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RISK LOADS FOR INSURERS

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AUTHOR'S REPLY TO DISCUSSION

Abstract

Insurance risk has moved to the forefront of the actuary's concerns. Three other papers on this topic by Fellows of the Casualty Actuarial Society, all written independently, have appeared at the same time as this one: Kreps [14], Venter [24], and Meyers [18]. Insurance risk is the foundation of the NAIC risk-based capital requirements (Hartman, et al. [11]; Kaufman and Liebers [12]). It is also the subject of prize paper competitions by the CAS Loss Reserve Committee and the CAS Risk Theory Committee.

It is appropriate, therefore, that two actuaries deeply involved in the current deliberations, Glenn Meyers and Stephen Philbrick, have written a discussion of, and articles closely related to, this paper. The following remarks from their articles, along with my response to their remarks, provide the reader with a more complete perspective on the issues.

1. THE ACTUARY AND THE DJINN

Stephen Philbrick [21] takes issue with the statement that "the standard deviation of the individual's loss distribution is no guide even to the process risk faced by the insurer." Philbrick notes that when a risk is added to an insurer's book of business, the increase in either the aggregate variance or the aggregate standard deviation of the insurance portfolio is proportional to the variance of the added risk. He concludes that the variance of the individual risk is indeed a guide to the insurer's process risk.

Shortly after the CAS meeting at which this paper was presented, Philbrick wrote a marvelous column for the *Actuarial Review* [22], which should help the reader understand both his criticism and the reply here. A magical djinn offers to replace a lackluster portfolio with a larger and more profitable one, if only the actuary can answer certain questions. The djinn and the actuary agree that surplus requirements should be proportional to the aggregate standard deviation of the portfolio, and the djinn then asks:

> "You can either write an additional risk of Type A or an additional risk of Type B. Risk A has an expected loss of \$1 million, a standard deviation of \$100,000, and a variance of 1 times 10 to the 10^{th}. Risk B is identical to Risk A except that each of the individual losses is exactly twice that associated with Risk A. Consequently, Risk B has expected losses of \$2 million, a standard deviation of \$200,000, and a variance of 4 times 10 to the 10^{th}. You can also assume that both of these risks are independent of the rest of the portfolio."
>
> . . . If you decide to add Risk B to your portfolio instead of Risk A, how much additional surplus would you require to write Risk B relative to the additional surplus you would require for Risk A?"

The answer is four times as much, since the increase in the aggregate standard deviation is proportional to the variance of the marginal risk, not to the standard deviation of the marginal risk.

Philbrick is correct that if a risk is added to a portfolio, the *relative* increase in aggregate standard deviation is proportional to the *relative* variance of the added risk. But the actual increase in aggregate standard deviation, either in absolute terms or relative to the standard deviation of the existing portfolio, is *not* proportional to the variance of the added risk.

Philbrick's example, reproduced and expanded in Exhibit 1, clarifies this. In this example, Risk A has an expected loss of $1,000, a standard deviation of $9,950, and a variance of 99 million; Risk B has an expected loss of $1,000, a standard deviation of $31,607, and a variance of 999 million.

Risk B has a standard deviation about three times greater than Risk A's and a variance about 10 times greater. Whether one begins with 10,000 risks of Type A or 5,000 risks of Type A, both the marginal variance and the marginal standard deviation are about ten times greater when one adds a risk of Type B than when one adds a risk of Type A. If a standard deviation method were used for risk loads, *and we knew the appropriate risk load for adding a Type A risk*, then we could derive the corresponding risk load for adding a Type B risk.[1]

But the argument in the paper is that the loss distributions of individual risks tell us neither the appropriate risk load for the portfolio nor the additional risk load for adding another risk. The ratio of the marginal standard deviation of the portfolio to the variance of the added risk depends on the composition of the portfolio. If the begin-

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[1] In general, the marginal standard deviation is approximately equal to the variance of the added risk divided by twice the standard deviation of the portfolio. Letting

Var_{bk} = the variance of the portfolio,

Var_{risk} = the variance of the added risk, and

SD_{bk} = the standard deviation of the portfolio,

we have

$$SD_{bk} = \sqrt{(Var_{bk})};$$
$$\partial\sqrt{(Var_{bk})}/\partial Var_{bk} = 1/[2\sqrt{(Var_{bk})}];$$
$$\Delta\sqrt{(Var_{bk})} \approx \Delta(Var_{bk})/[2\sqrt{(Var_{bk})}].$$

But $\Delta(Var_{bk}) = Var_{risk}$, and $SD_{bk} = \sqrt{Var_{bk}}$, so the marginal standard deviation $\approx Var_{risk}/[2\,(SD_{bk})]$. I am indebted to Dr. Eric Brosius for this formula as well as for explanations of these concepts.

EXHIBIT 1

MARGINAL STANDARD DEVIATION OF AN INSURANCE PORTFOLIO

	10,000 Type A	10,000 Type A + 1 Type A	10,000 Type A + 1 Type B
Expected Losses	10,000,000	10,001,000	10,001,000
Variance	990,000,000,000	990,099,000,000	990,999,000,000
Marginal Variance		99,000,000	999,000,000
Standard Deviation	994,987.44	995,037.19	995,489.33
Marginal Std. Dev.		49.75	501.89

	5,000 Type A	5,000 Type A + 1 Type A	5,000 Type A + 1 Type B
Expected Losses	5,000,000	5,001,000	5,001,000
Variance	495,000,000,000	495,099,000,000	495,999,000,000
Marginal Variance		99,000,000	999,000,000
Standard Deviation	703,562.36	703,632.72	704,271.96
Marginal Std. Dev.		70.36	709.60

ning portfolio consists of 10,000 Type A risks, the ratio is about 5×10^{-7} [$\approx 49.75/99{,}000{,}000 = 501.89/999{,}000{,}000$]. If the beginning portfolio consists of 5,000 Type A risks, the ratio is about 7×10^{-7} [$\approx 70.36/99{,}000{,}000 = 709.60/999{,}000{,}000$].

Philbrick is correct that if we use a standard deviation risk load method, then the relative variances of the additional risks are a guide to the relative increases in the aggregate risk load. But the variance of the additional risk does not tell us what the increase in the aggregate risk load should be.

2. UNDERWRITING RISK AND RESERVING RISK

Philbrick writes:

> "Because . . . the risk load in pricing is inextricably linked to the risk margins in reserving, this paper will also add to the literature on that important subject."

The coming implementation of risk-based capital requirements for property/casualty insurers highlights the need for careful analysis of risk, both pricing and reserving. Philbrick is correct: pricing and reserving risks are linked. A few comments may further clarify the relationship between the two.

Pricing risk is an economic risk. When the actuary prices a policy, the premium has not yet been earned nor the losses incurred. The risk load is the additional profit required to induce the insurer to underwrite the policy. The risk load is a market transaction: the insurer actually receives the risk load from the policyholder.

Reserving risk is primarily an accounting risk. When the reserve is booked, the loss has already occurred. The risk is that the insurer's reserve estimates are inaccurate. The reserve margin is the additional capital the insurer must hold to protect policyholders and to satisfy regulators that its reserves will suffice to settle the claims. The reserve margin is *not* a market transaction: no cash passes hands, and there is no profit or loss to the insurer.[2]

Yet a partial connection remains between pricing risk and reserving risk. Pricing risk reflects the uncertainty in operating ratios. Reserving risk reflects the uncertainty in reserve adequacy. Lines with highly volatile reserves have volatile operating ratios as well.

Some actuaries proceed further along this path and presume that duration of reserves is a suitable proxy for both reserving risk and pricing risk. This last statement is an oversimplification. The relationship between reserve duration, pricing risk, and reserving risk in four lines of business should clarify this.[3]

- *Property:* Large property exposures, as in earthquake insurance or commercial fire insurance, may have great pricing risks. (Note that commercial multi-peril has a high standard deviation of operating ratios and a large β.) But reserves are paid quickly, and there is generally little doubt about the insurer's liability once the accident occurs. Both reserve duration and reserving risk are low.
- *Products Liability:* Products liability includes asbestos and pollution exposures, in addition to other toxic torts and potentially harmful operations. Reserve duration is long, because liability is so uncertain. In fact, much of the litigation in environmental impairment issues has been on coverage disputes: who (if anyone) must pay the costs of clean-up? Similarly, pricing risk is great, because liability may be imposed, even retroactively, in contravention of underwriters' intent in issuing the insurance contract [10, 16]. Products liability fits the simple scheme: reserve duration is long, and both reserving risk and pricing risk are great.

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[2] As an anonymous referee for the *Proceedings* has pointed out, there are also instances in which reserve margins may affect pricing or cash transactions, such as where "individual risks are retrospectively rated, individual risks are experience rated, or underwriting acceptability is a function of experience."

[3] Actuaries have different opinions about the relative risks by line of business. The subsequent statements in the text are one perspective; other views are also possible.

- *Automobile:* personal automobile underwriting problems often stem from regulatory or statutory enactments. California's Proposition 103 changed a competitive insurance marketplace to one characterized by prior approval of rates, severe restrictions on underwriting freedom, prohibitions on cancellation or non-renewal of insureds, mandated classification systems with no allowance for various traditional dimensions, and rollbacks of rates. New Jersey insurance regulators have depressed rate levels, flattened classification systems, imposed penalties on servicing carriers for the involuntary market, and now seek to recoup Joint Underwriting Association losses from insurance companies. Massachusetts personal automobile regulation has been so onerous and unpredictable that many carriers have paid large fees simply to leave the state.

  Regulatory problems heighten pricing risk. But reserve duration is short (less than one year for all coverages combined), and there is little reserving risk.

- *Workers' Compensation:* fixed statutory benefits and an administered pricing system left workers' compensation with little pricing risk for indemnity coverage from the mid-1970s to the mid-1980s.[4] (The advent of open competition and a multiplicity of statutory "reforms" have increased pricing risk since the late 1980s.) Disability benefits are paid only as the income loss accrues; the benefits may extend over the injured worker's lifetime in permanent total disability cases. Workers' compensation reserve duration is the longest among all Annual Statement lines, except for medical malpractice and casualty excess-of-loss reinsurance. Yet reserving risk is moderate, since the slowest paying claims are often quite certain. In sum, workers' compensation has long dura-

[4] Medical costs for catastrophic cases, however, are hard to predict and pose greater pricing risk.

tion reserves, below average pricing risk, and below average reserving risk.[5]

There are many types of risk which the actuary must consider. Philbrick, of course, is well aware of the interrelationships between these risks. Other readers should be equally careful not to confuse them, but to separately measure each one.

3. CAPM AND RISK DIVERSIFICATION

The Capital Asset Pricing Model (CAPM) posits that only systematic risk, or non-diversifiable risk, is rewarded by higher expected returns. Firm-specific risk can be eliminated by diversification, and it is not rewarded by increased returns.

Glenn Meyers [19] reproduces a derivation of the CAPM from Copeland and Weston [4], which concludes that

$$E[R_j] = R_f + \lambda \operatorname{Cov}[R_j, R_m],$$

where

$\lambda = (E[R_m] - R_f)/\operatorname{Var}[R_m]$,

R_f is the risk-free return,

R_j is the rate of return on the j^{th} asset, and

R_m is the market rate of return.

Meyers then comments: "CAPM proponents claim that the market should not reward [diversifiable] risks. . . . The flaw in these

[5] Similarly, traditional whole life policies and fixed benefit life annuity contracts have long reserve durations, but little reserving risk. (Disintermediation risks, which are present in these contracts, are not applicable to workers compensation loss reserves.) Note also that the April 1991 NAIC risk-based capital reserving risk charges are high for other liability but are nil for workers compensation [15]. Other liability has had high and unpredictable adverse loss development in the 1980s. The implicit interest discount in the long duration workers compensation reserves outweighs the moderately adverse loss development. Subsequent developments have partially changed these relationships; see [8, 9].

statements can be addressed by the CAPM itself. Nowhere in the above development of the CAPM is one required to label a particular risk as being diversifiable or non-diversifiable."

On the contrary, the CAPM derivation indeed reflects the diversifiability of risk. A diversifiable risk refers to the portion of the return that is independent of the market return. That is, for diversifiable risk, Cov $[R_j, R_m] = 0$, so $\mathrm{E}[R_j] = R_f$ in the equation above. Systematic risk is not independent of the market, so Cov $[R_j, R_m] \neq 0$. If Cov $[R_j, R_m] > 0$, as is normally the case, then $\mathrm{E}[R_j] > R_f$.

Similarly, the formula provided in the text of the paper has

$$\mathrm{E}[R_j] = R_f + \beta\,(E[R_m] - R_f), \text{ where}$$

$$\beta = \mathrm{Cov}\,[R_j, R_m]/\mathrm{Var}\,[R_m].$$ [6]

Again, if the risk's return is independent of the market return, then $\beta = 0$, and $\mathrm{E}[R_j] = R_f$.

If the risk's return is positively correlated with that of the market, then $\beta > 0$ and $\mathrm{E}[R_j] > R_f$.

4. SURPLUS ALLOCATION

Meyers notes that an application of the CAPM to insurance operations requires an allocation of surplus. He argues that this allocation is inappropriate, and he quotes Charles McClenahan's remarks at the 1990 CAS Ratemaking Seminar.[7]

[6] Using Meyers's notation, $\beta = \lambda\,\mathrm{Cov}\,[R_j, R_m]\,/\,(E[R_m] - R_f)$. Both the "lambda" and the "beta" expressions may be found in the theoretical literature, although the latter is now more common.

[7] Other actuaries have expressed similar reservations. In testimony regarding California's Proposition 103, Bass [2] says: "By its fundamental nature, surplus is not allocatable, whether to line of business, to jurisdiction, or to any other segment of an insurer's operation" (page 231). After reviewing several allocation methods, Kneuer [13] concludes that not one "addresses the philosophical questions that underlie any attempt to allocate surplus" (page 224). Roth [23], in a discussion of Proposition 103, argues against surplus allocation and proposes an alternative measure of return.

The application of the CAPM to insurance operations does not require an allocation of surplus. The analysis in the text of the paper deals with operating ratios by line of business, not returns on equity. The propriety of surplus allocation has no bearing on the usefulness of the Capital Asset Pricing Model to estimate insurance risk loads.[8]

There remains much confusion on the issue of surplus allocation, so a few more comments may be worthwhile. Actuaries correctly argue that surplus should not be allocated for solvency examinations. This is the gist of McClenahan's statement that Meyers quotes: "The protection against solvency afforded by a $100 million surplus for a free-standing automobile insurance company is not comparable to the protection afforded by a multi-line insurance company with $100 million of surplus allocated to automobile insurance." McClenahan emphasizes: "The fact is that the entire surplus of an insurer stands behind each and every risk" [17, page 152].

But solvency is different from pricing. Many actuarial pricing methods relate net income to some measure of net worth, such as statutory surplus or GAAP equity (see [7, 5, 6]). As Murdza [20] notes, "allocation for ratemaking purposes only does not mean that surplus is actually allocated for solvency or other purposes." That is, the actuary uses the allocation procedure to measure profitability, not to limit the company's legal obligations. Similarly, Callaghan and Derrig [3] say:

> "A company's surplus is not in fact or in law allocated by line and state. A company's entire surplus is available to meet the losses on any line in any state. . . .
>
> "The fact that surplus is not actually allocated by line and state does not, however, mean that it need not be allocated for purposes of determining an appropriate underwriting

[8] One might argue that the determination of operating income uses the Insurance Expense Exhibit Formula for spreading investment income by line of business. But this is not an allocation of surplus. In fact, the end of the paper suggests methods of improving the analysis and notes that "...cash flow discounting should be used instead of spreading investment income to that line of business."

> profit provision for each line. As noted above . . . Massachusetts law requires the determination of rates by line. Thus it is not only appropriate but required that the ratemaker . . . consider surplus by line, just as other elements of the rate-making methodology must be considered by line.
>
> "Such consideration requires that surplus be allocated by line and state for purposes of rate-making, even though it is not allocated by line and state by law. Indeed, such allocation is unavoidable. Any profit methodology which purports to determine profit provisions by line assumes an allocation of surplus by line and state."

The issue of surplus allocation is vexing. But surplus allocation is not needed for applying the CAPM to insurance operations, so it is not germane to this discussion of risk loads.

5. CORRECTIONS

William Bailey has pointed out that the figures in Tables 4 and 5 for fire and commercial multi-peril are in error. The standard deviations in Table 4 should be 6.48 for fire and 13.49 for CMP, and the β in Table 5 should be 0.92 for fire and 2.79 for CMP. For a method of measuring the stability of the β estimates, see [1].

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