THE EFFECT OF TREND ON EXCESS OF LOSS COVERAGES

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Abstract

The subject of the effect of trend on excess of loss coverages has been addressed quite frequently in the Proceedings over the years. Several authors have made the point that with a fixed retention and uniform trend by size of loss, expected excess losses increase proportionally much more than indicated by the general rate of inflation. This is certainly true when considering uncapped excess losses, but it may not be true when considering a specific layer of excess losses. This is because just as the effect of inflation is leveraged at the retention, it is dampened at the upper limit of the layer.

This paper uses graphs to examine how the leveraging effect and dampening effect combine to affect layers of excess losses. This particular issue has historically received very little attention in the Proceedings. The paper begins by examining the excess layer trend factors of a typical loss distribution, and then proceeds to demonstrate how changing each of the two parameters of this distribution affects the trend factors. The paper then looks at the effect of changing the type of distribution. Finally, the paper examines the effect of introducing the concept of varying trend by size of loss.

1. INTRODUCTION

The subject of the effect of trend on excess of loss coverages has been addressed quite frequently in the Proceedings over the years. Several authors have made the point that with a fixed retention and uniform trend by size of loss, expected excess losses increase proportionally much more than indicated by the general rate of inflation ([3], [4], [7], and [8]). This is certainly true when we consider uncapped excess losses, but it may not be true when we consider a specific layer of excess losses. This is because just as the effect of inflation is leveraged at the retention, it is dampened at the upper limit of the layer. The dampening effect on layer losses at the upper limit is equivalent to the dampening effect on basic limit losses at the retention.

This paper will examine how the leveraging effect and dampening effect combine to affect layers of excess losses. From a reinsurer's point of view, it is much more meaningful to look at a layer of excess losses rather than at uncapped excess losses, since a reinsurer virtually never provides unlimited coverage excess of a retention. We will begin by examining the excess layer trend factors of a typical loss distribution, and we will proceed to observe how changing each of the two parameters of this distribution affects the trend factors. We will then look at the effect of changing the type of distribution. Finally, we will see what happens when we introduce the concept of varying trend by size of loss. Graphs will be used to illustrate the results. The formulas used in generating the graphs are shown in the Appendix. Also, it should be noted that this paper presupposes some familiarity with common loss distributions.

2. A TYPICAL DISTRIBUTION

We will first look at what might be considered a typical general liability loss distribution, a Pareto with parameters $B = 10{,}000$ and $Q = 1$.[1] We will assume a general rate of inflation of 10%. The trended distribution becomes a Pareto with $B = 11{,}000$ and $Q = 1$.[2] Exhibit 1 shows the effect of this inflation rate on excess layers. Note that the graph uses a double logarithmic scale with retention along the x-axis and layer width along the y-axis. Each contour line represents various retention-layer width combinations with equivalent multiplicative trend factors.[3] As expected, the trend factors increase as the retention and/or layer width increase. However, contrary to what one might expect, the trend

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[1] The Pareto has been chosen because it is the distribution currently used by ISO to generate increased limits factors.

[2] With uniform trend, the Pareto may be trended by simply trending the B parameter and leaving the Q parameter unchanged. For more information on trending loss distributions under the assumption of uniform trend, see Hogg and Klugman[5].

[3] The term "layer width" is used in place of the term "limit" so as to avoid confusion with the "limits" shown along the top and right side of the graph. Also, the contour lines on the graph are approximate. Although they do not appear completely smooth on the graph, in reality they are completely smooth.

factors never increase beyond 1.10, which represents the general rate of inflation. This can be seen by examining the limits shown at the top and right side of the graph. The limits along the top are the limits for various retentions as the layer width approaches infinity. The limits along the right side are the limits for various layer widths as the retention approaches infinity. The limit at the upper right corner is the limit as both retention and layer width approach infinity.[4]

3. CHANGING THE SCALE PARAMETER

Exhibits 2 and 3 show what happens when we change the scale parameter B. Exhibit 2 shows how the trend factors behave when $B = 1{,}000$ and $Q = 1$; Exhibit 3 shows trend factor behavior when $B = 100{,}000$ and $Q = 1$. Note that the only significant difference between these graphs and Exhibit 1 is that the graphs are displaced slightly. Decreasing the scale parameter B moves the contour lines closer to the origin; increasing the scale parameter B moves the contour lines further away from the origin. This type of behavior can also be observed with other distributions where one parameter can be used to change the scale.[5]

4. CHANGING THE SHAPE PARAMETER

Exhibits 4 and 5 show what happens when we change the shape parameter Q. Exhibit 4 shows how the trend factors behave when $B = 10{,}000$ and $Q = 0.5$; Exhibit 5 shows trend factor behavior when $B = 10{,}000$ and $Q = 1.5$. Note that the trend factors are smaller with the thicker-tailed distribution of Exhibit 4 and larger with the thinner-tailed distribution of Exhibit 5. Similarly, the limits on the graphs exhibit the same pattern.[6]

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[4] It is important to note that although for a fixed retention, the limit as the layer width approaches infinity exists, the multiplicative trend factor which would apply to uncapped losses excess of a fixed retention does not exist. This is because the mean does not exist for this distribution (and does not exist for any Pareto distribution with $Q \leq 1$).

[5] For example, μ is the scale parameter of the lognormal and λ is the scale parameter of the Weibull.

[6] For the Pareto in general, we can say that if a is the trend factor representing the general rate of inflation, the limits on the top will all be a^Q if $Q \leq 1$ and will progress from a to a^Q if $Q > 1$. The limits on the right side will always be a^Q, as will the limit at the upper right corner. As noted by Philbrick[11], for the Single Parameter Pareto, the trend factor is a^Q regardless of layer. Thus, for this distribution, the trend factors over the entire graph would be a^Q.

The fact that the trend factors are smaller with a thicker-tailed distribution and larger with a thinner-tailed distribution is intuitively reasonable. Since a thicker-tailed distribution falls off more slowly, there are more dollars which are subject to the dampening effect at the upper limit of the layer relative to the dollars which are subject to the leveraging effect at the retention than is the case with a thinner-tailed distribution. This type of behavior can also be observed with other distributions where one parameter can be used to change the shape.[7]

5. CHANGING THE TYPE OF DISTRIBUTION

Exhibit 6 shows the trend factors for a lognormal distribution such that E[X; \$1,000,000] and E[X^2; \$1,000,000] are the same as those for the Pareto distribution in Exhibit 1.[8] \$1,000,000 was chosen as the censorship point since this might very well be the point beyond which actual loss data is very sparse. Thus, it is conceivable that distributions similar to those in Exhibit 1 and Exhibit 6 could be fitted from the same set of data. The graph shows that the trend factors in lower layers are not too different from the corresponding trend factors in Exhibit 1. However, in higher layers, the trend factors of the lognormal are substantially greater than the trend factors of the Pareto. Note that the top limits of the lognormal progress from 1.10 to infinity, and the right side limits as well as the upper right corner limit are infinity. This is in stark contrast to the limits of the Pareto. This pattern occurs because the lognormal inherently has a thinner tail than the Pareto. Just as the shape parameter affects the thickness of the tail (and thus the magnitude of the trend factors) for any given type of distribution, the type of distribution itself also affects the thickness of the tail.

This example also provides an illustration of the hazards of extrapolating distributions. Although the behaviors of two different types of distributions may look rather similar over the portion of the distributions which contains the data used for fitting them, their behaviors in the tail beyond this area may be very different.

[7] For example, σ is the shape parameter of the lognormal and τ is the shape parameter of the Weibull.

[8] Moments cannot be matched over the complete uncensored distributions since for a Pareto with $Q \leq 1$, moments for the complete uncensored distribution do not exist.

Exhibit 7 shows the trend factors for a Weibull distribution such that E[X; \$1,000,000] and E[X^2; \$1,000,000] are the same as those for the Pareto distribution in Exhibit 1. The comments regarding the lognormal also apply here, though with the Weibull, the increase in trend factors at higher layers is much greater. This is because the Weibull inherently has an even thinner tail than the lognormal.[9]

6. VARYING TREND

Back in 1981, Rosenberg and Halpert presented the hypothesis that loss trend varies by claim size [12]. They asserted that trend is greater at larger claim sizes.[10] Rosenberg and Halpert concluded that the formula ax^b can be used to model trend, where x is the claim size, b is a constant which indicates the magnitude of the varying trend and a is a constant which can be adjusted to yield a desired overall trend for the entire distribution. A positive b indicates increasing trend by claim size, a negative b indicates decreasing trend by claim size, and a b of zero indicates uniform trend by claim size.

To examine the effect of varying trend on excess layer trend factors, we will trend the distribution in Exhibit 1 using a b of .02 and a b of $-.02$. We will choose a such that the overall trend of the distribution from \$0 to \$1,000,000 is 10%.[11] A Pareto which has varying trend applied to it becomes a Burr (or Transformed Pareto) distribution. For this example, the trended distributions were calculated and then Pareto

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[9] For more information on the relative thicknesses of the tails of various loss distributions, see Beard, Pentikäinen and Pesonen[1] and Hogg and Klugman[5].

[10] Recently, this issue has been the subject of some debate. For example, see Feldblum[2].

[11] With b = .02, trend ranges from −13.8% at \$1 to −5.5% at \$100 to 3.6% at \$10,000 to 13.6% at \$1,000,000 to 24.6% at \$100,000,000. With b = −.02, trend ranges from 40.2% at \$1 to 27.9% at \$100 to 16.6% at \$10,000 to 6.4% at \$1,000,000 to −3.0% at \$100,000,000. Since very small or negative trend is probably unrealistic in most cases, Rosenberg and Halpert[12] presented an enhancement to the varying trend model which assumes that trend is subject to some minimum value. This enhanced model is essentially a hybrid between the uniform trend model and the pure varying trend model. Thus, the graphs of trend factors which would be generated by this hybrid model would have characteristics of both the graphs generated by the uniform trend model and the graphs generated by the pure varying trend model. The ISO varying trend procedure makes use of this hybrid model. For a description of the ISO varying trend procedure, see Insurance Services Office[6].

distributions with E[X; trended value of \$1,000,000] and E[X^2; trended value of \$1,000,000] matching the trended distributions were derived.[12] With increasing varying trend, the Q parameter becomes smaller (and thus the tail thicker), while with decreasing varying trend, the Q parameter becomes larger (and thus the tail thinner).

Exhibit 8 shows the resulting trend factors with increasing varying trend. Instead of approaching a limit, the trend factors continue to increase. In fact, all the limits at the edge of the graph are now infinity. Exhibit 9 shows the resulting trend factors with decreasing varying trend. The trend factors increase for awhile, but eventually begin to decrease. All the limits at the edge of the graph are now zero. Somewhat similar effects can be expected when varying trend is applied to other types of distributions.

Exhibit 10 provides a concise summary of the results which have been presented in the first nine exhibits.

7. CAVEATS

At this point, a few words of caution are in order. The conclusions that can be drawn from the graphs are only as valid as the size of loss distributions and trend assumptions that underlie them. In addition, policy limits can exert a significant effect on observed loss data by censoring losses below their true values. Also, changing policy limit distributions can significantly affect the change in expected losses in excess layers from year to year. Furthermore, given any general rate of inflation, the impact of trend on any specific excess layer will change from year to year as the distribution changes. These are just a few of the many complicating factors which must be considered when analyzing the effect of trend on excess of loss coverages.

8. CONCLUSION

In this paper, we have examined the effect of trend on layers of excess losses, as opposed to uncapped excess losses. We have observed that expected losses in excess layers do not necessarily trend at a rate greater than that indicated by the general rate of inflation. We have seen

[12] The ISO varying trend procedure uses a similar idea. See Insurance Services Office[6].

that trend in excess layers is significantly affected by the values of the parameters of the loss distribution under consideration, the type of loss distribution employed, and the assumption that is made regarding the relationship of trend to claim size. Of particular note is that we have seen that excess layers of thinner-tailed distributions are more greatly affected by trend than excess layers of thicker-tailed distributions. Finally, we have taken heed of a few caveats which must be considered before drawing any conclusions from the results presented here. While this paper certainly leaves many questions unanswered and thus open for further investigation, hopefully it has given the reader a better understanding of the effect of trend on excess of loss coverages.

REFERENCES

[1] R. E. Beard, T. Pentikäinen and E. Pesonen, *Risk Theory: The Stochastic Basis of Insurance*, Third Edition, New York, Chapman and Hall, 1984.

[2] Sholom Feldblum, "Varying Trend Factors by Size of Loss," *Casualty Actuarial Society Forum*, Fall 1988, p. 11.

[3] Ronald E. Ferguson, "Nonproportional Reinsurance and the Index Clause," *PCAS* LXI, 1974, p. 141.

[4] Robert J. Finger, "A Note on Basic Limits Trend Factors," *PCAS* LXIII, 1976, p. 106.

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[8] Yoong-Sin Lee, "The Mathematics of Excess of Loss Coverages and Retrospective Rating—A Graphical Approach," *PCAS* LXXV, 1988, p. 49.

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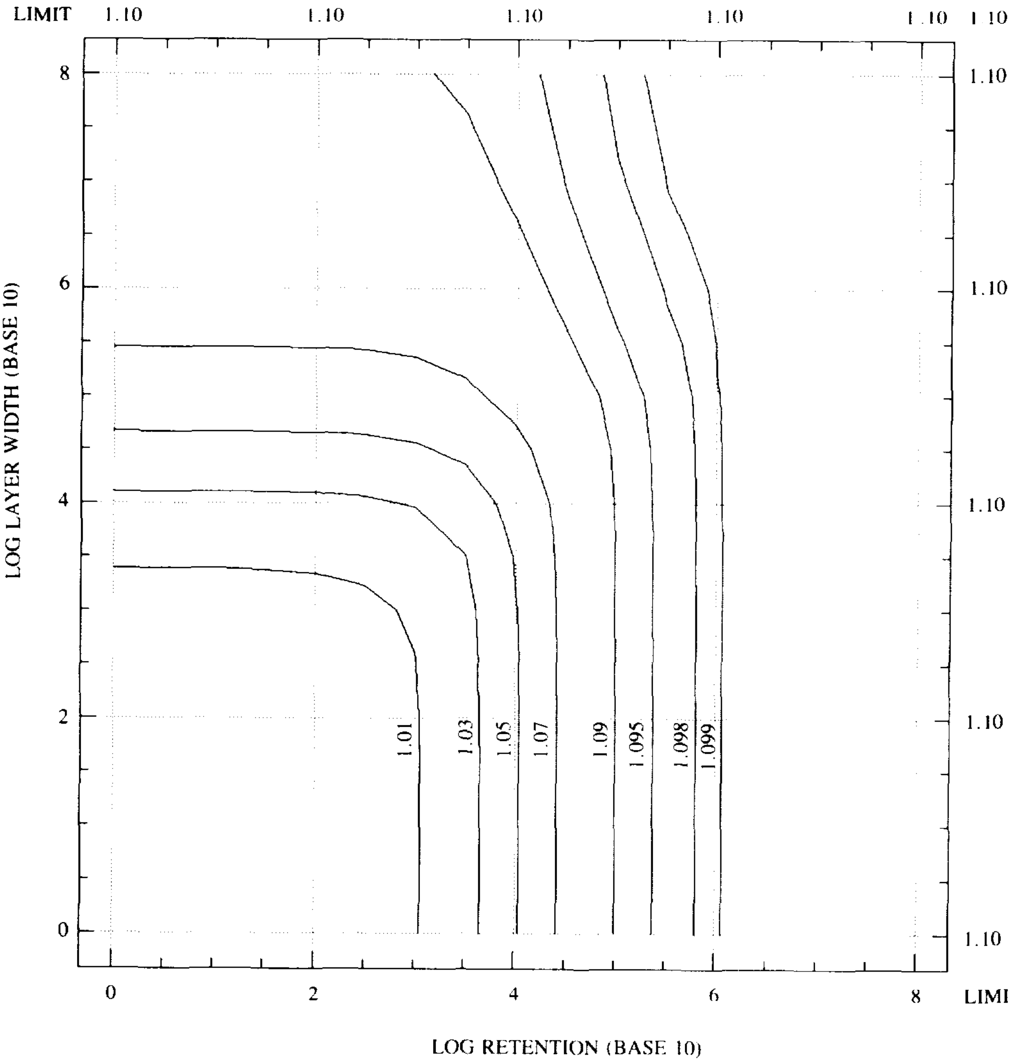
[10] Gary Patrik, "Estimating Casualty Insurance Loss Amount Distributions," *PCAS* LXVII, 1980, p. 57.

[11] Stephen W. Philbrick, "A Practical Guide to the Single Parameter Pareto Distribution," *PCAS* LXXII, 1985, p. 44.

[12] Sheldon Rosenberg and Aaron Halpert, "Adjusting Size of Loss Distributions for Trend," *Inflation Implications for Property-Casualty Insurance Products*, 1981 Casualty Actuarial Society Discussion Paper Program, p. 458.

EXHIBIT I

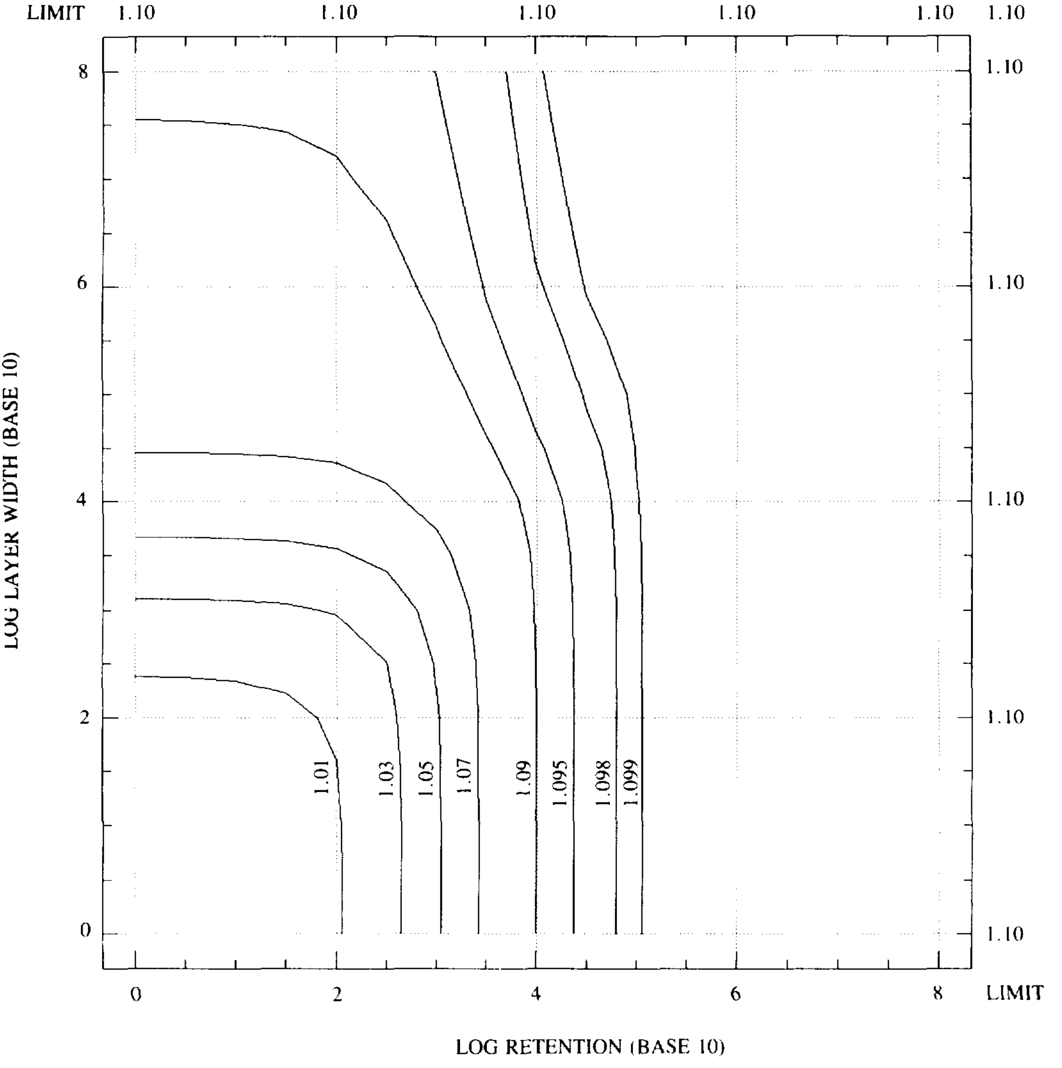
PARETO B = 10,000 Q = 1
EXCESS LAYER TREND FACTORS



10% UNIFORM TREND
TRENDED DISTRIBUTION: B = 10,000 × 1.10, Q = 1

EXHIBIT 2

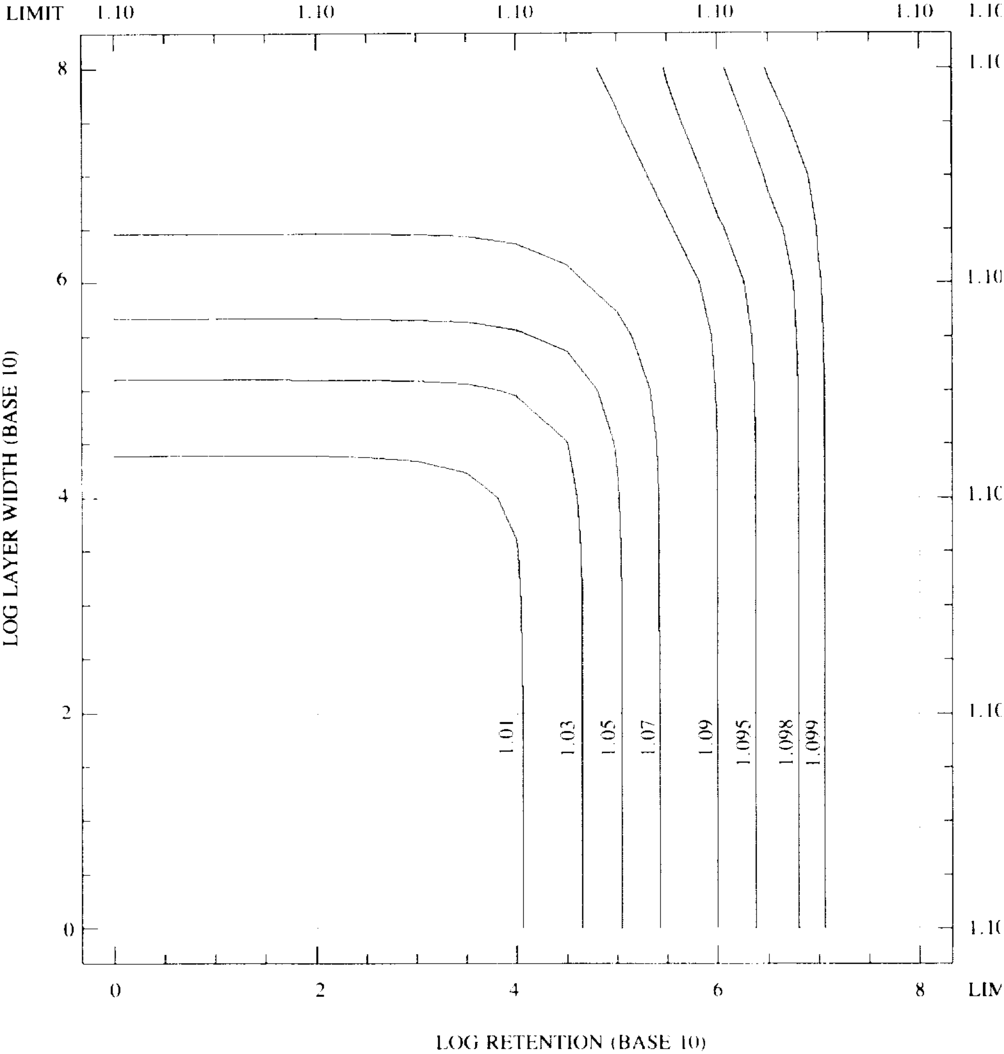
PARETO $B = 1{,}000$ $Q = 1$
EXCESS LAYER TREND FACTORS



10% UNIFORM TREND
TRENDED DISTRIBUTION: $B = 1{,}000 \times 1.10$, $Q = 1$

EXHIBIT 3

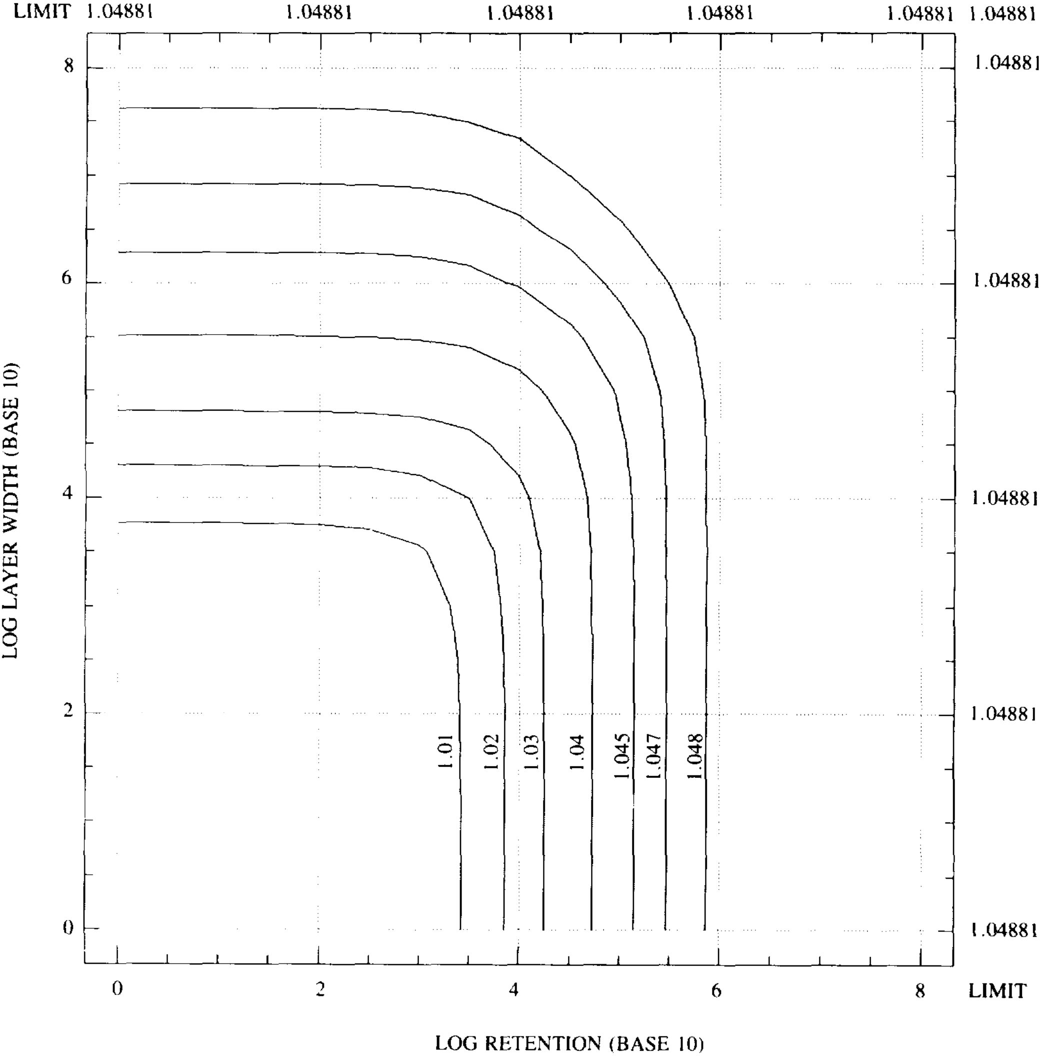
PARETO B = 100,000 Q = 1
EXCESS LAYER TREND FACTORS



10% UNIFORM TREND
TRENDED DISTRIBUTION: B = 100,000 × 1.10, Q = 1

EXHIBIT 4

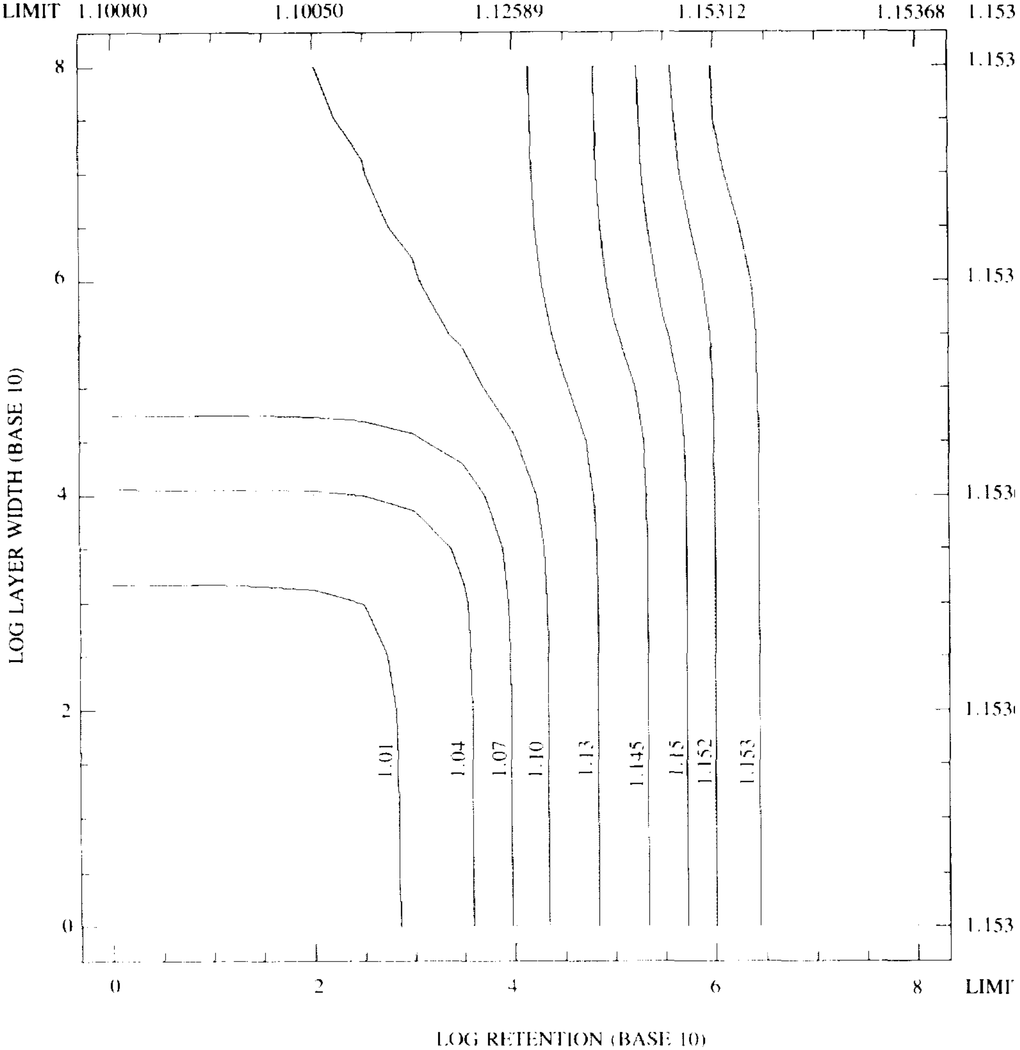
PARETO B = 10,000 Q = 0.5
EXCESS LAYER TREND FACTORS



10% UNIFORM TREND
TRENDED DISTRIBUTION: B = 10,000 × 1.10, Q = 0.5

EXHIBIT 5

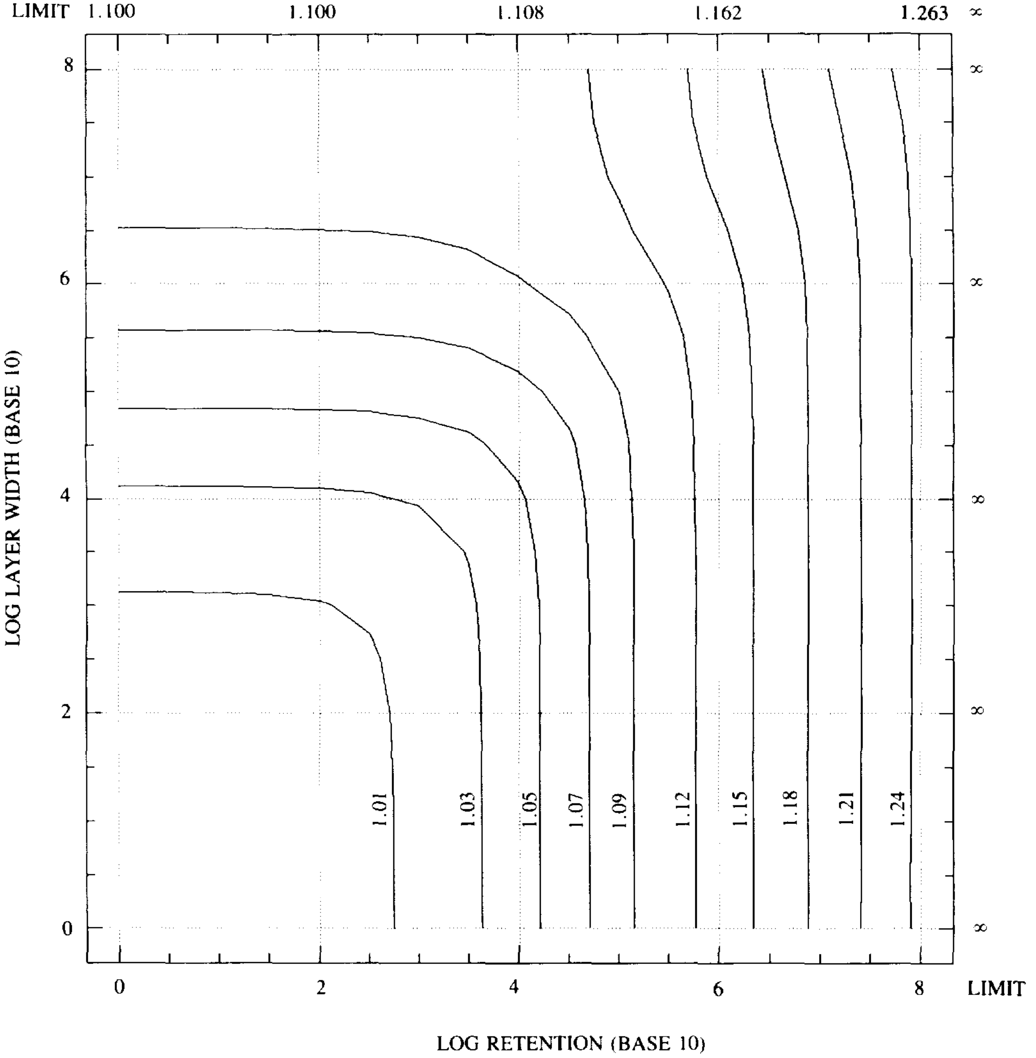
PARETO B = 10,000 Q = 1.5
EXCESS LAYER TREND FACTORS



10% UNIFORM TREND
TRENDED DISTRIBUTION: B = 10,000 × 1.10, Q = 1.5

EXHIBIT 6

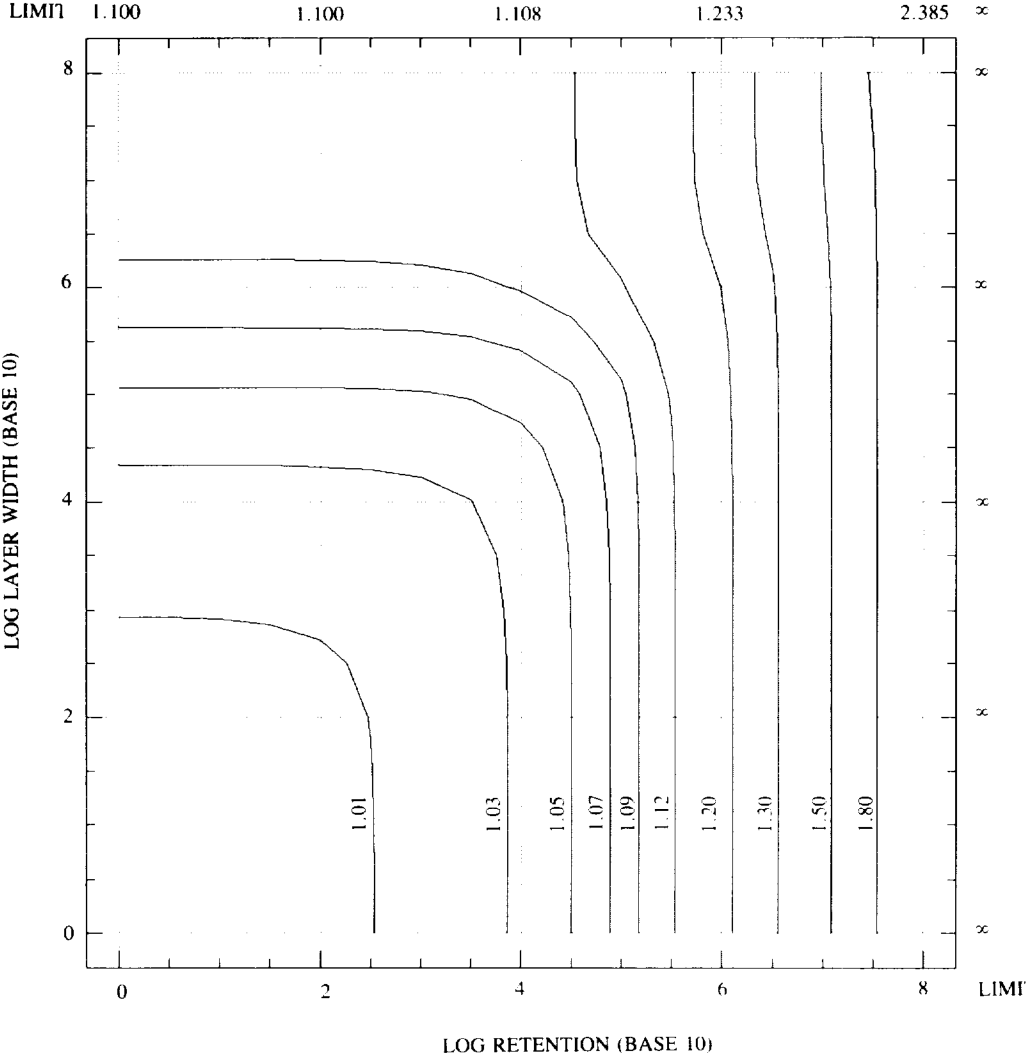
LOGNORMAL $\mu = 8.855$ $\sigma = 2.077$
EXCESS LAYER TREND FACTORS



10% UNIFORM TREND
TRENDED DISTRIBUTION: $\mu = 8.855 + \ln 1.10$, $\sigma = 2.077$

EXHIBIT 7

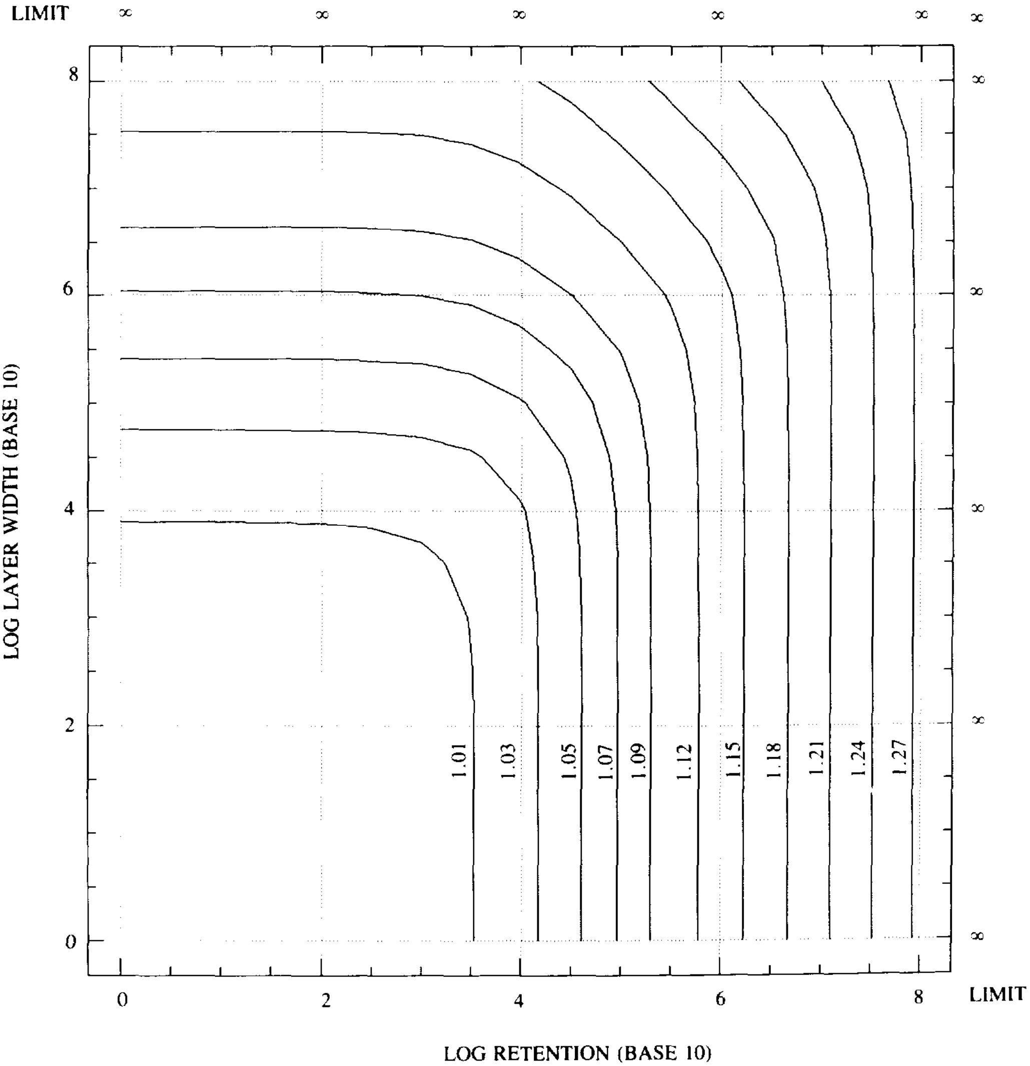
WEIBULL λ = .03818 τ = .3525
EXCESS LAYER TREND FACTORS



10% UNIFORM TREND
TRENDED DISTRIBUTION: $\lambda = .03818/(1.10)^{.3525}$, $\tau = .3525$

EXHIBIT 8

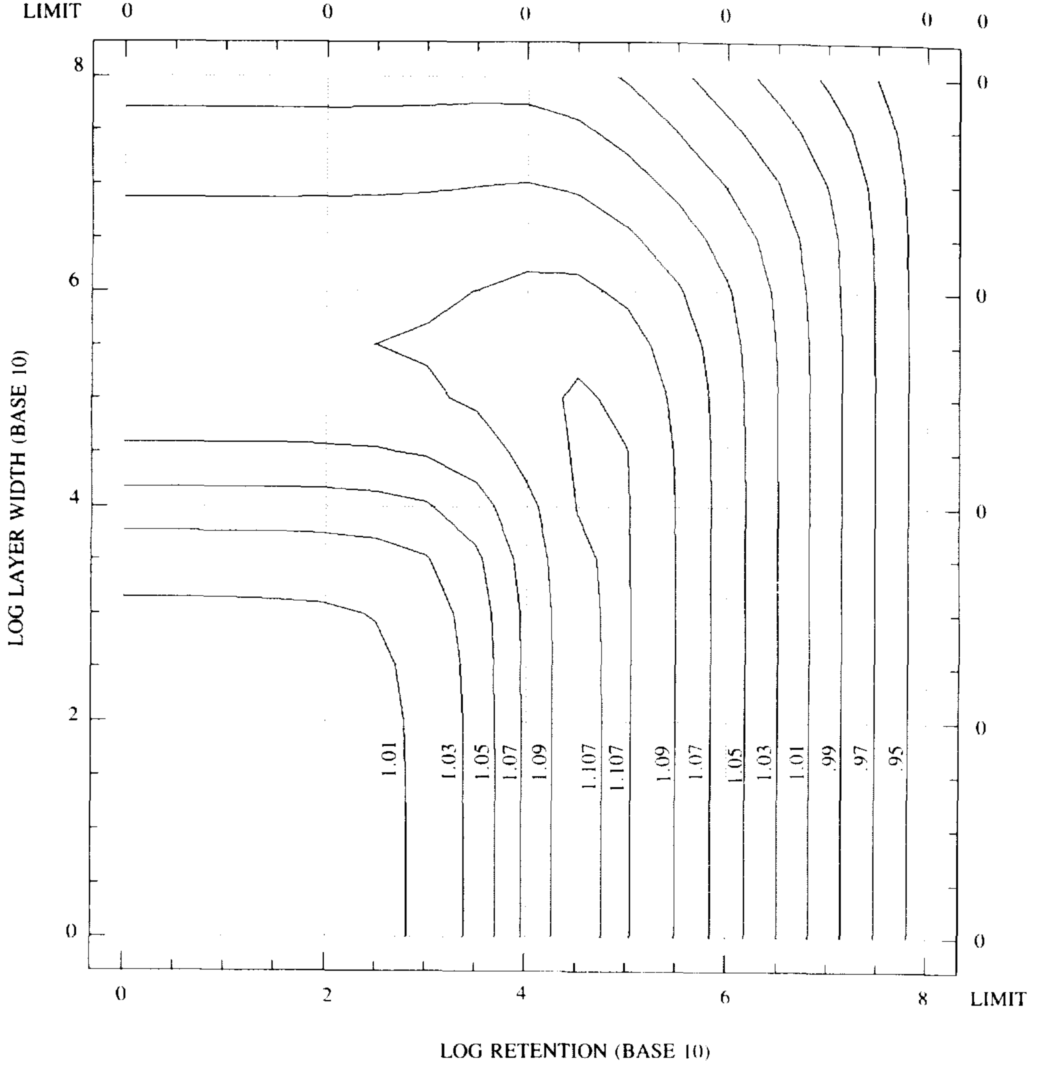
PARETO B = 10,000 Q = 1
EXCESS LAYER TREND FACTORS



INCREASING VARYING TREND: a = .8621, b = .02
TRENDED DISTRIBUTIONS: BURR $B = 10{,}000 \times (.8621)^{1/1.02}$, Q = 1, T = 1/1.02
matches first two moments of
PARETO B = 10,095, Q = .9746

EXHIBIT 9

PARETO B = 10,000 Q = 1
EXCESS LAYER TREND FACTORS



DECREASING VARYING TREND: a = 1.4023, b = −.02
TRENDED DISTRIBUTIONS: BURR $B = 10{,}000 \times (1.4023)^{1/.98}$, Q = 1, T = 1/.98
matches first two moments of
PARETO B = 11,995, Q = 1.0272

EXHIBIT 10

SUMMARY OF RESULTS

Exhibit	Change Made From Exhibit 1	Impact on Graph
2	Scale parameter decreased	Contour lines displaced toward origin
3	Scale parameter increased	Contour lines displaced away from origin
4	Shape parameter decreased	Trend factors decreased throughout
5	Shape parameter increased	Trend factors increased throughout
6	Lognormal used instead of Pareto	Trend factors increased in higher layers
7	Weibull used instead of Pareto	Trend factors increased in higher layers (more than with lognormal)
8	Increasing varying trend applied	Trend factors increase without bound (instead of toward a limit)
9	Decreasing varying trend applied	Trend factors initially increase, but then decrease toward zero

APPENDIX

The formulas which are used in generating the graphs are shown here.

$$G(x) = 1 - F(x)$$
$$R = \text{retention}$$
$$L = \text{limit (or layer width)}$$
$$S = R + L$$
$$\mathrm{E}\{h(X; R, L)\} = \text{expected layer loss}$$
$$\mathrm{E}(X; c) = \text{first moment of the distribution censored at } c$$
$$\mathrm{E}(X^2; c) = \text{second moment of the distribution censored at } c$$
$$\Gamma(\alpha) = \int_0^\infty y^{\alpha-1} e^{-y}\, dy$$
$$\Gamma(\alpha; k) = \frac{\int_0^k y^{\alpha-1} e^{-y}\, dy}{\Gamma(\alpha)} \text{ (where } k \text{ is a constant)}$$

The trend factor for any retention-layer width combination is computed by simply dividing the expected layer loss under the trended distribution by the expected layer loss under the original distribution.

I. PARETO

$$\mathrm{E}\{h(X; R, L)\} = \int_R^S G(x)\, dx = \int_R^S \left(\frac{B}{x + B}\right)^Q dx$$
$$= \frac{B^Q}{1 - Q} [(S + B)^{1-Q} - (R + B)^{1-Q}] \text{ if } Q \neq 1$$
$$= B \ln \left(\frac{S + B}{R + B}\right) \text{ if } Q = 1$$
$$\mathrm{E}(X; c) = \frac{B}{Q - 1} - Q \left(\frac{B}{c + B}\right)^Q \left[\frac{c + B}{Q - 1} - \frac{B}{Q}\right]$$
$$+ c \left(\frac{B}{c + B}\right)^Q \text{ if } Q \neq 1$$
$$= B \ln \left(\frac{c + B}{B}\right) \text{ if } Q = 1$$

$$\begin{aligned} E(X^2; c) = {} & \frac{2B^2}{(Q-1)(Q-2)} \\ & - Q\left(\frac{B}{c+B}\right)^Q \left[\frac{(c+B)^2}{Q-2} - \frac{2B(c+B)}{Q-1} + \frac{B^2}{Q}\right] \\ & + c^2\left(\frac{B}{c+B}\right)^Q \text{ if } Q \neq 1,2 \\ = {} & 2B\left[c - B \ln\left(\frac{c+B}{B}\right)\right] \text{ if } Q = 1 \end{aligned}$$

See Patrik [10] for a derivation of the moments of the Pareto.

II. LOGNORMAL

$$\begin{aligned} E\{h(X; R, L)\} = {} & \int_R^S x\, dF(x) + S\, G(S) - R\, G(R) \\ = {} & \left\{\int_0^S x\, dF(x) + S\, G(S)\right\} - \left\{\int_0^R x\, dF(x) + R\, G(R)\right\} \\ = {} & \left\{e^{\mu+\frac{\sigma^2}{2}} \Phi\left(-\sigma + \frac{\ln S - \mu}{\sigma}\right) + S\left[1 - \Phi\left(\frac{\ln S - \mu}{\sigma}\right)\right]\right\} \\ & - \left\{e^{\mu+\frac{\sigma^2}{2}} \Phi\left(-\sigma + \frac{\ln R - \mu}{\sigma}\right) + R\left[1 - \Phi\left(\frac{\ln R - \mu}{\sigma}\right)\right]\right\} \end{aligned}$$

$$E(X; c) = e^{\mu+\frac{\sigma^2}{2}} \Phi\left(-\sigma + \frac{\ln c - \mu}{\sigma}\right) + c\left[1 - \Phi\left(\frac{\ln c - \mu}{\sigma}\right)\right]$$

$$E(X^2; c) = e^{2\mu+2\sigma^2} \Phi\left(-2\sigma + \frac{\ln c - \mu}{\sigma}\right) + c^2\left[1 - \Phi\left(\frac{\ln c - \mu}{\sigma}\right)\right]$$

See Miccolis [9] for a derivation of $\int_0^k x\, dF(x)$ and $\int_0^k x^2\, dF(x)$ (where k is a constant).

III. WEIBULL

$$E\{h(X; R, L)\} = \int_R^S G(x)\, dx = \int_R^S e^{-\lambda x^\tau}\, dx$$

$$= \Gamma\left(1 + \frac{1}{\tau}\right) \frac{[\Gamma(1/\tau;\, \lambda S^{\tau}) - \Gamma(1/\tau;\, \lambda R^{\tau})]}{\lambda^{1/\tau}}$$

$$E(X;\, c) = \Gamma\left(1 + \frac{1}{\tau}\right) \frac{\Gamma(1 + 1/\tau;\, \lambda c^{\tau})}{\lambda^{1/\tau}} + c\, e^{\lambda c^{\tau}}$$

$$E(X^2;\, c) = \Gamma\left(1 + \frac{2}{\tau}\right) \frac{\Gamma(1 + 2/\tau;\, \lambda c^{\tau})}{\lambda^{2/\tau}} + c^2\, e^{-\lambda c^{\tau}}$$

See the appendix of Hogg and Klugman [5] for an illustration of the techniques used in these derivations.

IV. BURR

$$G(x) = \left(\frac{B}{B + x^T}\right)^Q$$

$$E(X;\, c) = Q\, B^{1/T} \int_{B/(B+c^T)}^{1} (1 - y)^{1/T}\, y^{Q - 1/T - 1}\, dy + c \left(\frac{B}{B + c^T}\right)^Q$$

$$E(X^2;\, c) = Q\, B^{2/T} \int_{B/(B+c^T)}^{1} (1 - y)^{2/T}\, y^{Q - 2/T - 1}\, dy + c^2 \left(\frac{B}{B + c^T}\right)^Q$$

See the appendix of Hogg and Klugman [5] for an illustration of the techniques used in these derivations.