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THE CALIFORNIA TABLE L

DAVID SKURNICK

DISCUSSION BY WILLIAM R. GILLAM

It's di Lemma, it's de limit, it's de lovely . . .

(apologies to Cole Porter)

"The California Table L" is as pertinent today as it was when it was published almost twenty years ago. It is well-constructed, rigorous and easy to follow.

The subject, Table L, provides great simplicity in calculating retrospectively rated plans with a prescribed individual accident limit. It enables a built-in correction for the overlap of the charge for the per-accident limit and the aggregate loss limit. In the days when retros were calculated by hand, the simplification was highly desirable.

THE NEED FOR GREATER FLEXIBILITY

But the plan based on Table L is not the reason for the enduring value of the paper. Since the charge for a pre-determined accident limit is built into the table, it cannot be used for alternate accident limits. The table must be updated regularly to account for changes in the incremental charge for that accident limit, as well as for changes in the aggregate loss distribution resulting from a fixed cap on accidents during a time of loss size inflation. Further, the need for calculations simple enough to do by hand has been obviated by the revolution in electronic data processing.

The need for a more flexible plan has been addressed recently in a more general manner. The Revised Retrospective Rating Plan[1] of the National Council on Compensation Insurance (NCCI) allows a degree of choice not available in the California plan. The plan is built around a Table of Insurance Charges (previously called Table M, as it will be in this review) which lists excess pure premium ratios for loss distributions when there is no per accident limit. Like Table L, Table M is indexed by entry ratio as well as size of risk. Setting $K = 0$ and removing the asterisks in Mr. Skurnick's definition of $\varphi^*(r)$ results in the Table M Charge $\varphi(r)$.[2]

Incremental charges for the accident limit (Excess Loss Factors or ELFs) are separate from Table M charges and are updated with each state rate filing. Table M is updated regularly, at least for claim size inflation, by changes in the Expected Loss Size Ranges used to select the appropriate column of the table for the specific insured. In rating a specific plan, the insurance charge is now calculated respective of any selected accident limit. Inherent in this calculation is the correction for overlap with the ELF; hence the mnemonic ICRLL for Insurance Charge Reflecting Loss Limitations.

The ICRLL equations equivalent to Skurnick's formulas (20) and (21) are easily derived if one realizes the actual losses subject to the plan are limited on a per accident basis.[3] Given the expected unlimited loss ratio, E, the expected limited loss ratio is

$$\hat{E} = E - ELF\ .$$

[1] Principles of this plan are described in "Overlap Revisited—the 'Insurance Charge Reflecting Loss Limitation' Procedure," by Ira Robbin, 1990 CAS Discussion Paper Program, *Pricing*, p. 809, as well as "Fundamendals of Individual Risk Rating," by William R. Gillam and Richard H. Snader, © 1992, National Council on Compensation Insurance.

[2] A nostalgic description of the construction of Table M may be found in "The 1965 Table M," by LeRoy J. Simon, *PCAS* LII, 1965, p. 1. A more generic, if less detailed, description may be found in "Fundamentals of Individual Risk Rating," op. cit.

[3] Fundamentals of Individual Risk Rating," op. cit.

Entry ratios $\hat{r}_H$ and $\hat{r}_G$ are ratios of actual limited to expected limited losses. Then

$$\hat{r}_H - \hat{r}_G = \frac{G - H}{c\hat{E}T} ,$$

and

$$\hat{\varphi}(\hat{r}_H) - \hat{\varphi}(\hat{r}_G) = \frac{P - PD - H}{c\hat{E}T} ,$$

where the reviewer has added a Tax Multiplier, T, and hats, ^ , to notation taken from the paper. NCCI uses discounted expense ratios, e, to Standard Premium, not including tax. Using $P - PD = T(e + E)$, the latter equation can be written:

$$\hat{\varphi}(\hat{r}_H) - \hat{\varphi}(\hat{r}_G) = \frac{e + E - H/T}{c\hat{E}} .$$

An absolutely correct ICRLL calculation would require multiple Limited Loss Tables M, i.e., one for each possible accident limit. Limited Loss Table M should be distinguished from Table L. The former lists excess pure premium ratios (charges) appropriate for the aggregate loss distribution of the insured risk when a per accident loss limit is elected, but includes no charge for the loss limit; the incremental charge for this limit must be included as a separate item in plans with such a limit.

The NCCI plan uses a formula shift in Table M columns to approximate a limited loss Table M. Specifically, the selection of a loss limit reduces the skewness of the claim size distribution and hence the loss ratio distribution. This can be modeled by a column of Table M for a larger size risk. The NCCI plan specifies a multiplier, K, to apply to standard expected losses to determine the Expected Loss Size Group (ELG) of the risk:

$$K = \frac{1 + (0.8)\,LER}{1 - LER}$$

$$= \frac{1 + (0.8)\,ELF/E}{1 - ELF/E} .$$

The loss elimination ratio, *LER*, for the selected accident limit is calculated by dividing the *ELF* by the expected loss ratio, *E*.

THE LEMMA

This reviewer would like to highlight a seemingly trivial portion of Skurnick's paper: Lemma 1. Skurnick uses this lemma the way one normally uses a lemma: To prove theorems. The theorems relate to the important relationships in Table L and the balance in the Retrospective Rating Plan. The longevity of the paper is due in part to the elegance of these proofs. But it is easy to overlook the power of the lemma.[4]

The lemma looks simple enough. Let A be a loss process with expectation, $E[A]$, and L the same loss process except that aggregate loss amounts are capped at $r_2 E$ and subject to a minimum value of $r_1 E$. In a retrospective rating plan with minimum and maximum premium factors, L would be the *ratable* losses. Then:

$$E[L] = E[A] \bullet (1 - \varphi(r_2) + \psi(r_1)),$$

where $\varphi(r)$ is the Table M (or L) charge for entry ratio r and $\psi(r)$ is the corresponding savings.

Skurnick uses the lemma to prove the useful formula, $r = 1 + \psi(r) - \varphi(r)$, as well as derive the balance equations used in the plan. This reviewer shows how to use the lemma to evaluate retrospective rating plans that *do not necessarily balance to guaranteed cost.*

Example 1

An example of such an application follows. (This is adapted from 1989 CAS Examination 9, question 26.) The question describes an

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[4] Others have recognized the value of the lemma. See, for instance, "The Mathematics of Excess of Loss Coverage and Retrospective Rating—A Graphical Approach," by Yoong-Sin Lee, *PCAS* LXXV, 1988, p. 67.

unbalanced retro plan such as used in the Residual Market in some states.

The workers' compensation assigned risk pool has promulgated a retrospective rating plan for assigned risks with $50,000 to $59,999 of Standard Premium.

	Retrospective Premium	$= 0.28 \times$ Standard Premium + $1.00 \times$ Incurred Losses
SUBJECT TO:	Minimum premium	= Standard Premium
	Maximum premium	= 1.50 times Standard Premium

Suppose that the expected standard loss ratio for these risks is 120% and the following Table M applies to this group of risks.

ENTRY RATIO	CHARGE	SAVINGS
0	1.00	0.00
0.20	0.80	0.00
0.40	0.62	0.02
0.60	0.46	0.06
0.80	0.32	0.12
1.00	0.18	0.18
1.20	0.10	0.30
1.40	0.06	0.46
1.60	0.04	0.64
1.80	0.02	0.82
2.00	0.00	1.00

a. In terms of Standard Premium, what is the expected ultimate premium for a risk in this group?

b. Assume that 28% of Standard Premium is needed for expense including loss adjustment expenses and taxes. Compute the maximum premium factor needed (instead of the 1.5 given above) so that the expected ultimate premium will be adequate for these risks.

It should be clear that the expected premium of this plan is at least the Standard Premium, which is, in turn, greater than guaranteed cost (assuming premium discounts would otherwise apply). We would say the plan does not *balance* to guaranteed cost.

Part b. of the question asks for a plan that does balance, not to guaranteed cost, but to expected losses and expenses.

The answer to Part a. may be obtained easily using the lemma. Using ratios to Standard Premium, the plan looks like the following:

$$1 \leq RP = 0.28 + A \leq 1.5$$

or

$$1 \leq RP = 0.28 + r\,\mathrm{E}[A] \leq 1.5$$

where RP is the retrospective premium, and r is the entry ratio of actual to expected actual losses.

The losses leading to the maximum premium result from entry ratio r_2.

$$0.28 + r_2\,\mathrm{E}[A] = 1.5$$

$$r_2\,\mathrm{E}[A] = 1.22$$

$$r_2 = \frac{1.22}{1.20}$$

$$r_2 \approx 1.00$$

Similarly, minimum losses are represented by r_1.

$$0.28 + r_1\,\mathrm{E}[A] = 1.00$$

$$r_1\,\mathrm{E}[A] = .72$$

$$\text{and } r_1 = 0.60$$

Now,

$$\begin{aligned} \mathrm{E}[\mathrm{RP}] &= 0.28 + \mathrm{E}[\mathrm{L}] \\ &= 0.28 + \mathrm{E}[A] \bullet (1 - \varphi(1.0) + \psi(0.6)) \\ &= 0.28 + 1.2\,(1 - (0.18) + (0.06)) \\ &= 1.336 \end{aligned}$$

The answer to Part a. is 133.6% of Standard Premium. The answer to Part b. requires finding a maximum premium factor leading to a maximum entry ratio where the charge is offset by the known savings for the minimum.

Example 2

The lemma can be used to answer another question of practical interest: what is the premium impact of an update to Expected Loss Size Ranges in the Retrospective Rating Plan? The Expected Loss Size Ranges are shown in a table that relates expected losses of a risk to columns of Table M. The expected losses of a risk are first adjusted by a factor based on its state and hazard group assignment, called the (state) Hazard Group Differential. Typically, an update accounts for one year's inflation in the average cost per case of workers' compensation claims. For the last several years, this has been about +10%, so the size range endpoints have increased by that amount. In order to estimate this impact, it would be extremely difficult—if not impossible—to check the results of policies actually retrospectively rated. It is particularly difficult to estimate the impact of loss development on individual insured loss ratios. Rather, it makes sense to assume Table M was adequate last year, and the proposed update is needed to keep the plan in balance.

An example will clarify the idea. Suppose an insured is rated according to 1992 size ranges. Assume the following values:

$E = 0.62$	Expected Loss Ratio (This is expected actual losses, E[A], as a ratio to adequate standard premium)
$T = 1.07$	Tax Multiplier
$e = 0.220$	Expense Ratio
$D = 0.101$	Premium Discount Factor
risk $ELG = 60$	Indicated column of Table M (Columns of Table M are indexed by

the charge at entry ratio 1.0, which in this case is 0.60).

A not atypical plan for a risk this size would be as follows:

$G = 1.20$ (Maximum Premium)

$H = 0.7$ (Minimum Premium)

$c = 1.125$ (Loss Conversion Factor)

We find a basic premium factor of $B = 0.576$, with $r_G = r_2 = 0.78$ and $r_H = r_1 = 0.11$ and an insurance charge.

$$E \bullet (\varphi(r_2) - \psi(r_1)) = (0.62)(\varphi(0.78) - \psi(0.11))$$
$$= (0.62)(0.653 - 0.031)$$
$$= 0.386$$

Expected ratable losses are given by

$$\mathrm{E[L]} = E \bullet (1 - \varphi(0.78) + \psi(0.11))$$
$$= 0.62\,(1 - 0.653 + 0.031)$$
$$= 0.234$$

It is no accident $0.386 + 0.234 = 0.62$, which is to say the plan is balanced with respect to loss. Thus,

$$\mathrm{E[RP]} = T(B + c\mathrm{E}[L])$$
$$= 1.07(0.576 + (1.125)(0.234))$$
$$= 0.898$$

if the 1992 size ranges apply. Notice that $0.898 \approx 1\text{-}0.101$, the Standard Premium minus Premium Discount Ratio.

In due course of time, the loss process is better described by the 1993 Expected Loss Size Ranges; the insured should be in *ELG* 61. We evaluate this 1992 *ELG* 60 plan according to the distributions underlying column 61 of Table M. First evaluate expected ratable losses.

$$\begin{aligned} E'[L] &= E \bullet \ (1 - \varphi'(0.78) + \psi'(0.11)\ ) \\ &= 0.62\ (1 - 0.662 + 0.032) \\ &= 0.229 \end{aligned}$$

The primes denote expectation according to the updated loss distribution.

Now

$$\begin{aligned} E'[RP] &= T\,(B + cE'[L]\ ) \\ &= 1.07(0.576 + (1.125)(0.229)\ ) \\ &= 0.892 \end{aligned}$$

The change in net expected retrospective premium will be the following amount.[5]

$$\begin{aligned} \frac{E'[RP] - E[RP]}{E[RP]} &= \frac{0.892 - 0.898}{0.898} \\ &= -\,0.007 \end{aligned}$$

This is not to say the update of size ranges *reduces* expected retrospectively rated premium, but rather *failure* to make the indicated update causes an expected 0.7% *shortfall,* at least on this specific plan.

We calculate an expected aggregate impact by grouping premium according to size of insured, calculating the expected impact on typical plans within each size range, and weighting these impacts by the respective premium volume. The actual volume of premium retro-

[5] The expected change as a percent of standard premium turns out to be nothing more than a factor times a difference in Table M values:

$$\begin{aligned} E'[RP] - E[RP] &= T(B + cE'[L]\ ) - T(B + cE[L]\ ) \\ &= Tc\ (E'[L] - E[L]\ ) \\ &= Tc\,(E((1 - \varphi'(r_2) + \psi'(r_1)) - E(1 - \varphi(r_2) + \psi'(r_1)\ )) \\ &= Tc\,E(\varphi(r_2) - \varphi'(r_2) - \psi(r_1) + \psi'(r_1)\ ) \end{aligned}$$

= Tax Multiplier × Loss Conversion Factor × Expected Loss Ratio × (Difference in Table M charges - Difference in Table M Savings) spectively rated within each size range must be estimated, but the average impact is not very sensitive to the weights. This is because the expected impact on typical plans within each size range, as calculated above, does not differ much between sizes.

The expected impact on any individual risk depends more on the number of columns shifted in Table M. Given an inflationary update of 10%, a risk may shift 0, 1 or 2 columns in the table. Most of the larger size risks likely to be written on retro shift only one group.

Exhibit 1 shows an application of this procedure to the Size Range Update effective July 1, 1993. This is done using a computer program we call "square peg, round hole," a name which reminds us that balance is not a foregone conclusion. The second line of the exhibit corresponds to the example above. Notice most risks shift one size group, but one of the smaller ones shifts two. We believe this is a fair depiction of the actual distribution of changes. A more accurate treatment of the shifting of size groups is described in the Appendix.

EXHIBIT 1
Part 1

CALCULATION OF THE PREMIUM IMPACT OF CHANGES IN RETRO PARAMETERS
SAMPLE PLANS-STATE X
(AVERAGE SHG RELATIVITY 0.774)

Risk Distributions			Effective Parameters				Hypothetical Plan Values		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Premium Range	Risks	Avg Std Prem	*E*	*T*	*D*	*e*	*c*	*H*	*G*
25,001-50,000	58	35,874	0.620	1.070	0.094	0.227	1.125	0.80	1.20
50,001-100,000	71	72,371	0.620	1.070	0.101	0.220	1.125	0.70	1.20
100,001-250,000	89	154,037	0.620	1.070	0.111	0.210	1.125	0.65	1.10
250,001-500,000	53	360,223	0.620	1.070	0.120	0.203	1.125	0.55	1.10
over 500,000	27	1,290,138	0.620	1.070	0.135	0.188	1.125	0.45	1.10
	298	**251,187**	**0.620**	**1.070**	**0.123**	**0.199**	**1.125**	**0.54**	**1.11**

EXHIBIT 1
Part 2

CALCULATION OF THE PREMIUM IMPACT OF CHANGES IN RETRO PARAMETERS
SAMPLE PLANS-STATE X
(AVERAGE SHG RELATIVITY 0.774)

Rating According To Current Plan									Evaluation With Indicated Changes					
(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)	(25)
ELG	*RG-RH*	*XH-XG*	*RG*	*RH*	*XG*	*SH*	*B*	EXP(*R*)	*E′*	*ELG′*	*XG′*	*SH′*	*EXP(R′)*	((24)-(19))/(19)
69	0.54	0.142	0.81	0.27	0.724	0.136	0.560	0.906	0.620	70	0.733	0.140	0.903	-0.003
60	0.67	0.266	0.78	0.11	0.653	0.031	0.576	0.898	0.620	61	0.662	0.032	0.892	-0.007
50	0.60	0.320	0.70	0.10	0.595	0.014	0.538	0.889	0.620	52	0.611	0.016	0.878	-0.012
39	0.74	0.442	0.87	0.13	0.435	0.009	0.422	0.880	0.620	40	0.444	0.010	0.874	-0.007
29	0.87	0.556	1.04	0.17	0.276	0.003	0.301	0.865	0.620	30	0.286	0.003	0.858	-0.008
								0.877	**0.620**				**0.869**	**-.009**

APPENDIX[6]

For a given update, the distribution of possible column movement (within each Expected Loss Size group or *ELG*) should be determined. Then impacts for 0, 1 or 2 columns can be weighted by the appropriate probabilities to obtain the expected impact.

A good example may be found by starting with a risk in 1992 *ELG* 40. In 1993, the risk may find itself in *ELG* 41 or *ELG* 42, with probabilities determined below:

1992 Group	1993 Group			Probabilities
1992 *ELG* 40 $153,035 to 165,308	1993	*ELG* 41	$156,077 to 168,490	75%
	1993	*ELG* 42	$144,650 to 156,076	25%

The probability of a 1992 *ELG* 40 risk arriving in *ELG* 42 is calculated as a ratio of intervals. For instance:

$$\frac{\text{The segment of old } ELG \text{ 40 in new } ELG \text{ 42}}{\text{old } ELG \text{ 40}} = \frac{156{,}077 - 153{,}035}{165{,}309 - 153{,}035} = 0.25 \ .$$

[6] This degree of care in the estimation was suggested by Howard Mahler.