# DISCUSSION OF PAPER PUBLISHED IN VOLUME LXXIV

## A NOTE ON THE GAP BETWEEN TARGET AND EXPECTED UNDERWRITING PROFIT MARGINS

EMILIO C. VENEZIAN

DISCUSSION BY SHOLOM FELDBLUM

This paper argues that if forecast and actual insurance costs are random variables, then the traditional actuarial ratemaking procedure produces an average underwriting profit margin lower than the target underwriting profit margin. The argument is correct in that the average profit margin, per policy or per book of business, will indeed be lower than the target profit margin. However, the expected total profit margin, for the insurer or for the industry as a whole, will not differ from the target profit margin. Observed differences between target and actual profit margins are due to marketplace competition, random forecasting errors, or unsustainable target margins, not to biases in the ratemaking procedures. The total profit margin for the insurer is the important figure, not the average margin per policy.

An illustration should clarify these comments. Suppose that:

1. The forecast insurance costs average $95 per policy, but they vary with equal probability among $80, $95, and $110 per policy.
2. The actual insurance costs have the same probability distribution.
3. The insurer uses a target underwriting profit margin of 5%.

With these probability distributions, the profit margins are as shown below.

TABLE 1

TARGET AND EXPECTED PROFIT MARGINS

| | Forecast Cost | Premium | Actual Cost | Profit Margin | Profit Dollars |
|---|---|---|---|---|---|
| | $ 80 | $ 84.21 | $ 80 | +5.00% | $ +4.21 |
| | 80 | 84.21 | 95 | −12.81 | −10.79 |
| | 80 | 84.21 | 110 | −30.63 | −25.79 |
| | 95 | 100.00 | 80 | +20.00 | +20.00 |
| | 95 | 100.00 | 95 | +5.00 | +5.00 |
| | 95 | 100.00 | 110 | −10.00 | −10.00 |
| | 110 | 115.79 | 80 | +30.91 | +35.79 |
| | 110 | 115.79 | 95 | +17.95 | +20.79 |
| | 110 | 115.79 | 110 | +5.00 | +5.79 |
| Average: | $ 95 | $100.00 | $ 95 | +3.38% | $ +5.00 |

The "average profit margin per policy," at 3.38%, is lower than the target profit margin. The total profit margin for the book of business, at 5.00%, is exactly equal to the target profit margin. Moreover, the *weighted average* of the profit margins per policy, where the weights are the premiums per policy, also equals 5.00%.

These observations are not restricted to the particular illustration used here. Equation (2.8) of the paper shows that the profit margin per policy is

$$A = \mathrm{E}(m) = 1 - (1 - T) \cdot \mathrm{E}\,((1 + y)/(1 + x)),$$

where

$A$ is the achieved underwriting profit margin;
$T$ is the target underwriting profit margin;

$y$ and $x$ are independent random variables with means of zero, measuring prediction errors and random cost fluctuations (see Equations (2.7) and (2.4));
E is an expected value operator; and,
$m$ is the underwriting profit margin.
$C$ is the cost per policy (see the following equations).

In other words, the profit margin for an individual policy $i$ is

$$A_i = 1 - (1 - T) \cdot ((1 + y_i)/(1 + x_i)).$$

The dollar profit for this policy is

$$\text{Dollars}_i = C_i \cdot ((1 + x_i) - (1 - T) \cdot (1 + y_i)).$$

The total underwriting profit margin is the ratio of total dollars of profit to total dollars of premium. Since the $x$ and $y$ random variables appear as numerators of separate terms, they both average to zero and the ratio simplifies to

$$(\text{Total Cost} \cdot [1 - (1 - T)]) / \text{Total Cost} = T.$$

The weighted average of the profit margins is also equal to $T$. The weighted average is

$$\Sigma\ (\text{premium}_i \cdot \text{profit margin}_i) / \Sigma\ (\text{premium}_i).$$

This is the total dollars of profit divided by the total premium, and so equals $T$, as shown above.

When he read his paper before the CAS convention in San Antonio, Professor Venezian indicated that a weighted average by premium is inappropriate, since we are concerned with returns on equity, not returns on premium. This does not alter the situation. Suppose first that the surplus (equity) supporting each policy is a fixed amount that does not vary with the premium, or that it is related to the average cost, not the "forecast" cost. The relevant profit margin is the dollars of profit divided by the surplus amount. The $x$ random variable never enters this ratio, and no "gap" is ever produced. Alternatively, suppose that the surplus supporting each policy varies with the premium charged—say, required surplus equals 50% of the premium. The $x$ random variable does enter

the denominator of the profit margin per policy. However, the weighted average profit margin uses surplus as the weights, so the $x$ random variable once more cancels out of the ratio.

The paper's thesis is that the average of ratios is not the ratio of the average. This is unrelated to actuarial pricing procedures. Consider any firm: suppose the costs of two products are \$50 and \$150, and the corresponding revenues for each are \$100. The profit margins are +100% and −33%, for an average of +33%. Yet the firm's total revenues are \$200 and total costs are \$200, for a profit margin of 0%. The latter figure is the important one, since it shows the true profitability of the firm. The former figure varies with the allocation of costs and revenues to products or product lines. Similarly, in the insurance example, if total profit margins are considered, then expected underwriting profits should equal target underwriting profits.

TABLE 2

AVERAGE PROFIT MARGINS

| Product | Quantity | Cost | Revenues | Profit |
|---|---|---|---|---|
| A | 1 | 50 | 100 | +100% |
| B | 1 | 150 | 100 | −33 |
| Total | 2 | 200 | 200 | 0% |

Since the paper focuses on average profit margins per policy, instead of the total profit margin, it does not address the original problem:

> *"Over extended periods of time the average underwriting profit margins achieved by the industry as a whole, or by individual firms, in most jurisdictions differ substantially from the targets ostensibly built into the rates."*

This "gap" is the total profit margin achieved by the insurer or by the industry minus the target profit margin. The argument advanced in the paper has no relation to this gap.