# RISK LOADS FOR INSURERS

SHOLOM FELDBLUM


## *Abstract*

*Insurance companies are risk averse, even as individuals are. Casualty actuaries have suggested several methods of calculating risk loads to compensate the insurer for the risk it accepts. Methods currently in use, and reviewed in this paper, consider (a) the standard deviation and variance of the loss distribution, (b) utility functions, (c) the probability of ruin, and (d) reinsurance costs.*

*These methods are theoretically unsound. They consider the wrong type of risk; they arbitrarily equate risk with a mathematically more tractable variable; and, they require equally arbitrary assumptions about an insurer's aversion to risk. More importantly, they concentrate on the size of loss distribution, though the true risk to the insurance company resides in profit fluctuations.*

*Modern portfolio theory measures the risk assumed by investors in securities. Systematic risk, the overall risk faced by a diversified stock portfolio, requires an additional premium. Firm-specific risk, or the fluctuations in an individual stock's price, can be eliminated by diversification and is not compensated for in security returns. Insurance equivalents to modern portfolio theory can be applied to insurance portfolios to determine risk premiums by line of business. Such analysis reveals the Commercial Liability lines to be highly risky and the Personal Property lines to be less risky. In sum, this method allows insurers to measure the true risk they face in each line of business.*


I am indebted to Richard Woll and Benjamin Lefkowitz, who made numerous corrections to an earlier version of this paper. The remaining errors, of course, are my own.

## 1. INTRODUCTION

Most persons are risk averse: they prefer a stable income to a fluctuating one, even if the two have equal expected values. Risk aversion is one of the foundations of insurance, for the insured trades the chance of a fortuitous but large loss for the payment of a fixed annual premium.

Insurers also are risk averse, although their large size masks their preference for a stable income. When faced with a large risk, an insurer may decline the application, seek reinsurance, or charge an additional premium, a "risk load." The third option is the most desirable, since declining the application reduces business volume, and buying reinsurance gives up potential profit on the ceded business.

Yet calculating risk loads is a complex task. On the one hand, insurers often incorporate "contingency" provisions in premium rates, whether for conflagration hazards in turn of the century fire rates or unanticipated liabilities in current General Liability rates. On the other hand, there is no established procedure for determining the size of the risk load.

So actuaries have devised numerous methods, which are grouped below into four categories:

(1) The risk load may vary with the random loss fluctuations of the individual risk; e.g., "standard deviation" and "variance" methods.

(2) The risk load may vary with the characteristics of the overall portfolio of risks; e.g., "utility function" and "probability of ruin" methods.

(3) The risk load may vary with the empirical costs of reducing risk; e.g., "reinsurance" method.

(4) The risk load may vary with fluctuations in profitability; e.g., "modern portfolio theory" methods.

Some methods are simple to implement but lack theoretical justification; others are mathematically elegant but difficult to apply. The advantages and deficiencies of each method are examined below. Only the last method, however, measures the true risk faced by insurers.

## 2. STANDARD DEVIATION AND VARIANCE METHODS

The simplest approach is to conceive of the insurer's risk in the same fashion as the insured's risk. Suppose an insurer sells a General Liability policy to a contractor, who has a 1% chance of being liable for a $100,000 loss, and a 99% chance of no loss. The expected value of the loss is $1,000, but the contractor may be willing to pay $2,000 to entirely avoid the risk of loss. Similarly, the insurer may require a pure premium of *more* than $1,000, to compensate it for the risk it assumes.

Suppose a second contractor also purchases an insurance policy. This insured has a 0.1% chance of a $1,000,000 loss, and a 99.9% chance of no loss. The expected value of the loss is again $1,000, but both the standard deviation and the variance of the loss are higher, as shown below.[1]

### TABLE 1

STANDARD DEVIATION AND VARIANCE OF LOSS

| Amount of Loss | Probability of Loss | Expected Value of Loss | Standard Deviation of Loss | Variance of Loss |
|---|---|---|---|---|
| $ 100,000 | 1.0% | $1,000 | $ 9,950 | $ 99,000,000 |
| 1,000,000 | 0.1 | 1,000 | 31,607 | 999,000,000 |

The loss distribution on the second policy has a standard deviation about three times as large and a variance about ten times as large as that for the first policy, though their expected losses are the same. If the risk load is proportional to the standard deviation or the variance of the losses, then the risk load for the second policy should be either three times or ten times as large as that for the first policy. The standard

[1] Consider the first contractor, with a 1% chance of a $100,000 loss. The variance of loss is $(0.01)(100{,}000^2) + (0.99)(0^2) - (1{,}000^2) = 99{,}000{,}000$ "dollars squared." The standard deviation is the square root of this, or $9,950.

deviation method, as currently applied by the Insurance Services Office,[2] would determine the pure premium as

Pure premium = expected loss + (constant · standard deviation).

If the constant is 0.5%, the pure premiums are \$1,050 for the first policy and \$1,158 for the second policy.[3] This method is now in vogue, as it requires information only about the loss distribution, not about other insurer characteristics. Therefore, it can be applied to all carriers equally.[4] The Insurance Services Office (ISO), the major U.S. rate making bureau for the non-Compensation lines of business, presently uses the standard deviation of the loss distribution to calculate risk loads for General Liability, Products Liability, and Commercial Automobile increased limits factors.[5] Until the mid-1980's, ISO used the variance of the loss distribution for this purpose, a method proposed by Robert S. Miccolis [38] in 1977.

Loss frequencies and severities vary by policy, and no insurer could estimate all the needed figures. As an approximation, one can determine the standard deviation or variance of the loss distribution for policies with a specified limit of liability. A General Liability policy with a limit of \$25,000 truncates all loss indemnification at that amount. The expected value, standard deviation, and variance of the loss distribution are all lower than those for a similar policy with a \$1,000,000 limit. Using the standard deviation method and a loss distribution modeled by a Pareto curve, ISO calculated the following risk loads and increased limits factors for one group of Premises/Operations risks:

[2] The probability of loss for any particular policy is indeterminate. Rather, ISO estimates the loss distribution for policies of a given limit of liability, and applies the resultant risk loads to the increased limits factors.

[3] For the first policy, \$1,000 + (0.005)(\$9,950) = \$1,050. For the second policy, \$1,000 + (0.005)(\$31,607) = \$1,158.

[4] This is particularly important for rating bureaus, which have information only about the size of loss distribution for the block of business.

[5] For details, see the memoranda of ISO's Actuarial Research Committee.

## TABLE 2

### RISK LOADS AND INCREASED LIMITS FACTORS FOR PREMISES/OPERATIONS RISKS (MEDIUM TABLE)

| Policy Limit | Average Severity | ALAE per Claim | ULAE per Claim | ILF without RL | Risk Load | ILF with RL |
|---|---|---|---|---|---|---|
| $ 25,000 | $ 4,039 | $2,325 | $ 477 | 1.00 | $ 521 | 1.00 |
| 50,000 | 5,314 | 2,325 | 573 | 1.20 | 797 | 1.22 |
| 100,000 | 6,698 | 2,325 | 677 | 1.42 | 1,179 | 1.48 |
| 200,000 | 8,135 | 2,325 | 784 | 1.64 | 1,706 | 1.86 |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |
| 50,000,000 | 14,828 | 2,325 | 1,287 | 2.70 | 8,503 | 3.66 |
| 100,000,000 | 16,227 | 2,325 | 1,391 | 2.92 | 11,943 | 4.33 |

ALAE: Allocated loss adjustment expense (ISO uses a constant dollar amount for each policy limit; although unrealistic, this simplifies the calculations).
ULAE: Unallocated loss adjustment expense (ISO determines the ULAE as 7.5% of expected loss plus ALAE for this line of business).
ILF: Increased limits factor.
RL: Risk load.

Unfortunately, this method has no theoretical justification, for several reasons. First, the *insurer's* risk is different from the *insured's* risk. The insured is more concerned about random loss fluctuations—which could ruin him financially—than about the accuracy of the expected loss estimate. But the insurer may have thousands of policies in each line of business. It is less concerned about random loss fluctuations, which even out over a large volume of risks, than about the accuracy of its expected loss estimate.

To illustrate this, suppose 10,000 insureds buy General Liability policies. Each insured has the same probability of a $100,000 loss. This probability is not known exactly, but is estimated to be between 0.5% and 1.5%. The expected value of the loss on each policy may be as low as $500 or as high as $1,500, but these figures are not the major concern of the insured. He seeks relief from worry, from the risk of possible bankruptcy. For him, the range of probable losses—for which actuaries use standard deviation and variance statistics—is the primary concern.

Suppose the insurer charges a \$2,000 premium for each policy. Its expected loss ratio lies between 25% and 75%, depending upon the true probability of loss. For example, if the probability of loss is actually 1%, then the expected loss for each policy is \$1,000 and the expected loss ratio is 50%. Random loss fluctuations will not cause the actual loss ratio to deviate much from the expected, since many homogeneous risks are covered. But the actual loss ratio *will* differ greatly from the forecasted loss ratio if the probability of loss is incorrectly estimated. A 0.5% chance of loss will bring large profits, while a 1.5% chance of loss will have the opposite effect. This is the "risk" that the insurer must guard against.

Actuaries use the terms "process risk" and "parameter risk" to denote these two causes of fluctuation in insurance losses. Process risk refers to random loss fluctuations about a stable mean; this is the major risk for the insured. Parameter risk refers to uncertainty in estimating the expected loss; this is the major risk for the insurer.[6]

If the standard deviation and variance methods capture process risk, not parameter risk, why are they used to calculate insurer risk loads for liability policies? After all, process risk and parameter risk are independent, so estimating one is of no help for the other. The usual explanation is that: "There is no easy method of estimating parameter risk. To satisfy their member companies, rating bureaus must somehow calculate risk loads. Basing the factors on Pareto curves and process risk is sophisticated enough that no further questions will be asked." Sophisticated it may be, but a satisfying explanation it is not.[7]

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[6] The actuarial use of the terms "process risk" and "parameter risk" is due to Robert L. Freifelder [25]. Freifelder speaks of the probability distribution function of the loss process (whence *process risk*) and of an *a priori* distribution of the unknown parameters of the loss distribution function (whence *parameter risk*).

[7] There are two problems in estimating parameter risk. One is to quantify the magnitude of this risk—e.g., the expected fluctuation in forecasted average pure premiums due to estimation errors. The second is to use these estimates of parameter risk to evaluate needed actuarial figures, such as Workers Compensation excess loss factors. This second part is a mathematical exercise, albeit a complex one. Philip Heckman and Glenn Meyers [30] outline a sophisticated method of solving this problem. Moreover, actuaries often *assume* an *a priori* distribution for the parameters of the loss function, and thereby "quantify" the parameter risk. However, they have yet to address the crucial first question noted above: *How does one estimate the true parameter risk*?

Even if one seeks to calculate process risk, one must measure the standard deviation of the insurance portfolio as a whole, not that of individual risks. The ratio of the standard deviation to the expected value decreases as additional homogeneous risks are added to the portfolio. Consider the first policy in Table 1. If the insurer issues a single policy, the ratio of standard deviation to expected loss is 9.950 (standard deviation of $9,950 divided by expected loss of $1,000). If the insurer issues two such policies, the expected loss is $2,000 and the standard deviation is $14,071, for a ratio of 7.036.[8] If the insurer issues one hundred such policies, the ratio is less than one. In other words, the standard deviation of the individual policy's loss distribution is no guide even to the process risk faced by the insurer.[9]

On first reflection, it might seem that using the variance of the loss distribution avoids this problem. After all, the ratio of the variance to the expected value does not change when similar risks are added to the portfolio. In truth, using the variance simply aggravates the problem. The process risk faced by the insurer does in fact decrease as additional

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[8] The probability of loss is 1% for each policy. Thus, the probability of a loss on both policies, for a total loss of $200,000, is $0.01^2$, or 0.0001. The probability of one loss of $100,000 is (2)(0.99)(0.01), or 0.0198. The probability of no loss is $(0.99)^2$, or 0.9801. The expected loss is (2)(0.01)($100,000), or $2,000. These figures, as well as the calculation of the variance and standard deviation, are shown below.

| Number of Losses | Total Loss (1) | Probability (2) | Expected Loss (3) | Variance Calculation: $(3) \cdot (2)^2$ |
|---|---|---|---|---|
| Two losses | 200,000 | 0.0001 | 20 | 4,000,000 |
| One loss | 100,000 | 0.0198 | 1,980 | 198,000,000 |
| No losses | 0 | 0.9801 | 0 | 0 |
| Total | | 1.0000 | 2,000 | 202,000,000 |

Variance = $202,000,000 - 2,000^2 = 198,000,000$ (dollars squared).
Standard deviation = $198,000,000^{0.5} = \$14,071$

The ratio of standard deviation to expected loss is $14,071 ÷ $2,000, or 7.036.

[9] David B. Houston [33] makes a similar distinction between an individual's and an insurer's risk. The individual is concerned with variations in outcomes of a particular action. The insurer is concerned with sampling error that affects the estimated mean pure premium. This distinction is similar to that in the text, except that Houston ascribes all parameter risk to sampling error.

policies are issued, but the ratio of the variance to the expected loss does not show this. The variance method ignores the problem; it does not solve it.[10]

The second theoretical failure of the standard deviation and variance methods is that they determine only relative risk, not absolute risk. The ISO exhibit for Premises/Operations risk loads (see Table 2 above) says that the risk load for a policy with a $50,000 limit should be about one and one half times that for a policy with a $25,000 limit. But how are the dollar amounts of the risk loads determined—or the ratio of risk load to expected loss?

The mathematics provide no answer. ISO simply chooses an overall risk load for the line of business, and then spreads this risk load by size of policy limit using the standard deviation or variance method. But determining the overall risk load is our primary concern, and an arbitrary choice is no solution.

The third theoretical failure is that these methods determine relative standard deviation, or relative variance, not relative risk. The simplified illustration of two General Liability risks in Table 1 provides different "risk loads" depending upon whether the standard deviation or variance method is used. The risk load for the second policy is either three times or ten times that for the first policy. There is no *a priori* reason to equate risk with either the standard deviation or the variance. These statistics are used because they are mathematically tractable. But the goal is to measure actual risk, not to equate risk with an appealing mathematical concept and then to measure the latter.

To sum up the standard deviation and variance methods: Parameter risk, the real concern, is too hard to measure, so process risk is substituted for it. The standard deviation is a tractable mathematical construct, so it replaces "risk." Then an overall portfolio risk load is chosen arbitrarily, and the standard deviation method spreads it over policies according to the size of the policy limit. Somehow, this hardly sounds like proper actuarial practice.

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[10] Advocates of exponential utility functions often cite the invariance of exponential utility to the wealth of the insurer as an advantage; see the quotation from Freifelder in footnote 16 below. Again, just the opposite is true. The risk does vary with the wealth of the insurer. A method which ignores this is defective.

## 3. UTILITY FUNCTIONS

Microeconomists have long used utility functions in consumer demand theory, and casualty actuaries have recently suggested using them to calculate risk loads. Utility functions allow the rate maker to vary the risk load on a policy with the composition of the entire insurance portfolio and with the insurer's attitude toward risk. Unfortunately, the mathematics required are complex and needed assumptions can only be guessed at, so this method is not popular.[11]

A utility function expresses the value of a given basket of assets to its owner. Utility functions provide an ordinal, not a cardinal, sequence of values. In other words, it is meaningless to speak of the absolute utility of a loaf of bread or a quart of milk to an individual. We can say only that the individual prefers a loaf of bread to a quart of milk, or vice versa.[12] Similarly, we cannot determine the absolute utility of a $2,000 premium for the insured, but we can say that he or she prefers paying this premium to suffering a 1% chance of a $100,000 loss.

The discussion below seems to imply cardinal values for utility. For instance, an exponential utility function assigns a cardinal value to a given basket of goods. This is not the intention, however. The implication is only that the utility is proportional to the value of the exponential function, not that it is equal to it. The same comment applies to all the utility functions discussed below.

Utility functions are an ideal tool for calculating risk loads, since they are the mathematical equivalent of the "attitude toward risk." Utility functions depend upon the insurer's degree of risk aversion, the composition of its insurance portfolio, and its corporate wealth.

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[11] On the use of utility functions in demand theory, see James M. Henderson and Richard E. Quandt [31] or Angus Deaton and John Muellbauer [20], pages 25–26.

[12] See, for example, Paul A. Samuelson [44], page 91: " . . . a cardinal measure of utility is in any case unnecessary; . . . only an *ordinal* preference, involving 'more' or 'less' but not 'how much,' is required for the analysis of consumer's behavior"; or Armen A. Alchian [1], page 39: "Any numbering sequence which gives the most preferred sure prospect the highest number, the second preferred sure prospect the second highest number, etc., will predict his choices according to 'utility maximization.' But any other sequence of numbers could be used so long as it is a *monotone transformation* of the first sequence. And this is exactly the meaning of the statement that utility is *ordinal* and not cardinal."

As a simple illustration, suppose the utility of an asset is proportional to the square root of its price: $U_x = Kx^{0.5}$.[13] If an insured has \$10,000 of assets, with a 1% chance of losing it all, and a 99% chance of no loss, his present utility is $(0.01)(0)K + (0.99)(10{,}000^{0.5})K = 99K$. This equals the utility of \$9,801 of assets. In other words, the insured would be willing to pay \$199 to avoid the risk of loss.

Suppose the insurer begins with \$1,000,000 of assets. If it accepts the risk of loss from the insured, its total utility is

$$(0.01)(990{,}000^{0.5})K + (0.99)(1{,}000{,}000^{0.5})K = 999.95K,$$

which is equal to the utility provided by assets of \$999,899.74. That is, it needs a pure premium of \$100.26, or a risk load of 0.26%. The larger the insurance portfolio, and the greater the surplus of the insurer, the smaller is the risk load needed.

But what is an appropriate utility function? Theoretical economists do not have this problem, since they use utility functions to prove mathematical theorems, not to solve practical problems. But actuaries desirous of using utility functions to calculate risk loads must first determine what utility functions are most realistic.

There are two considerations in determining an appropriate utility function.

First, the function should satisfy the mathematical properties needed for utility theory. Gary Venter [48] lists several such properties:[14]

1. *Utility is an increasing function of wealth*; that is, as wealth increases, the utility of that wealth increases.

2. *Actors are risk averse*; that is, each incremental increase in wealth yields progressively less incremental utility for the actor.

3. *Risk aversion decreases as wealth increases*; that is, the poor individual has greater absolute risk aversion than the wealthy individual has (on average).

[13] The constant "$K$" is a proportionality factor that transforms the utility function from a cardinal to an ordinal measure. This is not as general as Alchian's *monotone transformation* (see preceding footnote). A monotone transformation is appropriate for the theory, but it cannot generate the absolute risk loads needed in practice.

[14] Only the first three of Gary Venter's criteria are listed in the text. His last two, that the utility function be bounded from above and that the utility be equal to zero for negative amounts of wealth, are less commonly accepted by economists.

Gary Venter's criteria mirror reality, and many persons would agree with them.[15] But casualty actuaries have found that one of the simplest and most tractable utility functions, the exponential function, has a risk aversion level that is invariant with the wealth of the actor. With an exponential utility function, the utility of a portfolio of insurance contracts equals the sum of the utilities of each individual contract.

This attribute of exponential utility functions simplifies the mathematics of calculating risk loads, and it has made the exponential function the utility function of choice for calculating risk loads. But it does not accord with reality. The essence of insurance is that the insurance company, due to its large size, is less risk averse than each individual insured.[16]

ISO has noted that even if one posits a given family of curves for the utility function, such as the exponential family, varying the parameters of the family provides different risk loads for each size of risk. One can determine whatever risk load one wants, as well as various relationships among risk loads for different policies, simply by varying the parameters of the utility function.[17] To avoid this problem, John Cozzolino and Naomi Kleinman [15] have suggested using the reciprocal of the insurer's surplus as the parameter of the exponential utility function. This does indeed provide a simple formula for the parameter of the utility function. But what evidence is there that it accurately reflects differences in risk aversion among insurers of different sizes? Presum-

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[15] Venter's first criterion simply says that people prefer more wealth to less wealth. His second criterion seems realistic. It is not universally true, but it seems to hold for most persons in most situations. His third criterion has been formulated rigorously by Ken Arrow [2], though it is hardly amenable to a simple proof.

[16] See, for example, Robert Freifelder [25], *op. cit.*, as well as his shorter article [26]. Note his theorem 1 on page 75 of this article: "If premium rates are based on an exponential utility function, the total premium required for a class of independent contracts is equal to the sum of the premiums required for each of the contracts individually." His justification for the exponential utility function strikes the practical businessman as strange, but it fits well with a desire for elegant and tractable procedures: "There are no 'portfolio' or 'wealth' effects with an exponential utility function. What this means is that with an exponential utility theory ratemaking model, the decision maker does not have to know the exact characteristics of the company's portfolio or its wealth. In practical situations the above information is not generally available" (p. 74).

[17] Note the comment by J. David Cummins and David J. Nye [17], page 429: "Risk loadings and hence solvency are very sensitive to the choice of the risk aversion parameter when the expected utility approach is used."

ably, large insurers are less risk averse than small insurers are. But what evidence is there that the risk aversion varies directly with the reciprocal of the insurer's surplus?

The second problem in determining an appropriate utility function is equally serious. The utility function must model reality, or the risk load procedure becomes a sterile mathematical exercise. Is the risk aversion demonstrated by insurers indeed similar to that implied by an exponential utility function, or a square root utility function, or some other function? This question is difficult to answer, and no one has yet proposed a method of doing so.[18]

Utility function analysis translates the vague "attitude toward risk" into concrete mathematical expressions. But it provides no practical guidance towards measuring either risk aversion or utility. In other words, it restates the problem of determining risk loads; it does not solve it.

The earlier comments regarding process risk and parameter risk apply to utility function analysis as well. In the example above, parameter risk refers to the uncertainty regarding the probability of loss. It might be 1%; it might be 2%; it might be some other probability. If we knew the distribution of the probability of loss, we could incorporate this into utility function analysis. But utility function analysis provides no aid for measuring this distribution, so we are no better off than when we began.[19]

[18] The Society of Actuaries life contingencies textbook, *Actuarial Mathematics*, models insurance transactions between a risk averse insured and a risk neutral insurer. This is a standard economic model. Since insureds are more risk averse than insurers are, it also reflects reality (though imperfectly). Nevertheless, it leaves unanswered our question: "What is the appropriate risk load for insurers?" See Newton L. Bowers, Jr., et al., [8], pages 7–16.

[19] Freifelder, following Bühlmann, proposes one means of empirically measuring parameter risk. Using automobile accident data, he assumes a Poisson loss distribution for each driver, and an underlying Gamma distribution of the Poisson means in the population of drivers. Thus, one driver may have a 10% chance of an accident, so his loss distribution is Poisson with a mean of 10%. A second driver may have a 20% chance of an accident, so his loss distribution is Poisson with a mean of 20%. The Gamma distribution may be estimated by examining the moments of the empirical loss distribution. See Robert L. Freifelder [25], pages 83–84, Hans Bühlmann [11], and Lester B. Dropkin [21].

This procedure masks the true parameter risk; namely, that *the underlying distribution of means changes over time*. Richard Woll has pointed out that if the underlying distribution of means remained constant, then average loss frequencies for a large insurer would not vary from year to year. Yet they do vary. That is, the parameter risk is not just that the Poisson means are unknown, but that they change over time. For further discussion, see Richard Woll's review [50] of Cozzolino and Kleinman's paper [15], especially pages 21–22.

In sum, utility theory is no more promising than the "standard deviation" and "variance" methods discussed previously. For the theoretical economist, utility theory produces mathematical theorems. But no one has yet even suggested how to model an insurer's risk aversion. Instead, the theoreticians say: "Let us choose a simple and tractable utility function, regardless of its accuracy or applicability, and determine risk loads accordingly." This is hardly a suitable actuarial procedure.

## 4. PROBABILITY OF RUIN

European actuaries developed probability of ruin analysis to determine surplus requirements for insurers of different sizes and with different insurance portfolios. "The probability of ruin" is the probability that the insurer will become technically insolvent during a specified time period, such as the coming year. In other words, it is the probability that required reserves will exceed available assets sometime during the period.[20]

The analysis may concentrate on any of three variables: the probability, the assets, or the liabilities (required reserves). That is, one may formulate the problem in three ways: (1) What is the probability that an insurer with given assets and a given portfolio of risks will become technically insolvent? (2) For an insurer with a given portfolio of risks, how much assets (or surplus) are needed such that the probability of ruin is less than a given amount?[21] (3) For an insurer with given assets (or surplus) and a given insurance portfolio, what risk loading must be added to the premium such that the probability of ruin is less than a given value?

[20] See, for example, R. E. Beard, T. Pentikainen, and E. Pesonen [5], especially pages 132–159. For an American exposition, see Alfred E. Hofflander [32]. A stochastic cash flow probability of ruin model, which considers the availability of assets to pay claims instead of insurance regulatory requirements, is presented in C. D. Daykin, et al., [19], as well as in earlier papers by these authors.

[21] For example, Robert Cooper [14], pages 22–43, uses probability of ruin analysis to determine the *necessary invested capital* for an insurance company, though his "high degree of confidence" seems low.

At first glance, probability of ruin analysis seems to solve some of the problems associated with utility function analysis. Absolute risk loads are still not provided, since they require an assumption about an appropriate probability of ruin—one in a thousand? one in ten thousand? But one may calculate the relative risk load for any risk in a given insurance portfolio: it is the extra premium such that the addition of that risk does not change the overall probability of ruin.

An illustration should clarify this—and show the problems with this procedure as well. Suppose an insurer sells General Liability policies, and all its insureds have a 1% chance of a loss equal to the policy limit. The insurer has $50,000 of assets, and it may issue either two $100,000 policies to two independent insureds or one $200,000 policy to a single insured. Finally, the insurer demands that the probability of ruin be no more than one in one thousand.

The expected loss of either portfolio is $2,000. A pure premium of $2,000 brings total assets to $52,000. This leaves a chance of ruin of 1%, as any loss would exceed available assets. For the portfolio of two risks, the insurer needs $100,000, or $50,000 in addition to its original assets, to lower the probability of ruin to one in a thousand. Note that the chance of total loss on *both* policies is one in ten thousand, less than the probability of ruin set by the insurer. A pure premium of $25,000 is therefore needed for each policy, of which $1,000 is the expected loss and $24,000 is the risk load.

For the portfolio of one $200,000 risk, the insurer needs $200,000 to lower the probability of ruin to one in a thousand. Since it has original assets of $50,000, it requires a pure premium of $150,000. Of this amount, $2,000 is the expected loss, and $148,000 is the risk load. The single large risk needs a greater risk load than do the two small risks if the probability of ruin is to be equal.[22]

Unfortunately, probability of ruin analysis concentrates on the chance of technical insolvency. It does not balance this against the income from the additional premium. In practice, one must choose an extremely low probability of ruin (say, one in ten thousand) so that risk loads are needed to prevent insolvency. Suppose one determines that, to ensure a proba-

[22] This illustration is extreme; no insurer writes only one or two policies. The oversimplification is for heuristic purposes only. The same analysis may be applied to an insurer writing a thousand policies.

bility of ruin less than one in ten million, the needed risk loads are 10% of premium on a $100,000 premium policy and 50% of premium on a $1,000,000 premium policy.

Even if the marketplace allowed only a 20% risk load on the latter policy, almost all insurers would prefer the second policy to ten of the first. After all, the probability of ruin is low, and the additional risk load is extra income. In truth, the needed risk load for the second policy is between 1 time and 5 times that for the first policy. Somewhere between these two numbers, the additional profit makes up for the additional risk.

Probability of ruin analysis helps define the boundaries, or endpoints, for the needed risk load. It does not determine where within that interval the appropriate risk load lies. It is useful for solvency regulation, since only the endpoint is desired. It is useless for risk loads, since the actual load is needed.[23]

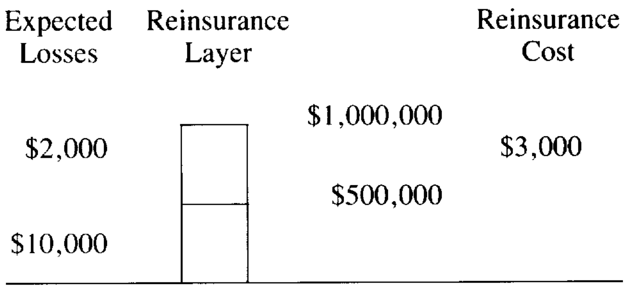
## 5. REINSURANCE METHOD

The risk for insurers is the possibility of unexpected losses either on an individual policy or on a book of business. To stabilize loss fluctuations, a primary insurer may enter into an excess of loss reinsurance treaty. Such protection is not costless. The reinsurance premium must cover not only costs but also the reinsurer's administrative expenses and profit margin. The primary insurer must balance the additional cost of reinsurance protection against the reduction in risk afforded by the treaty.[24]

[23] Stephen P. D'Arcy and Neil A. Doherty [18], page 3, present a similar argument in another context: "The ruin probability, no doubt, forms an important constraint on managerial decisions if only because insurers operate in a regulatory environment that focuses attention on solvency. However, constraints are not objectives. Additionally, as an objective, the probability of ruin is quite incomplete since no account is taken of the value of the equityholders', policyholders', and other parties' claims in the respective states of solvency and ruin. There is indeed a world of difference between surviving and prospering that is ignored by the probability of ruin objective." In other words, two policies may both pass the probability of ruin test set by the insurer. Nevertheless, the insurer may judge one of the policies to be more "risky" and require a higher risk load.

[24] Reinsurance involves various costs, such as underwriting profits and investment income received by the reinsurer and administrative and processing costs of the primary carrier. The *former* costs would be used to estimate the risk load. For a clear description of these costs, see Daniel A. Bailey [3].

The reinsurance treaty is the real-world counterpart of the theoretical risk load. Suppose the expected losses and expenses (i.e., not including a risk load) for a General Liability policy are \$10,000 for a \$500,000 limit and \$12,000 for a \$1,000,000 limit (that is, \$2,000 for the second \$500,000 layer). Suppose also that the charge for facultative per risk excess of loss reinsurance protection of \$500,000 over \$500,000 is \$3,000, as shown below.




The "empirical" risk load for the second \$500,000 of coverage is \$1,000: the reinsurer's charge minus the expected losses. The "empirical" risk load for the lower layer would be determined in the same manner (e.g., by examining the cost of facultative reinsurance of \$250,000 over \$250,000, then \$150,000 over \$100,000, and so forth). Since reinsurance underwriters vary their premium rates by the characteristics of the primary insurer, such as its financial stability, insurance portfolio, and underwriting stringency, the complete risk faced by the insurer is considered, not just the process risk on individual policies.[25]

Unfortunately, this method places the cart before the horse. Reinsurers need actuarial guidance as much as other insurers do. Risk theory is as much for their benefit as it is for that of primary insurers. Reinsurance underwriters evaluate risk as best they can: some succeed and some go bankrupt. Actuaries can help both primary insurers and reinsurers by recommending appropriate risk loads.

[25] Robert Butsic [12], analyzing the economic value of a loss reserve portfolio, compares the risk adjusted discount rate to a hypothetical loss reserve transfer to a reinsurer. The profit margin required by the reinsurer should equal the difference between present values of the loss reserves using a risk free versus a risk adjusted discount rate.

Moreover, reinsurance premiums are based on more than just evaluations of risk. If there is strong competition for a certain type of business, reinsurers cut rates. If some reinsurers leave a line of business, others raise rates. Marketplace pressures influence prices as much as risk characteristics do, and their independent influences cannot be easily distinguished.

The risk load may be subsumed under "profit and contingencies." In practice, the profit margin depends more on competitive pressures and marketplace constraints than it does on actuarial cost considerations. But insurers need the cost analysis as much as they need the marketing analysis, for they must continually decide whether to match competitors' prices. The question here is, "What is the appropriate cost of the additional risk to the insurer?"

## 6. LOSSES AND PROFITS

The risk load methods discussed above concentrate on insurance losses. But insurers do not just pay losses. They collect premiums as well, and they try to match premium rates to anticipated expenditures. Risk is a function of profitability, or net income, not just of loss payments.

Three examples should clarify this. Each illustration is idealized, but their combination provides a realistic portrayal of insurance operations.

(1) Suppose an insurer issues a retrospective rating plan, with no maximum or minimum premium, and no loss limit. In other words, the final premium is equal to the actual losses, with a loading for expenses and profit.

The variability of loss payments has no effect on the insurer's profit. The profit is set by the retrospective rating plan. It is not dependent upon random loss fluctuations or even "parameter risk."[26]

[26] The major risk for the insurer stems from the potential uncollectability of additional premiums. See Roy P. Livingston [36].

(2) Suppose two lines of business have the same size of loss distributions but different loss payout patterns. In one line, the average loss is paid out six months after the accident date; and, in the other line, the average loss is paid out four years after the accident date. Inflation and investment returns affect the second line much more than they do the first, so insurance profitability, or net after-tax operating income, will vary more for the second line. The size of loss distributions, however, do not show this.

Similarly, competitive pressures affect insurance profitability. Again, suppose two lines of business have the same size of loss distributions. One line earns a constant 10% return on equity. In the second line, however, fluctuating market conditions cause profitability to vary substantially from year to year. Clearly, there is more risk for the insurer in the second line of business.

(3) Size of loss distributions are only meaningful for determining risk loads when the risks insured are homogeneous and the premiums are the same for each of them. When the risks insured are *heterogeneous,* and the insurer, by its underwriting and pricing expertise, charges different premiums based upon the anticipated hazards, then size of loss distributions give no clue to the insurer's risk.

Gary Koupf illustrated this with a simple Commercial Liability example.[27] Suppose an insurer sells Commercial Automobile Bodily Injury and Property Damage coverages to a group of homogeneous insureds. Each insured incurs one Bodily Injury claim for $10,000 and one Property Damage claim for $1,000. The insurer charges $15,000 for the BI coverage and $1,500 for the PD coverage. Clearly, there is no risk for the insurer. Profitability is stable, and the size of loss distributions are degenerate for each coverage.

If one combines the Bodily Injury and Property Damage coverages, however, the size of loss distribution becomes highly variable. For a single insured, the average expected loss is $5,500, but the variance of the loss distribution is $20,250,000. The variance of the loss distribution depends upon the degree of heterogeneity of the coverages or of the risks insured. Yet the insurer's profitability remains stable, as long as appropriate premiums are charged for each coverage.

[27] Gary Koupf, comments at the ISO Actuarial Research Commmittee meeting, June 15, 1988.

The combination of these three examples portrays reality well:

(1) Underwriters vary premium rates with the anticipated hazards. Most policies are not retrospectively rated, but they are not purely random contracts either. Much of the variance in the size of loss distribution is reflected in premium rate differences.

(2) Many factors besides size of loss distributions affect insurer profitability: investment income, competitive pressures, and regulatory decisions. Insurers in many lines of business are comfortable with the statistical loss distributions. They are concerned, however, whether regulators will allow needed rate revisions, whether investment returns will match loss cost inflation, and whether competitive pressures will force them to cut rates in order to retain market share.

(3) Most Commercial Liability insureds are heterogeneous. Each has different loss characteristics, and each has its own hazards. Insurance underwriters adjust policy conditions, vary premiums, and select insureds to obtain a profitable book of business. If one ignores the insurance operations, and one examines only the size of loss distributions, one finds great variability. But much of this variability is neither "process risk" nor "parameter risk." It is the anticipated variability reflected by the different coverages and risks.

In sum, the size of loss distribution is but one influence on the insurer's profitability and risk—and not even the most important one. To appropriately determine the risk faced by insurers, one must examine overall profitability, not individual losses.

## 7. MODERN PORTFOLIO THEORY METHODS

Investors face risks similar to those of insurers. *Process risk* in insurance refers to the random fluctuations of actual losses about their expected values; *firm-specific risk* in financial theory refers to the random fluctuations of a specific stock's price that are unrelated to market movements. *Parameter risk* in insurance refers to the uncertainty of expected losses; *systematic risk* in financial theory refers to the unexpected movements of the stock market as a whole.[28] Diversifying an insurance portfolio smooths process risk but does not affect parameter risk. Diversifying a financial portfolio eliminates specific risk but has little effect on systematic risk.

Modern portfolio theory rests on two assumptions. First, the risk premium varies with systematic risk, not specific risk. Portfolio diversification eliminates specific risk, so the investor should receive no additional return for voluntarily assuming such risk. Second, the original formulation of modern portfolio theory (Markowitz) assumed that systematic risk varies as the standard deviation of returns on a diversified portfolio. Historical returns, on a weekly or monthly basis, can be used to measure the standard deviation. More recent approaches (Capital Asset Pricing Model) assume that systematic risk varies as the regression coefficient (termed "beta") of the diversified portfolio's return on the total market return.[29,30]

One can apply this method to determine insurance risk loads as well.

The risk load should depend upon fluctuations in overall insurance portfolio returns. It should not vary with the loss fluctuations of individual risks, since these can be reduced and often eliminated by proper diversification.

Fluctuations in insurance portfolio returns can be measured by the standard deviation of historical operating returns by line of business. Alternatively, they can be measured by the regression coefficient of the return from a particular line of business on the return of all lines combined.

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[28] Systematic risk is often termed *diversifiable* or *market* risk. Specific risk is also termed *unsystematic*, *residual*, *unique*, or *undiversifiable* risk. See Richard A. Brealey and Stewart C. Myers [9], page 132.

[29] A good introduction to modern portfolio theory is J. Fred Weston and Thomas E. Copeland [49], chapters 16 and 17. The development of the theory is due to William F. Sharpe [45] and John V. Lintner [35].

[30] Several technical assumptions used in the Capital Asset Pricing Model are more relevant to securities than to insurance products, such as costless financial transactions and the availability of various quantities of securities at a given market price. See below in the text for further discussion of these issues.

Of course, insurance policies do differ from financial investments, and modern portfolio theory is more applicable to the latter than to the former.

Financial investments can be broken down into small pieces. Even a small investor can diversify his portfolio by purchasing shares in a mutual fund. A portfolio of thirty or more unrelated stocks is well diversified, and most investors can afford such purchases. In contrast, insurance policies are discrete units. Distinct policies, if written by the same agency or branch office, may not be unrelated—just as one does not diversify a financial portfolio by purchasing a dozen oil stocks.

The price of a stock reflects not only current earnings but also investors' expectations for future earnings. A well-established but cyclical industry may show severe fluctuations in year to year profitability, but milder changes in stock prices. The insurance industry shows consistent "underwriting" or "profitability" cycles. The standard deviation of insurance returns may not accurately reflect investors' expectations of long term profitability.[31]

Neither of these problems is insurmountable. A small General Liability insurer faces not only systematic risk borne by the industry as a whole, but also some specific risk due to its particular book of business. This implies that the insurer must examine the standard deviation of historical returns on a book of the same size and quality, not on a fully diversified book, such as the industry book. The small- or moderate-size insurer needs a slightly larger risk load than that indicated by industry-wide experience.

The second difference mentioned above has the opposite effect. Since insurance profitability is cyclical, yearly operating ratios show greater fluctuation than investors' expectations do. In other words, the insurer

[31] Nahum Biger and Yehuda Kahane [7] state this as follows: " . . . underwriting profits reported by insurers are not necessarily equal to the way market participants assess those profits, their variability, and the systematic portion of the risk. It follows that evaluation of the systematic risk of underwriting, which is not based on market returns but on reported profits, may result in biased estimates of the coefficients."

needs a slightly smaller risk load than that indicated by the standard deviation of insurance operating ratios.[32]

Best's *Aggregates and Averages* shows industry-wide operating returns by line of business. There are two problems with these figures: (1) there is no adjustment for reserve deficiencies and redundancies and (2) operating income is determined by spreading net investment income to line of business, not by discounting all cash flows to a common date. Nevertheless, Best's figures are carefully compiled and widely available, and they are sufficient for the illustrative purposes of this paper.[33]

Best's determines operating ratios by line of business as:

(Losses + loss adjustment expenses incurred) / net premiums earned
+
(commissions, brokerage, and other underwriting expenses) /
net premiums written
+
(policyholder dividends − net investment income) /
net premiums earned.

[32] The mathematical derivation of the Capital Asset Pricing Model relies on the opportunity of borrowing or lending at the risk-free interest rate. This is not true for insurers, but it is not true for investors either. Both investors and insurers must pay a premium to borrow money.

The major difference between the financial and insurance markets is that investors can quickly modify their portfolios, whereas insurers are constrained by competitive pressures, high new business production costs, and higher pure premiums among new policyholders. (See Conning & Co. [13] and Sholom Feldblum [23] for further discussion of these costs.) Modern portfolio theory presumes that optimal portfolios are determined by risk and return. In truth, numerous other factors are also relevant.

Risk and return considerations are important, but they cannot—in isolation—argue for restructuring an insurance portfolio. J. D. Hammond and N. Shilling [29] note that the "efficient" insurance portfolios determined by their analysis consist mostly of minor lines of business. J. R. Ferrari [24] finds that an "efficient" insurance portfolio would require separation of automobile bodily injury from property damage, and separation of fire from extended coverage. As Ferrari notes, these other factors must be considered when structuring an insurance portfolio. See also the discussion of Ferrari's paper by Matthew Rodermund [42].

Thus, we do not determine "efficient" insurance portfolios, or recommend restructuring an insurer's writings to "optimize" risk-return relationships. Rather, we simply analyze the variance in insurance profitability by line of business to suggest the risk loading appropriate to each.

[33] A comprehensive model for determining discounted insurance profits by line of business is provided by Richard G. Woll [51]. For methods of examining insurance reserve adequacy, see Ruth E. Salzmann [43].

For instance, the 1988 operating ratios for the Fire (profitable) and Private Passenger Automobile Liability (unprofitable) lines of business are as follows:[34]

TABLE 3

INSURANCE OPERATING RATIOS

| | Loss Ratio | Expense Ratio | Dividends | Investment Income | Operating Ratio |
|---|---|---|---|---|---|
| Fire | 53.9% | 37.8% | 0.9% | 4.9% | 87.7% |
| Pers Auto Liab | 93.2 | 22.7 | 0.8 | 9.7 | 107.1 |

Instead of operating ratios, we use profit margins. That is, an 87.7% operating ratio is a profit margin of 12.3%, and a 107.1% operating ratio is a profit margin of −7.1%. Profit margins, and the standard deviations of profit margins by line of business over the past 10 years, are shown in Table 4.

The Commercial Liability lines of business—Commercial Multiple Peril, Other Liability, Medical Malpractice, and Commercial Auto Liability—are highly risky: the standard deviations of their profit margins average 12.5. The Personal Property lines of business—Homeowners and Private Passenger Auto Physical Damage—are less risky: the standard deviations of their profit margins average 3.8.[35]

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[34] Data from Best's *Aggregates and Averages*, Property-Casualty, 1989 Edition (Oldwick, NJ: A. M. Best Company, 1989), pages 96 and 98.

[35] Natural catastrophes, such as hurricanes and earthquakes, present the greatest risks in Homeowners insurance. During most years, Homeowners experience is favorable, but a major hurricane may cause enormous industry losses. U.S. catastrophe experience was mild in the late 1970s and in the 1980s, so operating ratios have been relatively stable. Hurricane Hugo and the California earthquake of 1989, which are not yet included in the data presented in the text, demonstrate the catastrophe potential in this line of business. Many climatologists believe that the experience of the early and mid-1980s has been exceptional, and we may expect more severe catastrophes in the future. If so, the Homeowners stability is deceptive. The risk may be hidden, but it is still there. See also footnote 37.

TABLE 4

PROFIT MARGINS AND THEIR STANDARD DEVIATIONS BY LINE OF BUSINESS (1979–1988)

| | 1979 | 1980 | 1981 | 1982 | 1983 | 1984 | 1985 | 1986 | 1987 | 1988 | Std. Dev. (79–88) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Fire | 8.2 | 5.1 | 5.6 | 5.6 | −0.6 | −6.6 | 8.8 | 12.7 | 7.2 | 12.3 | 6.48 |
| Allied | 1.5 | −1.4 | 7.8 | 3.2 | −5.6 | 0.1 | 2.0 | 22.7 | 24.0 | 22.5 | 10.58 |
| Farmowners | 9.5 | −9.1 | −7.9 | −16.3 | −7.3 | −14.8 | −7.4 | −4.3 | 8.6 | 4.9 | 8.66 |
| Homeowners | 4.0 | −2.0 | 1.8 | −0.1 | 0.5 | −1.8 | −6.6 | 1.6 | 8.0 | 4.8 | 3.87 |
| CMP | 11.4 | 6.6 | −0.5 | −9.1 | −14.8 | −25.1 | −12.2 | 11.1 | 6.5 | 3.1 | 13.49 |
| Ocean Marine | −1.1 | −6.9 | −1.6 | −3.3 | −2.4 | −2.4 | 5.5 | 8.5 | 9.4 | 2.6 | 5.14 |
| Inl. Marine | 8.2 | 1.2 | 1.5 | 1.5 | 0.9 | −1.8 | 7.4 | 21.8 | 22.1 | 18.0 | 8.76 |
| Group A&H | 4.1 | 2.5 | 0.6 | 1.9 | 1.8 | 7.4 | 2.3 | −10.6 | −6.8 | −2.5 | 5.05 |
| Other A&H | 4.1 | 1.8 | 1.3 | 3.3 | 1.9 | 8.1 | 9.4 | 6.8 | −0.1 | 1.9 | 3.03 |
| Work Comp | 6.3 | 9.3 | 10.2 | 11.1 | 3.7 | −5.3 | −3.8 | −7.4 | −4.8 | −5.7 | 7.08 |
| Other Liab | 14.0 | 7.3 | 3.5 | −6.4 | −13.8 | −25.1 | −25.8 | −2.5 | 3.7 | 8.5 | 13.22 |
| Med Mal | 8.0 | 0.2 | −1.4 | −9.8 | −8.9 | −18.3 | −29.5 | −8.7 | 8.1 | 16.0 | 12.84 |
| Aircraft | −3.1 | 4.6 | 7.6 | 10.0 | 5.6 | 4.6 | 6.7 | 16.4 | 17.8 | 9.2 | 5.71 |
| PPA Liab | 5.2 | 3.9 | −1.4 | −2.1 | −2.9 | −3.9 | −9.5 | −9.2 | −7.7 | −7.1 | 4.86 |
| CA Liab | 2.4 | −1.1 | −8.4 | −15.3 | −21.3 | −30.8 | −16.4 | −3.6 | 1.0 | 2.3 | 10.82 |
| PPA Phy Dam | 1.4 | 4.9 | 3.0 | 0.2 | 5.1 | 1.1 | 2.9 | 8.2 | 11.7 | 9.5 | 3.67 |
| CA Phy Dam | 8.2 | 5.9 | 2.1 | −4.2 | −4.0 | −8.5 | 5.5 | 19.9 | 22.7 | 22.4 | 10.80 |
| Fidelity | 21.1 | 15.4 | 10.6 | −3.2 | −11.9 | −6.2 | 16.1 | 32.0 | 36.1 | 35.9 | 16.53 |
| Surety | 13.2 | −1.2 | 18.3 | 15.4 | 18.4 | 12.3 | −2.2 | −7.6 | −0.6 | 17.2 | 9.51 |
| Burglary | 24.0 | 13.5 | 9.6 | 8.2 | 15.9 | 23.5 | 30.5 | 34.8 | 41.3 | 39.5 | 11.54 |
| Boiler & M | 15.6 | 13.1 | 11.2 | 3.5 | −3.7 | −10.6 | 24.1 | 11.8 | 18.1 | 4.8 | 9.88 |
| Reinsurance | 6.6 | 5.9 | 4.5 | −1.3 | −6.4 | −22.2 | −7.0 | 0.4 | −2.6 | 10.0 | 8.81 |
| Other Lines | −12.7 | −1.5 | −1.4 | −16.3 | 2.9 | 6.7 | 15.7 | −14.3 | −8.8 | −12.3 | 9.94 |
| Total | 5.9 | 4.1 | 2.4 | −0.5 | −2.3 | −7.4 | −6.3 | 0.9 | 4.4 | 4.3 | 4.39 |

These standard deviations reflect fluctuations in returns, not dispersion of the loss distribution. For instance, Ocean Marine has great random loss fluctuations on individual policies. But most Ocean Marine claims are small partial losses: the standard deviation of the profit margin is low (5.1), since there is not much uncertainty in the expected loss values.[36] Workers Compensation also has high variation in the size of loss distribution, since there is no limit on medical payments in the

[36] See Klaus Gerathewohl, et al. [28].

policy. But the statutory benefits and bureau rate making reduce the fluctuation in overall portfolio returns to a manageable level.[37]

Commercial Liability insurers must continually adjust their expected loss values as social conditions change. The proliferation of new causes of action hampers General Liability expected loss forecasts, while the increasing claims consciousness of the public frustrates Medical Malpractice loss forecasts. This is the risk which insurers face, and for which they need additional "risk loads."[38]

How stable are these results over time? Are the high standard deviations noted for the Commercial Liability lines characteristic of these types of risks or are they peculiar to the time period used?

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[37] David Appel, a research economist formerly with the National Council on Compensation Insurance, has pointed out to me an important difference in pricing strategies between Workers' Compensation and other Commercial lines of business. Dr. Appel's insights are correct, and they modify the conclusion in the text. Many carriers write "accounts," providing Commercial Automobile, General Liability, Commercial Property, and Workers Compensation coverages for the insured. During downturns of the underwriting cycle, insurers reduce their Commercial Auto and GL rates, or they provide large schedule modifications, to retain the business. Conversely, during upturns of the cycle, Commercial Auto and GL rates increase rapidly and schedule modifications diminish.

Workers Compensation rates, however, show less variation from year to year. Thus, the high variability in Commercial Automobile and General Liability profits may reflect on all the coverages marketed together, and does not necessarily indicate that these lines are more risky than Workers Compensation.

Similar business and competitive considerations apply to all the figures in this paper. Financial theory is abstract: it provides directions, but it does not offer decisions for concrete cases. The pricing actuary must temper the abstract theory with practical judgment to arrive at an equitable risk load for any line of business.

[38] Fluctuations in reported operating returns by line of business depend primarily on insurance risk, not on investment risk. The more stable investment returns, such as interest, dividends, rents, and realized capital gains are carried to the income statement. Unrealized capital gains and losses, which vary widely from year to year, are a direct charge to surplus. Thus, CMP, with a short average settlement lag but great insurance risk, has a high standard deviation and a high $\beta$ in Table 5, as well as in the studies by Hammond and Shilling and by Cummins and Nye (see following footnote). Workers Compensation, with a long settlement lag but less insurance risk, has lower standard deviations and $\beta$s in this paper and in the previous studies.

An alternative possibility for fluctuating insurance returns, that they are caused more by stock value variations than by insurance risk, is considered by Yehuda Kahane [34].

Hammond and Shilling [29] analyzed the standard deviations of *underwriting profits* by line of business for 1956–1970.[39] Among the major lines of business, they found high standard deviations for Commercial Multiple Peril and General Liability BI, and low standard deviations for Workers Compensation, similar to the results of the present analysis. However, they found a somewhat higher standard deviation for the Personal Property lines than for automobile liability.[40]

Modern portfolio theory considers the historical variance of returns of a single segment of a portfolio an incomplete approximation for risk. Equally important is the covariance of returns among securities.[41] Unfortunately, estimating covariances among securities or lines of insurance is an arduous task.[42]

The Capital Asset Pricing Model provides an elegant means of determining the risk on an individual security, composed of both the variance of its own returns and the covariances with the returns on other securities.[43] Returns from each security are regressed against the returns of the total market portfolio, thereby quantifying price fluctuations that cannot be reduced by diversification.

A prudent investor diversifies his financial holdings. Variances of return that can be eliminated by diversification should receive no reward for the additional risk undertaken. Variances of return that are correlated with total market fluctuations, however, cannot be eliminated by diversification. The CAPM posits that this "risk" is rewarded by a higher expected return.

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[39] Investment income by line was not readily available in the 1970s, so Hammond and Shilling [29] used the complement of the combined ratio. Interest rates were relatively stable from 1956 through 1970, so the standard deviations of underwriting income and operating profits should be similar.

Cummins and Nye [17] examined the variability of returns by line of business for one insurance company from 1958 to 1975 and found the same results for the major lines of business as in this paper: high variability for CMP and General Liability, low variability for Auto Physical Damage and Fire (which in the 1960s accounted for most of Personal Property insurance), low variability for Workers Compensation, and low to moderate variability for Automobile Liability.

[40] This accords with the more rigorous estimation method discussed below; see Table 5.

[41] See Harry Markowitz [37].

[42] Ferrari [24], Brubaker [10], and Cooper [14] emphasize the importance of covariance among lines of business. The development of the Capital Asset Pricing Model has obviated the need for quantifying covariances, so there has been little subsequent work on Ferrari's or Brubaker's methods.

[43] See William F. Sharpe [46].

Formally, the following regression model quantifies the undiversifiable or systematic risk ($\beta$):

security's return = $a + \beta$ (market return).

The $\beta$ is determined from historical returns. The current expected return, $R$, is

$$R = R_f + \beta(R_m - R_f),$$

where $R_f$ is the risk free rate, such as the rate on Treasury bills, and $R_m$ is the overall market return.[44]

Several writers have applied modern portfolio theory to the investment of stockholders in insurance firms.[45] The "risk" associated with insuring a given block of business is related to the covariance of return from that business with the diversified financial portfolios held by the investors in the insurance firm. These covariances, or the *underwriting betas* associated with writing a line of insurance, have generally been low and unstable.[46]

A stockholder chooses between investing his money in an insurance firm and investing it in other securities. The insurance firm itself does not have this option. Were it to invest part of its equity in securities, instead of using it to "support" insurance writings, it would subject its stockholders to double income taxation: the insurer pays taxes on its investment earnings and its stockholders pay taxes on dividends and

[44] This equation relies on the assumed availability of borrowing and lending at the risk free rate; see J. Fred Weston and Thomas E. Copeland [49] for a good summary. This assumption is unrealistic, but the agreement of the CAPM with empirical returns is the major justification of its use. Sharpe and Alexander [47] use a quotation from Milton Friedman [27] to clarify this issue: "The relevant question to ask about the 'assumptions' of a theory is not whether they are descriptively 'realistic,' for they never are, but whether they are sufficiently good approximations for the purpose at hand. And this question can be answered only by seeing whether the theory works, which means whether it yields sufficiently accurate predictions." On empirical testing of the CAPM, see D. W. Mullins, Jr. [39] and the references cited therein.

[45] See particularly William Fairley [22].

[46] Fairley [22], *op. cit.*, estimates an underwriting $\beta$ of $-0.21$. Biger and Kahane [7], *op. cit.*, conclude that "preliminary empirical evidence presented shows that the 'systematic risk' of underwriting profits approaches zero in most lines." J. David Cummins and Scott Harrington [16], using quarterly accounting data, find a highly unstable underwriting $\beta$ through the 1970s, averaging to $-0.03$ for 1970–1981 (or $-0.01$ for an annual data value).

capital gains. The insurer's stockholders would prefer to invest their monies directly in securities and pay income taxes only once.[47]

Thus, the traditional use of the Capital Asset Pricing Model for estimating underwriting $\beta$'s quantifies the risk faced by the investor in insurance stocks, not the risk of the insurer. Commenting on CAPM based pricing models, D'Arcy and Doherty say, "Notice that nowhere is there a direct relationship between the competitive underwriting profit and risk. The riskiness of the insurance operations *per se* is not at issue. Much of the risk can be diversified by the insurance company's own equityholders in the management of their personal portfolios. Only that component of risk that is not so diversifiable, the systematic risk, is reflected in the competitive underwriting profit. Thus the competitive price is related only to the beta, which picks up this systematic risk."[48]

An insurer chooses lines of insurance (or blocks of business) to maximize its expected return while minimizing its "risk." The market return $R_m$ in the CAPM model should be replaced by the return on a fully diversified insurance portfolio. The appropriate equation is

$$R = R_f + \beta(R_p - R_f),$$

where $R_p$ is the return on the all lines combined insurance portfolio.[49]

Operating returns from Best's *Aggregates and Averages* (see Table 3) are used to determine the $\beta$'s by line shown below. Note carefully: these do *not* reflect the risk to the investor in insurance stocks. Rather, they reflect the risk to the insurer of writing different lines of business.

The highest $\beta$'s occur in the Commercial Liability lines of business: Commercial Multiple Peril, General Liability, Medical Malpractice, and Commercial Auto Liability.[50] In other words, when the insurance industry as a whole does well, these lines show excellent returns; when the

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[47] Myers and Cohn [40] therefore argue that policyholders should compensate the insurer for federal income taxes on the investment income from surplus.

[48] Stephen P. D'Arcy and Neil A. Doherty [18], page 37.

[49] $\beta$ may be calculated either by a least squares regression or as $COV(R,R_p) \div VAR(R_p)$; see Simon Benninga [6]. I am indebted to Gabriel Baracat for aid in estimating the risk loads by line of business.

[50] The high $\beta$ for fidelity is due to the strong profits in this line during the most recent years. This is presumably a random event, due to the low premiums in this line and the U.S. economic prosperity, which reduces fidelity losses.

industry is less profitable, these lines fare even worse. The Personal Property lines of business—Homeowners and Private Passenger Auto Physical Damage—have low β's. These lines have smoother underwriting cycles than does the experience of all lines combined.

TABLE 5

β'S BY LINE OF INSURANCE
(BASED ON 1979–1988 EXPERIENCE)

| Line of Ins. | Beta | Line of Ins. | Beta | Line of Ins. | Beta |
|---|---|---|---|---|---|
| Fire | 0.92 | Work Comp | 0.46 | Fidelity | 2.32 |
| Allied Lines | 1.04 | General Liab | 2.98 | Surety | 0.04 |
| Farmowners | 1.35 | Med Mal | 2.65 | Burglary | 0.33 |
| Homeowners | 0.65 | Aircraft | 0.07 | Boiler & Mach | 0.87 |
| CMP | 2.78 | Pers Auto Liab | 0.45 | Reinsurance | 1.74 |
| Ocean Marine | 0.04 | Comm Auto Liab | 2.27 | Other Lines | −1.62 |
| Inland Marine | 0.88 | PPA Phy Dam | 0.37 | | |
| Group A&H | −0.46 | CA Phy Dam | 1.52 | Total | 1.00 |
| Other A&H | −0.51 | | | | |

Most accident and health insurance is sold by life companies. The profitability of these lines is unrelated to the Property/Casualty underwriting cycle; the historical correlation is negative. Much reinsurance is bought for Commercial Property and General Liability risks, and its profitability follows the returns of these primary lines.

Workers Compensation and Private Passenger Auto Liability have not been profitable lines in recent years, but their returns have been relatively stable. In Auto Liability, the large number of small risks smooths the fluctuations in insurance returns, though consumer complaints about high premium rates keep profits low. Administered rating and account pricing smooth the fluctuations in Workers Compensation returns. The divergence between state legislators, who mandate WC benefits (often in response to labor desires) and state regulators, who oversee rates (sometimes in response to employer needs), depresses profits.[51]

[51] See William Bailey [4].

To determine risk loads by line of business, one needs the risk free rate and the expected return for all lines combined. Actuaries and financial analysts regularly forecast returns for the Property/Liability insurance industry.[52] The risk free rate may be derived from returns on Treasury bills and bonds. Thus, if the risk free rate is 7% per annum, and the expected return for the industry as a whole is 14% per annum, the risk premium for Reinsurance is 12.2% per annum [ $= 1.74 \cdot (14 - 7)$ ].

This is a return on equity; it must be converted into a return on premium for the rate making calculation. In other words, one needs appropriate premium-surplus ratios by line of business.

Inasmuch as surplus is needed to support *insurance* risk, represented by fluctuations in reported operating ratios, the loadings discussed here compensate the insurer for the risk it undertakes. The return on equity can be directly converted to a return on premiums, and a relationship such as the Kenney rule is appropriate.[53]

If surplus is also needed to support asset value fluctuations, such as unrealized capital gains and losses, which do not flow through to the income statement, then additional surplus is needed for long-tailed lines of business. The ratemaking loadings would be slightly different from those shown here. In particular, CMP would have a somewhat lower load and Workers Compensation would have a higher load.[54]

But is modern portfolio theory correct even for financial investments? Financial analysts note two problems with the theory: First, different β's result when different experience periods or different statistical methods are used. Second, the "security market line,"—the empirical relationship of returns afforded by stocks and their historical β's—is less steeply sloped than the theoretical Capital Asset Pricing Model line predicts

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[52] Stock analysts estimate the average CAPM β for property/casualty insurers to be approximately unity.

[53] The β's determined here are based on operating ratios, which relate profits to premiums. The magnitude of the profit fluctuations is viewed relative to annual premium, not to loss reserves. Surplus allocation should vary with premium if these βs are used. If one allocates surplus relative to loss reserves, one should relate profit fluctuations to reserves. If so, the βs for the Commercial Liability lines of business would be lower, since their reserves are larger.

[54] On the functions of supporting surplus, see Alfred E. Hofflander [32]. See also the National Association of Insurance Commissioners [41], page 8: "In addition to providing protection against unusually large losses, surplus also provides a cushion against declines in the value of equity investments, such as common and preferred stocks."

(though the difference is not great). That is, the actual returns demanded by investors increase somewhat less rapidly with the increase in $\beta$ than is predicted by modern portfolio theory.

The actuary must be aware of these problems. Refinements of modern portfolio theory may lead to improvements in estimating risk loads by line of business. But this method at least quantifies the true risk faced by insurers, not some substitute that has no relationship to the insurer's risk.

How might this analysis be improved? First, cash flow discounting should be used instead of spreading investment income to line of business. Different growth rates by line of business cause the Insurance Expense Exhibit allocation of investment income by line to distort the true expected present values of insurance operations. Divergences between embedded yields and expected new money rates are also a problem. Second, if an insurer's writings are large enough, the historical returns on its own book of business should be used instead of industry totals, since one insurer's book may have different characteristics from that of another insurer. Third, quarterly returns by line of business should be examined over different time periods. Reinsurance had stable and favorable returns for 1979–1981, but highly variable profits in subsequent years. Quarterly returns for the most recent seven years may better reflect the risks in this line of business. Fourth, the expected return for the industry as a whole should be estimated by various methods and by type of insurer. For instance, the expected returns in Personal Automobile Liability insurance differ between agency companies and direct writers.

These are refinements, additional bells and whistles. Even without these enhancements, this method is superior to current "risk load" estimation procedures. It quantifies the true risk faced by insurers, not the "process risk" faced by insureds. It rests on the relationship found in financial investments of greater expected returns for portfolios with greater variance. It relies on the empirical risk aversion demonstrated by institutional investors, which is presumably similar to the risk aversion characteristic of insurers. In sum, it provides insurers with a measure of the true cost of insuring "risky" lines of business.

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