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TESTING FOR SHIFTS IN RESERVE ADEQUACY

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Abstract

This paper develops regression models that can be used to test for the effects of changes in reserving practices. The models include terms for exposure, trend, and loss development. A loss triangle of reported losses at annual valuation dates is used to estimate the parameters of the regression models. Dummy variables are introduced into the loss development factor terms of the models to test for shifts and trends in the loss development factor parameters. The expanded models are estimated, and the parameters associated with the shift and trend variables are tested for significance. If shifts in reserve adequacy are indicated, the models can be used to restate reported incurred losses for the early valuation dates on a basis that is consistent with recent valuation dates. Similar models can be used to test for changes in settlement rates that create changes in the paid loss development pattern. If a change is revealed, the models can be used to estimate the effects of the change.

1. INTRODUCTION

This paper develops regression models that can be used to test for the effects of changes in reserving practices. If shifts in reserve adequacy are indicated, the models can be used to restate reported incurred losses for the early valuation dates in the data sample on a basis that is consistent with recent valuation dates. This topic has been explored by Berquist and Sherman [1] and more recently by Fleming and Mayer [2]. The procedures advocated in both of those approaches rely on subjective estimates for some of the parameters. While actuarial methods rely on the judgment of professionals, the credibility of results is improved when it is possible to obtain objective confirmation of the subjective assessments.

The Berquist and Sherman procedure for testing for shifts in reserve adequacy is to compare, at each valuation, the rate of growth of the per claim reserve for open claims with the rate of growth of the per claim cost for closed claims. They calculate the rate of growth for both averages over the years in the experience period. If reserving practices are consistent, they contend that the rate of growth in average claim reserves should be equal, approximately, to the rate of growth in average closed claims. Unequal rates indicate a change in reserve adequacy over the experience period.

Given a shift in reserving practices, the Berquist-Sherman adjustment for the shift begins by obtaining the rate of inflation in average closed claims. Next, the average reserve at the most recent valuation date is calculated for each year. These average reserves are trended back to earlier valuation dates at the estimated trend rate to obtain the average reserve at each age for each year in the experience period. The computed average reserves are then multiplied by the number of open claims at each age to get the estimated cost of open claims. Cumulative claim payments are then added to get an estimate of incurred losses on a basis that is consistent with current reserving practice.

Fleming and Mayer observed that if there is an increase in the claim closing rate and if claims close at a cost that exceeds the amount reserved, there will be a change in the incurred loss development pattern. They present an addition to the Berquist-Sherman method that adjusts the data for this speed-up in claim settlement rates.

This paper presents a model for estimating reported incurred loss amounts that incorporates a loss development factor (LDF) function. The model is generalized to account for shifts or trends in the LDFs. If the shift or trend parameters are significant, the function can be used to restate incurred losses from prior valuation dates on a basis that is consistent with current levels of reserve adequacy.

2. A MODEL FOR REPORTED LOSS

To develop a regression model for estimating reported incurred losses at each valuation date, one begins by assuming the basic relationship that ultimate loss for year n, Y_n^o, is the product of the number of claims, F_n, and the average claim cost, X_n,

$$Y_n^o = F_n X_n . \tag{2.1}$$

An estimate of the ultimate cost is the reported amount as of a given valuation date, $Y_{n,k}$, times the to-ultimate loss development factor, D_k^, appropriate for the age, k, of the year n. Alternatively, the reported incurred loss can be expressed as the ultimate cost divided by the LDF,

$$Y_{n,k} = Y_n^o / D_k . \tag{2.2}$$

Substituting Equation 2.1 into Equation 2.2 gives

$$Y_{n,k} = F_n X_n / D_k . \tag{2.3}$$

A model is developed for each of these factors.

Before proceeding with further development of the model, a system for numbering the observations must be explained. The numbering system expresses the observation number, t, as a function of n and k. Expressing the matrix of loss data as an array is required when using most regression packages. In addition, the model will contain some variables that are functions of the numbering system. Assume there are N years of loss data with annual valuations of each year's losses.

The loss triangle is arranged as follows:

			Age (k)		
Year (n)	1 (t)	2 (t)	3 (t)	4 (t)	5 (t)
19 x 1	xx 1	xx 6	xx 10	xx 13	xx 15
19 x 2	xx 2	xx 7	xx 11	xx 14	
19 x 3	xx 3	xx 8	xx 12		
19 x 4	xx 4	xx 9			
19 x 5	xx 5				

There are N valuations of the earliest year; $N-1$ valuations of the next earliest. The number of valuations continues to decline until there is one valuation for the most recent year. Assume that the data are arranged such that the first valuations for each of the N years are listed in the first column; the second valuations for each of the $N-1$ years are listed in the second column; and so on. The observation number is

$$t = n + (k-1)(2N - k + 2)/2 \tag{2.4}$$

and $Y_{n,k}$ will be referred to as Y_t.

Specific forms for each of the factors in Equation 2.3 are now developed. The specification of the model for the number of claims assumes that the number of claims for each year is related to a measure of the exposure for that year, E_n. The specific form assumed for the relationship is

$$F_n = a_1 \, E_n^{\,B_0}. \tag{2.5}$$

The standard assumption is that $B_0 = 1$, and Equation 2.5 has the form $F_n = a_1 E_n$. Thus, this form is more general than the standard form. The parameters a_1 and B_0 will be estimated from the company's data.

The model for average claim amount assumes that the average claim size increases exponentially:

$$X_n = a_2\, e^{n B_1}. \tag{2.6}$$

This is the standard form assumed for the trend component of loss costs. Substituting Equations 2.5 and 2.6 into Equation 2.3 gives

$$Y_t = a_1\, E_n{}^{B_0}\, a_2\, e^{n B_1}/D_k\,. \tag{2.7}$$

The specification of the model for the loss development factors consists of two parts. The first part describes the LDF function for early valuations where the LDFs decline fairly rapidly. The second part describes the LDF function for relatively high ages, where the decay toward unity is slight from one valuation to the next. Both branches of the function are assumed to be a trend function with the general form

$$D_k = a_i\, k^{B_j}. \tag{2.8}$$

For the first m valuations, the equation is expressed as

$$D_k = a_3\, k^{B_2},\ \ k = 1, \ldots, m; \tag{2.9}$$

and, similarly, the second part of the function has the equation

$$D_k = a_4\, k^{B_3},\ \ \ k = m + 1, \ldots, N. \tag{2.10}$$

In order to express the LDF function in a more compact form that can be estimated by regression analysis, three additional variables are introduced. First, let $a_4 = a_3\, e^{B_4}$, and $d_1 = 1$ if $k \leq m$ or $d_1 = e$ if $k > m$. Also, let $k_1 = k$ if $k \leq m$ or $k_1 = 1$ and $k > m$. Similarly, $k_2 = 1$ if $k \leq m$ or $k_2 = k$ if $k > m$. Now, the LDF function can be written as

$$D_k = a_3\, d_1{}^{B_4}\, k_1{}^{B_2}\, k_2{}^{B_3}. \tag{2.11}$$

A brief analysis of this model indicates that it is equivalent to Equations 2.9 and 2.10. For the first m observations, d_1 and k_2 are one, and the expression reduces to Equation 2.9. For the last $N - m$ observations $d_1 = e$, $k_1 = 1$, and $k_2 = k$, and Equation 2.11 reduces to Equation 2.10.

When estimating this function, a decision has to be made concerning the size of m, i.e., at which age the function should be branched. This depends on the exposure that is being studied, but branching the function at an age of three to four years usually gives a good fit for casualty exposures.

The LDF function is central to the objective of this paper. Changes in reserving practices must be manifest in changes in the parameters of this function if they are to be detected. Therefore, it is important that the function be capable of providing an excellent fit to the observed development patterns. On the other hand, it is not important that the function be capable of extrapolation outside of the range of the data since its purpose is to identify and measure shifts within the data sample. The particular form used for the LDF function is flexible enough to fit regular loss development patterns, but it is not appropriate for extrapolation to ages outside the data range. For example, the LDF should approach one as the age of the loss data increases, but the LDF from the function specified above approaches zero if $B_3 < 0$. The assumed form of the LDF function has two positive features: its flexibility and its linearity when expressed in logarithmic form.

Substituting Equation 2.11 into Equation 2.7 and combining the a_i gives the expression

$$Y_t = \frac{a_0 E_n^{B_0} e^{n B_1}}{d_1^{B_4} k_1^{B_2} k_2^{B_3}}. \tag{2.12}$$

where $a_0 = a_1 a_2/a_3$. This model will be fit to the Berquist-Sherman data and used to test for a shift in reserve adequacy.

3. ESTIMATION OF THE MODEL

The model developed above is now applied to the Berquist-Sherman Medical Malpractice data. After estimating the model, it is reestimated in several forms that test for a shift in reserve adequacy. Each of the forms tests for a shift in one of the parameters. If the model indicates that a shift has occurred, the data is adjusted for the indicated shift.

The top section of Table 1 reproduces Exhibit A of Berquist-Sherman. The to-ultimate LDFs derived by Berquist and Sherman are labelled least squares estimates (L.S. Est.). Equation 2.11 is fit to this data with the branching occurring after the fourth valuation (48 months) for each year ($m = 4$). Logarithms of both sides of the equation are taken:

$$\ln(D_k) = \ln(a_3) + B_4 \ln(d_1) + B_2 \ln(k_1) + B_3 \ln(k_2). \qquad (3.1)$$

The bottom section of Table 1 gives the results of the least squares estimation of Equation 3.1. All of the coefficients have the anticipated signs and are significant at the 1% level. The coefficient of determination, R^2, is .998, which indicates an excellent fit. The residuals were tested for departures from randomness using the Durbin-Watson test and the von Neumann ratio test. The results of both tests did not indicate a rejection of the null hypothesis of randomness at the 5% level of significance. Autocorrelation in the residuals would be anticipated if the observations were ordered in time (time series data), or if one or more explanatory variables were not included in the model, or if the model being fit to the data had the wrong functional form. The loss development factors are not time dependent observations. Because the error terms exhibit random behavior, the form used to estimate the LDF function has an appropriate shape and includes appropriate explanatory variables.

The actual and estimated loss development factors are compared in Figure 1. The chart demonstrates that the form chosen for the LDF function can give a good fit to the empirical function. An accurate fit is essential if the function is to be used to test for reserve adequacy shifts in the incurred loss data.

The complete model is estimated using the natural logarithms of the reported incurred losses in Table 1. Unfortunately, the Berquist-Sherman paper does not give any exposure data nor the total number of reported claims. To complete the model, the number of claims is estimated from the data and is used as the exposure base for each loss year. Berquist and Sherman report the number of open claims as of each valuation date, and the number of closed claims has been estimated from two of their exhibits. Their Exhibit C gives the average cost of claims closed in the intervals between valuation dates. Their Exhibit E gives the cumulative paid losses

TABLE 1

Medical Malpractice
Incurred Losses (000s Omitted)

Months of Development

Accident Year	12	24	36	48	60	72	84	96	Projected Ultimate
1969	2,897	5,160	10,714	15,228	16,661	20,899	22,892	23,506	23,506
1970	4,828	10,707	16,907	22,840	26,211	31,970	32,316		33,183
1971	5,455	11,941	20,733	30,928	42,395	48,377			52,312
1972	8,732	18,633	32,143	57,196	61,163				79,700
1973	11,228	19,967	50,143	73,733					112,457
1974	8,706	33,459	63,477						145,490
1975	12,928	48,904							215,308
1976	15,791								176,051

Age-to-Age Development Factors

	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-Ult
1969	1.7812	2.0764	1.4213	1.0941	1.2544	1.0954	1.0268	1.0000
1970	2.2177	1.5791	1.3509	1.1476	1.2197	1.0108		
1971	2.1890	1.7363	1.4917	1.3708	1.1411			
1972	2.1339	1.7251	1.7794	1.0694				
1973	1.7783	2.5113	1.4705					
1974	3.8432	1.8972						
1975	3.7828							

Average Incurred Loss Development Factors

	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-Ult
Average	2.5323	1.9209	1.5028	1.1705	1.2051	1.0531	1.0268	1.0000
Cum.	11.1488	4.4027	2.2920	1.5252	1.3031	1.0813	1.0268	1.0000
L.S. Est.	11.3864	4.1892	2.3340	1.5412	1.2578	1.1369	1.0438	1.0000

Regression Output:

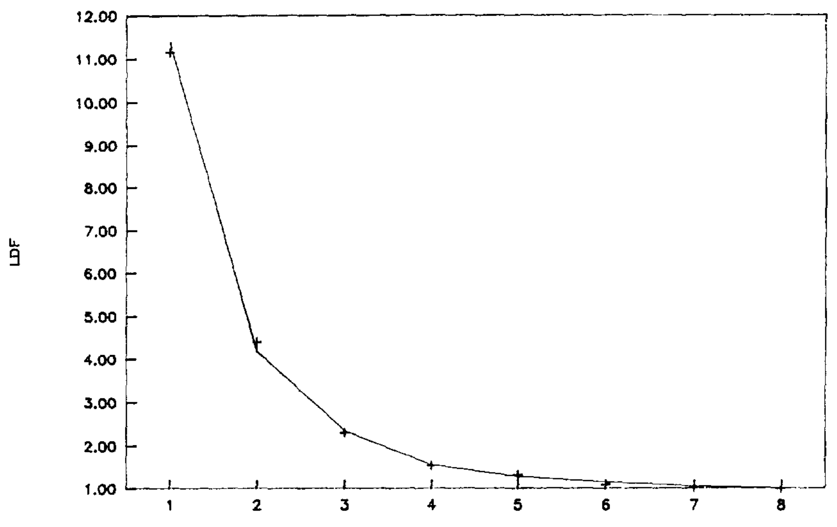
	$\ln(a_3)$	B_2	B_3	B_4	R^2
X Coefficient(s)	2.432	-1.443	-0.554	-1.311	0.998
Standard Error of Coefficient		0.044	0.130	0.247	

as of each valuation date. By subtraction, the amount paid between valuation dates is determined. Dividing the average claim payment into the total amount paid is used to approximate the number of claims closed during the period. These closed claim counts are accumulated from period to period. The open claims at each valuation are added to the total number closed to date to give the reported claim counts. The reported claim counts are developed to an estimated ultimate number of claims for each year. The estimated claim counts and their development are presented in Table 2.

Given the estimated claim count for each year, numbering the years from one to eight, and assigning d_1, k_1, and k_2 their values as defined above, Equation 2.12 is estimated by taking the natural logarithms of both sides and using least squares regression. The results of the estimation are reported on Table 3. The error terms are tested for autocorrelation using

FIGURE 1

LOSS DEVELOPMENT FACTORS—ACTUAL VERSUS ESTIMATED



Age (Years)

— Estimated + Actual

TABLE 2

MEDICAL MALPRACTICE
NUMBER OF REPORTED CLAIMS

Months of Development

Accident Year	12	24	36	48	60	72	84	96	Projected Ultimate
1969	1,060	1,672	2,182	2,566	2,555	2,579	2,608	2,625	2,625
1970	1,051	1,877	2,340	2,719	2,777	2,804	2,828		2,846
1971	1,296	2,511	3,138	3,743	3,859	3,909			3,973
1972	1,354	2,725	3,515	4,210	4,459				4,581
1973	1,382	2,828	3,671	4,665					4,921
1974	1,365	2,765	3,623						4,586
1975	1,544	2,785							4,524
1976	1,594								4,879

Age-to-Age Development Factors

	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-Ult
1969	1.578	1.305	1.176	0.996	1.009	1.011	1.007	1.000
1970	1.786	1.247	1.162	1.021	1.010	1.009		
1971	1.938	1.250	1.193	1.031	1.013			
1972	2.013	1.290	1.198	1.059				
1973	2.046	1.298	1.271					
1974	2.026	1.310						
1975	1.804							

Average Claim Count Development Factors

	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-Ult
Average	1.884	1.283	1.200	1.027	1.011	1.010	1.007	1.000
Cum.	3.061	1.624	1.266	1.055	1.027	1.016	1.007	1.000

TABLE 3

BASE MODEL
ESTIMATED LOSSES (000S OMITTED)

Accident Year	Months of Development 12	24	36	48	60	72	84	96
1969	2,571	6,617	11,503	17,029	19,676	21,606	23,383	25,041
1970	3,421	8,804	15,304	22,657	26,180	28,746	31,112	
1971	5,426	13,965	24,276	35,940	41,527	45,598		
1972	7,535	19,393	33,713	49,910	57,670			
1973	9,962	25,638	44,569	65,982				
1974	11,930	30,703	53,375					
1975	14,865	38,256						
1976	19,706							

Regression Output:

Constant	$\ln(a_0) =$ 2.149		$a_0 =$ 8.576		
Standard Error of Y Est.	0.163				
R^2	0.964				
Number of Observations	36				
Degrees of Freedom	30				
	B_0	B_1	B_2	B_3	B_4
X Coefficient(s)	0.695	0.229	-1.364	-0.513	-1.210
Standard Error of Coefficient	0.239	0.031	0.064	0.329	0.585
Durbin-Watson	$D = 1.916$				
Trend	$\text{Exp}(B_1) =$ 1.258				

the Durbin-Watson statistic, $D = 1.916$. This is very close to the expected value, and the hypothesis of independent error terms is accepted. The data are not a time series in the normal sense. The data are ordered such that the 12-month valuations for all years are grouped, then the 24-month valuations, etc. The presence of independent error terms indicates that the estimates at each age are neither too large nor too small.

The bottom section indicates that the fit is excellent with a coefficient of determination, R^2, of .964. All of the individual coefficients are significant at the 5% level, with the exception of B_3, which is about 1.56 standard errors above zero. Berquist and Sherman estimated a trend in average claim costs of about 30%, whereas this analysis indicates a trend of 25.8% in the average claim cost. The estimated development of incurred losses using the model is reported in the top portion of Table 3. These may be compared to the actual values which are reported in Table 1. Since the year-to-year development is variable, there are some substantial differences between the individual estimates and the observed values, but, on the whole, the fit is good. Thus, a model that gives good estimates of reported loss amounts has been developed. In the next section, the model will be modified to test changes in loss development patterns. If the revised models give superior results, reserving practices will have changed during the sample period.

4. TESTS FOR RESERVING CHANGES

If a shift has occurred in reserving patterns, it would be reflected in a change in the parameters of the LDF function, Equation 2.11. There are several parameters that might change with a shift in reserving. The coefficients a_3 and a_4 could be affected and/or the exponents B_2 and B_3 might change. These possibilities are explored beginning with testing for changes in the coefficients a_3 and a_4.

One procedure for testing for a shift in the parameters is to introduce a variable that has a value of unity for valuations that occurred prior to a certain date, and a value of e for valuations after that date. All of the reported losses on the last diagonal of the loss development triangle have the same valuation date. The diagonal elements of the loss development triangle have values of $t = k(2N - k + 1)/2$. Assume that the p most recent valuations reflect the change in reserving practices, then define $d_2 = 1$ if $t \leq k(2N - k + 1)/2 - p$, and $d_2 = e$ if $t > k(2N - k + 1)/2 - p$. Introducing d_2 into Equation 2.12 gives

$$Y_t = \frac{a_0 E_n{}^{B_0} e^{n B_1}}{d_1{}^{B_4} d_2{}^{B_5} k_1{}^{B_2} k_2{}^{B_3}} . \tag{4.1}$$

This equation has been estimated, and the results are given in Table 4.

The coefficient of determination increases slightly from .964 to .972 with the addition of the new variable. The Durbin-Watson statistic is 2.194, indicating a small, insignificant amount of negative autocorrelation in the error terms. The coefficient B_3 is substantially less significant than in Model 1; however, the coefficient for the shift variable, B_5, is highly significant, and indicates that the more recent reported incurred losses are 27.4% larger, on average, than the estimates at the earlier valuations. Also, the estimated trend has decreased from 25.8% to 18.5%. The trends estimated by Berquist and Sherman dropped from 30% to 15%.

The estimates obtained from this model can be used to restate the reported incurred losses for the earlier valuations on a basis consistent with the reported incurred losses for more recent valuations. The early valuations can be increased by 27.4%, to adjust for the indicated shift in the estimates that has occurred during the past two years. This adjustment has been made for the malpractice data, and the results are displayed in Table 5. The last two diagonals of Table 5 are the same as the corresponding numbers in Table 1. All of the numbers above the last two diagonals have been increased by the indicated 27.4%. The restatement results in lower loss development factors and substantially lower estimates of ultimate incurred losses for the more recent years.

To test for a shift in the exponents B_2 and B_3, two variables are added to Equation 2.12. The first variable, d_3, is assigned a value of unity for valuations before the cutoff date, and a value of k_1 after the cutoff date, i.e., for the p most recent valuations for each year. Thus, $d_3 = 1$ if $t \leq k(2N - k + 1)/2 - p$, and $d_3 = k_1$ if $t > k(2N - k + 1)/2 - p$. The second variable, d_4, is also assigned a value of unity for valuations before the

TABLE 4

MEDICAL MALPRACTICE
MODEL 2 ESTIMATED LOSSES (000S OMITTED)

Accident Year	Months of Development: 12	24	36	48	60	72	84	96
1969	2,874	7,005	11,796	17,073	18,631	18,936	24,456	24,748
1970	3,632	8,854	14,909	21,580	23,549	30,490	30,911	
1971	5,612	13,678	23,034	33,340	46,347	47,105		
1972	7,449	18,156	30,574	56,373	61,517			
1973	9,346	22,781	48,869	70,734				
1974	10,475	32,526	54,773					
1975	15,649	38,143						
1976	19,698							

Regression Output:

Constant	$\ln(a_0) =$ 1.543	$a_0 =$	4.677
Standard Error of Y Est.	0.147		
R^2	0.972		
Number of Observations	36		
Degrees of Freedom	29		

	B_0	B_1	B_2	B_3	B_4	B_5
X Coefficient(s)	0.794	0.170	-1.285	-0.089	-1.726	-0.242
Standard Error of Coefficient	0.218	0.035	0.064	0.332	0.559	0.086

Durbin-Watson	$D = 2.194$
Trend	$\text{Exp}(B_1) =$ 1.185
Shift	$\text{Exp}(B_5) =$ 1.274

TABLE 5

MEDICAL MALPRACTICE

B_5 ADJUSTED INCURRED LOSSES (000S OMITTED)

Months of Development

Accident Year	12	24	36	48	60	72	84	96	Projected Ultimate
1969	3,690	6,573	13,648	19,399	21,224	26,623	22,892	23,506	23,506
1970	6,150	13,639	21,537	29,095	33,390	31,970	32,316		33,183
1971	6,949	15,211	26,411	39,398	42,395	48,377			46,463
1972	11,123	23,736	40,946	57,196	61,163				65,654
1973	14,303	25,435	50,143	73,733					86,807
1974	11,090	33,459	63,477						106,587
1975	12,928	48,904							150,347
1976	15,791								117,204

Age-to-Age Development Factors

	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-Ult
1969	1.7812	2.0764	1.4213	1.0941	1.2544	0.8599	1.0268	1.0000
1970	2.2177	1.5791	1.3509	1.1476	0.9575	1.0108		
1971	2.1890	1.7363	1.4917	1.0761	1.1411			
1972	2.1339	1.7251	1.3969	1.0694				
1973	1.7783	1.9714	1.4705					
1974	3.0169	1.8972						
1975	3.7828							

Average Incurred Loss Development Factors

	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-Ult
Average	2.4143	1.8309	1.4263	1.0968	1.1177	0.9353	1.0268	1.0000
Cum.	7.4222	3.0743	1.6791	1.1773	1.0734	0.9604	1.0268	1.0000

cutoff date and a value of k_2 for valuations after the cutoff date. With these two variables included, the new equation becomes

$$Y_t = \frac{a_0 \, E_n^{\;B_0} \, e^{n B_1}}{d_1^{\;B_4} \, d_3^{\;B_6} \, d_4^{\;B_7} \, k_1^{\;B_2} \, k_2^{\;B_3}} . \tag{4.2}$$

Equation 4.2 has been fit to the Berquist-Sherman data, and the results are summarized in Table 6. The coefficient of determination is marginally higher than for Equation 4.1, and the Durbin-Watson statistic is 2.3887, indicating an insignificant ($\alpha = .05$) amount of negative autocorrelation in the error terms. As before, all of the coefficients are significant with the exception of B_3, and in this case, B_3 entered with the wrong sign. This model gives a higher estimate of the trend factor than the previous model by about 4.5 percentage points.

A small table has been inserted to indicate the average ratio of losses valued after the critical date to losses valued before the critical date. The ratios for this model vary with the age of the data at the valuation date. The ratios range from no adjustments for 12-month valuations to a 50.5% adjustment for 48-month valuations. These adjustments have been applied to the loss data, and the results are displayed in Table 7. As for the previous model, the adjusted estimates of ultimate incurred loss are considerably lower than for the unadjusted data.

A combined form of Equations 4.1 and 4.2 that included d_2, d_3, and d_4 was estimated. The variables d_3 and d_4 entered as significant, but d_2 was not significant. This indicates that Equation 4.2 is the appropriate model to describe the shift in the reserving practices for these data.

5. SUMMARY

A procedure that tests for changes in loss development patterns in an objective manner has been demonstrated. If a change is observed, the models developed can be used to restate the early valuations on a basis that is consistent with the current valuations. These models cannot replace the judgment of the actuary, but they do provide an additional tool with which to analyze this problem.

TABLE 6

MEDICAL MALPRACTICE
MODEL 3 ESTIMATED LOSSES (000S OMITTED)

Accident Year	Months of Development 12	24	36	48	60	72	84	96
1969	2,988	6,964	11,426	16,234	19,286	19,263	24,344	24,718
1970	3,841	8,953	14,688	20,870	24,794	30,750	31,296	
1971	5,670	13,217	21,684	30,810	44,460	45,396		
1972	7,539	17,574	28,832	61,647	59,116			
1973	9,643	22,479	50,984	78,853				
1974	11,411	32,632	60,334					
1975	13,932	39,840						
1976	17,860							

LOSS ADJUSTMENT MULTIPLIERS

k_1	$k_1^{(-B_6)}$	k_2	$k_2^{(-B_7)}$
1	1.000	5	1.215
2	1.227	6	1.242
3	1.382	7	1.265
4	1.505	8	1.286

Regression Output:
Constant $\ln(a_0) = 3.489$ $a_0 = 32.744$

Standard Error of Y Est.	0.133						
R^2	0.978						
Number of Observations	36						
Degrees of Freedom	28						
	B_0	B_1	B_2	B_3	B_4	B_6	B_7
X Coefficient(s)	0.5470	0.2070	-1.2210	0.0066	-1.8756	-0.2948	-0.1208
Standard Error of Coefficient	0.2053	0.0267	0.0637	0.3524	0.5695	0.0745	0.0659

Durbin-Watson $D = 2.3887$ Trend $\text{Exp}(B_1) =$ 1.2300

TABLE 7

MEDICAL MALPRACTICE
B_6 AND B_7 ADJUSTED INCURRED LOSSES (000S OMITTED)

Months of Development

Accident Year	12	24	36	48	60	72	84	96	Projected Ultimate
1969	2,897	6,330	14,812	22,916	20,238	25,951	22,892	23,506	23,506
1970	4,828	13,134	23,374	34,371	31,838	31,970	32,316		33,183
1971	5,455	14,648	28,663	46,542	42,395	48,377			47,016
1972	8,732	22,857	44,437	57,196	61,163				67,913
1973	11,228	24,494	50,143	73,733					77,567
1974	8,706	33,459	63,477						98,816
1975	12,928	48,904							151,813
1976	15,791								140,169

Age-to-Age Development Factors

	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-Ult
1969	2.1850	2.3400	1.5471	0.8831	1.2823	0.8821	1.0268	1.0000
1970	2.7205	1.7796	1.4705	0.9263	1.0041	1.0108		
1971	2.6853	1.9567	1.6238	0.9109	1.1411			
1972	2.6177	1.9441	1.2871	1.0694				
1973	2.1815	2.0472	1.4705					
1974	3.8432	1.8972						
1975	3.7828							

Average Incurred Loss Development Factors

Average	2.8594	1.9941	1.4798	0.9474	1.1425	0.9465	1.0268	1.0000
Cum.	8.8765	3.1043	1.5567	1.0520	1.1104	0.9719	1.0268	1.0000

The models that have been illustrated test for a change in reserving practices as of a specified date. Models that will detect a trend in the loss development factors, rather than an abrupt change in the factors as of the specified date, can also be employed. As above, one can test for a trend in the coefficients, a_3 and a_4, or in the exponents, B_2 and B_3. All data on the same diagonal of the loss development triangle have the same valuation date and are given the same time index of $g = n + k - 1$. This index numbers the diagonals beginning with one in the northwest corner of the matrix and increases by one for each diagonal added to the triangle. The LDF model that estimates and tests for a trend in the exponents is

$$D_t = a_3 \, k_1^{\,(B_2 + gB_8)} \, k_2^{\,(B_3 + gB_9)} \, d_1^{\,B_4}. \tag{5.1}$$

Finally, a model for the LDF function that includes a trend factor for the coefficients is

$$D_t = a_3 \, a_5^{\,g} \, k_1^{\,B_2} \, k_2^{\,B_3} \, d_1^{\,B_4}. \tag{5.2}$$

Both Equation 5.1 and Equation 5.2 have been fit to the Berquist-Sherman data, but the results were not as significant as the models that incorporated a jump in the parameters. The results of the estimation are not reported.

Similar models can be employed to test for changes in claim settlement rates that are reflected in changes in paid loss development factors. If paid losses are substituted for reported losses as the dependent variable and the loss development function is interpreted as the paid loss development function, the models can be used to test for parameter changes in the same manner.

This paper has shown how regression models can be used to estimate the effects of changes in reserving practices. Once the effects have been estimated, the appropriate adjustments can be made to past valuations to restate them on a basis consistent with current reserving practices. The models allow one to test for abrupt changes in reserving practices versus changes that emerge progressively. This procedure is flexible and objective.

REFERENCES

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