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PERSONAL AUTOMOBILE PREMIUMS: AN ASSET SHARE PRICING APPROACH FOR PROPERTY/CASUALTY INSURANCE

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Abstract

Asset share pricing models are used extensively in life and health insurance premium determination. In contrast, property/casualty ratemaking procedures consider only a single period of coverage. This is true for both traditional methods, such as loss ratio and pure premium ratemaking, and financial pricing models, such as discounted cash flow or internal rate of return models.

This paper provides a full discussion of property/casualty insurance asset share pricing procedures. Section 1 compares life insurance to casualty insurance pricing. It notes why asset share pricing is so important for the former, and how it applies to the latter as well. Section 2 describes the considerations essential for an asset share pricing model. Premiums, claim frequency, claim severity, expenses, and persistency rates must be examined by

time since inception of the policy. Appropriate discount rates must be selected for: (a) present values of the contract cash flows during each policy year, and (b) the present value of future earnings at the inception date of the policy.

Sections 3 through 7 present four illustrations of asset share pricing:

- *Section 3 is a general introduction.*
- *Section 4 illustrates pricing considerations for an expanding book of business. Since both loss costs and expense costs are higher for new business than for renewal business, traditional loss ratio or pure premium pricing methods show misleading rate indications.*
- *Section 5 discusses classification relativities. Since persistency rates and coverage combinations differ by classification, the traditional relativity analyses may be erroneous.*
- *Section 6 presents a competitive strategy illustration. Premium discounts and surcharges affect retention rates, particularly among policyholders who can obtain coverage elsewhere.*
- *Section 7 shows how underwriting cycle movements can be incorporated into pricing strategy. Expected future profits vary with the stage of the cycle; these future earnings and losses must be considered when setting premium rates.*

Section 8 discusses several types of profitability measures: returns on premium, returns on surplus or equity, internal rates of return, and the number of years until the policy becomes profitable. Traditional financial pricing models examine a single contract period and multiple loss payment periods. For asset share pricing, these models are expanded to consider multiple contract periods. For instance, the "return on premium" is the present value of future expected profits divided by the *present value of future expected premium, not the single period amounts used for operating ratios.*

Asset share models determine the long-run profitability of the insurance operations, the true task of the pricing actuary.

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The author is indebted to Richard Woll and Stephen D'Arcy for inspiration and criticism of this paper. Ten years ago, Richard Woll was examining the effects of business volume growth on accounting profitability versus true profitability, and he demonstrated the powerful effects on the "costs of new business" (compare the first illustration in the paper). At about the same time, Professor D'Arcy was writing his papers on the "Aging Phenomenon" and on "Adverse Selection, Private Information, and Lowballing," which deal with some of the same issues as this paper covers, though it treats them differently. Professor D'Arcy sent early drafts of his papers to the author, he provided helpful critiques of the author's previous papers on this subject at a CAS conference, and he sent written comments on an earlier draft of this paper. The contributions of Richard Woll and Stephen D'Arcy greatly improved this paper.

1. INTRODUCTION

Asset share pricing models have long been used for life and health insurance premium determination. These models examine the profitability of the complete insurance contract from its inception to its final termination, including all renewals of the policy. That is to say, the life insurance pricing actuary does not evaluate the profitability of a block of policies in a given calendar year, policy year, or calendar/accident year. Indeed, such a valuation would not be meaningful, since a whole life insurance policy is expected to lose money in the initial year of issue but to make up for the loss in subsequent years. Rather, the life insurance actuary sets policy premiums to achieve an appropriate profit over the lifetime of the policy. Similarly, this paper applies asset share pricing methods to property/casualty lines of business.

Asset share pricing is especially important when cash flows and reported income vary by policy year. For instance, a whole life policy issued to a standard-rated thirty-year-old insured shows:

- high expense costs the first year (often greater than the gross premium),
- low mortality costs the first several years,
- higher mortality costs in later years, as the policyholder ages and the underwriting selection "wears off," and
- statutory benefit reserves that are somewhat redundant after the second or third year because of the conservative valuation of mortality tables and interest rates; during the first several years, preliminary term reserves reduce the statutory liability.[1]

In property/casualty insurance, loss ratio and pure premium ratemaking methods predominate. Financial pricing models are often used to set underwriting profit targets, although these methods, like the traditional property/casualty rate making techniques, presume an insurance contract in effect for a single policy period. Most financial pricing models examine the duration of loss payments, but they do not consider the duration of the insurance contract.[2]

Life Versus Casualty Ratemaking

The differing ratemaking philosophies for life and health insurance versus property/casualty insurance stem from several

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[1]On asset share pricing models for life insurance, see Anderson [8], Huffman [95], and Atkinson [10]; for health insurance, see Bluhm and Koppel [25]. Menge and Fischer [131, p. 131] explain the term "asset share" as "the equitable share of the policyholders in the assets of the company." Similarly, Atkinson [11] explains the term as "the share of assets allocable to each surviving unit."

[2]On the traditional ratemaking techniques, see McClenahan [129] and Feldblum [75]. On the development of financial pricing models, see Hanson [89], Webb [162], and Derrig [64]. For examples of the major models, see Fairley [67], Hill [92], NAIC [136], Urrutia [155], Myers and Cohn [135], Mahler [124], Woll [169], Butsic and Lerwick [39], Bingham ([20], [22]), Robbin [144], Feldblum [71], and Mahler [126]. For analyses of these models, see Hill and Modigliani [93], Derrig [65], Ang and Lai [9], D'Arcy and Doherty [61], Garven [85], D'Arcy and Garven [62], Mahler [125], and Cummins ([48], [50], [51]).

factors:

- *Cancellation:* Few individual life or health insurance policies may be canceled or non-renewed by the insurer, except for non-payment of premium. In property/casualty insurance, particularly in the commercial lines, the carrier has the right to terminate the policy at the renewal date and often to cancel the policy in mid-term.[3]
- *Claim costs:* Life and health insurance claim costs vary by duration since policy inception, for two reasons:
  - Policyholder age: mortality and morbidity costs rise as the insured ages.
  - Underwriting selection: medical questionnaires and examinations for life and health insurance lead to lower average initial benefit costs for insured lives. The effects of underwriting selection "wear off" after several years (Jacobs [106, p. 5]; Dahlman [55, p. 5]).

  In property/casualty insurance, the relationship between expected losses and duration since policy inception is less apparent.
- *Expenses:* Expenses show a similar pattern. Whole life commission rates are high in the initial year but low for renewals.[4] For property/casualty companies using the independent agency distribution system, commission rates do not differ between the first year and renewal years.
- *Level premiums:* Much life insurance is provided by level premium contracts. The premium exceeds the anticipated benefits during the early policy years, when the insured is young and healthy. In later years, anticipated benefit costs exceed the

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[3]Renewability provisions in health insurance vary among contracts, though cancelable policies are proscribed in many jurisdictions (Barnhart [13]). Many states now proscribe mid-term cancellations of personal automobile policies; others, such as California or Massachusetts, prohibit even non-renewals.

[4]Lombardi and Wolfe [119]. Atkinson [11, p. 5] notes that traditional life insurance "acquisition costs usually exceed the first year premium by a wide margin. Acquisition costs may even exceed 200% of premium, especially for smaller policies."

premiums, and they are funded by the policy reserves built up in earlier years. In contrast, property/casualty insurance rates may be revised each year. No "policy reserves" are held to shift costs among accounting periods.

Developments in Casualty Insurance

These differences are valid, and asset share pricing is therefore more common for life and health insurance premium development. But property/casualty insurance is taking on several of the attributes that motivate asset share pricing.

- *Commissions:* Most personal lines insurance policies are now issued by direct writers, whose commission rates are higher in the first year than in renewal years.
- *Cancellations:* Although the insurer may have the right to cancel or non-renew the contract, it rarely does so. Profitability depends on the stability of the book of business, and carriers seek to strengthen policyholder loyalty.
- *Loss costs:* As will be discussed below, expected loss costs are greater for new business than for renewal business.[5]

The question faced by all insurers is the same: "*Is it profitable to write the insurance policy?*" A financially strong carrier does not focus on reported results or cash flows for the current year. Rather, it examines whether the stream of future profits, both from the original policy year and from renewal years, justifies underwriting the contract. Asset share pricing enables the actuary to provide quantitative estimates of long-term profitability.

2. ASSET SHARE COMPONENTS

Asset share pricing is not yet common in property/casualty insurance for several reasons:

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[5] Most actuarial studies of this phenomenon have concentrated on personal automobile insurance. Unpublished studies by the author and his colleagues show the same phenomenon in other lines, particularly for workers compensation.

- The data needed are not always available.
- Casualty pricing techniques do not always take into account long-term profit considerations.
- The casualty insurance policy allows great flexibility in premiums and benefit levels.
- Liability claim costs are uncertain, both in magnitude and in timing.

This section examines the qualitative influences on the asset share pricing components, to lay the groundwork for the quantitative model that follows.

A. Premiums

Premiums for whole life policies are set at policy inception, and they continue unchanged until the termination or forfeiture of the contract. Premiums for renewable term life policies are generally guaranteed for the first several years and illustrated for an additional ten or fifteen years. Similarly, policyholder dividends on participating contracts are often illustrated for the first twenty years.[6]

Property/casualty insurance premiums may be revised each year or half-year, and insurers do not illustrate the expected future premiums. In fact, premiums fluctuate widely from year to year for a variety of reasons:

- Inflation raises loss costs, and premiums are adjusted accordingly. Life insurance benefits, in contrast, are often fixed in nominal terms.
- Underwriting cycles raise and lower the premiums charged, whether by manual rate revisions or individual risk rating ad-

[6]The NAIC Life Insurance Solicitation Model Regulation requires that insurers illustrate surrender cost and net payment cost indices for ten and twenty year durations (Black and Skipper [23]; see also Jensen [107, pp. 449–450]). Premiums for some newer contracts, such as indeterminate premium and universal life policies, are harder to project for future years.

justments. Underwriting cycles are not found in individual life insurance.

- The insured's classification or exposure may change from year to year. The personal auto insured may marry, the workers compensation insured may expand its operations, and the commercial property risk may install fire protection equipment.[7] The classification of the individual life policyholder generally does not change after inception of the policy.[8]

In sum, the level premiums for traditional whole life insurance policies, versus the variable premiums for casualty products, have contributed to the greater reliance of life actuaries on asset share pricing methods.

B. Claims

Mortality rates are stable from year to year, and the influences on mortality are well documented. We may not fully understand why sex has such a strong influence on mortality, but given an individual's age, sex, and physical condition, we can provide a life expectancy (Berin, Stolnitz, and Teitlebaum [18]). At the inception of the insurance policy, the actuary can estimate mortality rates for the insured's lifetime. Barring major wars or epidemics, the estimates should be accurate.

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[7] See, for instance, Feldblum [70]: "… average loss costs vary over the life of a policy. For example, many young unmarried men are carefree drivers, less concerned with safety than with presenting a courageous image. Once they have married, begun careers, and borne children, they feel more responsibility, both individual and financial, for their families—and their driving habits improve accordingly. When their children become adolescents and start driving the family cars, auto insurance loss costs climb rapidly. But when the children leave home and the insured retires, the automobiles may be unused except for shopping trips and weekend vacations; automobile accidents become rare. Finally, when the driver enters his or her 70s, physiological health deteriorates and reactions are slowed. If the insured continues to drive, accident frequency increases." Similarly, Whitehead [167, p. 312] writes: "Changes in inherent risk over time—the typical 'life-cycle' of an insured with respect of individual private passenger automobile insurance is for the level of inherent risk to decline as the age of the insured and his level of driving experience and competence increases (at least until a relatively advanced age)."

[8] Minor exceptions exist. For instance, a substandard rated policyholder may be rerated after several years upon submission of evidence of insurability (Woodman [171]). Re-entry term insurance allows reclassification at the end of each select period (Galt [84]; Jacobs [106]).

B.1. Casualty Claim Rates

Claim rates in casualty insurance are more variable and less well understood. Why do urban drivers have higher personal auto claim frequencies than suburban residents? Is traffic density higher in cities than in rural areas? Are road conditions worse in urban areas? Are suburban residents, who are friendly with the neighboring children, more careful drivers? Are there more attorneys in cities, and do they encourage accident victims to file claims? Does the type and extent of medical treatment differ between urban and rural areas? Are rural residents more familiar with insurance agents and brokers and less inclined to seek compensation from "impersonal" corporations?[9]

Claim rates in workers compensation vary with economic conditions and with the operations of the insured. During recessions, when layoffs or plant closings are anticipated, many employees file workers compensation claims for minor, non-disabling injuries that they would ignore in more prosperous times (Borba [27]; Boden and Fleischman [26]; Victor and Fleischman [158]; Victor [157]; NJCIRB [139]). When a firm expands quickly, with young, inexperienced workers, accidental injuries are more common (Worrall, Appel, and Butler ([172], [173]); NCCI [137, p. 34]; Walters [160, p. 22]; ISO [102]).

In the commercial liability lines (other liability, products liability, medical malpractice, and professional liability), statu-

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[9]Casualty actuaries are just beginning to examine these issues. On traffic density in urban and suburban areas, and on the contribution of suburban drivers to urban traffic, see Brissman [29]. The importance of attorneys can be seen by comparing claims represented by attorneys and those not represented in urban and rural areas (AIRAC [5], [6]; Feldblum [75]; IRC [99]). The effects of "claims consciousness," or the proclivity to file insurance claims, can be measured by the ratio of bodily injury claims to property damage claims. The frequency of PD claims is primarily determined by the incidence of physical accidents. The frequency of BI claims is affected by claims consciousness and attorney involvement as well. The ratio of BI to PD claims varies by jurisdiction, and it is higher in cities than in rural areas (IRC [98], [100], [101]; Woll [169]; Cummins and Tennyson [53]). The type of medical practitioner, such as physician, chiropractor, or physical therapist, affects both claim frequency and severity (Marter and Weisberg [127], [128]; Weisberg and Derrig [163], [164], [165]). For the corresponding influences on workers compensation, see Feldblum [75].

tory enactments and judicial precedents affect the frequency of claims. Congressional passage of the CERCLA (Comprehensive Environmental Response, Compensation, and Liability Act) in 1980, with strict, joint, several, and retroactive liability, encouraged the filing of environmental impairment claims (Hamilton and Routman [88], Miller [132]; Kunreuther and Gowda [112]; ISO [105]). State legislation modifying the statute of limitations and setting caps on awards has affected the filing of medical malpractice claims.

The stability of life insurance benefits versus the variability of casualty insurance losses is a second reason for the greater use by life actuaries of asset share pricing methods. However, the fundamental issue is not the predictability of losses but the relationship of losses and expenses to persistency. The asset share model examines a particular policy and asks: "*Is this risk's expected profitability above or below the average for other insureds in its class?*" To answer this question, we examine three items: relative loss costs by policy year, expenses by policy year, and persistency rates by policy year and by classification.

B.2. Policy Duration and Claim Frequency

Policy duration has a strong influence on claim frequency, particularly in personal automobile, where new insureds have higher average loss ratios than renewal policyholders. Conning and Company [47, pp. 10–11], note that "Companies have acknowledged results which show new business loss ratios varying from 10% higher to more than 30% higher, depending on the line of business and the underwriting year."[10] Older drivers, with lower average claim frequencies and loss ratios, are more common in an insurer's renewal book than in its new business (Feldblum [70]). Several personal auto writers provide "renewal

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[10] So also Schraeder [149, p. 165]: "Experience has shown that new business, carefully underwritten, develops poorer overall results than that which has been reunderwritten, and the latter produces poorer results than that recorded by a seasoned or older book of underwriting risks."

discounts," which reflect the lower loss and expense costs after the first policy year.

B.3. Inexperience, Youth, Transience, and Vehicle Acquisition

The relationship between duration of the policy and expected claim frequency results from several factors. Drivers who apply for new auto insurance policies are likely to be inexperienced, young, or "transient" insureds. Also, they have often recently acquired the automobile itself, and they may be unaccustomed to the particular hazards of the vehicle.

- *Experience:* Good driving habits are acquired over time; safety precautions are "second nature" for the experienced driver. Many accidents result from carelessness, not recklessness, so inexperienced drivers have high claim frequencies (Bailey and Simon [12]).
- *Youth:* Young drivers, both male and female, have higher than average claim frequencies, even after adjusting for driving experience. Young drivers with their own residences or automobiles have relatively new auto insurance policies. (Adolescent drivers living at home may be insured on their parents' policies. Since these drivers have high average claim frequencies, they cause a temporary reversal in the generally inverse relationship of frequency with policy duration.[11])
- *Transience:* Many high risk drivers, such as young males, are "transient" insureds, in that they often drop their coverage with one carrier and purchase a policy from another. Termination rates for young male drivers are as high as 20–30% for several reasons:
    - Young male drivers are more likely to voluntarily cancel their policies, perhaps because they move to other locations,

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[11] In general, claim frequency declines as the policy ages. But when adolescent children obtain licenses, claim frequency on the parents' policy increases. This is an example of a classification change, which overwhelms the normal decline in claim frequency. See below in the text.

they get married and switch to their wives' insurers, or they drop their coverage after an accident.

- ◦ Company underwriters are more likely to cancel the coverage of a young male driver than that of an adult driver, since the young male driver is more likely to have caused an accident and be considered too risky to insure.
- ◦ Young male drivers are likely to experience financial difficulties and fail to pay the required premiums.
- ◦ Young male drivers with high premium payments have more incentive to shop around for cheaper coverage.[12]

Many low-risk insureds, such as retired drivers in their 60s and 70s, have termination rates as low as 3 or 4%. Retired drivers have less information about marketplace prices, which younger persons may hear about at the workplace.[13] These low-risk "stable" insureds reduce the claim frequencies of renewal business compared to new business.

- *Acquisition of the Vehicle:* The duration since the inception of the policy is correlated with the time since acquisition of the automobile. Accident frequency often decreases with time since acquisition, as the insured becomes accustomed to the hazards of the particular vehicle. For instance, the insured may have purchased a second hand vehicle during the summer, only to discover that the car skids on icy December roads.

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[12]See Feldblum [68], particularly Figure 7 and the accompanying discussion. Similarly, D'Arcy and Doherty [60, p. 38] speak of "poor risks that move from insurer to insurer as their true risk exposure is discovered." D'Arcy [56, p. 28] lists four reasons for the higher loss ratios of new business: "The inability to surcharge new insureds properly since less information is available, the higher loss potential of insurance shoppers who regularly shift from insurer to insurer in search of bargain coverage, the fact that new insureds include a high proportion of risks not wanted by other insurers, and the possibility that new insureds may be individuals unfamiliar with local driving conditions."

[13]Many policy "terminations" for older drivers result from death, poor health, or other reasons that prevent them from driving, not because they find a cheaper rate with another carrier. Thus, these drivers are not "transient" insureds.

The age of the vehicle (not the time since acquisition) is a classification dimension for physical damage coverages, since the value of the car declines over time.[14] The time since acquisition of the vehicle, not its age, is important for liability coverages. The two classification dimensions are the same only when the insured purchases a new automobile. Contrast a recently acquired five-year-old car with a new model car bought two years ago. The two-year-old car would have the higher physical damage relativity, and the five-year-old car would have the higher liability relativity.[15]

B.4. Reunderwriting

The relationship between loss ratios and the duration since policy inception may also be affected by the carrier's reunderwriting actions. D'Arcy and Doherty [60] suggest that "the accumulation of private information by the contracting insurer" causes declining loss ratios as the policy ages. The importance of this private information depends on the insurer's underwriting philosophy and on the power of this information to predict future loss costs.[16]

In workers compensation, the loss engineering services provided by the insurer, as well as its encouragement of a safe work environment, reduce claim frequency among persisting insureds. Loss control studies can be expensive, and the insurance carrier lacks the incentive to undertake them for "transient" risks.

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[14]This is true for the "age rating system" that was the predominant pricing procedure for automobile physical damage coverages in the 1960s and 1970s. The "model year rating" system pioneered by the major direct writers in the 1980s assumes that the decline in the value of the vehicle over time is offset by inflationary increases in repair costs. See Chernick [44, pp. 10–11].

[15]These are loss cost relativities, not rate relativities. When setting rates, an insurer must decide whether to use these relativities or other risk classification systems. For the differences between loss cost relativities and rate relativities, see Section 5.

[16]"Underwriting terminations" are less important than voluntary terminations in explaining the differences between young male and adult persistency rates in personal automobile insurance (Feldblum [68], Figure 8). However, underwriting terminations weed out the particularly poor risks, and so they may have a larger effect on the relationship between loss ratios and the duration since policy inception.

Similarly, a successful loss control program initiated by the carrier will encourage the insured employer to retain the coverage.[17]

C. Expenses

Insurance expenses are greater in the year the policy is first issued than in renewal years because underwriting and acquisition expenses are incurred predominantly at policy inception.[18] This is true for both "per policy" expenses, such as the costs of underwriting and setting up files, and "percentage of premium" expenses, such as commissions and premium taxes.

C.1. Life Insurance Expenses

Premium determination for life insurance policies incorporates these expense differences by policy year. For instance, Jordan [108, p. 133] gives the following illustration of a gross premium calculation (see also Neill [138, pp. 53–56]):

$$G\ddot{a}_x \approx 1005\left(1+\frac{i}{2}\right) A_x + .75G + .2G(\ddot{a}_{x:\overline{2}|} - \ddot{a}_{x:\overline{1}|}) + .1G(\ddot{a}_{x:\overline{6}|} - \ddot{a}_{x:\overline{2}|}) + .05G(\ddot{a}_x - \ddot{a}_{x:\overline{6}|}) + 10 + 2a_x,$$

where G is the annual gross premium for \$1000 of insurance, a_x, $\ddot{a}_x$, and A_x are the standard annuity and cost of insurance

[17] The relationship between claim frequency and "transient" risks is also applicable to workers compensation. Commenting on the unprofitability of small workers compensation risks, Kormes [110, pp. 49–50] says: " … this group of risks, which unfortunately float from carrier to carrier, has a great influence on the unsatisfactory small risk situation … "

Small enterprises that mushroom during prosperous years often fail when the economy sours. Since these firms lack the funds for needed workplace safety measures and their workforce often consists of inexperienced employees, their occupational injury rates are high. Those firms that fail face additional costs: Since the employee's alternative to insurance payments is unemployment, claim filings are high.

[18] Cf. Atkinson [11, p. 5]: "When a life insurance contract is sold, many expenses are incurred: marketing expenses, underwriting expenses, issue expenses, commissions and agent bonuses. These acquisition costs usually exceed the first year premium by a wide margin. Acquisition costs may even exceed 200% of premium, especially for smaller policies."

TABLE 1

ILLUSTRATIVE EXPENSE COSTS FOR A WHOLE LIFE POLICY

Policy Year	Percent of Premium Commissions	Percent of Premium Other	Percent of Face Value	Dollars per Policy
1	60%	5%	2.5%	$ 200
2	10	5	0.2	50
3	10	3	0.2	25
4	5	3	0.2	25

functions, and expenses are as follows:

per premium: 75% of the first premium, 20% of the second premium, 10% of the third through sixth premiums, and 5% of each premium thereafter;

per amount: $10 at the beginning of the first year, and $2 at the beginning of each subsequent year per $1,000 of insurance;

per claim: $5 per $1,000 of insurance as the cost of settlement.

An asset share pricing model uses a table of expense rates, which might begin as in Table 1 (Belth [15, pp. 22–24]).

C.2. Casualty Insurance Expenses

The loss ratio and pure premium methods that are used for casualty insurance ratemaking do not differentiate insurance expenses by policy year. An expected loss ratio is derived from company budgets (e.g., advertising), agency contracts (e.g., commissions), state statutes (e.g., premium taxes), and Insurance Expense Exhibit data (e.g., general expenses). The experience loss ratio, after trending, development, and similar adjustments, is compared to the expected loss ratio to determine the indicated rate change (Stern [151]; Lange [113]; Graves and Castillo [86]; McClenahan [129]; Brown [30]). This procedure treats all expenses identically, regardless of their actual incidence.

C.3. Policy Duration and Insurance Expenses

Property/casualty expense costs, like life insurance expense costs, are greater in the original year of issue than in renewal years.

- Underwriting expenses incurred predominantly in the first year include salaries, costs of policy issuance and underwriting reports (e.g., DMV reports for automobile insurance or credit reports for homeowners), and expenses allocated as overhead on salaries. Renewal underwriting may be only a perfunctory review of past loss experience.
- Loss control expenses incurred either at or before policy issuance include technical inspections (boiler and machinery), landfill inspections (environmental impairment), loss engineering services (workers compensation), financial analyses (mortgage guarantee), and building inspections (commercial fire). Few inspections are repeated at renewal dates. Those which are, such as some workplace safety inspections for workers compensation, are less comprehensive than the original underwriting inspection.
- Acquisition expenses for direct writers are greater in the first year than in renewal years. Three types of commission schedules are used in property/casualty insurance:
  - Independent agency companies pay level commissions, such as 15% or 20% of premium, in all years. The level commission structure is needed because the agent "owns the renewals" (*National Fire Insurance* case of 1904). That is, the insurer may not bypass the agent when renewing the policy. Rather, the agent may place the insurance with any carrier he or she represents, as long as the consumer agrees. A lower commission in renewal years would induce the agent

to move the policy to a competing insurer and obtain a "first year" commission.

The level commission structure does not reflect the actual incidence of acquisition expenses, since agents spend more effort writing new policies than renewing existing policies. Because of this (and other reasons), many economists consider the independent agency system to be inefficient.[19] In the personal lines of business, direct writers are steadily gaining market share, and the level commission structure is becoming less important. As the asset share pricing model shows, a level commission structure works well for risks that terminate quickly. It works poorly for risks that endure with the carrier. In other words, a level commission structure is inappropriate for the persisting and profitable risks.

- Many direct writers pay commissions that vary by policy year: high first year commissions (20% to 25%) and low renewal commissions (2% to 5%). Since the insurer, who is the agent's sole employer, owns the renewals, the agent has no opportunity to move the policyholder to a competing carrier.

- Some direct writers have either a salaried sales force or a sales force that is compensated partly by commission and partly by salary. The acquisition costs incurred by the insurer may be determined by the actual incidence of these expenses. For instance, suppose the agent receives salary and benefits of $100,000 a year, and spends 80% of his or her time obtaining $500,000 of new business a year and 20% of his or her time servicing $2 million of renewal business. The insurer is paying the equivalent of a 16% commission

[19]The primary "other reasons" are the relative ease of automating a captive agency compared to an independent agency and the ability of direct writers to integrate distribution strategy with underwriting strategy. The efficiency of insurance distribution systems is a disputed issue; see Joskow [109], Cummins and VanDerhei [54], Cather, Gustavson, and Trieschmann [43], and Berger, Cummins, and Weiss [17].

on new business and a 1% commission on renewal business.[20]

- Most "other acquisition expenses," such as advertising, subsidies for new agents, and development costs for expanding or automating distributions systems, are expended at or before the inception date of the policy.

Casualty actuaries often differentiate between "fixed" and "variable" expenses. Variable expenses are those that are directly proportional to premium. Fixed expenses do not vary directly with premium: some are "per policy" expenses, such as some underwriting expenses, and some are "sunk costs" related to the block of business as a whole, such as certain advertising costs. The appropriate treatment of fixed and variable expenses is discussed in Section 4.

D. Persistency

Persistency rates, or retention rates, are the crux of asset share pricing models. Independent insurers pay careful attention to personal automobile retention rates, though rating bureaus have yet to incorporate them into their ratemaking procedures.

D.1. Policy Duration and Profitability

Persistency rates are most important when the net insurance income varies by duration since inception of the policy. Consider first a whole life insurance policy.

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[20] Formally, if x is the first year commission rate and y is the renewal commission rate, then we have the following:

- The total salary and benefits earned by the agent equals the implicit commission rates times the premium volume, or

$$\$500{,}000(x) + \$2{,}000{,}000(y) = \$100{,}000.$$

- The implicit commissions earned on new and renewal business should be proportional to the amount of time spent on these two components of the business, or

$$0.80 \equiv 0.20 = \$500{,}000(x) \equiv \$2{,}000{,}000(y).$$

Solving these two equations yields $x = 16\%$ and $y = 1\%$.

Net insurance income

= (premium collected + net investment income)

– (benefits paid + increase in policy reserves

+ incurred expenses + federal income taxes).

The standard non-forfeiture laws of each state cause the expected value of

(premium + net investment income)

– (benefits paid + increase in reserves)

to be rather level each year, whether the policyholder persists or terminates.[21]

D.2. *Influences on Persistency Rates*

Persistency rates vary widely by company. In personal auto, for instance, State Farm has high retention rates because: it targets a suburban and rural insured population; it offers low premium rates; and it provides renewal discounts.[22] Many independent agency companies have low retention rates because: the agents, who are not beholden to any particular carrier, can move the insured to whichever company offers the lowest rates; and these carriers use little consumer advertising.[23] The typical personal auto direct writer has retention rates of about 90%, ranging from under 85% in the first policy year to about 95% after ten years. In other words, termination rates (lapse rates) are over

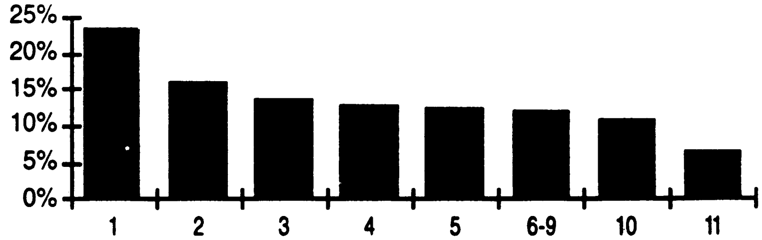
[21] The expected value will be level, but the actual value will vary, being lower in the year of death. Preliminary term policy reserves increase the value of net insurance income in the first policy year, though not enough to offset the higher underwriting and acquisition expenses.

[22] The terms "persistency rates" and "retention rates" are used interchangeably in this paper.

[23] Life insurance shows similar variability. With regard to whole life persistency, LIMRA [117, p. 286] notes: "Regardless of policy year, there is considerable variation in lapse experience across companies. For policy years one through ten, one quarter of the lapse rates are below ten percent. Another quarter of the lapse rates generally exceed twenty percent." See also Anderson [8, p. 373]; Winn et al. [168]; Moorehead [134, p. 295]; Belth [15, p. 19].

FIGURE 1

LONG-TERM ORDINARY LIFE LAPSE RATES



15% in the first policy year and decline to about 5% after ten years.

Persistency improves with duration since policy inception. Figure 1 shows industry-wide ordinary life insurance lapse rates (vertical axis) by policy year since inception (horizontal axis) (LIMRA [115, p. 338, Table 6]; Buck [33, p. 275]).

There is an intuitive relationship between duration and persistency for both life and casualty insurance. In the original year of issue, many policyholders are undecided about the relative value of the policy and the required premiums. Some insureds may decide that the insurance is not worthwhile; some may be dissatisfied with their carrier's service; some may believe the premium is too high and continue shopping for a lower rate; and some may be unable to afford any insurance. Thus, voluntary termination rates during the first year are high. In casualty lines of business, moreover, where underwriting terminations are permitted, carriers often reevaluate newly acquired risks that have had accidents in the first one or two policy years.

Once a policyholder has kept the policy for several years, it is likely that he or she will renew the contract for another year. The insured is probably satisfied with the carrier's service and finds the premiums reasonable and affordable. And unless the insured's classification changes, underwriting terminations are unlikely.[24]

D.3. Termination Rates and Probabilities of Termination

Persistency may be analyzed either by termination rates or by probabilities of termination. The *termination rate* is the number of terminations during a given renewal period divided by the sum of terminations during that period plus policies persisting through that period. The *probability of termination* is the number of terminations during a given renewal period divided by the number of originally issued policies in that cohort. (A cohort is a group of policies written in a given issue period.[25])

For instance, suppose an insurer writes 100 auto policies in 1990, 20 risks lapse the first year, 10 lapse the second year, and 5 lapse the third year. The termination rates are 20% [= 20 ≡ 100] the first year, 12.5% [= 10 ≡ 80] the second year, and 7.1% [= 5 ≡ 70] the third year. The probabilities of termination are 20% [= 20 ≡ 100] the first year, 10% [= 10 ≡ 100] the second year, and 5% [= 5 ≡ 100] the third year. Termination rates more clearly distinguish persistency patterns by classification.[26] Prob-

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[24]Classification changes are common in personal automobile. Most changes are from higher to lower rated classifications, such as a movement from youthful to adult driver, from unmarried to married driver, or from urban to suburban resident. These changes rarely provoke underwriting terminations. Some changes are to higher rated classifications: for example, an adolescent son may turn 16 and obtain a driver's license, the use of the car may switch from "pleasure" to "drive to work," or the insured may move from a low rated territory to a higher rated territory. These changes may lead to a re-evaluation of the risk. The most common impetus for re-underwriting, though, is not classification changes but poor claim experience, as noted in the text.

[25]Compare Huffman's distinction between asset shares and the asset fund [95, pp. 278, 279]. A_t is the "asset share per $1,000 unit of coverage *in force* at the end of policy year t." F_t is "the asset fund per I_0 *initially issued* units, accumulated at interest to duration t" (italics added). Huffman notes that "the asset share prorates funds among policyholders so that each gets its share; the asset fund does not, thereby measuring the accumulated funds held by the insurer."

[26]For instance, suppose 100 policies were issued to adult drivers and 100 policies were issued to young male drivers. By the fifth renewal, 20 of the adult drivers had lapsed, and 60 of the young male drivers had lapsed, leaving 80 adult drivers and 40 young male drivers. By the next renewal, an additional 5 adult drivers and 5 young male drivers terminate their coverage. The termination rates are 5 ≡ 80, or 6.25%, for adult drivers

abilities of termination, in certain analyses, provide a better portrayal of the insurer's profitability.[27]

D.4. Persistency by Classification

Persistency rates vary greatly by classification. In personal auto insurance, young male drivers have high termination rates, retired drivers have low termination rates, and middle-aged drivers are in between. Figure 2 shows average probabilities of termination for these three classifications.

The termination rate differences by classification, of course, are greater. The vertical axis in Figure 2 shows the probability of termination, and the horizontal axis shows the policy period since inception.[28]

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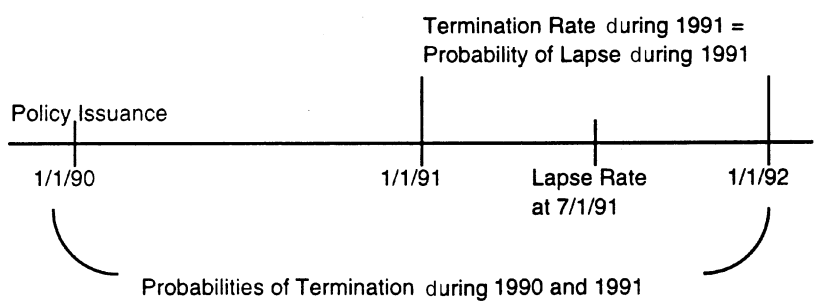
and $5 \equiv 40$, or 12.5%, for young male drivers. The probabilities of termination, however, are 5% for both groups of insureds.

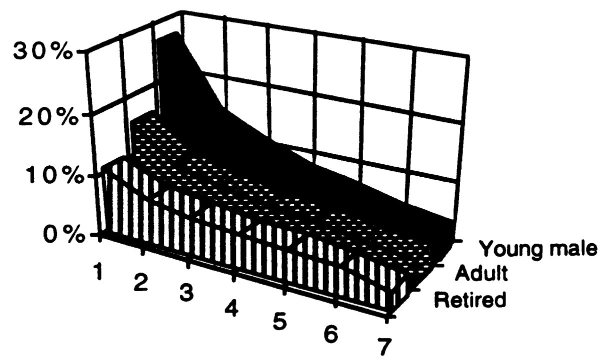
[27] The distinction between termination rates and probabilities of termination is taken from life insurance. The *mortality rate* is the annualized probability that an individual will die at a given time. The corresponding *probability* is the number of deaths at a given age divided by the number of insureds who have attained that age (Batten [14]; Atkinson [11, pp. 51–54]).

The use of these terms here is not identical to that in life insurance. The life insurance lapse rate pertains to a given moment of time. The life insurance probability of lapse is the percent of withdrawing policyholders during the year. The termination rate as used here is equivalent to the probability of lapse. The probability of termination as used here is the percent of original policyholders who terminate in a given year. The diagram below illustrates the use of these terms.



[28] See Feldblum [68] and [70]. LIMRA [116, Tables 8–10] shows similar relationships for long-term ordinary life insurance. Lapse rates for issue ages 20–29 are about double those for issue ages 50–59 at all policy durations.

FIGURE 2



Life insurance persistency patterns are analyzed by issue age, duration, interest rates, sex, rating (standard, preferred, and substandard), policy face amount, premium payment pattern (whole life versus limited payment life; annual, monthly, and payroll deduction), policy form (ordinary life, universal life, graded premium whole life, variable life, traditional term, select and ultimate term), distribution system (general agents, brokers, and branch offices), and numerous other variables.[29] Some of these dimensions are pertinent only to life insurance. For instance, if market interest rates rise faster than the credited rate on a universal life policy, lapse rates may increase. Other dimensions apply to casualty insurance as well. The relationship between the distri-

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LIMRA's most recent studies show lapse rates in the year of issue about 50 to 100% higher than those in the tenth and subsequent renewal years. Older persistency studies, such as Linton [118], Moorehead [133], and LIMRA's studies from the 1970s, show lapse rates in the year of issue about five times higher than those in the tenth and subsequent renewal years. (See LIMRA [117, p. 295, Table 2], for a comparison.) Persistency patterns are sensitive to external economic and social forces, so an unexamined extrapolation from historical experience may be misleading. Similar caution should be used when extrapolating from past personal auto experience.

[29]See Atkinson [10] and [11]. Belth [15, p. 18] notes additional dimensions, such as policyholder's income, occupation, previous ownership of life insurance, experience of the agent, and presence of policy loans. Bluhm and Koppel [25] discuss the variables affecting health insurance persistency patterns.

bution system and persistency patterns is particularly important for casualty insurance.

The dependence of persistency patterns on these dimensions warrants a careful analysis of the available experience. For an independent agency company to use persistency patterns derived from direct writers makes as much sense as for an insurer to use claim frequencies from adult drivers for young male insureds. Similarly, the persistency patterns between urban and rural territories may differ as much as loss costs differ between these territories. The termination rates used in Sections 4 through 7 are illustrative; only by coincidence would they be appropriate for a given company and a given block of policies.

E. Discount Rates

Asset share models examine cash flows and revenue streams over the lifetime of the policy. Future profits and losses of each policy year are discounted to the original issue date to determine present values.

E.1. Life Insurance Discount Rates

In non-participating whole life insurance contracts, both premiums and benefits are fixed at issue. Claims are paid soon after death, so there is no "settlement lag." The discount rate used to determine the present values of future premiums and benefits for statutory policy reserves is limited by the state's Standard Valuation Law. Life insurance policy reserves do not have the uncertainty of casualty insurance loss reserves, which are affected by inflation rates, court decisions, jury awards, and social expectations.

The life insurance actuary using an asset share model begins with known quantities: premium, death benefits, and policy reserves. With appropriate assumptions for mortality and withdrawal rates, he or she can determine statutory or GAAP book profits of each year. All that is needed is a discount rate to determine the present value of future earnings.

E.2. Casualty Insurance Issues

Casualty claims are not settled immediately after the accident. Under tort liability compensation systems, claim investigation, determination of liability, and legal negotiation and adjudication may delay settlements for months or years. In the no-fault lines of business, such as workers compensation and automobile personal injury protection, wage loss reimbursements are made only as the loss is accrued, so payments stretch out over years.

Property/liability insurance accounting, whether statutory or GAAP, records incurred losses on an undiscounted basis, resulting either in underwriting losses or in lower underwriting profits than if discounted loss reserves were held (Lowe and Philbrick [123]; Lowe [120], [122]). The investment income in the Annual Statement or in the Insurance Expense Exhibit—which may be viewed as offset to the underwriting loss—is the present investment income from the company's financial assets, not the investment income expected in the future (Feldblum [69], [74]; Bingham [19]). Property/liability insurance accounting, both statutory and GAAP, does not match the underwriting experience on a block of policies with the investment experience for the same block of policies. This matching, though, is essential for asset share pricing models. Several methods of matching underwriting and investment experience may be used:

- Record undiscounted incurred claims, but include an offsetting investment income account tied to the assets supporting the unpaid losses (Ferrari [77]; option three of Salzmann [147]; pricing models one and three of Robbin [144]).

- Record cash transactions, not the accounting statement incurred losses. The asset share model looks like an expanded (multi-period) internal rate of return model.[30]
- Record discounted loss reserves. The discount rates for unpaid losses may be market interest rates, risk-free rates, or "risk adjusted" rates.[31]

For simplicity, this paper uses the third method. The illustrations speak of "discounted incurred losses" without specifying the method of discounting. Note that the discount rate used to determine the present value of unpaid losses at the accident date need not be the same as the discount rate used to determine the present value of future earnings at the issue date.[32]

E.3. Rate Revisions and Rates

Casualty pricing methods often determine rate revisions and rate relativities, not actual rates. For instance, the actuary may determine that overall statewide rates should be increased 10%, or that the rate relativity for young male drivers should be changed from 1.750 to 1.850.

Asset share pricing determines rates, not rate revisions. Since there is no overall statewide rate, the actuary selects "pivotal" classifications for which an actual rate is determined. Interpola-

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[30] Internal rate of return and asset share pricing models, however, have different viewpoints. The internal rate of return model views the insurance transactions from the equityholder's perspective. It requires surplus commitment and equity flow assumptions (Feldblum [71]). The asset share model uses the insurance company's perspective and need not consider equity flows. For instance, Anderson [8] determines the ratio of the present value of profits to the present value of premium, not the return on investment or surplus. Thus, the asset share model is similar to a multi-period internal rate of return model in its construction, not in its perspective.

[31] Woll [170] and Bingham ([19], [20]) use risk-free rates. Fairley [67], Myers and Cohn [135], and Butsic [37] use risk adjusted rates, though they determine the adjustment differently. The need for risk margins is discussed in CAS Committee on Reserves ([40], [41]) and CAS Committee on the Theory of Risk [42]. See also D'Arcy ([57], [58]); Lowe [121]; FASB [80]; and Tiller, et al. [153].

[32] See Paquin [141] for a life insurance discussion of different discount rates for cash inflows and outflows. On the appropriate discount rate for determining present values of future uncertain profits, see also Shapiro [150].

tion and relativity analyses may be used for other (non-pivotal) classifications.

For instance, the life actuary may use an asset share model to determine whole life insurance rates for standard rated, non-smoking males at five year age intervals (e.g., ages 30, 35, 40). The mortality and persistency rates at these ages are derived from their own experience combined with the graduated experience for the entire insured population. Whole life insurance rates for a male aged 37 would be determined by interpolation of the rates for age 35 and age 40.

The same procedure is applicable to casualty ratemaking. Rates are determined for pivotal classifications, such as adult married drivers in a given group of territories, or young unmarried male drivers in an urban area.[33] To form the rates, one uses the experience of these classifications as well as the graduated experience of similar classifications. Rates are then determined for non-pivotal classifications by interpolation and relativity analyses.[34]

3. ASSET SHARE MODELING—FOUR ILLUSTRATIONS

Asset share modeling is particularly valuable when differences in termination rates influence expected profits. The first three illustrations in this paper show how an asset share model deals with such conditions. The fourth illustration shows how the movements of the underwriting cycle can be incorporated into policy pricing. The illustrations are as follows:

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[33] Thus, in appearance, asset share pricing is more akin to pure premium ratemaking than to loss ratio ratemaking. However, this similarity is deceptive. Both the pure premium approach and the loss ratio approach seek to estimate the expected loss costs during the future policy year. The asset share method assumes that the actuary has already estimated future loss costs, expense costs, and persistency rates, and now seeks to determine optimal premium rates.

[34] A similar procedure is used by Brubaker [31, pp. 107, 108]. Brubaker uses interpolation among "grid points" for geographic rating, similar to the interpolation among pivotal ages for asset share pricing.

1. *Business Expansion:* When an insurer begins writing in a new territory or policyholder classification, most risks are new business, with high loss and expense ratios. Traditional ratemaking procedures show high combined ratios, and the pricing actuary may conclude that the business is not profitable. But this is simply the cost of building an insurance portfolio. New business is generally "unprofitable," though the "loss" may be offset by the future profits in a stable renewal book. Asset share modeling helps the actuary determine the true profitability of the insurance writings.

2. *Classification Relativities:* Traditional ratemaking methods determine classification relativities from loss ratios, perhaps tempered with "expense flattening" procedures. Persistency differences among classifications can cause these methods to be misleading. If persistency is ignored, then rate relativities are too low for the poorly persisting classes and too high for the long-persisting classes. The illustration shows an asset share model determination of personal automobile classification relativities for young male drivers.

3. *Competitive Strategy:* Traditional ratemaking procedures match premiums to anticipated losses and expenses. They ignore the future profits and losses from expected renewals. Moreover, they ignore the effects of rate revisions on policyholder retention and new business production. A rate increase will reduce policyholder retention, particularly among the most profitable risks, who can obtain coverage from other carriers. Competitive pricing strategy is to raise or lower rates such that the expected changes in policyholder retention, new business production, and lifetime policy profits or losses will maximize long-term income. The illustration shows how asset share modeling determines the optimal retired driver discount in personal automobile insurance.

4. *Underwriting Cycles:* Market share and profit objectives are the linchpins of competitive strategy. Attempts to gain market share drive the soft phase of the cycle, and attempts to restore profits drive the hard phase. It is often unclear whether market share gains during the soft phase combined with profits on these policies during the hard phase will lead to satisfactory long-term income. Asset share modeling enables the actuary to quantify the effects of different pricing strategies on overall returns.

These illustrations demonstrate the power of the asset share pricing technique. Each illustration expands the scope of the issues being addressed:

- In the *business expansion* illustration, all the actuarial data are given. The rate levels, rate relativities, and classification scheme are predetermined. The pricing actuary uses the profitability measures provided by the asset share analysis to determine marketing strategy.
- In the *rate relativities* illustration, the classification scheme and business strategy are given, but not the rate levels or rate relativities. The pricing actuary uses the asset share analysis to determine class rates to achieve the desired profits from each group of insureds.
- In the *competitive strategy* illustration, neither the classification scheme nor the rate relativities are given. Rather, the pricing actuary uses the asset share analysis to determine the class groupings that will optimize the insurer's return.
- In the *underwriting cycles* illustrations, the issues are more general. The insurer must decide whether a particular line of insurance is expected to be profitable, and whether entry or exit from a given market is indicated.

4. ILLUSTRATION 1—BUSINESS EXPANSION

Company growth or contraction distorts reported financial results, particularly when the expected loss and expense ratios depend on the time since inception of the policy. Even without this dependence, business growth raises the statutory combined ratio, since loss reserves are held at undiscounted values and acquisition costs are written off when incurred. Deferring acquisition expenses and adding investment income, to give a "GAAP operating ratio," does not fully resolve the problem, since the investment income received in any calendar year derives from the business insured in the past. If the insurer is growing rapidly, the investment income received is smaller than the present value of the investment income expected from the current block of business.[35]

To circumvent this problem, the following illustrations assume that all figures are restated on a fully discounted basis. For instance, the $656 of the first policy year's losses in the "business expansion" illustration does not mean statutory incurred losses

[35]Because premiums, losses, and insurance industry assets grew faster than after-tax investment returns during the 1970s and 1980s, statutory operating ratios were overstated by about 2.2 percentage points (Feldblum [74]; see also Butsic [38]).

The effects of business growth on statutory operating ratios can be grasped most easily by an illustration. In a steady state environment, with no growth, the statutory operating ratio equals the "true" operating ratio. Suppose the insurer writes $100 million of premium each January 1, has no expenses, pays $100 million of losses three years later, and earns a 5% investment yield. Each year, it holds about $300 million of loss reserves, on which it earns $15 million of investment income. (For simplicity, we have not assumed compounding of the investment balances.) The statutory operating ratio is

$$\$100 \text{ M losses} - \$15 \text{ M investment income} \equiv \$100 \text{ M premium} = 85\%.$$

The present value of losses when the policy premium is collected is $85 million (again, assuming simple interest, not compound interest, for simplicity of illustration). The "true" operating ratio is also 85%.

What if the company's business volume expands? Consider the extreme case: what if the company begins writing the business this year? The "true" operating ratio is still 85%. But the company has only $100 million of reserves the first year, on which it earns $5 million of investment income, leading to a statutory operating ratio of 95%.

In practice, of course, the difference is not so great. But as long as a company is growing more quickly than the after-tax investment yield, the statutory operating ratios understate the company's true profitability.

of \$656, but fully discounted losses of \$656. Since the illustration uses a policy year model, not a calendar year model, there is no "property/casualty type" deferred acquisition cost. There is, of course, a "life insurance type" deferred acquisition cost, since underwriting and acquisition costs are higher in the original year of issue than in renewal years. The asset share pricing model incorporates this phenomenon, though without setting up an explicit asset.[36]

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[36]The difference between a "life insurance type" deferred policy acquisition cost (DPAC) and a "property/ casualty type" DPAC clarifies the workings of the asset share model. Suppose the insurer writes a personal auto policy on July 1, 1995, for a \$1,000 premium, and it expects to renew the policy four times. Deferrable acquisition costs, such as agency commissions, are 24% the initial year and 6% in renewal years.

- Property/casualty statutory accounting says that all acquisition costs are written off when incurred. On July 1, 1995, the company collects \$1,000 in premium, sets up an unearned premium reserve of \$1,000, pays \$240 in expenses, and shows an accounting loss of \$240. Over the next twelve months, as the premium is earned, the unearned premium reserve declines to \$0.
- Property/casualty GAAP statements show a deferred policy acquisition cost asset that is set up when the policy is issued and is taken down as the premium is earned. On July 1, 1995, the company collects \$1,000 in premium, sets up an unearned premium reserve of \$1,000, pays \$240 in expenses, sets up a DPAC asset of \$240, and shows no accounting loss or gain. Over the next twelve months, as the premium is earned, both the unearned premium reserve and the DPAC asset decline to \$0. For instance, on December 31, 1995, the earned premium is \$500, the unearned premium reserve is \$500, and the DPAC asset is \$120.

  On July 1, 1996, the company again collects \$1,000 in premium, sets up an unearned premium reserve of \$1,000, pays \$60 in expenses, sets up a DPAC asset of \$60, and shows no accounting loss or gain. The accounting follows the same procedures as in the initial policy year. There is no interaction between the initial year of issue and renewal years.
- Life insurance accounting, both statutory and GAAP, shows a DPAC asset that is set up when the policy is issued and is taken down over the lifetime of the policy. For simplicity, suppose that the company is certain that it will renew the policy exactly four times, and that the interest rate and inflation assumptions are both 0% per annum. The total acquisition expenses for this policy are $\$240 + 4 \times \$60 = \$480$. The policy persists five years, or 60 months, so these expenses must be amortized at \$8 a month. On July 1, 1995, the company pays \$240 in expenses and sets up a DPAC asset of \$240. It reduces this asset by \$8 a month, so on December 31, 1995, it has a DPAC asset of \$192, not \$120, and on June 30, 1996, the DPAC asset is \$144, not \$0. (In practice, of course, the amortization of the life insurance DPAC asset is more complex, depending on mortality and interest rate assumptions; see Tan [152].)

The asset share model is the pricing equivalent of the life insurance accounting system. It effectively "amortizes" the first year expenses over the lifetime of the policy when determining premium rates.

Growth in a New Territory

Suppose a profitable personal automobile direct writer expands into a new geographic area in 1992. To ensure an accurate financial appraisal of the expansion, all statistics on the new operation are separately recorded. "Fixed" costs peculiar to the expansion, such as subsidies for new agents, construction costs for a new branch office, and extra advertising expenses during the first year, are charged to a corporate account; they are not included in these statistics.

The insurer writes 10,000 policies in 1992, at an average annual premium of $800. The company is satisfied with the new business production, and 10,000 new policies are again written in 1993. In early 1994, the policy year 1992 results are tabulated and show a loss of $2.4 million after full discounting of loss reserves.

The insurer accepts the $2.4 million loss as "start-up" costs in addition to what it has budgeted to the corporate account, and it continues to add 10,000 new policies a year. But when policy year 1993 results, tabulated in early 1995, reveal an additional loss of $1.9 million, company management is concerned. In early 1996, policy year 1994 results show a further loss of $1.3 million. Company management concludes that it erred by expanding too rapidly, and the growth program is curtailed. The pricing actuary tries to explain about the cost of new business but is summarily dismissed.

Has the company indeed erred? The asset share model shows that the company is earning a 19% return on surplus, despite its inexperienced sales force and lack of name recognition in this area. The error lies in curtailing a successful program. Yet actuarial generalizations do not suffice. The true return and the cause of the reported losses must be clearly presented.[37]

[37] Brealey and Myers [28, pp. 272–275] present a similar illustration emphasizing the difference between economic (or true) earnings and book earnings.

Asset Share Assumptions

How can a 19% return on surplus be consistent with losses of $5.6 million in three years? Assume the following conditions for this block of business:

1. *Premiums:* The average policy premium is $800 in 1992. The loss cost trend is 10% per annum, and "fixed" expense costs are rising at 5% per annum. Regulators are not averse to insurers in this state, and the company expects average rate increases of 9% per annum.
2. *Losses:* The fully discounted loss ratio on new business is 82% in 1992, or an average of $656 a car. Loss costs are increasing at 10% per annum. The company expects the average loss costs on any policy to improve by 3% a year since policy inception, after adjusting for inflation. For example, the average loss cost for new business written in 1993 will be $\$656 \times 1.1 = \$722$. The average loss cost in 1993 for policies originally issued in 1992 will be $\$722 \equiv 1.03 = \$701$.[38]
3. *Expenses:* A direct writer has high expense costs the first year but low expense costs in renewal years. Simulated expense costs are shown in Table 2. Expenses which vary directly with premium (such as commissions and premium taxes) increase at the same rate as premium. We assume that "fixed" expenses, such as salaries and rent, increase at 5% per annum.
4. *Persistency:* Termination rates vary by company, geographic location, class of business, and various other dimensions. The pricing actuary has chosen termination rates based on prior experience, beginning at 15% in the

[38]A more realistic model would show a larger effect in the first few policy years and a smaller effect in later years. For instance, the improvement in average loss costs from policyholder persistency may be 7% in the first year, 5% in the next year, 4% in the next year, and gradually decline to 1% after ten years.

TABLE 2

ACQUISITION AND UNDERWRITING EXPENSES BY POLICY YEAR

	New Policies		Renewal Policies	
	Fixed Expense Provision	Variable Expense Provision	Fixed Expense Provision	Variable Expense Provision
Agency Commissions	0.0%	25.0%	0.0%	3.0%
Advertising and Other Acq.	5.0	0.0	0.0	0.0
General Expenses	12.0	3.0	3.0	1.0
Premium Tax	0.0	2.0	0.0	2.0
Taxes, Licenses, and Fees	0.8	0.2	0.8	0.2
Total Expenses	17.8%	30.2%	3.8%	6.2%

year the policy is originally issued and declining to 8% after 15 years.

5. *Present Values:* The company determines the present value of future earnings by discounting at its cost of capital, which is 12% in this illustration.

The Model

The asset share model is shown in Exhibit 1. The present values of current and future profits and premium are $480 and $5,012, respectively, for a return on sales of 9.6%. If the insurer has a premium to surplus ratio of two, then the return on surplus is 19.2%.[39]

[39]To estimate the total return on surplus, one must consider federal income taxes and the investment return on surplus funds. The investment return on surplus funds as a percentage of premiums depends on the premium to surplus ratio. Federal income taxes depend on a combination of tax strategy and investment strategy (see Almagro and Ghezzi [7] for details). To avoid additional complexities, the illustrations do not incorporate these items. In this example, the effects are largely offsetting. If the investment return on surplus funds is 9% per annum, and the marginal tax rate is 35%, then the before-tax return on surplus is 19.2% + 9.0% = 28.2%, and the after tax return is 65% × 28.2% = 18.3%. In general, however, the effects are not offsetting, and these items must be considered in pricing.

Let us consider each column in Exhibit 1.

1. Column 1 shows the year since the inception of the policy. The policy in this illustration was issued in 1992. The figures in the exhibit pertain to this cohort of policies only, not to policies issued previously or subsequently.

2. Column 2 shows the average premium: \$800 a car in 1992, increasing at 9% per annum.

3. Column 3 shows the average losses. The discounted loss ratio is 82% for new business, so 82% of \$800 is \$656. Losses increase at 10% per annum. At each renewal, loss experience is slightly better, because poor risks voluntarily terminate and reunderwriting efforts weed out unprofitable insureds. The illustration presumes that the average loss costs in any policy year are 3% lower than the average loss costs in the preceding policy year, after adjustment for loss cost trend.

   In this illustration, \$656 increased by 10% is \$722; \$722 decreased by 3% is \$701. Although the aggregate loss cost trend (10%) is greater than the premium trend (9%), the loss ratio for ten year old business (68% = 1,186 ≡ 1,738) is lower than the loss ratio for new business (82%).

4. Columns 4 through 7 show expenses. Expenses that vary directly with premium are 30.2% of premium in the year of issue and 6.2% in renewal years. Thus, 30.2% of \$800 is \$242, and 6.2% of \$872 is \$54.

   Fixed expenses average 17.8% of premium in the year of issue; 17.8% of \$800 is \$142. Fixed expenses for renewal years are now 3.8% of premium. Consider a policy first issued in a previous year having an \$800 premium this year. It would have fixed expenses of 3.8% × \$800 = \$30.40. This policy would have fixed expenses of $\$30.40 \times 1.05 = \$31.92$ next year; $\$31.92 \times 1.05 = \$33.52$ the succeeding year; and so forth.

Thus, in the asset share model, the renewal fixed expense column shows $0 in the initial year of issue, then $31.92 in the first renewal year, $33.52 in the second renewal year, and so forth (rounded).

5. Column 8 shows the expected persistency rate. The entries indicate that 85% of new policyholders persist into the second year; 86% of second year insureds persist into the third year; and so forth. The persistency rates in this illustration are low in the year of issue (85%) and increase gradually to 92% by the fifteenth year.
6. Column 9 shows the cumulative persistency rate, or the percentage of original insureds who persist into any policy year. For instance, 85% of original policyholders persist into the second year; 73.1% [$= 0.85 \times 0.86$] of original policyholders persist into the third year; and so forth.
7. Column 10 shows the profit in each policy year. The profit is the product of the cumulative persistency rate and the policy year income, where the income equals premiums minus discounted losses minus expenses. For instance, in the third year, policy year income is $950 – $748 – $59 – $34 = $109. But only 73.1% of original policyholders persist into the third year, so 73.1% of $109 is $80.[40]
8. Column 11 shows the discount factors for future earnings. The company's cost of capital in this illustration is 12%, so Column 11 is 12% compounded annually (e.g., $1.12^2 = 1.25$).
9. Column 12 shows the present value of future earnings, or Column 10 divided by Column 11. Similarly, Column 13

[40]Premiums are assumed to be collected and expenses are assumed to be paid at the beginning of each policy year. Losses are discounted to the beginning of each policy year.

TABLE 3

RESULTS BY YEAR OF ISSUE AND POLICY YEAR SINCE INCEPTION ($000)

	Year Policies are Originally Issued			
Policy Year of Earnings	1992	1993	1994	Total
1992	−2,400			−2,400
1993	726	−2,625		−1,899
1994	803	743	−2,873	−1,327

shows the present value of future premiums, or Column 2 times Column 9 divided by Column 11. The totals of Columns 12 and 13 are $480 and $5,012, respectively. In other words, for a policy issued in 1992, the company expects to earn profits with a present value of $480 over the next 15 years. The present value of the premiums charged this insured, during the same period and with the same discount rate, is $5,012.

Accounting Results and Long-Term Profitability

The company reported earnings of −$5.6 million for the first three policy years, even after full discounting of losses. This is the result that traditional actuarial pricing techniques would show. Calendar year statutory financial statements, which use undiscounted loss reserves and write off all underwriting and acquisition expenses when incurred, show worse results.

The dependence of loss and expense ratios on the year since the policy was first issued explains the difference between the $5.6 million loss shown by traditional pricing analyses and the 19% return on surplus shown by the asset share model. The results by year of issue and by policy year since inception appear in Table 3.

The entries in the "1992" column are taken from Column 10 of Exhibit 1. The entries in the "1993" column are derived from an asset share model beginning one year later. Premiums begin 9% higher, losses begin 10% higher, and "fixed" expenses begin 5% higher. The entry in the "1994" column is derived from an asset share model beginning two years later.

Federal Income Taxes

To simplify the presentation, federal income taxes are not considered in these illustrations. The simplest way of incorporating income taxes is to multiply the "profit" column in the illustrations by the marginal tax rate. Thus, the pre-tax loss of $240 in the year of issue is an after tax loss of $156 (assuming a marginal tax rate of 35%). The pre-tax profit of $72.6 in the second policy year is an after-tax profit of $47.2.

With this procedure, the discount rate used to determine the present value of losses in Column 3 at the beginning of the corresponding policy year should be a before-tax discount rate appropriate for losses, and the discount rate used to determine the present value of profits at the original policy writing date in Column 11 should be an after-tax discount rate. If federal income taxes are first applied to the present value of profits in Column 12, then the discount rate in Column 11 should be a before-tax discount rate. In addition, the federal income taxes must also be applied to the present value of premiums in Column 13.

Alternatively, one could use after-tax values of premiums (revenues), losses, and expenses in Columns 2 through 7. In other words, the $800 of premium in the year of issue would be replaced by an after-tax revenue of $520. If this procedure is followed, then the discount rates used in Columns 3 and 11 should be after-tax discount rates.

Profitability Measures

Different measures of profitability can be incorporated in an asset share model. The illustration discounts future earnings at the company's cost of capital, implying that profits should be measured with a return on equity. To avoid the complexities of converting statutory surplus to GAAP equity, the illustration assumes that surplus equals equity and that the insurer writes at a two to one premium to surplus ratio.[41] Alternatively, one can use the premium to GAAP equity ratio for this insurer to directly obtain a return on equity.

One could also use asset share modeling to determine the "break-even" point. The company may ask: "Is writing insurance policies more profitable than simply investing the equity in financial securities of similar risk?" Assume that securities of similar risk are yielding 10% per annum. The insurer would use a 10% discount rate in Columns 3 and 11, discount losses to the same date as premiums are collected, and determine whether the present value of the total in Column 12 is greater or less than zero.

One can incorporate asset share pricing into an internal rate of return model. Instead of the "present value of losses" in Column 3, one would show several columns of cash transactions: losses paid, investments made, and investment income received. One would combine the cash transactions from the insurance operations with assumed equity flows and determine the internal rate of return to the equity providers (see Feldblum [71]).

In sum, asset share pricing is not restricted to any particular measure of profitability. Rather, whatever measure is used should be applied to the entire life of the policy, not to a single policy year or a single calendar year.

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[41] In practice, GAAP equity is generally greater than statutory surplus, because of deferred acquisition costs, non-admitted statutory assets, reinsurance penalties for unauthorized and slow-paying reinsurers and for overdue reinsurance recoverables, Schedule P penalties, and differences in the carrying value of subsidiaries. Offsetting these are the non-recognition of deferred federal tax liabilities on unrealized capital gains and the amortization of investment grade bonds in good standing under statutory accounting. See Holman and Stroup [94] and AICPA [4] for comparisons of statutory and GAAP accounting. Rosenthal [145] estimates that average GAAP equity is 25% greater than statutory surplus for property/casualty insurers. In addition, the economic net worth of the insurer is generally greater than GAAP equity because of the unrecognized interest discount in the loss reserves and because of the "goodwill" value of the distribution system (see ASB [1]).

5. ILLUSTRATION 2—CLASSIFICATION RELATIVITIES

Traditional ratemaking procedures determine classification relativities by comparing relative loss ratios or pure premiums among groups of insureds (Conger [45], Stern [151], Hurley [97], Harwayne [91], Finger [81]). For instance, if adult drivers (the "base" class) have average losses of $400 a year, and young male drivers have average losses of $900 a year, then young male drivers are assigned a classification relativity of 2.25. Similarly, if urban residents, with a territorial relativity of 1.50, have an average loss ratio of 70%, and the average loss ratio of all drivers in the state is 75%, then the territorial relativity for urban drivers should be reduced to 1.40 [$= 1.50 \times 70\% \equiv 75\%$].

Persistency Effects on Ratemaking Assumptions

Classification ratemaking has been refined with expense flattening procedures that separate expenses into those that vary directly with premium, or "variable" expenses, and those that do not, or "fixed" expenses.[42] In the first example in the paragraph above, suppose that losses per driver average $500 a year, variable expenses average $150 a year, and fixed expenses average $100 a year. Variable expenses are $\$150 \equiv \$750$ (20.0%) of premium. Average losses are $400 for the base class and $900 for young male drivers, so the gross premiums are

Base class (adult drivers):

$$\text{premium} = \$400 + \$100 + 20\% \times \text{premium},$$

$$\text{or premium} = \$625.$$

Young male drivers:

$$\text{premium} = \$900 + \$100 + 20\% \times \text{premium},$$

$$\text{or premium} = \$1{,}250.$$

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[42] On expense flattening procedures, see ISO [103]; Hunt [96]; Childs and Currie [45]; Wade [159]; Nodulman [140]; McClenahan [129]. The ratemaking terms "fixed" and "variable" expenses are not the same as the corresponding financial terms. The "fixed"

The classification relativity for young male drivers is 2.00 [= 1,250 ≡ 625].

These procedures fail to incorporate differences in persistency patterns among classes of insureds, resulting in inaccurate (and either unprofitable or uncompetitive) classification relativities. In any policy year, fixed expenses, as a percentage of total premium, are lower for young male drivers than for adult drivers, and variable expenses, as a percentage of total premium, are equal for the two classes. But young male drivers have higher termination rates than adult drivers have. Because of the higher termination rates, the ratio of total expenses to total premium *over the lifetime of the policy* is generally greater for young male drivers.[43]

Similar considerations apply to losses. Average losses, adjusted for loss cost trends, decline as the policy matures. The "business expansion" illustration assumed that average losses (after adjustment for trend) decline by 3% in each renewal year. Insureds who terminate quickly have "new business" loss ratios, which are generally higher than "renewal business" loss ratios.[44]

A Heuristic Example

The effects of persistency patterns on relative loss ratios by class depends on the type of classification system used. A simple (albeit unrealistic) example should clarify this.[45] Suppose

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expenses in actuarial ratemaking do vary with volume. However, they generally vary most closely with the number of policies, not with the dollar amount of premium.

[43] See Feldblum [68]. The generalization in the text is more applicable to direct writing insurers than to independent agency companies. Compare also Buck [32, p. 9]: "It is more expensive to handle a policy for a young, single male in a given territory than an adult policy in the same territory. This difference can be attributed to such factors as more frequent policy changes and flat cancellations in the youthful male policies."

[44] The cause and effect relationships are unclear. Perhaps young male drivers, who have higher loss ratios, have poorer persistency, so higher loss ratios also appear on new business. Or perhaps persisting drivers have lower loss ratios, so young male drivers, who terminate frequently, have higher loss ratios. As Stephen D'Arcy has pointed out to me, one must take care not to double count these effects. See also the following paragraphs in the text.

[45] The example is deliberately constructed to show a result opposite to the major conclusions in this paper, to demonstrate that careful analysis of each situation must be

average losses for adult drivers [the base class] are $500 a year, average losses for 17-year-old drivers are $1,000 a year, and all insureds persist for ten years. In other words, the 17-year-old drivers have twice the average loss costs of adult drivers. If all expenses vary with premium (i.e., there are no fixed expenses), their classification relativity should be 2.00.

But suppose that new business risks have average loss costs 25% higher than renewal business. All of the 17-year-old drivers are new business, but only 10% of the adult drivers are new business.[46] The 17-year-old drivers' average losses will drop to $800 during renewal years, so the 2.00 classification relativity is too high. An insurer can profit in the long-run by reducing the classification relativity for 17-year-old drivers and increasing its market share.[47]

Determinants of Rate Relativities

The correct relativity depends on the classification system, the average losses and persistency rates by classification, and

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undertaken. In general, however, reality has been in stark opposition to previous actuarial studies. Most analyses of "expense flattening" imply that high risk drivers are often overpriced, because their expense costs as a percentage of premium are less than those of lower risk drivers. In truth, when persistency rates are taken into account, many of these high risk drivers are *underpriced*, because their expense ratios over the policy lifetime are a greater percentage of premium than those of lower risk drivers.

[46] Adult drivers persist for ten years, so (in a steady state) 10% are in their first policy year, 10% in the second policy year, and so forth. This would be correct were there no switching of classifications. Since there is switching—that is, some adult drivers were first insured as young drivers—less than 10% of adult drivers are new business. If 25 is the minimum age for adult drivers, then drivers first insured below age 25 spend some renewal years in the adult classification but spend their first policy years as young drivers.

[47] This illustration is simplified for heuristic purposes. The actual analysis not only is more complex but may even lead to the opposite conclusion for two reasons. First, renewal loss experience may be better than new business loss experience because the renewal book has fewer 17-year-old drivers (among other reasons). This does *not* mean that when a group of 17-year-old drivers renew their policies, their loss experience will improve. Second, the illustration assumes that 17-year-old drivers and adult drivers have the same persistency rates. In fact, as this section shows, the different persistency rates among these classes affects the appropriate premium rate relativity.

The point of the simplified illustration in the text is two-fold: (1) persistency patterns cannot be ignored in determining rate relativities, and (2) the effect of persistency patterns, whether to increase or decrease the relativity, is not always obvious without careful actuarial analysis.

the strength of loss ratio improvement by policy year.[48] Asset share pricing models enable the actuary to determine accurate and profitable relativity factors.

This illustration compares young male drivers with adult drivers to determine the classification relativity factors. We need the following information, of which the second and third are essential for the asset share model:

1. the dimensions of the classification system,
2. the relative average loss costs of these two groups of insureds,
3. the relative average persistency rates of these two groups of insureds,
4. the strength of loss ratio improvement by policy year for these insureds.

The Classification System

The expected losses, expenses, and the current year's premium do not depend on the shape of the classification system. Future years' premium, however, are affected by such factors as renewal discounts and age boundaries between driver classes.[49]

For instance, suppose an asset share model is being used for an 18-year-old unmarried male driver. If the insurer differentiates between "males aged 25 and under" and "adult drivers," then this driver will spend 8 years in the "young male" classification. Since average losses decline rapidly between ages 17 and 25, his premium is probably too low for the first three or four years and

[48]The interrelationships among these dimensions are complex. For instance, a 22-year-old unmarried male driver who just completed college may have high expected losses. But if he is beginning a stable job, is engaged to be married, and is buying a house in a quiet suburb, his expected losses may drop quickly. In contrast, a 40-year-old married woman may have low expected losses, but she may show no loss ratio improvement for the next ten years.

[49]Persistency rates, which are influenced by relative future prices between the current insurer and its peer companies, also depend on the classification system.

too high for the subsequent four or five years. Termination rates are high for young male drivers but decrease with duration of the policy, so his expected termination rate will start high but decline markedly over the next eight years. A renewal discount will improve persistency but reduce renewal gross premiums.

Ideally, the classification system should be designed from the results of an asset share model. In practice, the classification system may be a "given" for the pricing actuary. In this section, the classification system is given. In the "competitive strategy" illustration (the following section), the classification system is designed from the asset share model.

Coverage Mix

Two types of differences affect classification relativities even for single policy year costs (that is, not considering persistency effects). First, average losses for any coverage vary by classification. For instance, young male drivers have higher expected bodily injury losses than adult drivers have. Second, the coverage mix varies by classification. For instance, young male drivers are less likely to purchase physical damage coverages or excess limits for liability coverages than adult drivers are.

If the ratio of expenses to premium did not vary with the coverage mix, or with the average loss per policy, then classification relativities would be similar to loss cost relativities. But fixed expenses do not vary directly with premium. They remain fixed regardless of the number of coverages, limits of liability, or deductibles chosen (Childs and Currie [45, pp. 53–54]).

Policy Basis Versus Coverage Basis Rate Relativities

We can use an asset share pricing model to develop rate relativities on either a policy basis or a coverage basis. The policy basis model compares losses and expenses for all coverages combined among classes of insureds. The resultant rate relativities must then be allocated to coverages. For instance, if the policy basis rate relativity for young male drivers is 2.0, and the premium volumes for liability and physical damage coverages are equal, the rate relativities by coverage might be 2.5 for liability and 1.5 for physical damage. When the coverage mix differs by classification, the allocation of the rate relativities may be complex.

The coverage basis model compares losses and expenses for an individual coverage among classes of insureds. The fixed expenses must be allocated to coverage before the asset share pricing model is used. Since some expenses do not vary with the number of coverages, the premiums rates are not additive: that is, there should be a "multiple coverages" discount. For instance, if the indicated rates are $500 for liability and $300 for physical damage, the correct rates might be $535 for liability alone, $325 for physical damage alone, and $780 for all coverages combined. Even when these differences are too small for practical application, the pricing actuary should know whether the rates are over- or under-stated for each classification and coverage combination.

Policy Basis Loss Cost Relativities

Policy basis loss cost differences between young male drivers and adult drivers depend on three factors:

1. *Young male driver rate relativities by coverage:* Average rate relativities for young male drivers are approximately 2.5 compared with the base classification rate (adult pleasure use). The rate relativities vary among insurers, depending on the definition of young male drivers (e.g., "25 and under," "29 and under," and so forth) and the other classification dimensions, such as years of driving experience and past accident history. Some states, such as New York, require separate relativities for comprehensive coverage, and some insurers use separate relativities in other states as well. The total average young

male driver rate relativity to that of all drivers is approximately 2.0.[50]

2. *Physical damage coverage by classification:* Young male drivers are more likely than other drivers to have liability coverage but no physical damage coverage because their premiums are high, they drive less valuable automobiles, and they may be less able to afford insurance.

3. *Average liability increased limits and physical damage deductibles:* Young male drivers have lower average liability limits and higher average physical damage deductibles for a given type of automobile. The higher average premiums for young male drivers, the fewer assets they have to protect, and the reluctance of company underwriters to provide high liability limits or full physical damage coverage to high risk drivers are the major reasons for this (Aetna [2, p. 26]).

For the "classification ratemaking" illustration, we use a coverage based asset share pricing model. Since the average coverage basis rate relativities are greater than the average policy basis rate relativities (about 2.0 : 1 versus 1.5 : 1), and much of the fixed expenses relate to per policy expenses, not per coverage expenses, we must adjust the per coverage fixed expenses by classification, assigning a higher dollar amount to young male drivers than to adult drivers.

An illustration should clarify this. Suppose class A purchases both liability and physical damage coverages, while class B, with a similar number of insureds, purchases only liability coverage.

[50]See ISO [104, pp. G-10–G-13]. ISO classifies young male drivers as (i) under 25 years of age if married or not the owner or principal operator of the vehicle and (ii) under 30 years of age if unmarried and the owner or principal operator. Rate relativities range from 1.15 for a 21 through 24-year-old "good student" married male using the automobile for pleasure use to 3.75 for a 17-year-old unmarried male driving his car to work and not eligible for a good student credit. Several jurisdictions, such as Massachusetts and California, prohibit classification by age, sex, or marital status. In these states, rate relativities are determined along other dimensions.

Expected losses and variable expenses are $600 for each coverage and each classification, and per policy fixed expenses are $100 per policy.

The ratio of fixed expenses to gross premiums for the entire line of business is 10% [= ($100 + $100) ≡ ($600 + $600 + $600 + $100 + $100)].[51] Equivalently, fixed expenses are one ninth of losses plus variable expenses. If we used this ratio to assign fixed expenses by class, we would assign $133 [= ($600 + $600) ≡ 9] to class A and $67 [= $600 ≡ 9] to class B.

Similarly, if we first allocated fixed expenses by coverage, we would assign $133 to liability and $67 to physical damage, since liability has twice the "losses plus variable expenses" that physical damage has. Splitting the $133 equally between classes A and B gives the same result as before. The expense flattening procedure suggested by ISO [103] begins with fixed expenses by coverage, so it would not solve the problem outlined here.

But this allocation is not correct. Since class A has twice the premium per policy that coverage B has, the ratio of fixed expense to premium for class B should be twice that for class A. (This is an extended "expense flattening" procedure.) Thus, ($600 + $600)($x$) + ($600)($2x$) = $200, or x = 8.33%. For the liability coverage, the expense loadings should be ($600)(8.33%) = $50 for class A, and ($600)(2)(8.33%) = $100 for class B. For the physical damage coverages, the expense loading should be ($600)(8.33%) = $50 (for class A).

For the previous example in the text, adult drivers have about four thirds [2.0 ≡ 1.5] as much coverage per policy as young

[51] This ratio is (Class A fixed expenses + Class B fixed expenses) ≡ total premium, where total premium equals

  Class A liability loss costs plus variable expenses
+ Class A physical damage loss costs plus variable expenses
+ Class B liability loss costs plus variable expenses
+ Class A fixed expenses
+ Class B fixed expenses.

male drivers have. A precise quantification of the fixed expenses by class is difficult for several reasons.

- First, fixed expenses are not strictly "per policy" expenses. For example, underwriting efforts are greater for a policy with both liability and physical damage coverages than for a policy with only liability coverage.
- Second, many fixed expenses, such as underwriting expenses, vary with the quality and type of risk. Louis E. Buck, in summarizing the findings of the Aetna Automobile Insurance Affordability Task Force [32], said: "… there are differences by classification in the cost of handling policies. It is more expensive to handle a policy for a young, single male in a given territory than an adult policy in the same territory. This difference can be attributed to such factors as more frequent policy changes and flat cancellations in the youthful male policies." His accompanying statistics show policy processing costs to be 50% to 100% higher for youthful unmarried male drivers than for adult drivers. (See Aetna [2, p. 9].)

There is no rigorous quantification of fixed expenses by classification in this paper. However, the dollars of fixed expenses per coverage in each policy year in the asset share pricing model are higher for young male drivers than for adult drivers. Expense flattening procedures, which are incorporated automatically in the asset share pricing model, reduce the "proportional" fixed expense loading for young male drivers in each policy year. Persistency patterns raise the lifetime "proportional" fixed expense loading for these insureds compared to adult drivers. These effects can be seen in Exhibits 2 and 3.

Persistency by Classification

An insurer selling whole life coverage expects to show an accounting loss during the first policy year. For medically underwritten risks, the acquisition and underwriting costs generally exceed the first year premium. For guaranteed issue policies, adverse selection raises first year benefit costs. In either case, the loss turns into a profit as the policyholder persists.

Similarly, an insurer selling personal automobile coverage expects an accounting loss during the first policy year, since both expenses and loss costs are higher that year. As with life insurance, the loss turns into a profit as the policyholder persists.

Expected long-term profits depend upon the policyholder persistency rates, in addition to premium, loss, and expense levels. *Since persistency varies by classification, the rate relativities must consider persistency rates as well.*

Classification differences may be based on either current classification or original classification. In most lines of insurance, the classification does not change: a frame building does not develop into a masonry building (homeowners), a retailer does not become a manufacturer (workers compensation), an architect does not become a lawyer (professional liability). But personal automobile classifications do change, as young drivers become adults, as urban residents move to the suburbs, and as new cars age.

Young Male Drivers

Traditional ratemaking procedures consider current classifications. Premium rates decline when the young male driver marries or ages, not before. Asset share pricing models consider original classifications and expected future changes: if we write a policy now, what is the expected long-term income?[52]

Persistency rates by duration are most easily determined for current classifications, such as the percentage of young male

[52]Pricing decisions hinge on supply and demand considerations, though these factors are hard to include in traditional ratemaking methods. The insurer asks: "If we raise the premium, what happens to expected long-term income?" Raising premium helps the current year's income, but it lowers persistency. The next illustration, "competitive strategy," shows how asset share pricing models deal with this issue.

drivers in their fifth policy year who persist into their sixth year. But if the young male classification consists of male drivers under 25 years of age, the group considered in the previous sentence are drivers originally insured below 20 years of age. These drivers have different persistency rates from drivers originally insured from 22 to 24 years of age. The persistency of young male drivers in their fifth policy year does not tell us the expected fifth year persistency of young male drivers. We need persistency rates by original classification, not current classification.

Model Assumptions

For the asset share model, we begin with pivotal classifications: the adult pleasure use (the base class) and unmarried males aged 21 and 22 who drive to work. We need to know three differences by classification to form rate relativities: average loss costs, average fixed expense costs, and persistency rates. For this illustration, we assume the following differences; in actual pricing work, we would derive these from past experience:

1. Average liability loss costs are $400 per annum for adults and $1,000 per annum for young male drivers. Were all expenses proportional to premium, and were persistency rates the same for both classes, the rate relativity for young male drivers would be 2.5.

2. Average premium for all drivers is $550. Average first year fixed expenses are 17.8% of this, or $98. Adult drivers are less expensive to underwrite, especially per coverage. There are fewer underwriting rejections among adult drivers, and they purchase more coverages, so average fixed expenses per coverage is 10% less, or $88 per policy for the liability coverages. Conversely, young male drivers are more expensive to underwrite, especially per coverage. Underwriting rejections are more common, some applicants never remit the premiums, and many drivers purchase only basic limits liability cover-

TABLE 4

PERSISTENCY RATES BY DURATION AND CLASSIFICATION (AS PERCENTAGES)

Policy Year	1	2	3	4	5	6	7	8	9	10+
Young male	60	65	70	73	76	79	82	85	88	90
Adult	82	86	87	88	89	90	90	91	91	92

ages. Average fixed expenses for the liability coverages are 20% higher, or $117 per policy.[53]

3. Retention rates are higher for adult drivers than for young male drivers. We use the simulated rates in Table 4 to illustrate the asset share pricing model. Actual rates vary by insurer, distribution system, and classification plan, so these rates may not be appropriate for any given carrier.

The classification plan, average loss costs, average fixed expenses, and persistency rates are given. We assume that the insurer writes at a 2 : 1 premium to equity ratio and desires a pre-tax 14% return on equity from its insurance operations (i.e., excluding investment income on surplus funds). We use the asset share pricing model to determine a 7.0% return on premium for each class, and we then derive the rate relativities from the resulting premiums.

Exhibits 2 and 3 show the calculations. For each class, we select a starting gross premium and increase it 9% per annum, which determines the variable expenses in all future years. In the first year, fixed expenses are $88 for adults and $117 for

[53] See Aetna [2, p. 64]: "In considering how expenses should be allocated to policyholders, it must also be noted that the company must charge policyholders for the underwriting costs of *rejecting* applications. Thus, even if the actual costs of underwriting each accepted risk were known, the amount charged to a policyholder would have to exceed that actual cost to compensate for the costs associated with the applications of rejected applicants, from whom the company collects no premium."

young male drivers. We use the same ratio of renewal to first year fixed expenses as in the previous illustration, 3.8% to 17.8%, and increase the fixed expenses by 5% per annum. For adult drivers, $\$88 \times 3.8\% \equiv 17.8\% = \$19$; this is then increased by 5% per annum to give all the fixed expense entries.

As before, the loss costs shown in the exhibit are discounted to the beginning of the corresponding policy year. The present values of future profits and premiums at the original policy issuance date are determined at a 12% interest rate, which is the assumed cost of capital. The original premium has been selected such that the ratio of the present value of all future profits to the present value of all future premiums is 7.0% for both classes.

Asset Share Results

The indicated premiums are $475 for adults and $1,272 for young male drivers. Note that:

- The loss cost relativity is 2.50, or $\$1,000 \equiv \$400$.
- The fixed expense cost relativity is 1.33, or $\$117 \equiv \$88$.
- The rate relativity is 2.68, or $\$1,272 \equiv \$475$.

Pricing procedures used in the 1960s would have set the rate relativity equal to the loss cost relativity, or 2.50. Since the fixed expense relativity is only 1.33, expense flattening procedures would have reduced the rate relativity. But the persistency differences between the two classes show that even the loss cost relativity is too low. A premium rate relativity of 2.68 is needed to equalize the returns between these two classes.

6. ILLUSTRATION 3—COMPETITIVE STRATEGY

The "business expansion" illustration presented in Section 4 took the environment as given and asked, "Is the growth strategy profitable?" The illustration in Section 5, "classification relativities," took the insured population as given and asked: "What prices are equitable?"

This is the traditional ratemaking perspective: the actuary aligns premiums with anticipated losses and expenses for a given insured population. *Competitive strategy reverses the question: "How can the pricing structure create a more profitable consumer base?"*

Some insurers have excelled at this task. New products, such as package policies in the commercial lines; modifications to existing products, such as replacement cost coverage for homeowners insurance; and classification revisions, such as retired driver discounts in personal automobile insurance, have spurred sustained growth for these carriers.

Two considerations should be kept in mind when seeking to change the insured population:

1. Any strategy may affect new business production or retention rates. For instance, the introduction of various professional liability coverages created a new clientele ("new business production"), whereas the expansion of experience rating plans increased renewals among desirable insureds ("retention rates"). Some new products, such as universal life insurance, serve both functions: they are savings vehicles for investors otherwise uninterested in life insurance, and they are replacement vehicles for insureds who might drop inefficient whole life policies.

2. Traditional ratemaking procedures are cost-based. The pricing actuary equates premiums with anticipated losses and expenses, so economic profits are eliminated. In practice, insurers seek to optimize certain goals, such as profits or market share. The price elasticity of demand becomes a crucial determinant of optimal strategy. That is, premium rates and relativities affect consumer

demand and the mix of insureds, thereby affecting insurer profitability.

Cars and Courage

> Although courage is a splendid attribute in its place, its place is not at the wheel of an automobile.
>
> — Ambrose Ryder [1935]

Early classification schemes had surcharges for older drivers: reactions slow as the body ages, and senior citizens lack the quick reflexes of their sons and daughters. Insurance experience, however, eventually showed the effects of youthful intrepidity, as Ambrose Ryder notes. The physical limitations of older drivers make them less capable of escaping from dangerous situations. But their awareness of these limitations make them less likely of entering into dangerous situations in the first place.[54]

The exposure to road hazards declines as drivers age. Older drivers, particularly after retirement, spend less time behind the wheel (Buck [32, p. 6]). They less frequently drive to work, take kids to amusement parks, or attend late parties.[55] As a re-

[54]Ryder [146, p. 143] says: "The next question is whether a driver is a better risk because he reacts one-fifth of a second quicker than the average. Various devices have been on the market for testing the reaction times to danger signals. I think these are all very interesting and may possibly prove of value, but generally speaking the person who is quick on the trigger and who reacts very promptly is probably a less desirable risk than the more phlegmatic person who likes to think things over two or three times before he decides to do anything. The latter type will not react as quickly to the sudden danger that presents itself to his oncoming car but on the other hand neither will he be so likely to allow himself to get into a position where any sudden danger will arise that will require a one-tenth of a second reaction. Give me my choice and I will take the man who is not so quick on the trigger in everything he does in life.

"If the individual driver is going to be measured for his reactions to danger, it is even more important that he should be measured for his willingness to keep away from danger . . . . The timid soul is a much better risk than the daring young man who has the courage to drive his car at 90 miles per hour on a slippery road. The best type of risk, therefore, is the person who is really afraid to take unnecessary chances."

[55]Compare also IRC [99, p. 5], which examines auto injury rate by age of the victim: "The lowest percentage of injured persons fell into the oldest age groups, with eight percent age 55 to 64 and eight percent age 65 or older." Drivers make up a large percentage of auto accident victims, so the Insurance Research Council statistics are relevent for the analysis here, though the exact figures are not suitable.

sult, many insurers now provide discounts for older or retired drivers.

Older drivers not only have lower expected loss costs, they also have less impetus to price shop at renewal time. Younger drivers with high premiums have incentives to find lower cost coverage, and they hear about competing rates from friends at work. Older drivers, with lower premiums and often with less information about competing carriers, have less incentive and less opportunity to price shop.

This section examines the pricing of a retired driver discount. The relevant considerations for the asset share model include:

- expected loss costs by policyholder age,
- persistency rates by policyholder age and policy duration,
- price elasticity of demand: that is, the effects of price on retention rates.

An Illustration

The actual data used to price a retired driver discount are complex, though the principles are straightforward. To see their importance, let us consider a simple illustration, from both a traditional ratemaking perspective and from an asset share pricing perspective.

Suppose an automobile insurance policy is offered, with a life of five years. That is, each insured purchases coverage for six years, though not necessarily with the same carrier each year. Cost and persistency assumptions are as follows:

1. Expected loss plus expense costs, including a reasonable profit, are \$100 the first year, \$90 the second year, \$80 the third year, \$70 the fourth year, and \$60 the fifth and sixth years.
2. The market is competitive, and consumers are most sensitive to price at early durations. Your major competitor

is offering the same product for $90 each year. If you price below the competitor's rate, your insureds will renew their policies. Moreover, you will attract 50% of your competitor's insureds in the first policy year, 25% in the second policy year, and none in subsequent policy years. If you price above your competitor's rate, you will attract none of your competitor's business, and you will lose 50% of your first year insureds and 25% of your second year insureds. If you price at the same level as your competitor, you will neither attract your competitor's insureds nor lose your own business.

3. You and your competitor each begin with 200 potential insureds. That is, if you charge equal rates, you will each have 200 insureds each year.

4. For simplicity, there is no time value of money. That is, interest and inflation rates are both 0%, and future events are certain. (The actual asset share pricing model, of course, determines present values of future profits and losses.)

These assumptions are summarized in Table 5.

The traditional ratemaking philosophy says that premiums should correspond to expected costs: $100 the first year declining to $60 the fifth and sixth years. With these rates, you will lose 100, or 50%, of your potential insureds the first year. In subsequent years, you will neither lose nor gain insureds, since in the second policy year you and your competitor have the same rates, and in the following policy years, insureds are not price sensitive. You will earn "normal" profits on this book of 100 insureds for six years, and you will have a 50% loss of market share.

But suppose you price the policy at $85 each year.

- The first year you attract 100 of your competitor's insureds and lose $15 on each policy.

TABLE 5

COMPETITIVE PRICING ILLUSTRATION

Policy Year	Expected Cost	Competitor's Rate	Effect of Rate Level on Retention and Production
1	$100	$90	50%
2	90	90	25
3	80	90	0
4	70	90	0
5	60	90	0
6	60	90	0

- The second year you attract 25 of your competitor's insureds and lose $5 on each policy.
- You retain these 325 policyholders for the next four years and earn $5, $15, $25, and $25 per insured each year.

Your net profit is:

$$(300)(-\$15) + (325)(-\$5) + (325)(+\$5) + (325)(+\$15) + (325)(+\$25) + (325)(+\$25) = \$16,625.$$

The factors used in this illustrations are oversimplified. For instance, the effects of rate level differences on business retention depend on the magnitude of the difference, not just on which competitor has the lower rate. But the principle is clear, and it is directly applicable to actual pricing problems: Since future profits are embedded in business renewals, long-term profits may be increased by incurring short-term losses to gain good risks.

Retired Drivers

The characteristics of this illustration are equally applicable to retired driver discounts:

1. Average loss costs decrease markedly as the policyholder ages. At age 55, the insured drives to work each day and

is exposed to road hazards. At age 65, the insured makes less use of the automobile and loss costs drop.

2. The price elasticity of demand, or the extent of comparison shopping, decreases as the policyholder ages. (Equivalently, "consumer loyalty" increases as the policyholder ages.) A driver is more likely to switch carriers at age 55 than at age 65 to obtain a lower rate.

Optimal pricing strategy calls for underpricing insureds in their 50s to gain market share among this desirable group, then reaping the profits when the policyholders advance into their 60s and 70s. Since expected loss costs decline when the driver retires, a level rate, or even a slightly decreasing rate, will cause the transition from losses to gains as the policyholder ages.

The pricing mechanics will be shown with an asset share model. The task of the actuary is not simply bringing premium to current level or developing losses to ultimate, so as to estimate future costs. Rather, optimizing long-term profits requires offering a discount before short-term data seem to justify it. The actuary must determine the initial age of the retired driver discount and its optimal magnitude, based on competitor actions and market share implications:[56]

- *Age:* The appropriate age for the retired driver discount is before actual retirement and even before any substantial decline in losses. The optimal age depends on the relationship between policyholder age and persistency and on the discounts offered by competitors, in addition to expected loss costs by age. (In the illustration above, termination rates drop from 50% in the first policy year to 0% in the third policy year. Actual termination rate differences are hardly so extreme.)

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[56]Compare also Daykin, Pentikäinen, Pesonen [63, Chapter 14, Section 3], who use the theory of games in a multi-unit market model to simulate the effects of company rate changes, similar to the analysis in this paper. For the application of the theory of games to industrial economics, see Fudenberg and Tirole [83] or Tirole [154].

- *Magnitude:* The optimal size of the discount depends on the price elasticity of demand and the rate structures of peer companies, in addition to expected loss costs. In the illustration above, there is only one competitor, and demand is extremely elastic. In practice, one must examine the rate structures of one's competitors and estimate the effects of rate differences on retention rates and new business production.

Model Assumptions

To determine the optimal age and magnitude for the retired driver discount, the asset share pricing model requires two sets of assumptions. Some assumptions are grounded in empirical data; others must be projected by the actuary.

Loss Costs by Age of Policyholder

Many insurers examine loss costs by age of policyholder to support classification relativities. Table 6 shows loss ratio relativities by policyholder age, separately for new and renewal business.[57] The relativity shows the ratio of the loss ratio in that row to the average loss ratio for all rows combined.

The loss ratio relativities are similar to those in the heuristic illustration provided earlier: about unity for drivers below age 55, but dropping as low as 65% as the policyholder ages. The loss ratio differences are more pronounced for existing policyholders than for new insureds. For new business, the loss ratio relativities never dip below 82%. The loss ratio relativities for renewal policyholders are at or below this level from age 55 through age 74.

This difference makes sense, since the effects of aging differ among insureds. Some retired drivers drive less and drive more

[57] The data are shown for all coverages combined. Actual experience differs somewhat by coverage and between frequency and severity. We use loss ratio *relativities* because (i) absolute dollar expected loss costs vary with inflation, with coverage, and with the policyholder mix, and (ii) absolute loss ratios vary with the stage of the underwriting cycle and with pricing strategy, but (iii) loss ratio relativities are relatively stable over time.

TABLE 6

LOSS RATIO RELATIVITIES BY POLICYHOLDER AGE

Policyholder Age	New Business LR Relativity	Renewal Business LR Relativity
20–49	1.02	1.03
50–54	1.00	0.98
55–59	0.94	0.83
60–64	0.84	0.72
65–69	0.82	0.65
70–74	0.98	0.76
75 & older	1.10	0.98
Total	1.00	1.00

carefully; these are the best risks. Others find their responses dulled, but they do not change their driving habits; these are dangerous insureds.

Why would a 65-year-old driver be looking for a new auto insurance policy? Many retired persons own their own homes and have close friends in their neighborhoods. They are not inclined to move elsewhere and begin new lives or careers—the most common motive for switching insurers. Those who do move often do so because of failing health. They join retirement communities, enter old age homes, or live with their children. They are not usually seeking new auto policies.

Insurers frequently review the policies of drivers who have had recent accidents. If the insurer believes the driver is too risky, it may terminate the policy or "discourage" renewal (e.g., by indifferent customer service). Some of the retired drivers seeking new automobile insurance policies have been considered poor risks by their former insurers.

Exposure distributions by age of the principal operator for new and renewal business reflect this. Among existing policyholders, older drivers form a large percentage of the population and are generally good risks. Among new insureds, older drivers

TABLE 7

PERSISTENCY RATES BY POLICYHOLDER AGE

Policyholder Age	50	54	58	62	66	70	74	78
Persistency Rate (%)	96	95	94	92	90	88	85	80

form a smaller percentage of the population. Some of these insureds are good risks; others are dangerous drivers.

For the asset share model, we use the loss ratio relativities for renewal business. The indicated retired driver discounts are not necessarily appropriate for new business. The criteria for the discount should be both the age of the policyholder and the number of years since inception of the policy.

Persistency Rates for Older Drivers

Retention rates improve as the policy ages and as the policyholder ages. Sections 4 and 5 show simulated persistency rates by policy duration for all drivers, adult drivers, and young male drivers. Simulated persistency rates for older drivers are shown in Table 7.

These persistency rates differ in two respects from those illustrated for adult drivers and for young male drivers in Section 5. First, most insureds aged 50 and over are mature renewal business, similar to the 10+ policy year duration category in Table 4. Thus, the persistency rates for insureds aged 50 through 66 are high. Second, as policyholders advance into their 70s, many stop driving because of death or ill health, so persistency rates drop.

In practice, the persistency rates depend upon the premium discount that is offered. If a 60-year-old driver pays $500 in premium, and a competing carrier offers the same policy for $450, the driver is unlikely to switch carriers. That is to say, price elasticity of demand is low, or policyholder loyalty is high.

TABLE 8

PERSISTENCY RATES BY POLICYHOLDER AGE

Policyholder Age	50	54	58	62	66	70	74	78
Persistency:								
with discount	98	97	96	94	92	90	85	80
without discount	90	85	80	75	80	80	85	80

However, if the competing carrier's premium is also $500, but it advertises a retired driver discount of 10%, the insured is more likely to switch carriers. The qualified insured views the retired driver discount as equitable; a carrier who does not offer it is seen as unfair.

We must therefore replace the persistency rates in Table 7 with a set of rows, showing persistency rates with no discount, with a 5% discount, with a 10% discount, and so forth. But these persistency rates depend on the discounts offered by other carriers. In other words, there are no absolute expected rates, since the expected rates depend on other carriers' discounts.

The difficulty in forecasting persistency rates highlights the importance of good assumptions. The persistency rate assumptions are subjective, at least until one develops the experience to justify them or to amend them. But they are essential for determining optimal prices.

For the asset share model, we assume two sets of persistency rates. One set, with lower rates, assumes that no premium discount is offered to older or retired drivers. The other set, with higher rates, assumes a 7.5% discount, which is the "market discount" in Table 8.

The persistency rates illustrated in Table 8 assume that most competing carriers offer a retired (or older) driver discount to policyholders aged 60, but only some of them offer discounts to policyholders in their early or mid-50s. Thus, persistency rates in the "without discount" scenario decline as the policyholder ages from the early 50s to the mid 60s. However, if a full discount is offered even to policyholders in their 50s, few of them switch carriers.

Determining the optimal premium discount requires several runs of the asset share pricing model, since the results depend on the actuary's assumptions. For instance, what effect does a 7.5% discount have on persistency rates? What effect do persistency rates have on average loss costs?[58] For simplicity, we use three iterations:

1. No carrier offers a retired driver discount.
2. Many peer companies offer the discount, but your company does not.
3. Your company offers a 7.5% discount, which is the prevailing "market" discount.

In each case, we use a 15 year asset share model for a cohort of insureds aged 52. We assume that persistency rates depend on the premium discount offered, but average loss costs do not.

Iteration 1. No Carriers Offer Discounts

Exhibit 4 shows the asset share model results for a cohort of 52-year-old drivers, assuming the persistency patterns in Table 7 and the loss ratio relativities in Table 6. Note several differences from the asset share model results in Section 4:

- The Section 4 illustration models new business production, so new business expense ratios are used for the first policy year. The cohort of 52-year-old drivers in this section consists of existing insureds, so only renewal business expense ratios are used.

[58]In life and health insurance, higher termination rates generally lead to higher mortality and morbidity costs, since insureds in poor health are more likely to retain their coverage. Health insurance actuaries refer to this phenomenon as "cumulative antiselection," following Bluhm [24].

- Average loss costs decrease sharply in the first few policy years but then level out. Section 4 used a 3% decline in average loss costs per policy year; this section uses a 1% decline, since most business is mature. In addition, the loss ratio improvements by policyholder age already reflect part of the loss cost improvements as the policy ages.

The model begins with average losses of $500 in the first year and average premium of $600. Because these are existing "high-quality" insureds, with high persistency rates and declining loss costs, profitability is good. The present value of profits over the next 15 years is $1,107, and the present value of premiums is $5,505, for a return on sales of 20%. This is not unusual. The insurer has already paid the high costs of new business production and is now earning the profits in the renewal book. Similarly, if one excludes the high first year costs in the "business expansion" illustration in Section 4, the return on sales is over 17%.

A return on premium measure of profitability is reasonable when market shares remain steady, not when market shares are affected by the rate structure. For instance, suppose an insurer writes 10,000 risks at a premium rate of $1,000 apiece, with an average loss plus expense cost of $900 per risk. The return on premium is 10%, or $1 million. Suppose also that if the insurer raises rates 50%, it loses most of its business. Only 2,500 of the poorer risks remain, with an average loss plus expense cost of $1,300 per risk. The return on sales has improved to 13.3% = [$200 ≡ $1,500], but the dollar amount of profits has declined to $500,000. The insurer's results have deteriorated, not improved.[59]

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[59]If the decline in market share is not offset by increases elsewhere, the insurer's return on equity has decreased. For instance, if the insurer has $5 million in equity, then the return on equity is +20% before the rate revision and +10% after the rate revision. Some pricing actuaries are so used to "implied equity assumptions" that they presume that equity strictly follows the business volume. Alternatively, this assumes that equity is the major constraint on the volume of business written. In practice, other factors such as marketplace competition are more important constraints on business volume.

Iteration 2. Only Competitors Offer Discounts

The profitability of this business is good, so carriers seek to increase market share by offering retired driver discounts or older driver discounts. Your company wishes to retain its high profit margin, so it offers no discount.

Persistency rates drop sharply. Your insureds see the retired driver discounts offered by other carriers, and they perceive your stance as inequitable. Exhibit 5 shows the asset share pricing model results. The loss and expense ratios on any given policy have not changed, so the company retains the full profit margin. But retention rates are lower, as more insureds drop out each year. Although 42% of insureds persisted through the full 15 years in Iteration 1, now only 8% do so. The present value of future profits has declined from $1,107 per policy to $666 per policy.[60]

Iteration 3. You and Your Competitors Offer Discounts

To arrest the loss of market share, you offer a 7.5% discount to all drivers age 52 and over, which is the most common market discount (Exhibit 6). The premium discount pleases your insureds, so persistency rates are high. Expenses that are a function of premium, such as renewal commissions and premium taxes, also show a 7.5% decrease, but average loss costs and fixed expenses do not change.

The 7.5% discount cannot be justified on a short-term basis for drivers in their early to mid-50s. In fact, you show a loss of

[60]Since insureds in their 60s are more profitable than insureds in their 50s, the reduction in persistency has a greater effect on the present value of future profits than on the present value of future premiums. Thus, the return on premium declines from 20.1% to 16.7%.

The actual effects may be more adverse than the exhibits here imply. It may be that the better drivers are the ones most likely to find less expensive coverage elsewhere and therefore to terminate their policies. Bluhm [24] notes this for health insurance ("cumulative antiselection"). It is unclear how this affects personal automobile insurance.

$2 the first year and inadequate returns the next two years (4% on premium). But now 49% of insureds persist for 15 years, and the present value of future profits has increased to $797.

Other Advantages

Several other aspects of the retired driver discount have not been illustrated in Exhibits 4–6 but can be incorporated into the asset share pricing model.

1. The exhibits show only a 15 year illustration, as if all insureds terminated at age 67. But the insurer can expect another five or ten years of steady profits, so the difference between an 8% persistency rate in the no-discount case and a 49% persistency rate in the 7.5% discount case has a great effect on future earnings. Ideally, one should extend the pricing model until most business terminates.
2. The exhibits assume no change in the fixed expenses per policy regardless of market share. This is reasonable for premium collection costs, policy printing costs, and similar expenses. Corporate overhead expenses, however, increase as a percentage of premium (or on a per policy basis) when market share declines. Ideally, one should have three expense categories in the asset share pricing model: variable expenses, per policy expenses, and overhead expenses.
3. Several effects of policyholder satisfaction are difficult to quantify. If policyholders perceive the discount offered at age 52 and over as equitable, there may be fewer instances of fraudulent claims. In addition, persistency may improve slightly even for policyholders younger than 52, since they expect to eventually qualify for the discount.

These items should be considered when determining the optimal premium discount. Most important, though, is a structure that examines long-term profits and market share, such as an asset share model. Without it, the actuary is easily misled, unable to quantify the effects described in this section. With it, the actuary can project the true profitability of each risk.

7. ILLUSTRATION 4—UNDERWRITING CYCLES

Traditional ratemaking methods have no place for competitive pressures, marketplace prices, or consumer demand. Actuaries use volumes of data, established procedures for developing and trending losses, and careful analyses of required profit levels. Credibility formulas and actuarial judgment keep rates on a steady path, never deviating too far from either expected costs or past experience. And market prices seem to jump and skip in willful abandon.

The knowledgeable actuary does not expect market prices to adhere to rate recommendations. In a competitive industry, prices are set by the market. Actuaries tug at them, sometimes drawing them closer to costs, sometimes finding their efforts to be fruitless.

But the actuary also knows that rate recommendations must consider market prices. If competitors are charging $1,400 for a certain risk, few actuaries would recommend a rate of $1,100. If the insurer wishes to expand in this market, it might charge a rate of $1,300 and still earn profits on each risk. If the insurer believes that a rate cut will lead to matching cuts by competitors, it may continue with the $1,400 price.[61]

The actuary's rate recommendations are based on both expected costs and expected market prices. Market prices follow the course of the underwriting cycle. The future is not known with certainty, but its outline can be traced.

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[61] For the economic theory of pricing in anticipation of competitors' actions, see Tirole [154] and Scherer [148]. For the underlying mathematics, see Varian [156], Waterson [161], and Shapiro [150]. For a general business perspective, see Porter [143]. For applications to insurance, see Cummins, Harrington, and Klein [52] and Feldblum [76].

Indeed, its outline must be traced. Future losses are not known with certainty, so actuaries examine past claims, observed development patterns, and projected trends to estimate future costs. So too must actuaries consider competitive pressures and industry structure to project future marketplace prices.

Let us consider several illustrations. We begin with unrealistic assumptions, simply to clarify the themes. Suppose first that:

- Policyholder persistency is perfect: 100% retention rates each year.
- There is no time value of money. Alternatively, the expected annual increase in profits exactly matches the discount rate.[62]
- The course of the underwriting cycle is known with certainty.
- The industry alternates between soft (unprofitable) and hard (profitable) markets. The average profit exactly matches the insurer's target return.

Figure 3, which shows time along the horizontal axis and return on equity along the vertical axis, puts numbers on this illustration. The return on equity generated by this policy oscillates between 0% and 20%. The long-term return averages to 10%, regardless of when the policy is first issued.

The cycle has no effect on the insurer's underwriting decisions. The insurer may lose money in soft markets and make money in hard markets, but the long-term profits do not depend on when the policy is first written.

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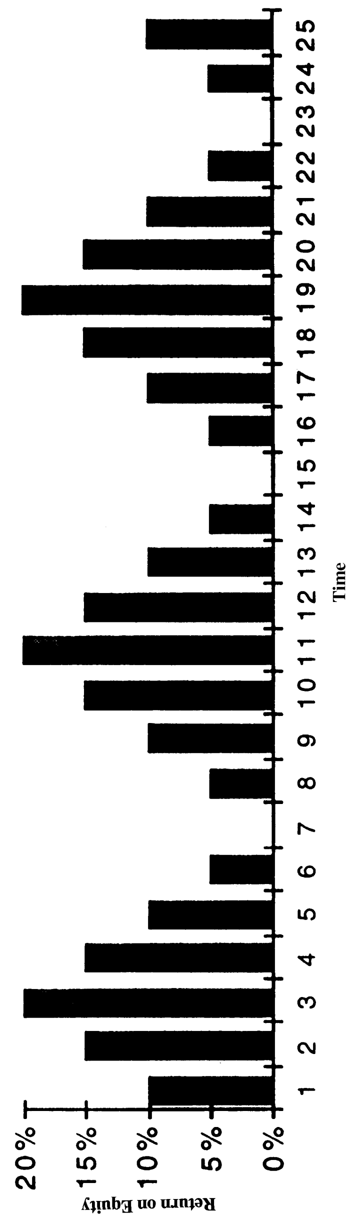
[62]In other words, suppose the financial analyst expects that all revenues and expenses will increase with inflation, but that all future profits should be discounted at the same rate. Modeling of the company's performance is simplified by assuming a 0% inflation rate. In practice, of course, the interest rate used for discounting the future profits is generally higher than the cost trends for revenues and expenses. The asset share exhibits therefore use distinct rates: the cost of capital for discounting future profits, loss cost trends, fixed expense cost trends, an expected rate of premium increases, and an implicit interest rate to determine the present value of losses.

FIGURE 3

UNDERWRITING CYCLE KNOWN AND NO DISCOUNTING



Traditional ratemaking procedures, which look at the future policy year in isolation, take no account of underwriting cycle movements. If underwriting results were poor during the experience period, a rate increase was “indicated.” It made no difference whether the poor results during the experience period stemmed from inadvertent underestimates of loss costs or from conscious decisions to reduce rate levels.

The asset share approach expands the perspective. If underwriting results are poor right now because the underwriting cycle is at a nadir and the industry as a whole is suppressing rate levels, but the long-term outlook for the line of business is good, the proper pricing recommendation is generally *not* an immediate rate increase. As discussed in the previous illustration, setting rates at the actuarially adequate level without taking cognizance of market constraints may simply cause a loss of market share and thereby a loss of future profits.

Two characteristics of underwriting cycles support the asset share pricing approach:

1. Underwriting cycles are industry phenomena, not company phenomena.[63] Underwriting cycle fluctuations are not caused by individual company ratemaking “errors,” which the pricing actuary should correct. On the contrary: “correction” of the “errors” simply prices the company out of the market. The prescient actuary “rides” the cycle; he or she does not swim against the current.

2. Following prices down in the underwriting cycle could be viewed as an effort to gain (or merely maintain) market share, and creating cyclical losses could be viewed as an effort to drive out new entrants, thereby protecting long-term profits. Underwriting cycles and asset share pricing techniques have similar underlying principles:

[63] See especially Daykin, Pentikäinen, and Pesonen [63, pp. 332–343]. Daykin, Pentikäinen, and Pesonen even provide a graph of six Finnish insurers, showing how the underwriting results of each insurer followed that of the five others.

> business decisions should be guided by long-term profits, not by short-term results.[64]

Let us now remove the unrealistic assumptions that we posited earlier:

- The retention rate is 90%. Expected profits decline each year because the insured may terminate the policy. The oscillatory pattern is dampened, as shown in Figure 4. The time value of money has two parts, which must also be incorporated.
- The insurer's cost of capital exceeds the expected (inflationary) increase in profits by five percentage points.[65]
- The course of the underwriting cycle is not certain. To offset the risk of uncertain future returns, the insurer discounts expected future returns by 5%.

The oscillatory pattern is further dampened, as shown in Figure 5. As one looks ten or twenty years into the future, most policyholders from the current cohort have terminated, and the profits actually achieved in those future years are deeply discounted.

In Figures 4 and 5, the point in the underwriting cycle at which the policy is issued affects the expected long-term return. The asset share model can be used to quantify the expected returns, using the same methods employed in the previous sections.

To model the effects of underwriting cycles, we begin with the standard asset share analysis shown in Exhibit 1. In Exhibit 1, premiums increase by 9% per annum. We now overlay an underwriting cycle pattern on the expected premiums. In Exhibit

[64]For more complete discussions of underwriting cycles and business strategies, see Feldblum [76] or Harrington and Danzon [90].

[65]For companies of average risk, we would expect the cost of capital to exceed the inflation rate by the sum of the market risk premium and the *real* interest rate on short-term risk-free securities, such as Treasury bills. The former is generally estimated at about six to eight percentage points, and the latter is about two percentage points, giving an eight to ten point spread.

FIGURE 4

RETENTION LESS THAN 100%

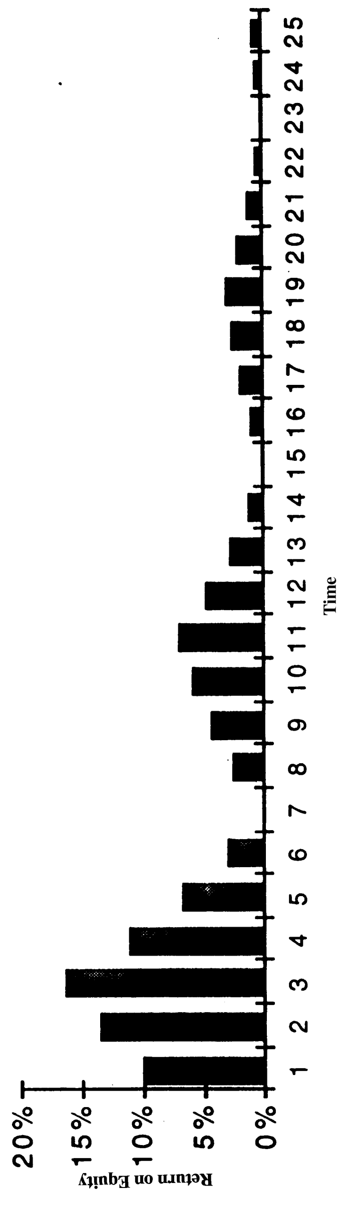
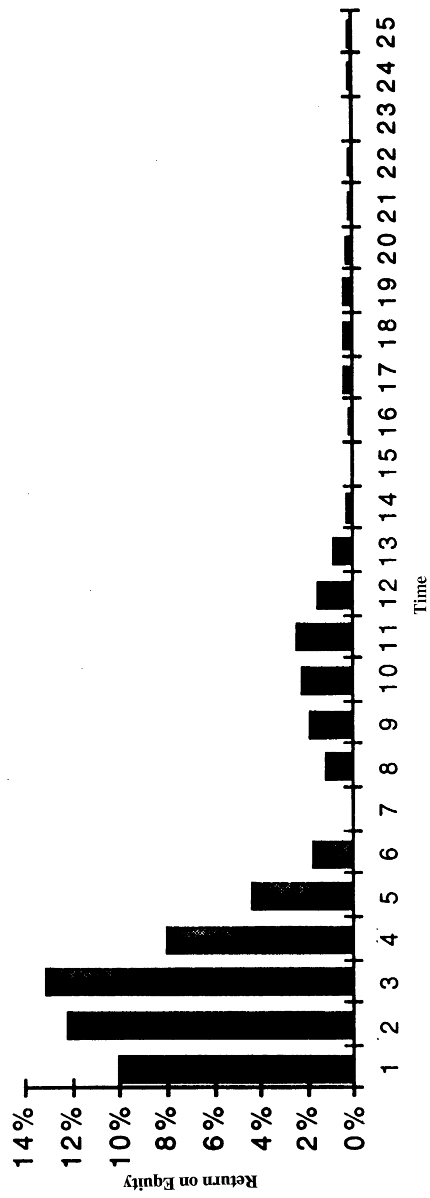


FIGURE 5

DISCOUNTING INTRODUCED



7, the pricing actuary presumes that the industry is now at the midpoint of the underwriting cycle, rates will increase to 30% above their long-term average (adjusted for inflation) over the next two years, then decrease to 30% below their long-term average over the next four years, and so forth. This is an eight year underwriting cycle, with the premiums in Exhibit 1 multiplied by the following factors:

Year	U/W Cycle Factor	Year	U/W Cycle Factor	Year	U/W Cycle Factor	Year	U/W Cycle Factor
1	1.00	5	1.00	9	1.00	13	1.00
2	1.15	6	0.85	10	1.15	14	0.85
3	1.30	7	0.70	11	1.30	15	0.70
4	1.15	8	0.85	12	1.15	16	0.85

Exhibit 8 shows an asset share exhibit with the same starting premium and assumptions for losses, expenses, persistency rates, and cost of capital, except now the company anticipates the underwriting cycle to be turning down. Exhibit 7 shows a "lifetime" return on premium of 14.0% [$730 ≡ $5,221], while Exhibit 8 shows a "lifetime" return on premium of 7.1% [$339 ≡ $4,803].[66]

The actuary does not try to change the course of the underwriting cycle; the solitary insurer cannot do this.[67] Rather, the pricing actuary sees underwriting cycles as constraints on the company's rate actions, and he or she sets premium levels, rate relativities, and various surcharges and discounts in that context.

8. PROFITABILITY MEASURES

Universally accepted standards for profit measurement in insurance do not exist. The traditional 5% or 2.5% underwriting

[66]An underwriting cycle with a premium swing of ∗ 30% is strong for personal auto insurance. It is mild compared to the general liability cycle of the early 1980s.

[67]However, "signaling" effects and market leadership movements can be potent; see Feldblum [76].

profit provision is no longer supported even by the NAIC, though a return on premium measure (in contrast to a return on equity measure) is advocated by several actuaries and economists (NAIC [136]; Woll [170]).

A common component of life insurance asset share profit measurement is the present value of future book profits (i.e., statutory profits). The rationale is that book profits determine the earnings available for stockholder dividends, so this measure is similar to financial measures of investor returns.[68]

Two differences between life and property/casualty insurers influence the optimal choice of profit measure:

1. Life insurers hold discounted policy reserves, with partial adjustment for deferred acquisition costs, so their book profits are similar to economic profits. Property/casualty insurers hold full value reserves with no offset for deferred acquisition costs, so book profits may differ greatly from economic profits.

2. The life insurance patterns of cash flows, adjusted for policyholder cash values, correspond to book profits. For

[68]See Anderson [8, p. 365]; Griffin, Jones, and Smith [87, p. 381]. See also Larner and Ryan [114, p. 448]: "The definition of economic or appraisal value as the present value of future net earnings streams taken at appropriate risk discount rates is generally accepted by actuaries and others as a natural one throughout the world in our experience . . . . Modern portfolio theory and other investment work provides a theoretical basis for the suggestion that the value of a company is the present value of its future net earnings." Actuarial Standard of Practice No. 19 concerning actuarial appraisals [1, p. 4, paragraph 5.2.1], notes the connection between book profits and investment returns: "*Distributable Earnings*—For insurance companies, statutory earnings form the basis for determining distributable earnings, since the availability of dividends to owners is constrained by the amount of accumulated earnings and minimum capital and surplus requirements, both of which must be determined on a statutory accounting basis . . . . Economic value generally is determined as the present value of future cash flows. Statutory accounting determines the earnings available to the owner. Hence, while future earnings calculated according to generally accepted accounting principles (GAAP) will often be of interest to the user of an actuarial appraisal, as may other patterns of earnings, the discounted present-value calculations contemplated within the definition of actuarial appraisal in this standard should be developed in consideration of statutory earnings, rather than some other basis."

instance, the first year "investment," corresponding to the first year book loss, is the first year cash outflow to agents and policyholders. Thus, investor returns correspond to book profits which correspond to actual patterns of cash flows and policyholder cash values.

Property/casualty insurance lacks this correspondence. First year cash flows are positive for the insurer. Capital to asset ratios, however, are high. The "investment" at the beginning of the insurance transaction is not simply the assets supporting the reserves, but also the investor capital "committed" to support the policy. In sum, the book profits for the insurer are not necessarily a good proxy for the implied equity transactions between the insurer and its stockholders.[69]

Measuring Rods

There are a variety of methods of adapting asset share profit measures for property/casualty operations. This paper uses economic profits instead of book profits by discounting the loss reserves. Profits may be measured in several ways:

- Profits may be measured as a return on surplus, using assumed premium to surplus (or reserves to surplus) leverage ratios (Butsic and Lerwick [39]; Bingham [19], [21]). This is the profit measure used in Section 4, the "business expansion" illustration. This is actually a return on sales measure, with an assumed turnover rate.
- Profits may be measured as the net present value of premiums minus the net present value of expenditures (losses, expenses,

[69] In contrast, life insurance capital to asset ratios are low, and surplus is needed more for asset risk and interest rate risk than for insurance risk. In other words, a "commitment of surplus" to support the insurance policy is less necessary. This difference can be seen most clearly in the risk-based capital formulas for life and property/casualty insurers. The property/casualty formula is dominated by underwriting risks (reserving risks and premium risks), whereas the life formula is dominated by asset risks (bond risks and equity risks); see Feldblum [73].

and taxes). Thus, Anderson [8], recommends that "the profit objective be defined by the criterion that the present value of the profits which will be received in the future be equal to the present value of the surplus depletion, with both present values based on a yield rate or yield rates which represent adequate return to the stockholders for the degree of risk incurred in expending surplus in the expectation of receiving future profits. That is, the present value of the entire series of profits and losses is zero." Surplus is relevant only for determining the taxes on investment income derived from capital (Myers and Cohn [135]).[70] This is similar to the dollar measure of profits in Section 6.

- Profits may be measured by a multi-period internal rate of return model, by showing:
  - the cash transaction between the insurer and its policyholders or claimants,
  - the investment transactions between the insurer and the financial markets, and
  - the implied equity transactions between the insurer and its stockholders (Cummins [50], [51]; Feldblum [71]).

  This procedure is the most accurate, since it determines the profit measure from all cash flows over the life of the policy. Other "multi-period" internal rate of return models show

[70] In other words, the surplus provided by equityholders is invested in financial markets and earns an appropriate return, which is returned to the equityholders. Were there no income taxes, there would be no need to consider the amount of surplus when pricing the policy. However, there are income taxes, and the investment income earned on equityholder supplied funds is taxed first at corporate rates before being returned to the equityholders. Equityholders would prefer to invest their funds themselves in the financial markets, rather than give them to an insurance company. Therefore, say Myers and Cohn, the policyholders must pay the tax on the investment income earned on policyholder supplied funds.

This argument by Myers and Cohn is true for all pricing models, not just for their risk-adjusted discounted cash flow procedure. The asset share exhibits shown in this paper are on a pre-tax basis. A major effect of putting the figures on a post-tax basis is the "double-taxation" of the investment income on equityholder supplied funds.

multiple periods from only one policy. This procedure shows multiple periods from each renewal. Nevertheless, its complexity may make this procedure less suitable for practical pricing work.

- Profits may be measured more simply, such as by the "discounted payback period," which is the number of years until the cumulative net present value of profits is positive (Atkinson [11, p. 18]). In the business expansion illustration, the cumulative net present value of profits is negative for the first four years and turns to a positive $11,000 in the fifth year. In other words, a policyholder must persist for at least five years before the transaction becomes profitable for the insurer.

Payback measures are sometimes criticized for their failure to consider the time value of money (Brealey and Myers [28]; Weston and Copeland [166]). This criticism is disingenuous: one need simply accumulate losses and profits at an appropriate interest rate to account for the time value of money. For instance, suppose a policy produces losses of $1,000 at the end of year 1, and then profits of $200 a year for the next ten years. Table 9 shows that the payback periods are six years at a 0% annual interest rate and nine years at a 10% interest rate.

9. CONCLUSION

Actuarial pricing must consider long-term profitability and market share objectives, not merely short-term accounting results. Considerations of persistency patterns, the variation of expected losses and expenses with the time since inception of the policy, and the use of a model that incorporates these effects are essential for accurate ratemaking.

This paper has presented the fundamentals of such an approach. It builds upon life insurance asset share techniques and adapts them for personal automobile business.

TABLE 9

PAYBACK PERIODS AT 0% AND 10% INTEREST RATES

Year	Cash Flow	Cumulative Cash Flow: 0% Interest		Cumulative Cash Flow: 10% Interest
1	−1,000	−1,000		−1,000
2	200	−800	$-1{,}000 \times 1.1 + 200 =$	−900
3	200	−600	$-900 \times 1.1 + 200 =$	−790
4	200	−400	$-790 \times 1.1 + 200 =$	−669
5	200	−200	$-669 \times 1.1 + 200 =$	−536
6	200	0	$-536 \times 1.1 + 200 =$	−389
7	200	200	$-389 \times 1.1 + 200 =$	−228
8	200	400	$-228 \times 1.1 + 200 =$	−51
9	200	600	$-51 \times 1.1 + 200 =$	144
10	200	800	$144 \times 1.1 + 200 =$	358
11	200	1,000	$358 \times 1.1 + 200 =$	594

Some of the specific techniques discussed above are new, but the underlying philosophy is not. Underwriters and salespersons of the major personal lines carriers base their marketing decisions upon intuitive estimates of long term results. Actuaries, seeking more accurate assessments, must strive to replace the intuition with facts.

REFERENCES

[1] Actuarial Standards Board, Actuarial Standard of Practice No. 19, "Actuarial Appraisals," American Academy of Actuaries, October 1991.

[2] Aetna Life and Casualty report, *Automobile Insurance Affordability*, March 1978.

[3] AICPA Committee on Insurance Accounting and Auditing, *Audits of Stock Life Insurance Companies*, Fourth Edition, New York: American Institute of Certified Public Accoutants, 1985.

[4] AICPA Insurance Companies Committee, *Audits of Fire and Casualty Insurance Companies*, New York: American Institute of Certified Public Accoutants, 1993.

[5] All-Industry Research Advisory Council, *Attorney Involvement in Auto Injury Claims*, Oak Brook, Illinois: All-Industry Research Advisory Council, 1988.

[6] All-Industry Research Advisory Council, *Compensation for Automobile Injuries in the United States*, Oak Brook, Illinois: AIRAC, March 1989.

[7] Almagro, Manuel, and Thomas L. Ghezzi, "Federal Income Taxes—Provisions Affecting Property/Casualty Insurers," *PCAS* LXXV, 1988, pp. 95–161.

[8] Anderson, James C. H., "Gross Premium Calculation and Profit Measurement for Nonparticipating Insurance," *Transactions of the Society of Actuaries* 11, 1959, pp. 357–394.

[9] Ang, James S., and Tsong-Yue Lai, "Insurance Premium Pricing and Ratemaking in Competitive Insurance and Capital Asset Markets," *Journal of Risk and Insurance* 54, 4, December 1987, pp. 767–779.

[10] Atkinson, David B., *Gross Premiums and Asset Shares*, Society of Actuaries Part 7 Study Note, 1987.

[11] Atkinson, David B., *Pricing Individual Life Insurance*, Society of Actuaries Course I-340 Study Note, 1989.

[12] Bailey, Robert A., and LeRoy J. Simon, "An Actuarial Note on the Credibility of Experience of a Single Private Passenger Car," *PCAS* XLVI, 1959, p. 159; discussion by W. J. Hazam, *PCAS* XLVII, 1960, p. 150.

[13] Barnhart, E. Paul, "Adjustment of Premiums Under Guaranteed Renewable Policies," *Transactions of the Society of Actuaries* 12, 1960, pp. 472–498; discussions by Walter Shur, pp. 499–501; Eduard H. Minor, pp. 501–504; E. L. Bartleson, pp. 504–506; Robert P. Coates, pp. 506–507; Ward Van B. Hart, pp. 507–509; Charles N. Walker, pp. 509–510; John H. Miller, pp. 510–511; author's review of discussions, pp. 511–525.

[14] Batten, R. W., *Mortality Table Construction*, Englewood Cliffs, NJ: Prentice-Hall, Inc., 1978.

[15] Belth, Joseph M., "The Impact of Lapse Rates on Life Insurance Prices," *Journal of Risk and Insurance* 35, 1, March 1968, pp. 17–34.

[16] Benjamin, Sidney, "Profit and Other Financial Concepts in Insurance," *Journal of the Institute of Actuaries* 103, 3, 424, December 1976, pp. 233–281; discussions by W. R. Rowland, pp. 282–285; R. E. Beard, pp. 285–286; M. S. Morris, pp. 286–288; D. H. Reid, pp. 288–290; A. A. Mason, pp. 290–291; J. Goford, page 291; M. Weinberg, pp. 291–292; G. B. Hey, pp. 292–294; G. H. Stacy, page 294; C. S. S. Lyon, pp. 294–295; P. N. Downing, pp. 295–296; L. D. Coe, pp. 296–297; T. G. Clarke, pp. 297–298; H. H. Scurfield, pp. 298–300; Gordon V. Bayley, pp. 300–301; author's reply, pp. 301–303; Harald Bohman, pp. 303–304; A. R. N. Ratcliff, pp. 304–305.

[17] Berger, Allen N., J. David Cummins, and Mary A. Weiss, "The Coexistence of Multiple Distribution Systems for Financial Services: The Case of Property-Liability Insurance," Finance and Economics Discussion Series, Washington, D.C.: Federal Reserve Board, May 1995.

[18] Berin, Barnet N., George J. Stolnitz, and Aaron Teitlebaum, "Women's Status: A Matter of Mortality," *Contingencies* 2, 1, January/February 1990, pp. 48–54.

[19] Bingham, Russ, "Discounted Return—Measuring Profitability and Setting Targets," *PCAS* LXXVII, 1990, pp. 124–159.

[20] Bingham, Russell E., "Rate of Return: Policyholder, Company and Shareholder Perspectives," *PCAS* LXXX, 1993, pp. 110–147.

[21] Bingham, Russell E., "Surplus—Concepts, Measures of Return, and its Determination," *Insurer Financial Solvency*, Casualty Actuarial Society Discussion Paper Program, 1992, Vol. I, pp. 179–230.

[22] Bingham, Russell E., "Surplus: Concepts, Measures of Return, and its Determination," *PCAS* LXXX, 1993, pp. 55–109.

[23] Black, Kenneth, Jr., and Harold D. Skipper, Jr., *Life Insurance*, Twelfth Edition, Englewood Cliffs, NJ: Prentice-Hall, 1993.

[24] Bluhm, William F., "Cumulative Antiselection Theory," *Transactions of the Society of Actuaries* 34, 1982, pp. 215–231; discussions by Claude Y. Paquin, pp. 233–234; Charles Habeck, pp. 234–240; Howard J. Bolnick, pp. 240–242; Francis T. O'Grady and Vincent Dooley, pp. 242–244; author's review of discussions, pp. 244–246.

[25] Bluhm, William F. and Spencer Koppel, "Individual Health Insurance Premiums," *Individual Health Insurance*, ed. Francis T. O'Grady, Schaumburg, Illinois: Society of Actuaries, 1988, chapter 4, pp. 57–91.

[26] Boden, Leslie I., and Charles A. Fleischman, *Medical Costs in Workers' Compensation: Trends and Interstate Comparisons*, Cambridge, Massachusetts: Workers Compensation Research Institute, 1989.

[27] Borba, Philip S., "Benefit Utilization," *NCCI Digest* 4, 4, December 1989, pp. 51–72.

[28] Brealey, Richard A. and Stewart C. Myers, *Principles of Corporate Finance*, Fourth Edition, New York: McGraw Hill Book Company, 1991.

[29] Brissman, Mark D., "The Effect of a Suburban Driving Population on Urban Auto Insurance Premiums," *Journal of Risk and Insurance* 47, 4, December 1980, pp. 636–659.

[30] Brown, Robert L., *Introduction to Ratemaking and Reserving for Property and Casualty Insurance*, Winsted, Connecticut: ACTEX Publications, Inc., 1993.

[31] Brubaker, Randall E., "Geographic Rating of Individual Risk Transfer Costs Without Territorial Boundaries," Casualty Actuarial Society *Forum*, Winter 1996, pp. 97–128.

[32] Buck, Louis E., "Statement before the National Association of Insurance Commissioners," in Aetna Life and Casualty, *Automobile Insurance Affordability*, March 1978, pp. 3–13.

[33] Buck, Norman F., "First Year Lapse and Default Rates," *Transactions of the Society of Actuaries* 12, 1960, pp. 258–293; discussions by Ernest J. Moorhead, pp. 294–295; James C. H. Anderson, pp. 295–296; Charles F. B. Richardson, pp. 296–302; E. James Morton, pp. 302–305; Neil W. MacIntyre, pp. 305–309; Maurice B. Roberts, pp. 309–310; author's review of discussions, pp. 311–314.

[34] Butler, Patrick, Twiss Butler, and Laurie L. Williams, "Sex-Divided Mileage, Accident, and Insurance Cost Data Show that Auto Insurers Overcharge Women," Part I in *Journal of Insurance Regulation* 6, 3, March 1988, pp. 243–284, and Part II in 6, 4, June 1988, pp. 382–420.

[35] Butler, Richard J., and John D. Worrall, "Labor Market Theory and the Distribution of Workers' Compensation Losses," *Workers' Compensation Insurance Pricing: Current Programs and Proposed Reforms*, eds. Philip S. Borba and David Appel, Boston: Kluwer Academic Publishers, 1988, pp. 19–34.

[36] Butler, Richard J., and John D. Worrall, "Work Injury Compensation and the Duration of Nonwork Spells," *The Economic Journal* 65, 4, September 1985, pp. 714–724.

[37] Butsic, Robert P., "Determining the Proper Interest Rate for Loss Reserve Discounting: An Economic Approach," *Evaluating Insurance Company Liabilities*, Casualty Actuarial Society Discussion Paper Program, 1988, pp. 147–188.

[38] Butsic, Robert P., "Property-Liability Rate of Return—(Can You Trust It?)," Part I in *The Actuarial Digest* 9, 2, April/May 1990, pp. 3 ff. and Part II in 9, 3, June/July 1990, pp. 7 ff.

[39] Butsic, Robert P., and Stuart Lerwick, "An Illustrated Guide to the Use of the Risk-Compensated Discounted Cash Flow Method," Casualty Actuarial Society *Forum*, Spring 1990, pp. 303–347.

[40] Casualty Actuarial Society, Committee on Reserves, "Position Paper on the Methodologies and Considerations Regarding Loss Reserve Discounting," Casualty Actuarial Society *Forum*, Fall 1987, pp. 212–228.

[41] Casualty Actuarial Society, Committee on Reserves, "Risk Margins for Discounted Loss Reserves," Casualty Actuarial Society *Forum*, Winter 1991, pp. 139–160.

[42] Casualty Actuarial Society, Committee on the Theory of Risk, "Risk Theoretic Issues in the Discounting of Loss Reserves," Casualty Actuarial Society *Forum*, Fall 1987, pp. 46–61.

[43] Cather, David A., Sandra G. Gustavson, and James S. Trieschmann, "A Profitability Analysis of Property-Liability Insurers Using Alternative Distribution Systems," *Journal of Risk and Insurance* 52, 2, June 1985, pp. 321–332.

[44] Chernick, David R., "Private Passenger Automobile Physical Damage Ratemaking," CAS Part 6 Examination Study Note.

[45] Childs, Diana, and Ross A. Currie, "Expense Allocation in Insurance Ratemaking," *Pricing Property and Casualty Insurance Products*, Casualty Actuarial Society Discussion Paper Program, 1980, pp. 32–66.

[46] Conger, Robert F., "The Construction of Automobile Rating Territories in Massachusetts," *PCAS* LXXIV, 1987, pp. 1–74.

[47] Conning & Company, *New Business versus Renewals: The Cost of New Business in a Soft Market*, Hartford, Connecticut: Conning & Co., June 1988.

[48] Cummins, J. David, "Discounted Cash-Flow Ratemaking Models in Property-Liability Insurance," *Benefits, Costs, and Cycles in Workers' Compensation*, eds. Philip S. Borba and David Appel, Boston: Kluwer Academic Publishers, 1990, pp. 163–182.

[49] Cummins, J. David, "Economies of Scale in Independent Insurance Agencies," *Journal of Risk and Insurance* 44, 4, December 1977, pp. 539–553.

[50] Cummins, J. David, "Multi-Period Discounted Cash Flow Ratemaking Models in Property-Liability Insurance," *Journal of Risk and Insurance* 57, 1, March 1990, pp. 79–109.

[51] Cummins, J. David, "Statistical and Financial Models of Insurance Pricing and the Insurance Firm," *Journal of Risk and Insurance* 58, 2, June 1991, pp. 261–302.

[52] Cummins, J. David, Scott E. Harrington, and Robert W. Klein (eds.), *Cycles and Crises in Property/Casualty Insurance: Causes and Implications for Public Policy*, National Association of Insurance Commissioners, 1991.

[53] Cummins, J. David, and Sharon Tennyson, "Controlling Automobile Insurance Costs," *Journal of Economic Perspectives* 6, 2, Spring 1992, pp. 95–115.

[54] Cummins, J. David, and Jack VanDerhei, "A Note on the Relative Efficiency of Property-Liability Insurance Distribution Systems," *The Bell Journal of Economics* 10, 1979, pp. 709–719.

[55] Dahlman, Gary E., "1980 Amendments to the Standard Valuation Law," Society of Actuaries Course I-340 Study Note, 1989.

[56] D'Arcy, Stephen P., "Application of Economic Theories of Regulation to the Property-Liability Insurance Industry," *Journal of Insurance Regulation* 7, 1, September 1988, pp. 19–51.

[57] D'Arcy, Stephen P., "Revisions in Loss Reserving Techniques Necessary to Discount Property-Liability Loss Reserves," *PCAS* LXXIV, 1987, pp. 75–100.

[58] D'Arcy, Stephen P., "Use of the CAPM to Discount Property-Liability Loss Reserves," *Journal of Risk and Insurance* 55, 3, September 1988, pp. 481–491.

[59] D'Arcy, Stephen P., and Neil A. Doherty, "Adverse Selection, Private Information and Lowballing in Insurance Markets," *Journal of Business* 63, April 1990, p. 145.

[60] D'Arcy, Stephen P., and Neil A. Doherty, "The Aging Phenomenon and Insurance Prices," *PCAS* LXXVI, 1989, pp. 24–44.

[61] D'Arcy, Stephen P., and Neil A. Doherty, *The Financial Theory of Pricing Property-Liability Insurance Contracts*, Homewood, Illinois: Richard D. Irwin, Inc., 1988.

[62] D'Arcy, Stephen P., and James R. Garven, "Property-Liability Insurance Pricing Models: An Empirical Evaluation," *Journal of Risk and Insurance* 57, 3, September 1990, pp. 391–419.

[63] Daykin, Chris D., Teivo Pentikäinen, and M. Pesonen, *Practical Risk Theory for Actuaries*, First Edition, Chapman and Hall, 1994.

[64] Derrig, Richard A., "The Development of Property-Liability Insurance Pricing Models in the United States 1969–1989," paper presented at the first AFIR International Colloquium, Paris, France: April 1990; reprinted in the Casualty Actuarial Society *Forum*, Fall 1991, pp. 19–46.

[65] Derrig, Richard A., "The Use of Investment Income in Massachusetts Private Passenger Automobile and Workers' Compensation Ratemaking," *Fair Rate of Return in Property-Liability Insurance*, eds. J. D. Cummins and S. E. Harrington, Boston: Kluwer*Nijhoff Publishing, 1987, pp. 119–146.

[66] Eckman, Michael V., "Additional Source of Earnings Analysis under FAS 97 Universal Life Accounting and Some Observations on the Effect of Unlocking Assumptions," *Transactions of the Society of Actuaries* 42, 1990, pp. 59–81; discussion by Joseph H. Tan, pp. 83–87; author's review of discussion, pp. 87–90.

[67] Fairley, William, "Investment Income and Profit Margins in Property-Liability Insurance: Theory and Empirical Results," *The Bell Journal of Economics* 10, Spring 1979, pp. 192–210; reprinted in *Fair Rate of Return in Property-Liability Insurance*, eds. J. D. Cummins and S. E. Harrington, Boston: Kluwer*Nijhoff Publishing, 1987, pp. 1–26.

[68] Feldblum, Sholom, "Expense Allocation and Policyholder Persistency," *Pricing*, Casualty Actuarial Society Discussion Paper Program, 1990, pp. 29–54.

[69] Feldblum, Sholom, "The Insurance Expense Exhibit and the Allocation of Investment Income," Casualty Actuarial Society Part 7 Examination Study Note, Third Edition, September 1995.

[70] Feldblum, Sholom, "Persistency and Profits," *Pricing*, Casualty Actuarial Society Discussion Paper Program, 1990, pp. 55–84.

[71] Feldblum, Sholom, "Pricing Insurance Policies: The Internal Rate of Return Model," Casualty Actuarial Society Part 10A Examination Study Note, 1992.

[72] Feldblum, Sholom, "Proposition 103: Text and Commentary," *The Actuarial Review*, February 1989, supplement.

[73] Feldblum, Sholom, "NAIC Property-Casualty Risk-Based Capital Requirements," *PCAS* LXXXIII, 1996, pp. 297–435.

[74] Feldblum, Sholom, "Statutory Returns on Surplus and the Cost of Equity Capital," Casualty Actuarial Society *Forum*, Summer 1993, pp. 203–224.

[75] Feldblum, Sholom, "Workers' Compensation Ratemaking," Casualty Actuarial Society Part 6 Study Note, September 1993.

[76] Feldblum, Sholom, "Underwriting Cycles and Insurance Solvency," *Insurer Financial Solvency*, Casualty Actuarial Society Discussion Paper Program, 1992, Vol. I, pp. 383–438.

[77] Ferrari, J. Robert, "The Relationship of Underwriting, Investments, Leverage, and Exposure to Total Return on Owners' Equity," *PCAS* LV, 1968, pp. 295–302; discussion by R. J. Balcarek, *PCAS* LVI, 1969, pp. 58–60; R. A. Bailey, *PCAS* LVI, 1969, pp. 60–62.

[78] Financial Accounting Standards Board, Statement of Financial Accounting Standards No. 60, "Accounting and Reporting by Insurance Enterprises," June 1982.

[79] Financial Accounting Standards Board, Statement of Financial Accounting Standards No. 97, "Accounting and Reporting by Insurance Enterprises for Certain Long-Duration Contracts and for Realized Gains and Losses from the Sale of Investments," December 1987.

[80] Financial Accounting Standards Board, Discussion Memorandum, *Present Value-Based Measurements in Accounting*, December 1990.

[81] Finger, Robert J., "Risk Classification," *Foundations of Casualty Actuarial Science*, ed. M. Rodermund, et al., Second Edition, New York: Casualty Actuarial Society, 1992, pp. 231–276.

[82] Freifelder, R. L., *A Decision Theoretic Approach to Insurance Ratemaking*, Homewood, Illinois: Richard D. Irwin, 1976.

[83] Fudenberg, Drew, and Jean Tirole, "Noncooperative Game Theory for Industrial Organization: An Introduction and Overview," *Handbook of Industrial Organization*, eds. R. Schmalensee and R. D. Willig, North-Holland: Elsevier Science Publishing Co., 1989, I, pp. 159–328.

[84] Galt, Helen, "Term Insurance," Society of Actuaries Course I-441U Study Note, 1989.

[85] Garven, James R., "On the Application of Finance Theory to the Insurance Firm," *Financial Models of Insurance Solvency*, eds. J. D. Cummins and Richard Derrig, Boston: Kluwer Academic Publishers, 1989, pp. 243–265.

[86] Graves, Nancy C., and Richard Castillo, "Commercial General Liability Ratemaking for Premises and Operations," *Pricing*, Casualty Actuarial Society Discussion Paper Program, 1990, II, pp. 631–696.

[87] Griffin, Dale C., Donald A. Jones, and Lee M. Smith, "Profit Measurement in Workers' Compensation Insurance," *Journal of Insurance Regulation* 1, 3, March 1983, pp. 378–397.

[88] Hamilton, Thomas M., and Eric L. Routman, "Cleaning Up America: Superfund and Its Impact on the Insurance Industry," *CPCU Journal* 41, 3, September 1988, pp. 172–184.

[89] Hanson, Jon S., "Measurement of Profitability and Treatment of Investment Income in Property and Liability Insurance," *Proceedings of the NAIC* IIA, 1970, pp. 719–951.

[90] Harrington, Scott E., and Patricia M. Danzon, "Price-Cutting in Liability Insurance Markets," *Cycles and Crises in Property/Casualty Insurance: Causes and Implications for Public Policy*, eds. J. D. Cummins, S. E. Harrington, and R. W. Klein, National Association of Insurance Commissioners, 1991, pp. 122–188.

[91] Harwayne, Frank, "Use of National Experience Indications in Workers' Compensation Classification Ratemaking," *PCAS* LXIV, 1977, pp. 74–84; discussions by James F. Golz, pp. 85–86; Lester B. Dropkin, pp. 87–92.

[92] Hill, Raymond, "Profit Regulation in Property-Liability Insurance," *The Bell Journal of Economics* 10, 1, Spring 1979, pp. 172–191.

[93] Hill, Raymond D., and Franco Modigliani, "The Massachusetts Model of Profit Regulation in Nonlife Insurance: An Appraisal and Extensions," *Fair Rate of Return in Property-Liability Insurance*, eds. J. D. Cummins and S. E. Harrington, Boston: Kluwer*Nijhoff Publishing, 1987, pp. 27–53.

[94] Holman, David L., and Chris C. Stroup, "Generally Accepted Accounting Principles," *Property-Liability Insurance Accounting*, Sixth Edition, Durham, NC: Insurance Accounting and Systems Association, Inc., July 1994, chapter 14.

[95] Huffman, Peyton J., "Asset Share Mathematics," *Transactions of the Society of Actuaries* 30, 1978, pp. 277–286; discussions by Pierre C. Chouinard, pp. 287–302; Mark D. J. Evans, pp. 302–305; Frank C. Metz, pp. 305–309; Robert R. Reitano, pp. 309–318; James A. Tilley, pp. 318–319; author's review of discussions, pp. 319–322.

[96] Hunt, James H., "The Allocation of Insurer Expenses Among Automobile Policyholders," *Automobile Insurance Risk Classification: Equity and Accuracy*, eds. A. F. Giffin, V. Travis, and W. Owen, Boston: Massachusetts Division of Insurance, 1978, pp. 121–143.

[97] Hurley, Robert L., "Commercial Fire Insurance Ratemaking Procedures," *PCAS* LX, 1973, pp. 208–257; discussions by Henry C. Schneiker, *PCAS* LXI, 1974, pp. 62–67; William P. Amlie, pp. 67–70; author's response, pp. 70–72.

[98] Insurance Research Council, *Auto Injuries: Claiming Behavior and Its Impact on Insurance Costs*, Oak Brook, Illinois, September 1994.

[99] Insurance Research Council, *Paying for Auto Injuries: A Consumer Panel Survey of Auto Accident Victims*, Oak Brook, Illinois, May 1994.

[100] Insurance Research Council, *Trends in Auto Bodily Injury Claims*, Oak Brook, Illinois, 1990.

[101] Insurance Research Council, *Trends in Auto Injury Claims*, Second Edition, Part One: Analysis of Claim Frequency, Wheaton, Illinois, February 1995.

[102] Insurance Services Office, *Changes in the Economic Environment: Insurance Implications*, New York: Insurance Services Office, October 1991.

[103] Insurance Services Office, "Expense Provisions in the Rates," CAS Part 6 Examination Study Note.

[104] Insurance Services Office, *Personal Auto Manual*, Third Edition, New York: Insurance Services Office, 1989.

[105] Insurance Services Office, *Superfund and the Insurance Issues Surrounding Abandoned Hazardous Waste Sites*, New York: Insurance Services Office, December 1995.

[106] Jacobs, Gregory D., "Pricing Non-Traditional Individual Life Products," Society of Actuaries Course I-340 Study Note, 1984.

[107] Jensen, Russell R., "Choice of Basis for Dividend Illustrations," *Transactions of the Society of Actuaries* 30, 1978, pp. 447–475; discussions by Peter F. Chapman, p. 477; John H. Harding, pp. 477–482; James P. Larkin, pp. 482–485; E. J. Moorhead, pp. 485–488; author's review of discussions, p. 488.

[108] Jordan, Chester Wallace, Jr., *Life Contingencies*, Second Edition, Chicago, Illinois: Society of Actuaries, 1975.

[109] Joskow, Paul L., "Cartels, Competition, and Regulation in the Property-Liability Insurance Industry," *The Bell Journal of Economics and Management Science* 4, 2, Autumn 1973, pp. 375–427.

[110] Kormes, Mark, "Small Risks versus Large Risks in Workmen's Compensation Insurance," *PCAS* XXIII, 1936–1937, pp. 46–76; discussions by Grady H. Hipp, pp. 257–261; G. F. Michelbacher, pp. 261–266; author's review of discussions, pp. 266–268.

[111] Kulp, C. A., and John W. Hall, *Casualty Insurance*, New York: John Wiley and Sons, 1968.

[112] Kunreuther, Howard, and M. V. Rajeev Gowda (eds.), *Integrating Insurance and Risk Management for Hazardous Wastes*, Boston: Kluwer Academic Publishers, 1990.

[113] Lange, Jeffrey T., "General Liability Insurance Ratemaking," *PCAS* LIII, 1966, p. 26; discussion by P. D. Presley, p. 53; S. C. DuRose, p. 56; author's response, p. 58.

[114] Larner, Ken, and John P. Ryan, "Appraisal Values—A Comparison of European and North American Practice," *International Topics: Global Insurance Pricing, Reserving and Coverage Issues*, Casualty Actuarial Society Discussion Paper Program, 1991, pp. 445–477.

[115] Life Insurance Marketing and Research Association, Inc., "1985–86 Long-Term Ordinary Lapse Survey in the United States," *Transactions of the Society of Actuaries*, 1984 Reports of Mortality, Morbidity and Other Experience, 1988, pp. 331–334.

[116] Life Insurance Marketing and Research Association, Inc., "1986–1987 Long-Term Ordinary Lapse Survey in the United States," *Transactions of the Society of Actuaries*, 1985–1986–1987 Reports of Mortality, Morbidity and Other Experience, 1990, pp. 265–284.

[117] Life Insurance Marketing and Research Association, Inc., "1983–1986 Whole Life Lapsation in the United States," *Transactions of the Society of Actuaries*, 1985–1986–1987 Reports of Mortality, Morbidity and Other Experience, 1990, pp. 285–302.

[118] Linton, M. A., "Returns Under Agency Contracts," *Record of the American Institute of Actuaries* 12, 1924, pp. 283–319.

[119] Lombardi, Lucian J., and Joel I. Wolfe, "Agent Compensation," *Marketing for Actuaries*, ed. E. Torlan, Hartford, Connecticut: Life Insurance Marketing and Research Association, 1986.

[120] Lowe, Stephen P., "GAAP and the Casualty Actuary," *Valuation Issues*, Casualty Actuarial Society Discussion Paper Program, 1989, pp. 259–278.

[121] Lowe, Stephen P., "A New Performance Measure for P/C Insurers," *Emphasis*, Summer 1988, pp. 8–11.

[122] Lowe, Stephen P., "Reserve Discounting, Rates of Return, and Profitability," *NCCI Digest* 2, 1, April 1987, pp. 19–26.

[123] Lowe, Stephen P., and Stephen W. Philbrick, "Issues Associated with the Discounting of Property/Casualty Loss Reserves," *Journal of Insurance Regulation* 4, 4, June 1986, pp. 72–102.

[124] Mahler, Howard C., "An Introduction to Underwriting Profit Models," *PCAS* LXXI, 1987, pp. 239–277.

[125] Mahler, Howard C., "Investment Income in Ratemaking in Massachusetts," Casualty Actuarial Society *Forum*, Fall 1991, pp. 197–216.

[126] Mahler, Howard C., "The Myers-Cohn Model, A Practical Application," manuscript, 1994.

[127] Marter, Sarah S., and Herbert I. Weisberg, "Medical Costs and Automobile Insurance: A Report on Bodily Injury Liability Claims in Massachusetts," *Journal of Insurance Regulation* 9, 3, March 1991, pp. 381–422.

[128] Marter, Sarah S., and Herbert I. Weisberg, "Medical Expenses and the Massachusetts Automobile Tort Reform Law: First Review of the Automobile Insurers' Bureau Study of 1989 Bodily Injury Liability Claims," Boston: Automobile Insurers Bureau of Massachusetts, July 1991.

[129] McClenahan, Charles L., "Ratemaking," *Foundations of Casualty Actuarial Science*, eds. M. Rodermund, et al., Second Edition, New York: Casualty Actuarial Society, 1992, pp. 25–90.

[130] McNamara, Daniel J., "Discrimination in Property-Liability Insurance Pricing," *Issues in Insurance*, eds. J. D. Long and E. D. Randall, Third Edition, Volume I, Malvern, Pennsylvania: American Institute for Property and Liability Underwriters, 1984, pp. 1–64.

[131] Menge, Walter O., and Carl H. Fischer, *The Mathematics of Life Insurance*, Second Edition, Ann Arbor, Michigan: Ulrich's Books, Inc., 1935.

[132] Miller, Lynne M., with Mary J. Mallonee (eds.), *Insurance Claims for Environmental Damages*, New York: Executive Enterprises, Inc., 1989.

[133] Moorehead, Ernest J., "The Construction of Persistency Tables," *Transactions of the Society of Actuaries* 12, 1960, pp. 545–563.

[134] Moorehead, Ernest J., Discussion of Buck: "First Year Lapse and Default Rates," *Transactions of the Society of Actuaries* 12, 1960, pp. 294–295.

[135] Myers, Stewart, and Richard Cohn, "A Discounted Cash Flow Approach to Property-Liability Insurance Rate Regulation," *Fair Rate of Return in Property-Liability Insurance*, eds. J. D. Cummins and S. E. Harrington, Boston: Kluwer*Nijhoff Publishing, 1987, pp. 55–78.

[136] National Association of Insurance Commissioners, "Report of the Investment Income Task Force to the National Association of Insurance Commissioners," *Proceedings of the NAIC* II, 1984 (Supplement), pp. 3–96; reprinted in *Issues in Insurance* II, eds. J. D. Long and E. D. Randall, Third Edition, Malvern, Pennsylvania: The American Institute for Property and Liability Underwriters, 1984, pp. 111–216; reprinted in the *Journal of Insurance Regulation* 3, 1, September 1984, pp. 39–112, and 3, 2, December 1984, pp. 153–181.

[137] National Council on Compensation Insurance, *Issues Report, 1991: A Summary of Issues Influencing Workers Compensation*, Boca Raton, Florida: National Council on Compensation Insurance, 1991.

[138] Neill, Alistair, *Life Contingencies*, London: Heinemann, 1977.

[139] New Jersey Compensation Rating and Inspection Bureau, *Annual Report 1991*, May 1992.

[140] Nodulman, Norman B., "Expense Analysis," Society of Actuaries Course I-340 Study Note, 1991.

[141] Paquin, Claude Y., "Cash Flow Analysis by the Prudent Banker's Method, or Discounting Turned on Its Head," *Transactions of the Society of Actuaries* 39, 1987, pp. 177–182; discussions by Donald R. Sondergeld, p. 183; Mark D. J. Evans, pp. 184–189; William L. Roach, pp. 189–192; Eric Seah and Elias S. W. Shiu, pp. 193–194; Roger E. Johnson, pp. 194–198; S. David Promislow, pp. 198–200; Bradley E. Barks, pp. 200–206; Courtland C. Smith, pp. 206–212; Thomas M. Maria, pp. 212–213; author's review of discussions, p. 214.

[142] Pentikäinen, Bonsdorff, Pesonen, Rantala, and Ruohonen, *Insurance Solvency and Financial Strength*, Helsinki, Finland: Finnish Insurance Training and Publishing Co., 1989.

[143] Porter, Michael, *Competitive Strategy*, New York: The Free Press, 1980.

[144] Robbin, Ira, "The Underwriting Profit Provision," Casualty Actuarial Society Part 6 Study Note, 1992.

[145] Rosenthal, Norman L., "Testimony on Behalf of Allstate Insurance Company before the Insurance Commissioner of the State of California," 1989.

[146] Ryder, Ambrose, "Informal Discussion: Automobile Liability Insurance," *PCAS* XXII, 1935–1936, pp. 143 ff.

[147] Salzmann, Ruth E., *Estimated Liabilities for Losses and Loss Adjustment Expenses*, West Nyack, NY: Prentice-Hall, 1984.

[148] Scherer, Frederick M., *Industrial Market Structure and Economic Performance*, Second Edition, Boston: Houghton Mifflin Company, 1980.

[149] Schraeder, Robert J., "An Overview of the Property/Casualty Financial Condition," *PCAS* LXVI, 1979, pp. 161–167.

[150] Shapiro, Carl, "Theories of Oligopoly Behavior," *Handbook of Industrial Organization*, eds. R. Schmalensee and R. D. Willig, North-Holland: Elsevier Science Publishing Co., 1989, I, pp. 329–414.

[151] Stern, Philipp K., "Ratemaking Procedures for Automobile Liability Insurance," *PCAS* LII, 1965, pp. 139–202; discussions by S. A. Dorf, *PCAS* LIII, 1966, pp. 190–192; J. F. Gill, pp. 192–194.

[152] Tan, Joseph H., "Source-of-Earnings Analysis Under FAS 97 Universal Life Accounting," *Transactions of the Society of Actuaries* 41, 1989, pp. 443–487; discussions by Bradley E. Barks, pp. 489–491; James E. Feldman, pp. 491–495; Mark Freedman, pp. 495–496; author's review of discussions, pp. 496–506.

[153] Tiller, Margaret W., Duane E. Allen, Kathryn G. Furr, William M. Mortimer, Marcus Ramsey, James A. Robertson, and Fames C. Vogt, "Solvency Issues in Discounting Loss Reserves," *CPCU Journal* 40, 1, March 1987, pp. 46–53.

[154] Tirole, Jean, *The Theory of Industrial Organization*, Cambridge, Massachusetts: The MIT Press, 1988.

[155] Urrutia, Jorge L., "The Capital Asset Pricing Model and the Determination of Fair Underwriting Returns for the Property-Liability Insurance Industry," *The Geneva Papers on Risk and Insurance* 11, 38, January 1986, pp. 44–60.

[156] Varian, Hal R., *Microeconomic Analysis*, Second Edition, New York: W. W. Norton and Company, 1984.

[157] Victor, Richard B., "Major Challenges Facing Workers' Compensation Systems in the 1990s," *Challenges for the 1990s*, ed. R. A. Victor, Cambridge, Massachusetts: Workers Compensation Research Institute, July 1990, pp. 9–23.

[158] Victor, Richard B., and Charles A. Fleischman, *How Choice of Provider and Recessions Affect Medical Costs in Workers' Compensation*, Cambridge, Massachusetts: Workers Compensation Research Institute, 1990.

[159] Wade, Roger C., "Expense Analysis in Ratemaking and Pricing," *PCAS* LX, 1973, pp. 1–10; discussions by Dale R. Comey, pp. 11–12; Orval E. Dahme, pp. 13–14; author's review of discussions, pp. 14–15.

[160] Walters, Mavis A., "Lessons Learned from Profits Lost," *Contingencies* 4, 6, November/December 1992, pp. 22 ff.

[161] Waterson, Michael, *Economic Theory of the Industry*, Cambridge: Cambridge University Press, 1984.

[162] Webb, Bernard I., "Investment Income in Insurance Ratemaking," *Journal of Insurance Regulation* 1, 1, September 1982, pp. 46–76.

[163] Weisberg, Herbert I., and Richard A. Derrig, "Fraud and Automobile Insurance: A First Report on Bodily Injury Liability Claims in Massachusetts," *Journal of Insurance Regulation* 9, 4, March 1991, pp. 497–541.

[164] Weisberg, Herbert I., and Richard A. Derrig, "Modelling the Payment of General Damages for Massachusetts Automobile Bodily Injury Liability Claims," Casualty Actuarial Society *Forum Special Edition: 1993 Ratemaking Call Papers*, Arlington, Virginia: Casualty Actuarial Society, February 1993.

[165] Weisberg, Herbert I., and Richard A. Derrig, "Preliminary Evaluation of Massachusetts Automobile Bodily Injury Tort Reform: A First Review of the AIB Study of 1989 BI Liability Claims," Boston: Automobile Insurers Bureau of Massachusetts, July 1991.

[166] Weston, J. Fred, and Thomas E. Copeland, *Managerial Finance*, Eighth Edition, Chicago: The Dryden Press, 1986.

[167] Whitehead, Guy H., "No-Claim Discount or Bonus/Malus Systems in Europe," *International Topics: Global Insurance Pricing, Reserving and Coverage Issues*, Casualty Actuarial Society Discussion Paper Program, 1991, pp. 305–386.

[168] Winn, Michael R., Leroy H. Christenson, A. Gordon Jardin, Philip K. Polkinghorn, and James L. Sweeney, "Effect of Lapse Rates of Profitability: Reinsurance View," *Record—Society of Actuaries* 15, 2, 1989, pp. 781–808.

[169] Woll, Richard G., "Auto Insurance and Territorial Rates," manuscript, May 1991.

[170] Woll, Richard G., "Insurance Profits: Keeping Score," *Financial Analysis of Insurance Companies*, Casualty Actuarial Society Discussion Paper Program, 1987, pp. 446–533.

[171] Woodman, Harry A., Jr., "Extra Premiums for Life Insurance on Substandard Risks," Society of Actuaries Course I-441U Study Note, 1989.

[172] Worrall, John D., David Appel, and Richard J. Butler, "Sex, Marital Status, and Medical Utilization by Injured Workers," *Journal of Risk and Insurance* 54, 1, March 1987, pp. 27–44.

[173] Worrall, John D., David Appel, and Richard J. Butler, "Sex, Marital Status, and Medical Utilization by Injured Workers," *NCCI Digest* 2, 1, April 1987, pp. 1–18.

[174] Worrall, John D., and Richard J. Butler, "Benefits and Claim Duration," *Workers' Compensation Benefits: Adequacy, Equity, and Efficiency*, eds. J. D. Worrall and D. Appel, Ithaca, NY: ILR Press, 1985, pp. 57–70.

EXHIBIT 1

ASSET SHARE MODEL FOR COMPANY GROWTH

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
Policy		PV of	Variable Expenses		Fixed Expenses		Persis-	Cumulative		Discount	PV of	PV of
Year	Premium	Losses	New	Renewal	New	Renewal	tency	Persistency	Profit	Factor	Profits	Premium
1	800	656	242	0	142	0	100%	100%	−240	1.00	−240	800
2	872	701	0	54	0	32	85%	85%	73	1.12	65	662
3	950	748	0	59	0	34	86%	73%	80	1.25	64	554
4	1,036	799	0	64	0	35	87%	64%	87	1.40	62	469
5	1,129	853	0	70	0	37	88%	56%	95	1.57	60	402
6	1,231	911	0	76	0	39	89%	50%	102	1.76	58	348
7	1,342	973	0	83	0	41	90%	45%	110	1.97	56	305
8	1,462	1,039	0	91	0	43	90%	40%	117	2.21	53	267
9	1,594	1,110	0	99	0	45	91%	37%	125	2.48	50	236
10	1,738	1,186	0	108	0	47	91%	33%	133	2.77	48	209
11	1,894	1,266	0	117	0	50	92%	31%	142	3.11	46	187
12	2,064	1,352	0	128	0	52	92%	38%	151	3.48	43	168
13	2,250	1,444	0	140	0	55	92%	26%	159	3.90	41	150
14	2,453	1,542	0	152	0	57	92%	24%	168	4.36	38	135
15	2,673	1,647	0	166	0	60	92%	22%	176	4.89	36	120
Total											$480	$5,012

Column 3, Present Value of Losses, is the present value at the beginning of that policy year.
Column 9, Cumulative Persistency, is the downward product of Column 8.
Column 10, Profit, equals Column 9 times {Column 2 minus the sum of Columns 3 through 7}.
Column 11, Discount Factor, is 12% a year compounded annually.
Column 12, Present Value of Profits, is Column 10 divided by Column 11.
Column 13, Present Value of Premium, is Column 2 times Column 9 divided by Column 11.

EXHIBIT 2

ADULT PLEASURE USE

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
Policy		PV of	Variable Expenses		Fixed Expenses		Persis-	Cumulative		Discount	PV of	PV of
Year	Premium	Losses	New	Renewal	New	Renewal	tency	Persistency	Profit	Factor	Profits	Premium
1	475	400	143	0	88	0	100%	100%	−157	1.00	−157	475
2	518	427	0	32	0	20	82%	82%	32	1.12	28	379
3	564	456	0	35	0	21	86%	71%	37	1.25	29	317
4	615	487	0	38	0	22	87%	61%	42	1.40	30	269
5	670	520	0	42	0	23	88%	54%	46	1.57	29	230
6	731	556	0	45	0	24	89%	48%	51	1.76	29	199
7	796	593	0	49	0	25	90%	43%	56	1.97	28	174
8	868	634	0	54	0	26	90%	39%	60	2.21	27	153
9	946	677	0	59	0	28	91%	35%	65	2.48	26	135
10	1,031	723	0	64	0	29	91%	32%	69	2.77	25	120
11	1,124	772	0	70	0	31	92%	30%	75	3.11	24	107
12	1,225	824	0	76	0	32	92%	27%	80	3.48	23	96
13	1,336	880	0	83	0	34	92%	25%	85	3.90	22	86
14	1,456	940	0	90	0	35	92%	23%	90	4.36	21	77
15	1,587	1,004	0	98	0	37	92%	21%	95	4.89	19	69
Total											$204	$2,887

Column 2: First year premium is chosen such that the present value of profits (Column 12 total) = 7.0% of the present value of premium (Column 13 total). Subsequently, premiums increase 9% per annum.
Column 3: First year losses average \$400; loss cost trend is +10% per annum; losses decrease 3% per annum as policy matures.
Columns 4 and 5: Variable expense ratio is 30.2% the first year and 6.2% in subsequent years.
Column 6: First year fixed expenses are \$98 per policy, or 17.8% of the average premium for all drivers (\$550). Fixed expenses for adult drivers are 10% lower, or \$88 per policy.
Column 7: Fixed expenses in the first renewal year are \$88x3.8% ÷ 17.8%. Subsequently, expenses increase 5% per annum.
Column 8: Assumed persistency rates for adult drivers. Column 9 = downward product of Column 8.
Column 10 = {Column 2 – sum (Columns 3 through 7)}xColumn 9.
Column 11: Discount factor reflecting annual 12% cost of capital; e.g., 1.25 = 1.12x1.12.
Column 12 = Column 10 ÷ Column 11.
Column 13 = Column 2xColumn 9 ÷ Column 11.

EXHIBIT 3

YOUNG MALE DRIVERS

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
Policy		PV of	Variable Expenses		Fixed Expenses		Persis-	Cumulative		Discount	PV of	PV of
Year	Premium	Losses	New	Renewal	New	Renewal	tency	Persistency	Profit	Factor	Profits	Premium
1	1,272	1,000	384	0	117	0	100%	100%	−230	1.00	−230	1272
2	1,386	1,068	0	86	0	26	60%	60%	124	1.12	110	743
3	1,511	1,141	0	94	0	28	65%	39%	97	1.25	78	470
4	1,647	1,218	0	102	0	29	70%	27%	81	1.40	58	320
5	1,796	1,301	0	111	0	30	73%	20%	70	1.57	45	227
6	1,957	1,389	0	121	0	32	76%	15%	63	1.76	36	168
7	2,133	1,484	0	132	0	34	79%	12%	58	1.97	29	129
8	2,325	1,584	0	144	0	35	82%	10%	55	2.21	25	103
9	2,535	1,692	0	157	0	37	85%	8%	54	2.48	22	85
10	2,763	1,807	0	171	0	39	88%	7%	55	2.77	20	73
11	3,011	1,930	0	187	0	41	90%	7%	56	3.11	18	64
12	3,282	2,061	0	204	0	43	90%	6%	58	3.48	17	56
13	3,578	2,201	0	222	0	45	90%	5%	59	3.90	15	49
14	3,900	2,351	0	242	0	47	90%	5%	61	4.36	14	43
15	4,251	2,511	0	264	0	50	90%	4%	62	4.89	13	38
Total											$269	$3,841

Column 2: First year premium is chosen such that the present value of profits (Column 12 total) = 7.0% of the present value of premium (Column 13 total). Subsequently, premiums increase 9% per annum.
Column 3: First year losses average $1,000; loss cost trend is +10% per annum; losses decrease 3% per annum as policy matures.
Columns 4 and 5: Variable expense ratio is 30.2% the first year and 6.2% in subsequent years.
Column 6: First year fixed expenses as $98 per policy, or 17.8% of the average premium for all drivers ($550). Fixed expenses for young male drivers are 20% higher, or $117 per policy.
Column 7: Fixed expenses in the first renewal year are $550x1.20x1.05x3.8%. Subsequently, expenses increase 5% per annum.
Column 8: Assumed persistency rates for young male drivers.
Column 9 = Downward product of Column 8.
Column 10 = {Column 2 − sum (Columns 3 through 7)}xColumn 9.
Column 11: Discount factor reflecting annual 12% cost of capital; e.g., 1.25 = 1.12x1.12.
Column 12 = Column 10 ÷ Column 11.
Column 13 = Column 2xColumn 9 ÷ Column 11.

EXHIBIT 4

NO CARRIERS OFFER DISCOUNTS

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
			Variable Expenses		Fixed Expenses								Loss
Policy Year	Premium	PV of Losses	New	Renewal	New	Renewal	Persis-tency	Cumulative Persistency	Profit	Discount Factor	PV of Profits	PV of Premium	Ratio Relativity
1	600	500	0	37	0	23	100%	100%	40	1.00	40	600	98%
2	654	528	0	41	0	24	96%	96%	59	1.12	53	561	95%
3	713	557	0	44	0	25	96%	92%	80	1.25	64	524	92%
4	777	586	0	48	0	26	95%	88%	102	1.40	72	484	89%
5	847	617	0	53	0	28	95%	83%	124	1.57	79	448	86%
6	923	649	0	57	0	29	95%	79%	149	1.76	84	414	83%
7	1,006	689	0	62	0	31	95%	75%	168	1.97	85	383	81%
8	1,097	732	0	68	0	32	95%	71%	189	2.21	85	354	79%
9	1,196	767	0	74	0	34	95%	68%	217	2.48	88	327	76%
10	1,303	813	0	81	0	35	94%	64%	238	2.77	86	299	74%
11	1,420	862	0	88	0	37	94%	60%	260	3.11	84	274	72%
12	1,548	912	0	96	0	39	93%	56%	279	3.48	80	248	70%
13	1,688	965	0	105	0	41	92%	51%	295	3.90	76	222	68%
14	1,839	1,036	0	114	0	43	91%	47%	301	4.36	69	196	67%
15	2,005	1,111	0	124	0	45	90%	42%	304	4.89	62	172	66%
Total											$1,107	$5,505	

Column 2: First year premium is set at $600; subsequent premiums increase 9% per annum.
Column 3: First year losses average $500; loss cost trend is +10% per annum; losses decrease 1% per annum as policy matures; and losses are adjusted by the change in Column 14 relativities. For instance, $528 = $500x1.1x0.99x0.95 ÷ 0.98.
Column 5: Variable expense ratio is 6.2% in renewal years.
Column 7: Fixed expenses in the first renewal year are $600x3.8% = $23. Subsequently, expenses increase 5% per annum.
Column 8: Assumed persistency rates for older drivers with mature policies.
Column 9 = Downward product of Column 8.
Column 10 = {Column 2 – sum (Columns 3 through 7)} x Column 9.
Column 11: Discount factor reflecting annual 12% cost of capital; e.g., 1.25 = 1.12x1.12.
Column 12 = Column 10 ÷ Column 11.
Column 13 = Column 2 x Column 9 ÷ Column 11.
Column 14: Loss ratio relativities by age of insured: 52 years old in first policy year shown and 66 years old in last policy year shown.

EXHIBIT 5

ONLY COMPETITORS OFFER DISCOUNTS

(1) Policy Year	(2) Premium	(3) PV of Losses	(4) Variable Expenses New	(5) Variable Expenses Renewal	(6) Fixed Expenses New	(7) Fixed Expenses Renewal	(8) Persis-tency	(9) Cumulative Persistency	(10) Profit	(11) Discount Factor	(12) PV of Profits	(13) PV of Premium	(14) Loss Ratio Relativity
1	600	500	0	37	0	23	100%	100%	40	1.00	40	600	98%
2	654	528	0	41	0	24	96%	96%	59	1.12	53	561	95%
3	713	557	0	44	0	25	94%	90%	78	1.25	62	513	92%
4	777	586	0	48	0	26	92%	83%	96	1.40	69	459	89%
5	847	617	0	53	0	28	90%	75%	112	1.57	71	402	86%
6	923	649	0	57	0	29	88%	66%	124	1.76	70	344	83%
7	1,006	689	0	62	0	31	85%	56%	125	1.97	63	285	81%
8	1,097	732	0	68	0	32	82%	46%	121	2.21	55	227	79%
9	1,196	767	0	74	0	34	80%	37%	118	2.48	47	177	76%
10	1,303	813	0	81	0	35	77%	28%	106	2.77	38	133	74%
11	1,420	862	0	88	0	37	75%	21%	92	3.11	30	97	72%
12	1,548	912	0	96	0	39	76%	16%	81	3.48	23	72	70%
13	1,688	965	0	105	0	41	77%	12%	71	3.90	18	54	68%
14	1,839	1,036	0	114	0	43	78%	10%	63	4.36	14	41	67%
15	2,005	1,111	0	124	0	45	80%	8%	56	4.89	11	32	66%
Total											$666	$3,996	

Column 2: First year premium is set at $600; subsequent premiums increase 9% per annum.
Column 3: First year losses average $500; loss cost trend is +10% per annum; losses decrease 1% per annum as policy matures; and losses are adjusted by the change in Column 14 relativities. For instance, $528 = $500x1.1x0.99x0.95 ÷ 0.98.
Column 5: Variable expense ratio is 6.2% in renewal years.
Column 7: Fixed expenses in the first renewal year are $600x3.8% = $23. Subsequently, expenses increase 5% per annum.
Column 8: Assumed persistency rates for older drivers with no premium discount.
Column 9 = Downward product of Column 8.
Column 10 = {Column 2 – sum (Columns 3 through 7)}xColumn 9.
Column 11: Discount factor reflecting annual 12% cost of capital; e.g., 1.25 = 1.12x1.12.
Column 12 = Column 10 ÷ Column 11.
Column 13 = Column 2xColumn 9 ÷ Column 11.
Column 14: Loss ratio relativities by age of insured: 52 years old in first policy year shown and 66 years old in last policy year shown.

EXHIBIT 6

ALL CARRIERS OFFER DISCOUNTS

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
			Variable Expenses		Fixed Expenses								Loss
Policy Year	Premium	PV of Losses	New	Renewal	New	Renewal	Persis-tency	Cumulative Persistency	Profit	Discount Factor	PV of Profits	PV of Premium	Ratio Relativity
1	555	500	0	34	0	23	100%	100%	−2	1.00	−2	555	98%
2	605	528	0	38	0	24	98%	98%	15	1.12	14	529	95%
3	659	557	0	41	0	25	98%	96%	35	1.25	28	505	92%
4	719	586	0	45	0	26	97%	93%	57	1.40	41	477	89%
5	783	617	0	49	0	28	97%	90%	81	1.57	52	450	86%
6	854	649	0	53	0	29	96%	87%	107	1.76	61	420	83%
7	931	689	0	58	0	31	96%	83%	128	1.97	65	393	81%
8	1,015	732	0	63	0	32	95%	79%	148	2.21	67	363	79%
9	1,106	767	0	69	0	34	95%	75%	178	2.48	72	336	76%
10	1,205	813	0	75	0	35	94%	71%	199	2.77	72	307	74%
11	1,314	862	0	81	0	37	94%	66%	222	3.11	71	281	72%
12	1,432	912	0	89	0	39	93%	62%	242	3.48	70	254	70%
13	1,561	965	0	97	0	41	93%	57%	263	3.90	68	230	68%
14	1,702	1,036	0	105	0	43	92%	53%	273	4.36	63	206	67%
15	1,855	1,111	0	115	0	45	92%	49%	284	4.89	58	184	66%
Total											$797	$5,491	

Column 2: First year premium is set at $600; subsequent premiums increase 9% per annum; 7.5% discount applied to all premiums.
Column 3: First year losses average $500; loss cost trend is +10% per annum; losses decrease 1% per annum as policy matures; and losses are adjusted by the change in Column 14 relativities. For instance, $528 = $500x1.1x0.99x0.95 ÷ 0.98.
Column 5: Variable expense ratio is 6.2% in renewal years.
Column 7: Fixed expenses in the first renewal year are $600x3.8% = $23. Subsequently, expenses increase 5% per annum. The 7.5% premium discount does not affect fixed expenses.
Column 8: Assumed persistency rates for older drivers with 7.5% premium discount.
Column 9 = Downward product of Column 8.
Column 10 = {Column 2 − sum (Columns 3 through 7)}xColumn 9.
Column 11: Discount factor reflecting annual 12% cost of capital; e.g., 1.25 = 1.12x1.12.
Column 12 = Column 10 ÷ Column 11.
Column 13 = Column 2xColumn 9 ÷ Column 11.
Column 14: Loss ratio relativities by age of insured: 52 years old in first policy year shown and 66 years old in last policy year shown.

EXHIBIT 7

UNDERWRITING CYCLE UPTURN

(1) Policy Year	(2) Premium	(3) PV of Losses	(4) Variable Expenses New	(5) Variable Expenses Renewal	(6) Fixed Expenses New	(7) Fixed Expenses Renewal	(8) Persis-tency	(9) Cumulative Persistency	(10) Profit	(11) Discount Factor	(12) PV of Profits	(13) PV of Premium	(14) U/W Cycle Factor
1	800	656	242	0	88	0	100%	100%	−186	1.00	−186	800	1.00
2	1,003	701	0	62	0	32	85%	85%	177	1.12	158	761	1.15
3	1,236	748	0	77	0	34	86%	73%	276	1.25	220	720	1.30
4	1,191	799	0	74	0	35	87%	64%	180	1.40	128	539	1.15
5	1,129	853	0	70	0	37	88%	56%	95	1.57	60	402	1.00
6	1,046	911	0	65	0	39	89%	50%	16	1.76	9	296	0.85
7	939	973	0	58	0	41	90%	45%	−60	1.97	−30	213	0.70
8	1,243	1,039	0	77	0	43	90%	40%	34	2.21	15	227	0.85
9	1,594	1,110	0	99	0	45	91%	37%	125	2.48	50	236	1.00
10	1,998	1,186	0	124	0	47	91%	33%	214	2.77	77	241	1.15
11	2,462	1,266	0	153	0	50	92%	31%	305	3.11	98	244	1.30
12	2,374	1,352	0	147	0	52	92%	28%	233	3.48	67	193	1.15
13	2,250	1,444	0	140	0	55	92%	26%	159	3.90	41	150	1.00
14	2,085	1,542	0	129	0	57	92%	24%	85	4.36	20	114	0.85
15	1,871	1,647	0	116	0	60	92%	22%	11	4.89	2	84	0.70
Total											$730	$5,221	

Column 2: First year premium is $800; subsequently, premiums increase 9% per annum. These premiums are then multiplied by the "underwriting cycle factor" in Column 14.
Column 3: First year discounted losses are $656, for an 82% discounted loss ratio; loss cost trend is +10% per annum; losses decrease 3% per annum as the policy matures.
Columns 4 and 5: Variable expense ratio is 30.2% the first year and 6.2% in subsequent years.
Column 6: First year fixed expenses are 17.8% of the premium.
Column 7: Fixed expenses in the first renewal year are 800 x 1.05 x 3.8%. Subsequently, expenses increase 5% per annum.
Column 8: Assumed persistency rates by policy duration.
Column 9 = Downward product of Column 8.
Column 10 = {Column 2 − sum (Columns 3 through 7)} x Column 9.
Column 11: Discount factor reflecting annual 12% cost of capital; e.g., 1.25 = 1.12 x 1.12.
Column 12 = Column 10 ÷ Column 11.
Column 13 = Column 2 x Column 9 ÷ Column 11.
Column 14: "Underwriting cycle factors," reflecting upward movement of an eight year cycle.

EXHIBIT 8

UNDERWRITING CYCLE DOWNTURN

(1) Policy Year	(2) Premium	(3) PV of Losses	(4) Variable Expenses New	(5) Variable Expenses Renewal	(6) Fixed Expenses New	(7) Fixed Expenses Renewal	(8) Persis-tency	(9) Cumulative Persistency	(10) Profit	(11) Discount Factor	(12) PV of Profits	(13) PV of Premium	(14) U/W Cycle Factor
1	800	656	242	0	88	0	100%	100%	−186	1.00	−186	800	1.00
2	741	701	0	46	0	32	85%	85%	−32	1.12	−28	563	0.85
3	665	748	0	41	0	34	86%	73%	−115	1.25	−92	388	0.70
4	881	799	0	55	0	35	87%	64%	−5	1.40	−4	399	0.85
5	1,129	853	0	70	0	37	88%	56%	95	1.57	60	402	1.00
6	1,416	911	0	88	0	39	89%	50%	188	1.76	107	400	1.15
7	1,744	973	0	108	0	41	90%	45%	279	1.97	141	396	1.30
8	1,682	1,039	0	104	0	43	90%	40%	200	2.21	90	307	1.15
9	1,594	1,110	0	99	0	45	91%	37%	125	2.48	50	236	1.00
10	1,477	1,186	0	92	0	47	91%	33%	51	2.77	18	178	0.85
11	1,326	1,266	0	82	0	50	92%	31%	−22	3.11	−7	131	0.70
12	1,755	1,352	0	109	0	52	92%	28%	68	3.48	20	143	0.85
13	2,250	1,444	0	140	0	55	92%	26%	159	3.90	41	150	1.00
14	2,821	1,542	0	175	0	57	92%	24%	250	4.36	57	155	1.15
15	3,475	1,647	0	215	0	60	92%	22%	342	4.89	70	156	1.30
Total											$339	$4,803	

Column 2: First year premium is $800; subsequently, premiums increase 9% per annum. These premiums are then multiplied by the "underwriting cycle factor" in Column 14.
Column 3: First year discounted losses are $656, for an 82% discounted loss ratio; loss cost trend is +10% per annum; losses decrease 3% per annum as the policy matures.
Columns 4 and 5: Variable expense ratio is 30.2% the first year and 6.2% in subsequent years.
Column 6: First year fixed expenses are 17.8% of the premium.
Column 7: Fixed expenses in the first renewal year are 800x1.05x3.8%. Subsequently, expenses increase 5% per annum.
Column 8: Assumed persistency rates by policy duration.
Column 9 = Downward product of Column 8.
Column 10 = {Column 2 − sum (Columns 3 through 7)}xColumn 9.
Column 11: Discount factor reflecting annual 12% cost of capital; e.g., 1.25 = 1.12x1.12.
Column 12 = Column 10 ÷ Column 11.
Column 13 = Column 2xColumn 9 ÷ Column 11.
Column 14: "Underwriting cycle factors," reflecting downward movement of an eight year cycle.