

## ALLOCATED LOSS ADJUSTMENT EXPENSE LIABILITIES

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### *Abstract*

*This paper sets forth a simple, practical, and straightforward method of establishing liabilities for allocated loss adjustment expenses (ALAE). With a minimum of judgment, the process flows smoothly from main frame computer input data, to the actuary's spreadsheet, to the answer. For this reason, a monthly update is easy to produce, which makes it possible to reflect changes in level earlier and less abruptly than with less frequent reviews. This fluid process produces total ALAE liabilities by coverage that recognize the monthly aging progression of the component liabilities by accident year (including the stub periods for the latest accident year).*

Most methodologies for quantifying ALAE liabilities are based upon measurable relationships between loss and ALAE; they are multiplicative processes.<sup>1</sup> These relationships are expressed as ratios of ALAE to losses by coverage by accident year, on either an incurred/incurred basis or an unpaid/unpaid basis. When incurred/incurred ratios are used, the ratios produce estimated ALAE incurred dollars, and the ALAE liabilities are derived by subtraction. When unpaid/unpaid ratios are used, the ratios produce the ALAE liabilities directly.

The underlying principle in these multiplicative processes is the following: "Because the smaller and easier claims (which are

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<sup>1</sup>Less common methods are these: (1) when the loss and ALAE liabilities are estimated on a combined basis, the combined liability is allocated between the two on a basis that is consistent with historical relationships, and (2) when individual ALAE claim-file estimates are available, the ALAE liabilities may be established independently, using reserving methodologies to derive the bulk estimates needed for unreported ALAE.

settled faster) require proportionately less ALAE, the ratio of paid ALAE to paid losses generally increases with age of development” [1, p.6]. For this reason, the ratios are applied by accident year. When incurred/incurred ratios are used, this principle is not evident in the ratios, but is apparent in the resulting liability comparisons. When unpaid/unpaid ratios are used, this principle governs the estimating process.

This paper is not a critique of methodologies for estimating ALAE liabilities. Its purpose is to introduce a simplified application of sound methodology. Simplified procedures generally enjoy the advantages of faster compilations or unsophisticated computer adaptations, or both, which make it easier to frequently update the estimates. This application has these advantages.

When unpaid/unpaid ratios are used to estimate the ALAE liabilities, the estimated ratios are generally derived in one of two ways: (1) using restated unpaid/unpaid ratios from prior accident years at the same age of development, or (2) using age-adjusted calendar year paid/paid ratios [1, pp. 98–111]. The latter basis is used in this simplified procedure. It is particularly appropriate for a simple procedure because there are no estimates in paid/paid ratios.

Age-adjusted calendar year paid/paid ratios are derived by adjusting calendar year paid data to reflect only payments *subsequent to* specified accident year ages. (In relatively mature operations, the mix by age in the age-adjusted calendar year data should approximate the expected mix by age in the liabilities.) The procedure in this paper derives the age-adjusted paid data through successive subtractions of data younger than the specified accident year ages. Remainders are produced after each accident year subtraction, starting with the latest (least mature) accident year and ending with the eleventh latest accident year. These eleven sets of “subsequent-to” remainders for loss and ALAE produce the age-adjusted paid/paid ratios that correspond to the expected mix by age in the respective liabilities.

Because data for the latest calendar period are used, these ratios reflect current ALAE/loss payout relationships. Barring unusual circumstances, the estimated unpaid/unpaid ratios should at least equal these levels. Otherwise, the ALAE/loss relationship in the liabilities would be less than current payment ratios. The use of lower ratios would be justified only when singular settlements distort the data. In this event, a better choice would be to adjust the paid data. The use of higher ratios may be justified under special situations as well. For ongoing situations, however, it is reasonable to assume a continuation of the current paid/paid relationships. If so, the unpaid/unpaid ratios will equal the age-adjusted paid/paid ratios, and the resulting ALAE liabilities will approximate the same level of adequacy that exists in the loss liabilities.

The use of age-adjusted paid/paid ratios is not a common methodology, probably due to the fact that the published material on their derivation is rather complicated [1, pp. 197–199]. This paper intends to change that. Exhibits 1 through 4 illustrate the calculation of age-adjusted paid/paid ratios and their use in estimating ALAE liabilities at both a year-end and interim evaluation date. A brief explanation of these exhibits follows:

1. Exhibit 1 shows the historical calendar year paid data in the accident year detail necessary to calculate age-adjusted paid/paid ratios as of July 31, 1994. (Because this exhibit includes the data needed as of December 31, 1993, a separate December 31, 1993 exhibit is unnecessary.) The exhibit includes data for the latest 36 months. Shorter calendar periods can be used if the data are sufficiently credible to do so. In the completion of each new exhibit, only the data for the latest calendar year are added; prior data are posted from the prior exhibits.
2. Exhibits 2 and 3 illustrate the calculation of the age-adjusted paid/paid ratios. Exhibit 2 shows the format used as of any year end. (December 31, 1993 is illus-

trated.) Exhibit 3 shows the format used as of any stub period. (July 31, 1994 is illustrated.) Line 1 includes the calendar year paid data for all accident years, producing the unadjusted paid/paid ratio for the latest 36 months. This ratio is informational, but it is interesting to compare this ratio with those that are age-adjusted. The subsequent lines illustrate the successive subtractions necessary to produce the age-adjusted paid/paid ratios. These ratios reflect the payment activity subsequent to the ages of the individual accident year components.

3. Exhibit 4 illustrates the calculation of ALAE liabilities as of December 31, 1993 and July 31, 1994. There is nothing new in this format. The ALAE liabilities are derived by multiplying the loss liability for each accident year by the appropriate unpaid/unpaid ratio. As discussed earlier, the assumption in this calculation is that current age-adjusted paid/paid relationships will continue. Thus the unpaid/unpaid ratios will be those produced in Exhibits 2 and 3. These ratios can be transferred to Exhibit 4 generally without adjustment. Adjustments are necessary only when the ratios are believed to be inconsistent with the underlying principle that paid/paid ratios should not decrease as the age of development increases. Strictly interpreted, the principle applies to paid accumulations on closed claims only. When paid accumulations on both open and closed claims are used, explainable decreases can result. Most decreases, however, are likely to be the random behavior of data that are not fully credible. Thus, unless there is a continuing pattern of decreasing ratios, it is prudent to apply the principle and override any decreases that occur.<sup>2</sup> Two such overrides were made in

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<sup>2</sup>For the purist who has data that include inventories of partial payments (ALAE and loss) on open claims, adjustments can be made to the paid data to produce aged paid-to-paid ratios on closed claims. These ratios are applied to gross loss reserves (which include partial payments), producing gross ALAE reserves. Net ALAE reserves are derived by subtracting partial ALAE payments on open claims.

Exhibit 4 and have been noted with an asterisk. After Exhibit 4 is completed, it is interesting to compare the liability/liability ratio for all accident years on the "Total" line with the unadjusted paid/paid ratio for all accident years on Line 1 in either Exhibit 2 or Exhibit 3. The difference, which is caused by the different mix by age in the two sets of data, emphasizes the importance of reflecting such differences when establishing the ALAE liabilities.

In conclusion, this paper provides a simple application of a sophisticated methodology for estimating ALAE liabilities. Because of its simplicity, the calculation can be made more frequently. The increased frequency creates a smooth change from evaluation date to evaluation date. By using updated data as frequently as monthly, one can see how easily this application could solve the problems of estimating the ALAE liabilities for the latest accident year as it progresses from January to December. Because of its simplicity, this application can also serve as a means of testing the sufficiency of ALAE liabilities produced from other methodologies.

## REFERENCE

- [1] Salzmann, Ruth E., *Estimating Liabilities for Loss and Loss Adjustment Expenses*, Prentice Hall, Englewood Cliffs, New Jersey, 1984.

**EXHIBIT 1**  
**PART 1**  
**PAID LOSS HISTORY**  
**GENERAL LIABILITY**

Accident Year	1991 Calendar Period		1992 Calendar Period		1993 Calendar Period		1994 Calendar Period
	7 Months	12 Months	7 Months	12 Months	7 Months	12 Months	
A/O	\$ 947,102	\$ 2,704,048	\$ (163,886)	\$ 823,924	\$ 984,700	\$ 2,906,899	\$456,752
1981	(41,296)	(37,594)					
1982	1,016,202	1,097,762	(38,070)	2,091	163,685	177,941	145,628
1983	968,230	1,099,984	(42,434)	(37,434)	178,477	1,047,694	52,548
1984	954,099	1,096,096	569,818	640,983	456,573	1,627,020	77,379
1985	839,299	1,471,917	401,645	1,371,431	380,483	408,226	844,479
1986	2,668,201	3,549,585	1,304,505	1,569,400	1,197,602	1,959,050	660,135
1987	2,315,101	5,132,868	2,719,031	4,317,898	2,510,750	3,158,500	907,105
1988	2,000,265	3,466,625	1,185,204	2,074,387	2,292,728	2,723,447	4,198,602
1989	(30,294)	5,355,822	1,791,761	7,875,311	2,136,841	3,181,337	4,632,950
1990	1,119,225	1,907,138	1,152,013	2,503,478	1,331,312	4,759,912	1,522,449
1991	270,784	758,151	718,508	1,745,973	556,904	1,162,908	1,049,460
1992			366,651	882,847	298,828	769,863	271,518
1993							
1994							
<b>TOTAL</b>	<b>\$13,026,918</b>	<b>\$27,602,402</b>	<b>\$9,964,746</b>	<b>\$23,770,289</b>	<b>\$12,488,883</b>	<b>\$23,882,797</b>	<b>\$14,819,005</b>

A/O = All Other Accident Years

**EXHIBIT 1**  
**PART 2**  
**PAID ALAE HISTORY**  
**GENERAL LIABILITY**

Accident Year	1991 Calendar Period		1992 Calendar Period		1993 Calendar Period		1994 Calendar Period
	7 Months	12 Months	7 Months	12 Months	7 Months	12 Months	
A/O	\$ 851,690	\$1,756,016	\$ 521,645	\$1,576,893	\$1,125,830	\$ 2,346,194	\$1,101,555
1981	79,814	184,644					
1982	122,812	211,142	(39,777)	(35,926)			
1983	463,919	571,911	(36,363)	18,848	91,738	173,262	155,395
1984	383,508	473,446	201,894	299,629	98,855	558,527	5,233
1985	487,820	837,936	337,515	665,911	404,721	744,480	41,968
1986	392,329	711,551	151,463	328,793	164,931	452,717	170,804
1987	672,324	1,324,508	529,592	917,211	408,359	696,751	387,995
1988	610,831	1,246,852	551,261	1,022,315	463,708	894,786	453,216
1989	764,654	1,458,817	1,033,379	1,761,475	742,136	1,221,764	544,522
1990	131,875	430,200	533,914	1,084,508	795,941	1,468,178	1,592,673
1991	19,256	71,604	150,818	597,953	1,154,530	2,021,536	437,834
1992			37,602	107,404	170,310	518,405	113,119
1993					32,034	72,476	17,170
1994							
<b>TOTAL</b>	<b>\$4,980,832</b>	<b>\$9,278,627</b>	<b>\$3,972,943</b>	<b>\$8,345,014</b>	<b>\$5,653,093</b>	<b>\$11,169,076</b>	<b>\$5,021,484</b>

A/O = All Other Accident Years



**EXHIBIT 2**  
**PART 1**  
**AGED CALENDAR YEAR PAID RATIOS**  
**ALAE DIVIDED BY LOSS**  
**GENERAL LIABILITY**  
**AS OF 12/31/93**

Accident Year	(a) 1991		(b) 1992		(c) 1993		(d) (a+b+c)		(e) 1991		(f) 1992		(g) 1993		(h) (e+f+g)	(i) Ratio (d)/(h)
	Calendar Year Paid ALAE (\$000)		Calendar Year Paid ALAE (\$000)		Calendar Year Paid ALAE (\$000)		Calendar Year Paid Loss (\$000)		Calendar Year Paid Loss (\$000)		Calendar Year Paid Loss (\$000)					
1 Total	9,279	8,345	11,169	28,793	27,602	23,770	23,883	75,255	0.383	detailed by accident year*						
2 n	72	107	72	251	758	883	770	2,411	0.392	detailed by accident year*						
3 Aged 1 Yr (1-2)	9,207	8,238	11,097	28,542	26,844	22,887	23,113	72,844		detailed by accident year*						
4 n-1	430	598	518	1,546	1,907	1,746	1,163	4,816		detailed by accident year*						
5 Aged 2 Yrs (3-4)	8,777	7,640	10,579	26,996	24,937	21,141	21,950	68,028	0.397	detailed by accident year*						
6 n-2	1,459	1,085	2,022	4,566	5,356	2,504	4,760	12,620	0.405	detailed by accident year*						
7 Aged 3 Yrs (5-6)	7,318	6,555	8,557	22,430	19,581	18,637	17,190	55,408		detailed by accident year*						
8 n-3	1,247	1,761	1,468	4,476	3,466	7,875	3,181	14,522	0.439	detailed by accident year*						
9 Aged 4 Yrs (7-8)	6,071	4,794	7,089	17,954	16,115	10,762	14,009	40,886		detailed by accident year*						
10 n-4	1,324	1,022	1,222	3,568	5,133	2,074	2,724	9,931	0.465	detailed by accident year*						
11 Aged 5 Yrs (9-10)	4,747	3,772	5,867	14,386	10,982	8,688	11,285	30,955		detailed by accident year*						

\* where n = latest accident year; n-1 = second latest accident year; etc.

**EXHIBIT 2**  
**PART 2**  
**AGED CALENDAR YEAR PAID RATIOS**  
**ALAE DIVIDED BY LOSS**  
**GENERAL LIABILITY**  
**AS OF 12/31/93**

	Accident Year	(a)		(b)		(c)		(d)		(e)		(f)		(g)		(h)	(i) Ratio (d)/(h)
		1991	1992	1992	1993	1993	(a+b+c)	1991	1992	1993	1992	1993	(e+f+g)				
12	n-5	712	917	895	2,524	3,550	4,318	3,159	11,027	0.595							
13	Aged 6 Yrs (11-12)	4,035	2,855	4,972	11,862	7,432	4,370	8,126	19,928	0.670							
14	n-6	838	329	697	1,864	1,472	1,569	1,959	5,000	0.697							
15	Aged 7 Yrs (13-14)	3,197	2,526	4,275	9,998	5,960	2,801	6,167	14,928	0.697							
16	n-7	473	666	453	1,592	1,096	1,371	408	2,875	0.697							
17	Aged 8 Yrs (15-16)	2,724	1,860	3,822	8,406	4,864	1,430	5,759	12,053	0.697							
18	n-8	572	300	745	1,617	1,100	641	1,627	3,368	0.782							
19	Aged 9 Yrs (17-18)	2,152	1,560	3,077	6,789	3,764	789	4,132	8,685	0.782							
20	n-9	211	19	558	788	1,098	(37)	1,047	2,108	0.912							
21	Aged 10 Yrs (19-20)	1,941	1,541	2,519	6,001	2,666	826	3,085	6,577	0.912							
22	n-10	185	(36)	173	322	(38)	2	178	142	0.883							
23	Aged 11 Yrs (21-22)	1,756	1,577	2,346	5,679	2,704	824	2,907	6,435	0.883							

\* where n = latest accident year; n-1 = second latest accident year; etc.

**EXHIBIT 3**  
**PART 1**  
**AGED CALENDAR YEAR PAID RATIOS**  
**ALAE DIVIDED BY LOSS**  
**GENERAL LIABILITY**  
**AS OF 7/31/94**

	Accident Year	(a)		(b)		(c)		(d)		(e)		(f)		(g)		(h)		(i)		(j)		(k)		(l)		(m)	(n)	(o)				
		1991		1992		1993		1994		1991		1992		1993		1994		1991		1992		1993		1994								
		last 5	first 7	last 5	first 7	last 5	first 7	last 5	first 7	last 5	first 7	last 5	first 7	last 5	first 7	last 5	first 7	last 5	first 7	last 5	first 7	last 5	first 7	last 5	first 7				last 5	first 7	last 5	Ratio (g)/(n)
1	Total	4,298	3,973	4,372	5,653	5,516	5,021	28,833																					0.374			
2	n																															
3	Aged 7 Mos (1-2)	4,298	3,936	4,372	5,621	5,516	5,004	28,747																								
4	n	53	151	70	170	40	113	163																								
5	n-1																															
6	Aged 19 Mos (3-4-5)	4,245	3,785	4,302	5,451	5,476	4,891	28,150																								
7	n-1	298	534	447	1,154	348	438	2,126																								
8	n-2																															
9	Aged 31 Mos (6-7-8)	3,947	3,251	3,855	4,297	5,128	4,453	24,931																								
10	n-2	694	1,033	551	796	867	1,593	3,422																								
11	n-3																															
12	Aged 43 Mos (9-10-11)	3,253	2,218	3,304	3,501	4,261	2,860	19,397																								
13	n-3	636	551	728	742	672	544	2,036																								
14	n-4																															
15	Aged 55 Mos (12-13-14)	2,617	1,667	2,576	2,759	3,589	2,316	15,524																								

\* where n = latest accident year; n-1 = second latest accident year; etc.

**EXHIBIT 3**  
**PART 2**  
**AGED CALENDAR YEAR PAID RATIOS**  
**ALAE DIVIDED BY LOSS**  
**GENERAL LIABILITY**  
**AS OF 7/31/94**

	Accident Year	(a)		(b)		(c)		(d)		(e)		(f)		(g)		(h)		(i)		(j)		(k)		(l)		(m)	(n)	(o)		
		1991		1992		1993		1994		1993		1994		1993		1994		1992		1993		1994		1993					1994	
		last 5	first 7	last 5	first 7	last 5	first 7	last 5	first 7	last 5	first 7	last 5	first 7	last 5	first 7	last 5	first 7	last 5	first 7	last 5	first 7	last 5	first 7	last 5	first 7				last 5	first 7
16	n-4	652	471	480	1,603	480	453	1,446	464	2,295	464	1,446	453	1,446	480	453	1,446	2,818	2,719	889	2,511	431	431	431	431	907	4,138			
17	n-5	1,965	1,138	2,105	2,295	3,109	1,863	12,475	2,295	3,109	1,863	12,475	2,295	3,109	3,630	2,031	3,938	3,630	2,031	3,938	3,361	5,413	2,237	2,237	2,237	2,237	20,610	0.605		
19	n-5	319	388	431	1,138	431	1,138	1,138	388	408	388	948	388	948	881	1,304	1,599	881	1,304	1,599	1,198	648	660	660	660	3,162	3,128			
20	n-6	1,646	986	1,717	1,887	2,678	1,475	10,389	1,887	2,678	1,475	10,389	1,887	2,678	2,749	727	2,339	2,749	727	2,339	2,163	4,765	1,577	1,577	1,577	14,320	0.725			
21	Aged 79 Mos (18-19-20)	350	177	289	816	289	171	674	177	289	171	674	177	289	633	401	265	633	401	265	380	761	761	761	761	1,659	1,659			
22	n-7	1,296	648	1,540	1,722	2,389	1,304	8,899	1,722	2,389	1,304	8,899	1,722	2,389	2,116	326	2,074	2,116	326	2,074	1,783	4,004	732	732	732	11,035	0.806			
24	Aged 91 Mos (21-22-23)	90	328	288	706	288	706	706	328	405	42	649	405	649	142	570	970	142	570	970	457	28	28	28	28	1,140	1,140			
25	n-7	1,206	446	1,212	1,317	2,101	1,262	7,544	1,317	2,101	1,262	7,544	1,317	2,101	1,974	(244)	1,104	1,974	(244)	1,104	1,326	3,976	655	655	655	8,791	0.858			
26	n-8	108	98	340	546	340	546	546	98	99	5	68	99	68	132	(42)	71	132	(42)	71	178	1,170	1,170	1,170	1,170	1,373	1,373			
27	Aged 103 Mos (24-25-26)	1,098	482	1,114	1,218	1,761	1,257	6,930	1,218	1,761	1,257	6,930	1,218	1,761	1,842	(202)	1,033	1,842	(202)	1,033	1,148	2,806	602	602	602	7,229	0.959			
28	n-8	89	55	459	603	459	603	603	55	92	155	207	92	207	81	(38)	5	81	(38)	5	163	869	869	869	869	955	955			
29	n-9	1,009	522	1,059	1,126	1,302	1,102	6,120	1,059	1,126	1,302	1,102	6,120	1,761	(164)	1,028	985	1,761	(164)	1,028	985	1,937	457	457	457	6,004	1.019			
30	Aged 115 Mos (27-28-29)																													
31	n-9																													
32	n-10																													
33	Aged 127 Mos (30-31-32)																													

\* where n = latest accident year; n-1 = second latest accident year; etc.

EXHIBIT 4  
ILLUSTRATIONS OF THE CALCULATION  
OF ALAE LIABILITIES  
GENERAL LIABILITY  
USING THE AGED PAID-TO-PAID RATIOS IN EXHIBITS 2 AND 3  
(\$000)

As of December 31, 1993				As of July 31, 1994			
Acc. Year	(1) Loss Liability	(2) Aged Ratio (Exh. 2)	(3) ALAE Liability (1)×(2)	Acc. Year	(4) Loss Liability	(5) Aged Ratio (Exh. 3)	(6) ALAE Liability (4)×(5)
≤1983	21,359	.912 *	19,479	≤1984	25,916	1.019	26,408
1984	4,446	.912	4,055	1985	5,585	.959	5,356
1985	5,490	.782	4,293	1986	6,581	.858	5,646
1986	6,099	.697	4,251	1987	7,446	.806	6,001
1987	8,068	.670	5,406	1988	10,095	.725	7,319
1988	9,302	.595	5,535	1989	14,348	.605	8,681
1989	15,308	.465	7,118	1990	15,511	.503	7,802
1990	19,656	.439	8,629	1991	18,186	.411	7,474
1991	21,730	.405	8,801	1992	18,666	.389 *	7,261
1992	22,337	.397	8,868	1993	19,657	.389	7,647
1993	<u>20,384</u>	.392	<u>7,991</u>	1994	<u>12,363</u>	.378	<u>4,673</u>
Total	154,179	.548 **	84,426	Total	154,354	.611 **	94,268

\* Manually adjusted so as not to be less than the next subsequent aged ratio.

\*\* Calculated after Totals are established.