DISCUSSION OF PAPER PUBLISHED IN VOLUME LXXV

RECENT DEVELOPMENTS IN RESERVING FOR LOSSES IN THE LONDON REINSURANCE MARKET

HAROLD E. CLARKE

DISCUSSION BY JOHN C. NARVELL

1. INTRODUCTION

Mr. Clarke has written a fine paper [1] which should be read by all actuaries who practice loss reserving. He has introduced two concepts that will broaden the horizons of actuaries in the U.S. and should provoke them to sharpen their skills.

The first concept is the introduction of a new curve for the fitting of loss development data. This "negative exponential" curve has the formula:

 $L(t) = A \times [1 - \exp(-[t/B]^{c})].$

The second concept is the use of regression techniques to provide a weighting scheme between Bornhuetter-Ferguson (B-F) and loss development factor (LDF) projections. Although it is not emphasized in the paper, these two concepts may be used separately or with other more traditional loss development techniques. In the course of the paper the two concepts are commingled, but the astute reader should be able to separate the two.

The first observation that the reader should make is that there are actuaries outside of North America who are developing skills in the property and casualty area. In many instances these actuaries are taking a fresh perspective to old problems and are producing novel solutions. This paper is an example of such innovation. The other item to note is that there are significant variations in terminology in the insurance industry, especially outside of North America. For example, in this paper the author uses data that is categorized by "account year." An account year is analogous to an underwriting year in reinsurance or a policy year for direct insurance. As stated in the paper, the techniques would be equally applicable to data categorized by underwriting year, policy year, accident year or even report year.

2. NEGATIVE EXPONENTIAL CURVE

The negative exponential curve is different from curves that many American actuaries use in that it fits loss development data instead of loss development factors. This curve models the upward growth of the losses as they asymptotically approach the ultimate losses (A) from below. Most development curves in the North American literature (e.g., Sherman's inverse power curve [2]) model loss development factors (LDFs) instead of losses.

By using the negative exponential curve, an entire step in the analysis process is bypassed; i.e., LDFs do not have to be calculated. There are numerous other advantages to the use of loss data instead of age-to-age LDFs.

The negative exponential curve ostensibly has three parameters, A, B and C. The A parameter of the curve is defined in the paper as the ultimate loss ratio. This could just as easily have been defined as the ultimate losses, since the ultimate loss ratio is a simple transform of the ultimate losses. The reason for the use of loss ratios is apparent in the latter part of the paper where loss ratios are used in the regression model.

Another alternative would be to set A = 1. This could be accomplished by dividing the historical losses by year by the estimated ultimate losses for each year. This alternative perspective shows the true nature of this loss development model. By defining each historical observation as a percentage of ultimate losses, the model may be thought of as a variation of a multiplicative LDF projection. If one takes the reciprocal of the percent of ultimate, then cumulative LDFs are easily produced.

In the original implementation of this curve, as introduced by David Craighead, a simple LDF implementation was advocated. On page 66 of his paper [3], he says:

"In practice, all that is necessary, given values of *B* and *t*, is to obtain values of $\exp(-(t/B)^2)/(1 - \exp[-t/B)^2]$) and then apply them to figures of claims paid plus claims outstanding in order to obtain figures of IBNR for each cell of business."

This is clearly a multiplicative LDF method. It is notable in the above equation that Craighead assumed a value of 2.0 for the C parameter. In fact Craighead's research "resulted in a conclusion to fix the value of C at 2.0" (page 54). He continues:

"The value of C is, in fact, too sensitive and has little effect on the overall results. It can be influenced easily by the random positioning of a few points at an early stage, where they are least reliable. It also appears to have a counteracting effect on the value of B.... Fixing C at 2.000 ... shows that, in most cases, the fit is almost as good as when C is allowed to float and, indeed, in several cases is actually better."

Mr. Clarke implicitly admits this aspect of the negative exponential curve in his paper (page 9) where "in this particular example C was set equal to 1.5 and only A and B were fitted." Thus it appears that some variation in the C parameter is allowed in practice but that C is usually fixed before the other parameters are fitted.

The reason for the counteracting effect on the B parameter is easily understood if one reexamines the form of the negative exponential curve. The equation may be rewritten as:

 $L(t) = A \times [1 - {B'}^{\ell}]$ where $B' = \exp[-1/(B^{\ell})]$.

As C gets bigger, B' has to get smaller. This alternative formula is easier to understand; the C parameter is unchanged and the B' parameter simplifies the form of the equation. B' is allowed to vary in the range from 0 to 1.

The difference between a simple LDF and the more sophisticated approach in this present paper is that the most current observation is not simply multiplied by the appropriate LDF to ultimate. Rather there is some consideration for a random error contained in the endpoint. This error is measured as the deviation of the endpoint away from a curve which is fit through the entire loss history for that year.

The best way to understand this is to consider an extreme example. Consider losses that exhibit sawtooth variations about a generally rising curve. The true underlying loss development pattern is in the middle with random variations about it. The negative exponential curve can extract the shape of the development curve and project it to ultimate. Effectively each historical data point is given equal credibility in the estimation of ultimate losses.

This is in contrast to traditional LDF projections where only the most recent observation is used; i.e., the endpoint is multiplied by a cumulative factor to ultimate to estimate the ultimate losses. Data observations prior to the endpoint do not affect the estimate. Further, as soon as a new endpoint is known, the previous end points are almost completely ignored. In the sawtooth example, the ultimate projections would fluctuate up and down from period to period in LDF projections but would be more stable in the smoothed negative exponential implementation.

This points out a major difference between the author's approach and the traditional LDF or B-F methods. When projecting a year from age t to ultimate, the negative exponential considers only the development patterns for the particular year before age t. In contrast, for most traditional LDF or B-F methods, only development data *after* age t (for other years) is considered.

Some LDF curve fitting techniques, such as the inverse power curve, can also look at all LDF data simultaneously to extract the true LDF patterns excluding randomness. In fact, the inverse power curve can be used in an analogous fashion to the negative exponential curve if sufficient credible data exists. LDFs for a single (accident) year can be used to estimate the remaining loss development tail for that year. However, one major limitation of the inverse power curve in its simplest implementation is that it cannot handle downward LDFs without performing some data smoothing which would destroy the true patterns. Also the data for a single year may produce unstable results similar to the fluctuating parameters for the negative exponential curves in the paper.

The theoretical advantages of the negative exponential curve form are numerous. First, the curve can handle many different data anomalies including downward development which, for example, the inverse power curve cannot. Also it is decomposable into as many time intervals as are available from the data. These time intervals do not need to be regularly spaced although the implementation in the paper imposes this limitation. Another major advantage is that the curve form naturally leads to graphical display and interpretation. Some of these advantages are not limited to the negative exponential curve but are true for curve fitting methods in general.

There is one potential disadvantage of the negative exponential method. For the curve fitting to be effective, many data points are required. Craighead states (page 65), "This will require the use of . . . loss ratios at least at quarterly intervals, preferably even monthly, and for at least two years, to give any meaningful result."

On the other hand, an advantage of the proposed methodology is that it is less subject to distortion arising from unintentional bias in the

selection of loss development factors. This is especially true in those cases where a mixture of upward and downward development is evident. Frequently, loss reserve analysts will exclude downward (and very small upward) development factors from consideration in the selection of loss development factors. In the curve fitting algorithm, there is less subjectivity in the selection of loss development factors both for the stages where there is historical data and for the "tail" development beyond the range of historical data. While interpretation and curve parameter selection will still be required of the loss reserve analyst, the use of such curve fitting techniques will introduce more science into the art of loss development projections.

3. PROJECTIONS BY INDIVIDUAL YEAR

An innovation in the model is the analysis of each year separately. The model does not require a large history of average loss development factors either for data within the range of the available loss development or for the calculation of a "tail" factor for development beyond the range of the available data. This is a very powerful advantage. However, as noted above, the method does require frequent observations (quarterly or monthly) for the curve to be well defined.

4. REGRESSION MODEL

In the worked example in the paper, the negative exponential curve was used for the older account years to project the loss development of the years individually. In contrast, the regression model ("line of best fit") was used for the three most recent account years. As noted in the last paragraph of Section 5, the regression model does not require the use of the negative exponential projections for the older years. The only data required for the line of best fit are: a) the historical loss ratios by year of account, and b) the estimated ultimate loss ratios for those years. It is immaterial how those ultimate loss ratios are calculated.

Regression analysis is not new to the members of the CAS. It has been used in many different contexts and is now on the *Syllabus of Examinations* (Part 3). Its use in this present paper should not be trivialized, however. The assumptions and procedures in this particular context merit review and consideration. The regression model projects losses (loss ratios) from their current status to ultimate in one step. The incremental steps of individual loss development factors are bypassed. This is accomplished by comparing the historical loss ratios at each age with the ultimate loss ratios for the previous account years.

For example, in order to project the 1982 year from 14 quarters to ultimate, a least squares regression of the 1971–81 years' loss ratios at 14 quarters with their corresponding estimated ultimate ratios is calculated. The resulting formula is used to project the loss ratio from 14 quarters to ultimate for the 1982 year. The 1971–81 years' loss ratios at 10 quarters are then regressed with their corresponding ultimate ratios to project the 1983 year from 10 quarters to ultimate. The 1984 year at 6 quarters is projected to ultimate in a similar fashion. In the worked example in the paper, some years are judgmentally excluded from the regression analysis.

In theory the number of years in the regression could have been increased. This is because the earlier years of account were regressed first; e.g., the 1982 year of account could have been included in the regression for the projection of the 1983 year. This would have permitted the inclusion of more observations including the most recent data.

A review of the regression formula shows that the ultimate loss ratio is equal to the immature loss ratio times some factor plus a constant. This may be contrasted with multiplicative loss development wherein the ultimate losses are simply a cumulative loss development factor (F)*times* the losses to date (L) with no additive constant.

Alternatively, a B-F model may be considered as an additive model; i.e., ultimate losses equal losses to date (L) plus some estimate of the remaining loss development. In the traditional B-F model, the estimate of the remaining future loss development (Fut) is equal to the percent unreported (or unpaid) times an initial loss estimate (E). The future percent is equal to the complement of the reciprocal of the LDF to ultimate (1-1/F).

The possibility of lack of fit exists with every regression model. This lack of fit may be thought of as lack of correlation or predictive ability of the independent versus the dependent variables. This element of variation necessitates the inclusion of an additional component in the process. This component is the naive estimate; i.e., a flat loss ratio. In

statistical terms, the loss ratio would be the mean value of the independent observations; i.e., the average ultimate loss ratio for all of the years included in the regression. In the case where the linear regression of the independent and dependent variables does not produce a significant fit, the ultimate loss would equal the initial loss estimate with no consideration of actual loss reporting to date. Statistical tests can be used to determine if the regression equation explains the variation about the mean ultimate loss ratio.

The regression model is a mixture of the multiplicative and additive models, subject to a least squares optimization. If one considers the regression to be a weighting formula between multiplicative (weight = W_1) and additive projections (weight = W_2) with consideration for the average ultimate loss ratio (weight = $1 - W_1 - W_2$), then the following formulas for the calculation of ultimate losses apply:

Model:	Multiplicative	Additive (B-F)	Naive
Formula:	$L \times F$	$L + [E \times (F-1)/F]$	Ε
Weight:	W_1	W_2	$(1 - W_1 - W_2)$
Weighted: $W_1LF + W_2L + W_2E[(F-1)/F] + E - W_1E - W_2E$			
C C	$= L \times (W_1F +$	W_2) + $E \times [W_2(F-1)]$	$F + 1 - W_1 - W_2$
	$= L \times (W_1F +$	W_2) + E × [1 - W_1	$-W_2/F$]

After rearranging the terms, the weighted formula may be interpreted as a restatement of the regression formula where $(W_1F + W_2)$ is the factor to be multiplied by the losses to date, and the additive constant equals the initial expected loss ratio times $(1-W_1 - W_2/F)$. This interpretation agrees with the observed data in that the factor coefficient can be less than unity without necessarily implying downward loss development.

With this restated formula, the assumptions in the regression analysis may be better analyzed. In the case where multiplicative loss development factors predict the ultimate losses exactly, then W_1 will be 1.00, W_2 will be 0 and the weighted formula will reduce to $Ult = L \times F$.

In the case where the loss development is perfectly explained by an additive process, then W_1 will be 0, W_2 will be 1.00 and the formula will reduce to $Ult = L + E \times 1 - 1/F$, which is exactly the B-F formula.

When neither the multiplicative nor additive model explains the loss development process and the historical losses are randomly scattered about the expected losses (E), then both W_1 and W_2 will be 0 and the weighted formula will reduce to Ult = E.

In the regression equation, the initial expected losses (E) will be the average ultimate loss ratio of the historical data included in the regression. The R^2 of the regression model measures the proportion of the variation about this mean that is explained by the independent variables (the immature loss ratios). In the case where the historical data is randomly scattered about the average ultimate (E), the R^2 is 0.

Although this reviewer has not derived a method for determining the respective weights, they, nonetheless, provide an attractive intuitive interpretation of the regression model. While the derivation of such weights is not required for the proper working of the regression model, further research into their calculation might produce interesting results.

For the regression the author proposes the inclusion of calculated observations from the fitted curves by year when the actual observations are either missing or are "very variable." This appears to violate the assumptions of the regression whereby a least squares weighting of the LDF, B-F and naive projections is desired.

The negative exponential curve fit produces an LDF projection with an additive offset for the random variance of the endpoint from the smooth curve. By removing the random variances of the endpoints of the historical data, more weight will be given in the regression to the LDF projections. The substitution of a smoothed negative exponential observation for an actual (or missing) observation will bias the regression and produce a weight (W_1) for the LDF that is artificially high. If the data are missing or "very variable," then exclusion of that year from the regression would be preferable to the LDF bias that would be introduced.

The only possible argument for the inclusion of such a smoothed observation is to try to include consideration of the curve parameters for that particular year. Since the negative exponential curve is fit for each year individually, the only time that all of the years are examined simultaneously is in the regression model.

Benjamin and Eagles suggest another variation of this application. In paragraph 22.3.7 of their paper [4], they advocate the use of projected loss ratios from the curves for more recent years to facilitate regression

equations for the earlier years. For example if there are nine years of data in the triangle (e.g., 1971–79), the curves for years 1974–77 would be projected to produce loss ratios at age 7. These values would be combined with the actual observations at age 7 for 1971 and 1972 to produce a regression equation to be used to project the 1973 year from age 7 to ultimate. The reason given for this sleight of hand is to enable the production of confidence intervals for the earlier years. One should question the meaning of confidence intervals calculated in such fashion.

5. ALL YEARS AT ONCE

Our British associates seem to prefer the examination of data one year at a time before subsequently looking at all years at once. This is in great contrast to the North American tradition of examining many years simultaneously. If one thinks of data in triangular form, this paper advocates a horizontal perspective instead of a vertical perspective. The growth patterns *within* a year are examined instead of average growth factors for previous years at the same age.

One troubling item in the paper is the instability of the B parameter in the negative exponential curve fits. For a single line of business, one would expect greater consistency from year to year. Perhaps the modeling of each year individually cuts the data too finely. In contrast, the grouping of all of the years for the regression model assumes the comparability of the various years. Similar grouping of the data in the negative exponential model would give greater stability to the B parameters.

The use of many years simultaneously in the regression appears to be in contradiction to the individual analysis of the account years for curve fitting; i.e., the combination of years assumes a degree of homogeneity and comparability among the years.

The question then naturally arises: Why are the curves not also determined on a multi-year basis? In fact one might argue that this procedure should be reversed; i.e., that the curves should be determined on a combined basis. The curve fitting of the B and C parameters would be more stable if more years were included. In fact, for any particular line of business, the loss development characteristics should remain relatively consistent over time.

In contrast, rate levels (and loss ratios) are subject to cyclical market pressures. Therefore loss development patterns, which are relatively immune to distortion resulting from rate movements, should be determined on a multi-year basis. In contrast, premium based loss ratio statistics may not be reliable over varying ups and downs of the market. One possible way to correct for this would be to introduce a rate adequacy adjustment vector to the premiums by year. While such an adjustment is open to arguments of subjectivity, it would nonetheless be more theoretically attractive.

The author discusses this latter option in the context of a case wherein the slope of the regression equation is not significantly different from zero. In this circumstance the average ultimate loss ratio (ULR) from the data is the best estimate for the ULR of the year to be projected. He states that "it would obviously be desirable to adjust the ULR's to allow for changes in premium rates that may have taken place." Such modification should be considered in all cases in order to extract the maximum amount of unbiased information from the data.

6. GRAPHS

In order to expand upon the graphical foundations of this paper, the historical loss ratios for the individual account years have been reproduced on a multi-year basis for this discussion. Years of account 1973–78 are shown together on Graph 1A, and years 1979–84 are shown on Graph 1B. When viewed simultaneously, the years 1973 through 1979 appear to exhibit a variety of differing paths in contrast to the five most recent years (80–84), which are closely packed on top of one another.

A better way to isolate the loss development patterns is to translate each curve onto a common vertical scale. It is recommended that the historical losses (loss ratios) be divided by the ultimates by year to produce curves that show percentages of ultimate losses. The various curves all approach a common horizontal asymptote of 100% of ultimate losses. Such calculations produced Graphs 2A, 2B and 2C. When viewed this way, there appears to be much greater stability and consistency in the loss development *patterns*.

However, two years, 1978 and 1979, still distinguish themselves as unusual. This is evidence that these two years ought to be examined in

further detail for possible exclusion from the development projections for the other years. The author was able to identify 1978 but was not able to differentiate 1979. The graphical examination of all years *simultaneously* allows for easier identification of data problems.

The graphs of the percentages of ultimate losses provide a quick and valuable test of reasonableness of the ultimate projections. By examining the endpoints of curves by year, one can visually compare current indicated reserves with hindsight indicated reserves. For example, for the 1978 year the endpoint is at 86.4%, indicating that 13.6% (100–86.4) of the ultimate losses need to be reserved. This percentage of ultimate is far outside the range of any historical observation for the preceding 7 years. Similar observations are applicable for other years, most notably 1977 and 1980. This indicates possible overreserving for these years.

Graphs 1C and 2C are the same as Graphs 1B and 2B but with expanded scale to show detail. This maximizes the advantages of graphical analysis and interpretation. In a similar way the graph of the regression from 7 to ultimate has been reproduced at larger scale and with the data points by year labelled (similar to the Benjamin and Eagles presentation).

From this alternative graph one can gain further insight. For example, it appears that the 1973, 1974 and 1979 years fall into a different cluster than the other years. This clustering might indicate that different consideration should be given to high loss ratio years versus low loss ratio years. The clustering was not as apparent when the data points were displayed in a small cramped section of an overscaled graph.

Another observation from the graph is that the range of validity of the line of best fit might be limited to the range of the minimum and maximum values used in the fit. Otherwise unexpected results may occur. For example in the line of best fit from 6 (22 quarters in the paper) to ultimate, for loss ratios above 84.1% the indicated reserve requirement is negative. While the prediction of downward development was appropriate for the one extreme case in the data history where the loss ratio was above 100%, the downward movement for that year appears more likely to have been caused by a calendar year miscoding of a reinsurance recovery to the 1978 year that was subsequently corrected into the 1979 year. Indeed, the general pattern of development after this age to ultimate is upward, indicating the impropriety of a negative reserve.

Theoretically, a negative reserve can result if the factor coefficient is less than unity or if the additive constant is negative.

7. COMPARISON WITH THE INVERSE POWER CURVE

A comparison of the negative exponential curve with the inverse power curve for LDFs reveals numerous differences. As mentioned previously, the negative exponential curve is an upward rising curve which is fit to the cumulative loss amounts that approaches a horizontal asymptote which is defined as the ultimate losses. In contrast, the inverse power curve fits incremental growth factors and is a downward sloping curve that approaches the horizontal asymptote of 1.00 as time goes to infinity. This aspect of the inverse power curve agrees with the empirical observation that loss development factors converge to unity as the losses eventually stop increasing.

On the other hand, one of the similarities of the two curves is their inability to fit loss data at early development stages. Clarke handles this problem by ignoring the early loss development; Craighead advocates a weighting of the errors using a time vector in addition to ignoring the first data observation.

A comparison of the curves produced by these two equations indicates that the inverse power curve is longer tailed than the negative exponential. When an inverse power curve is fit to a perfect negative exponential curve, the inverse power curve will project a longer tail than that which is contained in the negative exponential.

Attempts by this reviewer to fit the negative exponential curve to long-tailed casualty data from the United States have produced estimated ultimate losses that were unrealistically low. This indicates that the negative exponential curve may be too short tailed to fit truly long-tailed data. This may be due to differing underlying characteristics of the loss development data. In discussing the *B* curve parameter, Craighead notes (page 63):

"*B* is a measure of all the delay factors that affect premium or claim reporting, whether those delay factors arise from the fact that it is reinsurance business that is involved, or from the method of accounting, or from the length of the claims tail. Some of the values of *B*, for example, stem more from the method of accounting than from the length of the tail \ldots ."

Considering the reporting delays in the fragmented London reinsurance market, the loss development tail for those losses may well be better explained by the negative exponential curve. There are initial delays in reporting, followed by a steady stream of losses that then trail off relatively rapidly. The negative exponential starts slower but finishes faster. The general nature of the loss development delays for London market reinsurance are different than those of casualty exposures in North America, which, in contrast, are largely driven by social inflation forces such as increased claims consciousness and litigiousness.

8. REGULATORY OPTIONS FOR MINIMUM RESERVES

The current Schedule P formula for the "Excess of statutory reserves over statement reserves" is a crude means of mandating minimum reserves. These minimum loss ratios are even simpler than the current Lloyd's audit reserve calculations. For example, minimum loss ratios do not consider the actual reported loss experience to date. At least the current Lloyd's audit percentages produce varying minimum loss ratios for varying levels of reported paid losses.

The proposed mixed (multiplicative and additive) formula in the paper is a much more responsive means of establishing formula reserves. However, there are many issues that need to be addressed in the area of statutory minimum reserves. For example, should industry average factors be applicable to all companies? Or should it be the average minus one standard deviation in order to achieve the goal of merely producing a lower bound that will prevent ridiculously low reserves?

One option would be to use loss ratios for each individual company from Schedule P data. With the availability of ten years of data on Schedule P, it would be possible to include seven (or more) years of company data in a regression to produce minimum reserves for the three most recent accident years for the company. Actual investigation into this possibility should be easily performed by those who have computer access to a large volume of companies' Schedule P data.

9. RESERVE SETTING

One option not proposed in the paper is in the area of prospective reserve setting as opposed to retrospective reserve testing. With the availability of historical losses by quarter (or month), it would be possible to perform regressions with quarterly (or monthly) data to produce responsive formulas that could then be applied to the actual paid and/or incurred losses at the end of each period to produce periodic IBNR reserves. In fact, in the paper the reserves are calculated at midyear, although the regressions are ambiguously labelled as being at the end of the year.

10. SUMMARY

This paper has presented a fresh perspective to the challenge of loss reserving whereby loss development is not measured in stages but rather is projected in one jump from the current status to ultimate. This process has advantages in those cases where erratic up and down movements disguise the underlying development. The introduction of the negative exponential curve facilitates the author's approach.

However, this new alternative is not without problems. First of all, frequent data observations are required for the method to produce stable results. Even with frequent data points, the data may not produce stable curve parameters. In particular it may be necessary to fix the C parameter, thereby losing some of the predictive shape of the curve. And finally the shape of the negative exponential curve may be too short-tailed. Caution should be exercised by all actuaries who attempt to use this curve for casualty data from the United States.

REFERENCES

- [1] Harold E. Clarke, "Recent Developments in Reserving for Losses in the London Reinsurance Market," *PCAS* LXXV, 1988, p. 1.
- [2] Richard E. Sherman, "Extrapolating, Smoothing, and Interpolating Development Factors," PCAS LXXI, 1984, p. 122.
- [3] David Craighead, "The Financial Analysis of a Reinsurance Office," *Risk Research Group Ltd.*, London, England, 1982.
- [4] S. Benjamin and L. M. Eagles, "Reserves in Lloyd's and the London Market," J.I.A. Vol. 113, Part II, 1986.







210

REINSURANCE RESERVING









8

Ratio

214

REINSURANCE RESERVING