A NOTE ON THE GAP BETWEEN TARGET AND EXPECTED UNDERWRITING PROFIT MARGINS

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Abstract

Profit margins experienced by insurance companies are, on average, considerably lower than the "target" margins used to compute the premiums. The difference has been attributed to a variety of factors, ranging from errors in actuarial projections, to regulatory delays, to regulatory and competitive pressures. This note examines the potential impact of the procedure used to "mark up" the projected cost per policy on the gap between two quantities, the intended or "target" margin and the expected value of the realized profit margin.

The analysis shows that the practice of dividing the expected loss cost by a "permissible loss ratio" computed by deducting the anticipated expenses and a profit provision from unity will produce an expected underwriting profit margin that is, on average, lower than that built into the rates.

1. INTRODUCTION

A stylized view of actuarial ratemaking involves a provision for profit. Mathematically, the provision is made by dividing the expected cost of servicing an insurance contract by a number which represents unity minus the "target" profit margin.[1] The results of this computation may be used directly in the market, as in situations in which rates are promulgated by a department or bureau, or may be merely estimates of the marginal cost of providing insurance which guide management in its pricing policies. In either event, the result is a key input into the pricing decision.

Over extended periods of time in most jurisdictions, the average underwriting profit margins achieved by the industry as a whole, or by individual firms, differ substantially from the targets ostensibly built into the rates. This "gap" has been

[1] In practice the numerator may include the costs of losses and loss adjustment services and the denominator reflects other anticipated expenses (such as commissions, administrative expenses, and premium taxes) as well as the target profit margin.

emphasized by various authors [2,3,4]. One view is that the gap exists because the target profit built into the rates is excessive [2]. In this view, the gap represents the difference between improperly regulated prices and prices that would hold in a competitive economic system. Others attribute the gap to difficulties in ratemaking and to "cutbacks and delays in implementing rate increases" [3]. Still others remark on the existence and importance of these gaps but don't provide a rationale for their existence [4].

This note analyzes the gap in terms of the stylized procedure described above. The emphasis is on the difference between the underwriting margin that is incorporated into the ratemaking formula, here called the "target" margin, and the margin that would be expected on a statistical basis from the direct use of the formula. The analysis shows that the procedure used in developing premiums from actuarial projections is responsible for at least part of the difference between the target and expected profit margins, even if the projections used in making rates are unbiased.

2. A STYLIZED MODEL OF RATEMAKING AND PROFIT DETERMINATION

For our purposes, a very simple stylized model of actuarial ratemaking is adequate. The simple model presented here would be applicable directly to state-mandated rates or to bureau rates with no deviations. Trivial extensions would be needed for situations in which uniform deviations from promulgated rates are permissible. The analysis would also be applicable to rates developed through management discretion as long as the actuarial projections of needed rates are a major determinant of the rates ultimately adopted by management. We view ratemaking as consisting of the following steps:

1. The forecast cost per policy, F, is developed from past data.
2. The target margin, T, is determined.
3. The price, P, is calculated from

$$P = \frac{F}{1 - T} . \tag{2.1}$$

Policies are sold at this price and will, eventually, prove to involve a cost per policy of C. The underwriting profit per policy during the period in question will be the difference between the revenue, P, and the cost, C. The underwriting profit margin during the period will, accordingly, be

$$m = \frac{P - C}{P} . \tag{2.2}$$

Combining these equations, the underwriting profit margin may be expressed as

$$m = 1 - \frac{C}{F}(1 - T). \tag{2.3}$$

Denoting the true expected value of the cost per policy as C_T, the observed cost per policy will be a random variable whose expected value is C_T. Accordingly, we write

$$C = C_T(1 + y), \tag{2.4}$$

where y is a random variable. By definition the expected value of y is zero.

From these equations it follows that the achieved underwriting margin can be expressed as

$$A = \mathrm{E}(m) = 1 - (1 - T)\,\mathrm{E}\left(\frac{C}{F}\right) \tag{2.5}$$

$$= 1 - (1 - T)\,\mathrm{E}\left(\frac{C_T(1 + y)}{F}\right). \tag{2.6}$$

If we were to view the value of F as being identical to C_T, then we could use equation 2.6. In view of the fact that the expected value of y is zero, we would then conclude that the achieved margin is the same as the target margin. This appears to be the origin of the conventional wisdom that the two are equal in the absence of effects such as competition or regulatory lags. The value of F is, however, a forecast rather than the true value of the cost per policy. It is, accordingly, a random variable whose value depends on the unobservable value of the true cost. Assuming that F is fixed is tantamount to assuming that the actual cost per policy will tend to cluster around the forecast rather than around its true expected value.

In order to recognize the effect of forecast errors, we denote the forecast cost per policy as:

$$F = C_T(1 + x), \tag{2.7}$$

where x is a random variable measuring the prediction error. Since the premium is always greater than zero we can guarantee that $x > -1$. The expected value of x will be zero if the estimators used in ratemaking are unbiased, but this is not assured. We assume that the values of x and y are independent unless the errors in the forecast affect actual experience.[2]

Recognizing the random elements in the forecast we must write

$$A = \mathrm{E}(m) = 1 - (1 - T)\,\mathrm{E}\left(\frac{1 + y}{1 + x}\right). \tag{2.8}$$

In view of the independence of x and y, this can be simplified to

$$= 1 - (1 - T)\,\mathrm{E}(1 + y)\,\mathrm{E}\left(\frac{1}{1 + x}\right), \tag{2.9}$$

and since the expected value of y is zero

$$= 1 - (1 - T)\,\mathrm{E}\left(\frac{1}{1 + x}\right). \tag{2.10}$$

This can be written in a more suggestive form as

$$A = 1 - \frac{(1 - T)}{\mathrm{E}(1 + x)} - (1 - T)\left[\mathrm{E}\left(\frac{1}{1 + x}\right) - \frac{1}{\mathrm{E}(1 + x)}\right]. \tag{2.11}$$

If the actuarial estimates are unbiased, as they strive to be, the expected value of $1 + x$ will be one. The first two terms on the right hand side will, accordingly, be equal to T. The quantity in brackets in the third term will always be positive due to the fact that the harmonic mean[3] of a positive variable (in this case, $1 + x$) is always less than the arithmetic mean [1]. Since $1 - T$ is also positive and the third term has a negative sign, it follows that in general the expected value of the underwriting margin will be less than the target margin.

3. AN EXAMPLE

A concrete example may serve to illustrate the relationship. Let us consider the situation when the logarithm of variable $1 + x$ is normally distributed with mean m and standard deviation s. This is realistic in that it corresponds to a

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[2] Dependence can arise in a number of ways. Two deserve mention: the self-selection of the purchasers of insurance in response to changing effective prices and the easing or tightening claims settlement practices by management as profit margins change.

[3] The harmonic mean is defined as the reciprocal of $\mathrm{E}(1 + x)^{-1}$

situation in which rates are always positive and have lognormally distributed errors. In this case M_n, the expected value of $(1 + x)^n$, is given by

$$M_n = \exp(nm + \tfrac{1}{2}n^2s^2) \tag{3.1}$$

for any value of n.

Since the rate estimator is presumed to be unbiased, the expected value of $1 + x$ must be one. This requires that M_1 be one and, accordingly, that $m = -\tfrac{1}{2}s^2$.

Imposing this condition and obtaining M_{-1} from Equation 3.1, we find that Equation 2.11 may be written as

$$A = T - (1 - T)\,[\exp(s^2) - 1]\,. \tag{3.2}$$

It is worth noting that the factor multiplied by $1 - T$ is the variance of the relative error in the forecast. Thus, if the standard deviation of the relative forecast error is 10 percent, the bias in the underwriting profit margin will be very close to one percentage point. If the standard deviation of the relative forecast error were as high as 30 percent, the bias would be nine percentage points.

4. CONCLUSION

When premiums are set by marking up unbiased predictions of cost per policy by dividing them by one minus a target margin, it can be guaranteed that there will be a gap between the "target" and the "expected" underwriting profit margin. Mathematically, the gap is generated by the difference between the expected value of the reciprocal of a random variable and the reciprocal of the expected value of the variable. If projected loss ratios estimate the "true" expected loss ratios at current rates but are subject to random error, the same results apply when the premiums are derived by dividing the projected loss ratio at current rates by a "permissible loss ratio" that incorporates a target provision for underwriting profit.

This paper does not present estimates of the magnitude of the effect. Direct estimates of the difference could be calculated if there were records of the actual forecasts that could be compared with realized values. That data is not generally available. Even with that data, additional assumptions would be required in order to develop exact estimates. The example provided illustrates that the gap may be large. Extensive simulation based on distributions other than the lognormal and approximations based on publicly available data indicate that for workers' compensation insurance, the difference between intended and achieved margins attributable solely to the effect described in this paper is larger than one percentage point in most states, and may well reach five percentage points.

While errors of this magnitude are not uncommon, it must be remembered that this is a systematic, not a random effect. It is also important to keep in mind that the regulatory process and the rigors of competition may well result in estimators that are biased downward. In fact, if the estimates given above are correct, then for workers' compensation insurance, the effect of biased estimators may be three to four times larger than the statistical gap described in this note. Attempts to collect better data and refine the estimation procedure are in progress.

REFERENCES

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