AN ANALYSIS OF THE CAPITAL STRUCTURE OF AN INSURANCE COMPANY

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Abstract

This paper attempts to analyze the capital structure of an insurance company in a way that (1) views the insurance company as an ongoing enterprise and (2) allows for the stochastic nature of insurance business. A model is developed. This model is used to analyze the effect of uncertainty in the loss reserves, the underwriting cycle and the cost of insurance regulation to the consumer. The paper considers both the investor's and the regulator's points of view.

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1. INTRODUCTION

An insurance company is in the business of transferring risk. It does this by accepting premium from policyholders and paying claims. It can happen that the premium collected is less than the total amount paid for claims. If this is the case, the insurer is expected to pay for the claims from the capital[1] of the insurance company.

This paper addresses the following question.

How much capital will be invested in a given insurance company?

The owners of (or investors in) the insurance company are concerned with the return and the safety of their investment. The money they invest in the insurance company must be competitive with respect to the return and safety of alternative investments. The insurance regulator has a vital interest in this question. The concern is that the insurance company have enough money to fulfill its obligations to the policyholders.

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[1] We shall use the terms "capital" and "surplus" interchangeably to represent the owner's equity in the insurance company. In addition, for simplicity's sake, we shall ignore expenses, loss adjustment expenses, and investment income from the delayed payment of losses.

A deterministic analysis of the capital structure of an insurance company might proceed as follows.

Let

P = risk premium (or expected loss),
L = security (or profit) loading (we assume $L > 0$),
U = initial surplus, and
i_u = interest rate earned on the surplus.

The expected rate of return on the owner's equity, i, satisfies the following equation:

$$U \times i = P \times L + U \times i_u. \qquad (1.1)$$

If P, L and i_u are fixed, it is easily seen that lowering U will increase the rate of return, i. There are two forces that limit how low U will go. First, the rate of return may get sufficiently high to attract more capital. For example, let

P = \$20,000,000,
L = .025, and
i_u = .06.

Suppose the competitive rate of return is found to be i = 12%. We can solve equation 1.1 for U = \$8,333,333. If the surplus were to fall below \$8,333,333, then we assume that investors would supply new capital to this insurance company. Conversely, if the surplus were to go above \$8,333,333, the owners could invest the excess surplus elsewhere and obtain a greater return on their investment.

A second limiting force is that of regulation. Regulators are interested in assuring that the insurance company can fulfill its obligation to the policyholders. Putting a lower bound on U will help accomplish this purpose. However, it should be pointed out that this action is not without cost to the policyholders. Suppose, in the above example, the regulator decides to require a surplus of \$9,333,333. If the competitive rate of return remains at 12%, the insurance company will be forced to raise its profit loading, L. Solving equation 1.1 gives L = .028. Raising U by \$1,000,000 will cost the policyholders \$60,000 annually.

While this analysis captures some essential points of insurance company operations, there are many other factors that should be considered. These factors include the following.

1. An insurance company is an ongoing operation.
2. The amount paid for claims varies from year to year.
3. The insurance industry is very competitive. The profit loading varies from year to year in a fashion described as the "underwriting cycle."
4. The ultimate claim cost is not determined at the end of the policy year. The result is uncertainty in the liabilities, and hence in the surplus of the insurance company.

This paper analyzes the effect these factors will have on the capital structure of an insurance company. The analysis will consider the same questions as the deterministic analysis given above; namely, (1) what surplus will give a competitive rate of return to the insurance company owners, and (2) what is the cost to policyholders of minimum surplus regulation? We begin with a model that describes how claim amounts vary.

2. THE COLLECTIVE RISK MODEL

We shall use the collective risk model to describe the incurred losses, X_t, in year t. This model assumes separate claim severity distributions and claim count distributions for each line of insurance written by the insurer. We shall use the version of the model described by Heckman and Meyers [10] and Meyers and Schenker [12].

This version of the model can be described by the following algorithm.

1. Select β at random from an inverse gamma distribution with $E[1/\beta] = 1$ and $Var[1/\beta] = b$.
2. For each line of insurance, k, do the following.
   - 2.1 Select χ at random from a gamma distribution with $E[\chi] = 1$ and $Var[\chi] = c_k$.
   - 2.2 Select a random number of claims, N, from a Poisson distribution with mean $\chi \times \lambda_k$.
   - 2.3 Select N claims at random from the claim severity distribution for line of insurance k.

3. Set X_t equal to the sum of all claims selected in step 2, multiplied by β.

The parameter c_k, called the contagion parameter, is a measure of uncertainty in our estimate of the expected claim count, λ_k, for line k. The parameter b, called the mixing parameter, is a measure of uncertainty of the scale of the claim severity distributions. Note that the random scaling factor, β, acts on all claim severity distributions simultaneously.

For demonstration purposes, we have selected a comparatively small insurance company writing a single line of insurance. The claim severity distribution is a Pareto distribution with cumulative distribution function (CDF)

$$S(z) = 1 - (a/(a+z))^{\alpha} \tag{2.1}$$

where $a = 10{,}000$ and $\alpha = 2$. Each claim is subject to a \$500,000 limit.

The expected number of claims, λ, is set equal to 2039.544. The parameters b and c are set equal to 0 and .04 respectively. The resulting risk premium for this insurer is \$20,000,000.

Exhibit 1 shows the resulting aggregate loss distribution as calculated by the Heckman/Meyers algorithm [10]. We will refer to this example as the ABC Insurance Company in what follows.

3. THE DISTRIBUTION OF SURPLUS

We will view the insurance company as an ongoing operation. It collects premiums, pays claims, and pays dividends to the owners (or stockholders). Occasionally, the owners will be required to contribute additional capital in order to maintain the surplus at a level specified by the regulator.

The financial status of an insurance company is usually measured at year end. Accordingly, a discrete treatment of financial results is assumed; i.e., the state of a company's finances will be calculated at time $t = 0, 1, 2, \ldots$ where t is in years.

Let

P = risk premium (assumed constant for all years),

L_t = security loading for year t,

X_t = incurred loss during year t,
D_t = stockholder dividends paid at the end of year t,
R_t = additional capital contributed at the end of year t,
U_t = surplus at the end of year t, and
i_u = rate of return (assumed constant) earned on surplus.

Our model of insurance company operations can be described as follows. Given the surplus U_{t-1}, define the random variable V_t by

$$V_t = U_{t-1} \times (1+i_u) + P \times (1+L_t) - X_t. \tag{3.1}$$

Let $U_{\max}$ be the maximum surplus and $U_{\min}$ be the minimum surplus determined by the insurance company management and/or regulators. Then we define

$$D_t = \mathrm{MAX}(V_t - U_{\max}, 0), \tag{3.2}$$

$$R_t = \mathrm{MAX}(U_{\min} - V_t, 0), \text{ and} \tag{3.3}$$

$$U_t = V_t - D_t + R_t. \tag{3.4}$$

While the dividend and minimum surplus decisions are usually more complex, they should be reasonable for modeling purposes. This model is similar to that described by Beard, Pentikäinen and Pesonen [1, p. 215].

Let $F_t(v)$ be the CDF for V_t. Let $M = U_{\max} \times (1+i_u) + P \times (1+L_t) \cdot M$ represents the maximum value of V_t. $F_t(v) = 1$ for $v \geq M$.

Let u_t, d_t, and r_t represent the expected values of the surplus, U_t, the dividend, D_t, and the additional contributed capital, R_t, at time t respectively. We have

$$u_t = \int_{U_{\min}}^{U_{\max}} v \times dF_t(v) + U_{\max} \times (1 - F_t(U_{\max})) + U_{\min} \times F_t(U_{\min}); \tag{3.5}$$

$$d_t = \int_{U_{\max}}^{M} (v - U_{\max}) \times dF_t(v); \tag{3.6}$$

$$r_t = \int_{-\infty}^{U_{\min}} (U_{\min} - v) \times dF_t(v). \tag{3.7}$$

Note that $u_t + d_t - r_t = \mathrm{E}[V_t]$.

Also of interest are the probability of paying a dividend at time t, $\mathrm{P}_t(D)$, and the probability of having to contribute additional capital at time t, $\mathrm{P}_t(R)$.

The requirement that additional capital be contributed applies even when the surplus is negative. It is possible for reinsurance companies or guaranty funds to contribute money to raise the surplus to 0. Cummins [6] discusses a way to price this reinsurance.

Some notes on the history of this operating strategy are in order. This dividend-paying strategy originated in the risk theory literature in a paper by de Finetti [9]. It has been discussed by Bühlmann [4, p. 164] and Borch [3, p. 225]. A more general version of this strategy has been discussed by Tapiero, Zuckerman and Kahane [13]. They insert an additional level, U_{long}, between $U_{\min}$ and $U_{\max}$. When V_t goes above U_{long}, the amount, $V_t - U_{\text{long}}$, is put into long-term investments. Meyers [11] addresses the same questions addressed by this paper with an operating strategy that does not require the contribution of additional capital.

4. YIELD RATES

The yield rate of an investment is defined to be the interest rate at which the present value of the investments is equal to the present value of the returns.

Let T be the investor's time horizon. The investments consist of the initial surplus at time zero and the additional contributions to surplus at each time t. The returns consist of dividends payable at each time t, and the average surplus at time T. Of course, any yield rate calculation must reflect the probability that the payments are actually made.

Let i be the yield rate. The yield rate must satisfy the following equation.

$$u_0 + \sum_{t=1}^{T} r_t \times (1+i)^{-t} = \sum_{t=1}^{T} d_t \times (1+i)^{-t} + u_T \times (1+i)^{-T}. \tag{4.1}$$

This equation can be solved for i by the Newton-Raphson method.

The methodology described above has been incorporated into a computer program called the "Insurer Surplus Model." This program makes repeated use of the Heckman/Meyers algorithm.

Let us now consider the case of ABC Insurance Company. We make the following (debatable) assumptions.

1. The investors in ABC Insurance Company are risk neutral; i.e., they are interested only in the expected return on their investment.

2. The investors in ABC Insurance Company can easily shift their capital investments to seek the highest rate of return.

Suppose that the regulators require a minimum surplus of \$6,000,000, and that the market/regulators allow a security loading of .025. Suppose further that i_u = .06 and the investors select a time horizon of T = 25 years. The company management calculates the yields in Table 1 for varying levels of initial surplus (= maximum surplus).

TABLE 1

Surplus	Yield
\$12,000,000	10.80%
10,000,000	11.66
8,000,000	12.79

To continue our example, let us suppose that the yield on alternative investments is 12% for T = 25. It is a consequence of the above assumptions that the investors in ABC Insurance Company will adjust the surplus until a 12% yield is obtained. Thoughtful trial and error quickly gives an initial (= maximum) surplus of \$9,330,000. Note that the yield does vary with the time horizon, T, selected. The output of the Insurer Surplus Model for this initial surplus is given in Table 2.

TABLE 2

INSURER SURPLUS MODEL STANDARD ASSUMPTIONS

t	$P_t(R)$	r_t	u_t	d_t	$p_t(D)$	L_t	*Yield*
1	0.14518	371690	8,501,385	2,260,106	0.62482	0.02500	11.36%
2	0.20393	580,225	8,256,856	1,834,837	0.54171	0.02500	11.59
3	0.22181	644,846	8,183,506	1,713,608	0.51713	0.02500	11.72
4	0.22719	664,276	8,161,486	1,677,306	0.50976	0.02500	11.80
5	0.22880	670,109	8,154,875	1,666,409	0.50754	0.02500	11.85
6	0.22928	671,860	8,152,891	1,663,137	0.50688	0.02500	11.88
7	0.22943	672,386	8,152,295	1,662,155	0.50668	0.02500	11.91
8	0.22947	672,544	8,152,116	1,661,860	0.50662	0.02500	11.92
9	0.22949	672,591	8,152,062	1,661,772	0.50660	0.02500	11.94
10	0.22949	672,605	8,152,046	1,661,745	0.50660	0.02500	11.95
11	0.22949	672,610	8,152,041	1,661,737	0.50659	0.02500	11.96
12	0.22949	672,611	8,152,040	1,661,735	0.50659	0.02500	11.96
13	0.22949	672,611	8,152,040	1,661,734	0.50659	0.02500	11.97
14	0.22949	672,611	8,152,039	1,661,734	0.50659	0.02500	11.98
15	0.22949	672,611	8,152,039	1,661,734	0.50659	0.02500	11.98
16	0.22949	672,611	8,152,039	1,661,734	0.50659	0.02500	11.98
17	0.22949	672,611	8,152,039	1,661,734	0.50659	0.02500	11.99
18	0.22949	672,611	8,152,039	1,661,734	0.50659	0.02500	11.99
19	0.22949	672,611	8,152,039	1,661,734	0.50659	0.02500	11.99
20	0.22949	672,611	8,152,039	1,661,734	0.50659	0.02500	11.99
21	0.22949	672,611	8,152,039	1,661,734	0.50659	0.02500	11.99
22	0.22949	672,611	8,152,039	1,661,734	0.50659	0.02500	12.00
23	0.22949	672,611	8,152,039	1,661,734	0.50659	0.02500	12.00
24	0.22949	672,611	8,152,039	1,661,734	0.50659	0.02500	12.00
25	0.22949	672,611	8,152,039	1,661,734	0.50659	0.02500	12.00

One does not need the Insurer Surplus Model to find the yield for $T = 1$.

$$u_0 + r_1/(1+i) = (u_1+d_1)/(1+i) \tag{4.1}$$

$$(1+i) \times u_0 = u_1+d_1-r_1 = \mathrm{E}[V_1] \tag{4.2}$$

$$i = \mathrm{E}[V_1]/u_0 - 1. \tag{4.3}$$

Now:

$$\mathrm{E}[V_1] = u_0(1+i_u) + P \times L_1. \tag{4.4}$$

Thus:

$$i = i_u + P \times L_1/u_0. \tag{4.5}$$

Note that equation 4.5 can also be derived from equation 1.1.

5. UNCERTAINTY IN LOSS RESERVES

The time $t=0$ does not have to be the date the insurance company begins operation. The old advertising jingle "Today is the first day of the rest of your life" applies also to insurance companies. Applying the above approach to an ongoing insurance company presents a special problem which is discussed here.

Probably the largest and most uncertain liability for a property and casualty insurance firm is the loss reserve. This creates uncertainty in the initial surplus, u_0. We attempt to model this by making the additional assumption:

U_0 has a normal distribution with known mean and variance.

The debate concerning the variability of loss reserves has taken on new life within the last few years. Publications by the Casualty Actuarial Society Committee on the Theory of Risk [5], De Jong and Zehnwirth [7] and Taylor [14] deal with this problem extensively. Even so, the author considers the problem far from solved.

In our example, the ABC Insurance Company, we will use \$1,790,035 as the standard deviation of the loss reserve, i.e., the initial surplus. This figure was derived from the following assumptions.

1. The claim severity distribution is known.
2. Claims are paid out over a period of eight years. The paid to ultimate ratios are .05, .20, .40, .60, .75, .90, .96 and 1.00, respectively.
3. The smallest claims are settled first.

The details of this derivation are in the Appendix.

Using the Insurer Surplus Model we calculate that a value of \$9,340,000 for u_0 and $U_{\max}$ will result in a yield of 12% if all other inputs remain the same. Table 3 contains the output.

This example suggests that the uncertainty in loss reserves has little effect on surplus levels from the investor's point of view. More will be said about this later.

6. THE UNDERWRITING CYCLE

We now consider the case when the security loading varies from year to year in a cyclic manner. This is a well established phenomenon in casualty insurance which is felt, at least by the author, to be caused by intense competition from within the insurance industry. Berger [2] proposes a model whereby the underwriting cycle results from (1) the desire to maximize profits and (2) aversion to bankruptcy.

To model the underwriting cycle we assume that

$$L_t = L_0 + A \times \sin(\omega \times (t-1)+\phi). \tag{6.1}$$

This is a special case of the $AR(2)$ model considered by Beard, Pentikäinen and Pesonen [1, p. 202 and p. 388] for cyclic variation.

To demonstrate the effects of the underwriting cycle on the ABC Insurance Company we set $L_0 = .025$, $A = .02394$ and $\omega = \pi/4$. These parameters will produce an eight year cycle with a reasonable amount of variation.

TABLE 3

INSURER SURPLUS MODEL UNCERTAIN INITIAL SURPLUS

t	$P_t(R)$	r_t	u_t	d_t	$p_t(D)$	L_t	*Yield*
1	0.16524	458,453	8,444,587	2,414,266	0.61053	0.02500	11.35%
2	0.20838	596,836	8,244,475	1,803,622	0.53504	0.02500	11.61
3	0.22284	648,744	8,184,631	1,703,257	0.51494	0.02500	11.74
4	0.22722	664,559	8,166,623	1,673,646	0.50893	0.02500	11.81
5	0.22854	669,323	8,161,202	1,664,741	0.50712	0.02500	11.86
6	0.22893	670,757	8,159,570	1,662,061	0.50658	0.02500	11.89
7	0.22905	671,188	8,159,079	1,661,254	0.50641	0.02500	11.91
8	0.22909	671,318	8,158,931	1,661,011	0.50636	0.02500	11.93
9	0.22910	671,357	8,158,886	1,660,938	0.50635	0.02500	11.94
10	0.22910	671,369	8,158,873	1,660,916	0.50634	0.02500	11.95
11	0.22910	671,373	8,158,869	1,660,909	0.50634	0.02500	11.96
12	0.22910	671,374	8,158,868	1,660,907	0.50634	0.02500	11.97
13	0.22910	671,374	8,158,867	1,660,907	0.50634	0.02500	11.97
14	0.22911	671,374	8,158,867	1,660,906	0.50634	0.02500	11.98
15	0.22911	671,374	8,158,867	1,660,906	0.50634	0.02500	11.98
16	0.22911	671,374	8,158,867	1,660,906	0.50634	0.02500	11.98
17	0.22911	671,374	8,158,867	1,660,906	0.50634	0.02500	11.99
18	0.22911	671,374	8,158,867	1,660,906	0.50634	0.02500	11.99
19	0.22911	671,374	8,158,867	1,660,906	0.50634	0.02500	11.99
20	0.22911	671,374	8,158,867	1,660,906	0.50634	0.02500	11.99
21	0.22911	671,374	8,158,867	1,660,906	0.50634	0.02500	12.00
22	0.22911	671,374	8,158,867	1,660,906	0.50634	0.02500	12.00
23	0.22911	671,374	8,158,867	1,660,906	0.50634	0.02500	12.00
24	0.22911	671,374	8,158,867	1,660,906	0.50634	0.02500	12.00
25	0.22911	671,374	8,158,867	1,660,906	0.50634	0.02500	12.00

We first consider what happens when we catch the cycle on the way up. If we set $\phi = 0$ along with the assumptions stated immediately above and in Section 4, we calculate that a value of \$10,600,000 for u_0 and U_{max} will result in a yield of 12%. The results of the Insurer Surplus Model for this case are in Table 4.

Let us next consider what happens when we catch the cycle on the way down. If we set $\phi = \pi$ along with the assumptions stated immediately above and in Section 4, we calculate that a value of \$7,975,000 for u_0 and U_{max} will result in a yield of 12%. The results of the Insurer Surplus Model for this case are in Table 5.

7. RUIN THEORY

Thus far, our assumption has been that the investors in an insurance company will adjust the surplus so that the expected yield will be constant. An alternative to this assumption is provided by ruin theory. Ruin theory[2] makes the assumption that the investors in an insurance company will adjust the surplus so that the probability of insolvency (i.e., the probability of ruin) will remain constant. In this section, we shall demonstrate that these two assumptions imply quite different results.

It is sufficient to consider the probability of ruin for a one-year time span. Let ϵ be the selected probability of ruin. We have:

$$\Pr\{U_1 < 0\} = \epsilon \text{ if and only if } u_0(1+i_u) + P(1+L_1) = x_{1-\epsilon}.$$

where $x_{1-\epsilon}$ is the $1-\epsilon^{th}$ percentile of the random loss X. If ϵ is fixed, it can be seen that a reduction in L_1 should be accompanied by a corresponding increase in u_0, and conversely an increase in L_1 should be accompanied by a corresponding decrease in u_0.

Equation 4.5 indicates the opposite behavior. If i is fixed, it can be seen that L_1 and U_0 move in the same direction. This behavior also holds in the multiyear analysis of the underwriting cycle. If the cycle is on the way down, U_{max} also goes down and the insurance company's surplus is reduced. The opposite happens when the cycle is on the way up.

[2] See, for example, Beard, Pentikäinen and Pesonen [1, ch. 4].

TABLE 4

Insurer Surplus Model
Underwriting Cycle on the Way Up

t	$P_t(R)$	r_t	u_t	d_t	$p_t(D)$	L_t	*Yield*
1	0.09049	215,329	9,643,356	2,307,973	0.63153	0.02500	10.72%
2	0.13201	355,329	9,381,137	2,034,662	0.56686	0.04193	12.50
3	0.14135	387,016	9,323,605	1,986,147	0.55316	0.04894	13.58
4	0.15099	418,743	9,258,197	1,882,081	0.53551	0.04193	13.85
5	0.17115	486,594	9,124,016	1,676,267	0.49982	0.02500	13.50
6	0.19766	579,358	8,956,122	1,456,180	0.45728	0.00807	12.85
7	0.21712	649,582	8,838,445	1,325,894	0.42894	0.00106	12.23
8	0.21700	648,298	8,839,775	1,338,762	0.42969	0.00807	11.89
9	0.19766	577,898	8,958,231	1,489,828	0.45896	0.02500	11.87
10	0.17253	490,525	9,118,200	1,706,563	0.49970	0.04193	12.07
11	0.15641	436,580	9,224,283	1,856,320	0.52740	0.04894	12.29
12	0.15683	438,201	9,220,174	1,834,282	0.52578	0.04193	12.40
13	0.17355	494,854	9,108,970	1,659,268	0.49611	0.02500	12.37
14	0.19868	582,988	8,949,991	1,449,991	0.45582	0.00807	12.23
15	0.21755	651,133	8,835,919	1,323,475	0.42835	0.00106	12.08
16	0.21718	648,916	8,838,742	1,337,734	0.42944	0.00807	11.98
17	0.19773	578,126	8,957,820	1,489,373	0.45886	0.02500	11.97
18	0.17256	490,607	9,118,042	1,706,367	0.49966	0.04193	12.03
19	0.15642	436,610	9,224,224	1,856,242	0.52739	0.04894	12.11
20	0.15684	438,213	9,220,151	1,834,253	0.52577	0.04193	12.15
21	0.17355	494,859	9,108,961	1,659,258	0.49610	0.02500	12.14
22	0.19868	582,990	8,949,987	1,449,987	0.45582	0.00807	12.09
23	0.21755	651,134	8,835,917	1,323,473	0.42835	0.00106	12.04
24	0.21718	648,916	8,838,741	1,337,733	0.42944	0.00807	12.00
25	0.19773	587,126	8,957,820	1,489,372	0.45886	0.02500	12.00

TABLE 5

INSURER SURPLUS MODEL
UNDERWRITING CYCLE ON THE WAY DOWN

t	$P_t(R)$	r_t	u_t	d_t	$p_t(D)$	L_t	*Yield*
1	0.22753	636,215	7,380,114	2,209,600	0.61761	0.02500	12.27%
2	0.30233	922,729	7,214,041	1,693,095	0.52639	0.00807	10.41
3	0.32807	1,026,142	7,158,032	1,536,263	0.49623	0.00106	9.20
4	0.32173	1,000,594	7,171,898	1,577,697	0.50374	0.00807	9.00
5	0.29409	891,738	7,232,678	1,761,271	0.53683	0.02500	9.64
6	0.26412	778,592	7,299,746	1,983,999	0.57398	0.04193	10.64
7	0.24901	723,287	7,334,002	2,105,747	0.59320	0.04894	11.49
8	0.25610	748,834	7,317,772	2,043,617	0.58400	0.04193	11.95
9	0.28192	844,836	7,259,561	1,842,113	0.55153	0.02500	12.01
10	0.31286	964,985	7,191,167	1,630,440	0.51409	0.00807	11.81
11	0.33011	1,034,457	7,153,645	1,524,718	0.49389	0.00106	11.55
12	0.32211	1,002,136	7,171,065	1,575,421	0.50329	0.00807	11.42
13	0.29416	892,006	7,232,524	1,760,810	0.53674	0.02500	11.47
14	0.26413	778,637	7,299,718	1,983,909	0.57396	0.04193	11.66
15	0.24901	723,295	7,333,997	2,105,730	0.59319	0.04894	11.86
16	0.25610	748,835	7,317,771	2,043,614	0.58400	0.04193	11.98
17	0.28192	844,836	7,259,561	1,842,112	0.55153	0.02500	12.00
18	0.31286	964,985	7,191,166	1,630,440	0.51409	0.00807	11.94
19	0.33011	1,034,457	7,153,645	1,524,718	0.49389	0.00106	11.85
20	0.32211	1,002,136	7,171,065	1,575,421	0.50329	0.00807	11.81
21	0.29416	892,006	7,232,524	1,760,810	0.53674	0.02500	11.82
22	0.26413	778,637	7,299,718	1,983,909	0.57396	0.04193	11.88
23	0.24901	723,295	7,333,997	2,105,730	0.59319	0.04894	11.95
24	0.25610	748,835	7,317,771	2,043,614	0.58400	0.04193	11.99
25	0.28192	844,836	7,259,561	1,842,112	0.55153	0.02500	12.00

The two assumptions have different implications when we consider uncertainty in loss reserves. It was demonstrated in the example above that uncertainty in the loss reserves has little effect on the surplus. The surplus increases from \$9,330,000 to \$9,340,000. Suppose we are satisfied with the probability of ruin for the standard assumptions (Table 2). Using the Insurer Surplus Model with $U_{\min} = 0$, we calculate that the probability of ruin after one year is .0152. If the standard deviation of the loss reserve is \$1,790,035, as in Table 3, it requires a surplus of \$10,045,000 to maintain the probability of ruin of .0152 for the first year.

8. THE COST OF REGULATION

It is the regulator's job to impose standards that promote the solvency of insurance companies. One way of doing this is to impose a minimum surplus so that the probability of ruin is acceptably low. It was demonstrated in the last section that such a regulatory strategy may not be in accordance with the wishes of insurance company owners.

The owners don't have any choice in the matter. The regulators set the standards and the insurance companies comply with them. A higher minimum standard will result in a higher level of surplus in the industry as a whole, and a higher profit loading will be demanded. The purpose of this section is to find this additional cost of solvency regulation to insurance consumers.

Let us consider the example in Table 2. We will vary the minimum surplus and calculate the security loading that will result in a yield rate of 12% after 25 years. The results are in Table 6.

Note that if the minimum surplus goes above \$9,330,000 the minimum surplus becomes the maximum surplus, and the security loading can be obtained by solving equation 1.1.

The changes in the market conditions brought on by increasing the minimum surplus are clearly more complex than is assumed by the above example. However, this may be an indication that the cost of regulation is small if the minimum surplus is not too high.

TABLE 6

THE COST OF REGULATION

Minimum Surplus	Security Loading	Security Loading	Additional Security Loading
$6,000,000	2.500%	$500,000	—
7,000,000	2.583	516,600	$16,600
8,000,000	2.673	534,600	18,000
9,000,000	2.767	553,400	18,800
10,000,000	3.000	600,000	46,600
11,000,000	3.300	660,000	60,000

9. CONCLUDING REMARKS

This paper has attempted to analyze the capital structure of an insurance company in a way that

(1) viewed the insurance company as an ongoing enterprise, and

(2) allowed for the stochastic nature of the insurance business.

When one attempts a simple one-year deterministic analysis, as was done in the introduction, it is possible to comprehend the implications instantly. However, when given a complex computer program like the Insurer Surplus Model, the best one can do is to try some examples and draw tentative conclusions. This paper represents one such attempt. The main conclusions are listed below.

1. The underwriting cycle has a major effect on the amount of capital that will be invested in an insurance company. For example, an insurance company should lower its surplus in the down part of the cycle. In our examples, the goal was to obtain an expected yield of 12% over a 25-year period. One should not view this strategy as being shortsighted.

2. The uncertainty in loss reserves has little effect from the investor's point of view. This conclusion is very tentative since questions on

the variability of loss reserves still remain. However, uncertainty in loss reserves can have a substantial effect from the regulator's point of view.

3. Whether the investors like it or not, the regulators may require a minimum surplus. If this minimum is below what the investors would voluntarily allow, the cost to the policyholders is relatively small. As this regulatory minimum increases, the cost to the policyholders becomes substantial.

There are several items that should enter this analysis, but did not. A discussion of some of these items follows.

We assumed that the investor would seek the same expected yield in all circumstances. One could reasonably argue that the investor should seek a higher yield when the surplus is low because of the increased variability of the return. This is debatable. It is unlikely that the investor would invest all of his/her assets in a single enterprise, and so the investor's risk aversion should not be much of a factor. However, the author would like to keep the debate open.

The issue of asset risk has been omitted from this entire discussion. It could very well be as important as any of the items mentioned above. Any analysis of asset risk must include strategies for asset/liability matching. A good place for casualty actuaries to start would be the paper "Duration" by Ronald E. Ferguson [8]. Further research needs to be done in order to integrate asset risk into the above approach for analyzing the capital structure of an insurance company.

EXHIBIT 1

COLLECTIVE RISK MODEL
ABC INSURANCE COMPANY

Line	Expected Loss	Claim Severity Distribution	Contagion Parameter	Claim Count Mean	Claim Count Std. Dev.
1	20,000,000	Pareto	0.0400	2039.544	410.401

Mixing parameter	0.0000
Aggregate mean	20,000,000
Aggregate std. dev.	4,147,667

Aggregate Loss Amount	Cumulative Probability
10,000,000	0.0018
11,000,000	0.0056
12,000,000	0.0143
13,000,000	0.0315
14,000,000	0.0608
15,000,000	0.1055
16,000,000	0.1669
17,000,000	0.2440
18,000,000	0.3333
19,000,000	0.4295
20,000,000	0.5268
21,000,000	0.6195
22,000,000	0.7033
23,000,000	0.7756
24,000,000	0.8350
25,000,000	0.8821
26,000,000	0.9180
27,000,000	0.9444
28,000,000	0.9632
29,000,000	0.9762
30,000,000	0.9849
31,000,000	0.9907
32,000,000	0.9943
33,000,000	0.9966
34,000,000	0.9980
35,000,000	0.9989
36,000,000	0.9994
37,000,000	0.9996
38,000,000	0.9998
39,000,000	0.9999
40,000,000	0.9999
41,000,000	1.0000

APPENDIX

THE VARIABILITY OF LOSS RESERVES

In Section 5 we studied how the variability of loss reserves affected the surplus. We assumed that the loss reserves were normally distributed with a standard deviation of \$1,790,035. In this appendix we show how the standard deviation was derived.

Three assumptions were made.

1. The claim severity distribution is known.
2. Claims are paid out over a period of eight years. The paid to ultimate ratios are .05, .20, .40, .60, .75, .90, .96 and 1.00, respectively.
3. The smallest claims are settled first.

We used the Pareto distribution for the claim severity. The CDF is given by:

$$S(z) = 1 - (a/(a+z))^{\alpha}$$

with $a = 10{,}000$ and $\alpha = 2$.

Let:

$c(i)$ = maximum claim size settled in the i^{th} prior year; and

$n(i)$ = number of claims remaining to be settled.

We have

$$\frac{\int_0^{c(i)} z \cdot dS(z)}{\mathrm{E}[Z]} = \text{paid to ultimate ratio for prior year } i\text{; and}$$

$$n(i) = (1 - S(c(i))) \times 2039.544.$$

Recall that 2039.544 is the annual expected number of claims for the ABC Insurance Company.

We then calculate the following values.

i	$c(i)$	$n(i)$
1	2844	1236
2	7947	633
3	16754	285
4	32912	111
5	60172	41
6	154844	8
7	276340	2

For prior year i, $n(i)$ claims are selected at random from the claim severity distribution, $S(z)$, conditioned on each claim being above $c(i)$. The loss reserve is the total amount generated by this process. The distribution of loss reserves can be calculated by CRIMCALC, a computer program for the Heckman/Meyers algorithm. Exhibit 2 gives the output for CRIMCALC, and Exhibit 3 shows that the distribution of loss reserves can be approximated by a normal distribution.

EXHIBIT 2

Collective Risk Model

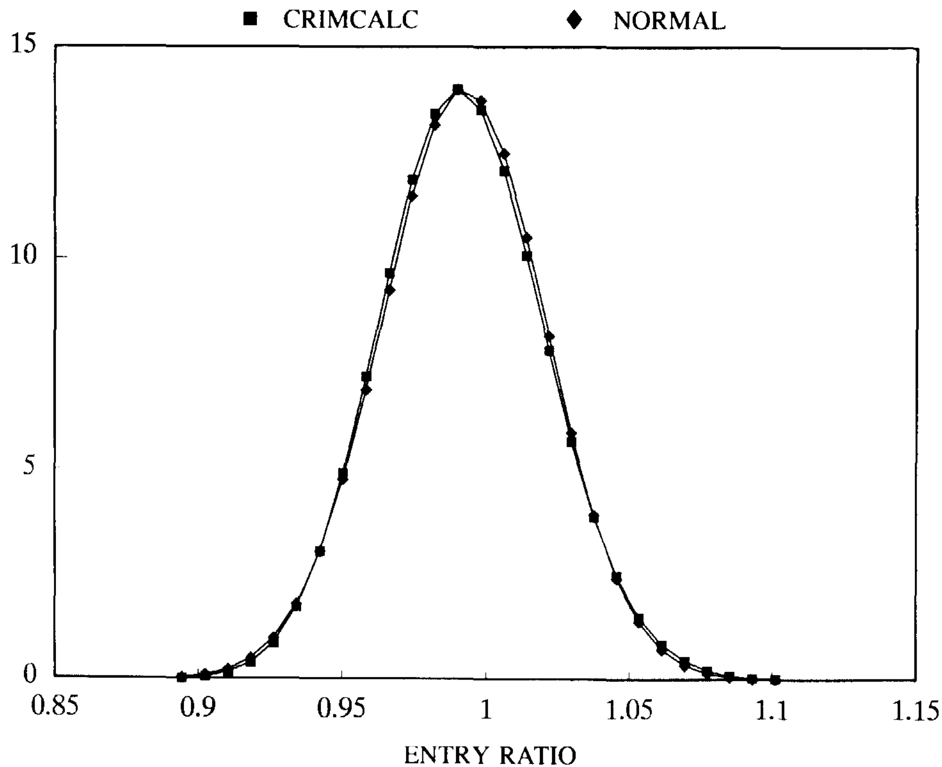
Reserve Risk

Line	Expected Loss	Claim Severity Distribution	Contagion Parameter	Claim Count Mean	Claim Count Std. Dev.
1	18,991,244	Prior year 1	−0.0008	1236.000	0.000
2	15,991,483	Prior year 2	−0.0016	633.000	0.000
3	12,001,336	Prior year 3	−0.0035	285.000	0.000
4	8,018,018	Prior year 4	−0.0090	111.000	0.000
5	4,949,249	Prior year 5	−0.0244	41.000	0.000
6	2,131,540	Prior year 6	−0.1250	8.000	0.000
7	803,876	Prior year 7	−0.5000	2.000	0.000

Mixing parameter	0.0000
Aggregate mean	62,886,746
Aggregate std. dev.	1,790,035

Aggregate Loss Amount	Cumulative Probability	Aggregate Loss Amount	Cumulative Probability
56,000,000	0.0000	63,500,000	0.6415
56,500,000	0.0001	64,000,000	0.7375
57,000,000	0.0002	64,500,000	0.8175
57,500,000	0.0006	65,000,000	0.8796
58,000,000	0.0019	65,500,000	0.9246
58,500,000	0.0050	66,000,000	0.9552
59,000,000	0.0117	66,500,000	0.9748
59,500,000	0.0252	67,000,000	0.9865
60,000,000	0.0492	67,500,000	0.9931
60,500,000	0.0881	68,000,000	0.9967
61,000,000	0.1452	68,500,000	0.9985
61,500,000	0.2219	69,000,000	0.9993
62,000,000	0.3162	69,500,000	0.9997
62,500,000	0.4229	70,000,000	0.9999
63,000,000	0.5341		

EXHIBIT 3



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