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# PROCEEDINGS

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# EVALUATING THE EFFECT OF REINSURANCE CONTRACT TERMS

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*Abstract*

*In many reinsurance pricing situations it is not possible to determine a "correct" absolute price without making a large number of tenuous assumptions. Even so, in order to maximize a company's profitability, it is important for the reinsurance actuary and underwriter to be able to choose the best contract terms among the achievable alternatives. Furthermore, being able to offer different but equivalent terms that better serve the needs of the cedant may help close an important deal.*

*This paper measures the efficiency of contract terms by estimating the distribution of the present value of cash flows. To do this, the paper examines paid and incurred aggregate distributions as a function of time over the life of a contract. Sensitivity of the results to changes in the parameters of the underlying loss model is investigated.*

The authors wish to thank Todd J. Hess for his patience in reading many drafts of this paper and suggesting numerous improvements. He also programmed the analytical model, verified the many cash flow formulas, and produced the accompanying exhibits and graphs.

## 1. INTRODUCTION

In many reinsurance pricing situations it is not possible to determine a "correct" *absolute* price without making a large number of tenuous assumptions. However, it is often advantageous to make some general statements about *relative* price adequacy. By *relative* price adequacy we mean statements (about a particular layer of subject business), such as:

1. Deal #1 is better than deal #2.
2. Deal #1 is equivalent to deal #2.
3. A deal is better than it was last year.
4. The reinsurer's side of a deal is better than the company's side.

Even if the underwriter cannot accurately estimate an adequate absolute price, consistently choosing the best contract terms among achievable alternatives is important to a company's profitability. Also, being able to offer different but equivalent terms that may better serve the needs of the cedant can help close a deal.

This paper will explore a method to compare *relative* prices for many types of reinsurance contracts, and look at how sensitive the results are to the parameters of the underlying model of losses.

Commonly used methods that utilize ultimate aggregate loss distributions can give some view of the relative price. However, this alone can sometimes lead to incorrect conclusions with regard to maximizing profitability. Additional insight into the relative prices can be seen by examining the distribution of cash flows and the accompanying investment income. To do this, the paper examines paid and incurred aggregate distributions as a function of time over the life of a contract.

Few papers in the casualty actuarial literature have dealt with the cash flow of a contract. For example, Meyers [6] includes investment income to determine the parameters of a primary retrospective rating plan which yields a desired operating profit. Lee [4] uses graphical techniques to lend insight into excess of loss coverages and retrospective rating. Bühlman and Jewell [1], Gerber [2], and Lemaire and Quairiere [5] consider optimal reinsurance and risk exchanges. However, these papers do not consider investment income and only deal with simplified reinsurance contract types (e.g., quota share contracts).

The procedure described herein uses a stochastic model to estimate the distribution of the present value of cash flows. The paper's emphasis will be to derive results that are applicable to real-life pricing decisions. The approach will be to summarize key information rather than to find the single "optimal" solution.

## 2. AN EXAMPLE

Imagine that it is December 28 and you are a Lloyds underwriter with a long queue of brokers waiting at your box. You are discussing a treaty reinsurance proposal for losses $250,000 excess of $250,000 per loss on a portfolio of long haul trucking liability business that generates a total premium of $5,000,000 (net of commissions). You are very familiar with this account; you have estimated the expected losses to the reinsurance layer as being $1,500,000 (30% of the total subject premium). You are the lead underwriter, so it is up to you to quote terms. After several days of back and forth discussions among you, the broker, and the company, the broker has summarized three types of proposals that he thinks will be acceptable to the company. He wants to know on which one(s) you will give a firm quotation. The alternatives are[1]:

1. Reinsurance premium = 10% of subject premium (sp).
   Aggregate deductible = 20% of sp.
   Aggregate limit = 400% of reinsurance premium.
2. Retrospectively rated contract.
   Provisional premium = 8% of sp.
   Maximum premium = 30% of sp.
   Premium adjusted monthly to 110% of paid losses plus 8% of sp.
   Aggregate limit = 200% of reinsurance premium.
3. Reinsurance premium = 27% of sp.
   Profit sharing after four years of 60% of reinsurer's profit after 10% deduction of reinsurance premium (i.e., 2.7% of sp), on a paid loss basis.
   Aggregate limit = 150% of reinsurance premium.

[1] The alternative contracts will be explained more fully in section 5. Further discussion of reinsurance contracts and terminology may be found in Lee [4], Patrik and John [7], and Reinarz [10].

## 3. NOTATION

The following notation will be used with respect to the reinsurance layer[2]:

1. $N$, random number of excess losses,
2. $P_t$, random variable denoting aggregate paid losses at time $t$,
3. $K_t$, random variable denoting aggregate known loss reserves at time $t$. Note that $P_t$ and $K_t$ can be viewed as the sum of a random number of individual paid or known reserved losses.
4. $R_t$, random variable denoting reinsurance premium at time $t$. This may be a function of paid or incurred losses.
5. $C_t$, random variable denoting the cumulative cash flow (positive and negative) for the reinsurance contract at time $t$. This is a function of the contract terms, $R_t$, $P_t$, and $K_t$.
6. $V$, random variable denoting the present value of the net cash flow to the reinsurer defined as:

$$V = \sum_t [C_t - C_{t-1}]v^{t-1}; \; v = \frac{1}{1+i}$$

In addition, it is assumed that losses occur mid-year; premium and loss transactions are made at mid-year; and, production and overhead expenses are ignored.

With this information, one can investigate properties of $V$ in order to judge what set of contract terms is most efficient over a broad range of reasonable assumptions.

## 4. CRITERIA FOR JUDGING THE EFFECT OF CONTRACT TERMS

There are three ways that a reinsurance contract affects a reinsurer:

*Economic Impact*: Present value of cash flows, $V$, from the transaction (pre-tax). The interest rate is assumed to be non-random and known in advance.

[2] Random variables are denoted by capital letters and non-random quantities are denoted with small letters.

*Accounting Impact*: An income statement and balance sheet are determined by the contract terms and $R_t$, $P_t$, and $K_t$. Two different reinsurance contracts can produce the same $C_t$'s and therefore have the same economic value, but have very different accounting effects.[3]

*Tax Impact*: The tax impact is determined from the accounting impact and affects the after-tax economic impact.

This paper considers only the economic impact.

## 5. DESCRIPTION OF COMMON CONTRACT TYPES

For the purposes of measuring their economic impact, many different types of reinsurance contracts (such as sliding scale commissions, retrospective rating plans, funded programs, aggregate caps, etc.) reduce to a few basic features.

The simplest types of contracts are those where $C_t$ is a function of only $P_t$, and the function does not vary over different ranges of $t$. For these, a useful first step in analyzing the economic effect is to graph $C$ as a function of $P$.

In other words, we are graphing the cumulative cash flow (prior to interest) to the reinsurer (through $t$) as a function of the underlying paid losses to the contract. The reinsurer prefers larger $C$'s and prefers $C$'s which are less than zero at $P$'s that have a low probability. We would normally expect $C$ to be a declining function of $P$ (as losses increase, the reinsurer's result deteriorates), but this is not always the case.

The following graphs illustrate the functioning of various contract terms; first for simple, then for more complicated types.

[3] As an example, assume that you are choosing among the following three plans described in section 5, to cover the same underlying risks: 1.b. paid loss retro, no minimum; 1.c. funded plan, with interest credit; and 1.d. aggregate deductible. Parameters can easily be chosen such that $C_t$ is the identical function of $P_t$ for all three plans. For those parameters, all three plans have the same economic impact. However, the definition of premium is different in each case. The profit or loss effect of each plan is the same, but the accounting entries producing that result differ.

1. Contracts of the form $C_t = \min(aP_t + b, r - P_t)$

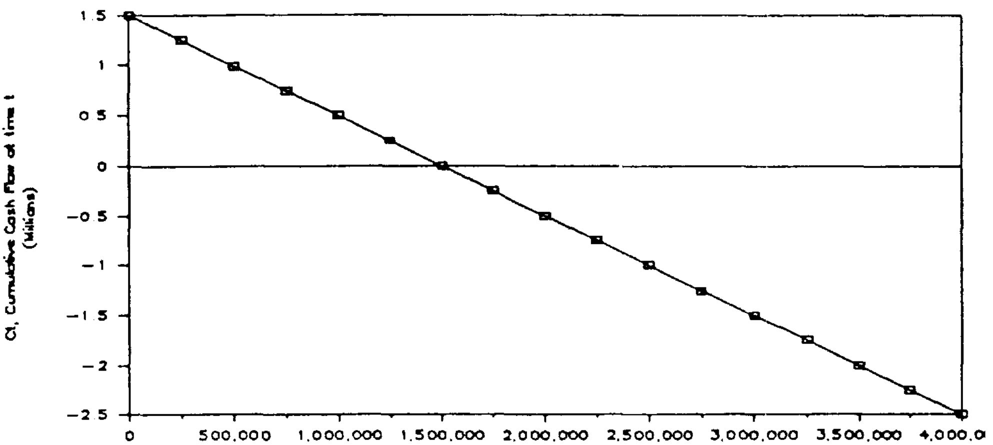
   a. Flat rated: The premium charged by the reinsurer is known in advance of the effective date and is fixed for the life of the contract. The premium is usually expressed as a percentage of the premiums charged by the ceding company on the business subject to the treaty (called subject premium).

$$C_t = r - P_t,$$

   where $r$ = premium.

   For example, let $r$ = \$1,500,000.

FLAT RATED
Cash Flow as a Function of Paid Loss



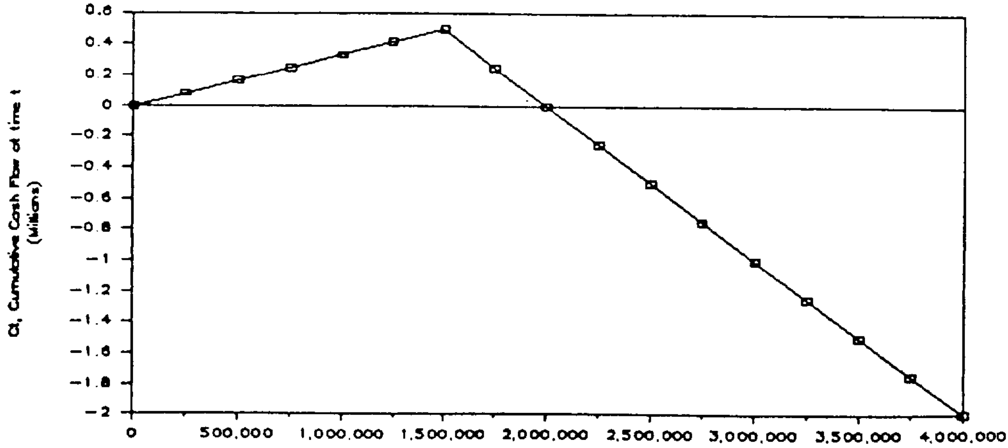

Pt, Paid Loss through time t

b. Paid loss retro, no minimum (sometimes called cash flow plans): The premium charged by the reinsurer is a function of the actual aggregate paid loss experience. In this case, the developed premium can increase to a maximum of $M$.

$$C_t = \min\,(aP_t + b,\ M - P_t).$$

For example, let $a = .333$, $b = 0$ and $M = \$2{,}000{,}000$.

PAID LOSS RETRO, NO MINIMUM
Cash Flow as a Function of Paid Loss




Pt, Paid Loss through time t

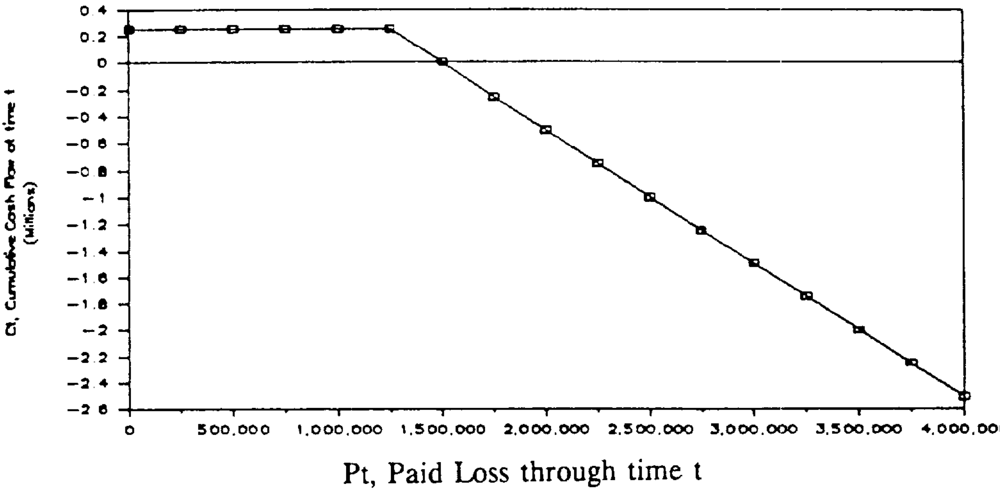
c. Funded plan, with interest credit: The premium less the reinsurer's margin is placed in a fund which accumulates interest at the credited amount and from which losses are paid. When the contract is commuted, the fund balance, if any, is returned to the cedant. The fund would normally be set at an amount sufficiently higher than expected losses to pay for actual losses in most years.

$$C_t = \min\,(r - f_0,\ r - P_t),$$

where $r = f_0 + \text{margin}$,
$f_0$ = fund at time.

For example, let $f_0$ = \$1,250,000 and margin = \$250,000.

FUNDED PLAN, WITH INTEREST CREDIT
Cash Flow as a Function of Paid Loss



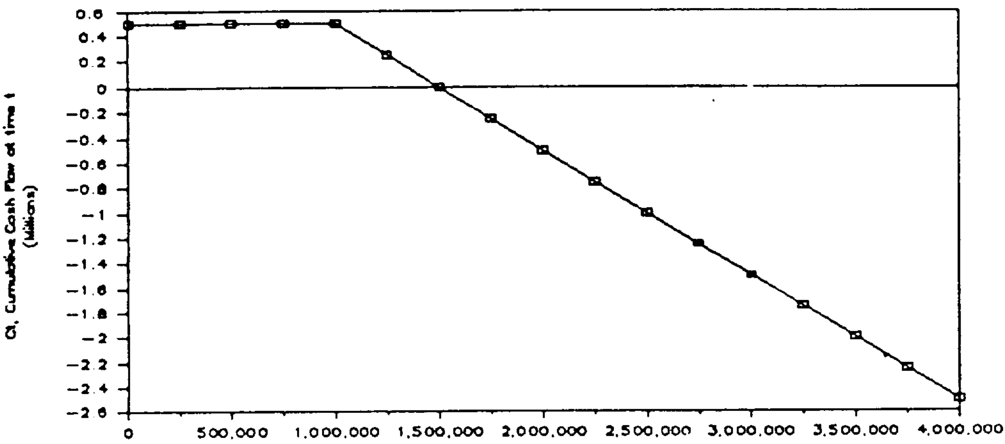
d. Aggregate deductible: For an aggregate deductible, the reinsurer pays no losses until the total losses to the excess layer exceed the deductible. Typically, the aggregate deductible is set lower than the total losses expected for the layer. The graph shows that the economic effect of an aggregate deductible is the same as a funded plan with interest (but the accounting effects are quite different).

$$C_t = \min(p - d, p - P_t),$$

where $p = r + d$,
$d$ = deductible,
$r$ = premium.

For example, let $r$ = \$500,000 and $d$ = \$1,000,000.

AGGREGATE DEDUCTIBLE
Cash Flow as a Function of Paid Loss




Pt, Paid Loss through time t

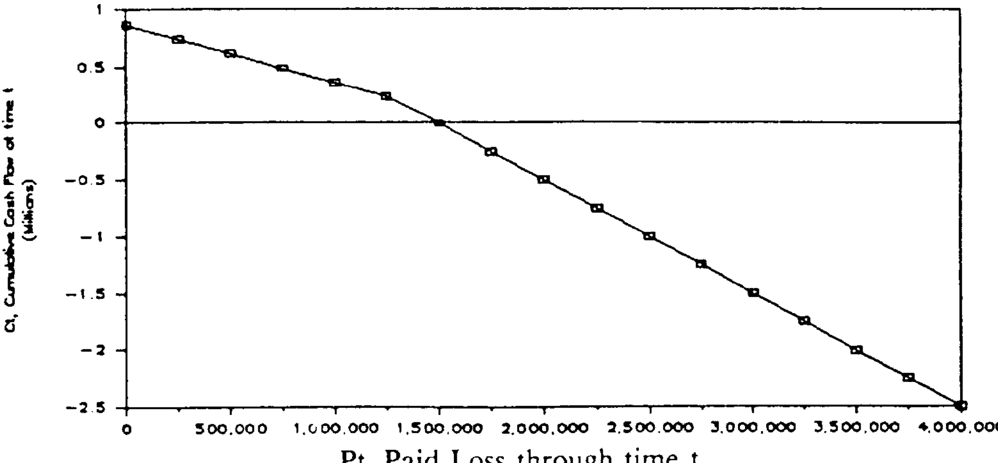
e. Profit Commission: In this plan the reinsurer returns a share of his profits to the cedant. Profit is defined to be premiums less losses and reinsurer's margin. Because actual profit will not be known for many years, profit commission could increase or decrease thereby requiring additional payments by the reinsurer or a return of profit commission by the cedant. However, the profit commission is never less than zero.

$$C_t = \min\left(-(1-h)P_t + r(1-h)(1-e),\ r - P_t\right),$$

where $h$ = profit sharing percent,
$e$ = reinsurer's percent margin,
$r$ = premium.

For example, let $h = .50$, $e = .15$ and $r = \$1,500,000$.

PROFIT COMMISSION
Cash Flow as a Function of Paid Loss



Pt, Paid Loss through time t

2. Contracts of the form $C_t = \max\,(aP_t + b,\ m - P_t)$

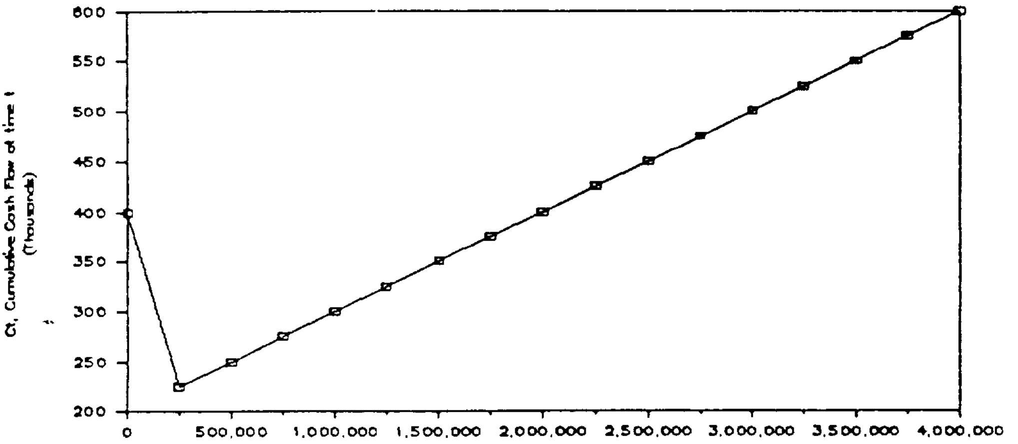
 a. Paid loss retro, no maximum: This is a pure cash flow plan which allows the cedant to spread his incurred loss experience and thereby smooth underwriting results. The cedant usually pays the reinsurer a provisional premium greater than or equal to the minimum, with the final premium based on actual paid losses plus loadings.

$$C_t = \max\,(aP_t + b,\ m - P_t),$$

where $a$ = multiplicative loading,
$b$ = additive loading,
$m$ = minimum premium.

For example, let $a = .10$, $b = \$200{,}000$ and $m = \$400{,}000$.

PAID LOSS RETRO, NO MAXIMUM
Cash Flow as a Function of Paid Loss




Pt, Paid Loss through time t

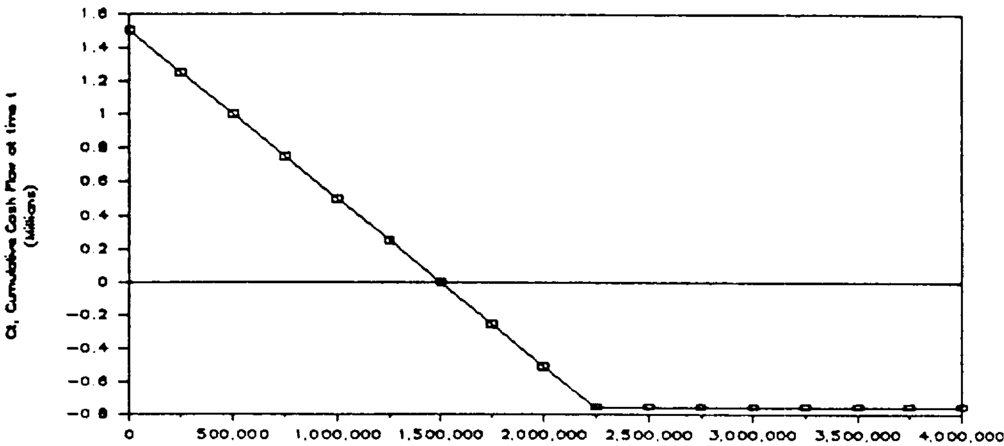
b. Loss ratio aggregate limit or "cap": The reinsurer's aggregate liability for losses is capped at a specific dollar amount expressed as a loss ratio or dollar limit. The loss ratio is usually against the reinsurer's net premiums.

$$C_t = \max\,(r - P_t,\ r - f),$$

where $r$ = premium,
$f$ = cap (in dollars).

For example, let $r$ = \$1,500,000 and $f$ = \$2,250,000.

LOSS RATIO CAP
Cash Flow as a Function of Paid Loss




Pt, Paid Loss though time t

3. Plans with both minimums and maximums.

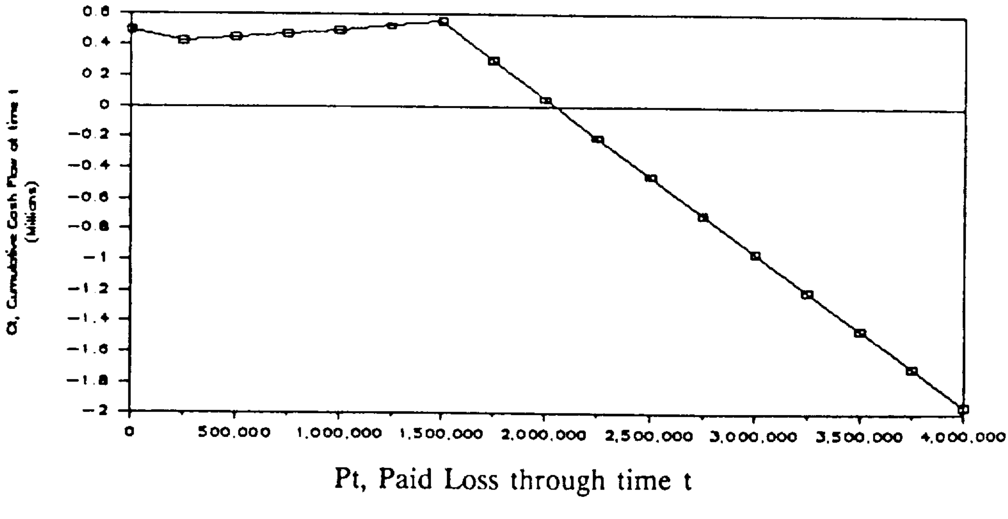
   a. Paid loss retro: This plan is a combination of plans 1b and 2a.

$$C_t = \min(\max(aP_t + b,\ m - P_t),\ M - P_t),$$

where $m$ = minimum premium,
$a$ = multiplicative loading,
$b$ = additive loading,
$M$ = maximum premium.

For example, let $a = .10$, $m = \$500{,}000$, $b = \$400{,}000$, and $M = \$2{,}050{,}000$.

PAID LOSS RETRO
Cash Flow as a Function of Paid Loss

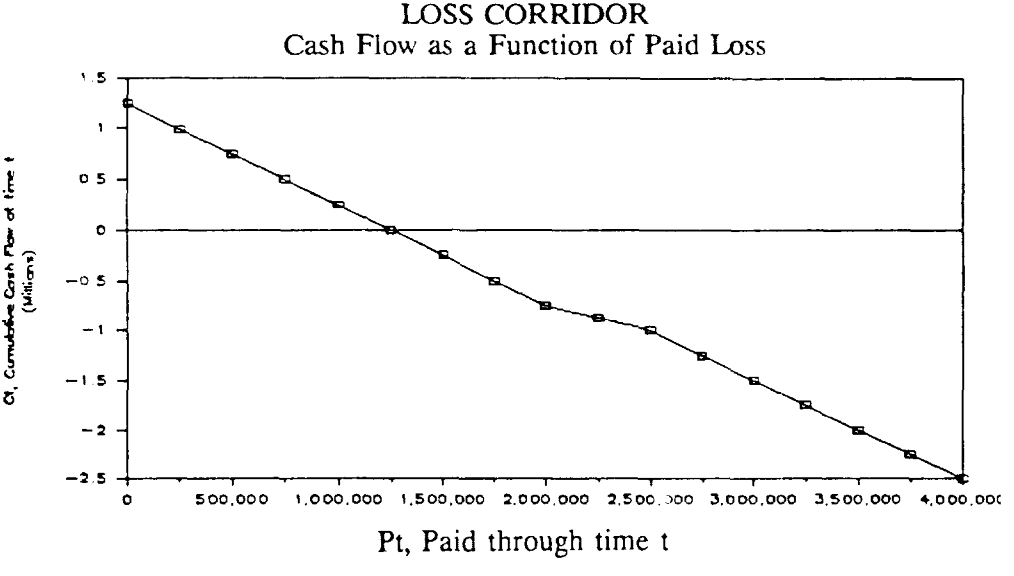


b. Loss corridor: In most loss corridor plans, the reinsurer pays 100% of the losses up to the beginning of the corridor, some share or fraction of the losses in the corridor, and 100% of the losses above the corridor. The corridor is usually expressed in terms of loss ratio points.

$$C_t = \min(\max(r - P_t,\ r - P_t + h(P_t - u)),\ r - P_t + h(v - u)),$$

where $h$ = fraction of corridor retained by reinsured,
$r$ = premium,
$u$ = beginning of corridor,
$v$ = end of corridor.

For example, let $h = .50$, $r = \$1{,}250{,}000$, $u = \$2{,}000{,}000$, and $v = \$2{,}500{,}000$.



4. $C_t$ depends on $t$.

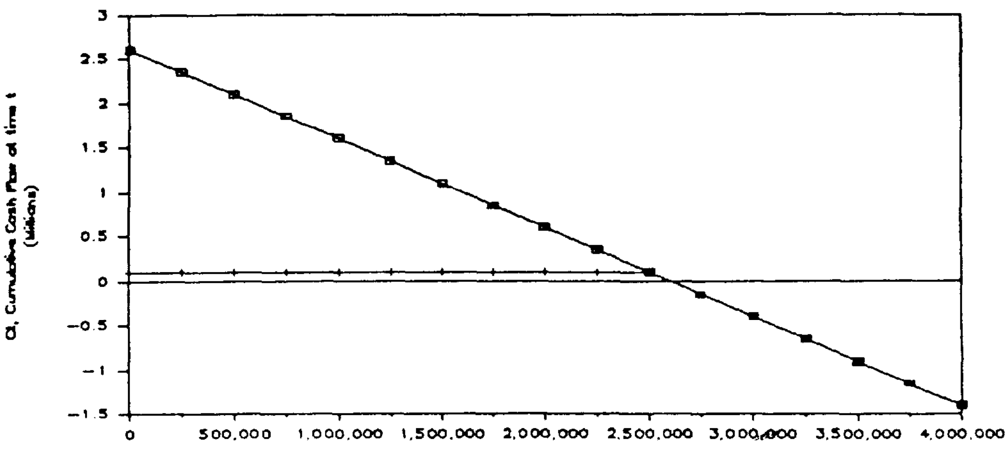
   a. Funded plan with no interest credit to cedant: Under such plans, the fund balance does not accumulate with interest; that is, the reinsurer keeps all interest earned for his own account. At time $t_0$, the fund, less paid losses and reinsurer's margin, is returned to the cedant provided this balance is positive. The cumulative cash flow at time $t_0$ is never greater than the margin, though the reinsurer does receive the benefit of full cash flow until the fund is returned.

$$C_t = \begin{cases} r - P_t & t < t_0, \\ \min\,(r - f,\, r - P_t) & t \geq t_0, \end{cases}$$

where $r$ = fund + margin,
$f$ = fund,
$t_0$ = date on which the fund is returned.

For example, let $f$ = \$2,500,000 and margin = \$100,000.

FUNDED PLAN WITH NO INTEREST CREDIT
Cash Flow as a Function of Paid Loss




Pt, Paid Loss through time t

□ t < $t_o$ + t > $t_o$

5. $C_t$ is a function of $K_t$ in addition to $P_t$.

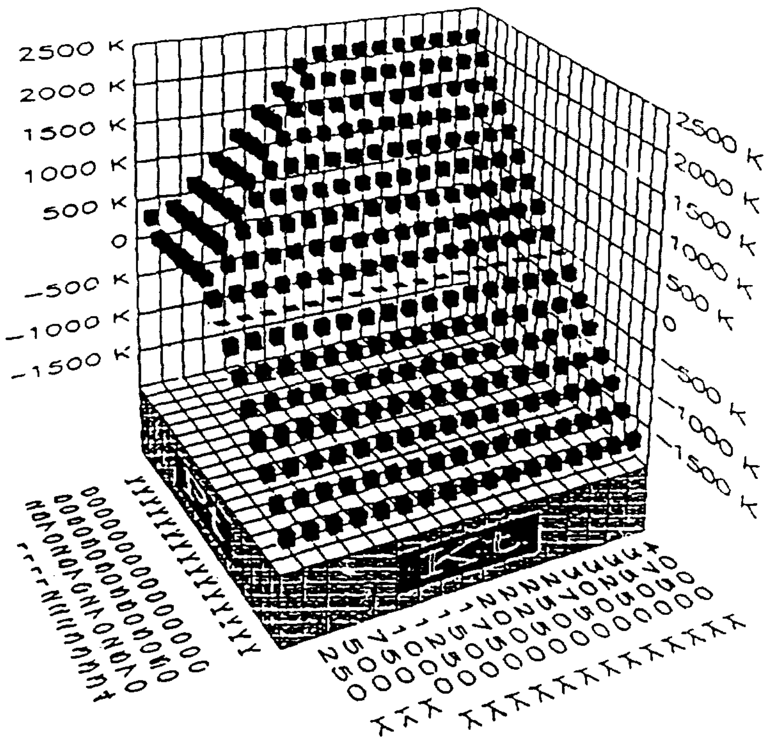
   a. Incurred loss retro: This is similar to a paid loss retro except that the reinsurer's premium, $R_t$, is a function of known incurred losses $(P_t + K_t)$, multiplied by a loading. The additive load, $b_t$, may include an IBNR provision that is a function of $t$.

$$C_t = \min\left(\max\left(aP_t + (a+1)\,K_t + b_t,\ m - P_t\right),\ M - P_t\right),$$

where $a$ = multiplicative loading,
$b$ = additive loading,
$M$ = maximum premium,
$m$ = minimum premium.

For example, let $a = .10$, $b = \$400{,}000$, $M = \$2{,}250{,}000$, and $m = \$400{,}000$. Note that this graph is three-dimensional because $C_t$ is a function of two variables, $P_t$ and $K_t$. In the prior examples $C_t$ was dependent upon one variable, $P_t$.

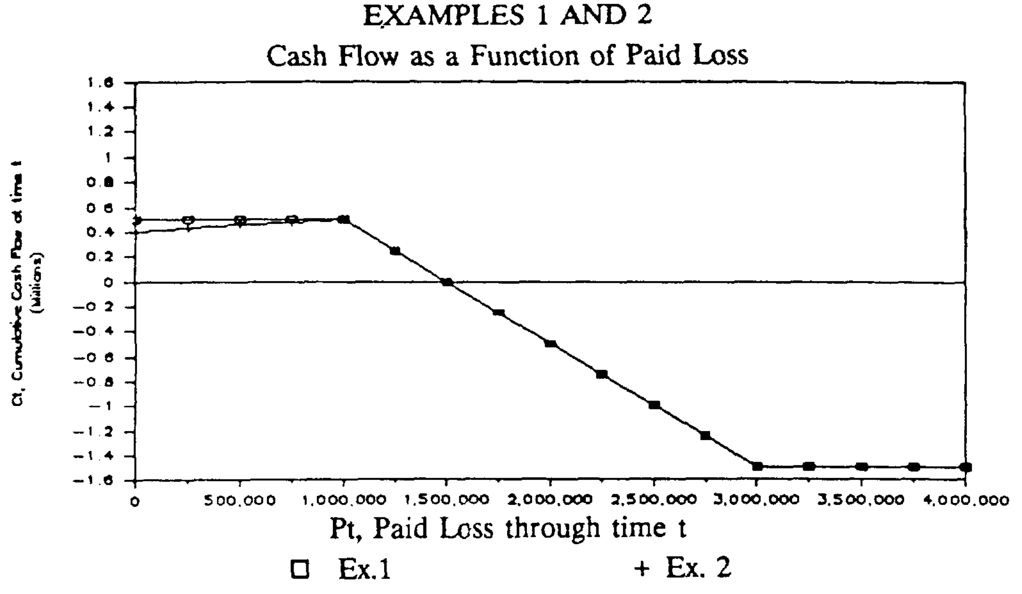
*Incurred Loss Retro*



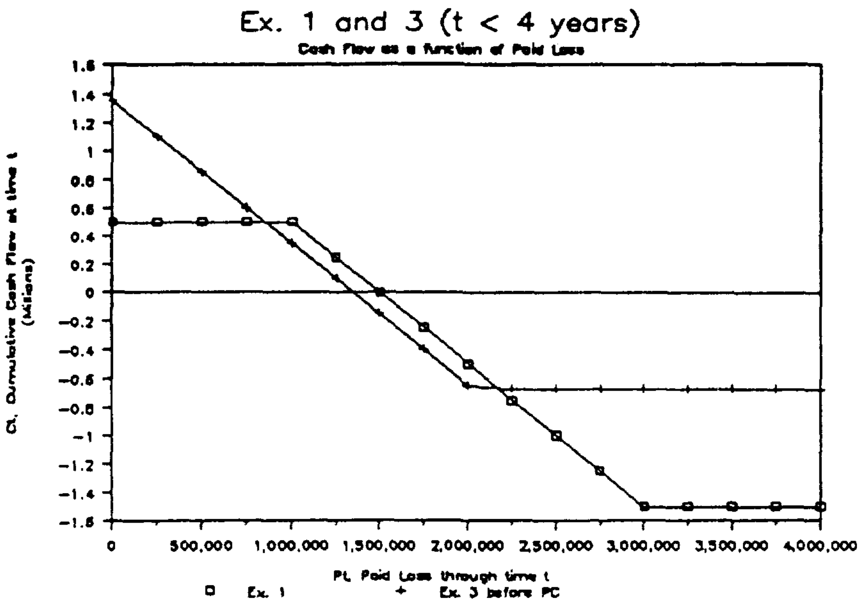
Actual contract terms are often a variation or mixture of the above types, such as the alternatives or the example presented in Section 2.

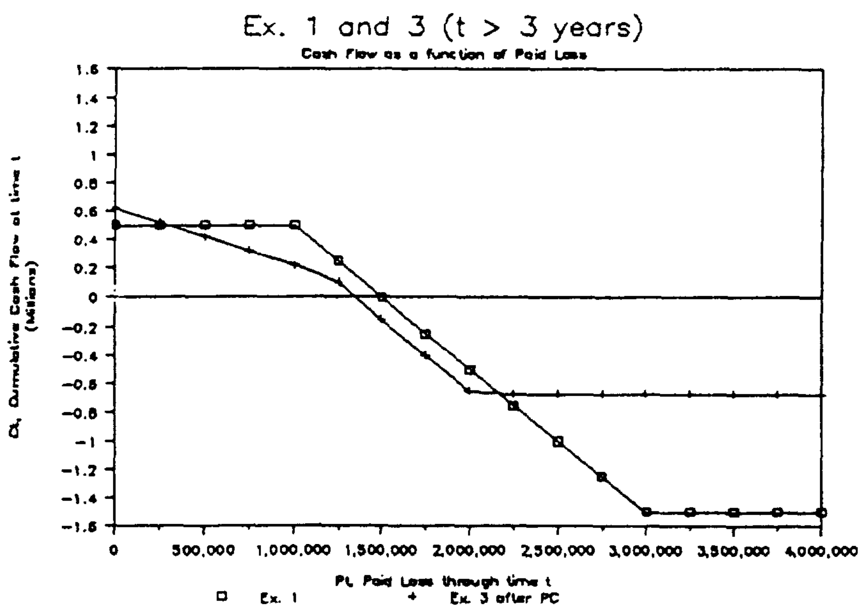
## 6. COMPARING GRAPHS FOR THE EXAMPLE

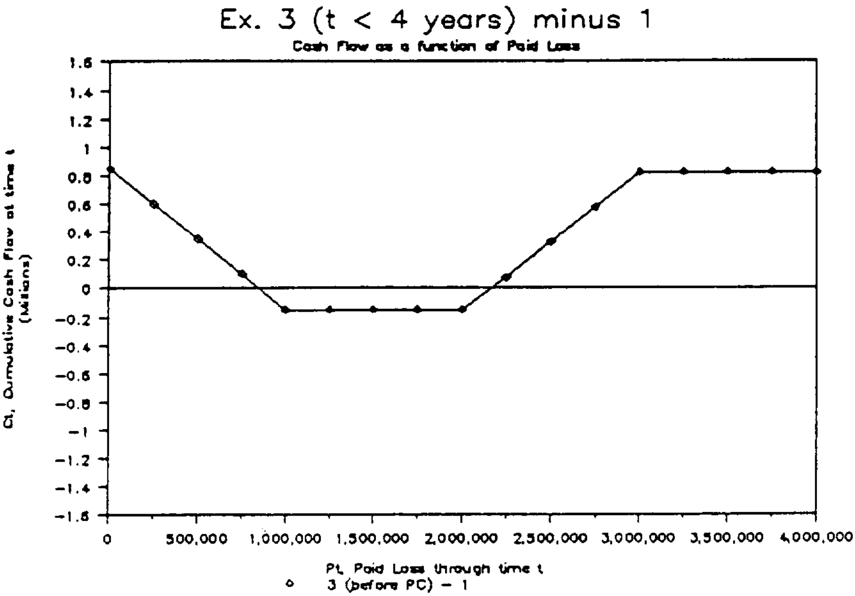
A first step in evaluating relative price adequacy is to examine the graphs of the various alternatives and to examine the graph of a hypothetical contract constructed as the difference between two deals. For the example in section 2, the graph below shows option #1, option #2, and option #1 minus option #2 (the "difference deal" is represented by the triangular region). The obvious conclusions are that the two options are very similar, but that #1 is better than or equal to #2 at all points. Therefore, reject option #2.

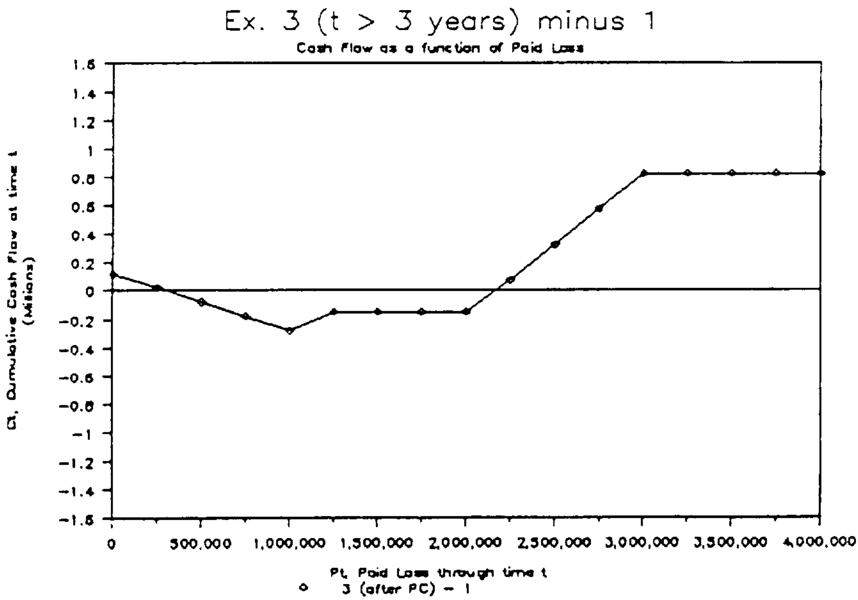



Comparing #1 and #3 is more complex because neither one dominated the other in all cases, and #3 varies with $t$. The graph of #3 and of #3 minus #1 (referred to as the "difference deal") are shown on the following pages.


Ex. 1 and 3 (t < 4 years)
Cash Flow as a function of Paid Loss
Ct, Cumulative Cash Flow at time t (Millions)
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
−0.8
−1
−1.2
−1.4
−1.6
0
500,000
1,000,000
1,500,000
2,000,000
2,500,000
3,000,000
3,500,000
4,000,000
Pt, Paid Loss through time t
Ex. 1
Ex. 3 before PC


Ex. 1 and 3 (t > 3 years)
Cash Flow as a function of Paid Loss
Ct, Cumulative Cash Flow at time t (Millions)
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
−0.8
−1
−1.2
−1.4
−1.6
0
500,000
1,000,000
1,500,000
2,000,000
2,500,000
3,000,000
3,500,000
4,000,000
Pt, Paid Loss through time t
Ex. 1
Ex. 3 after PC


Ex. 3 (t < 4 years) minus 1
Cash Flow as a function of Paid Loss
Ct, Cumulative Cash Flow at time t (Millions)
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1.2
-1.4
-1.6
0
500,000
1,000,000
1,500,000
2,000,000
2,500,000
3,000,000
3,500,000
4,000,000
Pt, Paid Loss through time t
3 (before PC) - 1


Ex. 3 (t > 3 years) minus 1
Cash Flow as a function of Paid Loss
Ct, Cumulative Cash Flow at time t (Millions)
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1.2
-1.4
-1.6
0
500,000
1,000,000
1,500,000
2,000,000
2,500,000
3,000,000
3,500,000
4,000,000
Pt, Paid Loss through time t
3 (after PC) - 1

## 7. DISTRIBUTION OF $V$

As a next step, it is helpful to compare the contracts over reasonable ranges of parameters of an underlying loss generation model. This will help to focus on the underlying conditions that must be true for one option to be superior to the other. The value $V$ of the difference deal is a random variable. How does the distribution of that variable change as the underlying loss model changes? One clear way to present this information is to look at a matrix (or 3-D graph) of the expected value of $V$ as two important parameters are varied.[4]

To do this one needs to estimate the aggregate distribution of incurred or paid losses. This can be accomplished using simulation or by calculating them directly from the frequency and severity distributions. Using a transformation discussed below, aggregate distributions for excess contracts that reflect the age of the contract can be determined. From this series of distributions, one can calculate the distribution of cash flows to the contract. The specific model of the loss process is based on distributions that are commonly used in casualty actuarial literature.

Consider the aggregate distribution for excess claims:

$$G(x) = \sum_{n=0}^{\infty} \text{Prob}[N = n]\ F(x)^{*N},$$

where $F(x)$ is the individual loss amount distribution. This represents the distribution of $P_t$ at ultimate. The Single Parameter Pareto (see Philbrick [9]) is used to model severity for its ease in estimating excess losses.

The model assumes a negative binomial frequency distribution defined as:

$$\text{Prob}[M = m] = \binom{m + \alpha - 1}{\alpha - 1} p^{\alpha} (1 - p)^{m},$$

where $M$ denotes the number of ground-up claims (i.e., claims from first dollar of loss).

[4] Although E[$V$] is probably the most important thing to look at, other information about the distribution of V, such as the Variance [$V$] and Probability [$V > 0$], can be examined in this format. Also, if you wish to postulate a utility function $U$ (on $V$), we can look at E[$U(V)$] as the parameters are varied.

It is interesting to see that if ground-up claims are negative binomial, NB($\alpha$, $p$), then the number of excess claims, $N$, excess of any retention $r$, is also negative binomial NB ($\alpha'$, $p'$)

$$\text{where } \alpha' = \alpha \text{ and } p' = \frac{p}{1 - F(r) + pF(r)} .$$

In addition, if one assumes that individual claim reporting (or payment) is independent of size[5], then the number of reported (or paid) claims is negative binomial NB ($\alpha_t'$, $p_t'$)

$$\text{where } \alpha_t' = \alpha', \ p_t' = \frac{p'}{w(t) + p'(1 - w(t))}$$

and $w(t)$ is the percent of claims reported (or paid) as of $t$ months from the average accident date. See Appendix D for a general proof of these relationships. Similar relationships hold for other common frequency distributions.

Some of the parameters in this model are "unimportant." That is, the conclusions drawn are insensitive to changes in these parameters. This is because alternative deals covering the same occurrence layer are being compared. (If one tried to compare a \$500,000 excess \$500,000 contract with a \$250,000 excess \$250,000 contract, the result would be much more sensitive to the choice of those "unimportant" parameters.)

The "important" parameters that significantly affect the distribution of $V$ are:

1. Average payment lag. The payment lag is the random delay between loss occurrence and loss payment.

2. Expected total losses to the occurrence layer.

[5] The authors do not believe that the independence assumption of claim reporting (or payment) from size is too restrictive because the claims being considered are already large on a ground up basis and their individual size is bound by the layer limit.

Two different applications of the method, producing equivalent results, were used to calculate E[$V$]. One was a Monte Carlo simulation described in Appendix B; the other was a calculation of the distribution of cash flows at each $t$ by Panjer's method described in Appendix C.[6]

The E[$V$] and difference matrices for each application are shown in Exhibits 1 and 2. As can be seen, the results are quite similar and would lead to the same conclusions. Displayed on Exhibits 3–7 are expected cash flow and underlying distributions for the case where E[total loss] equals $1,500,000 and E[lag] equals 36 months.

The E[$V$] of the difference deal is shown on the graph on the next page for each pair of parameters E[total losses] and E[lag]. Examining this graph shows that #1 is slightly superior if we are quite confident in our estimate of expected losses at $1,500,000 and if the average payment lag is short (less than 24 months). However, for a longer payout or for a misestimate of expected losses (either over or under), #3 is superior. The decision maker can use his subjective assessment of his own risk preferences in choosing between the deals. (The authors prefer deal #3.)

## 8. CONCLUSION

The methods outlined have the advantage of summarizing the many factors affecting the economic value of a reinsurance contract, first by graphing the contract terms, then by graphing E[$V$], to allow consistent choices among the alternatives. One can also construct contracts with equivalent terms in the sense that their E[$V$]'s are approximately equal. The model is general enough to handle most realistic contract types. Applying subjective probabilities to the range of the $V$ distributions corresponding to various lags and expected total losses could further summarize the results. Finally, the model can be made more general by using a random interest rate and applying utility functions to the cash flows. However, such extensions probably do not add much practical value.

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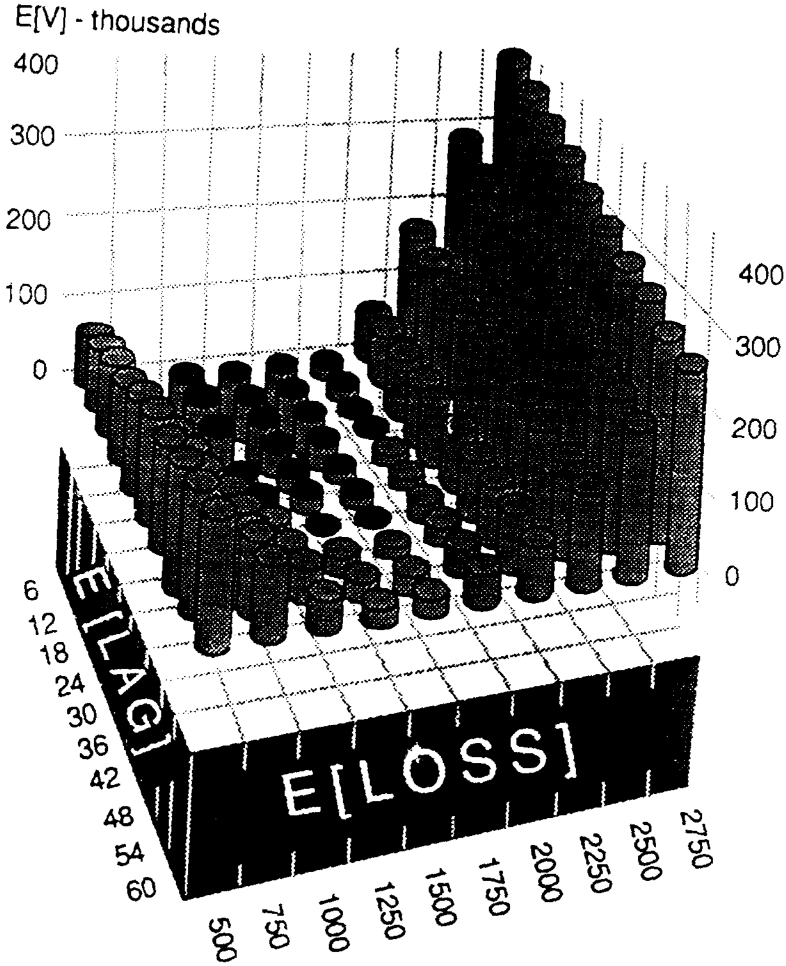
[6] The simulation has the advantage of producing the entire distribution of $V$; the Panjer method only gives E[$V$] easily. However, when E[$N$] is large, the Panjer method can be run much more quickly than the simulation.

EXAMPLE 3 MINUS
EXAMPLE 1



# EXHIBIT 1

## TABLE OF E[$V$], EXPECTED PRESENT VALUE OF THE CASH FLOWS
## PANJER'S METHOD

| Example | E[LOSS] | E[LAG] in months: 6 | 12 | 18 | 24 | 30 | 36 | 42 | 48 | 54 | 60 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 500 | 487 | 488 | 489 | 490 | 490 | 491 | 492 | 492 | 493 | 493 |
| 1 | 750 | 437 | 442 | 446 | 450 | 453 | 456 | 459 | 462 | 464 | 466 |
| 1 | 1,000 | 335 | 346 | 356 | 366 | 374 | 382 | 389 | 396 | 402 | 407 |
| 1 | 1,250 | 181 | 202 | 221 | 238 | 254 | 269 | 282 | 295 | 306 | 316 |
| 1 | 1,500 | (8) | 24 | 52 | 79 | 103 | 125 | 146 | 165 | 182 | 198 |
| 1 | 1,750 | (218) | (174) | (136) | (100) | (68) | (37) | (9) | 16 | 40 | 62 |
| 1 | 2,000 | (433) | (378) | (331) | (287) | (246) | (208) | (173) | (141) | (111) | (84) |
| 1 | 2,250 | (640) | (575) | (521) | (470) | (422) | (378) | (337) | (299) | (264) | (231) |
| 1 | 2,500 | (829) | (757) | (696) | (640) | (588) | (538) | (493) | (450) | (410) | (373) |
| 1 | 2,750 | (992) | (915) | (851) | (791) | (735) | (683) | (634) | (588) | (545) | (505) |
| 3 | 500 | 548 | 567 | 582 | 595 | 606 | 615 | 624 | 631 | 638 | 644 |
| 3 | 750 | 406 | 433 | 456 | 476 | 493 | 508 | 521 | 533 | 543 | 553 |
| 3 | 1,000 | 248 | 284 | 315 | 342 | 366 | 386 | 405 | 422 | 437 | 450 |
| 3 | 1,250 | 77 | 121 | 160 | 194 | 224 | 251 | 275 | 297 | 317 | 336 |
| 3 | 1,500 | (94) | (43) | 1 | 41 | 77 | 110 | 140 | 167 | 191 | 214 |
| 3 | 1,750 | (249) | (194) | (146) | (102) | (62) | (25) | 9 | 40 | 68 | 95 |
| 3 | 2,000 | (378) | (321) | (271) | (226) | (184) | (144) | (108) | (75) | (43) | (15) |
| 3 | 2,250 | (476) | (419) | (370) | (325) | (283) | (243) | (206) | (172) | (139) | (109) |
| 3 | 2,500 | (546) | (491) | (444) | (400) | (359) | (321) | (284) | (250) | (218) | (187) |
| 3 | 2,750 | (593) | (541) | (496) | (455) | (417) | (380) | (345) | (312) | (280) | (250) |
| 3-1 | 500 | 61 | 79 | 93 | 105 | 115 | 124 | 132 | 139 | 145 | 151 |
| 3-1 | 750 | (32) | (9) | 10 | 26 | 40 | 52 | 62 | 71 | 79 | 87 |
| 3-1 | 1,000 | (87) | (62) | (41) | (24) | (9) | 4 | 16 | 26 | 35 | 43 |
| 3-1 | 1,250 | (104) | (81) | (61) | (45) | (30) | (18) | (7) | 3 | 12 | 20 |
| 3-1 | 1,500 | (86) | (67) | (51) | (37) | (26) | (15) | (6) | 2 | 9 | 16 |
| 3-1 | 1,750 | (31) | (20) | (10) | (1) | 6 | 12 | 18 | 23 | 28 | 33 |
| 3-1 | 2,000 | 55 | 57 | 59 | 61 | 63 | 64 | 65 | 66 | 68 | 69 |
| 3-1 | 2,250 | 164 | 156 | 150 | 145 | 140 | 135 | 131 | 127 | 124 | 122 |
| 3-1 | 2,500 | 283 | 266 | 252 | 240 | 228 | 218 | 208 | 200 | 192 | 186 |
| 3-1 | 2,750 | 399 | 374 | 354 | 336 | 319 | 303 | 289 | 276 | 265 | 254 |

EXHIBIT 2

## TABLE OF E[V], EXPECTED PRESENT VALUE OF THE CASH FLOWS
## MONTE CARLO SIMULATION

| | | E[LAG] in months | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Example | E[LOSS] | 6 | 12 | 18 | 24 | 30 | 36 | 42 | 48 | 54 | 60 |
| 1 | 500 | 467 | 469 | 469 | 470 | 470 | 472 | 472 | 473 | 473 | 474 |
| 1 | 750 | 418 | 422 | 427 | 429 | 432 | 435 | 438 | 441 | 444 | 447 |
| 1 | 1,000 | 320 | 329 | 338 | 348 | 355 | 364 | 373 | 380 | 383 | 390 |
| 1 | 1,250 | 170 | 192 | 205 | 225 | 243 | 260 | 269 | 282 | 285 | 303 |
| 1 | 1,500 | (17) | 17 | 47 | 77 | 95 | 115 | 134 | 155 | 173 | 186 |
| 1 | 1,750 | (208) | (162) | (134) | (93) | (70) | (32) | (12) | 11 | 36 | 57 |
| 1 | 2,000 | (415) | (355) | (312) | (272) | (236) | (197) | (167) | (133) | (109) | (81) |
| 1 | 2,250 | (604) | (549) | (494) | (449) | (404) | (365) | (320) | (284) | (249) | (216) |
| 1 | 2,500 | (781) | (720) | (667) | (608) | (560) | (509) | (467) | (419) | (396) | (358) |
| 1 | 2,750 | (937) | (868) | (807) | (753) | (705) | (650) | (599) | (560) | (517) | (481) |
| 3 | 500 | 529 | 546 | 560 | 574 | 583 | 593 | 601 | 607 | 614 | 621 |
| 3 | 750 | 392 | 419 | 441 | 458 | 474 | 487 | 501 | 513 | 524 | 533 |
| 3 | 1,000 | 241 | 274 | 303 | 330 | 350 | 374 | 393 | 409 | 421 | 436 |
| 3 | 1,250 | 75 | 120 | 152 | 188 | 220 | 248 | 267 | 289 | 300 | 327 |
| 3 | 1,500 | (91) | (40) | 7 | 46 | 77 | 107 | 135 | 162 | 188 | 206 |
| 3 | 1,750 | (231) | (175) | (134) | (88) | (56) | 14 | 13 | 41 | 69 | 94 |
| 3 | 2,000 | (355) | (296) | (247) | (207) | (169) | (129) | (99) | (62) | (35) | (6) |
| 3 | 2,250 | (446) | (393) | (343) | (302) | (263) | (225) | (189) | (155) | (125) | (94) |
| 3 | 2,500 | (514) | (461) | (418) | (375) | (335) | (299) | (264) | (225) | (205) | (177) |
| 3 | 2,750 | (560) | (509) | (467) | (428) | (393) | (357) | (320) | (292) | (260) | (232) |
| 3-1 | 500 | 62 | 77 | 91 | 103 | 112 | 121 | 129 | 135 | 140 | 147 |
| 3-1 | 750 | (26) | (3) | 14 | 29 | 43 | 52 | 63 | 71 | 80 | 86 |
| 3-1 | 1,000 | (78) | (55) | (35) | (18) | (6) | 10 | 20 | 29 | 38 | 46 |
| 3-1 | 1,250 | (95) | (72) | (53) | (37) | (23) | (12) | (2) | 7 | 15 | 24 |
| 3-1 | 1,500 | (74) | (57) | (40) | (31) | (18) | (9) | 1 | 7 | 15 | 20 |
| 3-1 | 1,750 | (23) | (13) | (0) | 5 | 13 | 18 | 24 | 30 | 33 | 37 |
| 3-1 | 2,000 | 60 | 58 | 65 | 65 | 67 | 68 | 69 | 70 | 73 | 75 |
| 3-1 | 2,250 | 158 | 156 | 150 | 147 | 142 | 139 | 131 | 129 | 124 | 122 |
| 3-1 | 2,500 | 267 | 259 | 249 | 233 | 225 | 210 | 203 | 194 | 191 | 181 |
| 3-1 | 2,750 | 377 | 359 | 340 | 324 | 312 | 293 | 279 | 267 | 257 | 248 |

# EXHIBIT 3

## Expected Cash Flow Exhibit: Panjer's Method
## Example Number 1

### The 100% Expected Layer Losses = $1,500,000
### The Average Payment Lag is 36 Months

| Yrs | Expected Incurred | Expected Paid | Exp. Earned Premium | Expected Profit Commission | Incurred Loss Ratio | Expected Cash Flow | Present Value of E[CF] |
|---|---|---|---|---|---|---|---|
| 1 | 4,616 | 411 | 500,000 | 0 | 0.9% | 499,589 | 499,589 |
| 2 | 121,630 | 26,837 | 500,000 | 0 | 24.3% | −26,426 | −24,469 |
| 3 | 280,569 | 103,237 | 500,000 | 0 | 56.1% | −76,399 | −65,500 |
| 4 | 391,523 | 196,185 | 500,000 | 0 | 78.3% | −92,949 | −73,786 |
| 5 | 457,712 | 280,569 | 500,000 | 0 | 91.5% | −84,383 | −62,024 |
| 6 | 495,311 | 348,673 | 500,000 | 0 | 99.1% | −68,105 | −46,351 |
| 7 | 516,272 | 400,790 | 500,000 | 0 | 103.3% | −52,116 | −32,842 |
| 8 | 527,874 | 439,578 | 500,000 | 0 | 105.6% | −38,789 | −22,633 |
| 9 | 534,263 | 468,035 | 500,000 | 0 | 106.9% | −28,457 | −15,374 |
| 10 | 537,785 | 488,717 | 500,000 | 0 | 107.6% | −20,682 | −10,346 |
| 11 | 539,709 | 503,708 | 500,000 | 0 | 107.9% | −14,991 | −6,944 |
| 12 | 540,780 | 514,507 | 500,000 | 0 | 108.2% | 10,799 | −4,631 |
| 13 | 541,351 | 522,283 | 500,000 | 0 | 108.3% | −7,776 | −3,088 |
| 14 | 541,681 | 527,874 | 500,000 | 0 | 108.3% | −5,591 | −2,056 |
| 15 | 541,840 | 531,885 | 500,000 | 0 | 108.4% | 4,011 | −1,366 |
| 16 | 541,945 | 534,763 | 500,000 | 0 | 108.4% | −2,878 | −907 |
| 17 | 542,002 | 536,842 | 500,000 | 0 | 108.4% | −2,079 | −607 |
| 18 | 542,030 | 538,301 | 500,000 | 0 | 108.4% | −1,460 | −395 |
| 19 | 542,049 | 539,382 | 500,000 | 0 | 108.4% | 1,081 | −270 |
| 20 | 542,068 | 542,068 | 500,000 | 0 | 108.4% | −2,685 | −622 |
| TOTALS | | | | | | −42,068 | 125,378 |

Swing program minimum, provisional, and maximum are: .100, .100, .100.
The aggregate deductible and loss ratio cap percentage of subject premium are: .200, .400.
The aggregate deductible and loss ratio cap dollars are: $0 $0.

# EXHIBIT 4

## EXPECTED CASH FLOW EXHIBIT: PANJER'S METHOD
## EXAMPLE NUMBER 3

### THE 100% EXPECTED LAYER LOSSES = $1,500,000
### THE AVERAGE PAYMENT LAG IS 36 MONTHS

| Yrs | Expected Incurred | Expected Paid | Exp. Earned Premium | Expected Profit Commission | Incurred Loss Ratio | Expected Cash Flow | Present Value of E[CF] |
|---|---|---|---|---|---|---|---|
| 1 | 388,774 | 230,278 | 1,350,000 | 0 | 28.8% | 1,119,723 | 1,119,723 |
| 2 | 888,210 | 590,129 | 1,349,999 | 0 | 65.8% | −359,852 | −333,196 |
| 3 | 1,152,781 | 846,750 | 1,350,000 | 0 | 85.4% | −256,621 | −220,011 |
| 4 | 1,289,347 | 1,027,316 | 1,350,000 | 0 | 95.5% | −180,566 | −143,339 |
| 5 | 1,360,387 | 1,152,781 | 1,350,000 | 131,518 | 100.8% | −256,983 | −188,891 |
| 6 | 1,397,986 | 1,239,555 | 1,350,001 | 105,907 | 103.6% | −61,162 | −41,626 |
| 7 | 1,418,170 | 1,299,695 | 1,350,000 | 90,098 | 105.0% | −44,332 | −27,937 |
| 8 | 1,429,110 | 1,341,569 | 1,350,001 | 79,989 | 105.9% | −31,764 | −18,534 |
| 9 | 1,435,067 | 1,370,895 | 1,350,001 | 73,335 | 106.3% | −22,671 | −12,249 |
| 10 | 1,438,328 | 1,391,533 | 1,350,000 | 68,855 | 106.5% | −16,159 | −8,083 |
| 11 | 1,440,110 | 1,406,136 | 1,350,000 | 65,787 | 106.7% | −11,535 | −5,343 |
| 12 | 1,441,091 | 1,416,491 | 1,350,000 | 63,661 | 106.7% | −8,229 | −3,529 |
| 13 | 1,441,622 | 1,423,858 | 1,350,000 | 62,173 | 106.8% | −5,879 | −2,334 |
| 14 | 1,441,921 | 1,429,110 | 1,350,000 | 61,125 | 106.8% | −4,205 | −1,546 |
| 15 | 1,442,076 | 1,432,855 | 1,349,999 | 60,383 | 106.8% | −3,004 | −1,023 |
| 16 | 1,442,168 | 1,435,533 | 1,350,000 | 59,857 | 106.8% | −2,151 | −678 |
| 17 | 1,442,219 | 1,437,453 | 1,350,000 | 59,481 | 106.8% | −1,543 | −450 |
| 18 | 1,442,244 | 1,438,815 | 1,350,000 | 59,214 | 106.8% | −1,095 | −296 |
| 19 | 1,442,260 | 1,439,803 | 1,350,001 | 59,023 | 106.8% | −797 | −199 |
| 20 | 1,442,278 | 1,442,278 | 1,350,000 | 58,542 | 106.8% | −1,995 | −462 |
| TOTALS | | | | | | −150,820 | 109,996 |

Swing program minimum, provisional, and maximum are: .270, .270, .270.
The aggregate deductible and loss ratio cap percentage of subject premium are: .000, .405.
The aggregate deductible and loss ratio cap dollars are: $0, $0.
The profit commission equals .600 after .100 reinsurer's margin, but none until year 5.
Number of intervals tested = 191; largest calculated aggregate loss = 4,517,796

# EXHIBIT 5

## DISTRIBUTION OF $C_t$ USING PANJER'S METHOD

| | Example # | | | $t$ (years) | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 3 (before PC) | 3 (after PC) | | | | | | | | |
| $P_t$ | $C_t$ | $C_t$ | $C_t$ | 1 | 2 | 3 | 4 | 5 | 6 | 8 | 10 |
| 0 | 500,000 | 1,350,000 | 621,000 | 0.056 | 0.007 | 0.002 | 0.001 | 0.000 | 0.000 | 0.000 | 0.000 |
| 108,808 | 500,000 | 1,241,192 | 577,477 | 0.147 | 0.030 | 0.009 | 0.004 | 0.002 | 0.001 | 0.001 | 0.001 |
| 217,616 | 500,000 | 1,132,384 | 533,953 | 0.293 | 0.076 | 0.026 | 0.012 | 0.007 | 0.005 | 0.003 | 0.002 |
| 326,424 | 500,000 | 1,023,576 | 490,430 | 0.447 | 0.150 | 0.061 | 0.031 | 0.018 | 0.013 | 0.008 | 0.006 |
| 435,233 | 500,000 | 914,768 | 446,907 | 0.602 | 0.248 | 0.114 | 0.062 | 0.039 | 0.028 | 0.018 | 0.015 |
| 544,041 | 500,000 | 805,959 | 403,384 | 0.728 | 0.363 | 0.187 | 0.110 | 0.073 | 0.054 | 0.037 | 0.030 |
| 652,849 | 500,000 | 697,151 | 359,860 | 0.827 | 0.483 | 0.277 | 0.175 | 0.122 | 0.093 | 0.066 | 0.055 |
| 761,657 | 500,000 | 588,343 | 316,337 | 0.895 | 0.599 | 0.379 | 0.255 | 0.187 | 0.147 | 0.108 | 0.092 |
| 870,465 | 500,000 | 479,535 | 272,814 | 0.940 | 0.701 | 0.484 | 0.346 | 0.264 | 0.214 | 0.163 | 0.141 |
| 979,273 | 500,000 | 370,727 | 229,291 | 0.967 | 0.787 | 0.587 | 0.443 | 0.351 | 0.293 | 0.231 | 0.203 |
| 1,088,081 | 411,919 | 261,919 | 185,768 | 0.983 | 0.853 | 0.679 | 0.540 | 0.443 | 0.379 | 0.308 | 0.275 |
| 1,196,889 | 303,111 | 153,111 | 142,244 | 0.991 | 0.903 | 0.760 | 0.631 | 0.535 | 0.469 | 0.391 | 0.355 |
| 1,305,698 | 194,302 | 44,302 | 44,302 | 0.996 | 0.938 | 0.825 | 0.712 | 0.622 | 0.557 | 0.478 | 0.439 |
| 1,414,506 | 85,494 | (64,506) | (64,506) | 0.998 | 0.962 | 0.877 | 0.782 | 0.702 | 0.640 | 0.562 | 0.523 |
| 1,523,314 | (23,314) | (173,314) | (173,314) | 0.999 | 0.977 | 0.916 | 0.840 | 0.770 | 0.715 | 0.642 | 0.604 |
| 1,632,122 | (132,122) | (282,122) | (282,122) | 1.000 | 0.987 | 0.944 | 0.885 | 0.828 | 0.780 | 0.714 | 0.679 |
| 1,740,930 | (240,930) | (390,930) | (390,930) | 1.000 | 0.992 | 0.964 | 0.920 | 0.874 | 0.834 | 0.778 | 0.746 |
| 1,849,738 | (349,738) | (499,738) | (499,738) | 1.000 | 0.996 | 0.977 | 0.946 | 0.910 | 0.878 | 0.831 | 0.803 |
| 1,958,546 | (458,546) | (608,546) | (608,546) | 1.000 | 0.998 | 0.986 | 0.964 | 0.938 | 0.912 | 0.874 | 0.851 |
| 2,067,354 | (567,354) | (675,000) | (675,000) | 1.000 | 0.999 | 0.992 | 0.977 | 0.958 | 0.939 | 0.908 | 0.890 |
| 2,176,163 | (676,163) | (675,000) | (675,000) | 1.000 | 0.999 | 0.995 | 0.985 | 0.972 | 0.958 | 0.935 | 0.920 |
| 2,284,971 | (784,971) | (675,000) | (675,000) | 1.000 | 1.000 | 0.997 | 0.991 | 0.982 | 0.972 | 0.954 | 0.943 |
| 2,393,779 | (893,779) | (675,000) | (675,000) | 1.000 | 1.000 | 0.998 | 0.994 | 0.988 | 0.981 | 0.969 | 0.961 |
| 2,502,587 | (1,002,587) | (675,000) | (675,000) | 1.000 | 1.000 | 0.999 | 0.997 | 0.993 | 0.988 | 0.979 | 0.973 |
| 2,611,395 | (1,111,395) | (675,000) | (675,000) | 1.000 | 1.000 | 1.000 | 0.998 | 0.996 | 0.992 | 0.986 | 0.982 |
| 2,720,203 | (1,220,203) | (675,000) | (675,000) | 1.000 | 1.000 | 1.000 | 0.999 | 0.997 | 0.995 | 0.991 | 0.988 |
| 2,829,011 | (1,329,011) | (675,000) | (675,000) | 1.000 | 1.000 | 1.000 | 0.999 | 0.998 | 0.997 | 0.994 | 0.992 |
| 2,937,820 | (1,437,820) | (675,000) | (675,000) | 1.000 | 1.000 | 1.000 | 1.000 | 0.999 | 0.998 | 0.997 | 0.995 |
| 3,046,628 | (1,500,000) | (675,000) | (675,000) | 1.000 | 1.000 | 1.000 | 1.000 | 0.999 | 0.999 | 0.998 | 0.997 |
| 3,155,436 | (1,500,000) | (675,000) | (675,000) | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.999 | 0.999 | 0.998 |
| 3,264,244 | (1,500,000) | (675,000) | (675,000) | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.999 | 0.999 |
| 3,373,052 | (1,500,000) | (675,000) | (675,000) | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.999 |
| 3,481,860 | (1,500,000) | (675,000) | (675,000) | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 3,590,668 | (1,500,000) | (675,000) | (675,000) | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | | | Expected Cash Flow | | | | | | | | |
| | | | example 1 | 499,589 | 473,163 | 396,764 | 303,815 | 219,432 | 151,327 | 60,422 | 11,283 |
| | | | example 3 | 1,119,723 | 759,871 | 503,250 | 322,684 | 65,701 | 4,539 | (71,557) | (110,387) |

Note: PC denotes Profit Commission

# EXHIBIT 6

## Distribution of $C_t$ using Panjer's Method

| | Example # | | | t (years) | | | | |
|---|---|---|---|---|---|---|---|---|
| | 1 | 3 (before PC) | 3 (after PC) | | | | | |
| $P_t$ | $C_t$ | $C_t$ | $C_t$ | 12 | 14 | 16 | 18 | 20 |
| 0 | 500,000 | 1,350,000 | 621,000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 108,808 | 500,000 | 1,241,192 | 577,477 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 217,616 | 500,000 | 1,132,384 | 533,953 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 |
| 326,424 | 500,000 | 1,023,576 | 490,430 | 0.005 | 0.005 | 0.005 | 0.005 | 0.005 |
| 435,233 | 500,000 | 914,768 | 446,907 | 0.013 | 0.012 | 0.012 | 0.012 | 0.012 |
| 544,041 | 500,000 | 805,959 | 403,384 | 0.027 | 0.026 | 0.025 | 0.025 | 0.025 |
| 652,849 | 500,000 | 697,151 | 359,860 | 0.050 | 0.048 | 0.047 | 0.046 | 0.046 |
| 761,657 | 500,000 | 588,343 | 316,337 | 0.085 | 0.081 | 0.079 | 0.078 | 0.077 |
| 870,465 | 500,000 | 479,535 | 272,814 | 0.131 | 0.126 | 0.123 | 0.122 | 0.121 |
| 979,273 | 500,000 | 370,727 | 229,291 | 0.189 | 0.183 | 0.180 | 0.178 | 0.176 |
| 1,088,081 | 411,919 | 261,919 | 185,768 | 0.259 | 0.251 | 0.247 | 0.245 | 0.243 |
| 1,196,889 | 303,111 | 153,111 | 142,244 | 0.336 | 0.327 | 0.323 | 0.320 | 0.318 |
| 1,305,698 | 194,302 | 44,302 | 44,302 | 0.419 | 0.409 | 0.404 | 0.402 | 0.399 |
| 1,414,506 | 85,494 | (64,506) | (64,506) | 0.503 | 0.493 | 0.488 | 0.485 | 0.482 |
| 1,523,314 | (23,314) | (173,314) | (173,314) | 0.584 | 0.574 | 0.569 | 0.566 | 0.564 |
| 1,632,122 | (132,122) | (282,122) | (282,122) | 0.660 | 0.651 | 0.646 | 0.643 | 0.641 |
| 1,740,930 | (240,930) | (390,930) | (390,930) | 0.729 | 0.720 | 0.716 | 0.713 | 0.711 |
| 1,849,738 | (349,738) | (499,738) | (499,738) | 0.788 | 0.781 | 0.777 | 0.775 | 0.772 |
| 1,958,546 | (458,546) | (608,546) | (608,546) | 0.839 | 0.832 | 0.828 | 0.827 | 0.825 |
| 2,067,354 | (567,354) | (675,000) | (675,000) | 0.879 | 0.874 | 0.871 | 0.869 | 0.868 |
| 2,176,163 | (676,163) | (675,000) | (675,000) | 0.912 | 0.907 | 0.905 | 0.904 | 0.903 |
| 2,284,971 | (784,971) | (675,000) | (675,000) | 0.937 | 0.933 | 0.932 | 0.931 | 0.930 |
| 2,393,779 | (893,779) | (675,000) | (675,000) | 0.956 | 9.953 | 0.952 | 0.951 | 0.950 |
| 2,502,587 | (1,002,587) | (675,000) | (675,000) | 0.969 | 0.967 | 0.966 | 0.966 | 0.965 |
| 2,611,395 | (1,111,395) | (675,000) | (675,000) | 0.979 | 0.978 | 0.977 | 0.977 | 0.976 |
| 2,720,203 | (1,220,203) | (675,000) | (675,000) | 0.986 | 0.985 | 0.985 | 0.984 | 0.984 |
| 2,829,011 | (1,329,011) | (675,000) | (675,000) | 0.991 | 0.990 | 0.990 | 0.990 | 0.990 |
| 2,937,820 | (1,437,820) | (675,000) | (675,000) | 0.994 | 0.994 | 0.994 | 0.993 | 0.993 |
| 3,046,628 | (1,500,000) | (675,000) | (675,000) | 0.996 | 0.996 | 0.996 | 0.996 | 0.996 |
| 3,155,436 | (1,500,000) | (675,000) | (675,000) | 0.998 | 0.998 | 0.997 | 0.997 | 0.997 |
| 3,264,244 | (1,500,000) | (675,000) | (675,000) | 0.999 | 0.999 | 0.998 | 0.998 | 0.998 |
| 3,373,052 | (1,500,000) | (675,000) | (675,000) | 0.999 | 0.999 | 0.999 | 0.999 | 0.999 |
| 3,481,860 | (1,500,000) | (675,000) | (675,000) | 1.000 | 0.999 | 0.999 | 0.999 | 0.999 |
| 3,590,668 | (1,500,000) | (675,000) | (675,000) | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | | | Expected Cash Flow | | | | | |
| | | | example 1 | (14,507) | (27,874) | (34,763) | (38,302) | (42,068) |
| | | | example 3 | (130,151) | (140,235) | (145,390) | (148,028) | (150,820) |

Note: PC denotes Profit Commission

# EXHIBIT 7

## Distribution of Aggregate Paid Losses to Total Layer: Panjer's Method

Limit: 250,000
Retention: 250,000
Layer Expected Losses: 1,500,000
Single Parameter Pareto $q$: 1.5

The Mean of the Exponential Payment Lag: 36
Var[$N$]/E[$N$] for the Excess Layer at Ultimate: 1.032
Alpha = 320.083
Layer Severity = 146.447

| | $t$ (years) | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 |
| | Summary Statistics | | | | | | | | | | | | |
| $p'$ | 0.991 | 0.985 | 0.980 | 0.977 | 0.975 | 0.973 | 0.971 | 0.970 | 0.970 | 0.969 | 0.969 | 0.969 | 0.969 |
| E[# Paid] | 2.90 | 4.98 | 6.47 | 7.54 | 8.31 | 8.86 | 9.53 | 9.88 | 10.06 | 10.15 | 10.19 | 10.22 | 10.24 |
| E[$ Paid] | 425 | 730 | 948 | 1105 | 1217 | 1297 | 1396 | 1446 | 1473 | 1486 | 1493 | 1496 | 1500 |
| CV | 0.65 | 0.50 | 0.44 | 0.41 | 0.39 | 0.38 | 0.36 | 0.36 | 0.35 | 0.35 | 0.35 | 0.35 | 0.35 |
| SKW | 0.72 | 0.55 | 0.49 | 0.46 | 0.44 | 0.42 | 0.41 | 0.40 | 0.40 | 0.40 | 0.40 | 0.40 | 0.39 |
| Paid Dollars | Cumulative Distribution Function $P(L_t \le$ Paid Dollars) | | | | | | | | | | | | |
| 0 | 0.056 | 0.007 | 0.002 | 0.001 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 108,808 | 0.147 | 0.030 | 0.009 | 0.004 | 0.002 | 0.001 | 0.001 | 0.001 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 217,616 | 0.293 | 0.076 | 0.026 | 0.012 | 0.007 | 0.005 | 0.003 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 |
| 326,424 | 0.447 | 0.150 | 0.061 | 0.031 | 0.018 | 0.013 | 0.008 | 0.006 | 0.005 | 0.005 | 0.005 | 0.005 | 0.005 |
| 435,233 | 0.602 | 0.248 | 0.114 | 0.062 | 0.039 | 0.028 | 0.018 | 0.015 | 0.013 | 0.012 | 0.012 | 0.012 | 0.012 |
| 544,041 | 0.728 | 0.363 | 0.187 | 0.110 | 0.073 | 0.054 | 0.037 | 0.030 | 0.027 | 0.026 | 0.025 | 0.025 | 0.025 |
| 652,849 | 0.827 | 0.483 | 0.277 | 0.175 | 0.122 | 0.093 | 0.066 | 0.055 | 0.050 | 0.048 | 0.047 | 0.046 | 0.046 |

SKW = Coefficient of Skewness

# EXHIBIT 7 (continued)

| Paid Dollars | 1 | 2 | 3 | 4 | 5 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | *t* (years) | | | | | | | | | | | | |
| | Cumulative Distribution Function $P(L)$(= Paid Dollars) | | | | | | | | | | | | |
| 761,657 | 0.895 | 0.599 | 0.379 | 0.255 | 0.187 | 0.147 | 0.108 | 0.092 | 0.085 | 0.081 | 0.079 | 0.078 | 0.077 |
| 870,465 | 0.940 | 0.701 | 0.484 | 0.346 | 0.264 | 0.214 | 0.163 | 0.141 | 0.131 | 0.126 | 0.123 | 0.122 | 0.121 |
| 979,273 | 0.967 | 0.787 | 0.587 | 0.443 | 0.351 | 0.293 | 0.231 | 0.203 | 0.189 | 0.183 | 0.180 | 0.178 | 0.176 |
| 1,088,081 | 0.983 | 0.853 | 0.679 | 0.540 | 0.443 | 0.379 | 0.308 | 0.275 | 0.259 | 0.251 | 0.247 | 0.245 | 0.243 |
| 1,196,889 | 0.991 | 0.903 | 0.760 | 0.631 | 0.535 | 0.469 | 0.391 | 0.355 | 0.336 | 0.327 | 0.323 | 0.320 | 0.318 |
| 1,305,698 | 0.996 | 0.938 | 0.825 | 0.712 | 0.622 | 0.557 | 0.478 | 0.439 | 0.419 | 0.409 | 0.404 | 0.402 | 0.399 |
| 1,414,506 | 0.998 | 0.962 | 0.877 | 0.782 | 0.702 | 0.640 | 0.562 | 0.523 | 0.503 | 0.493 | 0.488 | 0.485 | 0.482 |
| 1,523,314 | 0.999 | 0.977 | 0.916 | 0.840 | 0.770 | 0.715 | 0.642 | 0.604 | 0.584 | 0.574 | 0.569 | 0.566 | 0.564 |
| 1,632,122 | 1.000 | 0.987 | 0.944 | 0.885 | 0.828 | 0.780 | 0.714 | 0.679 | 0.660 | 0.651 | 0.646 | 0.643 | 0.641 |
| 1,740,930 | 1.000 | 0.992 | 0.964 | 0.920 | 0.874 | 0.834 | 0.778 | 0.746 | 0.729 | 0.720 | 0.716 | 0.713 | 0.711 |
| 1,849,738 | 1.000 | 0.996 | 0.977 | 0.946 | 0.910 | 0.878 | 0.831 | 0.803 | 0.788 | 0.781 | 0.777 | 0.775 | 0.772 |
| 1,958,546 | 1.000 | 0.998 | 0.986 | 0.964 | 0.938 | 0.912 | 0.874 | 0.851 | 0.839 | 0.832 | 0.828 | 0.827 | 0.825 |
| 2,067,354 | 1.000 | 0.999 | 0.992 | 0.977 | 0.958 | 0.939 | 0.908 | 0.890 | 0.879 | 0.874 | 0.871 | 0.869 | 0.868 |
| 2,176,163 | 1.000 | 0.999 | 0.995 | 0.985 | 0.972 | 0.958 | 0.935 | 0.920 | 0.912 | 0.907 | 0.905 | 0.904 | 0.903 |
| 2,284,971 | 1.000 | 1.000 | 0.997 | 0.991 | 0.982 | 0.972 | 0.954 | 0.943 | 0.937 | 0.933 | 0.932 | 0.931 | 0.930 |
| 2,393,779 | 1.000 | 1.000 | 0.998 | 0.994 | 0.988 | 0.981 | 0.969 | 0.961 | 0.956 | 0.953 | 0.952 | 0.951 | 0.950 |
| 2,502,587 | 1.000 | 1.000 | 0.999 | 0.997 | 0.993 | 0.988 | 0.979 | 0.973 | 0.969 | 0.967 | 0.966 | 0.966 | 0.965 |
| 2,611,395 | 1.000 | 1.000 | 1.000 | 0.998 | 0.996 | 0.992 | 0.986 | 0.982 | 0.979 | 0.978 | 0.977 | 0.977 | 0.976 |
| 2,720,203 | 1.000 | 1.000 | 1.000 | 0.999 | 0.997 | 0.995 | 0.991 | 0.988 | 0.986 | 0.985 | 0.985 | 0.984 | 0.984 |
| 2,829,011 | 1.000 | 1.000 | 1.000 | 0.999 | 0.998 | 0.997 | 0.994 | 0.992 | 0.991 | 0.990 | 0.990 | 0.990 | 0.990 |
| 2,937,820 | 1.000 | 1.000 | 1.000 | 1.000 | 0.999 | 0.998 | 0.997 | 0.995 | 0.994 | 0.994 | 0.994 | 0.993 | 0.993 |
| 3,046,628 | 1.000 | 1.000 | 1.000 | 1.000 | 0.999 | 0.999 | 0.998 | 0.997 | 0.996 | 0.996 | 0.996 | 0.996 | 0.996 |
| 3,155,436 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.999 | 0.999 | 0.998 | 0.998 | 0.998 | 0.997 | 0.997 | 0.997 |
| 3,264,244 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.999 | 0.999 | 0.999 | 0.999 | 0.998 | 0.998 | 0.998 |
| 3,373,052 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.999 | 0.999 | 0.999 | 0.999 | 0.999 | 0.999 |
| 3,481,860 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.999 | 0.999 | 0.999 | 0.999 |
| 3,590,668 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |

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## APPENDIX A
## PARAMETERIZING THE MODEL

The two applications of the model described in Section 7 and Appendices B and C (simulation and Panjer's method) were parameterized using the following steps:

1. Select expected losses, E[$L$], where $L = P_t$ at $t$ equal to ultimate, to the occurrence layer. Several estimates using experience and exposure rating techniques should be applied. The consistency of the results will affect how extensive a range of E[$L$] estimates should be tested in the model.

2. Estimate the negative binomial (NB) parameter $p'$. The authors used the fact that the variance/mean, Var/E = $1/p$ for NB. Starting with an ISO ground up Var/E in the 2.0 to 3.0 range, and ISO increased limits severity distributions, one can translate $p$ (½ to ⅓) into $p'$ using the transformation $p' = p/[1 - F(r) + pF(r)]$, $r$ = retention. The examples herein use $p' = .969$ for excess of $250,000 long haul trucking. As mentioned in Appendix D, E[$V$] is not sensitive to changes in $p'$.

3. Estimate a severity distribution $F(x)$. The authors used a Single Parameter Pareto (SPP) with $q = 1.5$. This was suggested by Philbrick [9] as being appropriate for casualty lines. With an SPP severity, changes in the $q$ parameter do not cause significant changes in E[$V$]. The average severity for the layer ($250,000 excess of $250,000) is then calculated.

4. Dividing E[$L$] by the average severity produces an estimated number of excess claims, E[$N$]. One can then back into the NB $\alpha$ parameter as:

$$\frac{\mathrm{E}[N]p'}{1 - p'}$$

5. Select payment and/or report lag distributions. While this paper utilizes exponential lags, the results are equally valid for other distributions, although the parameter selection process would change.[7] The cash flows are fundamentally dependent on the payment lag distribution, so care should be taken in the selection of the distribution and its parameters. The extent of sensitivity testing is a function of one's confidence in the payment lag distribution. For a detailed discussion on estimating lag distributions, see Weissner [12]. John [3] discusses report lag distributions by reinsurance line of business.

6. Choose an interest rate to discount the cash flows. The examples used 8%.

[7] For example, if one were testing the sensitivity of results using a two parameter payment lag distribution, the coefficient of variation (CV) would be fixed, and scale parameter selected corresponding to several expected payment lags. This process would then be repeated for several different CV values.

## APPENDIX B
## MONTE CARLO SIMULATION

The simulation was programmed as follows: Single Parameter Pareto (SPP) $q$, expected layer losses, and exponential payment lag with lambda equal to 1/mean lag are selected and used to calculate Negative Binomial $p'$ and $\alpha$ parameters as shown in Appendix A. Then, for one iteration:

1. $N$ is drawn from a negative binomial NB $(\alpha, p')$.

2. For each of the $N$ claims, a paid loss amount is drawn from SPP and a payment lag is drawn from the exponential. It was assumed that claims occur mid-year and premium and loss transactions are made at mid-year.

3. The $P_t$ values are calculated by summing total payments in the appropriate time periods using the simulated lags.

4. The reinsurance contract terms were applied to the $P_t$'s to obtain the $C_t$'s.

5. $V$ is calculated $= \sum_{t=1}^{n} (C_t - C_{t-1})\, v^{t-1}$, then $V$ is stored.

The above was repeated for 20,000 iterations, then E[$V$], Variance [$V$] and Probability [$V > 0$] are calculated.

The program was written in HSFORTH and run on an IBM PS/2 with a 25 MH 80386 and 80387. For E[$N$] = 10, 20,000 iterations take 80 seconds, so a 10 × 10 matrix of parameters can be run in 2.3 hours.

## APPENDIX C
## PANJER'S METHOD

Recall that the aggregate loss distribution is a compound process formed by the infinite sum of $n$-fold convolutions of the frequency and severity distributions. Panjer [8] showed that this distribution can be estimated recursively provided that the frequency distribution satisfies the recursive relationship

$$p(n) = p(n - 1)(a + b/n) \qquad n = 1,2,3, \ldots,$$

where $p(n)$ denotes the probability of exactly $n$ claims occurring in a fixed time interval.

Sundt and Jewell [11] showed that the only distributions satisfying this condition are: Poisson, Negative Binomial, Binomial, and Geometric. In the case of the Negative Binomial,

$$p(n) = \binom{n + \alpha - 1}{\alpha - 1} p^{\alpha}(1 - p)^{n}, \; n = 0,1,2, \ldots;$$

and

$$a = 1 - p, \; b = (1 - \alpha)(1 - p), \; p(0) = p^{\alpha}.$$

Furthermore, if the severity distribution can be represented discretely then the recursive formula for the aggregate distribution $G(\cdot)$ is quite simple:

$$g_i = \sum_{j=1}^{i} (a + bj/i) f_j g_{i-j} \qquad i = 1,2,3, \ldots;$$

$$g_0 = p(0).$$

Section 7 and Appendix D show that if the number of ground-up claims is negative binomial NB $(\alpha, p)$, then the number of claims excess of a retention $r$, reported or paid at any time $t$, is also Negative Binomial with the appropriate transformations of the ground-up parameters.

This means that we can estimate the aggregate distribution of losses paid, $P_t$, or reported incurred, $L_t = P_t + K_t$, using Panjer's recursive formula.

For the layer being considered, the Single Parameter Pareto (SPP) severity distribution was discretized into equal intervals, with the number of intervals determined by the following formula subject to a maximum of 20:

$$\# \text{ intervals} = \frac{4}{\mathrm{E}[N]} + 10.$$

Increasing the number of intervals beyond these levels adds significantly to the run time without appreciable improvement in the results.

The above procedure can easily be used to estimate the expected value of $V$, the present value of net cash flows $C_t$. The variance of $V$, $\mathrm{Var}[V]$, on the other hand, is difficult to estimate. Obviously, the sequence of random variables $\{C_t\}$ is not independent. Not so obvious is the fact that, for even simple contract forms, the $C_t$'s do not have independent increments; that is, the sequence $\{C_t - C_{t-1}\}$ is not independent. This means that the $\mathrm{Var}[V]$ contains non-zero covariance terms. This fact is demonstrated by the example in this appendix.

If the decision maker would like to consider other properties besides $\mathrm{E}[V]$ (e.g., $\mathrm{Var}[V]$), then a covariance matrix can be produced, though simulation may be simpler. Aside from that, the Panjer analytical solution is relatively easy to implement.

*Example: $C_t$'s Do Not Have Independent Increments*

Consider the paid loss retro with no maximum, $C_t = C(P_t) = \max(aP_t + b, m - P_t)$, where $a$ = multiplicative loading, $b$ = additive loading and $m$ = minimum premium.

For convenience write $C(P_t)$ as:

$$C(P_t) = \begin{cases} m - P_t, & aP_t + b < m \\ aP_t + b, & aP_t + b > m. \end{cases}$$

Recall that the $P_t$'s are non-decreasing and consider the case $aP_1 + b < aP_2 + b < m < aP_3 + b$.

$$\begin{aligned}
&\mathrm{COV}[C(P_2) - C(P_1), C(P_3) - C(P_2)] \\
&\qquad = \mathrm{COV}[-(P_2 - P_1), aP_3 + b - (m - P_2)] \\
&\qquad = -a\,\mathrm{COV}[P_2 - P_1, P_3] + \mathrm{COV}[P_2 - P_1, m - P_2] \\
&\qquad = -a\,(\mathrm{COV}[P_2, P_3] - \mathrm{COV}[P_1, P_3]) - (\mathrm{COV}[P_2, P_2] \\
&\qquad\quad -\mathrm{COV}[P_1, P_2]) \\
&\qquad = (1 + a)\,(\mathrm{Var}[P_1] - \mathrm{Var}[P_2]).
\end{aligned}$$

Note: $\mathrm{COV}[P_{t-1}, P_t] = \mathrm{COV}[P_{t-1}, (P_t - P_{t-1}) + P_{t-1}]$

$$\begin{aligned}
&= \mathrm{COV}[P_{t-1}, P_t - P_{t-1}] + \mathrm{Var}[P_{t-1}] \\
&= \mathrm{Var}[P_{t-1}],
\end{aligned}$$

because $P_t$'s have independent increments.

## APPENDIX D

The following proof derives the transformations shown in Section 7.

*Theorem*: If $Y$ is negative binomial $(\alpha,p)$ and $X|y$ is binomial $(y,w)$, then $X$ is negative binomial $(\alpha',p')$ where $\alpha' = \alpha$,

$$\text{and } p' = \frac{p}{w + p\,(1 - w)}$$

*Proof*:

$$\Pr[X = x] = \sum_y \Pr[X = x|Y = y]\,\Pr[Y = y]$$

$$= \sum_y \binom{y}{x} w^x (1 - w)^{y-x} \binom{y + \alpha - 1}{\alpha - 1} p^\alpha (1 - p)^y$$

$$= \frac{w^x p^\alpha}{x!\,(\alpha - 1)!} (1 - p)^x \sum_y (1 - w)^{y-x} \frac{(y + \alpha - 1)!}{(y - x)!} (1 - p).^{y-x}$$

Multiplying numerator and denominator by $(x + \alpha - 1)!$ gives:

$$\binom{x + \alpha - 1}{\alpha - 1} [w(1 - p)]^x p^\alpha \sum_y \binom{y + \alpha - 1}{x + \alpha - 1} [(1 - p)\,(1 - w)].^{y-x}$$

Substituting $z = y - x$ and $1 - h = (1 - p)(1 - w) = 1 - (w + p - wp)$:

$$\binom{x + \alpha - 1}{\alpha - 1} \frac{[w\,(1 - p)]^x p^\alpha}{h^{x+\alpha}} \sum_z \binom{z + x + \alpha - 1}{x + \alpha - 1} h^{x+\alpha} (1 - h).^z$$

Notice that the summation over $z$ equals 1.

$$\binom{x + \alpha - 1}{\alpha - 1} \left(\frac{p}{w + p\,(1 - w)}\right)^\alpha \left(\frac{w\,(1 - p)}{w + p\,(1 - w)}\right)^x$$

$$= \binom{x + \alpha - 1}{\alpha - 1} p'^\alpha (1 - p')^x$$

$$= \mathrm{NB}(\alpha',p').$$

Comments:

(i) For claims excess of a retention, $r$, $w = 1 - F(r)$, where $F(x)$ is the ground-up severity distribution.

(ii) For reported or paid claims $w = w(t)$, where $w(t)$ is the percent of claims reported or paid as of $t$ months from the average accident date.

(iii) $$\frac{\mathrm{Var}[X]}{\mathrm{E}[X]} = \frac{1}{p'}$$
$$= \frac{w + p\,(1 - w)}{p}.$$

Note that as $w$ approaches zero (which is the case for excess claims as the retention, $r$, gets large), the variance/mean approaches 1.0. In fact, the variance/mean approaches 1.0 quite quickly as the following table shows:

VARIANCE/MEAN FOR EXCESS CLAIMS

| Ground-Up Var/E | Excess Claim Probability .25 | .10 | .05 | .01 |
|---|---|---|---|---|
| 2.0 | 1.25 | 1.10 | 1.05 | 1.01 |
| 3.0 | 1.50 | 1.20 | 1.10 | 1.02 |
| 4.0 | 1.75 | 1.30 | 1.15 | 1.03 |

This result is consistent with sensitivity tests of $p' = (\mathrm{Var}[X]/\mathrm{E}[X])^{-1}$ which showed that E[$V$], the expected present value cash flow, did not change significantly for large changes in $p'$.

Also, as $w$ approaches 1.0, $p'_t$ approaches $p'$, as expected.

(iv) The coefficient of variation, $\mathrm{CV}[X]$, is given as follows:

$$\mathrm{CV}[X]^2 = \frac{1}{\alpha(1 - p')}$$

$$= \mathrm{CV}[Y]^2 \frac{w + p\,(1 - w)}{w} \,.$$

While the variance/mean approaches 1.0 as $w$ approaches zero, the coefficient of variation gets increasingly large as the retention increases.