ADJUSTING LOSS DEVELOPMENT PATTERNS FOR GROWTH

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Abstract

This paper examines the impact of changes in exposure growth on loss development patterns. An adjustment methodology for use in cases where growth patterns have changed materially during the observation period is proposed and an example is presented.

1. INTRODUCTION

The vast majority of pricing and reserving analysis performed by casualty actuaries is based, at least in part, upon the construction of loss development triangles and the projection of "loss development factors" (or "link ratios"). Where these factors are based upon historical development patterns there is an underlying, and generally unstated, assumption that each historical exposure period at a given point of development represents a body of claim experience at a consistent average age. In practice, the average age of the exposure period may change over time as a result of variations in inflation, settlement practices, reporting patterns, and exposure growth. The purpose of this short paper is to examine the impact of exposure growth changes upon the development patterns and to propose a method for the adjustment of historical patterns where such impact is material.

While this paper deals with the impact of exposure growth upon the loss development patterns, an earlier paper by LeRoy J. Simon [1] deals with the specific impact of such growth patterns upon exposure-based IBNR factors.

2. GROWTH AND DEVELOPMENT PATTERNS

In order to understand the relationship between exposure growth and loss development, let us look at a highly simplified development pattern. We will assume that losses only occur on the first day of a month and are always reported on the first day of the month immediately following occurrence. Each claim has an associated indemnity benefit of $300 with $100 being paid on the first day of each of the three months immediately following reporting. Case reserves are assumed to be exactly adequate on an undiscounted basis. The following example will summarize the assumed pattern for a single claim occurring on 7/1/86:

Date	Cumulative Reported	Cumulative Paid	Case Reserve
7/1/86	$ 0	$ 0	$ 0
8/1/86	300	0	300
9/1/86	300	100	200
10/1/86	300	200	100
11/1/86	300	300	0

Let us now look at three companies, each having 156 claims occurring during accident year 1986. Company A has increasing exposure, and therefore increasing monthly claims. Company B has stable exposure and Company C has declining exposure. The assumed claim counts are as follows:

Accident Date	Company A	Company B	Company C
1/1/86	2	13	24
2/1/86	4	13	22
3/1/86	6	13	20
4/1/86	8	13	18
5/1/86	10	13	16
6/1/86	12	13	14
7/1/86	14	13	12
8/1/86	16	13	10
9/1/86	18	13	8
10/1/86	20	13	6
11/1/86	22	13	4
12/1/86	24	13	2
Total	156	156	156

For accident year 1986, the three companies have the following situations as of 12/31/86:

	Company A	Company B	Company C
Paid Loss	$27,200	$35,100	$43,000
Case Reserve	12,400	7,800	3,200
Case Incurred	39,600	42,900	46,200
IBNR	7,200	3,900	600
Ultimate Loss	46,800	46,800	46,800
Ultimate/Paid	1.721	1.333	1.088
Ultimate/Case Inc.	1.182	1.091	1.013

In practice, of course, the ultimate values will not be known with certainty at 12/31/86. For the sake of illustration we are assuming perfect knowledge.

Here we have three hypothetical companies writing the same line of business with identical accident year claim counts and very different accident year development patterns. The differences, of course, arise from the varying distributions of the claims in time over the accident year. The average age of claim at 12/31/86 is 4.67 months for Company A, 6.50 months for Company B, and 8.33 months for Company C. Inasmuch as claims growth can be generally expected to reflect exposure growth, the exposure growth pattern can be seen to have a potentially significant impact upon the loss development pattern.

This relationship between exposure growth and development pattern is not, in and of itself, a problem. Should either Company A or Company B continue to experience consistent exposure patterns, the indicated loss development patterns would produce reliable estimates for unpaid and for unreported losses. When exposure growth is inconsistent, however, an adjustment to historical indications may be warranted.

3. HYPOTHETICAL CASE STUDY

Appendix A contains the assumptions and data underlying a somewhat more complex example for a hypothetical company. A totally fictitious reporting pattern has been assumed along with uniform exponential pure premium trend. The exposure growth assumption is a period of uniform positive growth followed by a period of declining growth with the final exposure growth rate being negative. The observed loss development factors are as follows:

Accident Year	Age-to-Age Factors (Age in Years) 1-2	2-3	3-4
1983	1.8699	1.1144	1.0009
1984	1.8697	1.1143	
1985	1.8537		
Weighted Average	1.8635	1.1144	1.0009
To Ultimate:	2.0785	1.1154	1.0009

Using ultimate factors based upon observed weighted averages:

Accident Year	Reported 12/31/86	Ultimate Factor	Projected Ultimate	"Actual" Ultimate
1984	$1,469,650	1.0009	$1,470,973	$1,470,979
1985	1,542,366	1.1154	1,720,355	1,718,089
1986	875,722	2.0785	1,820,188	1,755,193

While it may be argued that the use of the weighted average factors is inappropriate in light of the observed "trend" in the 1-2 factors, it is unlikely that the selected factor for 1-2 would have been as low as the 1.7971 required to generate the "actual" ultimate value had the "trend" been projected to continue. Comparing the projected and "actual" IBNR needs:

Accident Year	Projected IBNR	"Actual" IBNR	Percent Error
1984	$ 1,323	$ 1,329	−0.5%
1985	177,989	175,723	1.3
1986	944,466	879,471	7.4
Total	$1,123,778	$1,056,523	6.4%

Since we have used a consistent monthly reporting pattern along with constant pure premium change, the error in projection, other than rounding error, is due entirely to our inability to accurately reflect the impact of the varying rate of exposure growth on the development pattern.

4. PROPOSED ADJUSTMENT TO DEVELOPMENT FACTORS

Assume that in a growth-free environment, observed losses at accident year age x are $1 - a^x$ of ultimate. (Note that if a is replaced with $e^{-\alpha}$ this becomes $1 - e^{-\alpha x}$, the standard single parameter exponential decay function. While the author does not contend that any single parameter function can be expected to provide a good fit to an entire development pattern, the assumption is sufficiently reasonable for use in calculating adjustment factors within the context of this paper. Appendix B contains information relating to the indicated values of a for various industry data.)

Further assume that exposure growth is at a rate of $100g\%$ per annum. Let us now define L_i^g to be the observed proportion of ultimate losses at accident year age i:

$$L_i^g = \int_{i-1}^{i} (1 + g)^{i-x}(1 - a^x)dx \qquad i \geq 1$$

$$= \frac{g}{\ln(1 + g)} + \frac{a^{i-1}(1 + g - a)}{\ln(a) - \ln(1 + g)} \qquad i \geq 1;\ g \neq 0 \tag{4.1}$$

If we now define the age-to-age development factor from age $i - 1$ to i as ${}_{i-1}F_i^g$:

$${}_{i-1}F_i^g = \frac{L_i^g}{L_{i-1}^g} \qquad i \geq 2;\ g \neq 0$$

$$= \frac{g\{\ln[(1 + g)/a]\} + \ln(1 + g)\{1 - [(1 + g)/a]\}a^i}{g\{\ln[(1 + g)/a]\} + \ln(1 + g)\{1 - [(1 + g)/a]\}a^{i-1}} \qquad i \geq 2;\ g \neq 0$$

Or, letting $c = g\{\ln[(1 + g)/a]\}$ and $b = -\ln(1 + g)\{1 - [(1 + g)/a]\}$,

$${}_{i-1}F_i^g = \frac{c - ba^i}{c - ba^{i-1}} \tag{4.2}$$

In the special case where $g = 0$:

$$L_i^0 = 1 + \frac{a^{i-1}(1 - a)}{\ln(a)} \qquad i \geq 1$$

$${}_{i-1}F_i^0 = \frac{\ln(a) + a^{i-1}(1 - a)}{\ln(a) + a^{i-2}(1 - a)} \qquad i \geq 2 \tag{4.3}$$

It is proposed that, where growth has been erratic, an attempt be made to estimate the value of a and that historical development patterns be adjusted to a growth-free basis. After selection of factors, growth would be re-introduced into the projected ultimates.

5. EXAMPLE OF PROCESS

Going back to the hypothetical case outlined in Appendix A, the first requirement is an estimate of the parameter a. Looking at the 1983 accident year, we note that at accident year age 1, .479 (589,380/1,229,203) of "ultimate" losses were observed. Using 1/83 to 1/84 earned exposure growth, the observed growth rate was .127 [(1,062/942) − 1]. Setting (4.1) equal to .479, and substituting .127 for g yields an estimate for a of .251. (Of course, we don't know the true ultimate losses in actual practice. The goal here is to attempt, by the best means available, to estimate the parameter a. By using a reasonably well-developed year, or group of years if available, where exposure growth is known or can be reasonably estimated, an approximate value for a can be derived.) Using (4.2) we can now generate the following:

Accident Year	a	g	b	c
1983	.251	.127	.417	.191
1984	.251	.126	.414	.189
1985	.251	.060	.188	.086
1986	.251	−.138	−.361	−.170

Accident Year	Theoretical Development Factors		
	1-2	2-3	3-4
1983	1.908	1.119	1.027
1984	1.915	1.120	1.027
1985	1.911	1.120	1.027
1986	1.855	1.116	1.026

Note that the growth factors (g) for 1984 through 1986 are based upon the December-to-December growth from Appendix A.

Application of (4.3) provides the following "growth-free" factors:

1-2	2-3	3-4
1.886	1.118	1.026

The following factors adjust to a "growth-free" basis:

Accident Year	1-2	2-3	3-4
1983	.988	.998	1.000
1984	.985	.998	
1985	.987		

The following factors adjust back to a "growth-inclusive" basis:

Accident Year	1-2	2-3	3-4
1984			1.000
1985		1.002	1.000
1986	.984	.998	1.000

Next we adjust the observed development factors to a "growth-free" basis and project the remainder of the development to ultimate (brackets indicate projected factors). In this example the projection is assumed to be the beginning-incurred-weighted "growth-free" factor:

Accident	Growth-Free Development Factors		
Year	1-2	2-3	3-4
1983	1.8475	1.1133	1.0009
1984	1.8417	1.1121	[1.0009]
1985	1.8296	[1.1126]	[1.0009]
1986	[1.8385]	[1.1126]	[1.0009]
Weighted Average	1.8385	1.1126	1.0009

Now we readjust the projected "growth-free" factors back to a "growth-inclusive" basis:

Accident Year	1-2	2-3	3-4	To Ultimate
1984			[1.0009]	[1.0009]
1985		[1.1148]	[1.0009]	[1.1158]
1986	[1.8072]	[1.1104]	[1.0009]	[2.0085]

Finally, we calculate the adjusted projected ultimate losses:

Accident Year	Reported 12/31/86	Ultimate Factor	Projected Ultimate
1984	$1,469,650	1.0009	$1,470,973
1985	1,542,366	1.1158	1,720,972
1986	875,722	2.0085	1,758,888
Total	$3,887,738		$4,950,833

Looking at the efficacy of the projections:

Accident Year	Adjusted IBNR	Actual IBNR	Percent Error
1984	$ 1,323	$ 1,329	−0.5%
1985	178,606	175,723	1.6
1986	883,166	879,471	0.4
Total	$1,063,095	$1,056,523	0.6%

Obviously this represents an improvement over the unadjusted error of 6.4%.

6. WHEN TO USE ADJUSTMENT PROCESS

The reader will have noted that where changes in growth are small or where development factors are close to unity there is little impact of the adjustment process. In order to help the user decide when it may be appropriate to utilize the proposed adjustment process, Appendix C contains "growth-free" adjustment factors for various values of a and g. Note how insensitive the factors are to the underlying value of a. In order to use this table, the appropriate factor for the "old" growth rate should be divided by the factor for the "new" growth rate. The resultant factor represents the approximate impact on the unadjusted age-to-age factor. For example:

Auto Liability-Paid Loss Development ($a = .600$)
Observed 1-2 Factor = 2.100
Growth Underlying Observation = +15% Per Year
Current Exposure Growth Rate = −5% Per Year
Approximate 1-2 Factor = 2.100 (.984/1.006) = 2.054

7. CONCLUSION

This method is intended to produce appropriate adjustments to indicated loss development factors in situations where there have been material changes in exposure growth patterns. While frequency and severity changes can produce variations in development patterns as well, this method does not address those situations. Where frequency and/or severity changes are observed concurrently with exposure growth changes, this method can be used to eliminate the impact of the exposure growth changes in order to facilitate the analysis of frequency and severity.

In most cases, exposure growth will have been sufficiently consistent to obviate the need for the approach outlined in this paper. For new lines of business or where rapid growth or withdrawal occur, however, this approach provides a relatively simple and efficacious basis for improving estimates of ultimate losses.

REFERENCE

[1] LeRoy J. Simon, "Distortion in IBNR Factors," *PCAS* LVII, 1970, p. 64.

APPENDIX A
HYPOTHETICAL REPORTED LOSS DEVELOPMENT

Assume the following loss reporting pattern (ages in months):

Age	Incremental Reports	Cumulative Reports
1	5.0%	5.0%
2	5.0	10.0
3	15.0	25.0
4	10.0	35.0
5	10.0	45.0
6	7.5	52.5
7	7.5	60.0
8	5.0	65.0
9	4.0	69.0
10	3.0	72.0
11	2.5	74.5
12	2.5	77.0
13	2.5	79.5
14	2.5	82.0
15	2.0	84.0
16	2.0	86.0
17	2.0	88.0
18	2.0	90.0
19	1.5	91.5
20	1.5	93.0
21	1.5	94.5
22	1.5	96.0
23	1.0	97.0
24	1.0	98.0
25	1.0	99.0
26	1.0	100.0

Assume further that exposure in force during January, 1983 was 942 units and that exposure grew between January, 1983 and December, 1984 at a monthly rate of 1.0% (12.7% per annum), and then grew at a declining rate such that growth was zero at December, 1985 and −25.0% per annum by December,

1986. Finally, assume that the January, 1983 pure premium per exposure unit was $100.00 and that pure premium grew between January, 1983 and December, 1986 at a monthly rate of 0.5% (6.2% per annum).

As detailed below, the observed reported loss development pattern would be as follows:

Accident Year	Age 12	Age 24	Age 36	Age 48
1983	$589,380	$1,102,063	$1,228,092	$1,229,203
1984	705,364	1,318,846	1,469,650	
1985	832,041	1,542,366		
1986	875,722			

HYPOTHETICAL REPORTED LOSS DEVELOPMENT

Month	Earned Exposure	Pure Premium	Ultimate Incurred	Reported Losses as of Date: 12/83	12/84	12/85	12/86
1/83	942	$100.00	$ 94,200	$ 72,534	$ 92,316	$ 94,200	$ 94,200
2/83	952	100.50	95,676	71,279	92,806	95,676	95,676
3/83	961	101.00	97,061	69,884	93,179	97,061	97,061
4/83	971	101.51	98,566	68,011	93,145	98,566	98,566
5/83	980	102.02	99,980	64,987	92,981	99,980	99,980
6/83	990	102.53	101,505	60,903	92,877	101,505	101,505
7/83	1,000	103.04	103,040	54,096	92,736	103,040	103,040
8/83	1,010	103.56	104,596	47,068	92,044	104,596	104,596
9/83	1,020	104.08	106,162	37,157	91,299	106,162	106,162
10/83	1,031	104.60	107,843	26,961	90,588	107,843	107,843
11/83	1,041	105.12	109,430	10,943	89,733	109,430	109,430
12/83	1,052	105.65	111,144	5,557	88,359	110,033	111,144
1/84	1,062	106.18	112,763		86,828	110,508	112,763
2/84	1,073	106.71	114,500		85,303	111,065	114,500
3/84	1,083	107.24	116,141		83,622	111,495	116,141
4/84	1,094	107.78	117,911		81,359	111,426	117,911
5/84	1,105	108.32	119,694		77,801	111,315	119,694
6/84	1,116	108.86	121,488		72,893	111,162	121,488
7/84	1,127	109.40	123,294		64,729	110,965	123,294
8/84	1,139	109.95	125,233		56,355	110,205	125,233
9/84	1,150	110.50	127,075		44,476	109,285	127,075
10/84	1,162	111.05	129,040		32,260	108,394	129,040
11/84	1,173	111.61	130,919		13,092	107,354	130,919
12/84	1,185	112.17	132,921		6,646	105,672	131,592
1/85	1,196	112.73	134,825			103,815	132,129
2/85	1,206	113.29	136,628			101,788	132,529
3/85	1,216	113.86	138,454			99,687	132,916
4/85	1,224	114.43	140,062			96,643	132,359
5/85	1,232	115.00	141,680			92,092	131,762
6/85	1,238	115.58	143,088			85,853	130,926
7/85	1,244	116.16	144,503			75,864	130,053
8/85	1,248	116.74	145,692			65,561	128,209
9/85	1,252	117.32	146,885			51,410	126,321
10/85	1,254	117.91	147,859			36,965	124,202
11/85	1,256	118.50	148,836			14,884	122,046
12/85	1,256	119.09	149,577			7,479	118,914
1/86	1,255	119.69	150,211				115,662
2/86	1,251	120.29	150,483				112,110
3/86	1,244	120.89	150,387				108,279
4/86	1,236	121.49	150,162				103,612
5/86	1,224	122.10	149,450				97,143
6/86	1,211	122.71	148,602				89,161
7/86	1,195	123.32	147,367				77,368
8/86	1,177	123.94	145,877				65,645
9/86	1,157	124.56	144,116				50,441
10/86	1,134	125.18	141,954				35,489
11/86	1,110	125.81	139,649				13,965
12/86	1,083	126.44	136,935				6,847
AY 83	11,950	$102.86	$1,229,203	$589,380	$1,102,063	$1,228,092	$1,229,203
AY 84	13,469	109.21	1,470,979		705,364	1,318,846	1,469,650
AY 85	14,822	115.91	1,718,089			832,041	1,542,366
AY 86	14,277	122.94	1,755,193				875,722

APPENDIX B

a VALUES IMPLIED BY INDUSTRY PAID LOSS AND LOSS EXPENSE DATA
A.M. BEST 200 COMPANY SCHEDULE P DATA AS OF 12/31/85

Accident Year	Auto Liability	Workers' Compensation	General Liability	Multi-Peril
		Paid-to-Incurred Percentage		
1976	99.12%	89.59%	87.96%	99.12%
1977	98.83	88.95	87.15	98.78
1978	98.55	87.47	85.05	98.08
1979	97.88	85.77	80.59	97.72
1980	96.65	83.86	75.40	96.65
1981	93.94	80.31	66.40	94.19
1982	89.18	75.81	55.11	91.14
1983	80.38	68.04	39.68	86.48
1984	65.28	54.66	24.94	79.15
1985	34.27	26.04	8.81	55.80
		Implied a to Generate Observed Cumulative Percentage		
1976	.6226	.7975	.8092	.6233
1977	.6097	.7829	.7961	.6131
1978	.5893	.7713	.7886	.6103
1979	.5768	.7569	.7912	.5826
1980	.5678	.7379	.7916	.5679
1981	.5709	.7225	.8040	.5660
1982	.5735	.7013	.8185	.5455
1983	.5811	.6837	.8449	.5133
1984	.5892	.6734	.8664	.4566
1985	.6573	.7396	.9119	.4420

Method: 1980 Workers' Compensation
1980 is age 6 years at 12/31/85
Set $1 - a^6 = .8386$ thus, $a = .7379$

APPENDIX C

FACTORS TO ADJUST TO "GROWTH-FREE" BASIS

	a = .250			a = .600			a = .800		
g	1-2	2-3	3-4	1-2	2-3	3-4	1-2	2-3	3-4
−.250	1.033	1.004	1.001	1.033	1.006	1.002	1.032	1.006	1.003
−.200	1.025	1.003	1.001	1.025	1.005	1.002	1.025	1.005	1.002
−.150	1.018	1.002	1.000	1.019	1.003	1.001	1.018	1.004	1.001
−.100	1.012	1.001	1.000	1.012	1.002	1.001	1.012	1.002	1.001
−.050	1.006	1.001	1.000	1.006	1.001	1.000	1.006	1.001	1.000
.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
.050	.994	.999	1.000	.994	.999	1.000	.994	.999	1.000
.100	.989	.999	1.000	.989	.998	.999	.989	.998	.999
.150	.984	.998	1.000	.984	.997	.999	.984	.997	.999
.200	.979	.998	.999	.979	.996	.999	.979	.996	.998
.250	.974	.997	.999	.974	.995	.998	.974	.995	.998
.300	.970	.996	.999	.969	.994	.998	.970	.994	.998
.350	.965	.996	.999	.965	.994	.998	.965	.993	.997
.400	.961	.995	.999	.961	.993	.997	.961	.993	.997
.450	.957	.995	.999	.956	.992	.997	.957	.992	.997
.500	.953	.994	.999	.952	.991	.997	.953	.991	.996