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RISK LOADS FOR INSURERS

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DISCUSSION BY STEPHEN W. PHILBRICK

1. INTRODUCTION

Mr. Feldblum has written a very interesting paper on the subject of risk loads. I am happy to see more written on this subject. His paper concentrates on risk load in the context of pricing. Because I believe that the risk load in pricing is inextricably linked to the risk margins in reserving, this paper will also add to the literature on that important subject.

However, I believe that Mr. Feldblum's enthusiasm to embrace Modern Portfolio Theory Methods has caused him to summarily dismiss other approaches a bit too quickly. It is only a slight overstatement to summarize Mr. Feldblum's paper as follows:

> There are five ways to calculate risk loads.
> Four are wrong; one is right.

I find that many of Mr. Feldblum's concerns are quite relevant and, to some degree, compelling. Many of the methodologies currently employed do suffer from incomplete theoretical justification. However, my opinion is that the conclusions are not nearly so black-and-white as Mr. Feldblum would have us believe.

I will offer my comments on each of the five methods as defined by Mr. Feldblum.

2. STANDARD DEVIATION AND VARIANCE METHODS

The most important comment (perhaps obvious to many) is that Mr. Feldblum's criticisms do not extend to *all* standard deviation and variance methods, but only to the specific methodology employed by the Insurance Services Office (ISO), which incorporates the process risk associated with the severity distribution. It is, of course, possible to incorporate frequency considerations into the calculations and, less easily, parameter risk considerations. One still might label these methods standard deviation and variance methods, and they might not suffer the same criticisms outlined by Mr. Feldblum.

I strongly share Mr. Feldblum's concern about the absence of parameter risk considerations in the risk load procedure. When the procedure was first implemented, I recall long conversations with a colleague where we attempted to determine whether the parameter risk might be even approximately coincident with the process risk. We concluded that parameter risk would be distributed across limits in a pattern differently than process risk; thus, the ISO procedure would not provide a surrogate for the total risk loading.

While I agree with Mr. Feldblum's concerns about parameter risk, I cannot agree with his statement, "In other words, the standard deviation of the individual's loss distribution is no guide even to the process risk faced by the insurer." He purports to show this by noting that the coefficient of variation (CV), or standard deviation divided by the mean, of 100 policies is vastly different than the CV of a single policy. This might be relevant if an insurer considered writing a single policy, but it does not.

The more relevant question is: If an insurer writes 100 policies and contemplates writing an additional policy, will the insurer's risk load requirements bear any relationship to the standard deviation or variance of the individual risk in question? This specific issue is explored in Rodney E. Kreps's recent paper [1].

The answer is yes, although the specific form of the answer surprised me. Suppose an insurer decides that its total risk load should be proportional to the variance of the aggregate distribution of its entire portfolio. Then it is reasonable to conclude that the risk load for an additional (marginal) insured should be proportional to the *marginal* increase in the aggregate variance. Assuming independence of risks, the marginal increase in aggregate variance is proportional to the variance of the individual risk. (This should hardly be surprising, as the marginal increase in aggregate variance is equivalent to the variance of the marginal risk.)

Alternatively, if the insurer decides that its total risk load should be proportional to the standard deviation of the aggregate distribution (a ruin theory approach), then the risk load for a marginal insured should be proportional to the marginal increase in the aggregate standard deviation. This increase is *also* proportional to the *variance* of the individual risk (not the standard deviation). (See Appendix for details.)

It should be noted that the calculations in the Appendix are done with the assumption of independence between risks, i.e., no covariance. The covariance term is incorporated in Mr. Kreps's paper [1]. The covariance terms should probably not be ignored in practice.

While Mr. Feldblum may be literally correct to say that the standard deviation of the individual risk is no guide to the insurer's process risk, the variance of the individual risk *is* such a guide.

3. UTILITY THEORY

I share all of the concerns laid out by Mr. Feldblum. While mathematically appealing, the practical problems are so difficult that I have never attempted to actually use utility theory in practice; nor have I read an exposition of such an attempt that satisfied *all* my concerns.

My only disagreement with Mr. Feldblum is his broad application of the concluding sentence of his introduction: "Only the last method, however, measures the true risk faced by insurers." Utility theory does, in fact, measure the true risk faced by insurers. Utility theory fails to be used commonly, not because it doesn't measure the true risk, but because of the practical problems associated with implementation.

4. PROBABILITY OF RUIN

Mr. Feldblum suggests that there are three ways to formulate the problem in the context of ruin theory. If one is limited to these three alternatives, one might indeed conclude that ruin theory is not up to the task of specifying risk loads. Let me suggest a fourth formulation of the problem that I believe falls within the sphere of ruin theory: For an insurer with a given portfolio of risks, what is the required amount of surplus plus risk loading necessary such that the probability of ruin is less than a given amount, and what is the proper relationship between the relative amounts of surplus and risk loading?

I believe that the above formulation may lead to practical solutions to the problem. (See Mr. Kreps's paper [1] for a specific exposition along this line.)

Mr. Feldblum's mathematical examples are not persuasive. In his first example, he is apparently attempting to prove that ruin theory applications would produce inappropriate or inconsistent risk loading requirements. While he concedes (in a footnote) that his examples are extreme, he suggests that similar conclusions will follow if one applies the analysis to "an insurer writing 1,000 policies." I disagree.

The calculations associated with an insurer writing only one or two risks are not a reliable guide to the calculations for an insurer that has already written 1,000 policies and is considering the addition of one of these two alternatives. Mr. Feldblum argued eloquently in his discussion of utility theory that wealth independence does not conform with reality. Mr. Feldblum should not then make the assumption he earlier refuted.

In his second mathematical example, he asks us to presume that a ruin theory calculation requires a risk load of "10% of premium on a $100,000 premium policy and 50% of premium on a $1,000,000 premium policy." Furthermore, the marketplace allows "only a 20% risk load on the latter policy." He then concludes that insurers would prefer the latter policy, thereby (apparently) proving that ruin theory is flawed. It is difficult to respond precisely without seeing the actual numbers that led to his required loads. I suspect that he may not be carefully distinguishing between required additional risk load and required additional surplus. His examples do not prove that ruin theory leads to inappropriate conclusions.

Ruin theory (in its present form) is not the solution to all problems. A personal concern is the overly simplistic binary division of the world into solvent and insolvent companies. Gradations of solvency are important, and not easily handled in ruin theory. Gradations of insolvency are also important, for a company that is "just barely" insolvent imposes a different burden on guaranty funds and society than a company that becomes insolvent by many millions of dollars.

5. REINSURANCE METHOD

The reinsurance method is far from perfect. For one reason, many reinsurance transactions are motivated in part by tax or regulatory considerations. This will distort the ability of the reinsurance transaction to provide a reliable guide to the appropriateness of risk loads. And, of course, Mr. Feldblum is technically correct in concluding that reinsurance does not "solve" the risk load problem; it merely transfers the problem to someone else. However, the reinsurance approach is valuable for two very different reasons:

1. It provides a powerful reality check for theoretically-based methods. I have seen a proposed theoretical method easily disproven by considering it in a reinsurance environment.

2. It provides real world answers in real situations. Consider a small insurer, wishing to issue policies with a $2,000,000 limit, but only able to retain $500,000 net. This insurer might set the price (including risk load) on the $1,500,000 excess of $500,000 equal to what its reinsurer is charging for that layer. It is not very meaningful to calculate a theoretical amount of risk load for that upper layer (unless the insurer can persuade the reinsurer to change its prices). The insurer is still left with the task of calculating the risk load on the net layer, but some of the original problem has been solved.

6. MODERN PORTFOLIO THEORY

Mr. Feldblum's discussion of this subject is a welcome addition to the actuarial literature. In the year that the CAS finally adds finance to its *Syllabus*, it is appropriate that we continue to explore the financial literature for useful tools. I am convinced that the Capital Asset Pricing Model (CAPM) is a useful tool for explaining concepts. I am less convinced that CAPM is the best tool for explicitly determining risk loads in practice.

For example, I am not yet ready to conclude that companies should write Aircraft and Surety (with betas of .07 and .04, respectively) at rates that generate returns equal to risk-free securities. Mr. Feldblum has provided us with much interesting and relevant background on CAPM, but he has left out the fact that major controversies arise over the actual application of the theory to specific problems, including insurance.

7. SUMMARY

Mr. Feldblum has given us much to think about regarding the subject of risk loads. He properly points out that some of the existing methodologies have various flaws, and a promising methodology (CAPM) should be explored further.

The subject of risk loads is critically important to the actuary. As a profession, actuaries need to refine, correct, or enhance all of our potential tools (including others not discussed here, such as option pricing theory). Eventually, we may settle on a single approach; but, at the present time, the choice is far from obvious.

APPENDIX

Assume an insurer considers writing one or the other of the two risks described by Mr. Feldblum in Table 1. The relevant information is repeated below:

Risk	Amount of Loss	Probability of Loss	Expected Value of Loss	Standard Deviation of Loss	Variance of Loss
A	$ 100,000	.01	$1,000	$ 9,950	$ 99,000,000
B	1,000,000	.001	1,000	31,607	999,000,000

Suppose that the insurer already writes 10,000 risks of type A. (The conclusions can also be made if the existing portfolio consists of type B risks, a mixture of each, or even a variety of different risks. I chose 10,000 type A risks to simplify the mathematics.) With 10,000 risks of type A, the aggregate parameters are as follows:

Expected Losses	$10,000,000
Variance	$990,000,000,000
Standard Deviation	$994,987
Coefficient of Variation	.099

If the insurer writes an additional risk of type A, obviously the total variance increases by $99,000,000. The standard deviation increases from $994,987 to $995,037, an increase of approximately $50.

If the insurer, instead, were to start with 10,000 risks of type A and write one additional risk of type B, the variance would increase by \$999,000,000 and the standard deviation would increase from \$994,987 to \$995,489, an increase of approximately \$502. This information is summarized as follows:

Risks	10,000 Type A	10,000 Type A Plus 1 Type A	10,000 Type A Plus 1 Type B
Expected Losses	\$10,000,000	\$10,001,000	\$10,001,000
Variance	\$990,000,000,000	\$990,099,000,000	\$990,999,000,000
Marginal Variance		\$99,000,00	\$999,000,000
Standard Deviation	\$994,987.44	\$995,037.19	\$995,489.33
Marginal Standard Deviation		\$49.75	\$501.89

Note that the increase in standard deviation associated with risk B compared to risk A is in the same proportion as the relative variance of the individual risk. In both cases, this ratio is 10.09.

REFERENCE

[1] Rodney E. Kreps, "Reinsurer Risk Loads from Marginal Surplus Requirements," *PCAS* LXXVII, 1990, p. 197.