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EXTENDED SERVICE CONTRACTS

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Abstract

Automobile extended service contracts (ESCs) have been in existence for many years. Due to the nature of the coverage, an insurer may not know the actual results for a particular book for some time after the book has been in place. This paper discusses this coverage and unique characteristics of ESCs that should be recognized when analyzing experience for an ESC program. The paper also discusses some approaches that have been used to address these problems and to derive reserve and rate estimates for ESC programs.

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1. INTRODUCTION

Automobile extended service contracts (ESCs) have been responsible for financial losses to more than one insurer. Although the impact of ESC programs on insurance industry profitability cannot be determined, some multiple line insurers have left the market after substantial ESC losses, and some specialty carriers have become insolvent. Witness, for example, the demise of Consumers Indemnity, a Washington based ESC writer, and the recent fortunes of both American Warranty and General Warranty, both very large ESC administrators. Because of the way ESC business is treated, data from published financial statements usually are not useful in identifying the effect of ESC business on particular insurers. However, the author is aware of several insurers that are no longer in this market, due largely to poor loss experience.

The cause of difficulties can usually be traced to inadequate pricing. Often though, misunderstanding key aspects of these contracts can be a major factor. In this paper, we discuss ESCs, identify areas that can lead to future financial problems, and describe some approaches for analyzing ESC experience.

2. BACKGROUND

Most people buying a new or used car from a dealer are presented the opportunity to purchase additional protection against mechanical breakdown of that car. This protection can be in the form of a policy purchased directly from an insurer with the dealer acting as an agent, or as a response to a direct mail appeal from an insurer. In a limited number of states, direct insurance is the only type of transaction allowed for such coverage. In this case of direct insurance, state insurance regulation including rate regulation, anti-rebate statutes, and agency licensing requirements usually applies.

A more common arrangement is a contract between the buyer and the dealer, often with an administrator or managing general agent providing administration of the program. The dealer then obtains insurance to cover the liability assumed under the contract or self-insures the risk. In this case, since state insurance laws often exclude service contracts and warranties, regulation usually applies to the transaction between the dealer and the insurer but not necessarily to the transaction between the car buyer and the dealer. In the latter transaction, the dealer knows the wholesale price for coverage received, and is free to set the retail price charged to the car buyer. The buyer can negotiate the price with the dealer, if the buyer is aware of the nature of this arrangement.

In either the direct insurance or service contract arrangement, the basic idea of protection is the same: In exchange for a sum of money, a promise is given to repair or replace covered parts that fail for specified causes during the term of coverage. This term is usually expressed in both time and mileage elapsed, although there are some contracts without any mileage limitation.

3. RESERVE CONSIDERATIONS FOR ESCs

Some believe, with justification, that this type of coverage is for physical damage to a vehicle, and, as such, loss reserves are not a significant item. This is often the case. Claims are usually reported quickly after they occur, repairs made soon after authorization is given, and payments made promptly. Thus the reserves for claims incurred, whether or not they have been reported, are often relatively small. Exceptions can occur, however, in cases where, because of processing features and possible batching of claims in a particular ESC program, there is a longer time lag between loss occurrence and final payment.

In this line of business, many program managers tend to rely on calendar year loss ratios calculated as losses incurred in the year divided by premiums earned in the year. The rationale is that, since loss reserves play a relatively minor role, calendar year experience is not materially affected by reserve movement. This is correct as far as it goes. Without consideration of the flow of premium, however, this reasoning can lead to disastrous results.

It is critically important in evaluating loss ratio results for an ESC program to recognize that losses generally cannot be expected to emerge uniformly during the life of a contract which can be in effect for several years. For this reason, the rate at which premiums are earned can have a significant impact on the loss ratios. Thus, when reviewing loss ratios for an ESC program it is important to know what "earning curve" is used to bring premiums into income. Until it is verified that the earning pattern accurately tracks loss emergence, loss ratios using premiums earned with that pattern should be suspect.

An approach that does not depend directly on the formula used to earn premiums would be to estimate ultimate losses for fixed groups of policies. If this approach is taken, the separation of the resulting unpaid amounts among the various reserve categories must still be considered for accounting purposes. For example, the ultimate losses for policy year 1990 as of December 31, 1992, will be composed of:

1. losses paid through December 31, 1992, on 1990 policies,
2. case reserves for open known claims as of December 31, 1992, if such reserves are set,
3. additional reserves for development of reserves on known claims including possible re-openings (development reserves),
4. reserves for claims that occurred before December 31, 1992, but are not yet reported (true IBNR reserves), and
5. amounts estimated to be paid for claims expected to arise after December 31, 1992 during the unexpired terms of current contracts.

Loss reserves would provide for items 2, 3 and 4. As noted above, claims usually are closed rather quickly (reducing case reserves), are usually reported quickly (reducing true IBNR), and are usually easy to evaluate (reducing development reserves), so true loss reserves are usually rather small.

The last item, the amount expected to be paid on unexpired portions of current contracts, is generally provided for in the unearned premium reserves. Unless the formula used to earn premiums matches the expected loss emergence, a mismatch between the unearned premium reserves and the amounts included in item 5 above can occur, even if the premiums charged are correct.

If premiums are earned more rapidly than losses are expected to emerge, and if incurred losses are compared to earned premiums, the resulting loss ratios will be understated at early ages. If, in addition, rates are inadequate and the program is growing, the "profitable" new business will offset the losses on the "unprofitable" old business, masking difficulties even further.

3. EXAMPLE OF TIMING MISMATCH

The following example, though hypothetical, does parallel the experience of more than one insurer with this type of business. For this example, assume that:

1. the insurer earns premium on a pro-rata basis over time,
2. all ESCs are on new cars for five years or 50,000 miles, whichever comes first, and
3. losses emerge during the life of a contract in the following pattern:

EXAMPLE LOSS EMERGENCE FOR ONE POLICY

Year	Percentage of Losses Incurred
1	5%
2	15
3	25
4	30
5	25

We used these assumptions, along with the simplifying one of uniform issuing of contracts through the year, with an assumed loss ratio of 150% to derive Exhibit 1. As can be seen from the resulting loss ratios, the mismatch between the emergence of losses and the premium earning is definitely misleading. The program starts out with a 38% loss ratio, and loss ratios do not exceed 100% until the end of the fourth year on a calendar year basis and not until the end of the fifth year on an inception-to-date basis. By this time the insurer is already committed to several years of very unprofitable business.

Though this example may seem somewhat extreme, the potential for mismatch exists in almost any earning formula used for this type of business. The actuary must be cautious in relying on loss ratios calculated as the ratio of incurred losses to earned premiums in ESC business, even if the incurred losses include proper provision for all claims that have already occurred.

If loss ratios based on earned premiums are to be used to make financial decisions regarding an ESC book, the match between loss emergence and earning should be checked. A pattern of increasing loss ratios over time, as shown on Part 3 of Exhibit 1, should give some warning that a mismatch may be occurring. However, the presence of newer policy years, contributing more to the earned premiums than to losses incurred, could mask that pattern, especially if there is growth in the business.

Rather, it would be better to look at a fixed group of policies and see how the earning formula has tracked with the historical emergence of losses. The following table shows the progression of the cumulative indicated loss ratios for policy year 1 from Exhibit 1:

LOSS RATIO EMERGENCE—POLICY YEAR 1

Calendar Year	Indicated Loss Ratio
1	38%
2	63
3	98
4	129
5	146
6	150

The increase shows up more quickly than in the inception-to-date or even calendar year loss ratios for the entire book. Any consistent pattern in the loss ratios for a fixed group of policies over time, either up or down, provides a warning that there may be a mismatch in timing between premium earning and loss emergence.

Several aspects of ESCs can influence the emergence of losses during the life of a contract. The following section deals with these characteristics and their potential effect on loss emergence.

4. CHARACTERISTICS OF ESCs

Several characteristics of ESCs make the analysis of ESC experience significantly different from that for many other types of insurance. First, the contracts themselves differ from many other insurance coverages. As noted above, the contract is often between an automobile dealer and an automobile purchaser, with insurance covering the dealer's liability assumed under the contract.

ESCs often run for many years and contract holders have limited rights to cancel coverage. Most ESCs come with mileage limitations, although unlimited mileage contracts have been issued. There will thus be contracts expiring before their time limit, as a result of exceeding the mileage limitation.

ESC coverages normally begin where manufacturer warranties end. They usually exclude anything covered under manufacturer warranties, and they sometimes provide coverage for items such as towing, car rental, and travel interruption expenses not covered under the original warranty. Generally very little loss is expected to be incurred by the ESC policies during the original manufacturer warranties. These warranties are usually at least one year or 12,000 miles and can be three years or 36,000 miles or even longer on many early 1990 models. Thus we would expect much less than one-fifth of all ESC losses to arise during the first year of a five-year policy.

Finally, most ESCs have a provision for the transfer of the contract in the event of the transfer of a covered car, after payment of a specified fee and after application to the insurer or dealer. Otherwise, coverage does not continue to the new owner. In most cases, contracts cannot be transferred if the car is sold to a dealer. It is therefore not unusual for cars to be sold without the contract being transferred to the new owner.

A second area of difference is in the nature of the hazard insured; that is, the cost of repair of certain covered parts that fail during the contract term and are not otherwise covered by manufacturer warranty. Thus, different manufacturer warranties will cause ESC losses on different vehicles to emerge differently. This will also cause proportionately less of the covered losses to emerge in the early stages of the contract, while the car is new and the manufacturer warranty is in effect, than in later stages as parts wear out and costs increase. Finally, we may even expect different makes, models, or even model years to experience different cost emergence patterns than others.

A third area of difference lies in the nature of the contract purchasers. The purchasers often have a choice of contract length and mileage limitations. Thus it is possible that selection will affect the characteristics of the contract holders of different contract terms and the rate at which mileage restrictions form the real limit on coverage. There may be other situations where the contract holders forget the coverage or sell the covered vehicle without transferring coverage. In addition, most ESCs require that the vehicle owner comply with certain service requirements. Different contract holders may have different attitudes toward such requirements.

These characteristics could lead one to conclude that, on the average, there is less exposure to loss at the end of the contract term measured by time than at the start. This is often the case for contracts sold on used cars; however, it is definitely not the case in most new car coverages. This line of reasoning ignores the fact that the more expensive claims tend to occur near the end of the policy term. In addition, the presence of manufacturer warranties tends to reduce costs in the early stages of new car contracts. The inescapable conclusion is that, in either new or used car coverages, losses cannot be expected to arise uniformly during the term of the contract.

5. AN AGGREGATE APPROACH TO LOSS ESTIMATION

Instead of concentrating on loss and unearned premium reserves separately, the actuary could take a unified approach in monitoring the profitability of an ESC program. Such an approach would focus on the ultimate forecast position of the program, rather than using earned premiums. Ultimate losses would be forecasted and compared with premiums to assess program profitability. In this way, the earning curve arises implicitly from the loss data and does not need to be specified beforehand. Separate analyses could then be performed to estimate the portion of the resulting estimated total unpaid amount attributable to claims that have already occurred. The remainder would provide an estimate of the amount necessary to fund for losses that have not yet occurred.

Usual actuarial projection methods making use of data triangles can also be used for forecasts in ESC programs. If losses are grouped by accident period (month, quarter, year), where an accident is defined to be the occurrence of a covered repair, the resulting projections will provide estimates for accidents that have occurred. These estimates can then be used to estimate the amount of loss and expense reserves necessary for claims that have occurred (items 2, 3 and 4 above). As mentioned above, we generally find the tail to be fairly short in these cases, often with 90% or more of losses paid within 12 months of the repair.

Many writers of ESC coverages have different coverage terms available with multiple choices of both length of time and mileage limitations. The lag from repair to payment, however, should not depend materially on those options but rather should relate to the operation of the individual ESC program. For this reason an actuary may be able to gain stability in projections by accident period by combining the data for several policy terms.

As with other lines of insurance, many factors can influence and change the lag from repair to final payment. One obvious factor is the structure of the particular program. Some programs require pre-authorization of repairs that exceed a certain amount, while others may require pre-approval on all repairs. Some programs may require frequent submission of claims from the dealer to the administrator, whereas the frequency may be much less in other programs. The administrator may also batch reportings to the insurer. Such batching affects the timing of information flow. Changes in these or other procedures can affect projections of ultimate losses on an accident period basis.

Another factor to be aware of is the presence of case reserves. In some cases the pre-authorized repair amounts are entered as incurred losses. As with other coverages, if such data are available, both incurred and paid loss forecasts are possible.

Projections using triangles organized by policy period will provide estimates of ultimate losses for all policies issued during a particular period. As noted above, this approach has the benefit of not relying on specific earning formulae to estimate the profitability of a book of ESC business. Rather, this approach uses the emergence patterns inherent in the program's own data. However, it brings with it all the difficulties inherent in estimating losses for longer tailed lines.

For five-year policies, a policy year will not have expired until six years from its start. There is an additional lag from when the last policy expires until the last payment is made, making the lag in the neighborhood of seven to eight years until all claims are settled. The percentage of losses emerging in early stages of development is further reduced by the presence of new car manufacturer warranties. It is not unusual for 2% to 5%, or even less, of the losses for a single five-year policy to emerge in its first year. Thus the experience for relatively green policy periods has the potential for substantial future development.

Some may prefer to analyze experience by model year. The benefit of such analysis is that it keeps the experience for similar vehicles together. It does extend the lag until a year is completely closed, since manufacturers may introduce next year's models relatively early in a year and have those cars in stock well into the next model year. It is conceivable that a model year 19xx could last from March 19xx-1 until March 19xx+1, or even longer.

The tail can be shortened a little by separately considering the lag from policy issue to claim occurrence versus the lag from claim occurrence to final settlement. The accident period development could be used first to develop policy period data to ultimate for claims already incurred. The resulting adjusted data would then have a maximum lag of one more period than the policy term. For example, adjusted policy year data for five-year policies would have a maximum six year lag from the beginning of the policy year until the policy year is fully closed. Similarly, for policy quarters, the maximum lag would be five and one-quarter years.

This two-step approach has another benefit. It separates lag characteristics that are under the direct control of the insurer or administrator (occurrence to settlement) from those that are less subject to their control (policy to occurrence). This latter pattern should be more dependent on the actual policy provisions, term, and mileage limitation and less dependent on specific characteristics of a particular ESC program and administrative structure. In this case, other data sources may also prove useful. If other sources are used, however, the actuary should consider the effects of potential differences in ESC provisions between programs.

Exhibits 2 through 7 provide an example of these concepts. These data are all hypothetical but present general characteristics of ESC programs. We assume that these data are for five-year contracts with the same mileage term.

Exhibit 2 shows accident year paid loss development, Exhibit 3 shows policy year paid loss development, and Exhibit 4 shows the distribution of paid losses by policy year and accident year. All these data are as of September 30, 1992, and the policy and accident years represent fiscal years ending September 30. Fiscal year was selected over calendar year due to the timing of new model roll-out by manufacturers, which typically takes place around October 1. As mentioned above, however, there are many exceptions to this general rule.

Exhibit 2 also shows the indicated development factors and resulting projections of ultimate losses by accident year. Given the relatively short tail inherent in these losses, development factor methods probably provide reasonably accurate forecasts of ultimate losses by accident year. The difference between these forecasts and the amounts paid to date can provide estimates of required total loss reserves by accident year. Although the tail is usually fairly short, this exhibit shows that true loss reserves cannot be completely ignored in these sample data.

Similar development factor projections are also shown in Exhibit 3 for losses sorted by fiscal policy year. In this case, the ultimate loss estimates include projections for future claims as well as for claims that have already occurred. Here, given the tail inherent in the development, development factor methods may not be sufficient to provide stable forecasts, especially in later policy years. Also shown in Exhibit 3 is an estimate for development after age 84 months. Though this represents time after all policies have expired, there is the potential for later development on payments. This estimate is based on projections from Exhibit 5.

The top portion of Exhibit 4 shows the distribution of loss payments as of September 30, 1992, by fiscal policy year and fiscal accident year. For example, of the $10,696,000 in payments to date for the policy year ending September 30, 1988, $43,000 arose from accidents occurring during the year ending September 30, 1988, $814,000 arose from accidents occurring during the year ending September 30, 1989, and so forth.

Since all of these amounts are valued as of September 30, 1992, the last diagonal represents accidents occurring during the year ending September 30, 1992, currently at 12 months of maturity. Similarly, the next older diagonal represents accidents occurring during the year ending September 30, 1991, currently at 24 months of maturity. We use the accident year development from Exhibit 2 to project these amounts to their estimated ultimate levels. These estimates are shown in the bottom portion of Exhibit 4. Here the 12 month factor is used to develop the losses along the last diagonal

$$41 = 29 \times 1.407,\ 1{,}373 = 976 \times 1.407, \text{ etc.},$$

the 24 month factor is used to develop losses along the next older diagonal

$$64 = 62 \times 1.035,\ 1{,}755 = 1{,}696 \times 1.035, \text{ etc.},$$

with similar calculations for the remaining estimates.

The amounts shown in the top portion of Exhibit 5 are the cumulative totals from the bottom portion of Exhibit 4. These amounts are estimates of the emergence of losses during the life of the particular contracts in contrast to the payment of losses during the life of the contracts as shown in Exhibit 3. We then use development factor methods to derive another set of ultimate loss estimates as shown in Exhibit 5.

As discussed in greater detail below, changes in manufacturer warranties can affect the development of losses for new car contracts. For this reason both Exhibits 3 and 5 show two sets of development factor selections. In this hypothetical case we assumed that changes in original manufacturer warranties were made for 1990 models. Thus, development for policy years ending September 30, 1990 and subsequent is expected to be different than that for earlier years. In these exhibits the different factors were judgmentally selected. Later in this paper we will describe some approaches that may assist in quantifying the effects of such changes.

Exhibit 6 summarizes the results of the projections from Exhibits 3 and 5. Also in that exhibit is a third forecast method that does not have the "leveraging" problem of development factor methods. This third method is akin to a Bornhuetter-Ferguson approach but uses adjusted and trended pure premiums based on development factor projections instead of loss ratios as its initial estimate. Column 3 shows the initial selections by policy year which are based on development factor projections shown in Columns 1 and 2. Column 5 is the pure premium indicated by these initial selections. The pure premiums in Column 6 are based on these initial pure premiums, taking into account both trend and an estimated 10% decrease because of changes in manufacturer warranties in 1990. Part 2 of Exhibit 6 shows the calculation of these smoothed pure premiums in more detail. We first adjust the pure premiums to a common warranty level, using the assumed 10%, and then trend the resulting pure premiums. Then we adjust the trended pure premiums to reflect the assumed effect of the warranty change.

The forecasts in Column 7 are the adjusted policy year/accident year losses from Exhibit 5 plus the product of expected losses [Column 6 x Column 4] and the proportion of losses expected to emerge in the future. This latter amount is [1 – 1/age-to-ultimate factor] using the development from Exhibit 5. These calculations are shown in more detail on Exhibit 6, Part 2.

The remainder of Exhibit 6, Part 1 shows the final selections, the resulting pure premiums and total unpaid losses by policy year. Also shown is the separation of that total unpaid amount between loss reserves and estimated unpaid amounts on unexpired terms of current policies.

Some contracts have provisions that allow car buyers to cancel for various reasons. It is also not unusual for new car contracts to be sold after the car purchase but before the expiration of the manufacturer's warranty. This latter situation is especially true for some insurers who market directly to the new car buyer after the sale. In this case, the effective date of the contract is often recorded as the date the car was put in service.

There can be development in premiums and contract counts over time. Analyses based on losses implicitly include this development. It should be recognized explicitly, however, in methods that consider average losses per contract or expected loss ratios. In this case, the actuary should consider the development of contracts and adjust the forecasts accordingly.

If we calculate loss ratios to monitor the experience in an ESC program, we can use these results to estimate the appropriate earning curves to use. For example, if the program has contingent commissions or some form of retrospective rating, we could use our earning curves to estimate earned premiums for a particular agent or dealer. Exhibit 7 shows the loss emergence implied by the analysis in Exhibits 2 through 6. However, because of the assumed changes in manufacturer warranties for 1990, we suggest using different emergence curves for 1989 and prior contracts versus 1990 and subsequent contracts for these specific calculations. In actual applications, the impact of changes in new car manufacturer warranties should be considered when reviewing earning curves, or equivalent development patterns. We include additional discussion of these adjustments in Section 8.

6. FORECASTS WITHOUT SUFFICIENT DATA

Often an insurer or administrator will not have sufficient experience to assemble complete development triangles needed for the analysis described above. Even with substantial experience available, changes in manufacturer warranties or in contract provisions may require adjustments before that experience can be used for projections.

An additional complication arises in ESC programs that have a large variety of available terms and mileage limitations. Some programs are designed as a cafeteria where a customer can choose among several mileage limitations within each of several time limitations. Though this is often cited as an advantageous sales feature, it further subdivides an already small data base. If there has not been a significant shift in the mix of mileages chosen within a particular time limitation, a combination of the mileages may provide a broader base upon which to make projections.

In the case of changes in manufacturer warranty or ESC provisions, the insurer or administrator may have sufficient data to recast past experience under the new manufacturer warranty or ESC provisions. This is the preferred approach.

In case sufficient data are not available, or if available data are too sparse, the actuary may need to develop estimates of future development from other sources. Currently, no central statistical organization collects and summarizes ESC data or provides other compilation services such as those performed by the Insurance Services Office (ISO) or the National Council on Compensation Insurance (NCCI). On the contrary, most administrators and many insurers hold their data very closely. Thus, modeling from other sources may be required.

One approach is to use Monte Carlo simulation to model the interaction of various aspects of ESCs to estimate the timing of loss emergence. The approach we discuss here concentrates on the loss emergence from policy issue to loss occurrence. This pattern should be less dependent on the activities of a particular insurer or administrator than the development of payments from occurrence to final payment. The latter lag could be estimated using data specific to the insurer or administrator.

The modeling approach described here considers the following aspects of the ESC under analysis:

1. contract term measured by time,
2. contract term measured by mileage,
3. treatment of transfers (vehicle re-sales) in contract,
4. cost of repairs by mileage,
5. inflation in repair and parts costs,
6. effect of manufacturer warranties on costs, and
7. effect of contract provisions on costs.

Exhibit 8 is a diagram that summarizes this approach. In this model we randomly select the mileage to be traveled by a particular car in each year of the contract. Based on the mileage driven in each year, we then estimate the total covered cost limited by various contract provisions. The modeling is then carried out for many cars, to determine relative loss emergence during the life of a contract. This relative emergence can then be used as a substitute for the factors derived in Exhibit 5.

As noted above, it is not unusual to have situations where hard data are not available to quantify various parameters of the simulation. In such cases, we must turn to publicly available sources of information, one of which is published information from the United States Department of Transportation (DOT).

Exhibits 9 through 11 present some such information that can be used in this exercise. Exhibit 9 presents distributions of annual mileage driven by year of ownership and shows that cars tend to be driven less as they age. Exhibit 10 presents data on vehicle retention patterns, and Exhibit 11 provides information regarding repair costs.

It would also seem reasonable that the mileage a particular car is driven in one year will not be independent of the mileage driven in other years. Thus we could make the selection of mileage in subsequent years dependent on the miles driven in earlier years.

There are many possible approaches to reflect this potential dependence. One is to select a Bayesian model wherein the mileage for an individual car follows some random distribution, with the parameters of the distribution being uncertain. Dependence from year to year can be reflected by similar selections of the uncertain parameters from year to year.

An inverse Burr distribution provides excellent fits to the annual mileage distributions shown in Exhibit 9. We then assume that the mileage for an individual car in a particular year has an inverse Weibull distribution; i.e., that such mileage has the cumulative density function:

$$F(x|\theta) = e^{-\theta x^{-\tau}}.$$

Here the parameter τ is assumed to be fixed and known, and the parameter θ is assumed to be unknown but has a Gamma distribution with probability density function given by:

$$g(\theta) = \frac{\lambda^{\alpha}\,\theta^{\alpha-1}\,e^{-\lambda\theta}}{\Gamma(\alpha)}\,.$$

In this case the posterior distribution of the annual mileage x is an inverse Burr distribution with parameters α, $\lambda^{-1/\tau}$, and τ. The proof of this is given in the appendix to this paper.

We model the annual mileage by first randomly selecting a probability level, p, for a particular simulated car. For each year in the life of the simulated car, we select the parameter θ as that value having p probability in the corresponding Gamma distribution. We select the mileage for that year using that value of θ as the parameter in the inverse Weibull distribution. This procedure maintains some dependence from one year to the next, in that the parameter θ is at the same probability level from one year to the next, but still maintains randomness in the mileage for individual cars.

We model the annual mileage by first randomly selecting a probability level, p, for a particular simulated car. For each year in the life of the simulated car, we select the parameter θ as that value having p probability in the corresponding Gamma distribution. We select the mileage for that year using that value of θ as the parameter in the

As mentioned above, if a car is sold to a party other than a car dealer, most service contracts provide for the transfer of the ESC with the payment of a fee, usually \$25, and the completion of the proper forms. However, many cars are sold without the necessary paperwork or are traded. Thus, sales can affect an ESC's exposure to loss, especially in the later years.

Exhibit 10 shows some retention data published by the DOT. Though this source is a bit old, it does show that vehicle sales can affect ESC loss development. Before these data are used, however, it should be noted that there may be selection in the purchase of ESCs. Those who buy an ESC may expect to own their cars longer. Direct use of the statistics in Exhibit 10 may tend to understate the level of losses in the latter stages of new car contracts. We caution that vehicle retention patterns may have changed significantly since the compilation of the data in Exhibit 10. Given current conditions, the amounts shown in Exhibit 10 should probably be considered as upper bounds for actual retention practices. To the extent that information specific to a particular program is available, it should be used to obtain better estimates of retention rates.

These two exhibits summarize information that can be used to estimate the "retention" of contracts, that is, to estimate the percentage limitations or were transferred. As mentioned above, however, losses generally cannot be expected to arise evenly over the life of ESCs. We still need to incorporate loss information in the model.

In addition to estimates of driving and ownership patterns, the DOT has also published data regarding the cost of owning and operating automobiles. Exhibit 11 presents a summary of the 1984 study. As can be seen in that exhibit, the cost per mile of the category "unscheduled repairs and maintenance" is not constant during the life of a car. It rises during the first 81,000 miles, then falls off to relatively low levels near the end.

The cost portion of our model combines the randomly generated total miles driven with this cost model to estimate total costs. For example, if one of the simulated cars were to travel 13,000 miles the first year and 9,500 miles the second, the first year costs, using the average from Exhibit 11, would be $10.40 (13,000 × $0.0008) and the cumulative costs through the second year, with a total of 22,500 miles, would be $40.40 (14,500 × $0.0008 + 8,000 × $0.0036). Thus the indicated second year costs would be $30.00. We then apply a selected inflation factor, derived at least in part from considering vehicle repair costs in the Consumer Price Index (CPI), to estimate losses to be paid in each year of a contract.

The estimates presented so far all assume that the coverage offered by an ESC matches the costs in "unscheduled repairs and maintenance" shown in Exhibit 11. There are several factors that can affect this assumption.

One factor is the particular ESC itself. Different ESCs have different exclusions of covered parts. If we assume that such exclusions affect the same proportion of costs at all mileage levels, then the data from Exhibit 11 can still be useful in estimating the *timing* of losses in contrast to the emergence of absolute dollar costs. If we expect that contract exclusions can have a substantial impact on these amounts, we could make adjustments before we simulate the results.

A more important influence on ESC costs is changes in new car manufacturer warranties. The data in Exhibit 11 were directed toward a 1984 model car. It is safe to assume that these are based on the existence of a new car warranty that covered virtually all failures in the first year or 12,000 miles, with little or no coverage after that. This was the predominant form of warranty at that time.

Currently, however, several different warranties exist. Almost every manufacturer offers "bumper-to-bumper" coverage for the first year or 12,000 miles, and most offer additional coverage on major components for a period after that. An example is Chrysler's "7/70" that extends coverage on the power train (portions of the engine, transmission, and differential or trans-axle) to the first seven years or 70,000 miles, after payment of a $100 deductible. General Motors' coverage for most 1992 models is three years or 36,000 miles on a "bumper-to-bumper" basis. As mentioned above, ESCs can still experience losses in this period.

In addition to variation in extended warranties among manufacturers, there is also variation within the same manufacturer. Often "high end" cars come with more complete extended manufacturer warranties than other cars from the same manufacturer. Even cars of the same make and model may have different manufacturer warranties. For example, Chrysler offered buyers of 1992 models a choice between the "7/70" option or "bumper-to-bumper" coverage for the first three years or 36,000 miles.

Estimates of loss emergence for particular manufacturers or vehicles could use loss estimates similar to those in Exhibit 11, after adjustment for changes in underlying manufacturer warranties. For example, if a manufacturer has a one year or 12,000 mile basic bumper-to-bumper warranty and a three year or 36,000 mile extended warranty on power train components, and if power train losses are assumed to be 60% of all losses, we could multiply losses between 12,000 and 36,000 miles in Exhibit 11 by 40% as an approximation of the effect of these changes.

In the rare situations where an ESC program covers only one manufacturer and when that manufacturer has modified the warranties on all its vehicles uniformly, this analysis may be sufficient.

Unfortunately, most programs cover vehicles from several, if not all, manufacturers, and different manufacturers have incorporated different changes in their underlying warranties at different times. Hence adjustment of emergence patterns for a more complex book of business tends to be much more complicated in practice.

Also complicating emergence patterns for ESCs is selection by the contract holders. The potential contract holder's perceptions of how he or she will use the car over the coming years may influence the choice of term and mileage limitation selected. For example, if the buyer plans to sell the car after five years, he or she will have little interest in six or seven year contracts. Similarly, if the buyer typically drives many miles per year, he or she would opt for high mileage limitations or even unlimited mileage coverage, if it is available.

Thus, different contract terms can have different underlying loss cost patterns, even if the underlying manufacturer warranties and ESC contracts are the same. This further complicates analysis for an immature program where such differences may not yet be apparent.

In addition, the emergence of losses for a program can be influenced by other factors such as the presence of "good" or "bad" models or model years as well as features unique to a particular ESC program. All of these factors should be considered when modeling in practice.

Exhibit 12 shows the results of the simulation of 50,000 cars using the unadjusted data from Exhibits 9 through 11 for a 5 year/100,000 mile contract, assuming that the Exhibit 11 data are for a basic manufacturer's warranty of one year/12,000 miles of bumper-to-bumper coverage with no extended manufacturer coverage. As can be seen, there is a relatively small portion of loss expected to emerge in the first year of the contract.

The results of this basic simulation are distributions of expected loss emergence in each year of a single contract. These estimates can be combined with assumptions or estimates regarding the writing of policies during a period of time (year, quarter) to derive estimates of loss emergence for individual policy periods. These loss emergence patterns can then be combined with estimates of lags from emergence to final payment to estimate payment lags for policy periods. An example of such a combination for a policy year is shown in Exhibit 13.

Exhibits 9 through 11 show published data that can be a source for estimated costs and mileage distributions for input to the simulation model. Of course, the closer the particular input assumptions are to the experience of the program, the better the model will estimate the emergence and cost patterns for the program. Even if the program is not fully mature, sufficient data may be available to refine the estimates from Exhibits 9 through 11.

Some may express concern regarding the costs of obtaining more detailed data relative to the benefits those data could provide. We have found that, in practice, the benefits of refined data usually outweigh the associated costs. As with other areas of practical actuarial work, this remains a valid consideration.

We caution that these estimates are *for example only*. In practice, actual loss emergence often differs from these model estimates. Thus these particular estimates should not be used without full verification that they are appropriate for the particular program.

7. INCORPORATION OF LIMITED PROGRAM DATA

It is not uncommon to have substantial development experience for a limited number of policy years. This, in some respects, is the worst of both worlds. There is too much real data to ignore but not enough to rely on completely.

In these cases we are able to test the appropriateness of the models against what is already present in the real data. Here again, it is very useful to separate the loss-to-payment lag from the policy-issue-to-loss-emergence lag. Since the first tends to be shorter and more dependent on individual insurer or administrator procedures, even relatively green programs have useful experience in this area.

Once these two lags are separated, the issue-to-emergence lags predicted by the model can then be compared with those actually present in the data, even if only for early emergence stages. If the two curves have similar shapes where real data are available, then we can make use of the emergence from the model to estimate the tail for immature policy periods.

If, however, there are differences, the reasons for those differences should be explored. It may be appropriate, after review, to adjust the emergence predicted by the model to reflect patterns apparent in the actual loss emergence.

In addition to these adjustments to the model emergence patterns, we can consider the appropriateness of the various model assumptions to the particular ESC program. As indicated in Section 6, the primary input data for the simulation model are:

1. the mileage distributions for each year in the life of the car;
2. the estimated costs of repair at various mileage points in the life of the car;
3. the estimated inflation between contract issuance and time of repair; and
4. the estimated rates of contract termination during the life of the contract.

Except for the inflation assumptions, Exhibits 9 through 11 provide examples of some of these estimates, though the data themselves may be somewhat dated.

For example, the average annual mileage for the distributions in Exhibit 9 roughly compares with the annual aggregates in Exhibit 11. However, use patterns change and average annual mileages which were appropriate for 1981 may not be reflective of current driving habits. In particular, a 1988 publication from the U.S. Department of Energy (DOE) titled "Household Vehicles Energy Consumption, 1988" indicates that during 1988, 1987 models averaged 13,400 miles, 1986 models averaged 12,600 miles, 1985 models averaged 12,100 miles, and so forth. This is a different annual mileage pattern than shown in Exhibit 11. In addition, the same 1988 DOE study indicated that the average number of miles driven per vehicle has increased from 9,399 in 1983 to 10,246 in 1988. The more recent information should be incorporated in forecasts of loss emergence.

Mileage distributions may also become important in quantifying selection by insureds between contracts of different terms. As noted above, it is possible that those selecting higher mileage contracts may expect to have a higher annual mileage than those selecting a lower mileage policy. To the extent that significant selection is expected, it may be beneficial to modify the mileage distributions used to model the loss emergence for different contract terms. In this case, we should increase the annual mileages used to model higher mileage contracts relative to those used to model lower mileage contracts.

Actual experience under an ESC program may also be useful in refining estimates of the cost curves used in the simulation model. Many ESC data bases capture mileage at time of repair. This can be very useful in estimating cost emergence.

It is usually a relatively simple task to sort loss payments into categories by mileage at time of repair. There is a difficulty in using these data directly. If the data are taken from policy periods that are not yet fully mature, we do not know the number of contracts expected to be exposed to potential loss in a particular mileage category. Thus, without some estimate of earned exposure we cannot estimate the true average cost for various mileage categories.

We again turn to our Monte Carlo simulation model to derive estimates of these earned exposures. In this case we will not concern ourselves with the costs but simply worry about the proportion of cars that can be expected to have various total mileages at various contract ages. Exhibit 14 is an example of such a distribution, derived from the Monte Carlo model using the assumptions from Exhibits 9 and 10.

The percentages shown in Exhibit 14 represent the estimated proportion of policies of a given age that will exceed the indicated mileage. We can use these estimates in conjunction with written exposures in a particular program to estimate the number of exposures generating losses in a particular mileage range.

An example of this calculation is shown in Exhibit 15. Part 2 displays the contract-years exposed to losses for the ages and mileage entries. These are the products of total contracts by contract age with the corresponding proportions in Exhibit 14. Thus, there are a total of 511,000 (200,000 + 150,000 + 100,000 + 50,000 + 10,000 + 1,000) contracts in this hypothetical program, as shown in Part 1 of Exhibit 15. The lower section of Part 1 shows the number of contracts exposed to losses in each year, given the simplifying assumption of uniform writing during a year. For example, by the end of 1992 all contracts issued from 1987 through 1991 generated a full year of exposure in their first year. With the simplifying assumption of even writings during the year, the 1992 contracts generated one-half of a year of exposure. Thus there were approximately 411,000 contracts contributing to losses in their first year. Similarly, 1987 through 1990 contracts and half, on average, of the 1991 contracts experienced second year exposure, for a total of 236,000, and so forth.

Given the percentages from Exhibit 14, all of these contracts could contribute to losses above 0 miles, but not all could contribute to losses above 6,000 miles. In fact, from Exhibit 14, an estimated 37.24% of the 411,000 first year exposures would contribute in this range (153,056 = .3724 × 411,000), 82.17% of the second year exposures (193,921 = .8217 × 236,000), and so forth. Part 2 thus provides estimates of the number of contracts having exposure in the various mileage bands.

Exhibit 16 provides an example of how these estimates can be used to obtain better estimates of costs per mile, even for an immature new car program. This exhibit shows hypothetical costs for repairs in various mileage intervals. In this case we have assumed that all costs have been adjusted to a common cost level before aggregation. The amounts in Column 3 are adjusted to reflect the cost per mile for an individual contract. We are assuming that these are five-year contracts; thus we divide the exposure count from Column 2 by 5 to calculate Column 3.

Though the costs themselves are hypothetical, the resulting cost-per-mile estimates in Column 3 do represent patterns that arise in practice. Note that the costs start quite low in early years. This is due primarily to the existence of manufacturer warranties covering losses in the first year or 12,000 miles. We could now use these averages in place of the estimates in Exhibit 11 in the Monte Carlo model to obtain a better picture of the loss emergence under a particular program.

8. OTHER USES FOR EMERGENCE MODEL FORECASTS

The primary value of these emergence models is that they can provide insight as to *relative* loss differences under various situations. One such application is in estimating the timing of loss emergence, as described in the previous section.

These models can also be useful in providing insight into the influence of various factors on the overall cost of ESCs. For example, we can use the model to estimate the relative cost difference between five year/50,000 and five year/100,000 mile contracts. This can be done by simply changing the mileage limitation in the model from 50,000 to 100,000. Better estimates of relative differences can be obtained by running the same random set of vehicles with both mileage limitations. Note that the resulting estimates implicitly assume that insureds for different terms will have the same inherent loss pattern. This ignores potential selection by insureds and should be recognized when reviewing results.

The model can also be used to estimate the impact of changes in manufacturer warranties on the costs covered by ESCs. In this case two different cost functions can be used on the same random set of vehicles. For example, we used the input assumptions from Exhibits 9 through 11 and an assumed cost inflation of 7% to derive Table 1, estimates for a five year/100,000 mile new-car ESC. In this case we assumed that the first manufacturer warranty was for one year/12,000 miles for all components, the second for three years/36,000 miles for all components, while the third was for one year/12,000 for all components with coverage for the power train for seven years/70,000 miles. In these calculations we assumed that power train repairs constituted 60% of total costs.

As the table shows, changes in the manufacturer warranty can have a noticeable effect on both the loss emergence and costs of ESCs. Both of the alternative warranties tend to lengthen the emergence curve even though they both reduce total ESC costs. This is of significance in practice. Unless adjusted, development methods based on older contracts with more limited manufacturer warranties may tend to understate losses on more recent contracts where manufacturer warranties cover more. Conversely, pure premium trends will be depressed by the introduction of longer manufacturer warranties.

TABLE 1

EXAMPLE EMERGENCE AND RELATIVE COSTS UNDER ALTERNATIVE MANUFACTURER WARRANTIES

	Manufacturer Warranty		
Contract Age	1/12	3/36	7/70
1	3.80%	2.60%	3.40%
2	20.30	15.80	16.80
3	49.40	44.50	43.70
4	78.40	76.40	75.00
5	100.00	100.00	100.00
Relative Cost	1.00	0.91	0.63

As noted above, these comparisons are based on the assumption that power train losses constitute a uniform 60% of all losses. It is likely that power train losses will experience a different emergence than non-power-train losses. We could use the methodology in Exhibit 16, applied separately to power train and non-power-train losses, to derive separate emergence curves to refine this rough assumption.

If the ESC program has only one make, we could use the revised curves to estimate the impact of changes in manufacturer warranties.

If, however, as in many ESC programs, there are many different underlying manufacturer warranties, then simply calculating separate cost-per-mile curves may not provide sufficient data to modify emergence curves for the program. In fact, the losses that would be used in Exhibit 16 are themselves reduced by existing manufacturer warranties, and these warranties themselves can change from one model year to the next.

Thus, the assumption that changes in warranties can be addressed by simple modifications of the cost-per-mile input data may not hold. In such a case we could develop separate cost curves for each major component of cost to a program. Such components could include those costs covered by the ESCs but not covered under the basic (often bumper-to-bumper) manufacturer warranty, those costs covered by the basic warranty but not covered by an extended (often power train) warranty, and those costs covered by the extended manufacturer warranty. Once these separate curves are estimated using the data for a particular program, we could refine estimates of the effects of changes in underlying warranties on the costs and emergence of losses in a program.

With sufficient data the approaches in Exhibits 14 through 16 may provide a means of separately identifying these separate cost curves *if credible data were available by loss component and mileage, and separately for different underlying manufacturer warranties.* Unfortunately sufficient data in this detail are seldom available. We must sometimes use curve fitting methods that consider the underlying mix of manufacturer warranties to estimate these components. Because of the complexity of this approach and the survey nature of this paper, it will not be discussed further here.

9. USED CAR COVERAGES

Although the above discussion focuses primarily on new car coverages, the same techniques can be applied to analyze the experience of used car programs. As opposed to new car coverages, used car contract terms are relatively short, running between one and three years. In addition, manufacturer warranties generally have less influence on experience for used cars than for new cars. This leads to a greater proportion of losses emerging in the earlier stages of used car contracts than in the later stages.

On the other hand, there may be greater moral hazard present in used car contracts than in those for new cars. The presence of ESCs on used cars can provide a dealer with incentives to recondition used cars at the cost of the ESC program. When analyzing experience for a particular program this possibility should be recognized; in addition, measures should be taken in the program to avoid such reconditioning.

10. LOSS RATIOS IN ESC PROGRAMS

As with many areas of insurance, loss ratios, calculated as incurred (or even paid) losses divided by earned premiums, are frequently used to monitor the profitability of an ESC program. Hopefully the foregoing discussion makes it clear that simple earning patterns probably do not provide sufficient match to expected loss emergence to be relied upon solely.

As mentioned above, we cannot expect losses to emerge uniformly during the life of an ESC. Except in the extremely rare case of unlimited contracts, mileage limitations, and to some extent ownership transfers, reduce the number of used car contracts able to generate losses in their later stages. For used car contracts we often expect that losses will emerge more quickly than pro-rata, and pro-rata earning in fact may provide a conservative basis on which to evaluate profitability.

On the other hand, losses for new car contracts can usually be expected to emerge more slowly than time limitations alone, at least in the early stages of the contract. This usually happens even in unlimited mileage contracts. Though the emergence curves used above are hypothetical, they do present general patterns that appear in new car contracts.

One thing is clear, however: Any formula that claims to apply to a broad range of contracts over a broad range of makes and model years is suspect. One would generally expect that more losses will emerge during the last year of a five year/100,000 mile contract than in the last year of a five year/50,000 mile contract, even though the two contracts experience similar loss experience in their first year or two, with all other variables held constant.

Similarly, one would expect different experience for similar contracts for different model years. In this case, changes in manufacturer warranties would influence the amount and timing of losses during the life of the contracts. Generally, one would expect that extending the manufacturer's warranty on a vehicle will lower the losses on a given ESC. However, this will also push proportionately more losses into the tail of the loss emergence curve. Thus, an earning formula that was appropriate before the introduction of extended manufacturer warranties may earn premiums too rapidly after such introduction.

If earned premiums are used to assess the profitability of an ESC program, we strongly recommend that they be calculated to match the expected flow of losses incurred and that this match be verified periodically. The methods used above can be used for the first calculation. Once they are calculated, emergence patterns should be tested regularly, if loss ratios are to be relied upon.

Probably the easiest way to periodically test the appropriateness of an earning curve is to test that curve against incurred losses (including IBNR) for a fixed policy period. If the resulting loss ratios show a consistent upward pattern as time progresses, we could suspect that the earning curve is pulling premiums into income faster than losses are emerging. Conversely, if the curve shows a consistent downward pattern as time progresses, then we could suspect conservative earning of premium. A match could be indicated by a loss ratio progression that seems to randomly move around a fixed level. However, it is possible that the emergence and earning patterns match over one interval, only to deviate over another. This too should be considered when reviewing loss ratios for an ESC program.

Loss ratios are often used in ratemaking applications, both for determining overall rate level change requirements and in estimating relativities among various classes. As with other lines of insurance, the selected pure premiums shown in Exhibit 6, or corresponding loss ratios, can be used to assess overall rate level adequacy.

We may also be able to use loss ratios to assist in determining the relative adequacy of class rates. One could use the earning curve determined from the aggregate book to estimate earned premium by class. We caution, however, that since classes are usually composed of similar vehicles, new car warranties may vary substantially by class. In this case the actuary could modify the earning curves to reflect the differences in manufacturer warranties and calculate loss ratios that should provide a better indication of relative loss potential among the various classes.

11. STRUCTURE OF ESC PROGRAMS

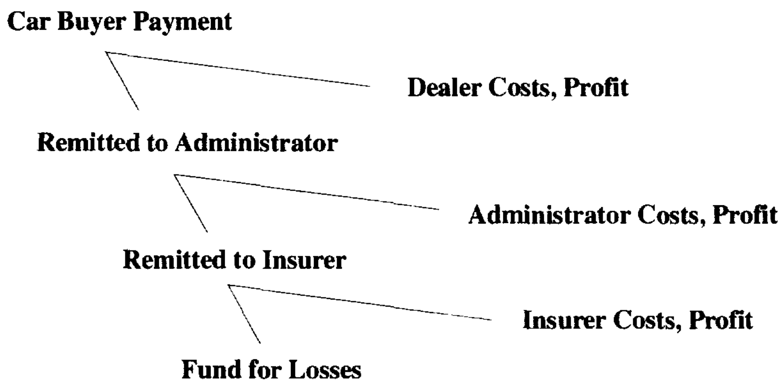
As indicated in Section 2, there are two common types of ESC programs. Other arrangements also exist. In some, a portion of the amounts collected by the dealer are put into a fund and an insurer provides coverage if that fund is depleted.

A common element in many ESC programs is a middleman. This role can be taken on by a managing general agent or a third party administrator. Usually this party supplies data processing, claims, marketing, and other services and sometimes determines rates and rate plans for the program in exchange for a fee. The fee can be a flat charge per contract, a percentage of insurance premiums, or even related to the loss experience of the program. This structure becomes important when evaluating permissible loss ratios. The structure also influences the amount required for loss reserves. The longer the pipeline between car buyer and insurer, the longer the expected lag can be between claim occurrence and final claim payment. This in turn would indicate proportionately larger loss reserve requirements.

12. HOW MUCH PREMIUM WAS CHARGED?

Given the many hands funds may flow through, the actuary must know the precise definition of premium. Not only does this impact the premium tax that the insurer pays, but it influences the permissible or expected loss ratio for the business. This expected loss ratio, along with additional loads, is then used to monitor rate level adequacy.

Briefly, the cash flow for an administered ESC program wherein the insurance transaction is between the dealer and the insurer may look like:



In this case it would not be unusual for the amount "Remitted to Insurer" to be considered premium. It is obvious that the insurer would then expect a relatively large portion of the premium to be available to fund losses. An expected or permissible loss ratio in this case could be in the neighborhood of 85%.

On the other hand, in a program where the car buyer is the insured, with the dealer acting as agent, the amount that the buyer pays may be considered premium. In this case, that premium should provide for commissions to the dealer, administrator fees, insurer costs, and profit. The permissible or expected loss ratio could be small, possibly as low as 30% or lower.

ESC programs are written in a highly competitive market. It is likely that auto dealers choose from more than one program. There is great incentive to sell the contracts with the lowest wholesale price to the car buyer in order to maximize dealer profit. In conjunction with potentially under-priced policies offered in some programs, this stiff competition makes it difficult to implement rate increases. On the other hand, dealers who have experienced the insolvency of one or more of their ESC carriers and now realize they are responsible for the repairs may be more selective in their choice of program.

13. OTHER IMPLICATIONS

Clearly, the rate at which premiums are earned impacts a company's financial position. *We are not advocating any formula or position as to how premiums are earned for financial statements.* An insurer should be aware, however, of the impact of an ESC program on its financial statements. If, after evaluation of an ESC program, an insurer finds that the total unpaid losses, including claims expected to arise from the unexpired terms of existing contracts, exceed the total of its loss and unearned premium reserves, it may need to post additional reserves.

Actuaries who prepare statutory opinions for companies with ESC exposure should be aware of the implications of this conclusion. All ESCs we have seen provide for the reimbursement of expenses for repair of a covered part that breaks down during the term of the contract, limited either by time or mileage. One may argue that the obligation to pay does not exist until a repair occurs and that the loss date is the date of the covered repair. If this position is taken, the loss and loss adjustment expense reserve would provide only for future payments for repairs that have already occurred. As a result, loss and loss adjustment expense reserves would include no provision for repairs that have not yet occurred.

We understand that additional reserves for deficiencies in the unearned premium reserve are required for statements prepared under generally accepted accounting principles (GAAP). However, under current statutory accounting standards, we understand that there is no requirement to book such deficiencies on statutory statements, nor are there explicit provisions for such deficiencies in current annual statements. In this case, an insurer may elect to include a write-in item or segregation of surplus to provide for such deficiencies.

If the amount of these indicated additional reserves is material in terms of a company's surplus, an actuary preparing a statutory statement of actuarial opinion on loss and loss adjustment expense reserves may face a dilemma. The loss and loss adjustment expense reserves may be adequately stated, but the actuary's analysis may imply that the financial solidity of the company is impaired due to future obligations under existing contracts. In addition, there does not appear to be a way to reflect this in statutory statements. Unfortunately, we do not have a solution to offer, but refer the actuary to the appropriate standards of practice.

This additional reserve may not be deductible for the purposes of federal income taxes until the losses are incurred. Thus, the insurer may be in the position of having to increase its reserves without the benefit of a corresponding tax deduction. Again, we do not have a solution to this dilemma, but raise it as a consideration in dealing with ESC programs.

This is not the only area where federal income tax laws come into play. In situations where the insurance policy is with the dealer, it may be considered as contractual liability and included under Other Liability in the annual statement. Thus the Internal Revenue Service (IRS) may require the use of Other Liability discount factors when calculating the deduction for incurred losses, even though the expected pay-out for loss reserves would generally be expected to be very short.

An exhaustive discussion of all aspects of an ESC program and their impact on an insurer is beyond the scope of this paper. The above discussion provides only a brief view of some of the hidden complexities of such a program.

14. CONCLUSION

An ESC program can provide a profitable book of business to an insurer. However, monitoring the profitability of that book presents unique problems. Contracts are often sold for multiple years with limited right to cancellation. Although the coverage generally has losses paid soon after the occurrence, the extended period of coverage heightens the role of the unearned premium reserve on the financial soundness of the program. Unlike most other insurance policies, losses cannot be expected to emerge uniformly throughout the term of the policy. Thus pro rata earning of premium usually does not provide a match between income and liabilities. This can result in inaccurate conclusions regarding the profitability of ESC programs. Insurers writing such programs should continually monitor the fit between premium earning and loss emergence, if loss ratios using earned premiums are to be used to monitor the profitability of an ESC program.

Probably the best way to assess profitability, however, is to compare forecasted total losses with premiums. In this way there are no assumptions regarding the timing of premium earning, and the actual loss experience can provide insight to the future emergence of losses for later policy periods.

The author is aware of only three other publications in the actuarial literature dealing with ESC programs, as presented in the attached bibliography. The concepts and approaches presented there, as well as those presented here, should be considered as starting points in the analysis of an ESC program. There remains much to be done in this area.

REFERENCES

[1] Cheng, Joseph S. and Stephen J. Bruce, "A Pricing Model for New Vehicle Extended Warranties," *Casualty Actuarial Society Forum,* Special Edition, Ratemaking Call Papers, 1993, pp. 1-24.

[2] Noonan, Simon J., "The Use of Simulation Techniques in Addressing Auto Warranty Pricing and Reserving Issues," *Casualty Actuarial Society Forum,* Special Edition, Ratemaking Call Papers, 1993, pp. 25-52.

[3] Schilling, Timothy L.,"The Challenge of Pricing Extended Warranties," *Pricing,* Casualty Actuarial Society Discussion Paper Program, 1990, pp. 369-393.

EXHIBIT 1
Part 1

EXAMPLE: LOSS RATIOS UNDER MISMATCHED EARNING

	Premiums Earned						
Calendar	Policy Year						Cumulative
Year	1	2	3	4	5	Total	Total
1	$1,000					$1,000	$1,000
2	2,000	$1,250				3,250	4,250
3	2,000	2,500	$1,563			6,063	10,313
4	2,000	2,500	3,125	$1,719		9,344	19,657
5	2,000	2,500	3,125	3,438	$1,719	12,782	32,439
6	1,000	2,500	3,125	3,438	3,438	13,501	45,940
7		1,250	3,125	3,438	3,438	11,251	57,191
8			1,562	3,438	3,438	8,438	65,629
9				1,717	3,438	5,155	70,784
10					1,717	1,717	72,501
Total	$10,000	$12,500	$15,625	$17,188	$17,188	$72,501	

NOTE: Dollar amounts are in thousands.

EXHIBIT 1
Part 2

EXAMPLE: LOSS RATIOS UNDER MISMATCHED EARNING

	Losses Incurred						
Calendar Year	Policy Year 1	2	3	4	5	Total	Cumulative Total
1	$375					$375	$375
2	1,500	$469				1,969	2,344
3	3,000	1,875	$586			5,461	7,805
4	4,125	3,750	2,344	$645		10,864	18,669
5	4,125	5,156	4,688	2,578	$645	17,192	35,861
6	1,875	5,156	6,445	5,156	2,578	21,210	57,071
7		2,344	6,445	7,090	5,156	21,035	78,106
8			2,930	7,090	7,090	17,110	95,216
9				3,223	7,090	10,313	105,529
10					3,223	3,223	108,752
Total	$15,000	$18,750	$23,438	$25,782	$25,782	$108,752	

NOTE: Dollar amounts are in thousands.

EXHIBIT 1
Part 3

EXAMPLE: LOSS RATIOS UNDER MISMATCHED EARNING

	Earned Premiums		Incurred Losses		Indicated Loss Ratios		
Calendar Year	Total	Cumulative Total	Total	Cumulative Total	Total	Cumulative Total	Cumulative Profit (Loss)
1	$1,000	$1,000	$375	$375	38%	38%	$625
2	3,250	4,250	1,969	2,344	61	55	1,906
3	6,063	10,313	5,461	7,805	90	76	2,508
4	9,344	19,657	10,864	18,669	116	95	988
5	12,782	32,439	17,192	35,861	135	111	(3,422)
6	13,501	45,940	21,210	57,071	157	124	(11,131)
7	11,251	57,191	21,035	78,106	187	137	(20,915)
8	8,438	65,629	17,110	95,216	203	145	(29,587)
9	5,155	70,784	10,313	105,529	200	149	(34,745)
10	1,717	72,501	3,223	108,752	188	150	(36,251)
Total	$72,501		$108,752		150%		

NOTE: Dollar amounts are in thousands.

EXHIBIT 1
Part 4

EXAMPLE: LOSS RATIOS UNDER MISMATCHED EARNING

Indicated Loss Ratios by Calendar and Policy Year

Calendar Year	Policy Year 1	2	3	4	5	Total	Cumulative Total
1	38%					38%	38%
2	75	38%				61	55
3	150	75	37%			90	76
4	206	150	75	38%		116	95
5	206	206	150	75	38%	135	111
6	188	206	206	150	75	157	124
7		188	206	206	150	187	137
8			188	206	206	203	145
9				188	206	200	149
10					188	188	150
Total	150%	150%	150%	150%	150%	150%	

EXHIBIT 2

SAMPLE FISCAL ACCIDENT YEAR PAID LOSS DEVELOPMENT

(AS OF SEPTEMBER 30, 1992)

Fiscal Accident Year Ending 9/30	Months of Development						Ultimate	Indicated Loss
	24	36	48	60	72	84	Forecast	Reserves
1986	$13	$14	$14	$14	$14	$14	$14	$0
1987	187	188	188	188	188		188	0
1988	1,362	1,383	1,394	1,394			1,394	0
1989	3,622	3,887	3,926				3,926	0
1990	8,616	8,687					8,765	78
1991	15,554						16,098	544
1992							23,677	6,849
							Total	7,471

DEVELOPMENT FACTORS

Fiscal Accident Year Ending 9/30	Months of Development						
	24/12	36/24	48/36	60/48	72/60	84/72	Ultimate/84
1986	1.625	1.077	1.000	1.000	1.000	1.000	
1987	1.520	1.005	1.000	1.000	1.000		
1988	1.684	1.015	1.008	1.000			
1989	1.945	1.073	1.010				
1990	1.386	1.008					
1991	1.237						
Selected	1.359	1.026	1.009	1.000	1.000	1.000	
Cumulative	1.407	1.035	1.009	1.000	1.000	1.000	1.000

NOTE: Dollar amounts are in thousands.

EXHIBIT 3

SAMPLE FISCAL POLICY YEAR PAID LOSS DEVELOPMENT

(AS OF SEPTEMBER 30, 1992)

Fiscal Policy Year Ending 9/30	Months of Development: 12	24	36	48	60	72	84	Ultimate Forecast	Indicated Loss Reserves
1986	$0	$114	$581	$1,294	$2,341	$3,102	$3,176	$3,201	$25
1987	18	406	1,754	4,701	7,198	8,547		8,821	274
1988	16	370	2,974	7,156	10,696			13,477	2,781
1989	26	1,439	7,743	16,588				32,164	15,576
1990	29	1,237	6,516					29,087	22,571
1991	32	1,038						24,688	23,650
1992	29							25,724	25,695
								Total	$90,572

DEVELOPMENT FACTORS

Fiscal Policy Year Ending 9/30	Months of Development: 24/12	36/24	48/36	60/48	72/60	84/72	Ultimate/84
1986	--	5.096	2.227	1.809	1.325	1.024	
1987	22.556	4.320	2.680	1.531	1.187		
1988	23.125	8.038	2.406	1.495			
1989	55.346	5.381	2.142				
1990	42.655	5.268					
1991	32.438						
Selected-1	--	--	--	1.539	1.221	1.024	
Cumulative	--	--	--	1.939	1.260	1.032	1.008
Selected-2	37.295	5.328	2.142	1.575	1.250	1.050	
Cumulative	887.024	23.784	4.464	2.084	1.323	1.058	1.008

NOTES: 1. Dollar amounts are in thousands.

2. Selected-1 is used to develop policy years 1989 and prior; Selected-2 is used for policy years 1990 and subsequent.

EXHIBIT 4

SAMPLE FISCAL POLICY YEAR DISTRIBUTION OF PAID LOSSES BY ACCIDENT YEAR

(AS OF SEPTEMBER 30, 1992)

Fiscal Policy Year	Accident Year						
Ending 9/30	PY	PY+1	PY+2	PY+3	PY+4	PY+5	PY+6
1986	$14	$145	$675	$868	$951	$522	$0
1987	43	676	2,112	2,622	2,170	925	
1988	43	814	2,941	4,142	2,756		
1989	132	2,096	6,961	7,399			
1990	77	1,696	4,743				
1991	62	976					
1992	29						

ACCIDENT YEAR DEVELOPMENT FACTORS

	Accident Year Age						
	12	24	36	48	60	72	84
Factor	1.407	1.035	1.009	1.000	1.000	1.000	1.000

ESTIMATED ULTIMATE POLICY YEAR/ACCIDENT YEAR LOSSES

Fiscal Policy	Accident Year						
Year Ending 9/30	PY	PY+1	PY+2	PY+3	PY+4	PY+5	PY+6
1986	$14	$145	$675	$868	$960	$540	$0
1987	43	676	2,112	2,646	2,246	1,301	
1988	43	814	2,967	4,287	3,878		
1989	132	2,115	7,205	10,410			
1990	78	1,755	6,673				
1991	64	1,373					
1992	41						

NOTES: 1. Dollar amounts are in thousands.
2. The Accident Year Development Factors are the cumulative factors from Exhibit 2.

EXHIBIT 5

SAMPLE FISCAL POLICY YEAR BY ACCIDENT YEAR PAID LOSS DEVELOPMENT

(AS OF SEPTEMBER 30, 1992)

Fiscal Policy Year	Number of Accident Years Emerged							Ultimate	Indicated Total
Ending 9/30	1	2	3	4	5	6	7	Forecast	Unpaid
1986	$14	$159	$834	$1,702	$2,662	$3,202	$3,202	$3,202	$26
1987	43	719	2,831	5,477	7,723	9,024		9,024	477
1988	43	857	3,824	8,111	11,989			14,111	3,415
1989	132	2,247	9,452	19,862				34,202	17,614
1990	78	1,833	8,506					32,008	25,492
1991	64	1,437						23,798	22,760
1992	41							15,636	15,607
								Total	$85,392

DEVELOPMENT FACTORS

Fiscal Policy Year	Number of Accident Years Emerged						
Ending 9/30	2/1	3/2	4/3	5/4	6/5	7/6	Ultimate/7
1986	11.357	5.245	2.041	1.564	1.203	1.000	
1987	16.721	3.937	1.935	1.410	1.168		
1988	19.930	4.462	2.121	1.478			
1989	17.023	4.206	2.101				
1990	23.500	4.640					
1991	22.453						
Selected-1	--	--	--	1.463	1.177	1.000	
Cumulative	--	--	--	1.722	1.177	1.000	1.000
Selected-2	23.028	4.401	2.101	1.480	1.210	1.000	
Cumulative	381.367	16.561	3.763	1.791	1.210	1.000	1.000

NOTES: 1. Dollar amounts are in thousands.

2. Selected-1 is used to develop policy years 1989 and prior; Selected-2 is used for policy years 1990 and subsequent.

EXHIBIT 6
Part 1

SUMMARY OF ULTIMATE LOSS FORECASTS
(AS OF SEPTEMBER 30, 1992)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Fiscal Policy Year Ending 9/30	Policy Year Development (Exhibit 3)	Policy-Accident Year Development (Exhibit 5)	Initial Selection	Total Written Contracts	Initial Indicated Pure Premium	Smoothed Pure Premium	Expected Pure Premium Method	Selected Ultimate Losses	Indicated Pure Premium	Losses Paid 9/30/92	Indicated Unpaid Losses
1986	$3,201	$3,202	$3,202	22,399	$143	$143	$3,202	$3,202	$143	$3,176	$26
1987	8,821	9,024	9,024	60,513	149	152	9,024	9,024	149	8,547	477
1988	13,477	14,111	13,953	85,716	163	161	14,065	14,028	164	10,696	3,332
1989	32,164	34,202	33,693	197,116	171	171	33,995	33,894	172	16,588	17,306
1990	29,087	32,008	31,278	186,064	168	163	30,776	30,943	166	6,516	24,427
1991	24,688	23,798	24,243	143,542	169	173	24,770	24,638	172	1,038	23,600
1992	25,724	15,636	---	149,963	---	184	27,562	27,562	184	29	27,533
Total								$143,291		$46,590	$96,701

Indicated Loss Reserves as of 9/30/92 $7,471

Estimated Losses on Future Claims as of 9/30/92 $89,230

NOTES:
1. Amounts in Columns (1), (2), (3), (7), (8), (10) and (11) are in thousands of dollars.
2. The derivation of Column (6) is shown in Column (5) of Part 2.
3. The derivation of Column (7) is shown in Column (9) of Part 2.

EXHIBIT 6
Part 2

DERIVATION OF COLUMNS (6) AND (7) OF EXHIBIT 6, PART 1
(AS OF SEPTEMBER 30, 1992)

Fiscal Policy Year Ending 9/30	(1) Initial Indicated Pure Premium	(2) Estimated New Car Warranty Change Factor	(3) Warranty Adjusted Pure Premium (1)/(2)	(4) Smoothed Warranty Adjusted Pure Premium	(5) Selected Smoothed Pure Premium (2) x (4)	(6) Total Written Contracts	(7) Estimated Ultimate Losses on Emerged Claims	(8) Estimated Percent of Losses Emerged	(9) Expected Pure Premium Method (7)+[1−(8)]×(5)×(6)
1986	$143	1.00	$143	$143	$143	22,399	$3,202	100.00%	$3,202
1987	149	1.00	149	152	152	60,513	9,024	100.00	9,024
1988	163	1.00	163	161	161	85,716	11,989	84.96	14,065
1989	171	1.00	171	171	171	197,116	19,862	58.07	33,995
1990	168	0.90	187	182	163	186,064	8,506	26.57	30,776
1991	169	0.90	188	193	173	143,542	1,437	6.04	24,770
1992	--	0.90	--	205	184	149,963	41	0.26	27,562

NOTES:
1. Column (1) is Column (5) from Part 1.
2. Column (2) is assumed, based on a separate analysis.
3. Column (4) is the result of an exponential fit on Column (3).
4. Column (7) is the last diagonal from Exhibit 5.
5. Column (8) is the reciprocal of the age-to-ultimate factors from Exhibit 5. The cumulative for Selected-1 is used for 1986-1989 and the cumulative for Selected-2 is used for 1990-1992.
6. Amounts in Columns (7) and (9) are in thousands of dollars.

EXHIBIT 7

LOSS EMERGENCE IMPLIED BY POLICY YEAR/ACCIDENT YEAR DISTRIBUTION AND SELECTED ULTIMATE LOSSES BY POLICY YEAR

(AS OF SEPTEMBER 30, 1992)

Fiscal Policy	Accident Year						
Year Ending 9/30	PY	PY+1	PY+2	PY+3	PY+4	PY+5	PY+6
1986	0.4%	5.0%	26.0%	53.2%	83.1%	100.0%	100.0%
1987	0.5	8.0	31.4	60.7	85.6	100.0	
1988	0.3	6.1	27.3	57.8	85.5		
1989	0.4	6.6	27.9	58.6			
1990	0.3	5.9	27.5				
1991	0.3	5.8					
1992	0.1						
Weighted Averages:							
1986-89	0.4%	6.6%	28.2%	58.4%	85.2%	100.0%	100.0%
1990-92	0.2	5.9	27.5	--	--	--	--

EXHIBIT 8

EXAMPLE FLOW OF MONTE CARLO SIMULATION MODEL

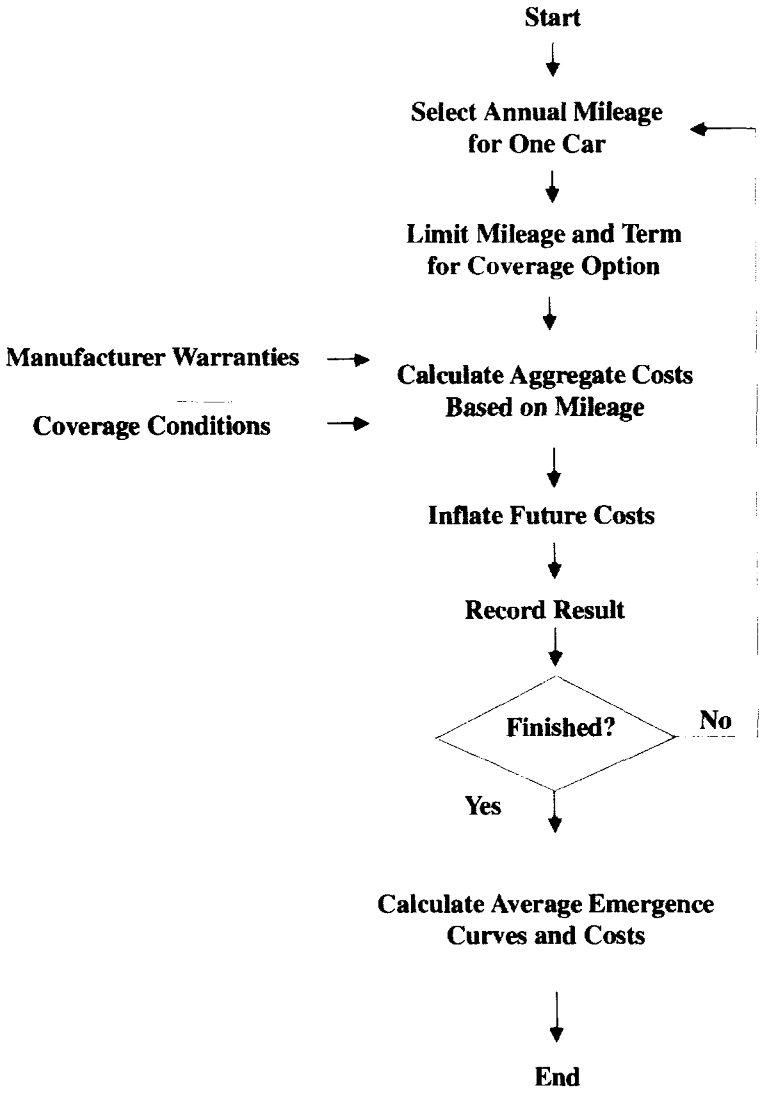


EXHIBIT 9

PERCENTAGE OF VEHICLES BY ANNUAL MILEAGE AND AGE

Annual Mileage	Vehicle Age											
	Under 1	1	2	3	4	5	6	7	8	9	10+	All
0-999	22.0%	4.9%	2.5%	3.6%	4.1%	4.5%	5.2%	5.4%	8.0%	9.0%	19.0%	8.2%
1,000-2,999	13.1	6.1	6.5	7.3	9.6	10.8	11.3	12.9	12.9	16.8	18.9	12.0
3,000-7,999	18.4	22.7	20.9	23.9	26.0	26.9	29.0	32.2	32.8	31.2	31.4	27.7
8,000-12,999	20.0	24.9	30.7	33.2	32.4	31.0	30.1	28.7	30.3	25.6	19.2	27.4
13,000-17,999	9.2	18.8	18.2	15.3	14.6	14.0	13.2	9.4	9.0	9.4	5.6	12.2
18,000-22,999	5.0	8.8	9.8	7.1	6.2	6.7	5.6	6.5	3.9	3.7	2.4	5.7
23,000-27,999	5.6	6.3	4.7	4.4	2.9	2.7	2.7	1.8	1.3	2.0	1.1	2.9
28,000-	6.7	7.5	6.7	5.2	4.2	3.4	2.9	3.1	1.8	2.3	2.4	3.9
Total	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%
Average Annual Mileage	11,268	13,498	13,562	12,261	11,497	10,694	10,624	9,655	8,757	8,714	7,085	10,368

Source: "Household Vehicle Utilization," U.S. Department of Transportation, Federal Highway Administration, Office of Highway Planning, April 1981.

EXHIBIT 10

PERCENTAGE OF AUTOMOBILES PURCHASED NEW AND USED BY AGE OF AUTO AND AUTO OWNERSHIP IN 1977

Age of Autos in	Vehicles Purchased New		Vehicles Purchased Used		Total	Percentage with
Years	Percentage	Number	Percentage	Number	Vehicles	Original Owner
Less than 1	2.0%	912	0.0%	0	912	100.0%
1	16.6	7,570	0.9	445	8,015	94.4
2	16.9	7,706	3.4	1,683	9,389	82.1
3	11.2	5,107	5.1	2,525	7,632	66.9
4	11.1	5,062	8.3	4,109	9,171	55.2
5	11.0	5,016	10.1	5,000	10,016	50.1
6	8.5	3,876	10.8	5,346	9,222	42.0
7	5.4	2,462	8.8	4,356	6,818	36.1
8	4.7	2,143	8.8	4,356	6,499	33.0
9	3.4	1,550	9.4	4,653	6,203	25.0
10 or more	9.2	4,196	34.4	17,027	21,223	19.8
Total		45,600		49,500	95,100	47.9%

Source: 1977 values from Table 32 in "Household Vehicle Utilization" published by the U.S. Department of Transportation.

EXHIBIT 11

ESTIMATED AVERAGE COSTS PER MILE FOR UNSCHEDULED REPAIRS AND MAINTENANCE

		Vehicle Size				
Year	Annual Miles	Large	Intermediate	Compact	Sub-Compact	Average
1	14,500	$0.0010	$0.0008	$0.0007	$0.0006	$0.0008
2	13,700	0.0045	0.0035	0.0033	0.0029	0.0036
3	12,500	0.0273	0.0291	0.0174	0.0255	0.0248
4	11,400	0.0318	0.0268	0.0198	0.0285	0.0267
5	10,300	0.1203	0.0871	0.0495	0.0664	0.0808
6	9,700	0.0722	0.0756	0.0632	0.1068	0.0795
7	9,200	0.1238	0.1197	0.1588	0.1401	0.1356
8	8,700	0.0751	0.0593	0.0645	0.0596	0.0646
9	8,200	0.0296	0.0292	0.0149	0.0242	0.0245
10	7,800	0.0023	0.0019	0.0013	0.0012	0.0017
11	7,300	0.0019	0.0015	0.0009	0.0008	0.0013
12	6,700	0.0021	0.0017	0.0010	0.0009	0.0014

Source: "Cost of Owning and Operating Automobiles and Vans, 1984," published by the U.S. Department of Transportation.

EXHIBIT 12

ESTIMATED LOSS EMERGENCE FOR ONE 5/100 CONTRACT BASED ON SIMULATION MODEL

Policy Age	Estimated Percentage Emerged
1	3.8%
2	20.2
3	49.3
4	78.3
5	100.0

EXHIBIT 13

ESTIMATED AGGREGATE POLICY YEAR PAYMENT PATTERN FOR 5/100 CONTRACT

			Policy Year Loss Emergence						
Development Year	Accident Year Loss Emergence	Calendar Year	Accident Year 1	Accident Year 2	Accident Year 3	Accident Year 4	Accident Year 5	Accident Year 6	Total
1	71.1%	1	1.4%						1.4%
2	96.6	2	0.5	7.2%					7.7
3	99.1	3	0.0	2.6	16.1%				18.7
4	100.0	4	0.0	0.3	5.8	20.6%			26.7
		5		0.1	0.6	7.4	18.0%		26.1
		6			0.2	0.7	6.5	7.7%	15.1
		7				0.3	0.6	2.8	3.7
		8					0.2	0.3	0.5
		9						0.1	0.1
Policy Year Loss Emergence			1.9%	10.1%	22.7%	29.1%	25.4%	10.8%	100.0%

EXHIBIT 14

ESTIMATED PERCENTAGE OF CONTRACTS HAVING EXPOSURE GREATER THAN INDICATED MILEAGE

	Age of Policy Year					
Mileage	1	2	3	4	5	6
0	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
6,000	37.24	82.17	90.46	92.41	93.13	93.49
12,000	24.79	63.08	79.67	84.23	85.94	86.71
18,000	14.26	45.54	68.09	75.35	78.11	79.47
24,000	7.50	32.07	56.66	66.60	70.56	72.39
30,000	3.97	22.40	45.76	57.96	63.20	65.70
36,000	2.16	15.36	35.95	49.68	56.06	59.19
42,000	1.26	10.49	27.70	41.85	49.24	52.97
48,000	0.83	7.03	20.86	34.70	42.83	47.14
54,000	0.56	4.67	15.55	28.34	36.90	41.70
60,000	0.38	3.14	11.47	22.85	31.42	36.59
66,000	0.26	2.14	8.36	18.14	26.56	31.89
72,000	0.19	1.47	6.04	14.21	22.23	27.66
78,000	0.13	1.02	4.33	11.02	18.40	23.79
84,000	0.09	0.73	3.16	8.45	14.99	20.39
90,000	0.08	0.54	2.29	6.45	12.22	17.30
96,000	0.06	0.40	1.71	4.92	9.75	14.50
100,000	0.05	0.31	1.40	4.14	8.43	12.93

NOTE: Estimates are derived from the Monte Carlo simulation model based on input assumptions from Exhibits 9 and 10.

EXHIBIT 15
Part 1

EXPECTED EXPOSURE BY POLICY AGE EXPECTED TO EXCEED INDICATED MILEAGE

WRITTEN CONTRACTS

Policy Year					
1992	1991	1990	1989	1988	1987
200,000	150,000	100,000	50,000	10,000	1,000

POTENTIAL CONTRACTS EXPOSED BY AGE

Policy	Age of Policy Year					
Year	1	2	3	4	5	6
1987	1,000	1,000	1,000	1,000	1,000	500
1988	10,000	10,000	10,000	10,000	5,000	
1989	50,000	50,000	50,000	25,000		
1990	100,000	100,000	50,000			
1991	150,000	75,000				
1992	100,000					
Total	411,000	236,000	111,000	36,000	6,000	500

EXHIBIT 15
Part 2

ESTIMATED CONTRACTS EXPOSED

	Age of Policy Year						
Mileage	1	2	3	4	5	6	Total
0	411,000	236,000	111,000	36,000	6,000	500	800,500
6,000	153,056	193,921	100,411	33,268	5,588	467	486,711
12,000	101,887	148,869	88,434	30,323	5,156	434	375,103
18,000	58,609	107,474	75,580	27,126	4,687	397	273,873
24,000	30,825	75,685	62,893	23,976	4,234	362	197,975
30,000	16,317	52,864	50,794	20,866	3,792	329	144,962
36,000	8,878	36,250	39,905	17,885	3,364	296	106,578
42,000	5,179	24,756	30,747	15,066	2,954	265	78,967
48,000	3,411	16,591	23,155	12,492	2,570	236	58,455
54,000	2,302	11,021	17,261	10,202	2,214	209	43,209
60,000	1,562	7,410	12,732	8,226	1,885	183	31,998
66,000	1,069	5,050	9,280	6,530	1,594	159	23,682
72,000	781	3,469	6,704	5,116	1,334	138	17,542
78,000	534	2,407	4,806	3,967	1,104	119	12,937
84,000	370	1,723	3,508	3,042	899	102	9,644
90,000	329	1,274	2,542	2,322	733	87	7,287
96,000	247	944	1,898	1,771	585	73	5,518
100,000	206	732	1,554	1,490	506	65	4,553

NOTE: Estimates of contracts exposed are based on the written contracts on the top portion of this exhibit and the percentages in Exhibit 14.

EXHIBIT 16

ESTIMATED COST PER MILE

Mileage at Time of Repair	(1) Total Costs	(2) Estimated Exposed Contracts	(3) Indicated Cost per Mile in Interval (1)/[(2)/5]
$0 < X \leq 6{,}000$	$10,000	800,500	0.0010¢
$6{,}000 < X \leq 12{,}000$	15,000	486,711	0.0026
$12{,}000 < X \leq 18{,}000$	50,000	375,103	0.0111
$18{,}000 < X \leq 24{,}000$	60,000	273,873	0.0183
$24{,}000 < X \leq 30{,}000$	75,000	197,975	0.0316
$30{,}000 < X \leq 36{,}000$	150,000	144,962	0.0862
$36{,}000 < X \leq 42{,}000$	160,000	106,570	0.1251
$42{,}000 < X \leq 48{,}000$	165,000	78,967	0.1741
$48{,}000 < X \leq 54{,}000$	150,000	58,455	0.2138
$54{,}000 < X \leq 60{,}000$	125,000	43,209	0.2411
$60{,}000 < X \leq 66{,}000$	100,000	31,998	0.2604
$66{,}000 < X \leq 72{,}000$	90,000	23,682	0.3167
$72{,}000 < X \leq 78{,}000$	75,000	17,542	0.3563
$78{,}000 < X \leq 84{,}000$	50,000	12,937	0.3221
$84{,}000 < X \leq 90{,}000$	30,000	9,644	0.2592
$90{,}000 < X \leq 96{,}000$	20,000	7,287	0.2287
$96{,}000 < X \leq 100{,}000$	10,000	5,518	0.2265

NOTES:
1. Amounts in Column (2) are from Exhibit 15.
2. The amounts in Column (1) here are hypothetical but could be determined from company loss experience by miles driven at time of repair.

APPENDIX

This appendix contains the proof that the mixing of the inverse Weibull and Gamma distributions produces an inverse Burr distribution. First suppose the variable x has an inverse Weibull distribution with parameters θ and τ with cumulative density function given by:

$$F(x|\theta) = e^{-\theta x^{-\tau}}.$$

This results in a probability density function given by:

$$f(x|\theta) = \theta\tau x^{-(\tau+1)}\, e^{-\theta x^{-\tau}}.$$

Suppose, further, that θ is itself unknown but has a Gamma distribution with the probability density function:

$$g(\theta) = \frac{\lambda^{\alpha}\,\theta^{\,\alpha-1} e^{-\lambda\theta}}{\Gamma(\alpha)}.$$

In this case the probability density function for the variable x is given by:

$$h(x) = \int_0^\infty f(x|\theta) g(\theta) d\theta$$

$$= \int_0^\infty \frac{\lambda^{\alpha}\,\theta^{\alpha-1}\, e^{-\lambda\theta}}{\Gamma(\alpha)}\, \theta\tau x^{-(\tau+1)} e^{-\theta x^{-\tau}}\, d\theta$$

$$= \frac{\lambda^{\alpha}\,\tau\, x^{-(\tau+1)}}{\Gamma(\alpha)} \int_0^\infty \theta^{\alpha}\, e^{-\lambda\theta - \theta x^{-\tau}}\, d\theta$$

$$= \frac{\lambda^{\alpha}\,\tau\,x^{-(\tau+1)}}{\Gamma(\alpha)}\int_0^{\infty}\theta^{\alpha}\,e^{-\theta(\lambda+x^{-\tau})}\,d\theta\,.$$

We now make the change of variables with $z = \theta\,(\lambda+x^{-\tau})$, so that

$$d\theta = \frac{1}{\lambda+x^{-\tau}}\,dz\,.$$

The equation now becomes:

$$h(x) = \frac{\tau\lambda^{\alpha}\,x^{-(\tau+1)}}{\Gamma(\alpha)(\lambda+x^{-\tau})}\int_0^{\infty}\left(\frac{z}{\lambda+x^{-\tau}}\right)^{\alpha} e^{-z}dz$$

$$= \frac{\tau\lambda^{\alpha}\,x^{-(\tau+1)}}{\Gamma(\alpha)(\lambda+x^{-\tau})^{\alpha+1}}\int_0^{\infty} z^{\alpha}\,e^{-z}dz$$

$$= \frac{\tau\lambda^{\alpha}\,x^{-(\tau+1)}}{\Gamma(\alpha)(\lambda+x^{-\tau})^{\alpha+1}}\,\Gamma(\alpha+1)$$

$$= \frac{\tau\lambda^{\alpha}\,x^{-(\tau+1)}}{\Gamma(\alpha)(\lambda+x^{-\tau})^{\alpha+1}}\,\alpha\Gamma(\alpha)$$

$$= \frac{\alpha\tau\lambda^{\alpha}\,x^{-(\tau+1)}}{(\lambda+x^{-\tau})^{\alpha+1}}$$

$$= \frac{\alpha\tau\lambda^{\alpha+1}\,x^{-(\alpha+1)\tau}\,x^{\alpha\tau-1}}{\lambda(\lambda+x^{-\tau})^{\alpha+1}}$$

$$= \frac{\alpha\tau x^{\alpha\tau-1}(\lambda x^{-\tau})^{\alpha+1}}{\lambda(\lambda+x^{-\tau})^{\alpha+1}}$$

$$= \frac{\alpha\tau x^{\alpha\tau-1}}{\lambda}\left(\frac{\lambda x^{-\tau}}{\lambda+x^{-\tau}}\right)^{\alpha+1}$$

$$= \frac{\alpha\tau x^{\alpha\tau-1}}{\lambda}\left(\frac{1}{\frac{\lambda}{\lambda x^{-\tau}}+\frac{x^{-\tau}}{\lambda x^{-\tau}}}\right)^{\alpha+1}$$

$$= \frac{\alpha\tau(1/\lambda)x^{\alpha\tau-1}}{((1/\lambda)+x^{\tau})^{\alpha+1}}.$$

This is the probability density function for an inverse Burr distribution with parameters α, $\lambda^{-1/\tau}$, and τ.