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EXPOSURE BASES REVISITED

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DISCUSSION BY CHRISTOPHER DIAMANTOUKOS

If ye continue in my word, then are ye my disciples indeed; and ye shall know the truth, and the truth shall make you free.

John VIII, 31–32

I am grateful to David E. A. Sanders for reviewing my thoughts and helping me appreciate the scope and the difficulty of the problem in trying to measure exposures.

The focus of this paper is on some fairly difficult and sophisticated concepts that may not become obvious to the reader upon an initial reading. One key concept is that of "true exposure" as presented by Bouska, and its proxy provided by an exposure base. There are no complicated formulae presented in the paper, yet Bouska's observations that the subject is indeed complicated and intricate were realized during the discussions at the CAS Annual Meeting concurrent session where this paper was presented.

1. TRUE EXPOSURE

What is "exposure," or, better yet, what is "true exposure?" The paper offers little explanation beyond the "exposure to loss" definition first presented by Dorweiler [1].

The true exposure is a complex and changing characteristic of a risk. Intuitively, the true exposure of a risk can be viewed as the summation (integration) over the term of the policy of the random variable "insurable losses." The risk's "exposure to loss," its inherent insurable loss (risk pure premium), can change at any time during the policy period.

The "exposure" represents a measurable physical characteristic of the risk that is a dimensional translation of the expected value (mean) of the true exposure. The dimensions are dollars, which measure the mean true exposure, and whatever dimension the exposure is measured in: square feet, car-years, etc. The exposure pure premium, measured with respect to units of the exposure base, is the scalar reflecting the translation. For example, a class of risks might use miles driven as the exposure for automobile liability coverage. The exposure pure premium, $5 per 1,000 miles driven (a scalar of 5), translates the mean of the true exposure, measured in dollars, to units of the exposure base, units of 1,000 miles driven. To simplify analysis, true exposure might best be measured based on "full coverage" insurable losses, where coverage is defined by a specific combination of insured perils. Exposure pure premium estimates make use of risk characteristics, usually reflecting the entire term of the policy, such as classification and geographical location.

The term "risk" was mentioned several times in the last two paragraphs. The concept of an individual risk can be difficult to define and understand. Its definition is essential in order to measure true exposure or to count the number of units of an exposure base. The insuring of a large number of independent risks, or the "pooling of risks," is fundamental to making an insurance process work. The ideal is to pool (insure) a population of homogeneous risks. This ideal is rarely achieved, even at the classification level. Exposures measure differences in risk size. The exposures of an individual risk are clearly not independent, although rates are measured with respect to the pooled exposures of many risks.

Conceptually, for primary insurance coverages, risks can be thought of as representing the indivisible and independent entities to which coverages can be attached. Attachment results from the association of the coverage and the risk through the actual wording of the coverage form (insurance policy), particularly the insuring agreement. The association might be through the specification of an insured location, or the type of insured business activity conducted at certain premises of the insured, or the services provided by an insured at various points in time during the policy period, etc. For example, automobile liability coverage is attached to a vehicle and not to the insured.

Risks are normally independent of each other and form the basis from which true exposure is measured. Examples of familiar risk definitions include a building for fire coverage, a restaurant for premises liability coverage, and a vehicle for various forms of automobile liability and physical damage coverages. Bouska addresses the limitations on using exposures for "measuring" large risks.

An individual claim, or loss, is normally associated with one, and only one, risk. Exceptions to this include long-term exposure injuries, such as those resulting from exposure to asbestos. The injured claimant might make one "claim" resulting in payments from the several policies in force during the claimant's exposure to asbestos. In some cases there may be a single risk giving rise to the claimant's injuries over the course of several consecutive policies.

David E. A. Sanders of Eagle Star Insurance Company, London, England, has suggested a quadrilateral that integrates the concepts of risk, exposure, true exposure, and expected losses. For a policy period for a given risk, he relates the concept through a diagram:

$$\begin{array}{ccc} \text{Risk Period} & \longrightarrow & \text{Expected Losses} \\ \downarrow & & \uparrow \\ \text{Exposure Period} & \longrightarrow & \text{True Exposure} \end{array}$$

Each arrow represents a function unique to each risk and policy period. These functions transform, or map, one variable to another as indicated in the diagram. Risk period is identical to policy period. Exposure period reflects the time during the risk period that the risk is "exposed" to loss, as well as the variation in that exposure over time. The aggregation of independent risks to create an insurance process through application of the Law of Large Numbers (Central Limit Theorem) presents a major problem because these functions are different for each risk.

In order to help focus on the concept of true exposure and why it is so difficult to measure, mathematical notation will be introduced for the variables of a policy that are germane. This will also provide the groundwork for a theoretical presentation of different types of exposure bases. Let $\Pi(T)_i$ represent the insurable losses for risk i from the inception of a policy until time T. This will be termed the risk's pure premium when T equals the expiration date of the policy. It is a random variable representing the dollar value of the aggregate losses during the policy period. Of particular interest is the change in the risk's pure premium with respect to time; i.e., its derivative. It can be integrated over the term of the policy to obtain the mean of the risk's pure premium.

Note that the characteristics of the risk affecting its pure premium can change during the policy period. These characteristics can also be considered as dependent on time. Not included in this discussion are those characteristics affecting pure premium that are external to the risk, such as the judicial climate affecting liability awards. Bouska discusses risk characteristics in the section of her paper entitled "Problems: Complexity of Hazard." If it is assumed that there is a finite number n of such characteristics, and letting the n-dimensional vector $X_i(t,n) = (\chi_1(t), \chi_2(t), \ldots, \chi_n(t))$ represent their respective values at time t, an equation defining the mean of the true exposure is:

$$\Pi(T)_i = \int_0^T \int_0^\infty \nabla_t \Pi(X_i(t,n)) f(\nabla_t \Pi(X_i(t,n)),t) d(\nabla_t \Pi_i) dt. \tag{1.1}$$

$\nabla_t \Pi_i$ represents the change in the pure premium for risk i with respect to time during the policy period. It is the partial derivative, or gradient, of the risk pure premium with respect to time. It is a random variable whose density function may also change over time.

This equation sheds light on Bouska's observation that one can never know the true exposure or make measurements of its mean or moments. The digital limitations of the most extensive data gathering mechanisms can only approximate the measurements of all the variables (characteristics) that range over continua needed to completely measure the true exposure for any given risk, or even for an entire homogeneous class of risks. Examples of variables ranging over continua include time (obviously) and economic activity as reflected to varying degrees in commercial fire and general liability classifications. It is left to actuaries to use all available information and experience, including measurements of exposures and classifications, to find a "best" approximation to true exposure.

A simplifying assumption that can be made is that the risk pure premium does not change. Time intervals of coverage could be made as small as necessary to achieve any desired degree of conformity to this assumption. The assumption of a constant risk pure premium might be stronger for seasonal coverages if time intervals focus on the operations or values at greatest risk. Examples include summer months for seashore amusement parks or hurricane season where windstorm coverages are sold. Let $\bar{f}$ represent this constant density function. The mean true exposure equation simplifies to:

$$\Pi(T)_i = \int_0^T \int_0^\infty \nabla_t \Pi(X_i(t,n)) \bar{f}(\nabla_t \Pi(X_i(t,n))) d(\nabla_t \Pi_i) dt; \qquad (1.2)$$

$$\Pi(T)_i = \int_0^T \mu[\nabla_t \Pi(X_i(t,n))] dt. \qquad (1.3)$$

At any point in time, the mean pure premium gradient can be decomposed into the product of the means of its frequency (N) and severity (δ) components. If the dimensions of dollars of insurable losses, number of claims, and time are reflected properly, equation (1.3) becomes:

$$\Pi(T)_i = \int_0^T \mu[\nabla_t N(X_i(t,n))] \mu[\delta(X_i(t,n))] dt. \qquad (1.4)$$

This equation represents the integration over time of the product of the change in the number of claims with the mean claim severity. Equation (1.4) should be compared with Bouska's equation (4):

> (number of exposure base units) × (expected number of losses per exposure base unit) × (expected dollars per loss) = expected losses

The integration over time has been substituted by the number of exposure base units in Bouska's equation (4). This is the result of the simplification offered by the use of exposures, discussed in the next section.

2. THE EXPOSURE SIMPLIFICATION

As mentioned earlier, the simplification to estimating the mean true exposure offered by the use of exposure bases is to determine some physically measurable risk characteristic that is directly proportional to the mean of the true exposure and use it as the exposure base. Note that the exposure base should ideally be susceptible to very accurate measurement and is not a transformation of the random variable true exposure. After classifying a risk, multiplying the exposure pure premium by the number of units of the exposure base for the policy period gives the expected pure premium for the risk. The chosen unit of the exposure base provides a numerical scaling to the values measured. It also determines the scalar value reflected in the dimensional translation between the exposure base and the true exposure.

To help explain the meaning of the simplification just presented, some assumptions will be made. First, the period of coverage will be taken to be one unit time period. Second, the characteristics of the risk are assumed to be fixed over the period of coverage. Third, the partial derivative of the risk pure premium with respect to time does not change over the period of coverage; i.e., it is a constant. Equation (1.4) then reduces to:

$$\Pi(1)_i = \mu[N(X_i)]\mu[\delta(X_i)] = \mu[\Pi(X_i)]. \tag{2.1}$$

Consider several examples of exposure bases. First, consider fire insurance, where the exposure is taken to be amount of insurance. In practice, the exposure is amount of insurance-years to take into account policy term and insured values that change significantly over time, for example, policies written on a reporting form basis. From a theoretical perspective, if full insurance to value is assumed (100% coinsurance) to be full coverage, then, at any point in time, current actual value is a better exposure base. The actual value should affect the mean severity of loss and not the frequency of loss. A normalized severity distribution of the type discussed by G. L. Head [2] could be used. The normalized severity ($\bar{\delta}$) ranges over the interval [0,1]. It represents severity as a percent of value. Multiplying by the number of units of the exposure base (ε) representing a constant actual value yields the expected severity.

By invoking the exposure simplification, the mean true exposure equation would then take the form:

$$\Pi(T)_i = \mu[N(X_i)]\mu[\tilde{\delta}(X_i)]\varepsilon_i. \qquad (2.2)$$

For liability coverages, increased limits factors are used to adjust base rates to reflect the expected increased severity of the higher limits purchased. Claim frequency is reflected by measuring units of the exposure base. One class of liability exposures involves a gradient of the exposure base with respect to time, such as sales per unit time. Integration will yield the total earned exposure over the policy period. Given an estimate of the mean frequency of claims per unit exposure and a constant gradient of the exposure with respect to time, the liability form of the mean true exposure equation for this first class of liability risks becomes:

$$\Pi(T)_i = \mu[\nu(X_i)]\mu[\delta(X_i)]\varepsilon_i. \qquad (2.3)$$

An example of an exposure in this case might be sales in dollars over a period of one year. Note that the mean claim frequency per unit exposure is $\mu[\nu(X_i)]$.

A second class of liability exposures involves bases such as square feet. These are constant level exposures represented as annualized values. In such cases, the gradient of the exposure with respect to time is zero. An example of a normalized exposure in this second class is number of square-feet-years. The actual normalized exposure reflects the annualized exposure basis used, such as per 1,000 square-feet-years. The analysis is similar to that of fire, substituting *frequency* proportional to the exposure base instead of *severity* being proportional to the exposure base as was the case in fire. The form is identical to equation (2.3) with the added simplification that the gradient of the frequency with respect to the constant level in force exposure is constant.

Finally, for workers compensation indemnity coverage, payroll can be considered as being directly proportional to the pure premium itself. The severity of indemnity losses increases with payroll per worker while the frequency of loss increases with workers per risk. Using payroll as the exposure base also employs a gradient of payroll with respect to time. The simplified mean true exposure equation becomes:

$$\Pi(1)_i = \mu[\pi(X_i)]\varepsilon_i. \tag{2.4}$$

Note that the mean pure premium per unit of exposure is represented by $\mu[\pi(X_i)]$. It is instructive to review Section 4 from the paper in light of the development above.

3. PROBLEMS IN CHOOSING AN EXPOSURE BASE

When Bouska presented the paper, she spoke of an emerging exposure base problem in aviation liability insurance. The paper's discussion of the workers compensation exposure base problem presents a theme similar to the aviation problem. Rate inequities in aviation are perceived due to the use of revenue passenger-miles as the exposure. Many causes of losses are related to takeoffs and landings. Hence, some commuter airlines, due to increased flight frequency, are thought to generate greater "exposure to loss" than might be indicated by using the existing exposure base. In addition, the growth of such commuter airlines results in a change to the underlying population upon which aviation liability rates are based. The previous underlying population that was used to set rates had a greater proportion of "long-distance" airlines.

This brief discussion of aviation liability insurance echoes observations made when the paper was presented: the measurement of true exposure must recognize frequency *and* severity components. Some exposure bases may be more responsive to frequency than they are to severity, or vice versa. The best solution to approximating the true exposure in some cases might be to utilize more than one exposure base. Two exposure bases might be used, one for frequency and the other for severity. For example, in the case of aviation liability insurance, the number of flights may be directly related to the frequency of losses while the number of passengers per flight relates to severity. Capturing the actual distribution of passengers per flight would focus even more sharply on true exposure.

The "dividing line" between exposure bases and rating variables that Bouska speaks of may need to be "crossed" sometimes. Workers compensation may have "solved" an "exposure base question" by addressing other parts of the rating system. Perhaps some changes in classification structure might solve some of the problems for some aviation liability risks. There is precedence in automobile liability where miles driven, a potential exposure base, perhaps a theoretically superior one in some respects, is reflected in classification values. There may be more than one acceptable solution for separating risk characteristics between potential exposure bases and rating variables.

Time was introduced as a continuum over which the change in pure premium could be measured. Some other continua were mentioned earlier. One discussed in the paper is size as measured in units of the exposure base. Size is a risk characteristic and there theoretically exists a function relating size to true exposure, or at least relating size to the mean true exposure. The paper discusses how less weight is put on the manual premium as the size of a risk increases. Size in the paper is measured by the mean of the risk's pure premium, which in turn depends on the true exposure. Bouska's observation of a decreased dependency of the charged premium (reflecting true exposure) with an increase in size of risk means that the number of units of the exposure base becomes less important. Greater credibility is given to a risk's experience as its actual loss (observed pure premium) experience increases.

The calculation of pure premiums, frequencies, and severities should still be of value in analyzing the true exposure of the large risk. The mean true exposure as a function of size does not necessarily have to approach a limiting value as size increases without bound. An alternative hypothesis is that the variation in the relationship between size and the mean true exposure becomes large enough to warrant giving increased credibility to the individual risk's loss experience, and less credibility to the exposure pure premium of the classification to which the risk belongs. Use of an exposure size function, one that measures the dependence of a risk's pure premium on its exposure size, could reinforce the use of exposures in the analysis of the very large risks. It would also be of interest if other risk characteristics have greater influence on the true exposure than size.

Another candidate for a continuum gradient is the rate of interest to discount loss payments if insurable losses are measured on a present value basis. This could be of particular interest in the long-tailed lines.

One of the problems discussed in Section 6 of the paper is that of temporal mismatch. The first example presented is that of claims-made policies. The problem involves the influence of the coverage provided on the measurement and choice of an exposure base. A careful description of coverage/policy forms is essential to clearly distinguish what is meant by coverage and what are considered coverage limitations, conditions, and exclusions. For the sake of this discussion, coverage refers to the underlying perils insured against without "limitations."

The temporal mismatch of claims-made coverage is caused by reporting limitations on covered claims. Another type of limitation that could cause temporal mismatch involves policy limits and aggregates. Recently introduced claims-made tail coverages allow the reinstatement of aggregate limits. Reinstatements have their greatest impact on rates, but they also affect understanding true exposure and how to select exposure bases. True "full coverage" would provide for unlimited reinstatements; but, in today's environment, aggregate limitations have become very important. The discussion by Bouska of products liability in this same section of the paper offers the "products in use during the year" alternative exposure base. This represents another case where true exposure could become a function of coverage limitations. The amount of "products used" for which liability coverage is provided could be limited.

Yet another example of temporal mismatch is provided in building fire coverage. If a building is significantly damaged by fire, there is clearly reduced value at risk until such time as the building is repaired. But what would happen if the building is fully restored before the end of the policy period? It would appear that coverage is reinstated for the duration of the policy and that a true exposure exists reflecting the restored value rather than the original insured value. The coverage rate might also be used to reflect this possibility.

The detailed mathematical presentation earlier dealt solely with approximations to the mean true exposure. There are density functions associated with the random variables used in the development that will affect the density functions of the true exposure. The estimation of the density function of aggregate losses is another approach that recognizes such randomness in the process. The approach is complicated if it must derive an estimate for a risk with an inherent risk pure premium that changes over time.

At the end of the subsection "General Liability: Area vs. Receipts," the author reflects on the passage of time as the answer to the question of how to derive a better exposure base. I agree; it would take considerable experience on a risk by risk basis, with the collection of several exposure base candidates, to answer the question. This was not done for general liability during the recent ISO conversion process.

Size is a key consideration when determining a good exposure base, or when choosing between exposure base candidates. Experience by individual risk over time is perhaps more important than experience by size groupings for making this determination. This is particularly true if the insured population included in an accident year evaluation of a size grouping changes from one year to the next. The focus would be on the relationship of size in units of the exposure base to the true exposure (or its mean). It would take extensive experience on a risk-by-risk basis to make such a determination because of the skewed nature and high variance of the distributions of risk pure premiums. However, any attempt at an analysis by risk over time would be hampered if the importance of exposures is played down because the individual "large" risk's influence on its expected pure premium increases.

The validity of basing experience rating for large risks on class pure premiums seems questionable if large risks are excluded from the underlying population of their "class." Furthermore, the appropriateness of class pure premiums can also be called into question when the underlying risk population is changing or eroding (depopulating) over time. Such erosion may have already happened in some classifications as a result of the migration of general liability risks and class-rated fire risks to the indivisible businessowners types of policies.

The use of an inflation-sensitive exposure can make it harder to understand the relationships over time between the exposure and the true exposure, pure premiums, or claim frequencies. For example, unadjusted claim frequencies per amounts of insurance over time for fire insurance should not be used at face value. The effects of trend (i.e., exposure trend reflecting increasing property values) must be factored out to allow for a consistent time series analysis. However, one can never tell exactly how much an exposure change for an individual risk is due to inflation, which affects all risks, and how much is due to a real increase in risk size. Some exposure bases directly reflect monetary inflation (at a minimum); e.g., "current" actual value.

However, a physical, measurable exposure would vary directly with the inherent pure premium of the risk as long as the insurance environment does not change drastically. Bouska speaks to this issue indirectly in her discussion of the workers compensation exposure base problem. Using physical exposure bases can separate exposure changes due to size from those due to economic or monetary inflation. Their use focuses on physical relationships between exposure and claim frequencies or severities. Applicable inflation can still be measured and reflected separately, at a minimum, through claim severities. Actual value in fire might be considered physical and measurable, but its current value is fairly subjective and susceptible to measurement error at policy inception.

The exposure base is not a transformation of the random variable true exposure as noted at the beginning of Section 2. This causes a large difference in the variances relative to the means (coefficients of variation, or CV) between the exposure (small CV) and the aggregate losses (large CV) of an individual risk. Hence, the appropriateness of an accurately measurable exposure base should be measured against the mean risk pure premium rather than trying to consider variations in one versus variations in the other.

The effect of exposures on credibility was not addressed in the paper. In an ideal model of an insurance process, pure premiums and claim frequencies are built up from homogenous risk populations. In practice, rates are generally determined per unit of exposure, not per risk. If increases in exposure truly translate to increases in insurable losses, then exposures will affect any credibility formulation of pure premiums. This can be addressed directly through the construction of an exposure-based credibility model or by separately considering the effect of changes in size of risk as measured by the exposure base. Exposures would be substituted for risks in the first case, along with a method of reflecting correlation among exposure units at the individual risk level. The second alternative focuses on risks as the population of the probability space, not exposures, for statistical inferences.

Each exposure unit is not independent of all other units whenever a risk can be greater than one exposure unit in size. This must be recognized as distinct from effects on statistical measurements caused by scaling alone through the selection of the unit employed for the exposure base. This is not an easy task. For example, to change an exposure basis from $1,000 to $100 of sales does not change the nature of risks ($1,000 of sales) that were formerly one but are now ten exposure units. If these risks were now treated as ten independent exposure units, estimates of classification pure premiums would be more credible and their coefficients of variation would decrease. It is also worth noting the difficulty in combining experience from classes that have different exposure bases within a coverage.

Briefly turning attention to reinsurance and excess of loss contracts, David E. A. Sanders notes that the underlying primary risks are not always independent. Catastrophes do indeed create a contagion effect, geographically causing many risks to suffer claims and losses. A new dimension must be addressed for these contracts making the measurement of true exposure and the choice of an exposure base a much more difficult task.

4. ARE EXPOSURES NECESSARY?

In the final analysis of the utility of exposures, it is their convenient physical reflection of increased true exposure rather than their use in calculating premiums from classification rates that is key. Were it not for exposures, rates would be stated as the average per risk without a convenient or reliable reflection of size differences except those afforded by experience rating. All variations would need to be measured on a risk-by-risk basis or through classification characteristics (variables). The use of an objective, physical measurement of size is also appealing in that it provides a convenient way to measure changes in aggregate pure premium without ignoring changes in coverage and business mix. There may be situations where it is worthwhile to separate policy experience into intervals smaller than one year in order to reflect any significant changes in true exposure.

There will continue to be other meanings employed for the term "exposure;" e.g., the total amount at risk in a hurricane-prone area or its use in specific fire rating schedules. However, it is the support of measuring "size of risk," which is synonymous with risk pure premium, that will continue to be the focal point of actuarial discussions.

The process of "rate times exposure" also has something more important about it than appearances might suggest. Measuring the *components* is important; i.e., the exposure and the rate, and not just the end product—premium. This helps assure that the process of setting rates is supported by the application of these same rates; that the way estimates of rates are determined is well founded. Bouska causes us to consider the critical role of exposures in property and casualty insurance. It is the *exposures* that ultimately affect the aggregate premiums and losses, to which so much attention is paid.

The paper takes casualty actuaries back to their roots to discuss a subject that may have been taken for granted. Bouska's discussions of the concept of "true exposure," the case of the large risks, and the types of problems exhibited by temporal mismatch, clearly focus on today's insurance environment. The research and experimentation needed to test theories and various exposure measures are very extensive and perhaps not easily supported by the statistical data gathering mechanisms currently in place. It is with interest that I await to see how the issues Bouska has raised play out over time.

REFERENCES

[1] P. Dorweiler, "Notes on Exposure and Premium Bases," *PCAS* XVI, 1929, p. 319; reprinted *PCAS* LVIII, 1971, p. 59.

[2] G. L. Head, *Insurance to Value*, Richard D. Irwin, Inc., Homewood, Illinois, 1971.