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SURPLUS—CONCEPTS, MEASURES OF RETURN, AND DETERMINATION

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1. INTRODUCTION

If the purpose of policyholder surplus is to provide a cushion against possible errors in the estimation of balance sheet assets and liabilities for an insurance company, then surplus is required wherever estimation errors might exist, regardless of their source. In particular, the balance sheet contains estimates of liabilities due to the runoff of previously written policies as well as due to current business. For that reason, the required or *benchmark* surplus that appears on a given balance sheet should be allocated to the exposure period (e.g., policy year, accident year, contract year, etc.) that gives rise to the uncertainty.

Russell Bingham advocates such a decomposition of balance sheet surplus and income statement flows into the contributing accident years. Because a given exposure period frequently impacts many annual statements, this decomposition results in the formation of historical supporting surplus triangles that are analogous to the loss development triangles used in the analysis of reserve level adequacy. Once the supporting surplus and income flows for each exposure period are known, the overall return on the supporting surplus can be determined. When evaluating the return earned by a particular product line, it is this long term investment of surplus that must be considered.

This is in sharp contrast to calendar year measures in which it is assumed that all of the company surplus supports the currently written exposure. The long-term commitment of supporting surplus to each accident year and the corresponding reflection of that commitment is the major idea presented in Bingham's paper.

To illustrate the segregation of surplus and income flows and the formation of insurance company balance sheet triangles on a present value basis, Bingham presents a simplified example. Some aspects of the example are more complicated than need be to illustrate the basic concepts (e.g., the explicit consideration of federal income tax), whereas other aspects are deceptively simple (e.g., the adoption of a constant leverage ratio). Several issues are left unresolved if the single example is to be used as the springboard to a more comprehensive return on equity (i.e., return on benchmark supporting surplus) model. In the course of this discussion, a more transparent illustrative model for the determination of the return on equity is described. Additional levels of complexity are introduced to the model as the previously unresolved issues are considered.

By means of the more transparent example, the essential features of Bingham's methodology are summarized and the invariant nature of Bingham's present value ratio of total return to supporting surplus is demonstrated. Two refinements to the model are then introduced. The first refinement involves changing the basis for determining the benchmark surplus from nominal loss reserves to discounted loss reserves. This allows for a reflection of both ultimate loss *amount* risk and payout *timing* risk. The second refinement involves replacing the constant reserve-to-surplus ratio with a variable leverage ratio. Both of these refinements are compatible with the agreement inherent in Bingham's scheme for releasing operating gain (i.e., internal rate of return = average annual return on supporting surplus = Bingham's present value ratio).

An examination of the behavior of Bingham's methodology in two extreme pricing situations (severe rate inadequacy and severe rate redundancy) discloses that the simplified model does not produce reasonable results under these extreme conditions unless the leverage ratio is a function of the expected retained operating gain. Determining a functional relationship while maintaining the advantages of Bingham's release scheme is shown to be a nontrivial exercise.

Bingham's example assumes that, at each point in time, events which were *expected* to have occurred previously actually *did* occur. Because of that, the earned investment income and retained operating gain at any given time are exactly what they were expected to be when the supporting surplus requirement for that time was originally determined. This discussion considers whether or not supporting surplus to be carried during the runoff should be modified if the actual history is not what was expected a priori. Resolution of this issue affects both the prospective and retrospective determination of the return on equity for an insurance product.

Two appendices serve to flesh out the discussion. The first appendix provides a rigorous proof that Bingham's timing of the release of the insurance operating earnings always leads to agreement among the internal rate of return (IRR), average annual return on equity (ROE), and present value ratio, regardless of the level of sophistication introduced into the insurance model (e.g., the reflection of federal income tax, policyholder dividend payment, etc.) or the nature of the reserve-to-surplus leverage ratio (e.g., dependence upon the number of open claims, the expected retained operating gain, etc.). It is this proof that allows simplified models to be used to illustrate the methodology. The second appendix provides evidence that, contrary to common wisdom, a *decreasing* leverage ratio may be appropriate even for a line such as workers compensation with lifetime pension cases.

2. OVERVIEW OF BINGHAM'S METHODOLOGY

The world can be divided into three parts. These are the insurance product itself, shareholder funds, and everything that is external to the other two parts.

The *insurance product* can be narrowly defined as a single contract (e.g., a primary company policy or single reinsurance treaty, etc.), or the definition can be broadened to include a portfolio of similar contracts. In the extreme case, a portfolio could encompass all of the writings of a company.

Bingham refers to the second division as *shareholder funds*. While this designation works well for stock companies, the more general name, *surplus account*, allows us to extend the discourse to encompass mutual companies. The surplus account consists of two types of surplus, the surplus that is required to support the particular insurance operation (Bingham's benchmark surplus) and free surplus or *surplus* surplus. Surplus surplus is available to pay stockholder dividends, back new insurance operations, or simply remain idle with a return equal to that of the company's investment portfolio.

The third division includes everything external to the insurance company such as the policyholders, the stock market, and the Internal Revenue Service. Elements of the third division are relevant only to the extent that their existence results in cash flows either into or out of the other two divisions.

Having (implicitly) assumed this division, Bingham states that the purpose of supporting surplus is to act as a buffer to ensure an acceptably low probability of ruin. The buffer must be made available because of uncertainty that gives rise to both investment risk and underwriting risk. He, therefore, concludes that *supporting surplus must be allocated to the insurance product as long as any uncertainty exists.* A logical corollary to this is that supporting surplus can be released to the surplus surplus portion of the surplus account only as uncertainties are resolved and the corresponding probability of ruin decreases. This implies that supporting surplus must be a function of all of the stochastic asset and liability variables, not simply a function of one year's written premium as many surplus allocation formulae dictate. At any point in time, surplus is required to support not only the uncertainty associated with the current exposure period (accident year, policy year, contract year, etc.), but also the uncertainty associated with the runoff of prior exposure periods as well.

With that in mind, Bingham turns his attention to a single accident year and its contribution to each subsequent balance sheet. Bingham goes on to observe that *the timing of the release of the supporting surplus and operating gain*[1] *from an insurance product to the surplus account affects the measured return on equity for that accident year*. This observation is the second major point raised in Bingham's paper. While reserves and supporting surplus are clearly identified as "belonging" to the insurance product, the time at which other funds that arise from the insurance product are released to the surplus account is somewhat arbitrary. As a result of the sensitivity of the ROE to the arbitrary identification of these funds as insurance funds vs. surplus funds, the calculated return on supporting surplus can be manipulated by users of these models to produce a wide range of values purporting to be *the* ROE. Bingham proposes a timing scheme which, while still arbitrary, has a logical foundation.

By means of a simple example, Bingham illustrates the consequences of releasing supporting surplus and operating earnings as uncertainty is resolved and releasing investment income on supporting surplus as it is earned. A significant observation is that, under this release scheme, the annual return on supporting

[1]Operating gain is usually thought of as a calendar year concept. In this context, the operating gain associated with a particular exposure period is the amount by which the present value of the premium income exceeds the present value of the loss payments and other expenses (including federal income tax and policyholder dividends). It reflects both the underwriting result and investment income on underwriting funds. In contrast to a calendar year concept, operating gain in this context applies to the entire history of a particular exposure period.

surplus, the internal rate of return of the flows to and from the surplus account, and the present value of the flows to the surplus account divided by the present value of the supporting surplus (over the life of the product) are all equal.

The aesthetically pleasing agreement of the three measures of return on equity that results from Bingham's scheme for the release of surplus, investment income, and operating earnings is a strong argument in favor of following Bingham's lead. An additional argument in support of this scheme is that not only is the required amount of supporting surplus kept available to act as a buffer against insolvency risk, but a portion of the operating gain is also retained to serve as an additional buffer against the possibility of ruin. Both the supporting surplus and the retained operating gain are released as the uncertainties regarding occurrences that could lead to ruin (insolvency) are resolved.

Another strong argument supporting the resulting measure of the ROE is that Bingham's present value ratio measure is an *invariant* measure of the rate of return corresponding to an entire class of models, regardless of when and how the operating earnings are actually released to the surplus account.

In order to illustrate the relationship between the various measures of the return on surplus, Bingham presents a simplified example. As simple as the example is, more details concerning the workings of the insurance product were described than were necessary. In particular, only the operating profit associated with the insurance product and the time at which reserves (with the associated uncertainties) are present need to be known in order to determine the return on surplus. This is not to say that such issues as expenses and federal income tax timing are not important; rather, these aspects of the insurance product can be left inside the "black box" that determines the operating profit and establishes reserves. Leaving them out of the example serves to make the illustration more transparent. To that end, an even more simplified illustrative example is presented in this discussion.

On day one of this example (denoted as the last day of year zero in all of the exhibits), \$400 of premium is collected. Payments of \$264, \$96, \$32, \$8, and \$4 are made at the ends of years one through five, respectively. The series of payments may be thought of as either claim payments or as the aggregation of claim, expense, and tax payments. Only the magnitude and timing of the payments, together with the establishment of a liability in recognition of future payments, are germane to this discussion. So as not to obscure the basic concepts with the unnecessary details concerning how federal tax law would apply to the hypothetical situation, it will be assumed that there are no federal income taxes or other expenses. The payments, therefore, may be thought of as claim (loss) payments. Any reference to *losses* or *loss ratio* is equally valid for *losses*, *expenses*, and *taxes* together with the corresponding *combined ratio*. Investment income is assumed to be earned at a 5% annual effective rate.

Ruin occurs whenever there are insufficient funds available with which to make payments (loss, expense, and tax) as they become due. Sources of available funds include policyholder premium, investment income on underwriting funds, and supporting surplus. It is usually assumed that premiums and investment income on underwriting funds provide sufficient funds to cover all of the *expected* payments as they become due. Unexpected events such as unexpected loss payments (both with respect to amount and timing) are a major source of potential ruin. Supporting surplus (surplus allocated to the insurance product) provides additional funds to cushion against possible unexpected events. The more supporting surplus that is allocated to the insurance product, the more extreme the unexpected event would have to be in order to cause ruin. Assume that, for the insurance product under consideration, the probability of ruin can be kept to an acceptable level (e.g., less than 2%) by supporting each dol-

TABLE 1

Year End	Paid to Date Loss	Nominal[2] O/S Loss	Discounted[3] O/S Loss	Supporting Surplus
0	$0.00	$404.00	$375.86	$202.00
1	264.00	140.00	130.66	70.00
2	360.00	44.00	41.19	22.00
3	392.00	12.00	11.25	6.00
4	400.00	4.00	3.81	2.00
5	404.00	0.00	0.00	0.00

lar of outstanding loss reserve with $0.50 of surplus (i.e., a 2 : 1 reserve to surplus leverage ratio). This assumption presupposes that a rigorous determination of the appropriate leverage ratio has been conducted and that the result was the 2 : 1 ratio. As was the case in Bingham's paper, the details of this determination fall beyond the scope of this discussion.

The payment and leverage ratio assumptions are almost identical to the situation presented by the NAIC as an illustration of an IRR model [2]. Table 1 shows the loss and supporting surplus under the assumption of a constant reserve-to-surplus leverage ratio and payment pattern.

Exhibit 1 displays the essential features of the situation. The insurance product has an operating gain equal to $24.14 (present value of the premium on day one less the present value of the loss payments on day one). In this particular example, the entire operating gain was allowed to accrue interest as part of the insurance product account for a year before it was released to the

[2]"Nominal O/S" is the (estimated) sum of all future claim payments whether or not the claims have been reported to the carrier at the time that the estimate is made. This outstanding amount includes carried reserves and bulk reserves such as true IBNR and the less restrictive IBNE (Incurred But Not Enough).

[3]"Discounted O/S" is the present value of the expected flows that make up the nominal outstanding amount. As such, it may include a provision for the present value of claim payments that are expected to be made on claims that have not yet been reported. Discounted outstanding is more inclusive than the present value of the carried reserves.

surplus account. The accrued value of \$24.14 after one year is \$25.34.

Because the reserves along with their associated uncertainty remained constant during the first year, the supporting surplus was held constant at \$202.00 for the entire year. The total return on this supporting surplus during the first year consisted of the \$25.34 accrued operating gain from the insurance product and the \$10.10 of investment income earned by the supporting surplus. The total, \$35.44, represents a 17.55% return on the \$202.00 surplus investment.

During subsequent years there was no contribution to the total return on supporting surplus arising from the insurance product. Investment income that was earned on underwriting funds during each calendar year was exactly sufficient to establish the year-end discounted loss reserve after all of the calendar year loss payments were made. In this respect, the insurance product did not participate in any further fund transfers between its own account and the surplus account after the end of the first year. Regardless of this, the fact that there was uncertainty regarding the ultimate loss outcome during each subsequent year led to the requirement that some surplus had to be allocated to the insurance product. During these subsequent years there was a 5% annual return on the supporting surplus. This is the same return as would have been earned had the surplus been idle (i.e., not supporting an insurance product). This surplus was, however, committed to supporting uncertainties during the runoff and was *not* available to support *new* writings. The average annual return on supporting surplus [Σ(released operating gain plus investment income on the supporting surplus)/Σ(supporting surplus), where the summation is over all years] was 13.39%.

It would not be correct to consider only the first year and to report a 17.55% return on equity for the product. Doing so would ignore the commitment of surplus during the subsequent four years. To see that this is precisely what is done when calendar year earnings are compared to average calendar year surplus amounts, consider what the calendar year return would be if the carrier wrote only a single contract during its lifetime. The first year-end balance sheet would indicate an average surplus amount equal to $202.00, while the statement of income would reflect $35.44, giving us a 17.55% return on surplus for the calendar year.

It can be argued that a carrier's calendar year return on equity would be equal to the average annual return on equity if the book of business were to be repeatedly renewed until a steady state had been achieved. Mathematically, this is a true statement. In the case of the single contract described above, repeated renewal for four or more years would result in an annual commitment of $302 of surplus ($202 for the most recently renewed contract, $70 for the contract written one year before, etc.) and annual income equal to $40.44 ($35.44 for the most recently renewed contract, $3.50 for the contract written one year before, etc.), which would yield a 13.39% return on supporting surplus for the calendar year.

Conceptually, the two measures are very different. While the average annual return measure relates to a single contract, the calendar year measure requires identical contracts to be written year after year. If the mix of business changes from year to year or if all of the company surplus is not being used to support the runoff of previously written contracts, then the equality no longer holds.

The conceptual difference between calendar year and average annual return on surplus is similar to the one that exists between accident year (or policy year) loss ratio and calendar year loss ratio. Here, too, the two measures are numerically equal once a steady state situation has been achieved. Each age to age development that is observed in the accident year (or policy year) triangle would be contributed to the calendar year experience by different accident year contracts in the corresponding stages of development. Just as one would not rely upon this equality when estimating the ultimate loss ratio for a single accident year, Bingham advocates determining the return on surplus associated with each underwriting period separately. Since the activity associated with a single underwriting period often spans several calendar years, balance sheet triangles arise in a manner that is analogous with loss and premium development triangles.

A second measure of the rate of return is the IRR implied by the flows to and from the surplus account. For this purpose, the flows of invested supporting surplus to and from the surplus surplus (surplus that is not supporting an insurance product) account must be reflected as well as the release of the operating gain and investment income on the supporting surplus. The total flows (initial supporting surplus investment, return of supporting surplus as it is released, investment income on the supporting surplus as it is earned, and the accrued operating gain as it is released) are displayed in the IRR column in Exhibit 1. For this example, the IRR is 13.87%.

A third measure of the return on equity is the ratio of the present value of the flows to surplus (the released operating gains and the investment income on the supporting surplus) to the present value of the *year-end* supporting surplus, 13.58% in this example. This measure is similar to the average return except that the present values of the numerator and denominator have been taken prior to forming the ratio. Of the three measures of return, only the present value ratio appears to lack an intuitively satisfying context. Taking the present value of a year-end surplus amount, which does not represent a discrete cash flow at year-end, contributes to the initial uneasiness with this measure.

The last three columns in Exhibit 1 are for reference. They display the *retained earnings*, the *investment balance*, and insurance product *overfund*. The retained earnings represent the accrued underwriting gain or loss at each year-end. In a way, the retained earnings reflect the impact of statutory accounting requirements on the surplus account. The investment balance at any point is the amount of insurance product funds that are available for investment (accrued premium less paid losses and released operating gain). The overfund is the amount by which the investment balance exceeds the discounted outstanding loss reserve. The overfund represents the portion of the operating gain that has been retained to act as an additional buffer against insolvency risk. No model is acceptable that does not come to an end with exactly zero overfund and zero investment balance. Any alternative to closing out the insurance product account after the last claim has been paid would result in allocating a portion of surplus to the insurance product (or floating it a loan) long after all claims had closed and all uncertainties had been resolved.

Exhibit 2 represents the same situation but with a withdrawal of operating gain at the opposite extreme from that which was depicted in Exhibit 1. In Exhibit 2, the operating gain is retained within the insurance product until all claims have been paid. As long as the operating gain is retained within the insurance product account, all interest accrued on it will be attributed to the insurance product. At the end of the fifth year, when the accrued operating gain is finally released to the surplus account, it carries with it \$6.67 of accrued interest, all of which is considered as part of the total return on supporting surplus.

Once funds are released to the surplus surplus account, subsequent investment income earned by them is not attributed to the insurance product. Because the operating gain was released to the surplus account later than in Exhibit 1, more of the investment income earned on these funds was attributed to the insurance product (\$6.67 vs. \$1.20). As a result of the difference between these two arbitrary segregations of funds—between the insurance product account and surplus account—the average annual return on supporting surplus increases from 13.39% as displayed in Exhibit 1 to 15.20% under the operating gain release timing of Exhibit 2. Whereas reflecting more dollars of investment income causes the average return on surplus to increase, it has the opposite effect on the IRR. The IRR corresponding to 13.87% of Exhibit 1 is 11.84% in Exhibit 2.

If the insurance company had set a 13.5% target for its return on equity and used these measures of return to evaluate this product, it would have found the product to fall short of the target if it had adopted the average return measure under the Exhibit 1 scenario, but to be acceptable under the Exhibit 2 scenario. Just the opposite conclusions would be reached if the IRR measures were used. Now, simply earmarking funds as belonging to a particular company account (insurance product account or surplus account) does not affect the overall well-being of the company.[4] There must be something misleading about a model that produces different results for different earmarkings. While the present value measure of the return on equity remained equal to 13.58% for both alternatives, invariance alone does not provide sufficient support for it to be adopted as the true measure of the return on equity.

Exhibit 3 provides that support. This example begins by specifying how the operating gain is to be released to the surplus account. In this alternative, the operating gain is released Bingham's way, as uncertainty is resolved (i.e., under the same criteria that the supporting surplus is released). This timing results in releasing the operating gain in such a way that the ratio of released dollars to the invested surplus remains constant. In symbolic form, if $S(j)$ is the invested surplus during the jth year, and $O(j)$ is the accrued operating gain that is released at the end of the jth year, then the set $\{O(j)\}$ must satisfy two conditions:

1. $PV[\{O(j)\}]$ = the operating gain, and
2. $O(j)/S(j)$ = constant for all years, independent of j.

The bottom of Exhibit 3 displays the detailed calculation of the set $\{O(j)\}$ corresponding to this example.

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[4] While actions taken as a result of this earmarking, such as the declaration of stockholder dividends, can affect the overall well-being of the company, the *act* of earmarking funds cannot affect the company's well-being.

As promised by Bingham, the three measures of the return on equity are equal when his release of operating gain is adopted. This is more than a coincidence. Appendix A presents a general proof that Condition 2 is sufficient to force the three measurements into agreement.

The invariant measure is equal to the average annual return on equity and to the internal rate of return corresponding to the case in which operating earnings are released in the same manner as supporting surplus, as uncertainty is resolved. The invariant measure does have a context.

It should be emphasized that the only feature of the insurance product cash flow that is explicitly reflected in the determination of the return on surplus is the present value of the operating gain. Increasing the degree of sophistication of the insurance product model (e.g., reflecting federal income tax, other expenses, policyholder dividends, etc.) almost certainly will change the numerical value of the operating gain but will not alter any of the concepts that have been discussed. Once the operating gain is determined, the manner in which it is released remains unchanged (i.e., according to the two conditions), and the agreement among the IRR, average annual return, and the model invariant continues to hold.

3. DOES THE BINGHAM METHODOLOGY LEAD TO REASONABLE RESULTS?

All of the exhibits thus far have been based upon a situation that generates an operating profit. Furthermore, while the Bingham invariant ratio, average annual return on equity, and internal rate of return produce different measures of the return on equity, they do not differ significantly in the absence of the Bingham release scheme. For the purpose of a reasonableness check, a new example will be presented. The longer payout period accentuates the differences between the three measures of ROE when the

TABLE 2

ELEMENTS COMMON TO EXHIBITS 4–6

Year End	Paid to Date Loss	Nominal O/S Loss	Discounted O/S Loss	Supporting Surplus	Required Funds
0	$0.00	$2,000.00	$961.38	$1,000.00	$1,961.38
1	0.00	2,000.00	1,009.45	1,000.00	2,009.45
2	0.00	2,000.00	1,059.92	1,000.00	2,059.92
3	6.00	1,994.00	1,106.92	997.00	2,103.92
4	34.00	1,966.00	1,134.26	983.00	2,117.26
5	120.00	1,880.00	1,104.98	940.00	2,044.98
6	184.00	1,816.00	1,096.23	908.00	2,004.23
7	258.00	1,742.00	1,077.04	871.00	1,948.04
8	332.00	1,668.00	1,056.89	834.00	1,890.89
9	404.00	1,596.00	1,037.73	798.00	1,835.73
10	474.00	1,526.00	1,019.62	763.00	1,782.62
11	526.00	1,474.00	1,018.60	737.00	1,755.60
12	574.00	1,426.00	1,021.53	713.00	1,734.53
13	618.00	1,382.00	1,028.61	691.00	1,719.61
14	660.00	1,340.00	1,038.04	670.00	1,708.04
15	693.00	1,304.00	1,053.94	652.00	1,705.94
16	730.00	1,270.00	1,072.64	635.00	1,707.64
17	760.00	1,240.00	1,096.27	620.00	1,716.27
18	788.00	1,212.00	1,123.08	606.00	1,729.08
19	1,312.00	688.00	655.24	344.00	999.24
20	2,000.00	0.00	0.00	0.00	0.00

release of operating gain does not follow the resolution of uncertainty.

The longer payout period of this example is similar to that of high attachment point workers compensation excess of loss reinsurance. By the end of the 18th year, less than 40% of the ultimate loss is expected to have been paid. Table 2 displays the elements that are common to Exhibits 4 through 6.

The Required Funds column consists of the funds that must be allocated to the insurance product, an amount equal to the discounted outstanding loss plus the supporting surplus. Any additional funds may be released to the surplus account at any time.

The reasonableness check begins with a simple observation. If the operating gain associated with the insurance product is exactly zero (i.e., the premium is just sufficient to fund the discounted loss reserves), then there can be no net flow from the insurance product to or from the surplus account. Supporting surplus must be allocated, but all that can be earned on the supporting surplus is the 5% return that could be earned on idle surplus. No release of the operating gain (i.e., a set of non-zero flows that have a present value equal to zero) that results in a return on equity other than 5% is reasonable.

Exhibits 4A and 4B present just such a zero operating gain situation. With $961.38 of premium and $2,000 of expected loss, the underwriting loss would be $1,038.62 and the incurred loss ratio would be 208%. A premium equal to $961.38, paid on day one, exactly funds the discounted outstanding loss reserve. With no funds to spare, the operating gain is exactly zero. While supporting surplus is required during the 20 year runoff, its return will be exactly the same as if the insurance product had not been written, 5%. No measurement of the return on surplus other than 5% would be reasonable for this situation.

A quick glance at Exhibit 4A discloses that Bingham's invariant ratio passes the test, whereas the average annual return, at 8.1%, clearly fails the test.

The rather peculiar looking release of operating gain, $\{O(j)\}$, mimics the requirements of statutory accounting (SAP). Under SAP, the carrier must fund the nominal reserves rather than the discounted reserves. As a result of this requirement, the $961.38 premium falls short by $1,038.62. Consistent with the SAP requirement, $1,038.62 must be transferred *from* surplus surplus to the product on day one. The equivalent year-end transfer is displayed on Exhibit 4A. The $990.55 transfer can be thought of as the day one transfer of $1,038.62 plus interest (totaling $51.93) less the interest earned on the $2,000 nominal reserve (a total of $100.00).

While this set of operating gain flows is allowable (their present value is zero and they produce a zero investment balance by the end of the 20 year runoff), the corresponding 8.1% average annual return on supporting surplus is, clearly, unreasonable. This paradoxical result is an example of the type of manipulation that Bingham's release scheme is designed to prevent.

This manipulation was previously encountered in the first example for which the operating gain was greater than zero and for which all flows were positive. In that example, it was noted that once a flow is released to the surplus account, no further investment income earned on this money is credited to the insurance product. The longer the operating gain is retained as part of the insurance product, the more of its earned investment income is credited to the insurance product. Interest earned on surplus surplus is ignored, regardless of its source.

Likewise, when some of the flows are negative, the interest that is not earned (lost) by the surplus surplus is ignored. The insurance product, rather than surplus surplus, receives credit for the earned investment income. The $485.65 of nominal gain (sum of the stream of $O(j)$ flows) that appears to have been generated by the insurance product was at the (unrecognized) expense of the surplus surplus account.

If the average annual return on surplus is viewed as being the calendar year return once a steady state situation has been achieved, then the identification of the source of the additional $485.65 return is somewhat different. Under a steady state interpretation, the flows from year-ends 1 through 20 represent the contribution of previously written policies to the current calendar year. Under this interpretation, the policies in runoff do provide sufficient funds to establish the initial reserve on newly written policies and provide the missing 3.1% return on the steady state supporting surplus. What is missing in this interpretation is the cost of establishing the steady state (transferring funds from surplus to establish the first twenty years of writings). The additional 3.1% return is exactly equal to the investment income being lost by the surplus surplus account as a result of funding the underwriting loss for the first 20 years.

The final columns of Exhibit 4A display the calendar year return on equity during each year, if the SAP release scheme were to be followed. During the first 20 years, the runoff from successively more accident years is reflected in each annual statement. Eventually by year 20, the statement ROE reflects one year-end ROE for each of the 20 accident years. Growth from year to year affects the relative amount of each maturity that is reflected in the year-end ROE. Only when the exposure growth rate is 5% does the calendar year ROE approach the reasonable 5% figure.

Exhibit 4B looks at the same situation from the insurance carrier's perspective rather than from the perspective of a stockholder who is focused on the surplus account. In this representation, no distinction is made between supporting surplus and discounted loss reserves. All of the funds belong to the insurance carrier. Funds are released to the general surplus account as soon as they are not required to support the insurance product.

There is an initial investment of $1,000 from general surplus which, together with the premium, leaves $1,961.38 to be invested at 5% per year. The required funds are also equal to $1,961.38. At the end of a year, the invested funds will have accrued to $2,059.45 (there having been no loss payments). Only $2,009.45 is required by the insurance carrier to fund the discounted outstanding loss amount and supporting surplus, so the $50.00 difference can be released to surplus. Continuing in this fashion results in cash flows to the surplus account that have an internal rate of return equal to 5%.

With both the Bingham invariant ratio and the insurance carrier perspective treatment having passed the first reasonableness test, a new situation (depicted in Exhibits 5A and 5B) is considered. In these exhibits, less premium is collected. This results in a net operating loss for the product. Clearly, the supporting surplus must earn less than if it were not supporting this product.

As can be seen in Exhibit 5A, Bingham's invariant ratio represents a reasonable measure of the return on surplus; it is less than that of idle surplus. The statutory accounting model, again, fails the test because its ROE is greater than that of idle surplus.

From the insurance carrier's perspective (Exhibit 5B), \$1,961.38 is required to support the product, but only \$600.00 is received in the form of premium. The additional \$1,361.38 must be supplied from the surplus account. While not producing the same ROE as Bingham's scheme does, this measure is reasonable.

Both the Bingham scheme and the insurance carrier perspective agree that surplus would increase faster if this product, with its 333% loss ratio, were not written. The statutory accounting model does not agree.

A reasonable model should report an ROE that is greater than that of idle surplus if there is an operating gain produced by the insurance product. The purpose of supporting surplus is to cushion against uncertainty. If the premium is sufficient to fund the discounted loss reserve for the expected losses and to provide the required cushion against uncertainty, then no contribution of supporting surplus should be required. As the premium approaches this "no risk to the carrier" amount, the ROE should increase without bound. This expectation provides another test of a model's behavior.

Exhibit 6A displays the first portion of the reasonableness test when there is a net operating profit. With \$1,700 of premium, there is a \$738.62 operating gain. All three measures of ROE are greater than that of idle surplus (5%).

Both Bingham and the statutory model allocate the same amount of supporting surplus that they did in the other two cases. From the insurance carrier perspective (Exhibit 6B), only \$261.38 of surplus is needed in addition to the premium in order to fully fund the discounted outstanding loss and supply the required amount of cushion against uncertainty.

TABLE 3

RETURN ON EQUITY FOR EACH MEASURE

Premium	Operating Gain	Loss Ratio	SAP Measure	Bingham Invariant Measure	Insurance Carrier IRR Measure
$ 500.00	$ −461.38	400.0%	5.0%	0.6%	1.6%
600.00	−361.38	333.3%	5.7%	1.5%	2.2%
700.00	−261.38	285.7%	6.3%	2.5%	2.9%
800.00	−161.38	250.0%	7.0%	3.4%	3.6%
900.00	−61.38	222.2%	7.7%	4.4%	4.4%
961.38	0.00	208.0%	8.1%	5.0%	5.0%
1,000.00	38.62	200.0%	8.3%	5.4%	5.4%
1,100.00	138.62	181.8%	9.0%	6.3%	6.5%
1,200.00	238.62	166.7%	9.7%	7.3%	7.8%
1,300.00	338.62	153.8%	10.3%	8.3%	9.4%
1,400.00	438.62	142.9%	11.0%	9.2%	11.4%
1,500.00	538.62	133.3%	11.7%	10.2%	14.1%
1,600.00	638.62	125.0%	12.3%	11.1%	18.0%
1,700.00	738.62	117.6%	13.0%	12.1%	24.4%
1,800.00	838.62	111.1%	13.7%	13.1%	37.5%
1,900.00	938.62	105.3%	14.3%	14.0%	88.3%
1,920.00	958.62	104.2%	14.5%	14.2%	126.6%
1,940.00	978.62	103.1%	14.6%	14.4%	237.3%
1,950.00	988.62	102.6%	14.7%	14.5%	441.1%
1,960.00	998.62	102.0%	14.7%	14.6%	3,620.8%

For the second part of the reasonableness test, allow the premium to increase until it is sufficient to fund the $961.38 discounted loss reserve and to supply the required $1,000 cushion against adversity. At that point, zero surplus is required and the return on equity should become undefined. Table 3 displays the resulting returns on equity for each of the measures (average annual return under a release dictated by statutory accounting, Bingham's present value ratio, and the insurance carrier's IRR) as the zero risk extreme is approached.

The shaded sections of the table indicate regions in which the model fails a reasonableness test. The Bingham invariant ratio appears to fail the test at the high operating profit extreme because the required supporting surplus does not reflect the fact that the retained operating funds provide an additional (unquantified) cushion against uncertainty.

If the required surplus were to be reduced in recognition of the retained operating gain, with the sum of the supporting surplus and retained operating gain providing the required cushion (at a 2 : 1 reserve to cushion ratio), then insurance products with a larger expected operating gain would require less surplus. As the expected operating gain approached the required cushion amount, the required surplus would approach zero, and the resulting return on surplus would increase without bound as the expected operating gain approached this no risk situation.

Were it not for the Bingham requirement that the operating gain be released so as to maintain a constant return on the supporting surplus, reducing the surplus in recognition of the retained operating gain would be a trivial exercise. Difficulty arises because the set of release flows, $\{O(j)\}$, depends upon the year-end surplus amounts, $\{S(j)\}$, which in turn depend upon the set of retained operating gains, $\{R(j)\}$. These retained gains depend upon what has been previously released, the set $\{O(j)\}$.

Attempting to find a set of flows and surplus amounts that satisfy the two relations,

$$O(j)/S(j-1) = k, \quad \text{independent of } j, \text{ and}$$

$$R(j) + S(j) = \text{Reserves at year-end } j \text{ divided by the reserves-to-cushion ratio}$$

is not a trivial matter.

Solving this linked set of equations in closed form requires solving a polynomial of degree 20 for a product with a 20 year runoff.[5] Attempting to solve the system by an iterative technique

[5]The polynomial arises as the result of an attempt to determine the operating gain to be released at each year-end. As demonstrated in Appendix A, in the absence of modifying the supporting surplus to reflect the cushioning effect of the retained operating gain, a set

requires the imposition of additional conditions that are not specified by Bingham.[6]

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of linear equations in k, the constant annual return on equity resulting from the release of accrued operating gain,

$$O(j) = k * S(j-1),$$

must be solved. Because the $\{S(j)\}$ are independent of the $\{O(j)\}$, the set of n equations in k is linear.

When the supporting surplus is a function of the retained operating gain, as it is when the amount of supporting surplus is reduced in recognition of the operating gain that has been retained, $S(j-1)$ becomes a function of the previously released operating gain. Each $O(j)$ is, itself, a linear function of k. As a result,

$$O(j) = k * F(\{O(n)\}), \quad \text{where } n \text{ ranges from zero to } j-1.$$

It is this functional dependence of $O(j)$ upon the $O(n)$ that introduces increasingly higher powers of k as j increases.

To be more concrete, let

OP be the expected operating gain for the product (i.e., the present value at time zero),
$\{C(j)\}$ be the required amount of cushion at year-end j,
$\{R(j)\}$ be the retained operating gain at year-end j, and
i be the investment income rate.

During the first year, the required cushion, $C(0)$, consists of the sum of the operating gain, OP, and a contribution from surplus, $S(0)$. At the end of the year, $O(1)$ will be released such that

$$O(1)/S(0) = k.$$

With the exception of the fact that $S(0)$ is not equal to $C(0)$, this equation is identical to the first equation in the set of linear equations.

During the second year, the required cushion is $C(1)$. This is supplied by the retained operating gain, $(1+i)*OP - O(1)$ together with a contribution from surplus, $S(1)$, where

$$\begin{aligned} S(1) &= C(1) - (1+i)*OP - O(1) \\ &= C(1) - (1+i)*OP - k*S(0) \\ &= C(1) - (1+i)*OP - k*[C(0) - OP]. \end{aligned}$$

The condition that

$$O(2)/S(1) = k$$

becomes

$$O(2)/[C(1) - (1+i)*OP - k*[C(0) - OP]] = k$$

which is quadratic in k. Each additional year that is reflected introduces another power of k into the polynomial.

[6]The iterative solution begins with an initial solution that sets

$$S(j)_0 = C(j).$$

4. REFINEMENTS TO THE BINGHAM METHODOLOGY I (NOMINAL VS. DISCOUNTED RESERVES)

While several sources of uncertainty are enumerated in Bingham's paper, his example deals with only one of these sources, the uncertainty associated with the ultimate loss *amount*. As a result, his supporting surplus is a function of the nominal outstanding loss reserve. Bingham does not describe how the risk associated with the *timing* of loss payments would influence the amount of supporting surplus, nor does he discuss the effect of investment risk on the amount of supporting surplus that would be required.

A minor change is required to reflect not only the uncertainty in the ultimate *amount* but also *timing risk* and a portion of the *investment rate* uncertainty as well. The change involves applying the leverage ratio to the discounted reserves rather than to the nominal reserves. The variance of the expected discounted reserves can be modeled to reflect the uncertainty in the ultimate

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For this solution, a set of $O(j)_0$ are determined. Using these $O(j)_0$, the set of retained operating gains, $\{R(j)_0\}$, can be determined at each year-end.

The next iteration begins by setting

$$S(j)_1 = C(j) - R(j)_0$$

and completing another cycle.

The iteration is said to converge if, for all n greater than a fixed N, $S(j)_n - S(j)_N$ is not material.

When applied to the 20 year payout example, the iterative procedure ran into problems (failed to converge) when the premium was sufficient to cause

$$S(j)_m = C(j) - R(j)_{m-1} < 0 \qquad \text{for some } j, \text{ on the } m\text{th iteration.}$$

A logical additional condition to impose upon $S(j)$ is that it be greater than or equal to zero.

At even larger premium amounts (above $1,800), multiple $S(j)$s "zeroed out." Again, the iteration failed to converge to a single accumulation point, as $S(j)$s that were previously equal to zero became positive at the next iteration.

A determination of the conditions that must be imposed upon the iteration in order to make it converge for all premium amounts is beyond the scope of this discussion. It is very interesting to note that, when the procedure did converge, the indicated rate of return on surplus was numerically equal to the IRR produced by looking at the process from the insurance carrier perspective. Finding a logical set of constraints that would insure (proven rigorously) this equality at all premium levels would be a significant contribution to the literature.

amount, uncertainty in the cash flow timing, and uncertainty in the investment income rate as well. A description of how one would determine the leverage ratio that would cushion against variation of the expected discounted reserve around its mean is beyond the scope of this discussion.

When the role of the cushion is restricted to covering the *ultimate amount* at risk, it is still appropriate to apply a leverage ratio to discounted reserves. Even if the actual future loss payments are greater than expected, only the present value of the unexpected payments needs to be available now. It will accrue to the required amount by the time it must be used.

Returning to the original example, the nominal loss reserve is \$44.00 at the end of year two. The 2 : 1 reserve to surplus ratio[7] implies that if \$66.00 is made available to pay losses (\$44.00 of loss reserve and \$22.00 of supporting surplus), then the probability of ruin can be kept below some pre-established amount (e.g., less than 0.02). If there is no uncertainty regarding the timing of the future payments (i.e., the percentages of the actual ultimate loss to be paid by each year end are exactly those which were expected), then each future loss payment will be 50% higher than expected in this worst case scenario. If the \$41.19 discounted loss reserve accrues to pay the expected future losses, then an additional \$20.60 (50% of the discounted outstanding loss amount) should be sufficient to make the unexpected payments if they become due.

Differences between the expected and actual timing of loss payments have no impact upon the nominal loss reserves that should be carried but do affect the amount of discounted loss reserve that should be carried at any point in time. It is logical to cushion against the timing uncertainty that increases the

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[7] It has been assumed that the original 2 : 1 leverage ratio does not reflect any implicit discounting for interest.

variance in the discounted loss reserve by adopting the discounted reserve as the surplus allocation base.

5. REFINEMENTS TO THE BINGHAM METHODOLOGY II (A DECREASING LEVERAGE RATIO)

Bingham assumes that a constant reserve-to-surplus leverage ratio results when supporting surplus is established to maintain a constant probability of ruin. While this assumption is consistent with the other simplifications that he adopted for illustrative purposes, it must be emphasized that it is neither required to achieve an invariant ratio, nor is it realistic. Many models that allocate surplus over the life of a product assume that a constant leverage ratio is appropriate. Some models even allow the leverage ratio to increase over time. For many circumstances, the leverage ratio must decrease over the long run if a constant probability of ruin is to be maintained. This is not to say that a short term increase in the ratio of reserves to surplus is impossible, but that such a short term increase will be followed by a long term decrease as the runoff becomes increasingly more volatile.

For illustrative purposes, consider the hypothetical case of excess of loss casualty reinsurance with a very high attachment point. Because of the high attachment point, assume that small claims will be eliminated. Assume, further, that those claims that remain can be modeled by one of the more common distributions (e.g., the lognormal or Pareto distribution). A suitably high attachment point assures us that all of the possible claims will fall in the relatively flat tail of the severity distribution. This means that the likelihood of any particular claim size is almost equal to that of any other size claim. If each claim closes with a single payment and this payment does not depend upon how long the claim remained open before being settled, then the ultimate closing amount on each open claim can be represented by a stochastic variable where the same underlying distribution applies to all of the open claims.

When there are exactly N independent open claims,[8] the best estimate of the outstanding loss is

$$\text{Nominal outstanding loss reserve} = Ns,$$

where s is the mean severity from the single claim severity distribution.[9] Likewise, the variance of the possible loss outcomes for the group of N claims is given by

$$\begin{array}{l}\text{Variance of the aggregate ultimate}\\ \qquad\text{loss around the expected} = N\sigma^2,\end{array}$$

where σ is the standard deviation of the single claim severity distribution. If N is sufficiently large, the aggregate loss distribution will be approximately normal. The ultimate loss outcome will be less than

$$\text{98th percentile ultimate loss} = Ns + 2.06\sqrt{N}\sigma$$

98% of the time. If, for every Ns of expected loss, $2.06\sqrt{N}\sigma$ of supporting surplus is allocated, then the probability of ruin can be maintained at 2%. Here ruin means that more funds are required than are available. If only a single contract is being considered, ruin may be less catastrophic than company insolvency. The corresponding leverage ratio to cushion against this single contract ruin is given by

$$Ns : 2.06\sqrt{N}\sigma$$

or

$$\sqrt{N}s/2.06\sigma : 1.$$

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[8] Here, N may reflect not only the known open claim count but also an estimate of the IBNR claim count as well.

[9] The claim severity distribution is that which describes losses in the layer of reinsurance. For excess of loss reinsurance, this would not be the same distribution as the ground up severity distribution.

TABLE 4

Year End	Paid to Date Loss	Nominal O/S Loss	Discounted O/S Loss	Leverage Ratio	Supporting Surplus
0	$0.00	$404.00	$375.86	2.00 : 1.00	$187.93
1	264.00	140.00	130.66	1.18 : 1.00	110.97
2	360.00	44.00	41.19	0.66 : 1.00	62.40
3	392.00	12.00	11.25	0.34 : 1.00	32.63
4	400.00	4.00	3.81	0.20 : 1.00	19.14
5	404.00	0.00	0.00	N/A	0.00

As claims close, N, the number of open claims, decreases. As shown above, the leverage decreases in proportion to the square root of N.

If there are insufficient open claims to warrant the normal approximation, then the 98th percentile would have to be determined by means of some other aggregate loss modeling technique. The important point is that as the number of open claims decreases, the *relative* uncertainty increases as a function of the expected loss amount. In other words, the *absolute* amount of surplus may decrease, but the *relative* amount increases.

If claims are closed with a single payment and the same severity distribution can represent each claim, then the percentage of ultimate loss that is paid at any point in time is a measure of the number of claims that have been paid. Assuming that a 2 : 1 reserve-to-surplus ratio is appropriate at time zero, when none of the claims are closed, then the appropriate reserve-to-surplus ratio would become $2\sqrt{.75}$: 1 when 25% of the claims have closed. Returning to the original example with a five year runoff, and introducing both the modified leverage ratio and the discounted outstanding loss reserve as the base, the supporting surplus amounts shown in Table 4 are required at each year-end.

The *initial* supporting surplus, $187.93, is less than the $202.00 of supporting surplus for the unmodified case. This quickly changes as the leverage ratio decreases (i.e., more surplus is required to support a dollar of loss reserve as the number of open claims decreases and the proportionate volatility increases) and the offsetting loss discount unwinds.

Exhibit 7 displays the correspondingly modified Bingham model. Notice that the average return on surplus and internal rate of return remain equal to the Bingham invariant ratio after the modification. Because the modification involves changing the amount of supporting surplus, the invariant ratio is not equal to the corresponding invariant ratio displayed on the other exhibits. Such agreement would not be expected.

Exhibits 8A and 8B apply the modifications to the second example. While the invariant ratio is numerically equal to the internal rate of return from the perspective of the insurance carrier, this is simply a coincidence produced by rounding errors. Table 5 provides a comparison of the three models under the modified surplus determination.

6. OTHER ISSUES

There are a number of issues that fall outside the scope of this discussion paper. They are briefly mentioned in the hope that they may encourage further discussion.

1. How can the other sources of insurance product uncertainty be reflected?
2. How can the appropriate leverage ratios for a selected probability of ruin be determined empirically?
3. As presented, the model produces a *point estimate* of the return on equity. Expected loss amounts and payout timing are all that have been reflected in the determination of the return on equity. If $\{L_t\}$ represents the actual loss payments at times $\{t\}$, then the return on equity that has been determined is $\mathrm{ROE}(\{\langle L_t \rangle\})$ rather than

TABLE 5

RESULTS UNDER MODIFIED SURPLUS DETERMINATION

Premium	Operating Gain	Loss Ratio	Statutory Measure	Invariant Measure	Insurance Carrier IRR Measure
$500	$ −461.38	400.0%	5.0%	−2.1%	0.4%
600	−361.38	333.3%	6.0%	−0.6%	1.1%
700	−261.38	285.7%	7.0%	1.0%	1.9%
800	−161.38	250.0%	8.0%	2.5%	2.9%
900	−61.38	222.2%	9.1%	4.1%	4.1%
961	0.00	208.0%	9.7%	5.0%	5.0%
1,000	38.62	200.0%	10.1%	5.6%	5.6%
1,100	138.62	181.8%	11.1%	7.1%	7.6%
1,200	238.62	166.7%	12.1%	8.7%	10.6%
1,300	338.62	153.8%	13.1%	10.2%	15.8%
1,400	438.62	142.9%	14.1%	11.7%	32.6%
1,410	448.62	141.8%	14.2%	11.9%	37.7%
1,420	458.62	140.8%	14.3%	12.0%	45.6%
1,430	468.62	139.9%	14.4%	12.2%	60.7%
1,440	478.62	138.9%	14.5%	12.4%	123.9%
1,442	480.62	138.7%	14.5%	12.4%	381.8%

$\langle \text{ROE}(\{L_t\}) \rangle$, where $\langle \cdots \rangle$ denotes taking the expected value of the quantity that is enclosed. If the model is not a linear function of the $\{L_t\}$ then the two averages need not be equal. There are many possible sets of loss payments that *may* be made. Out of this population, only one set of payments *will* occur. Prior to their occurrence, the best estimate of what *will* occur is $\{\langle L_t \rangle\}$. Each of the possible $\{L_t\}$ will result in a different return on supporting surplus. There is no guarantee that the expected return is equal to the return corresponding to the expected loss payments. Even for our simple example, whether or not the two averages are equal depends upon how the next issue is resolved.

4. At a particular point in time there is an expected outstanding loss reserve. A corresponding amount of sur-

plus will be allocated in such a way that the probability of ruin is less than some predetermined amount. The estimate of future payments will, undoubtedly, change over time. After several years have elapsed and the first few years of actual payments have been made, as details concerning the actual open claims become known, and as IBNR emerges, expectations regarding payments yet to be made will probably not be the same as they were in the beginning. The question is whether or not these changed expectations of future loss payments should result in a modification of the supporting surplus during future periods.

In the first example, the a priori expected reserve at the end of year two is $44.00. Based upon this expectation, $22.00 of supporting surplus is considered to be an adequate cushion against ruin. Together, there will be enough funds available to cover $66.00 of future loss payments. But $44.00 is the a priori (at time zero) expected loss to be paid after the end of year two. What if the best estimate of the future loss payout is $60.00 when the end of year two actually arrives? Certainly, the reserve would be changed to reflect this additional information. Should the cushion at year-end two and subsequent periods be adjusted accordingly?

There appear to be three alternative ways in which to determine the required supporting surplus for future periods under this scenario.

- Assume that $16.00 of the $22.00 safety margin has been used to establish the originally unanticipated additional outstanding loss reserve. The remaining $6.00 of surplus continues to provide an adequate safety net. This approach assumes that the a priori outstanding amount defines the distribution of possible outcomes, and that the safety margin is always measured against the a priori expectation. Regardless of what the actual estimate is, the supporting surplus cushions against the a priori estimate of the worst case scenario. Under this alternative, all differences between the expected values and the actual values are attributed to process variance. There is no cushion provided for parameter errors contained in the a priori expectations.

In a sense, this method is analogous to a loss ratio reserving methodology in which IBNR reserves are established equal to the difference between the a priori loss ratio and the reported loss ratio. Only if the difference becomes negative (i.e., reported amounts exceed expected amounts) is the a priori assumption questioned. A negative difference means that ruin has occurred.

- Assume that the a priori outstanding loss amount defines the size of the uncertainty, \$22.00. Even when year two ends and the outstanding loss estimate (and it is still just an estimate as of year-end two) is \$60.00 rather than the expected \$44.00, \$22.00 of surplus provides the necessary safety margin.

  This alternative is analogous to the Bornhuetter/Ferguson loss reserving methodology. Future development (and uncertainty) depends upon an a priori assumption which is not modified to reflect current information.

- Assume that the \$60.00 estimate contains the same percentage of uncertainty as did the \$44.00 a priori estimate. In this case, the supporting surplus must be increased from \$22.00 to \$30.00. Intuitively, this approach is less than satisfying because it appears to imply that the claim department's opinion at the end of year two not only does nothing to decrease the uncertainty over the a priori estimate that was available at the beginning of year zero but actually increases the dollar amount of uncertainty. This alternative assumes that the a priori estimate was based upon so much parameter error as to be worthless once additional information becomes available.

This alternative is analogous to the chain ladder reserving methodology which is 100% responsive to the current information.

A resolution of how to deal with *actual estimates* vs. *a priori expectations* will be necessary in order to determine whether or not the point estimate, $\mathrm{ROE}(\{\langle L_t \rangle\})$, will be equal to the ensemble average (i.e., $\mathrm{ROE}(\{L_t\})$ run for each of the $\{L_t\}$ and then weighted by the probability of occurrence), $\langle \mathrm{ROE}(\{L_t\}) \rangle$. If the two estimates are not equal, then even a prospective evaluation of the rate of return must be performed on an ensemble of possible insurance product outcomes rather than a single expected value outcome.

For our simple example, the second alternative results in a linear model whereas the other two do not. This can easily be demonstrated by running several possible loss outcomes through the Bingham model. The expected loss for the example is \$404.00. Without changing the payout pattern (i.e., the percentage of ultimate loss paid at any particular point in time), consider Table 6, the possible loss outcomes and their corresponding probabilities of occurrence.

Note that Alternatives 1 and 3 produce deviations from the point estimate that are in opposite directions. The more volatile the loss distribution (the larger the variance), the more pronounced the deviation between the ensemble and point estimates will be for non-linear models.

While linearity makes the calculations easier, computational difficulty should not be the only criterion that is used in the selection of an alternative.

TABLE 6

OUTCOMES AND THEIR PROBABILITIES

Probability of Occurrence	$\sum_{t=0}^{5} L_t$	Alternative 1	Alternative 2	Alternative 3
0.35	\$380.00	19.76%	21.51%	22.56%
0.14	392.00	16.84%	17.55%	17.93%
0.02	404.00	13.58%	13.58%	13.58%
0.14	416.00	9.90%	9.61%	9.48%
0.35	428.00	5.73%	5.64%	5.61%
Ensemble Average	404.00	12.94%	13.58%	13.97%
Point Average	404.00	13.58%	13.58%	13.58%

5. Closely related to the ensemble vs. point estimate discussion is the appropriate allocation of surplus when a policy year is analyzed *retrospectively* to determine the actual return on surplus. Since the a priori expectations are rarely realized, how much supporting surplus should be reflected? When actual results deviate from expected results, actual outstanding loss reserves will deviate from those that were expected. At what point in the retrospective determination of the return on surplus should the actual reserves be reflected? Should it reflect *carried* reserves or what *should have been carried* at any point in time?

7. SUMMARY AND CONCLUSIONS

Russell Bingham made a significant contribution to the literature concerning the allocation of surplus and determination of the rate of return on that surplus. His advocacy of keeping the results of each exposure period separate so that the long-term commitment of surplus can be appropriately reflected is right on target.

In the process of taking the present values of the insurance flows and supporting surplus, Bingham has produced an invariant measure of the return on surplus.

The difference between the commonly used *calendar* year determination of the return on surplus and Bingham's accident year approach can be illustrated by the following two descriptions of the same investment:

- Calendar Year Approach: A carrier invests $1,000 of surplus and receives a $400 return, so the return on surplus is 40%;
- Accident Year Approach: A carrier invests $1,000 of surplus *for 10 years* and receives an *average annual return* equal to 3.4% on its investment.

The second approach takes into account the time over which the surplus funds are invested (until all of the uncertainties are resolved). This time horizon is well beyond the time that premiums are in force for most insurance products.

If a given probability of ruin is to be maintained by cushioning funds, then there must be some recognition of the cushion afforded by the premium provision for expected operating profit. Otherwise, the probability of ruin will vary with premium in a manner that is difficult to rationalize. This would appear to imply that ruin occurs whenever the expected operating profit is not achieved rather than whenever there are insufficient funds to meet the unexpected losses and expenses. The latter definition seems to be a more logical way to define ruin. This is an area that warrants further investigation.

Two modifications that can be made to enhance Bingham's model have been proposed. In actual practice, the leverage ratio will vary, but not in such a simple manner as suggested by the square root rule. A more detailed investigation of the characteristics of a particular line must be undertaken in order to establish actual leverage ratios for the runoff of a maturing policy year.

While not exhaustive, a list of additional considerations provides issues that must be addressed. In particular, the idea of averaging the returns over an ensemble of possible loss outcomes forces us to refine our ideas concerning the role of surplus as it cushions against ruin.

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EXHIBIT 1

A NOT COMPLETELY BINGHAM MODEL

	INSURANCE PRODUCT				SURPLUS ACCOUNT					IRR	INSURANCE PRODUCT		
End of Year	Written Premium	Paid Loss	Operating Gain	$O(n) \Rightarrow$ Funds Released	Supporting Surplus S	Funds Released to Surplus	Idle Surplus Investment Income	Idle Surplus Investment Income	Total Return on Supporting Surplus	Total Surplus Flows	Retained Earnings	Investment Balance	Overfund
0	$400.00	$0.00								$-202.00	$-4.00	$400.00	$24.14
1		264.00		$25.34	$202.00	12.55%	$10.10	5.0%	17.55%	167.44	-9.34	130.66	0.00
2		96.00		0.00	70.00	0.00%	3.50	5.0%	5.00%	51.50	-2.81	41.19	0.00
3		32.00		0.00	22.00	0.00%	1.10	5.0%	5.00%	17.10	-0.75	11.25	0.00
4		8.00		0.00	6.00	0.00%	0.30	5.0%	5.00%	4.30	-0.19	3.81	0.00
5		4.00		0.00	2.00	0.00%	0.10	5.0%	5.00%	2.10	0.00	0.00	0.00
NPV	400.00	375.86	$24.14	24.14	281.38	8.58%	14.07	5.0%	13.58%				
									Average = 13.39%	IRR = 13.87%			

EXHIBIT 2

ANOTHER NOT COMPLETELY BINGHAM MODEL

	INSURANCE PRODUCT				SURPLUS ACCOUNT					IRR	INSURANCE PRODUCT		
End of Year	Written Premium	Paid Loss	Operating Gain	$O(n) \Rightarrow$ Funds Released	Supporting Surplus S	Funds Released to Surplus	Idle Surplus Investment Income	Idle Surplus Investment Income	Total Return on Supporting Surplus	Total Surplus Flows	Retained Earnings	Investment Balance	Overfund
0	$400.00	$0.00								$-202.00	$-4.00	$400.00	$24.14
1		264.00		$0.00	$202.00	0.00%	$10.10	5.0%	5.00%	142.10	16.00	156.00	25.34
2		96.00		0.00	70.00	0.00%	3.50	5.0%	5.00%	51.50	23.80	67.80	26.61
3		32.00		0.00	22.00	0.00%	1.10	5.0%	5.00%	17.10	27.19	39.19	27.94
4		8.00		0.00	6.00	0.00%	0.30	5.0%	5.00%	4.30	29.15	33.15	29.34
5		4.00		30.81	2.00	1,540.35%	0.10	5.0%	1,545.35%	32.91	0.00	0.00	0.00
NPV	400.00	375.86	$24.14	24.14	281.38	8.58%	14.07	5.0%	13.58%				
									Average = 15.20%	IRR = 11.84%			

EXHIBIT 3

THE BINGHAM MODEL

	INSURANCE PRODUCT				SURPLUS ACCOUNT					IRR	INSURANCE PRODUCT		
End of Year	Written Premium	Paid Loss	Operating Gain	$O(n) \Rightarrow$ Funds Released	Supporting Surplus S	Funds Released to Surplus	Idle Surplus Investment Income	Idle Surplus Investment Income	Total Return on Supporting Surplus	Total Surplus Flows	Retained Earnings	Investment Balance	Overfund
0	$400.00	$0.00								$-202.00	$-4.00	$400.00	$24.14
1		264.00		$17.33	$202.00	8.58%	$10.10	5.0%	13.58%	159.43	−1.33	138.67	8.02
2		96.00		6.00	70.00	8.58%	3.50	5.0%	13.58%	57.50	−0.40	43.60	2.41
3		32.00		1.89	22.00	8.58%	1.10	5.0%	13.58%	18.99	−0.11	11.89	0.65
4		8.00		0.51	6.00	8.58%	0.30	5.0%	13.58%	4.81	−0.03	3.97	0.16
5		4.00		0.17	2.00	8.58%	0.10	5.0%	13.58%	2.27	0.00	0.00	0.00
NPV	400.00	375.86	$24.14	24.14	281.38	8.58%	14.07	5.0%	13.58%				
								Average =	13.58%	IRR = 13.58%			

Determination of the $\{O(n)\}$

1. Constant annual ROE $\Rightarrow O(n)/S(n) = k$, or $O(n) = k * S(n)$
2. $\text{NPV}(\{O(n)\}) = k * \text{NPV}(\{S(n)\}) \Rightarrow k = \text{NPV}(\{O(n)\})/\text{NPV}(\{S(n)\})$

n	$S(n)$		$O(n)$
1	$S(1) =$	$202.00	$17.33
2	$S(2) =$	70.00	6.00
3	$S(3) =$	22.00	1.89
4	$S(4) =$	6.00	0.51
5	$S(5) =$	2.00	0.17
	$\text{NPV}(\{S(n)\}) =$	$281.38	
	$\text{NPV}(\{O(n)\}) =$	$ 24.14	$24.14
	$k =$	0.085784	

EXHIBIT 4A

STATUTORY ACCOUNTING MODEL—ZERO OPERATING GAIN

	INSURANCE PRODUCT				SURPLUS ACCOUNT							ROE Under Statutory Accounting With a Growth Rate equal to		
End of Year	Written Premium	Paid Loss	Operating Gain	$O(n) \Rightarrow$ Funds Released	Supporting Surplus S	Funds Released to Surplus	Idle Surplus Investment Income	Idle Surplus Investment Income	Total Return on Supporting Surplus	Insurance Investment Balance	Year	0.0%	5.0%	10.0%
0	$961.38	$0.00								$961				
1		0.00		$ − 990.55	$1,000.00	−99.1%	$50.00	5.0%	−94.1%	2,000	1	−94.1%	−94.1%	−94.1%
2		0.00		100.00	1,000.00	10.0%	50.00	5.0%	15.0%	2,000	2	−39.5%	−40.9%	−42.1%
3		6.00		100.00	1,000.00	10.0%	50.00	5.0%	15.0%	1,994	3	−21.4%	−23.1%	−24.9%
4		28.00		99.70	997.00	10.0%	49.85	5.0%	15.0%	1,966	4	−12.3%	−14.3%	−16.3%
5		86.00		98.30	983.00	10.0%	49.15	5.0%	15.0%	1,880	5	−6.9%	−9.1%	−11.2%
6		64.00		94.00	940.00	10.0%	47.00	5.0%	15.0%	1,816	6	−3.4%	−5.7%	−8.0%
7		74.00		90.80	908.00	10.0%	45.40	5.0%	15.0%	1,742	7	−1.0%	−3.3%	−5.8%
8		74.00		87.10	871.00	10.0%	43.55	5.0%	15.0%	1,668	8	0.8%	−1.6%	−4.1%
9		72.00		83.40	834.00	10.0%	41.70	5.0%	15.0%	1,596	9	2.2%	−0.3%	−2.9%
10		70.00		79.80	798.00	10.0%	39.90	5.0%	15.0%	1,526	10	3.3%	0.7%	−2.0%
11		52.00		76.30	763.00	10.0%	38.15	5.0%	15.0%	1,474	11	4.2%	1.6%	−1.2%
12		48.00		73.70	737.00	10.0%	36.85	5.0%	15.0%	1,426	12	4.9%	2.2%	−0.6%
13		44.00		71.30	713.00	10.0%	35.65	5.0%	15.0%	1,382	13	5.6%	2.8%	−0.1%
14		42.00		69.10	691.00	10.0%	34.55	5.0%	15.0%	1,340	14	6.1%	3.3%	0.3%
15		36.00		67.00	670.00	10.0%	33.50	5.0%	15.0%	1,304	15	6.5%	3.7%	0.6%
16		34.00		65.20	652.00	10.0%	32.60	5.0%	15.0%	1,270	16	7.0%	4.0%	0.9%
17		30.00		63.50	635.00	10.0%	31.75	5.0%	15.0%	1,240	17	7.3%	4.4%	1.2%
18		28.00		62.00	620.00	10.0%	31.00	5.0%	15.0%	1,212	18	7.6%	4.6%	1.4%
19		524.00		60.60	606.00	10.0%	30.30	5.0%	15.0%	688	19	7.9%	4.9%	1.6%
20		688.00		34.40	344.00	10.0%	17.20	5.0%	15.0%	0	20	8.1%	5.0%	1.6%
											21	8.1%	5.0%	1.6%
Total	961.38	2,000.00		485.65	15,762.00	3.1%	788.10	5.0%	8.1%		etc.	8.1%	5.0%	1.6%
PV	961.38	961.38	$0.00	0.00	10,386.00	0.0%	519.31	5.0%	5.0%					

EXHIBIT 4B

ZERO OPERATING GAIN FROM THE INSURANCE CARRIER PERSPECTIVE

				Funds Released as they Become Available			
End of Year	Invested Surplus	Invested Premium	Paid Loss	Required Funds	Invested Funds	Released to Surplus	Flows for IRR
0	$1,000.00	$961.38	$0.00	$1,961.38	$1,961.38	$0.00	$ – 1,000.00
1			0.00	2,009.45	2,059.45	50.00	50.00
2			0.00	2,059.92	2,109.92	50.00	50.00
3			6.00	2,103.92	2,156.92	53.00	53.00
4			28.00	2,117.26	2,181.12	63.86	63.86
5			86.00	2,044.98	2,137.12	92.14	92.14
6			64.00	2,004.23	2,083.23	79.00	79.00
7			74.00	1,948.04	2,030.44	82.40	82.40
8			74.00	1,890.89	1,971.44	80.55	80.55
9			72.00	1,835.73	1,913.43	77.70	77.70
10			70.00	1,782.62	1,857.52	74.90	74.90
11			52.00	1,755.60	1,819.75	64.15	64.15
12			48.00	1,734.53	1,795.38	60.85	60.85
13			44.00	1,719.61	1,777.26	57.65	57.65
14			42.00	1,708.04	1,763.59	55.55	55.55
15			36.00	1,705.94	1,757.44	51.50	51.50
16			34.00	1,707.64	1,757.24	49.60	49.60
17			30.00	1,716.27	1,763.02	46.75	46.75
18			28.00	1,729.08	1,774.08	45.00	45.00
19			524.00	999.24	1,291.53	292.29	292.29
20			688.00	0.00	361.20	361.20	361.20
							IRR = 5.0%

EXHIBIT 5A

STATUTORY ACCOUNTING MODEL—NEGATIVE OPERATING GAIN (I.E., A LOSS)

	INSURANCE PRODUCT				SURPLUS ACCOUNT							ROE Under Statutory Accounting With a Growth Rate equal to		
End of Year	Written Premium	Paid Loss	Operating Gain	$O(n) \Rightarrow$ Funds Released	Supporting Surplus S	Funds Released to Surplus	Idle Surplus Investment Income	Idle Surplus Investment Income	Total Return on Supporting Surplus	Insurance Investment Balance	Year	0.0%	5.0%	10.0%
0	$600.00	$0.00								$600				
1		0.00		$ – 1,370.00	$1,000.00	–137.0%	$50.00	5.0%	–132.0%	2,000	1	–132.0%	–132.0%	–132.0%
2		0.00		100.00	1,000.00	10.0%	50.00	5.0%	15.0%	2,000	2	–58.5%	–60.3%	–62.0%
3		6.00		100.00	1,000.00	10.0%	50.00	5.0%	15.0%	1,994	3	–34.0%	–36.4%	–38.7%
4		28.00		99.70	997.00	10.0%	49.85	5.0%	15.0%	1,966	4	–21.8%	–24.5%	–27.2%
5		86.00		98.30	983.00	10.0%	49.15	5.0%	15.0%	1,880	5	–14.5%	–17.5%	–20.4%
6		64.00		94.00	940.00	10.0%	47.00	5.0%	15.0%	1,816	6	–9.8%	–12.9%	–16.0%
7		74.00		90.80	908.00	10.0%	45.40	5.0%	15.0%	1,742	7	–6.5%	–9.7%	–13.0%
8		74.00		87.10	871.00	10.0%	43.55	5.0%	15.0%	1,668	8	–4.1%	–7.4%	–10.8%
9		72.00		83.40	834.00	10.0%	41.70	5.0%	15.0%	1,596	9	–2.2%	–5.6%	–9.1%
10		70.00		79.80	798.00	10.0%	39.90	5.0%	15.0%	1,526	10	–0.8%	–4.2%	–7.9%
11		52.00		76.30	763.00	10.0%	38.15	5.0%	15.0%	1,474	11	0.4%	–3.1%	–6.9%
12		48.00		73.70	737.00	10.0%	36.85	5.0%	15.0%	1,426	12	1.4%	–2.2%	–6.1%
13		44.00		71.30	713.00	10.0%	35.65	5.0%	15.0%	1,382	13	2.3%	–1.4%	–5.4%
14		42.00		69.10	691.00	10.0%	34.55	5.0%	15.0%	1,340	14	3.0%	–0.8%	–4.8%
15		36.00		67.00	670.00	10.0%	33.50	5.0%	15.0%	1,304	15	3.6%	–0.2%	–4.4%
16		34.00		65.20	652.00	10.0%	32.60	5.0%	15.0%	1,270	16	4.2%	0.2%	–4.0%
17		30.00		63.50	635.00	10.0%	31.75	5.0%	15.0%	1,240	17	4.6%	0.7%	–3.7%
18		28.00		62.00	620.00	10.0%	31.00	5.0%	15.0%	1,212	18	5.1%	1.0%	–3.4%
19		524.00		60.60	606.00	10.0%	30.30	5.0%	15.0%	688	19	5.5%	1.4%	–3.1%
20		688.00		34.40	344.00	10.0%	17.20	5.0%	15.0%	0	20	5.7%	1.5%	–3.0%
											21	5.7%	1.5%	–3.0%
Total	600.00	2,000.00		106.20	15,762.00	0.0	788.10	5.0%	5.7%		etc.	5.7%	1.5%	–3.0%
PV	600.00	961.38	$ – 361.38	– 361.38	10,386.19	–3.5%	519.31	5.0%	1.5%					

EXHIBIT 5B

NEGATIVE OPERATING GAIN FROM THE INSURANCE CARRIER PERSPECTIVE

				Funds Released as they Become Available			
End of Year	Invested Surplus	Invested Premium	Paid Loss	Required Funds	Invested Funds	Released to Surplus	Flows for IRR
0	$1,361.38	$600.00	$0.00	$1,961.38	$1,961.38	$0.00	$ – 1,361.38
1			0.00	2,009.45	2,059.45	50.00	50.00
2			0.00	2,059.92	2,109.92	50.00	50.00
3			6.00	2,103.92	2,156.92	53.00	53.00
4			28.00	2,117.26	2,181.12	63.86	63.86
5			86.00	2,044.98	2,137.12	92.14	92.14
6			64.00	2,004.23	2,083.23	79.00	79.00
7			74.00	1,948.04	2,030.44	82.40	82.40
8			74.00	1,890.89	1,971.44	80.55	80.55
9			72.00	1,835.73	1,913.43	77.70	77.70
10			70.00	1,782.62	1,857.52	74.90	74.90
11			52.00	1,755.60	1,819.75	64.15	64.15
12			48.00	1,734.53	1,795.38	60.85	60.85
13			44.00	1,719.61	1,777.26	57.65	57.65
14			42.00	1,708.04	1,763.59	55.55	55.55
15			36.00	1,705.94	1,757.44	51.50	51.50
16			34.00	1,707.64	1,757.24	49.60	49.60
17			30.00	1,716.27	1,763.02	46.75	46.75
18			28.00	1,729.08	1,774.08	45.00	45.00
19			524.00	999.24	1,291.53	292.29	292.29
20			688.00	0.00	361.20	361.20	361.20
							IRR = 2.2%

EXHIBIT 6A

STATUTORY ACCOUNTING MODEL—POSITIVE OPERATING GAIN

	INSURANCE PRODUCT			SURPLUS ACCOUNT								ROE Under Statutory Accounting With a Growth Rate equal to		
End of Year	Written Premium	Paid Loss	Operating Gain	$O(n) \Rightarrow$ Funds Released	Supporting Surplus S	Funds Released to Surplus	Idle Surplus Investment Income	Idle Surplus Investment Income	Total Return on Supporting Surplus	Insurance Investment Balance	Year	0.0%	5.0%	10.0%
0	$1,700.00	$0.00								$1,700				
1		0.00		$ − 215.00	$1,000.00	−21.5%	$50.00	5.0%	−16.5%	2,000	1	−16.5%	−16.5%	−16.5%
2		0.00		100.00	1,000.00	10.0%	50.00	5.0%	15.0%	2,000	2	−0.8%	−1.1%	−1.5%
3		6.00		100.00	1,000.00	10.0%	50.00	5.0%	15.0%	1,994	3	4.5%	4.0%	3.5%
4		28.00		99.70	997.00	10.0%	49.85	5.0%	15.0%	1,966	4	7.1%	6.5%	6.0%
5		86.00		98.30	983.00	10.0%	49.15	5.0%	15.0%	1,880	5	8.7%	8.0%	7.4%
6		64.00		94.00	940.00	10.0%	47.00	5.0%	15.0%	1,816	6	9.7%	9.0%	8.4%
7		74.00		90.80	908.00	10.0%	45.40	5.0%	15.0%	1,742	7	10.4%	9.7%	9.0%
8		74.00		87.10	871.00	10.0%	43.55	5.0%	15.0%	1,668	8	10.9%	10.2%	9.5%
9		72.00		83.40	834.00	10.0%	41.70	5.0%	15.0%	1,596	9	11.3%	10.6%	9.8%
10		70.00		79.80	798.00	10.0%	39.90	5.0%	15.0%	1,526	10	11.6%	10.9%	10.1%
11		52.00		76.30	763.00	10.0%	38.15	5.0%	15.0%	1,474	11	11.9%	11.1%	10.3%
12		48.00		73.70	737.00	10.0%	36.85	5.0%	15.0%	1,426	12	12.1%	11.3%	10.5%
13		44.00		71.30	713.00	10.0%	35.65	5.0%	15.0%	1,382	13	12.3%	11.5%	10.6%
14		42.00		69.10	691.00	10.0%	34.55	5.0%	15.0%	1,340	14	12.4%	11.6%	10.7%
15		36.00		67.00	670.00	10.0%	33.50	5.0%	15.0%	1,304	15	12.6%	11.7%	10.8%
16		34.00		65.20	652.00	10.0%	32.60	5.0%	15.0%	1,270	16	12.7%	11.8%	10.9%
17		30.00		63.50	635.00	10.0%	31.75	5.0%	15.0%	1,240	17	12.8%	11.9%	11.0%
18		28.00		62.00	620.00	10.0%	31.00	5.0%	15.0%	1,212	18	12.9%	12.0%	11.1%
19		524.00		60.60	606.00	10.0%	30.30	5.0%	15.0%	688	19	13.0%	12.1%	11.1%
20		688.00		34.40	344.00	10.0%	17.20	5.0%	15.0%	0	20	13.0%	12.1%	11.1%
											21	13.0%	12.1%	11.1%
Total	1,700.00	2,000.00		1,261.20	15,762.00	8.0%	788.10	5.0%	13.0%		etc.	13.0%	12.1%	11.1%
PV	1,700.00	961.38	$738.62	738.62	10,386.19	7.1%	519.31	5.0%	12.1%					

EXHIBIT 6B

POSITIVE OPERATING GAIN FROM THE INSURANCE CARRIER PERSPECTIVE

				Funds Released as they Become Available			
End of Year	Invested Surplus	Invested Premium	Paid Loss	Required Funds	Invested Funds	Released to Surplus	Flows for IRR
0	$261.38	$1,700.00	$0.00	$1,961.38	$1,961.38	$0.00	$ – 261.38
1			0.00	2,009.45	2,059.45	50.00	50.00
2			0.00	2,059.92	2,109.92	50.00	50.00
3			6.00	2,103.92	2,156.92	53.00	53.00
4			28.00	2,117.26	2,181.12	63.86	63.86
5			86.00	2,044.98	2,137.12	92.14	92.14
6			64.00	2,004.23	2,083.23	79.00	79.00
7			74.00	1,948.04	2,030.44	82.40	82.40
8			74.00	1,890.89	1,971.44	80.55	80.55
9			72.00	1,835.73	1,913.43	77.70	77.70
10			70.00	1,782.62	1,857.52	74.90	74.90
11			52.00	1,755.60	1,819.75	64.15	64.15
12			48.00	1,734.53	1,795.38	60.85	60.85
13			44.00	1,719.61	1,777.26	57.65	57.65
14			42.00	1,708.04	1,763.59	55.55	55.55
15			36.00	1,705.94	1,757.44	51.50	51.50
16			34.00	1,707.64	1,757.24	49.60	49.60
17			30.00	1,716.27	1,763.02	46.75	46.75
18			28.00	1,729.08	1,774.08	45.00	45.00
19			524.00	999.24	1,291.53	292.29	292.29
20			688.00	0.00	361.20	361.20	361.20
							IRR = 24.4%

EXHIBIT 7

The Modified Bingham Model

	INSURANCE PRODUCT				SURPLUS ACCOUNT					IRR	INSURANCE PRODUCT		
End of Year	Written Premium	Paid Loss	Operating Gain	$O(n) \Rightarrow$ Funds Released	Supporting Surplus S	Funds Released to Surplus	Idle Surplus Investment Income	Idle Surplus Investment Income	Total Return on Supporting Surplus	Total Surplus Flows	Retained Earnings	Investment Balance	Overfund
0	$400.00	$0.00								$−187.93	$−4.00	$400.00	$24.14
1		264.00		$12.08	$187.93	6.43%	$9.40	5.0%	11.43%	98.44	3.92	143.92	13.26
2		96.00		7.14	110.97	6.43%	5.55	5.0%	11.43%	61.25	3.98	47.98	6.79
3		32.00		4.01	62.40	6.43%	3.12	5.0%	11.43%	36.90	2.36	14.36	3.11
4		8.00		2.10	32.63	6.43%	1.63	5.0%	11.43%	17.22	0.98	4.98	1.17
5		4.00		1.23	19.14	6.43%	0.96	5.0%	11.43%	21.33	0.00	0.00	0.00
NPV	400.00	375.86	$24.14	24.14	375.38	6.43%	18.77	5.0%	11.43%				
								Average = 11.43%		IRR = 11.43%			

Determination of the $\{O(n)\}$

1. Constant annual ROE $\Rightarrow O(n)/S(n) = k$, or $O(n) = k * S(n)$
2. $\text{NPV}(\{O(n)\}) = k * \text{NPV}(\{S(n)\}) \Rightarrow k = \text{NPV}(\{O(n)\})/\text{NPV}(\{S(n)\})$

n	$S(n)$		$O(n)$
1	$S(1) =$	$187.93	$12.08
2	$S(2) =$	110.97	7.14
3	$S(3) =$	62.40	4.01
4	$S(4) =$	32.63	2.10
5	$S(5) =$	19.14	1.23
	$\text{NPV}(\{S(n)\}) =$	$375.38	
	$\text{NPV}(\{O(n)\}) =$	$ 24.14	$24.14
	$k =$	0.064303	

EXHIBIT 8A

Statutory Accounting Model with Variable R : S Leverage Ratio

	INSURANCE PRODUCT				SURPLUS ACCOUNT							ROE Under Statutory Accounting With a Growth Rate equal to		
End of Year	Written Premium	Paid Loss	Operating Gain	$O(n) \Rightarrow$ Funds Released	Supporting Surplus S	Funds Released to Surplus	Idle Surplus Investment Income	Idle Surplus Investment Income	Total Return on Supporting Surplus	Insurance Investment Balance	Year	0.0%	5.0%	10.0%
0	$1,000.00	$0.00								$1,000				
1		0.00		$ − 950.00	$480.69	−2.0%	$24.03	0.0%	−1.9%	2,000	1	−1.9%	−1.9%	−1.9%
2		0.00		100.00	504.73	0.2%	25.24	0.1%	0.2%	2,000	2	−0.8%	−0.8%	−0.9%
3		6.00		100.00	529.96	0.2%	26.50	0.1%	0.2%	1,994	3	−0.4%	−0.5%	−0.5%
4		28.00		99.70	553.46	0.2%	27.67	0.0%	0.2%	1,966	4	−0.3%	−0.3%	−0.3%
5		86.00		98.30	567.13	0.2%	28.36	0.1%	0.2%	1,880	5	−0.2%	−0.2%	−0.2%
6		64.00		94.00	552.49	0.2%	27.62	0.0%	0.2%	1,816	6	−0.1%	−0.1%	−0.2%
7		74.00		90.80	548.12	0.2%	27.41	0.1%	0.2%	1,742	7	−0.0%	−0.1%	−0.1%
8		74.00		87.10	538.52	0.2%	26.93	0.1%	0.2%	1,668	8	−0.0%	−0.1%	−0.1%
9		72.00		83.40	528.45	0.2%	26.42	0.0%	0.2%	1,596	9	0.0%	−0.0%	−0.1%
10		70.00		79.80	518.87	0.2%	25.94	0.0%	0.2%	1,526	10	0.0%	−0.0%	−0.1%
11		52.00		76.30	509.81	0.1%	25.49	0.0%	0.2%	1,474	11	0.0%	−0.0%	−0.1%
12		48.00		73.70	509.30	0.1%	25.47	0.1%	0.2%	1,426	12	0.1%	0.0%	−0.0%
13		44.00		71.30	510.77	0.1%	25.54	0.1%	0.2%	1,382	13	0.1%	0.0%	−0.0%
14		42.00		69.10	514.31	0.1%	25.72	0.1%	0.2%	1,340	14	0.1%	0.0%	−0.0%
15		36.00		67.00	519.02	0.1%	25.95	0.0%	0.2%	1,304	15	0.1%	0.0%	−0.0%
16		34.00		65.20	526.97	0.1%	26.35	0.1%	0.2%	1,270	16	0.1%	0.0%	−0.0%
17		30.00		63.50	536.32	0.1%	26.82	0.1%	0.2%	1,240	17	0.1%	0.0%	−0.0%
18		28.00		62.00	548.14	0.1%	27.41	0.1%	0.2%	1,212	18	0.1%	0.1%	−0.0%
19		524.00		60.60	561.54	0.1%	28.08	0.1%	0.2%	688	19	0.1%	0.1%	0.0%
20		688.00		34.40	327.62	0.1%	16.38	0.0%	0.2%	0	20	0.1%	0.1%	0.0%
											21	0.1%	0.1%	0.0%
Total	1,000.00	2,000.00		526.20	10,386.22	0.1%	519.31	0.0%	0.1%		etc.	0.1%	0.1%	0.0%
PV	1,000.00	961.38	$38.62	38.62	6,509.42	0.1%	325.47	0.0%	0.1%					

EXHIBIT 8B

INSURANCE CARRIER PERSPECTIVE WITH A VARIABLE R : S LEVERAGE RATIO

			Funds Released as they Become Available				
End of Year	Invested Surplus	Invested Premium	Paid Loss	Required Funds	Invested Funds	Released to Surplus	Flows for IRR
0	$442.07	$1,000.00	$0.00	$1,442.07	$1,442.07	$0.00	$ − 442.07
1			0.00	1,514.18	1,514.17	−0.01	− 0.01
2			0.00	1,589.88	1,589.89	0.01	0.01
3			6.00	1,660.38	1,663.37	2.99	2.99
4			28.00	1,701.39	1,715.40	14.01	14.01
5			86.00	1,657.47	1,700.46	42.99	42.99
6			64.00	1,644.35	1,676.34	31.99	31.99
7			74.00	1,615.56	1,652.57	37.01	37.01
8			74.00	1,585.34	1,622.34	37.00	37.00
9			72.00	1,556.60	1,592.61	36.01	36.01
10			70.00	1,529.43	1,564.43	35.00	35.00
11			52.00	1,527.90	1,553.90	26.00	26.00
12			48.00	1,532.30	1,556.30	24.00	24.00
13			44.00	1,542.92	1,564.92	22.00	22.00
14			42.00	1,557.06	1,578.07	21.01	21.01
15			36.00	1,580.91	1,598.91	18.00	18.00
16			34.00	1,608.96	1,625.96	17.00	17.00
17			30.00	1,644.41	1,659.41	15.00	15.00
18			28.00	1,684.62	1,698.63	14.01	14.01
19			524.00	982.86	1,244.85	261.99	261.99
20			688.00	0.00	344.00	344.00	344.00
							IRR = 0.1%

APPENDIX A

PROOF OF THE CONSISTENCY OF THE THREE RATE OF RETURN MEASURES

In Bingham's paper and in this discussion, it is demonstrated that releasing both surplus and operating gain as uncertainty regarding the outstanding loss amounts is resolved results in agreement among the three measures of return on equity. This appendix presents a rigorous proof that when this release scheme is adopted, the internal rate of return, annual return on surplus, and the invariant ratio are equal.

Begin the proof with the following variable designations. Let:

$S(j)$ be the supporting surplus at year-end j,
$I(j)$ be the investment income earned on the supporting surplus during the jth year,
$O(j)$ be the operating return that is released at year-end j,
i be the annual effective investment income rate on invested assets, and
$v_i = 1/(1 + i)$, the discounting factor at interest rate i.

With these definitions, the investment income earned by the supporting surplus during the jth year can be expressed as

$$I(j) = i * S(j-1), \qquad \text{(A.1)}$$

where it is assumed that the supporting surplus remains unchanged during the course of a year. This is consistent with Bingham's assumption that losses are paid at year-end.

Bingham's release scheme dictates that accrued operating earnings be released as uncertainty is resolved (i.e., as losses are paid). More specifically, the ratio of the released accrued operating gain at any year-end j to the supporting surplus during the jth year must be a constant independent of the particular year. Symbolically,

$$O(j)/S(j-1) = k, \quad \text{a constant } \forall j, \quad 1 \leq j \leq \omega, \qquad \text{(A.2)}$$

where ω denotes the end of the last year during which there are open (or IBNR) claims. Note that a second requirement placed upon the set of released operating gains is that the present value of the $\{O(j)\}$ equals the present value of the operating gain. While this requirement insures releasing an amount exactly equal to the accrued operating gain, it is not a necessary condition for agreement of the three measures of ROE.

Upon solving Equation A.2 for $O(j)$, we obtain the released operating earnings at the end of the jth year in terms of the supporting surplus that was allocated at the end of the previous year,

$$O(j) = k * S(j - 1). \tag{A.3}$$

During the jth year, the supporting surplus is $S(j - 1)$ and the return on that surplus is the investment income on that surplus, $I(j)$, plus the released operating gain, $O(j)$, or

$$I(j) + O(j) = (i + k) * S(j - 1). \tag{A.4}$$

Dividing by the invested surplus, $S(j - 1)$, gives the *average return on surplus during the jth year*, $(i + k)$. This expression is independent of j, making it a constant for all years.

Taking the present value of the total return on supporting surplus using *any* interest rate gives

$$\text{NPV}[I(j) + O(j)] = (i + k) * \text{NPV}[S(j - 1)]. \tag{A.5}$$

Divide the present value of the total return by the present value of the year-end supporting surplus, to obtain Bingham's present value ratio,

$$\text{NPV}[I(j) + O(j)]/\text{NPV}[S(j - 1)] = (i + k). \tag{A.6}$$

Note that the present value ratio is equal to the average return on surplus.

To show that the IRR of the surplus flows is also equal to the present value ratio and average return on surplus, begin by observing that, for a set of cash flows $\{CF_j\}$, the internal rate of return is defined as the interest rate that satisfies the equation

$$\mathrm{NPV}[\{CF_j\}] = \sum_{j=0}^{\omega} v^j * CF_j = 0. \tag{A.7}$$

In addition to the total return on the supporting surplus during the jth year ($j \geq 1$), the surplus flow also includes the return of supporting surplus as it is released,

$$\Delta S(j) = S(j-1) - S(j). \tag{A.8}$$

$S(\omega)$, the supporting surplus at the *end* of the last year in which there are any carried reserves, is zero.

Combining Equations A.4 and A.8 and remembering that at the end of year zero, $S(0)$ is transferred *out of* the surplus account,

$$CF_j = \begin{cases} -S(0), & \text{if} \quad j = 0, \\ (1+i+k)*S(j-1) - S(j), & \text{if} \quad j \neq 0. \end{cases} \tag{A.9}$$

At the internal rate of return,

$$\mathrm{NPV}_{\mathrm{IRR}}[\{CF_j\}] = -S(0) + (1+i+k)\sum_{j=1}^{\omega+1} v_{\mathrm{IRR}}^{t} * S(j-1) - \sum_{j=1}^{\omega+1} v_{\mathrm{IRR}}^{t} \cdot S(j) = 0. \tag{A.10}$$

To test the average annual return as a possible solution, substitute

$$v_{\mathrm{IRR}} = \frac{1}{(1+i+k)}, \tag{A.11}$$

then

$$\mathrm{NPV}[\{CF_j\}] = -S(0) + \sum_{j=1}^{\omega+1} v^{j-1} * S(j-1) - \sum_{j=1}^{\omega+1} v^j * S(j). \tag{A.12}$$

With a change of the dummy variable in the first sum, (A.12) becomes

$$\mathrm{NPV}[\{CF_j\}] = -S(0) + \sum_{j=0}^{\omega} v^j * S(j) - \sum_{j=1}^{\omega+1} v^j * S(j) = -S(\omega + 1), \tag{A.13}$$

but $S(\omega + 1)$ is zero because all uncertainties will have been resolved by the end of the last year, ω. Therefore,

$$\mathrm{NPV}[\{CF_j\}] = 0, \tag{A.14}$$

which proves that $(i + k)$ is an internal rate of return for the surplus flows.

It has been proven that, as a result of the release of operating gain scheme, the average total return on invested supporting surplus, the internal rate of return of the surplus flows, and the ratio of the present value of the total returns to the present value of the supporting surplus are all equal.

Nothing in this proof depends upon the specific relationship between the supporting surplus and the insurance product. In fact, $\{S(j)\}$ could be been selected at random (as long as all of the $S(j)$s are equal to zero after all of the uncertainty is resolved). None of the details leading to a determination of the insurance product operating gain, in fact not even its numerical value, enter into the proof. The conclusion that can be drawn from this is that no additional level of sophistication in the determination of the operating gain (e.g., the reflection of federal income tax, expenses, and policyholder dividends) or refinement in the selection of a reserve to surplus leverage ratio will invalidate the conclusions that have been proven in this appendix. The Bingham release scheme automatically insures the equality of the three measures of return.

APPENDIX B

EVIDENCE FOR A DECREASING LEVERAGE RATIO

Workers compensation is often cited as a line of business in which the uncertainty in the outstanding loss reserve decreases rapidly because of the highly predictable nature of lifetime pension cases. The conventional wisdom is that once the more volatile minor cases have been resolved, all that remains are claimants with lifetime benefits. As soon as the open claims consist only of lifetime pension cases, supporting surplus can be released rapidly. In particular, as a result of a decision of the hearing officer during the workers compensation rating hearing for rates to become effective January 1, 1988 in Massachusetts, the leverage ratio *increases* uniformly from the end of the fifth quarter until all claims are closed [3]. The pension case argument has been used to support the accelerated release of surplus.

Workers compensation claims probably arise from several underlying distributions. Clearly, minor cuts and bruises cannot be described by the same severity distribution that would apply to more serious injuries of the type that can lead to long term disability. As groups of claims close, the remaining open claims may be of a more homogeneous nature. This, in itself, may decrease the relative uncertainty in the open claim reserves. Initially, at least, as certain classes of claims close, an increase in the leverage ratio may be possible.

Once the population of open claims consists of nothing but lifetime pension cases, the long term behavior of the leverage ratio manifests itself. For a reasonable example, it can be demonstrated that the leverage ratio must decrease over time. This is not a rigorous proof that the leverage ratio for workers compensation coverage *must always* decrease but, rather, it is evidence that one cannot assume that once pension cases dominate the open claim reserves, the leverage ratio *will always* increase. This appendix serves as a counter example, disproving the common contention.

It is an indication that even in the case of workers compensation runoff, additional research is necessary.

For the example, assume that there are exactly 100 open claims at a given point in time. To simplify matters, assume that each of these claims involves a 40 year old claimant who is receiving a $5,000 annual amount paid in weekly installments. There are no cost of living adjustments. Benefits terminate upon death of the claimant.

Further assume that the 1979–1981 U.S. Decennial Life Mortality table for the Total Population (that adopted by the National Council on Compensation Insurance for Unit Statistical Plan reporting) reflects the life expectancies of these claimants. The aggregate nominal outstanding loss reserve for these claimants would be $18,392,500 (100 claimants * $5,000 per year per claimant * 36.785 years per claimant on the average). Exhibit B-1 displays a section of the mortality table and the life expectancy calculation. The $18,392,500 reserve is only a point estimate. The actual amount paid to these claimants could be significantly more or less than this amount.

The mortality table shows a 0.014 probability that a claimant could die within five years rather than living the expected 36.785 years. Likewise, there is approximately a .02 probability that the claimant could live 58 more years rather than the expected number of years. If each claim were reserved to a 98% confidence level, the leverage ratio would be approximately 1.73 : 1.00; for every $1.00 of reserves, $0.58 of surplus would have to be allocated (i.e., of the 58 years that must be provided for, 36.785 would be provided for in the form of loss reserves with the remaining 21.215 coming from supporting surplus). Alternatively, a dollar of surplus can support $1.73 of reserves.

Of course, 100 times the individual claim supporting surplus is not necessary to maintain a 98% confidence level in the aggregate. The single claimant loss distribution was input into an aggregate loss model, such as the one described by Heckman and Meyers [1], to determine that the appropriate reserve-to-surplus ratio for a 2% probability of ruin would be 14.015 : 1.000 (i.e., the 98th percentile occurs at 1.07135 times the expected mean, so $0.07135 of surplus is required to support every $1.00 of reserves). Exhibit B-2 displays the cumulative probability corresponding to various aggregate loss amounts where the entry ratio is the ratio of the selected aggregate loss to the mean aggregate loss.

Ten years later, if everything has gone as was expected, there will be 96 open claims (consisting of lifetime pension cases for 50 year olds). At that time, there will be approximately a 2% probability of living at least 48 more years (almost no difference between the probability of a 40 year old living to 98, 0.0230, and the probability of a 50 year old living to 98, 0.0239). With a 27.939 year life expectancy, the individual claim leverage ratio for the 50 year old claimants is 1.39 : 1.00, which represents a decrease from 1.73. The 96 claim aggregate leverage ratio is 11.521 : 1.000, also a decrease from the 14.015 leverage ratio. Exhibit B-2 displays the aggregate loss distribution for 96.418 claims (96 being the result of rounding to whole numbers for the sake of the narrative).

By the time the claimants are 60 years of age, the individual claimant leverage ratio will have fallen to 1.11 : 1.00 (with a 20.019 year life expectancy and approximately a 2% probability of living to 98 years of age or longer, almost equal amounts of surplus and reserves are required). Of the original 100 claimants, 88 (88.2 claims were used in the aggregate loss model) are expected to reach age 60. The aggregate leverage ratio for these 88 living 60 year old claimants would be 9.25 : 1.00.

Unless there is another group of claims that both increases the variability of the open claim reserves in total and closes rapidly enough to more than offset the increasing variability of the pension claims, the leverage ratio for open workers compensation claims must *decrease* in the long run. The preceding example does not constitute a proof that the leverage ratio decreases; rather, it makes the conventional wisdom less obvious. The appropriate leverage ratio for any line of business must be the result of an investigation of the underlying volatility of its open claims at any point in time.

EXHIBIT B-1

PART 1

79/81 U.S. DECENNIAL LIFE MORTALITY TABLE

		$x = 40$				$x = 50$				$x = 60$			
Age, x	$l(x)$	n	$p(n)$	$np(n)$	Sum[$p(n)$]	n	$p(n)$	$np(n)$	Sum[$p(n)$]	n	$p(n)$	$np(n)$	Sum[$p(n)$]
38	95,317												
39	95,129												
40	94,926	0.5	0.00232	0.001	0.002								
41	94,706	1.5	0.00254	0.004	0.005								
42	94,465	2.5	0.00278	0.007	0.008								
43	94,201	3.5	0.00303	0.011	0.011								
44	93,913	4.5	0.00331	0.015	0.014								
45	93,599	5.5	0.00361	0.020	0.018								
46	93,256	6.5	0.00394	0.026	0.022								
47	92,882	7.5	0.00432	0.032	0.026								
48	92,472	8.5	0.00475	0.040	0.031								
49	92,021	9.5	0.00521	0.050	0.036								
50	91,526	10.5	0.00569	0.060	0.042	0.5	0.00590	0.003	0.006				
51	90,986	11.5	0.00615	0.071	0.048	1.5	0.00638	0.010	0.012				
52	90,402	12.5	0.00665	0.083	0.054	2.5	0.00689	0.017	0.019				
53	89,771	13.5	0.00721	0.097	0.062	3.5	0.00747	0.026	0.027				
54	89,087	14.5	0.00779	0.113	0.069	4.5	0.00807	0.036	0.035				
55	88,348	15.5	0.00840	0.130	0.078	5.5	0.00871	0.048	0.043				
56	87,551	16.5	0.00902	0.149	0.087	6.5	0.00935	0.061	0.053				
57	86,695	17.5	0.00968	0.169	0.096	7.5	0.01004	0.075	0.063				
58	85,776	18.5	0.01040	0.192	0.107	8.5	0.01078	0.092	0.074				
59	84,789	19.5	0.01120	0.218	0.118	9.5	0.01161	0.110	0.085				
60	83,726	20.5	0.01206	0.247	0.130	10.5	0.01251	0.131	0.098	0.5	0.01368	0.007	0.014
61	82,581	21.5	0.01299	0.279	0.143	11.5	0.01347	0.155	0.111	1.5	0.01473	0.022	0.028

EXHIBIT B-1

PART 2

79/81 U.S. DECENNIAL LIFE MORTALITY TABLE

		x = 40				x = 50				x = 60			
Age, x	$l(x)$	n	$p(n)$	$np(n)$	Sum[$p(n)$]	n	$p(n)$	$np(n)$	Sum[$p(n)$]	n	$p(n)$	$np(n)$	Sum[$p(n)$]
62	81,348	22.5	0.01395	0.314	0.157	12.5	0.01447	0.181	0.126	2.5	0.01581	0.040	0.044
63	80,024	23.5	0.01491	0.350	0.172	13.5	0.01546	0.209	0.141	3.5	0.01690	0.059	0.061
64	78,609	24.5	0.01582	0.388	0.188	14.5	0.01641	0.238	0.158	4.5	0.01794	0.081	0.079
65	77,107	25.5	0.01672	0.426	0.204	15.5	0.01734	0.269	0.175	5.5	0.01895	0.104	0.098
66	75,520	26.5	0.01763	0.467	0.222	16.5	0.01829	0.302	0.193	6.5	0.01999	0.130	0.118
67	73,846	27.5	0.01858	0.511	0.241	17.5	0.01927	0.337	0.212	7.5	0.02107	0.158	0.139
68	72,082	28.5	0.01964	0.560	0.260	18.5	0.02037	0.377	0.233	8.5	0.02226	0.189	0.161
69	70,218	29.5	0.02075	0.612	0.281	19.5	0.02152	0.420	0.254	9.5	0.02353	0.224	0.185
70	68,248	30.5	0.02194	0.669	0.303	20.5	0.02276	0.467	0.277	10.5	0.02488	0.261	0.210
71	66,165	31.5	0.02310	0.728	0.326	21.5	0.02396	0.515	0.301	11.5	0.02619	0.301	0.236
72	63,972	32.5	0.02422	0.787	0.350	22.5	0.02512	0.565	0.326	12.5	0.02746	0.343	0.263
73	61,673	33.5	0.02522	0.845	0.376	23.5	0.02616	0.615	0.352	13.5	0.02859	0.386	0.292
74	59,279	34.5	0.02613	0.901	0.402	24.5	0.02710	0.664	0.379	14.5	0.02962	0.429	0.322
75	56,799	35.5	0.02697	0.957	0.429	25.5	0.02797	0.713	0.407	15.5	0.03058	0.474	0.352
76	54,239	36.5	0.02781	1.015	0.456	26.5	0.02884	0.764	0.436	16.5	0.03153	0.520	0.384
77	51,599	37.5	0.02866	1.075	0.485	27.5	0.02973	0.818	0.466	17.5	0.03250	0.569	0.416
78	48,878	38.5	0.02957	1.138	0.515	28.5	0.03067	0.874	0.497	18.5	0.03353	0.620	0.450
79	46,071	39.5	0.03046	1.203	0.545	29.5	0.03159	0.932	0.528	19.5	0.03453	0.673	0.484
80	43,180	40.5	0.03131	1.268	0.576	30.5	0.03247	0.990	0.561	20.5	0.03550	0.728	0.520
81	40,208	41.5	0.03198	1.327	0.608	31.5	0.03317	1.045	0.594	21.5	0.03626	0.780	0.556
82	37,172	42.5	0.03241	1.378	0.641	32.5	0.03362	1.093	0.627	22.5	0.03675	0.827	0.593
83	34,095	43.5	0.03248	1.413	0.673	33.5	0.03368	1.128	0.661	23.5	0.03682	0.865	0.630
84	31,012	44.5	0.03215	1.431	0.705	34.5	0.03335	1.150	0.695	24.5	0.03645	0.893	0.666
85	27,960	45.5	0.03159	1.437	0.737	35.5	0.03277	1.163	0.727	25.5	0.03582	0.913	0.702
86	24,961	46.5	0.03079	1.432	0.768	36.5	0.03194	1.166	0.759	26.5	0.03491	0.925	0.737
87	22,038	47.5	0.02953	1.403	0.797	37.5	0.03063	1.148	0.790	27.5	0.03348	0.921	0.770
88	19,235	48.5	0.02778	1.347	0.825	38.5	0.02881	1.109	0.819	28.5	0.03150	0.898	0.802
89	16,598	49.5	0.02575	1.274	0.851	39.5	0.02670	1.055	0.845	29.5	0.02919	0.861	0.831

EXHIBIT B-1

PART 3

79/81 U.S. DECENNIAL LIFE MORTALITY TABLE

		$x = 40$				$x = 50$				$x = 60$			
Age, x	$l(x)$	n	$p(n)$	$np(n)$	Sum[$p(n)$]	n	$p(n)$	$np(n)$	Sum[$p(n)$]	n	$p(n)$	$np(n)$	Sum[$p(n)$]
90	14,154	50.5	0.02366	1.195	0.875	40.5	0.02454	0.994	0.870	30.5	0.02683	0.818	0.858
91	11,908	51.5	0.02154	1.109	0.896	41.5	0.02234	0.927	0.892	31.5	0.02442	0.769	0.882
92	9,863	52.5	0.01929	1.013	0.915	42.5	0.02001	0.850	0.912	32.5	0.02187	0.711	0.904
93	8,032	53.5	0.01694	0.906	0.932	43.5	0.01757	0.764	0.930	33.5	0.01921	0.643	0.923
94	6,424	54.5	0.01455	0.793	0.947	44.5	0.01509	0.671	0.945	34.5	0.01649	0.569	0.940
95	5,043	55.5	0.01221	0.678	0.959	45.5	0.01266	0.576	0.958	35.5	0.01384	0.491	0.954
96	3,884	56.5	0.00996	0.562	0.969	46.5	0.01032	0.480	0.968	36.5	0.01129	0.412	0.965
97	2,939	57.5	0.00794	0.457	0.977	47.5	0.00824	0.391	0.976	37.5	0.00901	0.338	0.974
98	2,185	58.5	0.00618	0.362	0.983	48.5	0.00641	0.311	0.983	38.5	0.00701	0.270	0.981
99	1,598	59.5	0.00472	0.281	0.988	49.5	0.00489	0.242	0.987	39.5	0.00535	0.211	0.986
100	1,150	60.5	0.00353	0.214	0.991	50.5	0.00366	0.185	0.991	40.5	0.00400	0.162	0.990
101	815	61.5	0.00258	0.159	0.994	51.5	0.00268	0.138	0.994	41.5	0.00293	0.121	0.993
102	570	62.5	0.00186	0.117	0.996	52.5	0.00193	0.102	0.996	42.5	0.00211	0.090	0.995
103	393	63.5	0.00133	0.084	0.997	53.5	0.00138	0.074	0.997	43.5	0.00150	0.065	0.997
104	267	64.5	0.00093	0.060	0.998	54.5	0.00096	0.052	0.998	44.5	0.00105	0.047	0.998
105	179	65.5	0.00063	0.041	0.999	55.5	0.00066	0.036	0.999	45.5	0.00072	0.033	0.999
106	119	66.5	0.00043	0.029	0.999	56.5	0.00045	0.025	0.999	46.5	0.00049	0.023	0.999
107	78	67.5	0.00028	0.019	0.999	57.5	0.00029	0.017	0.999	47.5	0.00032	0.015	0.999
108	51	68.5	0.00019	0.013	1.000	58.5	0.00020	0.012	1.000	48.5	0.00021	0.010	1.000
109	33	69.5	0.00035	0.024	1.000	59.5	0.00036	0.021	1.000	49.5	0.00039	0.020	1.000
110	0	70.5	0.00000	0.000	1.000	60.5	0.00000	0.000	1.000	50.5	0.00000	0.000	1.000
		Total	1.00000	36.785			1.00000	27.939			1.00000	20.019	

EXHIBIT B-2

PART 1

AGGREGATE LOSS DISTRIBUTION

100 Forty Year Old Claimants			96 Fifty Year Old Claimants			88 Sixty Year Old Claimants		
Entry Ratio	Aggregate Loss	Cumulative Probability	Entry Ratio	Aggregate Loss	Cumulative Probability	Entry Ratio	Aggregate Loss	Cumulative Probability
1.00000	18,393,470	0.4967	1.0750	14,480,151	0.9617	1.0800	9,535,527	0.9356
1.00625	18,508,429	0.5673	1.0800	14,547,501	0.9706	1.0900	9,623,818	0.9563
1.01250	18,623,388	0.6359	1.0860	14,628,320	0.9790	1.1000	9,712,110	0.9713
1.01875	18,738,347	0.7006	1.0861	14,629,667	0.9791	1.1010	9,720,940	0.9725
1.02500	18,853,306	0.7597	1.0862	14,631,014	0.9792	1.1020	9,729,769	0.9737
1.03125	18,968,266	0.8120	1.0863	14,632,361	0.9794	1.1030	9,738,598	0.9748
1.03750	19,083,225	0.8567	1.0864	14,633,708	0.9795	1.1040	9,747,427	0.9759
1.04375	19,198,184	0.8937	1.0865	14,635,055	0.9796	1.1050	9,756,256	0.9770
1.05000	19,313,143	0.9234	1.0866	14,636,402	0.9797	1.1060	9,765,086	0.9780
1.05625	19,428,102	0.9464	1.0867	14,637,749	0.9798	1.1061	9,765,968	0.9781
1.06250	19,543,062	0.9635	1.0868	14,639,096	0.9800	1.1062	9,766,851	0.9782
1.06860	19,655,262	0.9757	1.0869	14,640,443	0.9801	1.1063	9,767,734	0.9783
1.06870	19,657,101	0.9759	1.0870	14,641,790	0.9802	1.1064	9,768,617	0.9784
1.06880	19,658,940	0.9761	1.0871	14,643,137	0.9803	1.1065	9,769,500	0.9785
1.06890	19,660,780	0.9762	1.0872	14,644,484	0.9804	1.1066	9,770,383	0.9786
1.06900	19,662,619	0.9764	1.0873	14,645,831	0.9805	1.1067	9,771,266	0.9787
1.06910	19,664,458	0.9765	1.0874	14,647,178	0.9806	1.1068	9,772,149	0.9788
1.06920	19,666,298	0.9767	1.0875	14,648,525	0.9808	1.1069	9,773,032	0.9789
1.06930	19,668,137	0.9769	1.0876	14,649,872	0.9809	1.1070	9,773,915	0.9790
1.06940	19,669,976	0.9770	1.0877	14,651,219	0.9810	1.1071	9,774,798	0.9791
1.06950	19,671,816	0.9772	1.0878	14,652,566	0.9811	1.1072	9,775,681	0.9792

EXHIBIT B-2

PART 2

AGGREGATE LOSS DISTRIBUTION

100 Forty Year Old Claimants			96 Fifty Year Old Claimants			88 Sixty Year Old Claimants		
Entry Ratio	Aggregate Loss	Cumulative Probability	Entry Ratio	Aggregate Loss	Cumulative Probability	Entry Ratio	Aggregate Loss	Cumulative Probability
1.06960	19,673,655	0.9773	1.0879	14,653,913	0.9812	1.1073	9,776,563	0.9793
1.06970	19,675,494	0.9775	1.0880	14,655,260	0.9813	1.1074	9,777,446	0.9794
1.06980	19,677,334	0.9777	1.0881	14,656,607	0.9814	1.1075	9,778,329	0.9795
1.06990	19,679,173	0.9778	1.0882	14,657,954	0.9815	1.1076	9,779,212	0.9796
1.07000	19,681,013	0.9780	1.0883	14,659,301	0.9816	1.1077	9,780,095	0.9797
1.07010	19,682,852	0.9781	1.0884	14,660,648	0.9817	1.1078	9,780,978	0.9798
1.07020	19,684,691	0.9783	1.0885	14,661,995	0.9819	1.1079	9,781,861	0.9799
1.07030	19,686,531	0.9784	1.0886	14,663,342	0.9820	1.1080	9,782,744	0.9799
1.07040	19,688,370	0.9786	1.0887	14,664,689	0.9821	1.1081	9,783,627	0.9800
1.07050	19,690,209	0.9787	1.0888	14,666,036	0.9822	1.1082	9,784,510	0.9801
1.07060	19,692,049	0.9789	1.0889	14,667,383	0.9823	1.1083	9,785,393	0.9802
1.07070	19,693,888	0.9790	1.0890	14,668,730	0.9824	1.1084	9,786,276	0.9803
1.07080	19,695,727	0.9792	1.0891	14,670,077	0.9825	1.1085	9,787,158	0.9804
1.07090	19,697,567	0.9793	1.0892	14,671,424	0.9826	1.1086	9,788,041	0.9805
1.07100	19,699,406	0.9795	1.0893	14,672,771	0.9827	1.1087	9,788,924	0.9806
1.07110	19,701,245	0.9796	1.0894	14,674,118	0.9828	1.1088	9,789,807	0.9807
1.07120	19,703,085	0.9798	1.0895	14,675,465	0.9829	1.1089	9,790,690	0.9808
1.07130	19,704,924	0.9799	1.0896	14,676,812	0.9830	1.1090	9,791,573	0.9809
1.07135	19,705,844	0.9800	1.0897	14,678,159	0.9831	1.1091	9,792,456	0.9809
1.07140	19,706,763	0.9801	1.0898	14,679,506	0.9832	1.1092	9,793,339	0.9810
1.07150	19,708,603	0.9802	1.0899	14,680,853	0.9833	1.1093	9,794,222	0.9811

EXHIBIT B-2

PART 3

AGGREGATE LOSS DISTRIBUTION

100 Forty Year Old Claimants			96 Fifty Year Old Claimants			88 Sixty Year Old Claimants		
Entry Ratio	Aggregate Loss	Cumulative Probability	Entry Ratio	Aggregate Loss	Cumulative Probability	Entry Ratio	Aggregate Loss	Cumulative Probability
1.07160	19,710,442	0.9803	1.0900	14,682,200	0.9834	1.1094	9,795,105	0.9812
1.07170	19,712,281	0.9805	1.0901	14,683,547	0.9835	1.1095	9,795,988	0.9813
1.07180	19,714,121	0.9806	1.0902	14,684,894	0.9836	1.1096	9,796,871	0.9814
1.07190	19,715,960	0.9808	1.0903	14,686,241	0.9837	1.1097	9,797,754	0.9815
1.07200	19,717,799	0.9809	1.0904	14,687,588	0.9838	1.1098	9,798,636	0.9816
1.07210	19,719,639	0.9810	1.0905	14,688,935	0.9839	1.1099	9,799,519	0.9816
1.07220	19,721,478	0.9812	1.0906	14,690,282	0.9840	1.1100	9,800,402	0.9817
1.07230	19,723,318	0.9813	1.0907	14,691,629	0.9841	1.1200	9,888,694	0.9887
1.07240	19,725,157	0.9814	1.0908	14,692,976	0.9842	1.1300	9,976,986	0.9933
1.07250	19,726,996	0.9816	1.0909	14,694,323	0.9843	1.1400	10,065,278	0.9961
1.07260	19,728,836	0.9817	1.0910	14,695,670	0.9844	1.1500	10,153,570	0.9978
1.07270	19,730,675	0.9818	1.0911	14,697,017	0.9845	1.1600	10,241,862	0.9988
1.07280	19,732,514	0.9820	1.0912	14,698,364	0.9846	1.1700	10,330,154	0.9994
1.07290	19,734,354	0.9821	1.0913	14,699,711	0.9846	1.1800	10,418,446	0.9997
1.07300	19,736,193	0.9822	1.0914	14,701,058	0.9847	1.1900	10,506,738	0.9999