08)(.35)}{1.04}$$

$$- \frac{(40 - 40/1.08)(.35)}{(1.04)^2} + \frac{(50 + P - 20)(.07)(.35)}{1.07}$$

$$+ \frac{(50(.5) + P - 20 - 40)(.07)(.35)}{(1.07)^2}$$

$$P = 38.462 + 36.982 + 11.449 + 10 + .327P - 7.016$$

$$- 25.926 - .959 + .687 + .023P - .749 + .021P$$

$$P = 62.930/.629 = \$100.05$$

$$UPM = 1 - \frac{80}{100.05} - \frac{20}{100.05} = 0.05\%.$$

The prepayment of expenses increases the indicated premium level. Failure to reflect the fact that many expenses are actually expended before the premium is received leads to an understating of the premiums determined by the risk adjusted discounted cash flow models.

The risk adjusted discounted cash flow models are often adjusted to reflect the fact that premiums are not received at the inception of the policy term. The method developed by the Insurance Services Office, termed the ISO State X calculation, includes this adjustment. These delays may be several months, especially if an agent is given a certain amount of time before being expected to submit the premiums. However, if a representative of the company, such as an agent, has collected the premiums but not remitted them to the insurer, it is incorrect to reflect this delay by discounting the premiums for this lag. This delay reflects a form of agent compensation and should be reflected as an expense rather than as a discounted premium. If the policyholder has paid the premiums, then the insurance rates should not be increased because the insurer has not invested the funds.

As most insurance policies include grace periods, though, it is not unusual for premiums to be submitted after the coverage is in effect. Thus, reflecting the lag in collecting premiums is proper in these circumstances. To illustrate this effect, Example 5 assumes that premiums are paid, either to an agent or the company, one month after policy inception. The calculation becomes:

$$\frac{P}{(1.07)^{1/12}} = \frac{40}{1.04} + \frac{40}{(1.04)^2} + 10(1.07)^2 + 10$$

$$+ \frac{(P - 10(1.07^2) - 10)(.35)}{1.07} - \frac{(40 + 40/1.08)(.35)}{1.04}$$

$$- \frac{(40 - 40/1.08)(.35)}{(1.04)^2} + \frac{(50 + P - 20)(.07)(.35)}{1.07}$$

$$+ \frac{(50(.5) + P - 20 - 40)(.07)(.35)}{(1.07)^2}$$

$$.994P = 38.462 + 36.982 + 11.449 + 10 + .327P - 7.016$$

$$- 25.926 - .959 + .687 + .023P - .749 + .021P$$

$$P = 62.930/.623 = \$101.01$$

$$UPM = 1 - \frac{80}{101.01} - \frac{20}{101.01} = 1.00\%.$$

The effect of assuming a one month delay in the policyholders' payment of premiums is to increase the indicated premium level by one percentage point. In this example, the delay of premium payment generates a positive underwriting profit margin for the insurer. This is not, in itself, a more favorable financial position for the insurer than the prior underwriting loss or breakeven indications. In all cases, premiums simply equal the risk adjusted cash flows emanating from writing the policy. The underwriting profit margin is irrelevant to this method and is shown here only as a frame of reference with traditional insurance accounting conventions.

The general formula for the Risk Adjusted Discount Technique, as illustrated by the above examples, can be written as:

$$
\begin{aligned}
P\sum_{i=0}^{N}\frac{a_i}{(1+R_f)^i} &= L\sum_{i=0}^{N}\frac{b_i}{(1+R_L)^i} + E\sum_{i=-M}^{N}\frac{c_i}{(1+R_f)^i} \\
&\quad + \frac{\left(P - E\displaystyle\sum_{i=-M}^{N}\frac{c_i}{(1+R_f)^i}\right)t}{1+R_f} \\
&\quad - Lt\left(\frac{\displaystyle\sum_{i=1}^{N}\frac{b_i}{(1+R_T)^{i-1}}}{1+R_L} + \sum_{j=2}^{N}\frac{\displaystyle\sum_{i=j}^{N}\frac{R_T b_i}{(1+R_T)^{i-j+1}}}{(1+R_L)^j}\right) \\
&\quad + R_f t\left(\sum_{j=1}^{N}\left[\frac{S\left(\displaystyle\sum_{i=j}^{N} b_i\right) + P - E - L\displaystyle\sum_{i=0}^{j-1} b_i}{(1+R_f)^j}\right]\right),
\end{aligned}
\tag{6.2}
$$

where

a_i = fraction of premium received in time period i,

b_i = fraction of losses paid in time period i,

c_i = fraction of expenses paid in time period i,

S = owners' equity in insurer,

P = premiums,

L = losses and loss adjustment expenses,

E = underwriting expenses,

t = tax rate,

R_T = discount rate required for tax purposes,

R_f = risk-free rate,

R_L = risk adjusted rate for losses,

M = number of time periods before policy effective date that the first prepaid expenses are paid, and

N = number of time periods after policy effective date that the last loss payment is made.

Equation 6.2 applies the same methodology as Equation 6.1. The present value of premiums is set equal to the sum of the present values of losses, expenses, and taxes. In this formulation, expenses are allowed to be paid before the policy is written. Taxes on underwriting income are based on the provisions of the Tax Reform Act of 1986, in which outstanding reserves each year are discounted at a mandated rate and losses are incurred for tax purposes each year reflecting the loss payments compared to the discounted reserves and the fact that as time elapses, outstanding reserves are discounted for shorter periods of time. The tax on investment income reflects an equity allocation based on the percent of losses that are still unpaid.

In this example, expenses are listed as known values that do not depend on premiums. Some, but not all, expenses could more properly be stated as a percentage of premiums. Commissions and premium taxes tend to be percentages of premiums, and could be shown accordingly in Equation 6.2. Other expenses, such as administration, systems development, employee training, underwriting, and overhead may not depend on the premium level and should be treated as given values in the same way that losses and equity are independent of the premium level. For example, if an increase in taxes results in a higher premium loading, it is not appropriate to increase the full expense loading proportionally, as many expenses will not change.

This formula has other shortcomings as well. The present value of premiums is determined based on the risk-free rate. However, the lag in premium collection is not equivalent to an investment in a risk-free security. Some premiums are never paid, and the insurer is forced to cancel coverage. Other premiums are paid within the grace period, but only after losses have occurred; those that do not have losses simply do not pay the premiums, in essence obtaining free insurance protection for which the insurer must pass along the cost to other insureds.

The shortcomings described above relating to expenses proportional to premiums and a risk adjustment for premium delays could be accounted for by revising the formula to reflect these items. However, more serious drawbacks to the Risk Adjusted Discount Technique exist that cannot be so easily corrected. One major problem is the proper determination of the risk adjusted discount rate. In the examples, this rate was set at 4 percent. However, no widely accepted approach for setting this rate has yet been determined. In the original development of this technique, Myers and Cohn use the Capital Asset Pricing Model to determine the appropriate rate. Recent research suggests that the CAPM does not provide a large enough risk margin for insurance transactions. Also, research in finance has raised serious questions about the validity of the CAPM to investment returns in general. Not knowing how to select the appropriate risk adjusted rate is a serious flaw in this technique.

Another major problem relates to the allocation of equity to policies. Since the taxes incurred on investment income allocated to equity supporting a given line are included in the premium determination, knowing how much and how long equity is allocated are of critical importance. The traditional consideration of premium to surplus measures is inappropriate because, as described earlier, surplus is needed to protect against losses exceeding expected values. Thus, at least some equity must continue to be committed to a policy until all the losses are paid. In the approach outlined above, equity is released proportionately with loss payments. As expense payments have very little risk of exceeding expected values, only loss payments are considered in releasing equity.

A key decision in the Risk Adjusted Discount Technique is how much equity should be allocated to a given policy. The allocation may consider such items as the degree of variability in losses, the length of time loss payments will be made, covariability among different lines of insurance, or other factors. Perhaps an insurer should be required to maintain a higher level of equity for property coverages during tornado and hurricane seasons than at other times. A stop loss reinsurance contract would reduce the need for equity for a covered line. Liability lines may need additional equity in times of judicial instability, more than when the doctrine of *stare decisis* is likely to be applied. A consensus on the proper equity determination has not yet been reached. The approach applied in the examples, in which equity is predetermined, perhaps in proportion to expected losses but not as a function of premiums, is reasonable, but is not the only approach.

Another serious drawback to the Risk Adjusted Discount Technique is the fact that it considers only one policy term. The profitability of insurance policies depends on how many renewal cycles the policy has been through. New business tends to be unprofitable, but long term business becomes increasingly profitable. This tendency is termed the aging phenomenon and appears to occur for all insurers and for all lines of business. Thus, in determining a proper premium level, the aging phenomenon should be recognized. The cash flows emanating not only from the current policy but also from future renewals of the policy should be considered. This would be a multidimensional risk adjusted discounting approach that has not yet been developed.

In summary, the Risk Adjusted Discount Technique establishes the premium level for a policy by equating the present value of premiums to the present value of losses, expenses, and taxes on both underwriting and investment income. If appropriate values for the cash flows and the discount rates could be determined, this approach should generate valid premium levels. The technique is illustrated by simplistic examples. More realistic examples become increasingly complex, but still follow the same logic. The major difficulties in applying the Risk Adjusted Discount Technique revolve around selecting the appropriate discount rate and the equity allocation. Unless these values are correct, the premium levels resulting from this approach will not be valid.

7. OPTION PRICING

Option Mechanics

An option is termed a derivative security, one that derives its value based on the price of another asset. Typical options are traded on stocks, bonds, commodities, and stock indices. The owner of an option has the right to trade the underlying asset at a specified price by or on a given date. However, the owner does not have to exercise this right. Two types of options exist. A call option gives the owner the right to buy the asset at the specified price, which is called the strike or exercise price. A put option gives the owner the right to sell the asset. The seller of the option, called the writer of the option, has the obligation to sell (in the case of a call option) or to buy (in the case of a put option) the underlying asset at the exercise price if the buyer elects to exercise the option.

Options are also classified according to when they can be exercised. A European option can only be exercised on the expiration date. An American option can be exercised at any time up until expiration.

To illustrate how options work, let us examine the decision of the owners of European options on the expiration date. Assume an investor owns a call option on ABC stock with an exercise price of \$50, and on the expiration date ABC stock sells for \$60. The owner has to decide whether the call option should be exercised. If the option is exercised, the investor would pay \$50 per share for ABC stock which currently sells for \$60 per share. Once the investor exercises the call option for \$50 per share, he or she can sell the stock for \$60 a share, thereby making a profit of \$10 per share. Alternatively, the investor can simply keep the stock that was purchased for a bargain price. If the investor did not exercise the call option, the option would expire worthless, and the investor would receive nothing. Obviously, the call option owner should exercise the option in this case.

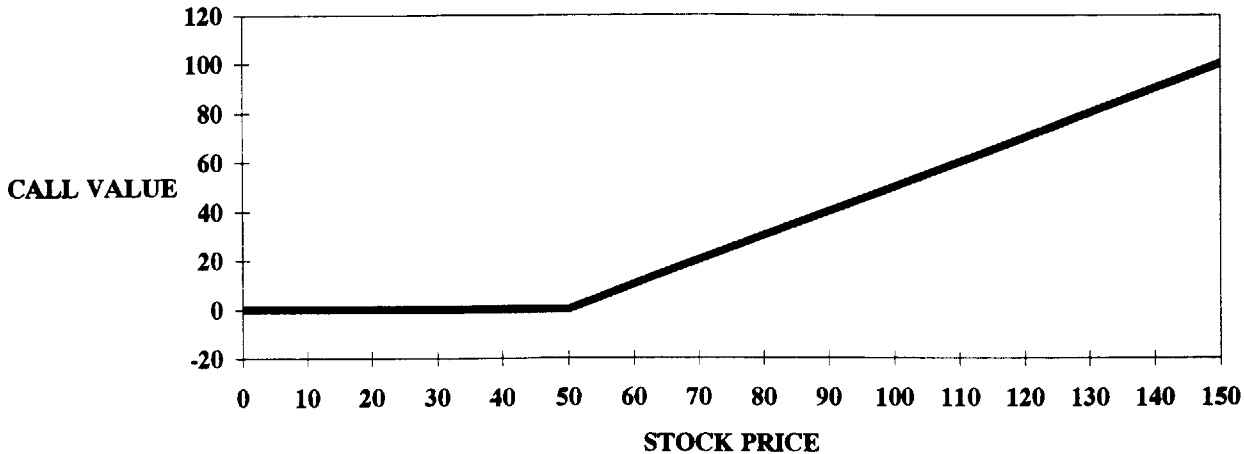
Now, using the same \$50 exercise price call option example, assume that the price of ABC stock is \$40 per share on the expiration date. If the call option owner exercised the option in this case, the owner would pay \$50 per share for a stock that could be purchased for only \$40 per share. If the call holder wanted to own ABC stock, the holder should let the option expire worthless and buy the stock for \$40 per share. The value at expiration of the call option in this situation would be zero.

From these two simple call option examples, a pattern emerges. The owner of a European call option should exercise the option if the underlying stock or asset price is greater than the exercise price at expiration. The value at expiration of the call option is the higher of the stock price minus the exercise price or zero. This payoff can be seen in Figure 7 for our ABC stock example and expressed generally in the following formula:

$$C = \max[S - X, 0], \tag{7.1}$$

FIGURE 7

CALL OPTION PAYOFF AT EXPIRATION



where

C = the value of the call option at expiration,

S = the price of the underlying asset, and

X = the exercise price of the option.

For the first example where the stock price was \$60 and the exercise price was \$50, Equation 7.1 would be:

$$C = \max[60 - 50, 0] = \$10.$$

Equation 7.1 for the second example where the stock price was \$40 and the exercise price was \$50 would be:

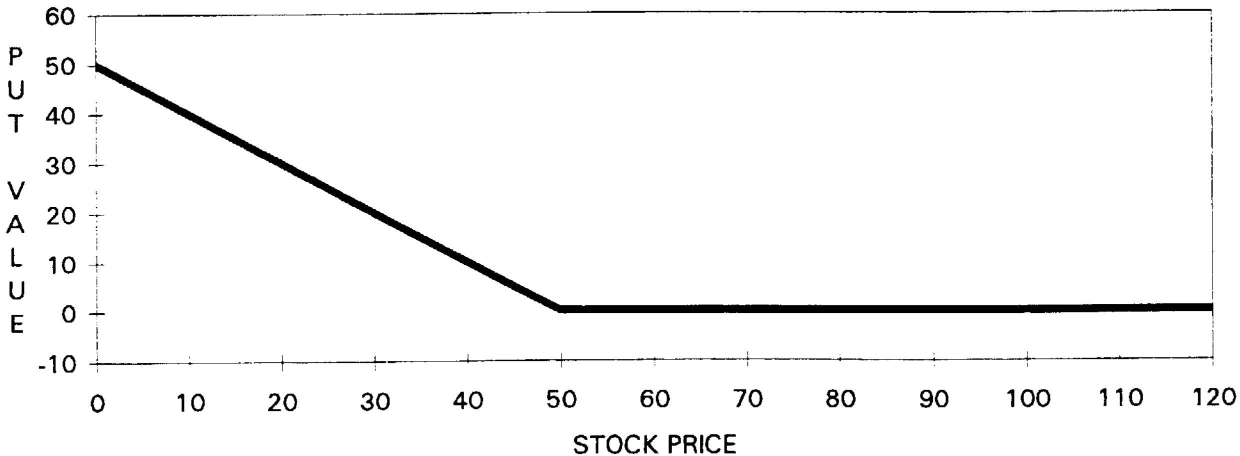
$$C = \max[40 - 50, 0] = 0.$$

Now consider the owner of a put option on ABC stock with an exercise price of \$50. At expiration, the price of ABC stock is \$35, for example. If the owner of the put option exercises the option, the put owner must first acquire the stock in order to sell the stock to the writer of the put option. So, the put owner buys ABC stock for \$35 per share in the market and completes the transaction by exercising the option to sell the stock for \$50 per share, which nets the option holder \$15 per share. Alternatively, if the put holder already owned ABC stock, the stock could be sold to the put writer for the higher price of \$50 per share, rather than the market price of \$35. However, if the price of the stock were \$55 per share at expiration, the put option owner would not exercise the option to sell the stock for \$50 per share even if the holder already owned the stock. Therefore, the decision rule for an owner of a put option is to exercise the option at expiration only if the stock price is less than the exercise price. The payoff at expiration for a put option is the larger of the exercise price minus the price of the underlying asset or zero. The payoff for our put option example is represented graphically in Figure 8 and given by the following general formula:

$$P = \max[X - S, 0], \tag{7.2}$$

FIGURE 8

PUT OPTION PAYOFF AT EXPIRATION



where

P = the value of a put option at expiration,

X = the exercise price of the option, and

S = the price of the underlying asset.

Verifying the first put option example where the stock price was \$35 and the exercise price was \$50, Equation 7.2 would be:

$$P = \max[50 - 35, 0] = \$15.$$

The second example where the stock price was \$55 and the exercise price was \$50 yields the following for Equation 7.2:

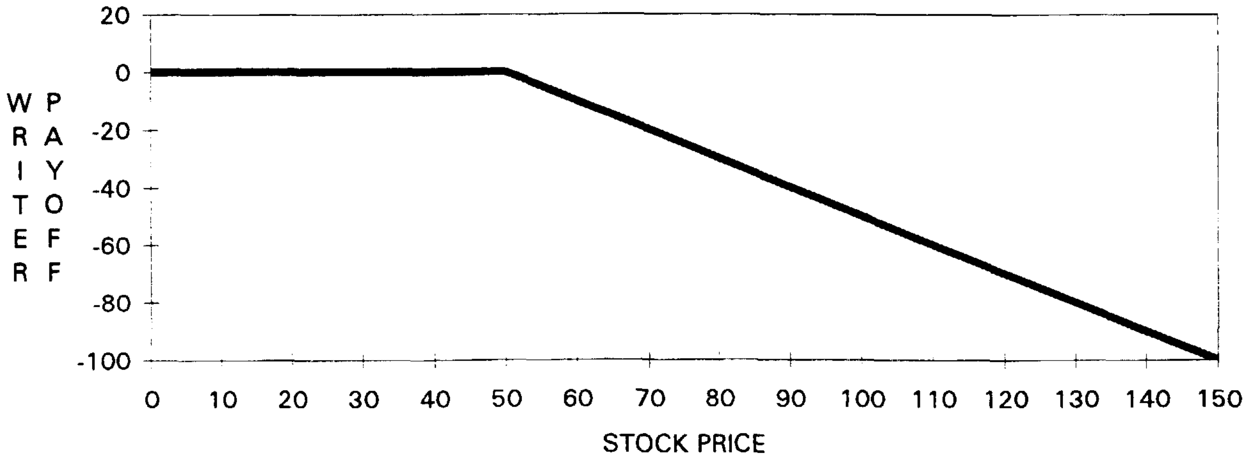
$$P = \max[50 - 55, 0] = 0.$$

One may wonder what happens to the writer of these options while all this action occurs at expiration. Going back to the first call option example, where the stock price was \$60 and the exercise price was \$50, the writer of this call option has to sell the stock to the owner of the call option who is exercising the option to buy ABC stock for \$50 per share. This means the writer of the call has to buy the stock if he or she does not own the stock already. In this situation the call writer has to buy the stock for \$60 and then sell it to the owner of the call for \$50 per share, incurring a loss of \$10 per share. In the second call option example, where the price of the stock was \$40 and the exercise price was \$50, the holder of the call would not exercise the option, which means the writer of the call would have a payoff of zero at expiration. In option terminology, even though the writer's expiration value is zero or negative, this expiration value is still called a payoff. The payoff to the writer of a call option at expiration can be expressed by the following equations and graphically in Figure 9:

$$W_c = \min[-(S - X), 0] \tag{7.3}$$

FIGURE 9

CALL WRITER PAYOFF AT EXPIRATION



or

$$W_c = \min[X - S, 0], \tag{7.4}$$

where

W_c = payoff at expiration to call option writer,

S = the underlying stock or asset price, and

X = the exercise price of the option.

Using Equation 7.4 to verify the previous call option examples yields the following in the first case where the stock price was \$60 and the exercise price was \$50:

$$W_c = \min[50 - 60, 0] = -\$10.$$

For the second call example where the stock price was \$40 and the exercise price was \$50, Equation 7.4 yields the following:

$$W_c = \min[50 - 40, 0] = 0.$$

The writer of a put option has to buy the stock at the exercise price if the option is exercised at expiration. Returning to the first put option example, where the stock price is \$35 and the exercise price is \$50, the owner of the put would exercise the option to sell the stock to the put writer for \$50 per share. The put option writer would have to raise \$50 per share to buy the stock that sells for \$35 per share in the market. If the put writer then sells the stock in the market for \$35 per share, a loss of \$15 per share is realized immediately. In the second put option example, where the stock price is \$55 and the exercise price is \$50, the owner of the put would not exercise the option, and the writer of the put will have a payoff of zero at expiration. In the put option case, as in the call option case, the put option writer's payoff at expiration is a loss equal to the put option owner's gain if the option is exercised and zero if the option is not exercised. The put option writer's payoff at expiration is given in the following equations:

$$W_p = \min[-(X - S), 0], \qquad \text{or} \tag{7.5}$$

$$W_p = \min[S - X, 0]. \tag{7.6}$$

For put option Example 1, $S = \$35$ and $X = \$50$, so Equation 7.6 yields:

$$W_p = \min[35 - 50, 0] = -\$15,$$

and for Example 2, $S = \$55$ and $X = \$50$, so:

$$W_p = \min[55 - 50, 0] = 0.$$

The above examples can also be seen graphically in Figure 10.

Given the maximum payoff of zero for writers of options in the previous examples, why would anyone want to write an option? The answer lies in the one important variable omitted from the discussion thus far: the original selling price of the option. This selling price will now be integrated into the discussion.

Let C_0 be the original selling price of a call option and P_0 be the original selling price of a put option. The writer of the option initially receives either C_0 or P_0 from the buyer of the option depending on the type of option sold. Given this fact, Equations 7.1 through 7.6 can be rewritten for the total payoff at expiration for the owners and writers of options as the following equations:

$$C = \max[S - X - C_0, -C_0], \tag{7.7}$$

$$P = \max[X - S - P_0, -P_0], \tag{7.8}$$

$$W_c = \min[C_0 - (S - X), C_0] \quad \text{or} \tag{7.9}$$

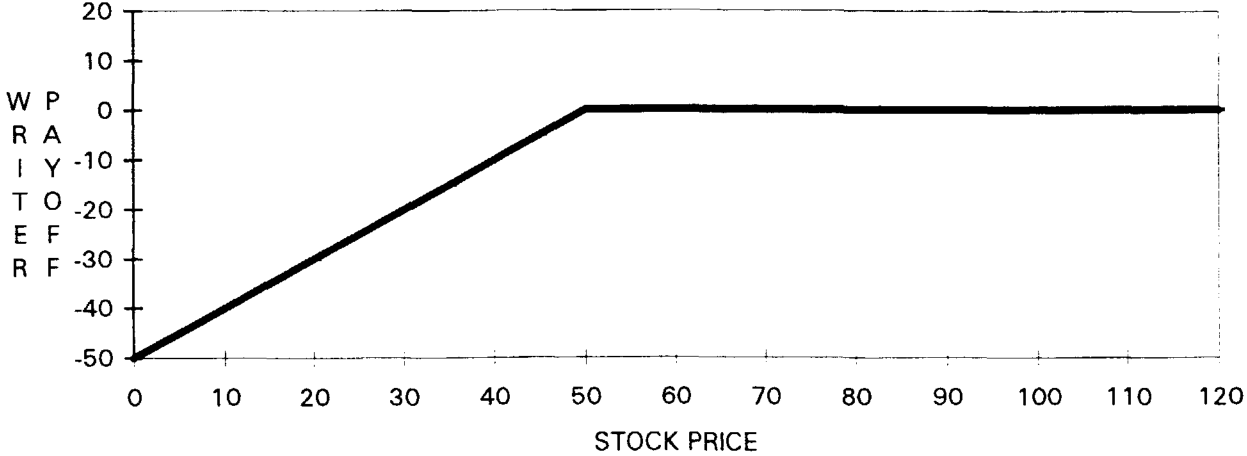
$$= \min[C_0 + X - S, C_0], \tag{7.10}$$

$$W_p = \min[P_0 - (X - S), P_0] \quad \text{or} \tag{7.11}$$

$$= \min[P_0 + S - X, P_0]. \tag{7.12}$$

FIGURE 10

PUT WRITER PAYOFF AT EXPIRATION



For ease of terminology, the time value of money concept of the price for the option being paid at the beginning of the period and the expiration payoff coming at the end of the period will be ignored for now. Equations 7.7 and 7.8 are the payoffs at expiration for the owner of a call option and put option, respectively, and Equations 7.9 and 7.11 are the payoffs at expiration for the writers of call and put options, respectively. With the integration of the option selling price and Equations 7.7 through 7.12 in the discussion, the motives of both the writers and buyers of options become more apparent. The buyer of a call option buys the option with the hope or belief that the price of the underlying asset will go up, and the writer sells the call option with the belief that the price of the underlying asset will go down. If the writer of the call option is correct and the stock or asset price is lower than the exercise price at expiration, the option will not be exercised, and the call option writer will pocket the original selling price of the call option.

To illustrate this point consider the case where a person decides to write a call option on INS stock that expires in three months with a current stock price of \$30 and an exercise price of \$30 and sells this call option to a buyer for \$3 per option. Assume three months later INS's stock price is \$25. The owner of the call option will not exercise the option; therefore, the call option owner's net payoff is negative \$3, the cost of the option. This value can be verified by Equation 7.7 and seen graphically in Figure 11:

$$\begin{aligned} C &= \max[S - X - C_0, -C_0] \\ &= \max[25 - 30 - 3, -3] = -\$3. \end{aligned}$$

The call option writer's net profit (or loss) is given by Equation 7.9 and graphically in Figure 12:

$$\begin{aligned} W_c &= \min[C_0 - (S - X), C_0] \\ &= \min[3 - (25 - 30), 3] = \$3, \end{aligned}$$

FIGURE 11

CALL OWNER NET PAYOFF AT EXPIRATION

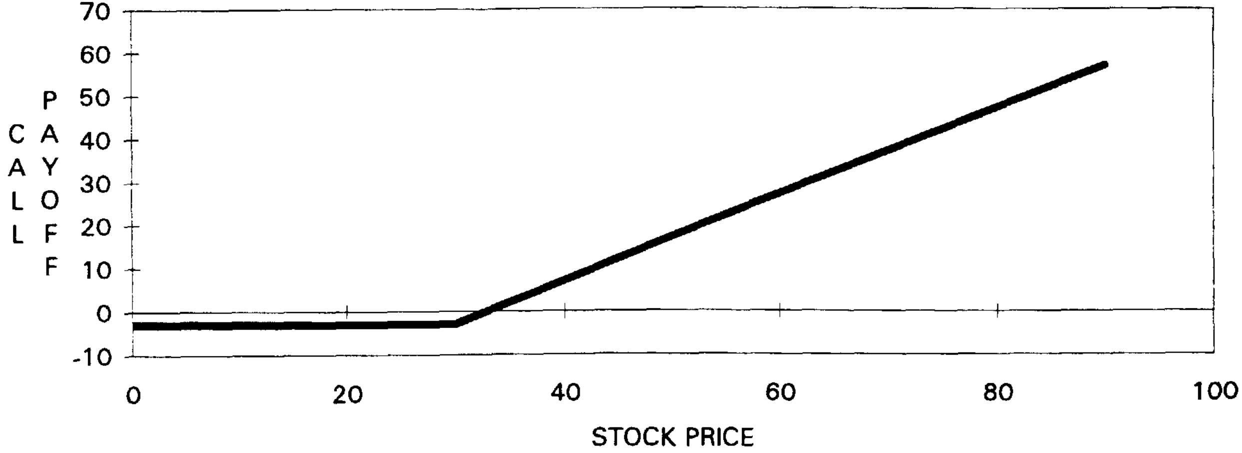
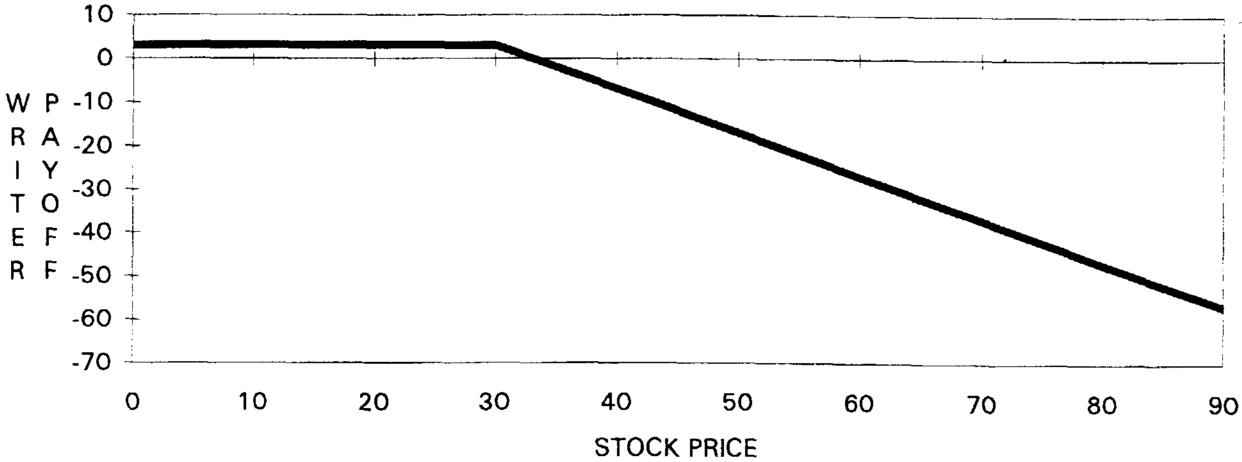


FIGURE 12

CALL WRITER NET PAYOFF AT EXPIRATION



or by Equation 7.10,

$$\begin{aligned} W_c &= \min[C_0 + X - S, C_0] \\ &= \min[3 + 30 - 25, 3] = \$3. \end{aligned}$$

Now suppose the stock price at the end of three months is \$32. Will the owner of this call option exercise the option? Consider the previous example, where the stock price was \$25, and the call option owner did not exercise the option and had a net loss of \$3 per option. As Equation 7.7 shows, the most the owner of a call option can lose is the amount paid for the option, which is the case when the owner does not exercise the option. Using this equation for the new example where the stock price is now \$32 and the exercise price is \$30, gives the following payoff to the owner of the call option:

$$C = \max[32 - 30 - 3, -3] = -\$1.$$

This example shows that the owner of the call option should exercise the option, even though the payoff is negative, because exercising the option results in a smaller net loss than letting the option expire worthless. The example also verifies the decision rule stated earlier of always exercising a call option if the stock price is greater than the exercise price. The writer of the call option in this case would have a payoff of \$1, as verified by Equation 7.10:

$$W_c = \min[3 + 30 - 32, 3] = \$1.$$

A writer of a put option sells the put in the belief that the stock price will go up, and the buyer of a put option buys the option in the belief the stock price will go down. Consider the following example: a person writes a put option that expires in three months with an exercise price of \$40 and a current stock price of \$40 and sells this option for \$5. Three months later at expiration the stock price is \$48. Since the stock price is greater than the exercise price, the owner of this put option will not exercise the option to sell the stock for the exercise price of \$40.

The put owner's payoff, or net profit (or loss), is equal to the amount paid for the option as given by Equation 7.8 below:

$$P = \max[X - S - P_0, -P_0]$$
$$= \max[40 - 48 - 5, -5] = -\$5.$$

The writer of the put option will keep the amount paid for the option in this case because the put option is not exercised. This put writer's payoff is expressed by Equation 7.12 below:

$$W_p = \min[P_0 + S - X, P_0]$$
$$= \min[5 + 48 - 40, 5] = \$5.$$

Now assume the stock price is \$37 instead of \$48 on the expiration date. The owner of the put would exercise the option because the exercise price is greater than the stock price. This would give the owner of the put option a net loss of \$2 as given below by Equation 7.8, but this \$2 loss is better than the \$5 loss derived in the previous example where the put option was not exercised:

$$P = \max[40 - 37 - 5, -5] = -\$2.$$

The writer of the put in this case would buy the stock for \$40 per share and then sell the stock for \$37 in the market resulting in a net gain of \$2 per option, as shown in Equation 7.12 below:

$$W_p = \min[5 + 37 - 40, 5] = \$2.$$

See Figures 13 and 14 for a graphical representation of the payoffs to the owner and writer of the put option in the above example.

The owner of a call option has the potential for an unlimited gain as there is no theoretical upper limit to the price of a stock. The most an owner of a call option can lose is the price paid for the option. Therefore, a call option owner has unlimited upside potential and limited downside potential. The writer of a call option has no limit on the amount of the potential loss, but the gain is limited to the selling price of the option..

FIGURE 13

PUT OWNER NET PAYOFF AT EXPIRATION

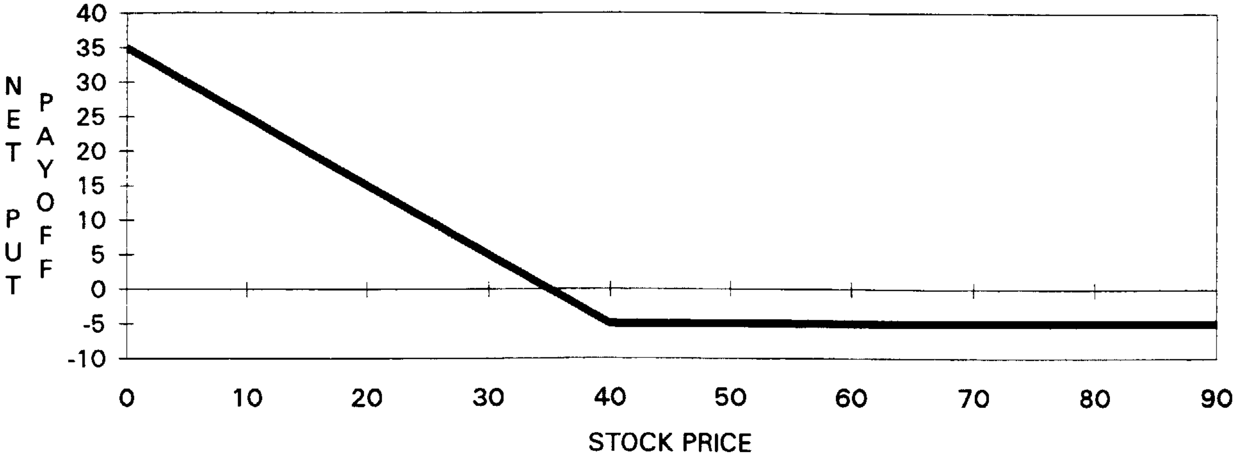
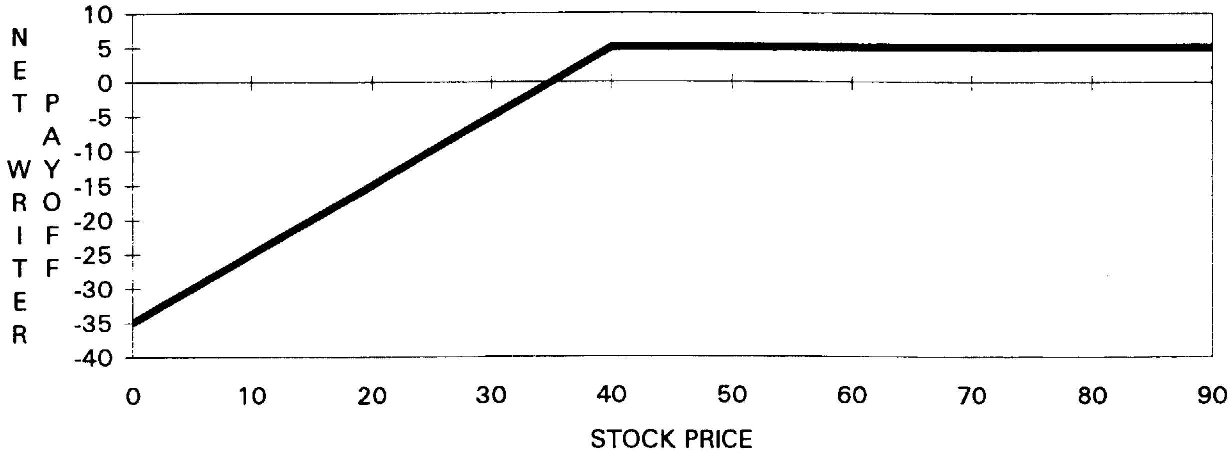


FIGURE 14

PUT WRITER NET PAYOFF AT EXPIRATION



The best possible outcome for the owner of a put option is that the underlying asset becomes worthless at expiration. This means the put option owner's maximum possible gain is the exercise price less the price paid for the option. The most a put option owner can lose is the price paid for the option. The writer of a put option has a maximum possible loss equal to the original selling price minus the exercise price of the option, and the gain of the put writer is limited to the selling price of the option.

This concludes the discussion of option mechanics at expiration. This discussion is important because the payoff at expiration for calls and puts given by Equations 7.1 and 7.2 represents the absolute lowest price of an option at any time and shows that the price for an option is directly related to the option's underlying asset price and exercise price.

Uses of Options

Investors trade options primarily for two reasons. One reason is a speculative motive in that an investor can profit (or lose) from the price movements of the underlying asset for a fraction of the cost of buying the asset itself. An investor who thinks the price of a $50 stock will increase could buy the stock at $50 per share or could buy a call option on that stock with an exercise price of $50 for a much lower price. Buying call options allows the investor to control the price appreciation potential of more shares of stock and still receive at least the same dollar gain in the option price that occurs in the stock price. For example, if the options were valued at $5, for $500 an investor could buy call options on 100 shares with an exercise price of $50 for the stock described above or 10 shares of the actual stock. If the stock price rose to $60 before option expiration, the price of the call option should rise from $5 to at least $10. If the investor bought options on 100 shares of stock at $5 per option, he or she could sell the options now for $10 each and receive a gain of $500 on the original $500 option investment. The owner of

10 shares of stock could sell the stock for $60 per share for a gain of $10 per share which results in a total gain of $100 on the 10 shares. The investor in options in this example was able to receive a much higher return in comparison to the investor in the underlying stock, highlighting the speculative motive for buying options.

However, a down side to this strategy exists. Assume in the above example that the stock price remains stable. At expiration, the call options will be worthless, resulting in a loss of $500 to the call option holder. At the same time, the owner of the stock will still have 10 shares of stock worth $50 per share, resulting in no loss of value to this investor.

The second reason for buying options is to hedge a position taken in the underlying asset. For example, an investor buys a stock in the hope that the price of the stock will rise above the original price paid for the stock. However, the investor may be concerned about the price of the stock falling below the original purchase price and want to minimize this possible loss, while still receiving a gain if the stock price rises. To hedge the stock position, the investor could buy a put option on the underlying stock. The put option increases in value when the price of the underlying stock decreases. For example, assume the investor bought 100 shares of XYZ stock for $50 per share and bought put options with an exercise price of $50 on 100 shares of XYZ stock for $3 per option. If the price of XYZ stock is $40 when the options expire, the investor will exercise the put options to sell the stock for $50 per share. Alternatively, if XYZ stock sells for $60, the investor would let the put options expire worthless and either sell XYZ for $60 or hold the stock. The expiration value of this hedged stock position is equal to:

$$S - X + \max[X - S - P_0, -P_0], \tag{7.13}$$

which is

$$S - X + X - S - P_0 = -P_0, \tag{7.14}$$

if S is less than X, and

$$S - X - P_0, \tag{7.15}$$

if S is greater than X, where

S = the stock price at expiration,

X = the exercise price of the option and the original purchase price of the stock, and

P_0 = the cost of the put option.

In the first example with XYZ stock selling for \$40 on the option expiration date, Equation 7.13 becomes:

$$40 - 50 + \max[50 - 40 - 3 = 7, -3] = 40 - 50 + 7 = -\$3,$$

$$\text{or} \qquad 40 - 50 + 50 - 40 - 3 = -\$3.$$

In the second example with XYZ stock selling for \$60 on the option expiration date, Equation 7.13 becomes:

$$60 - 50 + \max[50 - 60 - 3 = -13, -3] = 60 - 50 - 3 = \$7.$$

This hedging strategy limits the investor's loss to the cost of the put option in the first case and reduces the gain by the cost of the put option in the latter case.

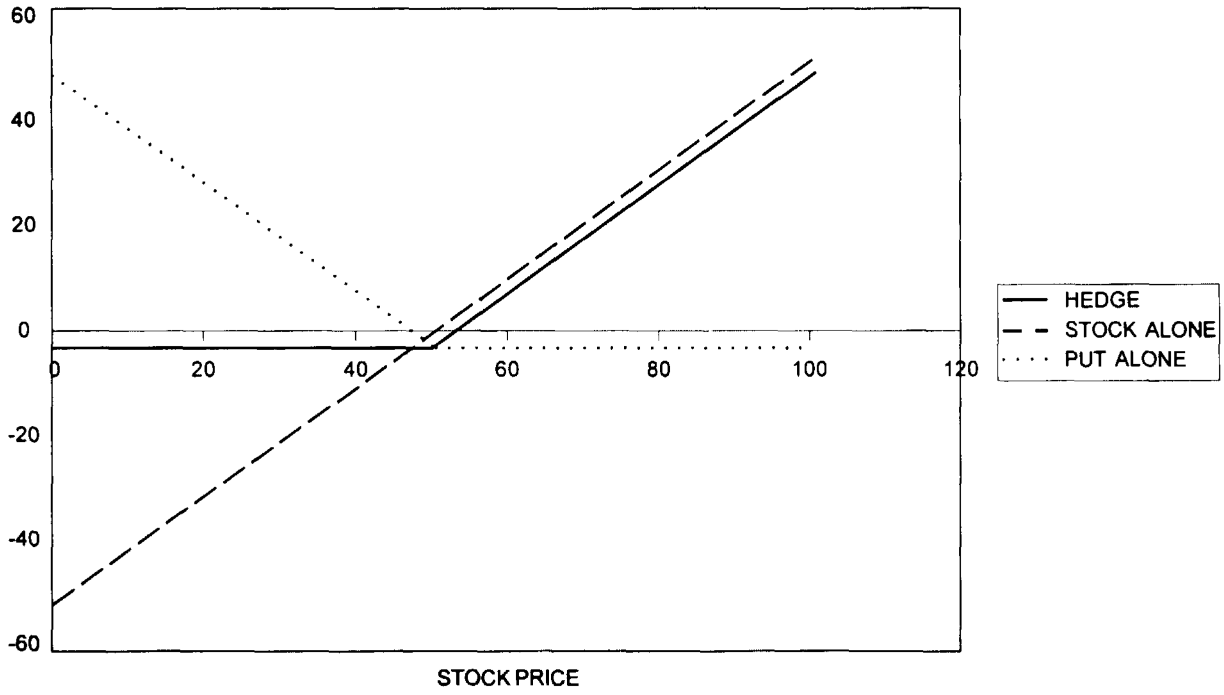
The net value of the hedged stock-put position is illustrated in Figure 15. The diagram of the hedged stock position looks exactly like the payoff of owning a call option in Figure 11 in shape and direction. The net value of the hedged position of owning a stock and a put option has the same characteristics as owning a call option, and in essence a call option has been created as a result of this hedged position. This can be seen by combining Equations 7.14 and 7.15 into one equation:

$$\text{Hedge payoff} = \max[S - X - P_0, -P_0]. \tag{7.16}$$

A put option can be created in a similar fashion. The creation of a put involves buying a call option to hedge a short stock

FIGURE 15

PAYOFF OF HEDGED STOCK-PUT POSITION



position. A short stock position is when an investor borrows shares of stock from someone else and sells them at the current price and then agrees to buy back the stock later in order to return the stock to the lender. A person in a short stock position profits if the stock price falls below the short sale price and loses if the stock price rises after the sale. Therefore, the short stock investor can hedge the downside potential of a rising stock price by buying a call option with an exercise price equal to the sale price of the stock, which increases in value when the price of the underlying stock rises. This position can be called a short hedge. A short hedge has a loss limited to the cost of the call option if the stock price rises above the original selling price of the stock, because the investor would exercise the call option of buying the underlying stock at the same price at which the stock was originally sold. If the stock price is below the original sale price of the stock at expiration, the call option will be allowed to expire worthless and the investor's payoff is the original stock sale price or exercise price minus the expiration stock price minus the cost of the call option. This short hedge net position can be expressed in the following formula:

$$SH = X - S + \max[S - X - C_0, -C_0], \tag{7.17}$$

which is

$$SH = X - S + S - X - C_0 = -C_0, \tag{7.18}$$

if the stock price is greater than the exercise price and

$$SH = X - S - C_0, \tag{7.19}$$

if the stock price is less than the exercise price. Rewriting Equation 7.17 by combining 7.18 and 7.19 leads to a payoff structure identical to owning a put option

$$SH = \max[X - S - C_0, -C_0].$$

This hedging idea was used in the development of one of the most popular option pricing models, the Black–Scholes option pricing model developed in 1973. To highlight the importance and popularity of the Black–Scholes model, secondary option markets were organized after the development of the Black–Scholes model, and traders used the model to set market option prices in the early stages of the secondary stock option markets.

The Black–Scholes option pricing model takes into account five variables that affect option prices:

1. the underlying stock price,
2. the exercise price,
3. the time to expiration of the option,
4. the volatility of price movements in the underlying stock, and
5. the risk-free rate of interest.

The derivation of the model is based on the idea that, if an investor is able to continuously maintain a perfect hedge using an option on the underlying stock or asset, and borrow or lend at the risk-free rate (borrowing at the risk-free rate to raise the money to buy the underlying asset or lending the funds generated from the short sale of an asset at the risk-free rate), then this hedging portfolio must yield the risk-free rate of return to the investor. The Black–Scholes model uses continuous time compounding of interest and a lognormal distribution of asset or stock prices. The lognormal distribution of asset prices is used because an asset cannot sell for a price less than zero, and a lognormal distribution model of asset price satisfies the reality of non-negative asset prices.

The formula for the Black–Scholes option pricing model is as follows:

$$C = S \times N(d_1) - Xe^{-R_f t} \times N(d_2), \tag{7.20}$$

where

C = the model price for a European call option,

S = the price of the underlying stock or asset,

X = the exercise price of the option,

$N(*)$ = the normal distribution function evaluated at $*$,

R_f = the risk-free rate of return,

t = the time to expiration,

$$d_1 = \frac{\ln(S/X) + (R_f + .5\sigma^2)t}{\sigma t^{1/2}},$$

$d_2 = d_1 - \sigma t^{1/2}$, and

σ = the standard deviation of the continuously compounded returns of the underlying asset.

The Black–Scholes model in Equation 7.20, although appearing quite complex, is fairly easy to use. All variables except the standard deviation are readily observable. The standard deviation can be estimated from historical asset return data, or derived by setting the current market call option price equal to Equation 7.20 and solving for the standard deviation.

Consider this example. Find the Black–Scholes call value for a call option with an exercise price of \$70, stock price of \$90, risk-free rate of .08 per year, time to expiration of 0.5 years, and standard deviation of the stock price returns of .25. The first step is to find d_1 and d_2:

$$d_1 = [\ln(90/70) + (.08 + .5(.25)^2).5]/[.25(.5)^{1/2}] = 1.7363$$

$$d_2 = 1.7363 - .25(.5)^{1/2} = 1.5595.$$

The values of d_1 and d_2 are substituted into Equation 7.20 which yields the following:

$$C = 90 \times N(1.7363) - 70e^{-.08(.5)}N(1.5595)$$
$$= 90(.9588) - 70(.9608)(.9405) = 23.04.$$

The values of the normal distribution functions at d_1 and d_2 are interpolated from the normal z table reproduced in the appendix.

From observation of the model, relationships between the variables and call option prices can be described. A positive relationship exists between the stock price and the call price. A negative relationship exists between the exercise price and the call price. Also, positive relationships exist between the call price and the remaining variables: time to expiration, the risk-free rate, and standard deviation.

Other Applications of Options

Besides options on stocks, bonds, and other financial assets, other financial instruments and insurance have characteristics of options and can be priced by option pricing models. For example, the value of a corporation has the characteristics of a European call option.

The value of a corporation's equity, E, is equal to the value of its assets, A, less the value of its debts or liabilities, D. Assume at the end of the period the corporation will liquidate, and the equityholders will receive the difference between the corporation's assets and liabilities if assets are greater than the liabilities or nothing if assets are less than liabilities. This relationship can be expressed by the following equation:

$$E = \max[A - D, 0]. \tag{7.21}$$

This end of period value of equity relationship in Equation 7.21 is the same as the payoff of a European call option at expiration where the value of the assets is the stock or asset price and the value of liabilities is the exercise price. The debtholders receive the full face value of their claims, D, or the value of the assets, A, if the corporation's assets are less than its liabilities at the end of the period. The end of period value of the debtholders' claims, V_D, can be written as the following:

$$V_D = \min[D, A]. \tag{7.22}$$

The debtholders have, in effect, written a put option whose maximum value is the face value of their claims, D, if the value of the corporation's assets, A, is greater than or equal to D, and whose minimum value is 0 if the corporation's assets are worthless at the end of the period.

An insurance contract is another example of a financial asset that has option characteristics. Assume an insurance company writes a single period policy with a premium, P, a deductible amount, B, and an unknown loss amount, L. Ignoring the time value of money for simplicity, the insurer's end of period policy value (V_p) would be written as the following:

$$V_p = \min[P, P-(L-B)] \qquad \text{or} \qquad \min[P, P-L+B]. \tag{7.23}$$

The insurer would net the premium if no loss occurred or if the loss did not exceed the deductible. If the loss were greater than the deductible, the insurer's income would be reduced by the difference between the loss and the deductible. This expression in Equation 7.23 is very similar to the payoff at expiration to the writer of a European call option. The insurer has in effect written a European call option with an exercise price of the deductible amount. In this case, the policyholder can be thought of as owning a European call option with an exercise price of the deductible amount. The value of the policyholder's claim (V_h) can be written as the following:

$$V_h = \max[L-B-P, -P]. \tag{7.24}$$

These straightforward examples of options have been used to determine the fair rate of return in pricing insurance with an option pricing framework. This application of option pricing to insurance will be discussed in the following section.

8. APPLICATION OF OPTION PRICING MODELS TO PRICING INSURANCE

The application of option pricing to insurance pricing was developed by Doherty and Garven [14]. Their formulation assumes a single-period insurer with initial equity of S_0 and premiums collected (net of expenses) of P_0. The aim of the model is to find the premium that gives the insurer an adequate or "fair" rate of return on equity. This is done by setting the present value of the expected end of period market value of equity equal to the beginning of period amount of equity.

The sum of initial equity and premiums represents the insurer's initial cash flow or asset portfolio of Y_0:

$$Y_0 = S_0 + P_0. \tag{8.1}$$

The insurer has this initial asset portfolio available to invest at rate R. All of the equity can be invested for the entire period and the premiums can be invested for a portion of the period because of the time lag between receipt of premiums and payment of losses. The time lag between receipt of premiums and payment of losses is called the funds generating coefficient and will be denoted by k. The end of period asset portfolio available to the insurer is the initial asset portfolio Y_0 plus the income generated from the investment of the initial asset portfolio at rate R which is written as the following:

$$Y_1 = S_0 + P_0 + (S_0 + kP_0)R. \tag{8.2}$$

The insurer has this end of period asset portfolio available to pay claimholders. The claimholders include policyholders who expect to have their losses paid and the government that expects taxes to be paid.

The policyholders hope the insurer has an adequate end of period asset value to pay their losses of amount L. If the insurer's end of period asset value is greater than or equal to L, the policyholders receive L. If the insurer does not have adequate asset value to cover the losses, the policyholders receive the amount of the insurer's end of period asset value Y_1. The policyholders' end of period claim, H_1, is represented by the following:

$$H_1 = \max\{\min[L, Y_1], 0\}. \tag{8.3}$$

This is equivalent to the expiration payoff to the owner of a European call option with an exercise price of L.

The government holds a similar type of call option based on whether the insurer makes a payoff. If the insurer has positive taxable income, the government receives taxes from the insurer based on the amount of the insurer's profits. If the insurer does not make a profit, the government receives no tax revenue. The value of the government's end of period tax claim, T_1, can be written as the following:

$$T_1 = \max\{t[i(Y_1 - Y_0) + P_0 - L], 0\}, \tag{8.4}$$

where

t = the insurer's corporate tax rate, and

i = the portion of investment income that is taxable.

The term $Y_1 - Y_0$ represents the insurer's investment income which may come from investments such as tax exempt bonds, corporate dividend income, and capital gains which may have differential tax rates from ordinary income.

Any portion of the asset portfolio remaining after the policyholders and taxes are paid reverts to the equityholders. Therefore, the end of period value of equity, V_e, is:

$$V_e = Y_1 - H_1 - T_1. \tag{8.5}$$

However, the end of period values in the previous equation are not known with certainty at the beginning of the period. The present value of the expected equity value must be found to begin the process of deriving a premium value that yields a "fair" rate of return on equity.

The present value of the policyholders' claim and the government's claim can be written as the following:

$$H_0 = V(Y_1) - C[Y_0; \mathrm{E}(L)], \tag{8.6}$$

$$T_0 = tC[i(Y_1 - Y_0) + P_0; \mathrm{E}(L)], \tag{8.7}$$

where

$V(Y_1)$ = the market value of the insurer's asset portfolio,

$C[A;B]$ = the current value of a European call option with exercise price B written on an asset with a value of A, and

$\mathrm{E}(L)$ = the expected losses and loss adjustment expenses during the period.

The market value of equity can now be written as:

$$\begin{aligned} V_e &= V(Y_1) - H_0 - T_0 \\ &= C[Y_0; \mathrm{E}(L)] - tC[i(Y_i - Y_0) + P_0; \mathrm{E}(L)] \\ &= C_1 - tC_2. \end{aligned} \tag{8.8}$$

For example, assume that an insurer is in the following situation (figures are in millions):

Initial Equity	\$100
Premiums Written	200
Expenses	40
Expected Losses	150
Standard Deviation of Investment Returns	0.5
Standard Deviation of Losses	0.0
Risk-Free Interest Rate	4.0%
Funds Generating Coefficient	1.0

In the context of the Doherty–Garven option pricing notation, S_0 is \$100, P_0 is \$160 (\$200 in premium less \$40 in expenses), E(L) is \$150 and there is no uncertainty about the value of these losses, and k is 1 year, at which time the insurer will pay the losses out of available assets.

Assuming no taxes initially, the value of the stockholders' interest in this insurer is, based on Equation 8.8 and the Black–Scholes option pricing model:

$$C[Y_0;\mathrm{E}(L)] = C[100+200-40;150] = C[260;150]$$

$$d_1 = \frac{\ln\left(\dfrac{260}{150}\right) + (.04+.5(.5)^2)1}{.5(1)^{1/2}}$$

$$= 1.43$$

$$d_2 = 1.43 - .5(1)^{1/2} = .93$$

$$C = 260N(1.43) - 150e^{-.04(1)}N(.93)$$

$$= 260(.9236) - 150(.9608)(.8238)$$

$$= 121.41.$$

Thus, the value of this insurer, based on the option pricing methodology, is \$121.41 million if taxes are ignored. This is higher than would be anticipated if the only consideration were given to the initial equity of \$100 million and the underwriting profit of \$10 million (premiums of \$200 less expenses of \$40 and losses of \$150). Adding the initial equity to the underwriting profit totals \$110 million. The reason for the much higher value based on the option pricing methodology is that the model considers the default option. If the end of period assets are less than \$150 million, the policyholders bear the loss, but the stockholders incur all the gains over that level.

Now taxes will be added to the calculation. Assume that all investment income is fully taxable, so the i in Equations 8.7 and 8.8 is 1.0. The insurer's tax rate, t, is 35 percent. The end of period asset portfolio is calculated based on Equation 8.2:

$$Y_1 = 100 + 160 + (100 + 1.0(160))(.04) = 270.4.$$

The value of the government's tax claim on the insurer is:

$$\begin{aligned}
T_0 &= .35C[(270.4 - 260) + 160; 150] \\
&= .35C[170.4; 150] \\
d_1 &= \frac{\ln\left(\frac{170.4}{150}\right) + (.04 + .5(.5)^2)1}{.5(1)^{1/2}} \\
&= .5850 \\
d_2 &= .5850 - .5(1)^{1/2} = .0850 \\
C &= 170.4N(.5850) - 150e^{-.04(1)}N(.0850) \\
&= 170.4(.7207) - 150(.9608)(.5339) \\
&= 45.86 \\
T_0 &= .35C = 16.05.
\end{aligned}$$

This value of the government's tax claim of \$16.05 million also may seem high, given that the insurer has investment income, based on the risk-free rate, of \$10.4 million and an underwriting profit of \$10 million. However, taxes are asymmetric, with the government collecting 35 percent of any gains, but not sharing in any losses. (In this model, tax loss carryforwards and carrybacks are ignored. In reality, taxes are much more complicated than the model provides.)

Considering taxes in determining the stockholders' value of the insurer described in this example yields the following, based on Equation 8.8:

$$V_e = 121.41 - 16.05 = 105.36.$$

Since V_e exceeds the initial equity value of \$100, the insurer gains value by writing insurance at this premium level. In this example, the expected losses are assumed to have no uncertainty.

When losses are allowed to vary, then, in essence, the exercise prices of the options for the stockholders and government are random variables. This variation can be accounted for, but complicates the calculation.

Doherty and Garven next use this methodology, including allowing losses to vary, to find the appropriate premium the insurer should charge. The premium should be set so that the market value of equity is equal to the initial equity amount of S_0 and yields a "fair" rate of return to shareholders. The values of Y_0 and Y_1 are functions of the "fair" premium of P^* as are the call options in Equation 8.8, rewritten as:

$$\begin{aligned} V_e &= C[Y_1(P^*);\mathrm{E}(L)] - tC[i(Y_1(P^*) - Y_0(P^*) + P^*;\mathrm{E}(L)] \\ &= C_1^* - tC_2^* \\ &= S_0. \end{aligned} \tag{8.9}$$

The insurer's fair underwriting profit margin is given by:

$$UPM = [P^* - \mathrm{E}(L)]/P^*. \tag{8.10}$$

The call option values are found by an option pricing model based on the Black–Scholes model. Doherty and Garven use two different option pricing models to price the options in Equation 8.9. These two models are arrived at by different assumptions about investor risk preferences and asset price distributions. One model is based on constant absolute risk aversion (CARA) and a normal distribution of asset prices, and the other assumes constant relative risk aversion (CRRA) and a lognormal distribution of asset prices similar to the Black–Scholes model. Since the models do not have closed form solutions, P^* is found by trial and error from properly parameterized versions of the models. Parameter estimates needed for the models are the initial equity level, standard deviation of claim costs and investment returns, and the correlation between claim costs and investment returns. The general results of that research indicate that the appropriate underwriting profit margins are higher under the option pricing model than under the CAPM.

This option pricing approach to pricing insurance is more complex than the CAPM or Discounted Cash Flow approaches, but it avoids many of the problems, such as estimating betas and market risk premiums, of the CAPM-based models. Also, the option model is different in that it uses the total risk of the insurer's investment portfolio and underwriting operations, rather than systematic risk.

The option pricing model has also been applied to insurance solvency considerations. Cummins [8] calculates the appropriate guaranty fund charge by using a diffusion process for assets and liabilities similar to the Black–Scholes option pricing model. Through the use of realistic parameters, Cummins is able to obtain a guaranty fund premium in line with past experience. Boyle and Kemna [5] use the option pricing model to examine incentives for cooperating behavior and for assuming excessive risk under the risk sharing arrangement inherent in guaranty funds.

One problem in applying the Black–Scholes option pricing model to insurance cases is the documented tendency of this model to underprice options in which the stock price is well above the exercise price (see [22], [23]). These options, termed in-the-money options, are exactly the type of option that is used in applications of the option pricing model to insurance, since the expected terminal value of the insurer's assets is generally much higher than expected losses. Thus, although the option pricing model has significant advantages over other valuation models, the bias inherent in the model needs to be taken into consideration.

9. CONCLUSION

The insurance industry, including regulators, insurers, and researchers, has grappled with the issue of a profit provision for over 70 years. The issue is as yet unresolved. The easy rules of thumb are based on invalid techniques. The valid techniques require input values that may not be possible to measure accurately.

Efforts to refine the techniques and advance the use of appropriate methods are continuing. Some results are quite promising, but additional work is necessary.

An analogy to loss reserving is appropriate. No actuary uses one method to set loss reserves, as no single method is applicable in all cases. The paid loss development method is very accurate, but takes a long time to reflect changes. Incurred loss methods reflect changes more quickly, but are sensitive to changes in case reserve adequacy. Data availability problems sometimes require reliance on less robust techniques. As discussed in Berquist and Sherman [3], the proper approach to establishing loss reserves is to use a variety of techniques, analyze the distribution of indications, try to determine if outliers are caused by errors or reflect early warnings of shifts in development patterns, and then use actuarial judgment to arrive at the best figure.

Ratemaking should be no different. Actuaries should apply a variety of ratemaking techniques to see what the various indications turn out to be. Some methods rely on parameters that are difficult to measure. Other methods are not responsive to changes in risk, interest rates, or other economic conditions. The techniques described in this paper can provide useful information about rate levels, but they should not be expected to determine, under all circumstances, the correct rate level. Within a portfolio of ratemaking techniques, each can contribute some value.

Actuaries have played, and will continue to play, a key role in the effort to determine appropriate profit provisions. However, since the mid-1970s, the playing field for investigation of these issues has shifted into relatively unfamiliar terrain for actuaries, the field of financial economics. Actuaries need to master this area in order to continue to play an influential role. The addition of finance to the Casualty Actuarial Society *Syllabus* is a useful step in developing this expertise. Hopefully, this paper will also be useful as an educational, or reference, tool.

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APPENDIX

NORMAL DISTRIBUTION FUNCTION
(FOR NEGATIVE X, $N(X) = 1 -$ VALUE LISTED)

X	.00	.01	.02	.03	.04	.05	.06	07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8079	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986