AGGREGATE RETROSPECTIVE PREMIUM RATIO AS A FUNCTION OF THE AGGREGATE INCURRED LOSS RATIO

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Abstract

The aggregate premium returned to a group of individual risks that are subject to retrospective rating depends upon the retrospective rating formula, the aggregate loss ratio of the risks, and the distribution of the individual risks' loss ratios around the aggregate. As the aggregate incurred loss ratio for a group of risks increases, the aggregate returned premium decreases, but not as rapidly as the loss ratio increases.

In this paper a simple equation is developed for the relationship between the aggregate incurred loss ratio and the aggregate retrospective return premium. The equation relies on the tabular charges and savings of Table M, thereby eliminating the need to perform Monte Carlo style simulations.

Using the relationship expressed in terms of Table M values, the response of several retrospective rating formulas to changes in the aggregate incurred loss ratio is determined.

ACKNOWLEDGMENTS

The author wishes to thank Nancy R. Treitel and William A. Bailey for reviewing earlier versions of this paper and for making several helpful suggestions regarding its content.

A balanced individual risk retrospective rating plan is one in which the aggregate premium retained for all risks is equal to the aggregate premium that would have been collected if all of the risks had been written on a guaranteed cost basis. While charging an amount equal to the guaranteed cost premium in the aggregate, the retrospectively determined individual risk premiums are allowed to vary (within limits) as a function of the individual risk's actual loss experience.

An attempt is made to anticipate and reflect all of the possible individual risk loss outcomes of the guaranteed cost rates. However, only those outcomes which produce retrospective premiums that lie between the specified minimum and maximum premiums enter the formula explicitly. Those loss outcomes that produce retrospective premiums less than the minimum premium have the same effect on the aggregate retrospective premium as those which yield the minimum premium exactly. Likewise, risks that produce retrospective formula premiums greater than the specified maximum contribute no more premium to the aggregate than those with losses that exactly produce the maximum premium.

The loss "capping" effect of the minimum and maximum retrospective premium constraints makes achieving a balance with the corresponding guaranteed cost rates a non-trivial exercise. The rather well known device by which a balance can be achieved is the insurance charge. The insurance charge is used to modify the retrospective premium formula in such a way that the aggregate retrospective and guaranteed cost premiums become equal. The mechanics of how one determines the appropriate insurance charge can be found in John Stafford's monograph [7] as well as the Retrospective Rating Plan Manuals for both the National Council on Compensation Insurance (NCCI) [4] and the Insurance Services Office, Inc. [3]. More theoretical treatments can be found in several monographs and papers (see [2], [5], and [6], for example). Both the guaranteed cost (GC) rates and the individual risk retrospective rating (IRRR) formula (together with the specified minimum and maximum premiums) are established prospectively. Only the individual risk premiums are determined retrospectively. In order to determine the GC rates and the insurance charge component in the IRRR formula, one must forecast the incurred loss ratio (*ILR*) for the aggregation of all policies to be written under these rates. A look at recent rate filings for workers' compensation shows that, in many jurisdictions, the *ILR* anticipated in the original filing was quite different from the *ILR* actually experienced.

Guaranteed cost rates offer no immunity to the insurer from the effects of missing the "target" loss ratio. Retrospective rating, on the other hand, does possess the ability to offset some of these effects by increasing or decreasing the aggregate retrospectively determined premiums in response to the error in the estimated *ILR*. The maximum and minimum premium constraints, however, place limits upon the degree to which a retrospective rating plan can respond to changes in the aggregate *ILR*.

It is desirable to have a quantitative measure to prospectively determine the degree to which a particular retrospective rating plan will respond to differences between the underlying expected aggregate loss ratio and the actual aggregate loss ratio. Section 1 provides a theoretical treatment of the problem. It concludes with the derivation of a formula expressing the ratio (*RP*) of aggregate retrospectively determined return premium to standard premium as a function of the aggregate *ILR* for all retro-rated policies. Rather than being explicitly dependent on a distribution about the mean of individual policy *ILR*s, the functional relationship is expressed in terms of Table M charges and savings. Appendix A displays a tractable, albeit unrealistic, numerical illustration of the theory that is introduced in Section 1.

In Section 2, the results of Section 1 together with Table M insurance charges are used to obtain sets of ordered pairs of aggregate *RP*s and *ILR*s for a set of risks that will be subject to retrospective rating. Using this set of ordered pairs, the sensitivity of a retrospective rating plan's *RP* to changing *ILR*s is examined. In particular, the influence of four factors (the individual risk *ILR* distribution, the plan loss conversion factor (*LCF*), the plan minimum premium ratio, and the plan maximum premium ratio) is discussed. Section 2 continues with some remarks about curve fitting. Appendix B provides the details of one of the simulations that is presented in Section 2.

Section 3 concludes by summarizing the results of Section 2 and suggesting practical applications of the theory to the evaluation of residual market retro plans. The establishment of a retrospective unearned premium liability is also briefly discussed.

1. THE FUNCTIONAL RELATIONSHIP BETWEEN *ILR* AND *RP*

In this section a functional relationship is derived for the aggregate *ILR* for a group of individual risks defined by a particular loss ratio density function, $f(s)$, and the resulting aggregate retrospective premium returned ratio *RP*. The distribution, f, will be defined by two moments, the familiar charge, $X(r)$, and savings, $Y(r)$, of Table M or Table L. (See [5] and [6].)

For simplicity, assume that the retrospective rating plan does not involve any per claim (or occurrence) loss limit, nor does it incorporate retrospective development factors (either of these could be handled within the theoretical framework that follows, but neither would add to the exposition). A retrospective rating plan consists of a retrospective rating *formula*

$$R = [e*S + c*I*S + c*L]*TM\,, \tag{1.1}$$

subject to the limiting *constraint* that

$$H*S \leq R \leq G*S, \tag{1.2}$$

where:

R	is the retrospectively determined premium;
S	is the standard premium for the risk;
e	is the ratio of non-loss-based expenses to S;
c	is the loss conversion factor (*LCF*) which consists of unity plus a provision for any loss-based expenses;
I	is the net insurance charge as a ratio to S;
TM	is the tax multiplier, $TM = 1/(1-\text{tax rate})$;

L is the actual incurred loss for the policy;

$H*S$ is the agreed upon minimum retrospective premium; and

$G*S$ is the agreed upon maximum retrospective premium.

Dividing both sides of Equation 1.1 by S gives

$$R/S = [e + c*I + c*ILR]*TM, \tag{1.3}$$

where

$$ILR = L/S \tag{1.4}$$

is the incurred loss ratio for the policy. Even if we limit our discussion to a priori identical policies that have the same individual risk expected loss amount $<L>$ and expected loss ratio $<ILR>$ (where the brackets, <...>, denote the expected value of the variable that they enclose), the individual risk loss ratios, ILR, can be expected to differ from the expected one.

Following the notational conventions that are used with the Table M of insurance charges, we define an individual policy entry ratio, s, as follows

$$s = L/<L> = ILR/<ILR>. \tag{1.5}$$

Assume that the probability density, f, is such that $f(s)ds$ gives the probability of finding an individual risk with an entry ratio between s and $s + ds$. The function f can, and in Table M does, vary with $<L>$. We note that $f(s)$ need not correspond to any published Table M. "Table M" is used in a generic sense to describe a set of ILR distributions and the charges and savings that are implied by them.

Given $f(s)$, we define two functions,

$$X(r) = \int_r^\infty (s - r) f(s) ds, \tag{1.6}$$

and

$$Y(r) = \int_0^r (r - s) f(s) ds, \tag{1.7}$$

which are the familiar charge and savings, respectively, of Table M or Table L (depending upon the particular density f and the definition of ILR).

Returning to Equations 1.2 and 1.3 we find that the minimum entry ratio, $r_{\min}$, is given by

$$r_{\min} = [H/TM - e - c*I\,]/(c*\langle ILR\rangle), \tag{1.8}$$

and the maximum entry ratio, $r_{\max}$, is given by

$$r_{\max} = [G/TM - e - c*I\,]/(c*\langle ILR\rangle)\,. \tag{1.9}$$

If we define a *capped* incurred loss ratio, ilr, as follows:

$$ilr = \langle ILR\rangle * \begin{cases} r_{\min} & \text{for } s < r_{\min} \\ s & \text{for } r_{\min} \le s \le r_{\max} \\ r_{\max} & \text{for } r_{\max} < s \end{cases} \tag{1.10}$$

then Equation 1.3 can be recast into a form that does not require the explicit constraint condition, namely:

$$R/S = [e + c*I + c*ilr]*TM. \tag{1.11}$$

The average ratio of the retrospective premium to the standard premium, over all policies described by f, is given by

$$\langle R/S\rangle = [e + c*I + c*\langle ilr\rangle]*TM, \tag{1.12}$$

where

$$\langle ilr \rangle = r_{\min} \int_{0}^{r_{\min}} f(s)ds + \int_{r_{\min}}^{r_{\max}} sf(s)ds + r_{\max} \int_{r_{\max}}^{\infty} f(s)ds. \qquad (1.13)$$

Equation 1.13 can be recast into the following form:

$$\langle ilr \rangle = \langle ILR \rangle * [1 + Y(r_{\min}) - X(r_{\max})], \qquad (1.14)$$

in which X and Y appear, implicitly representing all of the necessary details contained in $f(s)$.

The interpretation of Equation 1.14 is that the expected capped loss ratio, the one "seen by" the retrospective rating plan, differs from the uncapped expected value by the addition of some losses from risks with formula premiums below the minimum, $\langle ILR \rangle * Y(r_{\min})$, and by the removal of some losses for risks that produce formula premiums above the maximum premium, $\langle ILR \rangle * X(r_{\max})$.

If we require that

$$I + \langle ilr \rangle = \langle ILR \rangle, \qquad (1.15)$$

then $\langle R/S \rangle$ will, indeed, balance to the guaranteed cost premium. The determination of the insurance charge, I, is not as trivial as it appears. Solving Equation 1.15 for I gives

$$I = \langle ILR \rangle - \langle ilr \rangle, \qquad (1.16)$$

but $\langle ilr \rangle$, itself, depends upon I because the $r_{\min}$ and $r_{\max}$ depend on I. For the purpose of this paper, we can assume that a solution has been obtained, although nothing in what follows depends on a balance being achieved. The interested reader can refer to any of references [2] - [7] to see how the trial and error procedure to determine I, given a table of $X(r)$ and $Y(r)$, is usually performed. Even if we do not impose the requirement that R be balanced to the guaranteed cost premium, Equation 1.14 can be substituted into Equation 1.12 to determine the ordered pair ($\langle ILR \rangle$, RP).

Regardless of whether or not $<R/S>$ balances to the guaranteed cost premium, it can be assumed that increasing values of $<ILR>$ for *a fixed insurance charge* will produce increasing values of $<ilr>$ and hence $<R/S>$. The question is: By how much will $<ilr>$ increase when $<ILR>$ increases? That depends, of course, on the percentage of risks that are either no longer subject to the minimum constraint or are now subject to the maximum constraint. A change in the aggregate *ILR* after the retrospective plan has been established will have no effect on the minimum and maximum loss ratios to be seen by the plan. Only the corresponding entry ratios, r_{max} and r_{min}, will differ from those originally anticipated.

If $<ILR>_1$ is the actual aggregate loss ratio for a portfolio of risks, and

$$<ILR>_1 = g*<ILR>_0 , \tag{1.17}$$

where the constant, g, is defined by Equation 1.17, then, in terms of the actual distribution f_1,

$$r_{min1} = r_{min0}/g \text{ , and} \tag{1.18}$$

$$r_{max1} = r_{max0}/g. \tag{1.19}$$

We have adopted the indicator 0 for the originally assumed distribution parameters and density and 1 for the actual distribution parameters and density. The parameters indicated with zeroes may alternatively be thought of as being based upon a priori estimates.

As in Equation 1.13, the actual average loss ratio seen by the retrospective rating plan, given the actual density f_1 and the actual mean $<ILR>_1$, will be:

$$<ilr> = g*<ILR>_0*\left[(r_{min}/g)*\int_0^{r_{min}/g} sf_1(s)ds \right.$$

$$+ \int_{r_{\min}/g}^{r_{\max}/g} sf_1(s)ds + (r_{\max}/g) * \int_{r_{\max}/g}^{\infty} f_1(s)ds \Bigg]$$

$$= g*<ILR>_0*[1 + Y_1(r_{\min}/g) - X_1(r_{\max}/g)]. \quad (1.20)$$

Upon subtracting Equation 1.20 from 1.14, we find that the difference between the actual and estimated aggregate loss ratio as seen by the retrospective rating plan is given by

$$\Delta<ilr> = <ILR>_0 - <ILR>_0*[g*X_1(r_{\max}/g) - X_0(r_{\max})]$$

$$+ <ILR>_0*[g*Y_1(r_{\min}/g) - Y_0(r_{\min})]. \quad (1.21)$$

The corresponding change in the retrospective premium ratio is found by substituting Equation 1.21 into 1.12 as follows:

$$\Delta<R/S> = c*\Delta<ILR>_0*TM$$

$$- c*<ILR>_0*[g*X_1(r_{\max}/g) - X_0(r_{\max})]*TM$$

$$+ c*<ILR>_0*g*Y_1(r_{\min}/g) - Y_0(r_{\min})]*TM. \quad (1.22)$$

Because most retrospective rating plans are designed to return premium to policyholders in the aggregate, we shall refer to return premium ratios (RP), given by

$$RP = 1 - R/S \quad (1.23)$$

and

$$\Delta RP = -c\Delta<ILR>_0*TM$$

$$+ c*<ILR>_0*[g*X_1(r_{\max}/g) - X_0(r_{\max})]*TM$$

$$-c*<ILR>_0*[g*Y_1(r_{\min}/g) - Y_0(r_{\min})]*TM. \quad (1.24)$$

Appendix A provides a detailed analysis of the significance of the three terms that appear in Equation 1.24.

2. SENSITIVITY TESTING BY MEANS OF SIMULATION

By means of the simulation model, Equation 1.24, we may test the sensitivity of a particular retro plan to changes in any of its parameters. We shall begin by considering a set of risks with an expected loss ratio of 60 percent of standard premium. Furthermore, we shall assume that the individual risk loss ratios have the same distribution as expected loss group 60 of the NCCI Table M. Using the July 1, 1991 expected loss groupings, this expected loss group corresponds to a set of risks with approximately $53,000 of standard premium. If the expense provision in the rates (excluding taxes but including profit) is 26 percent of standard premium (about midway between the NCCI Table XIV stock and non-stock expense provision for this policy size), and taxes are 4.4 percent of collected premium (i.e., the taxes which lead to a 1.046 tax multiplier), then 10 percent of the standard premium will be available for retrospective premium returns. Using the standard Table M algorithms, and the information given above, the insurance charge for any c, G, and H can be determined. Once those items have been specified, the retrospective rating formula and constraints will be known. Equation 1.24 can be used to generate a set of simulation points for the particular plan. (See Appendix B for the details of one such simulation.)

To quantify the sensitivity of a retro plan to changes in any parameter, we must have a measure of the retro plan's response to changing $\langle ILR \rangle$s. As described in Appendix B, three curves were fit to each simulation: linear, geometric, and exponential. Even when the geometric or exponential model produced a better fit to the data, the linear model for the RP as a function of the $\langle ILR \rangle$ was a close runner-up (as measured by the mean squared error), and the linear model frequently performed best. Because of its simplicity, the slope of the linear curve has been selected as the best measure of a retro plan's response to changes in the $\langle ILR \rangle$. A slope of -.25, for example, means that an eight point increase in the aggregate *ILR* over and above the expected *ILR* results in a two point decrease in the *RP*.

The remarkable feature of our simulations using Table M loss group 60 is how small the slopes are. In other words, large changes in the <*ILR*> do not have a large impact on the *RP* for any of the retrospective rating formulas that were tested.

Table 1 provides us with a summary of the sensitivity of the generic formula,

$$R/S = [.260 + 1.000*I(G) + 1.000*ILR]*1.046, \tag{2.1a}$$

subject to

$$.70 \leq R/S \leq G, \tag{2.1b}$$

where H, the ratio of minimum premium to standard premium, has been set equal to .70.

TABLE 1

RESPONSE (SLOPE) AS A FUNCTION OF THE MAXIMUM PREMIUM RATIO, *G*
[For Table M Expected Loss Group 60 and 60 Percent Expected Loss Ratio]

			MSE (x 1,000,000)			
LCF	*G*	*H*	Linear	Geometric	Exponential	Slope
1.000	1.15	0.70	6	4	1	-0.10
1.000	1.20	0.70	6	14	2	-0.12
1.000	1.25	0.70	9	24	4	-0.13
1.000	1.30	0.70	10	36	8	-0.14
1.000	1.35	0.70	5	94	36	-0.16
1.000	1.40	0.70	8	111	43	-0.17
1.000	1.45	0.70	9	177	80	-0.18

By varying the maximum premium, we are able to test the formula's sensitivity to changes in G, the maximum premium. Here, $I(G)$ is the net insurance charge that places R/S into balance with the corresponding guaranteed cost rates for an <*ILR*> of 60 percent. Intuitively, we expect that as G increases, fewer risks will "max" out, and Equation 2.1a should reflect a greater portion of the actual losses. As expected, the slope does become larger (in absolute value) as the maximum premium increases from 1.15 to 1.45 times the standard premium.

Our intuitive notion concerning the shape of the best model is also confirmed. While both the linear and the exponential model fit the first four Gs well, the exponential model with its negative first *and* second derivatives fits better. The negative second derivatives are indicative of the law of diminishing returns, which is consistent with the upper bound on R/S. For maximum premiums above 130 percent of standard premium, the capping effect is less noticeable, and the linear model produces a better fit. It is interesting to note that, while the slope as a function of G has the expected monotonic behavior, even with G equal to 145 percent of standard premium, the slope is a modest 18 percent of the change in <*ILR*>. In the limit as G goes to infinity, one would expect the slope to approach -1.00 (i.e., equal to the *LCF*). Obviously 1.45 is not anywhere near being effectively infinite for this group of risks with its rather widely spread out $f(s)$.

In Table 2 we freeze G at 135 percent of standard premium and attempt to achieve greater response by varying the *LCF*. At the individual risk level, a greater *LCF* will make the formula more responsive to changes in the *ILR*, unless the risk is pinned to the minimum or maximum premium. Even with a 1.35 maximum, the aggregate response for *LCF*s between .700 and 1.200 is essentially flat at 16 percent. While the choice of the *LCF* is a significant factor as far as individual rate equity is concerned, it has almost no effect in providing a cushion for the carrier against missing the aggregate <*ILR*> target!

TABLE 2

RESPONSE (SLOPE) AS A FUNCTION OF THE LOSS CONVERSION FACTOR, *LCF*
[For Table M Expected Loss Group 60 and 60 Percent Expected Loss Ratio]

			MSE (x 1,000,000)			
LCF	*G*	*H*	Linear	Geometric	Exponential	Slope
0.700	1.35	0.70	6	46	12	-0.15
0.800	1.35	0.70	8	52	14	-0.15
0.900	1.35	0.70	8	56	16	-0.15
1.000	1.35	0.70	5	94	36	-0.16
1.100	1.35	0.70	7	75	25	-0.16
1.150	1.35	0.70	10	71	23	-0.16
1.200	1.35	0.70	10	79	26	-0.16

Table 3 is identical to Table 2 except that there is no stated minimum for the retro plan. No stated minimum implies a minimum that is equal to the basic premium, $B = e + c*I$, times the tax multiplier, TM. While a $B \times TM$ plan is slightly more responsive for large *LCF*s than the corresponding $H = .70$ plan, the slope as a function of *LCF* is still very flat. The additional responsiveness for the $B \times TM$ plans can be attributed to changes in R for risks which have an R that falls below 70 percent of the standard premium (and would have been pinned to the minimum for those loss ratios under the $H = .70$ plan).

Table 4 is a larger version of Table 1, but for $B \times TM$ plans. The response has been determined for various maximum premium factors, G. Plans with $G < 1.15$ are possible for $H = B \times TM$ but because their $B \times TM$ is greater than .70, they could not be considered in Table 1. A quick comparison of the slopes for the plans that are common to Tables 1 and 4 shows that there is no significant difference in response between a "no stated minimum" and a ".70 minimum" plan for NCCI loss group 60.

TABLE 3

RESPONSE (SLOPE) AS A FUNCTION OF THE LOSS CONVERSION FACTOR, *LCF*, WITH NO SPECIFIED MINIMUM PREMIUM RATIO, *H* [For Table M Expected Loss Group 60 and 60 Percent Expected Loss Ratio]

			MSE (x 1,000,000)			
LCF	*G*	*H*	Linear	Geometric	Exponential	Slope
0.500	1.35	$B \times TM$	5	24	4	-0.13
0.600	1.35	$B \times TM$	6	32	7	-0.14
0.700	1.35	$B \times TM$	8	38	8	-0.15
0.800	1.35	$B \times TM$	6	58	17	-0.15
0.900	1.35	$B \times TM$	9	67	20	-0.16
1.000	1.35	$B \times TM$	10	79	24	-0.16
1.100	1.35	$B \times TM$	8	125	49	-0.17
1.150	1.35	$B \times TM$	10	119	46	-0.17
1.200	1.35	$B \times TM$	11	115	42	-0.17

TABLE 4

RESPONSE (SLOPE) AS A FUNCTION OF THE MAXIMUM PREMIUM RATIO, *G* WITH NO SPECIFIED MINIMUM PREMIUM RATIO, *H* [For Table M Expected Loss Group 60 and 60 Percent Expected Loss Ratio]

			MSE (x 1,000,000)			
LCF	*G*	*H*	Linear	Geometric	Exponential	Slope
1.000	0.95	$B \times TM$	2	2	2	-0.02
1.000	1.00	$B \times TM$	1	1	1	-0.05
1.000	1.05	$B \times TM$	4	1	2	-0.06
1.000	1.10	$B \times TM$	6	2	2	-0.08
1.000	1.15	$B \times TM$	4	7	1	-0.11
1.000	1.20	$B \times TM$	9	9	1	-0.12
1.000	1.25	$B \times TM$	7	22	3	-0.13
1.000	1.30	$B \times TM$	8	45	11	-0.15
1.000	1.35	$B \times TM$	10	79	24	-0.16

The more compact the incurred loss ratio distribution is, that is, the smaller its variance is, the more responsive a retrospective rating formula should be for a given value of G. That is because a smaller percentage of its risks should have loss ratios that pin R to G. As the Table M expected loss group numbers decrease, the underlying distributions become more compact.

Table 5 tests the responsiveness of a $H = B \times TM$, $LCF = 1.000$, $G = 1.35$ retro plan for different Table M expected loss groups (i.e., different premium sizes). Our intuitive notion is supported by the resulting slopes. Again, the striking feature of the slopes as a function of expected loss group is how small (in absolute value) they are. Even for group 40, only 37 percent of the change in *ILR* translates into a change in *RP*.

TABLE 5

RESPONSE (SLOPE) AS A FUNCTION OF EXPECTED LOSS GROUP, WITH ALL OTHER PARAMETERS HELD CONSTANT
[Expected Loss Ratio Equals 60 percent]

			Loss	Standard		MSE (x 1,000,000)		
LCF	*G*	*H*	Group	Premium	Slope	Linear	Geometric	Exponential
1.000	1.35	$B \times TM$	70	24,000	-0.12	4	9	0
1.000	1.35	$B \times TM$	60	53,000	-0.17	10	28	3
1.000	1.35	$B \times TM$	50	113,000	-0.25	26	233	80
1.000	1.35	$B \times TM$	40	240,000	-0.37	44	1,344	792
1.000	1.35	$B \times TM$	30	860,000	-0.57	21	890	552
1.000	1.35	$B \times TM$	20	4,832,000	-0.78	8	682	430
1.000	1.35	$B \times TM$	15	14,773,000	-0.85	21	3,032	2,379
1.000	1.35	$B \times TM$	10	95,486,000	-0.94	3	753	512

Finally, we investigate the relationship between the shape of the distribution as characterized by $f(s)$ and the *RP* for a fixed premium size and constant <*ILR*>. Table 6 shows, for example, that if a set of risks were initially priced as if they had the loss ratio distribution corresponding to Table M's group 60, but they turned out to actually have the distribution of group 73 (i.e., $53,000 accounts turn out behaving like $18,000 accounts), the returned premiums would be 6.8 percent of standard premium more than originally intended. The expected *RP* for group 60 is 10 percent; whereas, one should have expected 16.84 percent. Table 6 makes use of Equation 2.24 when f_0 is not equal to f_1.

TABLE 6

RETURNED PREMIUM (*RP*) AS A FUNCTION OF THE AGGREGATE LOSS DISTRIBUTION AS IDENTIFIED BY THE TABLE M GROUP NUMBER [Insurance Charge Based Upon Group 60 For All Cases]

Group	*LCF*	*G*	*H*	*ILR*	*RP*
57	1.000	1.35	$B \times TM$	60.0%	8.43%
58	1.000	1.35	$B \times TM$	60.0	8.93
59	1.000	1.35	$B \times TM$	60.0	9.50
60	1.000	1.35	$B \times TM$	60.0	10.00
61	1.000	1.35	$B \times TM$	60.0	10.56
62	1.000	1.35	$B \times TM$	60.0	11.13
63	1.000	1.35	$B \times TM$	60.0	11.63
64	1.000	1.35	$B \times TM$	60.0	12.13
65	1.000	1.35	$B \times TM$	60.0	12.64
66	1.000	1.35	$B \times TM$	60.0	13.20
67	1.000	1.35	$B \times TM$	60.0	13.64
68	1.000	1.35	$B \times TM$	60.0	14.08
69	1.000	1.35	$B \times TM$	60.0	14.64
70	1.000	1.35	$B \times TM$	60.0	15.24
71	1.000	1.35	$B \times TM$	60.0	15.77
72	1.000	1.35	$B \times TM$	60.0	16.34
73	1.000	1.35	$B \times TM$	60.0	16.84

3. CONCLUDING REMARKS

While individual risk retrospective rating plans can be very responsive to individual risk experience for risks of any size, the responsiveness of the aggregate returned premium ($RP*S$) to changes in the aggregate loss ratio ($<ILR>$) for a portfolio of risks is rather weak for all but the so called "jumbo" accounts.

Risks typically written under the NCCI retrospective rating plans have standard premiums less than $1,000,000. Even with a plan maximum as high as 135 percent of the standard premium, the response (slope) ranged from a low of -.12 for risks that were near the lower limit of retrospective rating eligibility to a high of -.57 for risks that have almost $1,000,000 of standard premium.

The NCCI plans are significantly more responsive for the larger risks. Many jurisdictions now permit rating these "jumbo" accounts using retrospective rating formulas that do not strictly adhere to the NCCI parameters. In particular, the NCCI tabular expense provisions and NCCI expected loss ratios need not be used. Obviously, if one is free to select an extremely high maximum and free to load all of the expenses via a loss conversion factor, then even greater response could be expected.

This freedom is not available for the smaller accounts. Because of the ability to achieve very high responsiveness for "jumbo" accounts, and their ability to dominate any empirical study of industry-wide responsiveness, we developed our simulation for the evaluation of responsiveness for portfolios that consist of smaller policies. The discussion that follows is, therefore, confined to portfolios consisting of small and medium size policies.

For these risks, an unanticipated rate deficiency (such as one mandated by a regulator), or a uniform increase in all loss ratios for some other reason, can be expected to change the <*ILR*> without changing the distribution, *f(s)*, of *ILR*s around the *ILR*. When this occurs, only a small fraction of the loss ratio increase is reflected by a lower aggregate *RP*. As a result of this, mandatory retros for the residual market, while restoring some equity between risks, cannot be expected to compensate for uniform deterioration in the loss experience. Even retros with a high maximum (e.g., 150 percent of standard premium) provide little in the way of a safety valve.

If, in addition to missing the target <*ILR*>, the effects of inflation on Table M expected loss groups are not adequately reflected, additional *RP* will be generated, thereby increasing the "bottom line" loss. This was illustrated in Table 6.

If one could quantify the relationship between the reported <*ILR*> and its *f(s)* as losses mature (i.e., how the *ILR* distribution the retro parameters). If the *<ILR>* could be developed to its ultimate value, and the resulting $f(s)$ was known, then the ultimate *RP* could be estimated. This would be an enhancement to the method presented in Berry's 1980 paper [1]. The research that would be necessary to determine this functional relationship, $f(s)_{<ILR>,\, maturity}$, is well beyond the scope of this paper, but presents us with a challenge for further research.

Another application of Equation 1.24 is in the establishment of safety margins in retrospective rating. If the distribution of possible *<ILR>*s about the mean *<ILR>* is known (this is not the same distribution as f, which involves *ILR*s about *<ILR>*), then the expected profit could be calculated for a particular retro formula. For each *<ILR>* there would be an *RP* and these *RP*s could be averaged using the *<ILR>* distribution. By varying the insurance charge, I, the probability of achieving a profit of less than some number, α, could be reduced below some selected value, β. The details of this investigation are the subject of a future paper that relates the expected return on equity for a portfolio of retrospectively rated risks to this modified insurance charge.

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[7] Stafford, John R., *Retrospective Rating*, J&M Publications, Palatine, Illinois, 1981.

APPENDIX A

Section 1 presented a theoretical relationship between changes in the aggregate incurred loss ratio for a group of individual risks and the corresponding change in the retrospectively determined return premium (additional premium is, simply, negative return premium). This appendix presents a qualitative graphical interpretation of Equation 1.24 as well as a numerical example.

Figure A-1 presents the expected distribution of loss ratios (to standard premium) for a set of 400 a priori identical risks that are to be rated retrospectively. Without providing details concerning the retrospective rating formula, we assume that the plan minimum causes all risks with loss ratios that are less than 30 percent (all risks to the left of the *min retro prem* line on the graph) to pay the minimum retrospective premium. All risks with loss ratios that are greater than 100 percent (risks that lie to the right of the *max retro prem* line on the graph) pay the maximum retrospective premium. Risks with loss ratios between 30 percent and 100 percent are charged a retrospective premium that depends on their respective losses. The three terms of Equation 1.24 deal with the three regions of the graph.

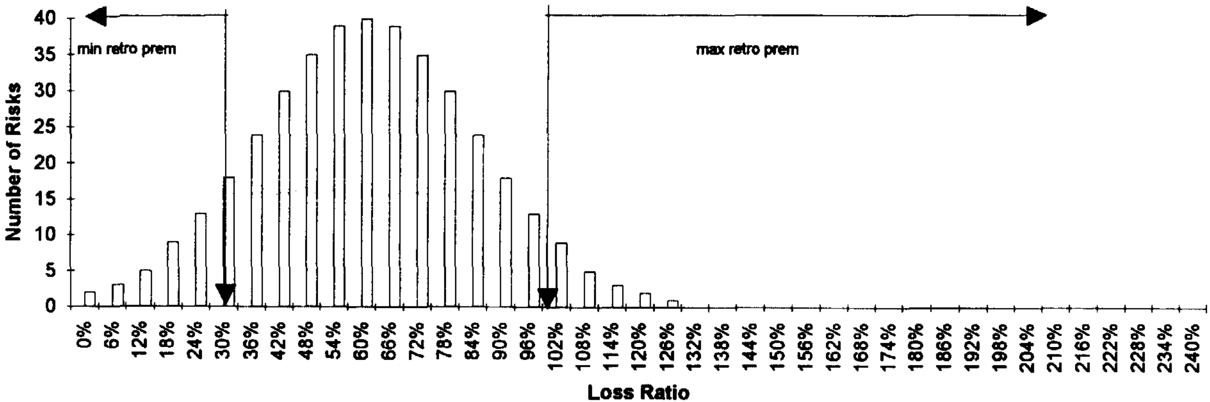
If every loss is 50 percent greater than expected (i.e., $g = 1.5$), then every risk in the first graph is shifted to the right, as shown in Figure A-2. For a given retrospective rating formula, the minimum retro premium and maximum retro premium lines remain unchanged by the difference between the expected and actual distribution.

The first term of Equation 1.24 assumes that every additional dollar of loss will result in a reduction in the aggregate premium that is returned. In particular, each additional dollar of loss is multiplied by the loss conversion factor, c, and the tax multiplier, TM, to determine the reduction in returned premium. The first term reflects the linear responsiveness of the retrospective rating formula. If every risk were to lie between the minimum and maximum lines, then the first term would accurately describe the entire situation.

FIGURE A-1

THE EXPECTED DISTRIBUTION

DISTRIBUTION OF RISKS BY LOSS RATIO



Risks with loss ratios that are less than the minimum pay the retrospective premium that corresponds to the minimum, regardless of the actual loss ratio.

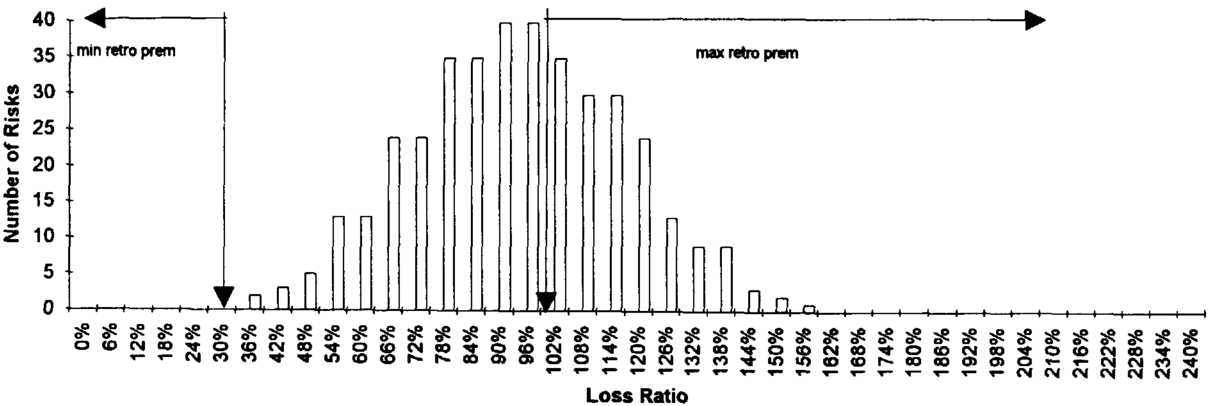
Risks with loss ratios between the minimum and maximum pay the retrospective premium indicated by the retrospective rating formula.

Risks with loss ratios that are greater than the maximum pay the retrospective premium that corresponds to the maximum, regardless of the actual loss ratio.

FIGURE A-2

THE CORRESPONDING DISTRIBUTION WITH $g = 1.5$

DISTRIBUTION OF RISKS BY LOSS RATIO



Risks with loss ratios that are less than the minimum pay the retrospective premium that corresponds to the minimum, regardless of the actual loss ratio.
Risks with loss ratios between the minimum and maximum pay the retrospective premium indicated by the retrospective rating formula.
Risks with loss ratios that are greater than the maximum pay the retrospective premium that corresponds to the maximum, regardless of the actual loss ratio.

Those risks that lie to the right of the maximum premium line would produce no change in the aggregate retrospective premium. The second term in Equation 1.24 deals with those risks that were expected to lie between the two extremes but which actually lie to the right of the maximum line. The losses are g times as large as expected (which explains the factor of g). The old maximum, $r_{\max}$, was expressed in terms of the expected loss ratio. In terms of the actual aggregate loss ratio, it is only $1/g$ as large. In other words, the new situation is the same as if the old distribution had been realized, but the maximum retrospective premium had been shifted to the left (which explains the argument of X_1). The net effect of the additional risks in the right hand tail is to mitigate the decrease in the aggregate returned retrospective premium.

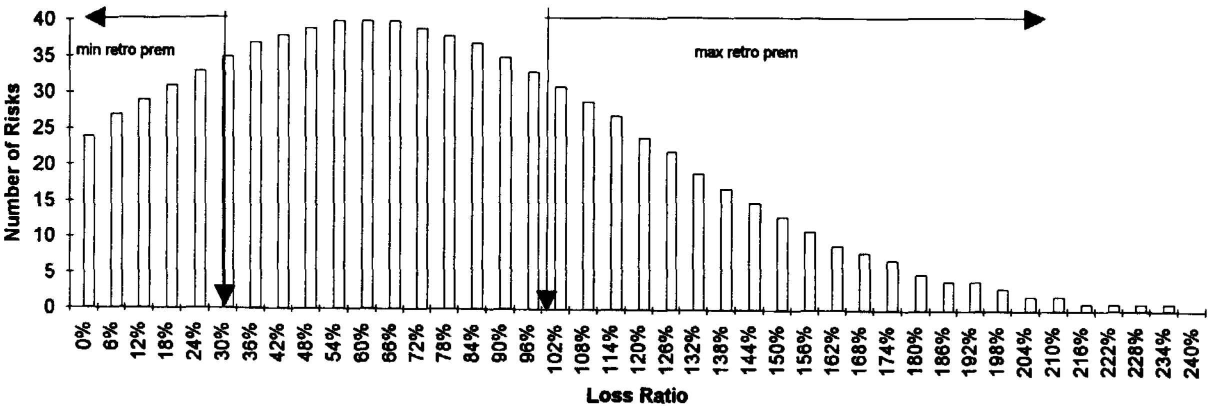
An offsetting effect occurs at the left side of the distribution. Here, some of the risks that were expected to pay the minimum retrospective premium now cross the line and become loss sensitive. The third term in Equation 1.24 represents the correction for the additional premium (reduction in the aggregate returned premium) resulting from those risks that cross the minimum line.

For any particular g, the magnitude of the two correction terms depends on the shape of the loss ratio distribution and relative location of the minimum and maximum premium lines. If we assume that the distribution in Figures A-1 and A-2 is typical of a large account, then the distribution shown in Figures A-3 and A-4 could represent a smaller account with its higher expected variance. (Smaller accounts can be expected to have higher probabilities for extreme loss ratios.) As with Figure A-1, the mean loss ratio of Figure A-3 is 60 percent. The same 50 percent increase in losses (Figure A-4) pins a much larger percentage of the individual risks to the maximum premium, so the retrospective rating formula is less responsive to the shift. Jumbo accounts, on the other hand, would be expected to have very compact loss ratio distributions. (Their loss ratios do not vary much from year to year.) With a fairly high maximum premium, one would expect most of the risks to remain between the two extremes, which would cause the first term in Equation 1.24 to dominate.

FIGURE A-3

THE EXPECTED DISTRIBUTION

DISTRIBUTION OF RISKS BY LOSS RATIO

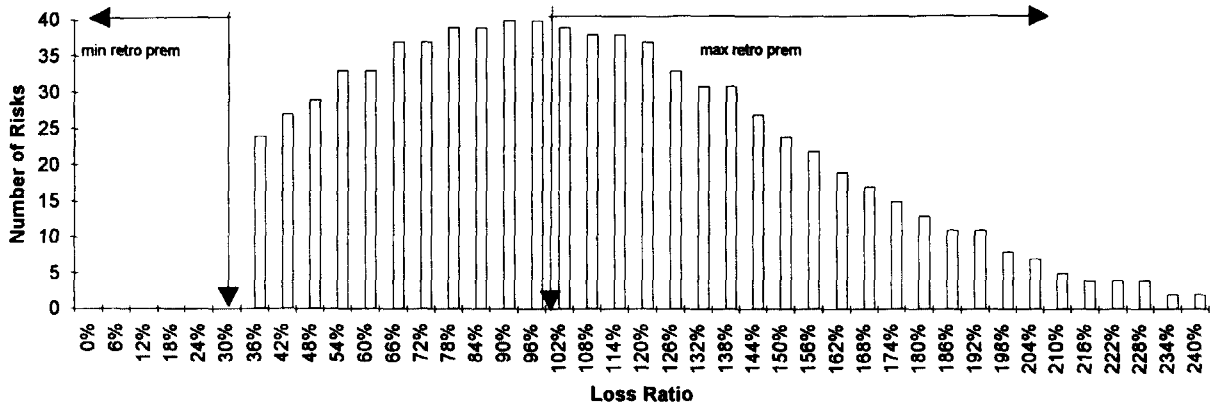


Risks with loss ratios that are less than the minimum pay the retrospective premium that corresponds to the minimum, regardless of the actual loss ratio.
Risks with loss ratios between the minimum and maximum pay the retrospective premium indicated by the retrospective rating formula.
Risks with loss ratios that are greater than the maximum pay the retrospective premium that corresponds to the maximum, regardless of the actual loss ratio.

FIGURE A-4

THE CORRESPONDING DISTRIBUTION WITH $g = 1.5$

DISTRIBUTION OF RISKS BY LOSS RATIO



Risks with loss ratios that are less than the minimum pay the retrospective premium that corresponds to the minimum, regardless of the actual loss ratio.

Risks with loss ratios between the minimum and maximum pay the retrospective premium indicated by the retrospective rating formula.

Risks with loss ratios that are greater than the maximum pay the retrospective premium that corresponds to the maximum, regardless of the actual loss ratio.

To provide a numerical example of Equation 1.24, consider the rather flat (hypothetical) distribution of loss ratios displayed in Figure A-5 and Figure A-6. The distribution appears to change shape only because we have grouped the loss ratios into bins that are five percent wide, and the 30 percent increase causes some of the groupings to change. The essential features are identical with those of the previous four graphs. A significant feature is the large spike that is expected to lie between the two extremes, but which actually lies to the right of the maximum premium line.

FIGURE A-5

THE EXPECTED DISTRIBUTION

DISTRIBUTION OF RISKS BY LOSS RATIO

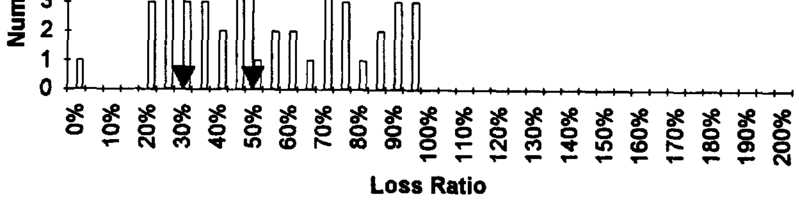


FIGURE A-6

THE CORRESPONDING DISTRIBUTION WITH $g = 1.3$

DISTRIBUTION OF RISKS BY LOSS RATIO

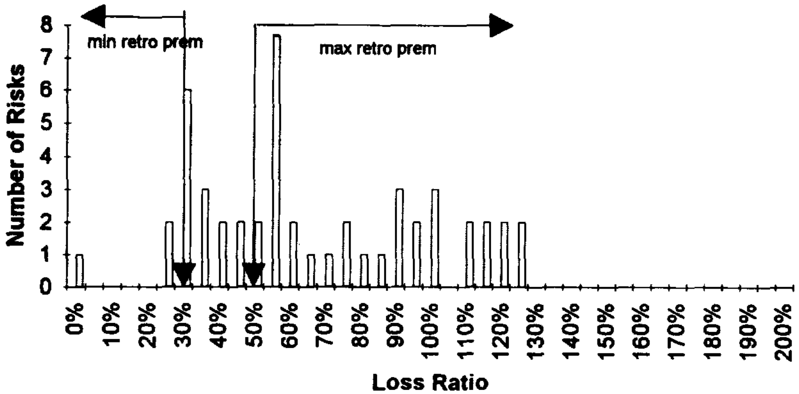


Exhibit A-1 introduces the numerical data corresponding to this group of 50 a priori identical risks. The expected average loss is equal to \$491.96. Individual policy loss ratios are distributed about the mean in the arbitrary (and perhaps a bit unrealistic) distribution that is displayed in Figure A-5. The individual policy standard premium was arbitrarily selected to be \$922.63. While the standard premium was selected at a level that provides for the expected incurred losses, the incurred expenses (including taxes), a reasonable profit, and a margin from which to pay a net retrospective premium return, the details behind the calculation need not be known in order to apply the results of Section 1. A knowledge of the aggregate premium returned under the established retrospective rating plan is required.

The retrospective premium for each of the 50 retrospectively rated risks will be determined by means of Equations A.1 and A.2:

$$R = (e*S + c*I*S + c*L)*TM, \tag{A.1}$$

subject to

$$H*S \leq R \leq G*S, \tag{A.2}$$

where

$$(e*S + c*I*S) = \$382.60;$$

$$c = 1.120;$$

$$TM = 1.031;$$

$$H*S = \$738.10;$$

$$G*S = \$968.76.$$

From Exhibit A-1 we see that the average retrospectively rated premium for the 50 risks is \$873.39, which may or may not be in balance with the guaranteed cost rates. The retro plan will be in balance *if and only if* the average premium discount is equal to 5.3 percent of the standard premium, the average amount of returned premium under the retro plan specified above. Whether or not the original retro plan is in balance, the relations derived in Section 1 hold. For that reason, we will not provide any support for the expense and insurance charge (e and I) components of the rating formula. Exhibit A-1 provides the necessary information: For this set of risks with their common a priori loss ratio distribution, an aggregate incurred loss ratio equal to 53.3 percent of standard premium produces an aggregate retrospective *return* premium equal to 5.3 percent of standard premium.

A direct substitution of the aggregate average loss, \$491.96, into the retrospective rating formula given by Equations A.1 and A.2 produces a retrospective premium equal to \$962.54, or an *additional* premium equal to \$39.91 (i.e., a -4.3% *RP*). The reason why 5.3 percent of standard premium was returned when the aggregate average loss produces 4.3 percent additional premium lies in the way in which losses for risks 30-50 are treated in the formula. While these losses are fully reflected in the aggregate average loss, only the first \$497.35 of loss is reflected in the retrospective premium. This is, precisely, the capping effect which leads to the requirement of an insurance charge. To see how this capping limits the responsiveness of the plan, we apply the equations derived in Section 1.

To apply the equations derived in Section 1, we must know $X(r_{max})$, $Y(r_{min})$, $X(r_{max}/g)$, and $Y(r_{min}/g)$. The maximum premium, \$968.76, corresponds to a loss of \$497.35 which implies that $r_{max} = 1.01(= 497.35/491.96)$. The minimum premium, \$738.10 corresponds to a loss of \$297.60 which implies that $r_{min} = 0.60(= 297.60/491.96)$. Exhibit A-1 displays $X(1.01)$ and $Y(0.60)$ for the 50 risks, where the discrete forms of Equations 1.6 and 1.7 have been used:

$$X(1.01) = \sum_{1}^{50} \text{Max}\,(0, L_i - 497.35)/491.96 = .1944, \qquad \text{(A.3)}$$

and

$$Y(0.60) = \sum_{1}^{50} \text{Max}\,(0, 297.60 - L_i)/491.96 = .0378. \qquad \text{(A.4)}$$

If the same retrospective rating formula were to be applied to a different set of risks that have an expected loss of \$639.55 (130 percent of the original group's expected loss), which are similarly distributed about the mean, $f(s)$ will be unchanged. (Remember that s measures each loss against the mean, so shifting the mean leaves s

unchanged.) The original distribution can, therefore, be used to determine

$$X(r_{\max}/g) = X(1.01/1.30) = X(0.78) = .3113, \qquad \text{(A.5)}$$

and

$$Y(r_{\min}/g) = Y(0.60/1.30) = Y(0.46) = .0116, \qquad \text{(A.6)}$$

as shown in the last two columns of Exhibit A-1.

Had the new $f(s)$ been different, Equations A.5 and A.6 would have been calculated using the new distributions.

The second set of risks, which are displayed in Exhibit A-2, has an incurred loss ratio equal to 69.3 percent of standard premium (i.e., 30 percent higher than that of the original group of policies). Using Equation 1.24 we can predict the corresponding aggregate retrospectively determined return premium, which is observed (see bottom of Exhibit A-2) to be 1.3 percent of standard premium.

From Equation 1.24,

$$\begin{aligned} RP_0 + \Delta RP = 5.3\% & \\ & - (1.12)(69.3 - 53.3)(1.031) \\ & + (1.12)(53.3)(1.3 * .3113 - .1944)(1.031) \\ & - (1.12)(53.3)(1.3 * .0116 - .0378)(1.031) \\ & = 5.3\% - 18.5\% + 13.0\% - (-1.4\%) \\ & = 1.2\% \end{aligned}$$

is approximately equal to the aggregate retrospective returned premium. The error (1.2 percent vs. 1.3 percent) is due to rounding errors introduced by the discrete nature of the distribution.

The rather weak response of this retrospective rating formula (a 4 point decrease in the return premium corresponding to a 16 point increase in the aggregate incurred loss ratio) was due to the effects of capping. Not only were the increased losses in the previously capped 20 risks not reflected, but a portion of the increased loss from risks 19-29 has been capped away.

We must emphasize that the distribution of risks used in this example was selected to accentuate the effect of capping and to illustrate the method, not to produce a realistic model of the $f(s)$ for a set of retrospectively rated risks.

EXHIBIT A-1
THE EXPECTED DISTRIBUTION

Risk #	Loss $	S	$(e+cI)S$	$c*L$	R	RP	$X(1.01)$	$Y(0.60)$	$X(0.78)$	$Y(0.46)$
1	0.00	922.63	382.60	0.00	738.10	184.53	0.00	297.90	0.00	229.15
2	200.00	922.63	382.60	224.00	738.10	184.53	0.00	97.90	0.00	29.15
3	210.00	922.63	382.60	235.20	738.10	184.53	0.00	87.90	0.00	19.15
4	221.00	922.63	382.60	247.52	738.10	184.53	0.00	76.90	0.00	8.15
5	232.00	922.63	382.60	259.84	738.10	184.53	0.00	65.90	0.00	0.00
6	244.00	922.63	382.60	273.28	738.10	184.53	0.00	53.90	0.00	0.00
7	244.00	922.63	382.60	273.28	738.10	184.53	0.00	53.90	0.00	0.00
8	244.00	922.63	382.60	273.28	738.10	184.53	0.00	53.90	0.00	0.00
9	244.00	922.63	382.60	273.28	738.10	184.53	0.00	53.90	0.00	0.00
10	256.00	922.63	382.60	286.72	738.10	184.53	0.00	41.90	0.00	0.00
11	269.00	922.63	382.60	301.28	738.10	184.53	0.00	28.90	0.00	0.00
12	282.00	922.63	382.60	315.84	738.10	184.53	0.00	15.90	0.00	0.00
13	296.00	922.63	382.60	331.52	738.10	184.53	0.00	1.90	0.00	0.00
14	311.00	922.63	382.60	348.32	753.58	169.05	0.00	0.00	0.00	0.00
15	327.00	922.63	382.60	366.24	772.05	150.58	0.00	0.00	0.00	0.00
16	343.00	922.63	382.60	384.16	790.53	132.10	0.00	0.00	0.00	0.00
17	360.00	922.63	382.60	403.20	810.16	112.47	0.00	0.00	0.00	0.00
18	378.00	922.63	382.60	423.36	830.94	91.69	0.00	0.00	0.00	0.00
19	397.00	922.63	382.60	444.64	852.88	69.75	0.00	0.00	13.27	0.00
20	417.00	922.63	382.60	467.04	875.98	46.65	0.00	0.00	33.27	0.00
21	418.00	922.63	382.60	468.16	877.13	45.50	0.00	0.00	34.27	0.00
22	418.00	922.63	382.60	468.16	877.13	45.50	0.00	0.00	34.27	0.00
23	418.00	922.63	382.60	468.16	877.13	45.50	0.00	0.00	34.27	0.00
24	418.00	922.63	382.60	468.16	877.13	45.50	0.00	0.00	34.27	0.00
25	418.00	922.63	382.60	468.16	877.13	45.50	0.00	0.00	34.27	0.00
26	418.00	922.63	382.60	468.16	877.13	45.50	0.00	0.00	34.27	0.00
27	439.00	922.63	382.60	491.68	901.38	21.25	0.00	0.00	55.27	0.00
28	461.00	922.63	382.60	516.32	926.79	-4.16	0.00	0.00	77.27	0.00
29	484.00	922.63	382.60	542.08	953.35	-30.72	0.00	0.00	100.27	0.00
30	508.00	922.63	382.60	568.96	968.76	-46.13	10.38	0.00	124.27	0.00
31	533.00	922.63	382.60	596.96	968.76	-46.13	35.38	0.00	149.27	0.00
32	560.00	922.63	382.60	627.20	968.76	-46.13	62.38	0.00	176.27	0.00
33	588.00	922.63	382.60	658.56	968.76	-46.13	90.38	0.00	204.27	0.00
34	617.00	922.63	382.60	691.04	968.76	-46.13	119.38	0.00	233.27	0.00
35	648.00	922.63	382.60	725.76	968.76	-46.13	150.38	0.00	264.27	0.00
36	661.00	922.63	382.60	740.32	968.76	-46.13	163.38	0.00	277.27	0.00
37	674.00	922.63	382.60	754.88	968.76	-46.13	176.38	0.00	290.27	0.00
38	687.00	922.63	382.60	769.44	968.76	-46.13	189.38	0.00	303.27	0.00
39	701.00	922.63	382.60	785.12	968.76	-46.13	203.38	0.00	317.27	0.00
40	715.00	922.63	382.60	800.80	968.76	-46.13	217.38	0.00	331.27	0.00
41	729.00	922.63	382.60	816.48	968.76	-46.13	231.38	0.00	345.27	0.00
42	744.00	922.63	382.60	833.28	968.76	-46.13	246.38	0.00	360.27	0.00
43	800.00	922.63	382.60	896.00	968.76	-46.13	302.38	0.00	416.27	0.00
44	816.00	922.63	382.60	913.92	968.76	-46.13	318.38	0.00	432.27	0.00
45	832.00	922.63	382.60	931.84	968.76	-46.13	334.38	0.00	448.27	0.00
46	849.00	922.63	382.60	950.88	968.76	-46.13	351.38	0.00	465.27	0.00
47	866.00	922.63	382.60	969.92	968.76	-46.13	368.38	0.00	482.27	0.00
48	883.00	922.63	382.60	988.96	968.76	-46.13	385.38	0.00	499.27	0.00
49	901.00	922.63	382.60	1,009.12	968.76	-46.13	403.38	0.00	517.27	0.00
50	919.00	922.63	382.60	1,029.28	968.76	-46.13	421.38	0.00	535.27	0.00
Total	24,598.00	46,131.50	19,130.00	27,549.76	43,669.71	2,461.79	4,781.03	930.70	7,657.68	285.62
Average	491.96	922.63	382.60	551.00	873.39	49.24	95.62	18.61	153.15	5.71
%Std Prem	53.3%	100.0%	41.5%	59.7%	94.7%	5.3%				
%Avg Loss							19.44%	3.78%	31.13%	1.16%

EXHIBIT A-2

THE CORRESPONDING DISTRIBUTION WITH $g = 1.3$

Risk #	Loss $	S	$(e + cI)S$	c^*L	R	RP
1	0.00	922.63	382.60	0.00	738.10	184.53
2	260.00	922.63	382.60	291.20	738.10	184.53
3	273.00	922.63	382.60	305.76	738.10	184.53
4	287.30	922.63	382.60	321.78	738.10	184.53
5	301.60	922.63	382.60	337.79	742.72	179.91
6	317.20	922.63	382.60	355.26	760.73	161.90
7	317.20	922.63	382.60	355.26	760.73	161.90
8	317.20	922.63	382.60	355.26	760.73	161.90
9	317.20	922.63	382.60	355.26	760.73	161.90
10	332.80	922.63	382.60	372.74	778.76	143.87
11	349.70	922.63	382.60	391.66	798.26	124.37
12	366.60	922.63	382.60	410.59	817.78	104.85
13	384.80	922.63	382.60	430.98	838.80	83.83
14	404.30	922.63	382.60	452.82	861.32	61.31
15	425.10	922.63	382.60	476.11	885.33	37.30
16	445.90	922.63	382.60	499.41	909.35	13.28
17	468.00	922.63	382.60	524.16	934.87	-12.24
18	491.40	922.63	382.60	550.37	961.89	-39.26
19	516.10	922.63	382.60	578.03	968.76	-46.13
20	542.10	922.63	382.60	607.15	968.76	-46.13
21	543.40	922.63	382.60	608.61	968.76	-46.13
22	543.40	922.63	382.60	608.61	968.76	-46.13
23	543.40	922.63	382.60	608.61	968.76	-46.13
24	543.40	922.63	382.60	608.61	968.76	-46.13
25	543.40	922.63	382.60	608.61	968.76	-46.13
26	543.40	922.63	382.60	608.61	968.76	-46.13
27	570.70	922.63	382.60	639.18	968.76	-46.13
28	599.30	922.63	382.60	671.22	968.76	-46.13
29	629.20	922.63	382.60	704.70	968.76	-46.13
30	660.40	922.63	382.60	739.65	968.76	-46.13
31	692.90	922.63	382.60	776.05	968.76	-46.13
32	728.00	922.63	382.60	815.36	968.76	-46.13
33	764.40	922.63	382.60	856.13	968.76	-46.13
34	802.10	922.63	382.60	898.35	968.76	-46.13
35	842.40	922.63	382.60	943.49	968.76	-46.13
36	859.30	922.63	382.60	962.42	968.76	-46.13
37	876.20	922.63	382.60	981.34	968.76	-46.13
38	893.10	922.63	382.60	1,000.27	968.76	-46.13
39	911.30	922.63	382.60	1,020.66	968.76	-46.13
40	929.50	922.63	382.60	1,041.04	968.76	-46.13
41	947.70	922.63	382.60	1,061.42	968.76	-46.13
42	967.20	922.63	382.60	1,083.26	968.76	-46.13
43	1,040.00	922.63	382.60	1,164.80	968.76	-46.13
44	1,060.80	922.63	382.60	1,188.10	968.76	-46.13
45	1,081.60	922.63	382.60	1,211.39	968.76	-46.13
46	1,103.70	922.63	382.60	1,236.14	968.76	-46.13
47	1,125.80	922.63	382.60	1,260.90	968.76	-46.13
48	1,147.90	922.63	382.60	1,285.65	968.76	-46.13
49	1,171.30	922.63	382.60	1,311.86	968.76	-46.13
50	1,194.70	922.63	382.60	1,338.06	968.76	-46.13
Total	31,977.40	46,131.50	19,130.00	35,814.69	45,524.74	606.76
Average	639.55	922.63	382.60	716.29	910.49	12.14
% Std Prem	69.3%	100.0%	41.5%	77.6%	98.7%	1.3%

APPENDIX B

Once a particular retrospective rating plan (formula and limits) has been specified, an aggregate incurred loss ratio and corresponding aggregate retrospective premium could be determined for a set of risks with a known incurred loss ratio distribution by means of simulation. Using the cumulative density function of the distribution and a random number generator, individual risk *ILR*s would be selected and then subjected to the retrospective rating formula. After a sufficiently large number of repetitions, an aggregate (*<ILR>*, *RP*) pair could be generated. In Section 1, we showed that the same result could be found if two functions of the *ILR* distribution, the charge and savings, are known. While not a simulation in the usual sense, we shall refer to points that are generated by means of Equation 1.24 as the results of a simulation.

For all of the simulations, we assumed that the particular set of risks can be described by one of the *ILR* distributions that underlie the NCCI's Table M; and that their standard premiums are such that 26 percent of the standard premium is used to meet expenses (excluding premium taxes, but including a provision for profit); 60 percent of the standard premium is needed for the expected aggregate losses; and that premium taxes give rise to a tax multiplier that is equal to 1.046. These assumptions imply that 10 percent of the standard premium is available for an aggregate retrospective premium return. Given these assumptions, the point (.60, .10) is common to all of our simulations, regardless of the individual retrospective rating plan *LCF*, *G* or *H*. For each simulation, Table M was used to establish an insurance charge that contemplated (.60, .10) as its target.

A total of 17 simulation points were generated for each retrospective rating plan. The points had *ILR*s that began with a low of 51.84 percent and ran to a high of 113.28 percent with each successive *ILR* being 5 percent higher than the previous one. With (.60, .10) being a "given," Equation 1.24 was used to generate the *RP* component of each other aggregate (*ILR*, *RP*) pair.

Exhibit B-1 displays the results of a simulation for the following retrospective rating formula:

$$R = [.260 + 1.000*.265 + 1.000*L]*1.046, \qquad \text{(B.1)}$$

subject to

$$.70*S \leq R \leq 1.40*S, \qquad \text{(B.2)}$$

with all risks assumed to have the *ILR* distribution that underlies Table M loss group 60. The net insurance charge, 0.265, is the one that results from imposing the requirement that the retrospective rating plan be in balance with the corresponding guaranteed cost rates. In terms of r, the minimum loss ratio reflected in the retrospective rating is .24 times the expected loss ratio, and the maximum loss ratio is 1.360 times the expected *ILR*.

To this set of simulation points we fit three curves,

1. Linear Model: $RP = A + B*ILR,$ (B.3a)

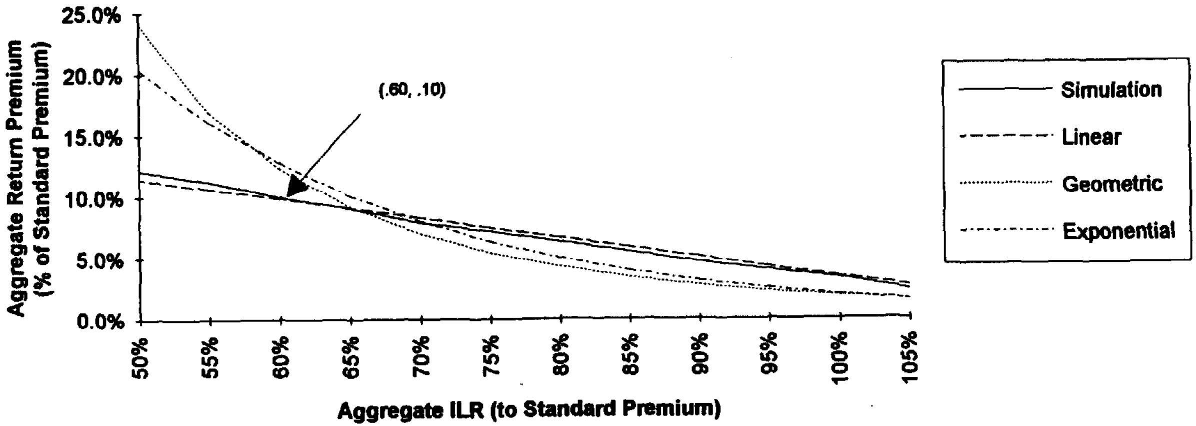
2. Geometric Model: $RP = A*(ILR^B)$, and (B.3b)

3. Exponential Model: $RP = A*e^{B*ILR}.$ (B.3c)

For each of these, we determined the t statistics, for A and B, and the mean squared error (MSE) of the model using the 17 points. Because the MSEs for each model were so small, we have multiplied them by 1,000,000 (for example, the Geometric Model MSE of 0.000111 is, therefore, displayed as 111). Figure B-1 displays a graph of the simulation points and the three model curves.

FIGURE B-1

AGGREGATE *ILR* VS. AGGREGATE RETURN PREMIUM



As with the example that is displayed in Exhibit B-1 all of the simulations were described by models which had coefficients that were significant at the 99.95 percent confidence level. Under these circumstances, one would usually select the model that produced a curve with the least MSE.

With the exception of the simulations displayed in Table 5, all of the retrospective rating plans were based upon an expense and profit provision ratio equal to 26.0 percent of standard premium. There was nothing special about its selection. It lies about midway between the NCCI retrospective rating stock and non-stock company expense provisions for risks with standard premiums near $53,000. (See NCCI Tables XIV-A and XIV-B.) The 60 percent *ILR* was selected because it is typical of the NCCI expected loss ratios that are to be used for retrospective rating.

Table 5 involved risks of various sizes. As a result, we felt that the NCCI expense graduations should be reflected. For each premium size, we established the $(e + c*I)$ term using the appropriate expense provision, e, from the NCCI stock company expense table and the insurance charge corresponding to the appropriate Table M grouping (using the July 1, 1991 NCCI expected loss ranges).

While one could argue that the expected loss ratios for a group of risks that use identical manual rates should reflect a size of risk dependency, we adopted a common expected loss ratio for all of the risks. This is consistent with the way in which the NCCI retrospective rating plan is applied.

Intuitively, one would expect to select a curve with a negative first derivative (i.e., an increase in the aggregate *ILR* results in a decrease in the aggregate *RP*), but with a positive second derivative (i.e., as more and more individual risks "max" out, additional increases in the aggregate *ILR* have less of a decreasing effect on the aggregate *RP*). As long as B is negative, all three models possess negative first derivatives. Only the linear model fails to exhibit the intuitively required positive second derivative. The linear model has performed better than the other two (using the minimum MSE criterion) more often than not for the various plans considered here. Because the range of *ILR*s is expected to encompass almost any realistic situation, the danger associated with extrapolating too steep a curve (one that doesn't "pull up" for high values of the *ILR*) was considered minimal. The lack of an intuitively correct second derivative for the linear model was not considered to be a serious defect, and is largely outweighed by the simple interpretation of its slope (the *B* coefficient) as the fraction of the *ILR* increase that impacts the returned premium.

Depending on the particular application, one might wish to use the actual simulation points, the best fitting curve, or the better fitting curve which satisfies the intuitive requirements.

EXHIBIT B-1

ILR	Simulation	Linear	Geometric	Exponential	
51.84 %	11.79%	11.34 %	14.68%	13.46%	
54.42	11.44	10.91	13.12	12.44	
57.12	10.67	10.46	11.72	11.46	
60.00	10.00	9.98	10.46	10.51	<==Target
63.00	9.30	9.48	9.34	9.59	
66.18	8.74	8.95	8.33	8.71	
69.48	8.16	8.40	7.44	7.88	
72.96	7.46	7.81	6.65	7.09	
76.62	6.93	7.20	5.93	6.35	
80.46	6.28	6.56	5.30	5.65	
84.48	5.58	5.89	4.73	5.00	
88.68	5.10	5.19	4.23	4.40	
93.12	4.37	4.45	3.77	3.85	
97.80	3.65	3.67	3.37	3.34	
102.72	3.07	2.85	3.01	2.87	
107.88	2.20	1.99	2.68	2.46	
113.28	1.48	1.09	2.40	2.09	

			Value	*t*-Statistic
Linear Model	$A + B*ILR$	$A =$	0.1998386	64.726797
		$B =$	-0.1667903	-43.773508
		$MSE =$	8	
Geometric Model	$A*(ILR**B)$	$A =$	0.0319978	-51.081205
		$B =$	-2.3187566	-12.325123
		$MSE =$	111	
Exponential Model	$A*Exp(B*ILR)$	$A =$	0.6484224	-3.2121784
		$B =$	-3.0334045	-18.224868
		$MSE =$	43	