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REINSURER RISK LOADS FROM MARGINAL SURPLUS REQUIREMENTS

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1. INTRODUCTION

Rodney Kreps's paper contains some useful formulae, and the central idea is an important one. That idea is to determine risk loads by estimating the additional surplus that is required to write an additional contract, and then requiring premium such that the return on additional surplus equals some rate selected by management. The amount of additional surplus is such that writing the contract does not change the probability that the losses from the book of business will cause surplus to fall below zero within the year. The required additional surplus is estimated using the formula

$$\mathrm{Var}(L_1 + L_2) = \mathrm{Var}(L_1) + \mathrm{Var}(L_2) + 2\mathrm{Cov}(L_1, L_2)\,, \tag{1.1}$$

where L_1 is a random variable equal to the ultimate losses from the contract, and L_2 is a random variable equal to the ultimate losses from the rest of the book of business for the next accident year.

I believe this approach could be useful, but it would require some modifications.

2. MARGINAL SURPLUS

If risk loads are estimated based on a required yield on marginal expected value lower than the required yield. An example may be the best way to point this out.

Suppose that a reinsurance company with surplus S has 1,000 contracts on its books, each with standard deviation of losses σ. If the losses from each contract are independent, then the standard deviation of the total losses is $\sqrt{1{,}000}\,\sigma$. Suppose a new contract, independent of the others and with standard deviation σ, is added to the books. The standard deviation of total losses becomes $\sqrt{1{,}001}\,\sigma$. Therefore, if r (in Kreps's terminology) and the desired yield on marginal surplus are small, the marginal surplus is approximately $((\sqrt{1{,}001} - \sqrt{1{,}000})/\sqrt{1{,}000})S$, i.e., about $(1/2{,}000)S$.

So, if Kreps's method is applied to each of the other 1,000 contracts as they are renewed, the sum of the marginal surplus amounts for each will equal approximately half the total surplus and the required yield on marginal surplus will be approximately twice the yield on total surplus.

The "marginal surplus required" as defined in Kreps's paper is related to the increase in the standard deviation of the book of business caused by the additional contract. However, if all the contracts in the book of business were ordered from first to n^{th}, then $\sum_{k=1}^{n} (\sigma_{1,k} - \sigma_{1,k-1}) = \sigma_{1,n}$, where $\sigma_{1,i}$ is the standard deviation of the set of the first i contracts. Kreps's method, however, estimates the effect on the standard deviation of each contract as if it were added at the end of the list, when the marginal effect is less.

If C_k is the k^{th} contract in the above type of ordering and $\Delta_k\sigma$ is the increase in the total standard deviation caused by the addition of the k^{th} contract, then $\sigma_k \geq \Delta_k\sigma \geq \sigma_k^{\text{l}}$ where σ_k is the standard deviation of the k^{th} contract and σ_k^{l} is the effect of the k^{th} contract if it is added at the end of the list, as in Kreps's method. Therefore, there is some W such that, if n is the total number of contracts,

$$\sum_{k=1}^{n} (W\sigma_k + (1 - W)\sigma_k^{\text{l}}) = \text{total standard deviation of the book.} \qquad (2.1)$$

If the contribution of the k^{th} contract to the total standard deviation of the book is estimated as $W\sigma_k + (1 - W)\sigma_k^1$, then the sum of the individual estimates equals the total standard deviation, as it should, and each estimate uses the same weighting.

3. VARIANCE OF LOSS RESERVE

A portion of surplus is needed to support the variance of the loss reserve, and, therefore, some amount may be required for many years to support a new contract. This affects the yield rate, but the paper's discussion of the yield rate on surplus does not address this complication. The method of allocating surplus to contracts based on their effect on total standard deviation could be used for allocating surplus to the various subdivisions of the loss reserves, as well as to contracts. The effect of the standard deviation of the run-off of reserves one year later on the total standard deviation of surplus could be used together with a method for contracts which will be suggested below.

4. DISCOUNTING OF LOSSES

For simplicity, the expected return is defined in the paper as premium less losses and expenses; but in practice, some decision has to be made on discounting losses to correctly reflect economic values in the risk loads. The yield rate on surplus in the paper is based on undiscounted losses without reflecting the time value of money.

5. SUGGESTED METHOD FOR SELECTING RISK LOADS

The two problems mentioned above, i.e., the need for discounted losses and the need for surplus to support loss reserves, can be dealt with simultaneously.

Butsic [1] explains the need to discount loss reserves at a rate lower than the risk-free rate. In this way, when the value of the liabilities is invested at the risk-free rate, there is an expected profit and thus a reward for risk. Myers and Cohn [2] use the Capital Asset Pricing Model (CAPM) to compute this discount rate.

The term "the value of the loss reserve" means that the value is discounted at the above rate. For contract i, let the random variable X_i be the present value at the risk-free rate, on the effective date of the contract, of the losses to be paid in the next year plus the value of the loss reserve at the end of the year. Let σ_i be the standard deviation of X_i . Surplus could be allocated to each contract i by using Kreps's method with the above formula $W\sigma_i + (1 - W)\sigma_i^1$ used in place of σ_i^1 .

The risk load which must be added to $E(X_i)$ to provide the required yield on surplus may then be determined. After the end of the year, the required yield on the portion of surplus allocated to the loss reserves of the contract is provided for by the rate at which loss reserves are discounted, as mentioned above. Also, a portion of surplus should be allocated to assets as well as loss reserves in order to reflect the fact that they are not risk-free. In addition, when surplus is allocated to contracts, loss reserves, and assets, the effect of each on the total risk of the company is considered. Therefore, the covariance between a contract's risk and the entire remainder of the insurer's risk must be considered in formula (1.1).

Kreps's approach is only a way of relating the required return on a new contract to its effect on total standard deviation. There is no consideration of systematic versus unsystematic risk (in the terminology of modern financial theory). The author states that market pricing is consistent with his approach, but financial theorists generally believe that the covariance of stock market returns and the rate of return on surplus must be considered in explaining market pricing. (See Myers and Cohn [2] and Cummins [3].)

The necessary risk load for a high layer may be much different for a small company than for a larger company, using Kreps's method. The larger company is able to diversify away much of the risk. If a company insures a high layer that it is going to reinsure, it would be reasonable for it to charge for that layer based on the actual cost of reinsurance rather than to apply Kreps's method to the gross losses.

REFERENCES

[1] Butsic, Robert P., "Determining the Proper Interest Rate for Loss Reserve Discounting: An Economic Approach," *Evaluating Insurance Company Liabilities*, Casualty Actuarial Society Discussion Paper Program, 1988, p. 147.

[2] Myers, Stewart C. and Cohn, Richard A., "A Discounted Cash Flow Approach to Property-Liability Insurance Rate Regulation," *Fair Rate of Return in Property-Liability Insurance*, 1987, p. 55.

[3] Cummins, J. David, "Asset Pricing Models and Insurance Ratemaking," *ASTIN Bulletin*, Vol. 20, No. 2, 1990, p. 125.