DISCUSSION BY DANIEL F. GOGOL

Mr. McClenahan presents a method that can be very useful when there has been a recent, significant change in exposure. The average accident date in the most recent accident year may be considerably different from what it was for previous accident years at the same stage of development. Both loss reserving and pricing decisions can benefit greatly from an accurate estimate of the effect of changing exposure on loss development patterns. For a method that is so simple, the one presented by Mr. McClenahan seems to apply very well to a fairly large portion of the exposure and development patterns encountered in practice.

The mathematical derivation of the method applies to a development pattern that has the property that for some $a < 1$, the observed losses at accident age x are $1 - a^x$ of ultimate. Actual development patterns sometimes poorly fit curves of this form. Exposure changes during an accident year are represented in the paper by the function $(1 + g)^x$, and it also may poorly fit actual patterns.

Another problem is the following. Mr. McClenahan defines the observed proportion of ultimate losses at accident year age i, if exposure growth is at a rate of $100g\%$ per annum, by:

$$L_i^g = \int_{i-1}^{i} (1 + g)^{i-x}(1 - a^x)dx \qquad i \geq 1$$

If $1 - a^x$ is the proportion of ultimate losses at accident age x (not accident year age x), then $L_i^g \div \int_{i-1}^{i} (1 + g)^{i-x}dx$ would be the proportion of ultimate losses at accident year age i. The divisor was omitted from Mr. McClenahan's expression. This does not affect the development factors since they are of the form $L_i^g \div L_{i-1}^g$ and the factor $\int_{i-1}^{i} (1 + g)^{i-x}dx$ cancels out. But the curve $1 - a^x$ should represent the proportion of ultimate losses at accident age x, and the curve that is calculated in Appendix B represents the proportion at accident year age x instead. The proportion at accident age x is closer to the proportion at accident year age $x + .5$, since the average accident is approximately one-half year old at the end of accident year age 1. In order to produce a curve, $1 - a^x$, that would be a good fit for the recent accident years, which are generally the most important in loss reserving, it would probably be better to use a much smaller value for x than 6, which is the value Mr. McClenahan uses in Appendix B.

A method will be presented that avoids the necessity of assuming that:

1. The development pattern, from accident age zero to ultimate, can be satisfactorily fitted by a curve of the form $1 - a^x$.
2. The pattern of change in exposure during the accident year can be satisfactorily fitted by a curve of the form $(1 + g)^x$.

Development patterns for accident quarters or accident months are a by-product of the method to be presented. These development patterns can be helpful in projecting losses during an accident year.

AN ALTERNATIVE METHOD

Let x_j represent the portion of an accident quarter's ultimate losses that has been reported as of j quarters after it begins. (The algebra that follows would apply equally well if the word "losses" above was replaced throughout by "number of claims," if "reported" was replaced by "paid," or if "quarter" was replaced by "month.")

If e_i is the portion of an accident year's ultimate losses that is ultimately produced by accidents in the i^{th} quarter, and A_k is the portion of an accident year's ultimate losses that has been reported as of k quarters, then the following series of equations is satisfied:

$$e_1x_4 + e_2x_3 + e_3x_2 + e_4x_1 = A_4$$
$$e_1x_5 + e_2x_4 + e_3x_3 + e_4x_2 = A_5$$
$$\cdots\cdots\cdots\cdots\cdots\cdots$$

No matter how many of these equations are listed, there are more unknowns than equations. However, if we assume that, for some n, $x_j = 1$ for $j > n$, then the equations

$$e_1x_4 + e_2x_3 + e_3x_2 + e_4x_1 = A_4$$
$$e_1x_5 + e_2x_4 + e_3x_3 + e_4x_2 = A_5$$
$$\cdots\cdots\cdots\cdots\cdots\cdots$$
$$e_1x_{n+3} + e_2x_{n+2} + e_3x_{n+1} + e_4x_n = A_{n+3}$$

can be used to solve for $x_n, x_{n-1}, \ldots, x_1$, respectively, by the use of the formula

$$x_j = (A_{j+3} - e_1x_{j+3} - e_2x_{j+2} - e_3x_{j+1}) \div e_4 \qquad 1 \leq j \leq n$$

The numbers e_i can be chosen to reflect changes in exposure, frequency (e.g., seasonal changes), and severity (e.g., due to claim cost inflation) that are estimated to be representative of the loss development data. The numbers A_k can be based on the loss development data and whatever curve fitting seems appropriate. The numbers x_j that are derived from the e_i's and A_k's can then be used to produce yearly development patterns resulting from a different pattern of change in exposure, frequency, and severity (i.e., a different e_1, e_2, e_3, e_4). By subdividing the year into quarters or months, the problem of variable expected losses between quarters or months is dealt with, but not the variability during quarters or months. However, the overall variability is decreased by subdividing. Two different patterns of exposure during a year can theoretically cause a difference of almost twelve months between the expected average accident dates. But, if the two patterns have the same total amount of exposure during each quarter, or during each month, then the difference between the expected average accident dates must be less than three months, or less than one month, respectively.

In order to use the method presented, it is necessary to choose some n such that $A_k = 1$ for $k > n$. This can be done for some n that is not so large as to be impractical. Some adjustment to the actual estimates of some of the later A_k may be necessary, but it does not have to significantly affect the early development factors derived from the method. These early factors are the ones that are most significantly affected by changes in exposure during an accident year.

Example

Suppose that an insurance company has started writing a new line of business and that the line's estimated ultimate losses for the year's accident quarters are .05, .12, .27, and .56, respectively, of the estimated ultimate losses for the accident year. Suppose reported loss development factors at the end of the year are based on industry-wide data for the line, and that the estimated average industry losses for the four quarters of the accident years on which the data is based are .238, .246, .254, and .262, respectively, of the estimated average accident year losses. Also, assume that the following smoothed progression is selected as a good fit to the industry data: $A_4 = .662$, $A_5 = .832$, $A_6 = .935$, $A_7 = .987$, $A_8 = 1.000$.

Since $A_8 = 1.000$, it is assumed that $x_j = 1.000$ for $j \geq 5$. Therefore, the equations

$$\begin{aligned}
.238\,x_4 + .246\,x_3 + .254\,x_2 + .262\,x_1 &= .662\\
.238\,x_5 + .246\,x_4 + .254\,x_3 + .262\,x_2 &= .832\\
.238\,x_6 + .246\,x_5 + .254\,x_4 + .262\,x_3 &= .935\\
.238\,x_7 + .246\,x_6 + .254\,x_5 + .262\,x_4 &= .987
\end{aligned}$$

can be solved, giving $x_1 = .330$, $x_2 = .600$, $x_3 = .800$, $x_4 = .950$. So the portion of ultimate accident year losses for the company's new line of business that is reported as of the end of the year is estimate by

$$.05(.95) + .12(.80) + .27(.60) + .56(.33) = .490.$$

So the development factor to ultimate for the company's new line is estimated as 2.041 (i.e., 1/.490) as compared to 1.511 from the industry data.