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THE CONSTRUCTION OF AUTOMOBILE RATING TERRITORIES IN MASSACHUSETTS

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Abstract

In Massachusetts, the past ten years have witnessed the evolution of an increasingly sophisticated system of methodologies for determining the definitions of rating territories for private passenger automobile insurance. In contrast to territory schemes in other states, which tend to group geographically contiguous towns, these Massachusetts methodologies have had as their goal the grouping of towns with similar expected losses per exposure, regardless of the geographic contiguity or non-contiguity of the grouped towns. This paper describes the evolving Massachusetts methodologies during that ten year period.

The paper includes the latest methodology, which was employed to establish rating territories for use in Massachusetts in 1986. That methodology evaluates by-town claim frequency and by-town claim severity separately and then combines the results. The claim frequency approach is to compile detailed insurance data by town, and to compare those actual observations to an a priori *model of the expected insurance losses in each town. The model and the actual observations are blended using empirical Bayesian credibility procedures. The claim severity analysis* *uses a two layer hierarchical empirical Bayesian method in which countywide and statewide severity data supplement less than fully credible town severity data. The combined results of the frequency and severity analyses serve as the basis for ranking the towns according to expected losses per exposure and for placing the towns into rating groups.*

The evolution of the current Massachusetts methodology chronicled in this paper has involved the development and exchange of ideas by a number of individuals in addition to the author. The other principal players have been Richard Derrig, William DuMouchel, Howard Mahler, Stefan Peters, Peter Siczewicz, and Richard Woll. Credit is due also to Ronald Dennis and Lesley Phipps, who played invaluable supporting roles.

1. INTRODUCTION

Classification of risks, including classification of risks by territory, plays an important part in the determination of private passenger automobile insurance premiums in the United States (Stern [29]). In Massachusetts, for example, an experienced driver in Boston may pay more than $400 for a package of compulsory liability coverages costing less than $200 in the territory with the lowest rates. In addition to the magnitudes of the premium differences that depend on risk classification, there are significant public policy issues related to classification (or categorization) of the driving public. As a consequence, private passenger automobile insurance risk classifications have long been a focus of debate in Massachusetts (Massachusetts Division of Insurance [16]) and elsewhere (SRI International [28]). As long ago as 1950, the electorate of Massachusetts specifically voted on a classification issue; in that year, a proposal to eliminate automobile insurance territorial rate variations was placed on the ballot as a referendum question (but was defeated). While the maintenance and pricing of automobile rating territories is just one of many classification issues, it is a very important one.

In recent years the debate about Massachusetts automobile insurance territories has shifted to the technical arena. Mathematicians, statisticians, and actuaries have labored to develop procedures that are practical and workable, and that produce territories best satisfying the criteria suggested nine years ago

(State Rating Bureau (SRB) [23]): equity, homogeneity, discrimination, reliability, stability and compatibility. Briefly, in the context of territory definitions, these criteria were defined as follows:

Equity	The costs of insurance should be distributed fairly among different classes of insureds. "Statistical" equity refers to pricing in accordance with expected losses, while social equity refers to public policy concepts of "fairness." The latter concept is viewed as a series of constraints that perhaps would require recombining statistically justifiable classification separations.
Homogeneity	All the towns in a territory should have approximately the same expected insured losses per car.
Discrimination	The probability of a town being placed in the wrong territory should be minimized.
Reliability	The index used to categorize a town should be a good estimator of the expected insured losses per car in the town.
Stability	The assignments of towns to territories should not change dramatically over time.
Compatibility	A single set of territory definitions should be established so as to be reasonably appropriate for each of the insurance coverages.

The satisfaction of these criteria generally has been sought through efforts to develop an effective way to estimate the expected insured losses per car in each town. These estimates provide a basis for identifying towns in which expected losses are similar, and for grouping towns which are as homogeneous as practicable. The evolution has yielded a territory review methodology that has several interesting features.

- A regular review, typically biennial, of all territory definitions.
- The use of detailed insurance data, by town, as the basic information underlying the determination of the town groupings.
- The development of a model that predicts variations in claim frequency among towns, and the use of empirical Bayesian credibility procedures to combine the predicted claim frequency patterns with the actual by-town claim frequencies.

- The implementation of an empirical Bayesian credibility procedure that estimates the average claim severity in a town by credibility weighting (a) the observed claim severity in the town with (b) the claim severity in the town's county, and (c) the statewide claim severity.

- The development of several measures of the homogeneity of various groupings of towns into territories.

This paper describes the latest territory review methodology and describes the development of that methodology. All of the evolutionary steps described in this paper reflect methods evaluated for and/or included in actual filed recommendations to the Massachusetts Commissioner of Insurance. Thus, while various methodological advances have been accomplished, the parties necessarily have observed a constraint that any methodology be sufficiently practical to include in a Massachusetts rate filing.

The details of the latest methodology, which is described in this paper, are set forth in a rate filing of the Massachusetts Automobile Rating and Accident Prevention Bureau[1] (MARB [14]) and in the resulting decision of the Commissioner (Massachusetts Division of Insurance [20]). This paper relies in part on that Bureau filing, which is quoted or paraphrased without specific attribution at several points in this paper (see Appendices A and B).

2. HISTORY

Automobile insurance rates have varied according to location of garaging for many years. Shortly after the turn of the century, automobile insurers recognized variations in accident frequency from one area to another and divided the United States into two rating territories (All-Industry Research Advisory

[1] The reader may be assisted by a brief description of the regulatory process that governs Massachusetts private passenger automobile insurance and a brief description of the parties involved. The Commissioner of Insurance, who is the state regulator, affirmatively establishes rates, territories, rating procedures, and so forth, effective January 1 each year. The Commissioner has statutory authorization to allow insurance companies to set rates competitively, but has chosen to retain the rate setting authority himself in each of the recent years, following a brief experiment with competitive rating in 1977. In establishing the various rating components, the Commissioner must rely on recommendations from participants in the annual rate hearing process. With regard to the establishment of territory definitions, three participants have offered the principal recommendations. First, the Massachusetts Automobile Rating and Accident Prevention Bureau (MARB), also known as the Massachusetts Rating Bureau, represents the insurance industry. Second, the State Rating Bureau (SRB), which is an arm of the Division of Insurance, the state regulatory body on insurance matters, participates routinely. Third, the Attorney General (AG) intervenes in the hearing process, ostensibly on behalf of the motoring public.

Council (AIRAC) [1]): Greater New York, Boston and Chicago; and Remainder of the United States.

By 1917, the country was divided into eleven rating territories (DuMouchel [3]): Greater New York; Chicago and St. Louis; Boston; Philadelphia; Providence; Baltimore, Washington, D.C., and Pittsburgh; Detroit, Indianapolis, and Milwaukee; Minneapolis and St. Paul; Alabama, Kentucky, Tennessee; Arkansas and portions of other states; Arizona and other states.

Over the years, the system of territories proliferated, and as the patterns of state definition of automobile insurance laws and state regulation of automobile insurance rates solidified by 1950, it became clearly appropriate for each state to have unique rates. In addition, most states were subdivided into a number of territories, as is the case today; the average number of territories per state is fourteen (AIRAC [1]).

The early territory definitions apparently were established largely by judgment, but typically many rating territories were subdivided into two or more statistical territories, so that possible alterations to the existing scheme of rating territories could be studied in a systematic fashion.

In recent years, various methods have been used in different states to review and revise territory definitions. Those methods are beyond the scope of this paper but are described in other sources (California Department of Insurance [2]; McDonald and Thornton (Texas) [24]; New Jersey Department of Insurance [25]; Rhode Island Ad Hoc Committee on Territorial Rating [26]; AIRAC [1]).

3. THE EVOLUTION OF MASSACHUSETTS METHODOLOGIES

Perhaps nowhere has the problem of establishing territory definitions been subjected to the frequent review and pace of methodological development that have occurred in Massachusetts over the past ten years. Several factors have contributed to this history, including:

- The availability of a long and continuing history of detailed insurance data by town, for each of the 351 cities and towns that comprise Massachusetts.[2] This data base provides ready building blocks for alternative territory schemes, and the continuity of reporting of town data facilitates regular reviews and revisions of such schemes.

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[2] In Massachusetts, unlike some states, all land falls inside the boundaries of cities and towns. Note that references below to 360 "towns" include a subdivision of Boston into ten "towns" for automobile rating purposes.

- Regulatory and statutory pressures to flatten rate differentials between territories, which have led to an increased interest by insurers in at least knowing the indicated rates for each geographic cell of the state.
- Regulatory demands for "scientific" approaches to all aspects of ratemaking.

Although the start of the evolution of the current territorial review process was stimulated by the revision of territories that took effect in 1977, some mechanisms for the regular review of territories were in place prior to 1977.

As recently as 1976, two sets of Massachusetts automobile rating territories existed, one set for liability coverages and one set for physical damage coverages.[3] As if the existence of two sets of territories were not sufficiently confusing, liability territory 1 was charged the *highest* liability rates, while physical damage territory 1 was charged the *lowest* physical damage rates. Since 1977, the various parties have unanimously agreed that a single set of territories should apply to all coverages (the SRB's "compatibility" criterion), and that the potential marginal actuarial precision to be gained by maintaining separate territories did not merit the accompanying additional administrative costs and confusion. This position is supported by the fact that most drivers purchase physical damage coverages and increased limits liability coverage in addition to compulsory liability coverages.

Prior to the 1977 rate revision, the *methodologies* used for devising liability and physical damage rating territories also were independent (SRB [23]). For physical damage coverages, twenty-four territories had been established on a geographic basis similar to that used in other states currently. For liability coverages, towns were grouped together into six territories based on the similarity of historical loss pure premiums[4] for the two principal compulsory injury

[3] The existence of separate territory definitions for different coverages was due, at least in part, to the fact that the two different sublines were under the jurisdiction of two different insurance industry rating bureaus in that era.

[4] Loss pure premium is defined as: (a) the claims dollars associated with claims against policies insuring cars in the town, divided by (b) the number of exposures, or insured cars, in the town. All data—exposures, claim counts, and losses—are coded to the town in which the car is garaged (not, for example, to the town in which the accident occurs). As is fairly common in the actuarial techniques applied to classification issues, loss development and trend are ignored on the assumption that they will not have measurably different effects in the different towns.

coverages, no-fault and liability; the two coverages were combined into a single pure premium in a somewhat complicated fashion that is beyond the scope of this paper.[5] A classical credibility factor was assigned to each town's data, based on a full credibility standard of 1000 claims. For any town with less than full credibility, the historical town pure premium was credibility weighted against the underlying pure premium for the territory to which the town had been assigned previously. The resulting "formula pure premiums" were used to rank the towns and to group each town with other towns having similar formula pure premiums, so as to produce six territories. Finally, various constraints were imposed to prevent a town from moving too many territories in any one revision or reversing direction from its movement in the previous revision.

The term "territory" in the 1976 liability methodology, and in all methodologies adopted since then, was purely an historical convention; no geographical constraints were imposed on the selection of towns to be included in a territory. Thus, each of the six territories could contain a variety of non-contiguous towns from all parts of the state. This approach is potentially somewhat confusing to the motoring public, who might hold a more geographically-based concept of territory; in recent years, the Commissioner has been offered proposals for partial imposition of geographic constraints (Massachusetts Division of Insurance [17], AG [10]). Each of the reviews since 1977, however, has indicated substantial variations in pure premiums among neighboring towns. Thus, imposition of geographical constraints would carry a cost: a reduction in the claims-experience homogeneity of the resulting territories. The Commissioner, since 1977, has maintained the freedom from geographic constraint in grouping towns into territories, and many of the territories include towns from all corners of the state.

The 1977 Revision of Territories

The review of territories for 1977 (SRB [23]) indicated that the historical methodologies were failing to produce homogeneous territories comprised of towns having similar pure premiums; rather, the town pure premiums within a territory varied widely. Several methodological sources of the inadequacy of traditional review techniques were identified:

- Excessive reliance on geographical factors in the establishment of physical damage territories;

[5] No use was made of data for the property damage liability coverage. The bodily injury territory definitions applied to this coverage as well. The exclusion of PDL data apparently was attributable in large part to the frequent enactment of statutes changing the nature of this coverage.

- Reliance on a subset of liability coverages to formulate liability territories, particularly since the subset used (bodily injury coverages) was perceived in 1976 as being subject to relatively great volatility in claim severity;
- Inadequate credibility treatment; and
- Excessive application of constraints on town movements. The constraints applied included both direct constraints—actual restrictions on town movements—and indirect constraints, such as assigning the complement of credibility to the town's former territory.

For 1977, an entirely new algorithm was introduced by the Massachusetts State Rating Bureau (SRB [23]). The new approach diverged from past methods in several respects.

First, claim frequency[6] rather than pure premium data were used. The exclusion of claim severity data was justified on the basis of the relatively great variability that the SRB perceived in such data, and the difficulty of studying a phenomenon whose distributions are "poorly known, badly skewed, and difficult to estimate from samples of actual experience" (SRB [23]). Although the possibility of systematic variations in claim severity from town to town was not denied, apparently the value of any information in the historical severity data was believed to be overwhelmed by the instability introduced by the use of such data. Preliminary tests underlying the 1977 review indicated to the actuaries at the State Rating Bureau that the use of claim frequency alone produced satisfactorily discriminatory territories. Subsequent reviews (SRB [22]); MARB [13]; MARB [14]; see below) have developed methodologies for extracting claim severity information from the historical data without also capturing undesirable chance variations in severity. These reviews have indicated that, with the benefit of the new methodologies, claim severity patterns are quite significant and should be reflected in the analysis of town data; but these new methodologies had not been developed by 1976.

Second, the review for 1977 relied on claim frequencies for the physical damage coverages (comprehensive and collision) only; no liability data were used, even though the resulting territories applied to all coverages.[7] A combined

[6] Claim frequency is defined as: (a) the number of claims against policies insuring cars in the town, divided by (b) the number of exposures, or insured cars, in the town. See also the definition of loss pure premium, above.

[7] By 1977, a single insurance industry rating bureau (MARB) had jurisdiction over all coverages; as a result, a unified approach to territories could be implemented more readily than in prior years.

"frequency" was constructed as the sum of comprehensive and collision claim counts, divided by comprehensive exposures. Concerns with the stability of bodily injury data, particularly for small towns, apparently contributed to the decision to exclude these data; the impact of this concern was amplified by the difficulty at that time of identifying an appropriate data element to which the complement of credibility could be applied systematically. The property damage liability (PDL) data, as in earlier years, was tainted by the effects of numerous statutory coverage changes, and thus was excluded from the methodology. The SRB analysts tested the performance of the constructed frequency, however, and concluded that this constructed physical damage frequency could be used to establish a single set of territories that would be acceptably homogeneous for every coverage. Later analyses reached a different conclusion and developed approaches that could successfully employ data from all coverages.

Third, the graduated credibility approach used prior to 1977 was replaced by a decision to assign zero credibility to the 72 smallest towns (based on their exposure volume) and *full* credibility to all larger towns. The small towns were assigned judgmentally to the same territory as a nearby larger "mother" town having similar demographic, economic, and industrial characteristics. This approach represented a rejection of the former complement rather than a rejection of the former credibility formula itself. In prior liability reviews, the complement of credibility was assigned to the data for a town's existing territory. For 1977, the existing territories were seen as being too out-dated to perform this function. Further, the existing territories for physical damage had been based on geographical contiguity, and thus did not necessarily provide an appropriate point of departure for the development of territories based on expected losses. Finally, the prior approach was seen as being structurally too restrictive on town movements. The 1977 resolution of the credibility issue was not entirely satisfactory, however, in that it provided no partial credibility and provided no systematic basis for the treatment of the "noncredible" towns. These issues were the focus of considerable analysis in subsequent reviews.

Fourth, as in the review for 1976 liability territories, the review for 1977 ranked towns according to the selected data element (in this case, the constructed physical damage claim frequency) and then towns having similar values were combined into territories. The review for 1977, however, introduced a more systematic method (which is beyond the scope of this paper) for deciding where to make the cutoff between one group of towns and the next. The result was one set of twenty-four territories used for all coverages[8] in 1977.

[8] Except for a few coverages that have rates not varying by territory.

Finally, the numerous constraints on town movements were removed, and as a result many towns were affected sharply by the territory reassignments. In later reviews, constraints were reimposed. These constraints were intended primarily to avoid sudden rate changes.

The method used for 1977, while lacking many of the important features of the later methodologies, can be credited with four significant achievements. First, it produced territories that were more homogeneous than the predecessor territories. Second, it highlighted the potential perils of including claim severity results in an assessment of the claims experience of smaller towns. Third, it pointed to the need for a credibility procedure that could deal with the small towns. Finally, and more generally, by dislodging the embedded process, the review of 1977 served to stimulate the ongoing research that followed.

The MARB Review for 1981 Territories

The 1977 territories remained intact through 1980. During 1980, the staff of the Massachusetts Automobile Rating and Accident Prevention Bureau (MARB), working with the Class-Territory Subcommittee of the Bureau's Private Passenger Actuarial Committee, conducted an extensive review of the data that had emerged since the 1977 revision, a review of the methods used in the 1977 revision, and research into possible methodological improvements. That research and review culminated in a filing (MARB [11]) that recommended a revision to the territory definitions based on a method that addressed some of the perceived shortcomings of the techniques used to construct the 1977 territories and that utilized the latest data. The key aspects of that proposed method are discussed below.

The MARB proposal for 1981 continued to rely on town-to-town differences in claim frequency rather than on town-to-town pure premium patterns. For each coverage, each town was assigned a severity equal to the statewide severity. A synthetic pure premium for the town was calculated as the product of (a) the town claim frequency, and (b) the statewide claim severity:

$$PP_{t,c} = Y_{t,c} \times X_c,$$

where $Y_{t,c}$ is the claim frequency for town t, coverage c;

X_c is the statewide average claim severity for coverage c; and,

$PP_{t,c}$ is the synthetic pure premium for town t, coverage c.

The inclusion of the statewide average claim severity served only to introduce a measure of the relative importance to overall premium of the various coverages. This approach, then, continued to ignore any town-to-town differences in claim severity. As in the 1977 review, the practitioners at this time believed that claim frequency effects explained most of the significant variation in pure premiums. The exclusion of the severity information was also based on concerns about the instability of the severity data, and on the absence of a credibility or modeling approach capable of separating information from noise in the severity data. While later reviews filled this void and indicated the significance of severity differences between towns, the later reviews also confirmed that the claim frequency effects were the dominant elements in defining town-to-town differences in pure premiums.

A major difference between the MARB proposal for 1981 and its predecessor methodologies was the inclusion of data for all the coverages for which rates varied by territory.[9] The use of data for all coverages has been retained in subsequent territory reviews. The MARB cited several reasons for this change in approach. First, public policy considerations seem to indicate, *a priori*, that the motorists in a town ought to bear *more* responsibility, not less, for the at-fault (liability) claims than for the physical damage claims; thus, the liability coverages ought to be returned to the territory review process. Second, the review for 1981 indicated that liability claim frequency patterns among towns did *not* parallel physical damage claim frequency patterns (contrary to the conclusions implicit in the preceding methodology), and thus that physical damage data could not be used as a proxy for all coverages. Third, the review indicated that, contrary to prior expectations, instability in liability claim frequencies was not a serious problem, so that there was no need to exclude them. Fourth, the statutory definition of PDL had finally stabilized (in 1977), so a usable data series for that coverage could, at last, be compiled. Fifth, the liability coverages are too large a component of overall rates to be ignored. Finally, the MARB review for 1981 introduced an empirical Bayesian credibility procedure that seemed to be capable of accommodating any inherent variations in claim frequencies. The several coverages were incorporated in the territory review process for 1981 by creating an overall average synthetic pure premium for each town that is simply the exposure-weighted average of the synthetic pure premium for each coverage:

$$I_t = \frac{\sum_c E_{t,c} \times PP_{t,c}}{\sum_c E_{t,c}},$$

[9] Compulsory Bodily Injury Liability (known as coverage A-1), compulsory No-Fault BI (A-2), compulsory Property Damage Liability (PDL), Collision, and Comprehensive.

where $E_{t,c}$ is insured exposures for town t, coverage c;

$PP_{t,c}$ is the synthetic pure premium for town t, coverage c (see above); and,

I_t is the all-coverages synthetic pure premium for town t.

This formula not only returns liability data to the analysis; it actually accords them dominant weight (since the insured exposures are greater for the compulsory liability coverages than for the optional physical damage coverages). This weighting scheme simply reflects the contribution of each coverage to overall premium rather than any conclusion that liability data are inherently more suitable for territory analyses.

A major area reviewed for 1981 was the treatment of credibility and the element to which the complement of credibility is assigned. Concerns with these aspects of the 1977 review included the absence of any systematic basis for assigning a complement to non-credible towns; the determination of a point of full credibility; and, the absence of any partial credibility treatment.

The MARB review for 1981 introduced a significant new element to supplement actual town data, to the extent town data were judged to be less than fully credible. As described above, the 1976 procedure assigned the complement of the town credibility to data from the town's previous territory, and the 1977 procedure judgmentally assigned the indications from a nearby "mother" town to a town whose data was judged not credible. In its proposal for 1981, the MARB introduced a claim frequency *model* that was assigned the complement of the town's credibility. This model estimated the claim frequency (or, more properly, the all-coverages synthetic pure premiums, I_t) in each town as a linear function of traffic density[10] in each town. For each town, the model M_t was calculated as:

$$M_t = aD_t + b,$$

where D_t is the town "traffic density"

$= E_t \div$ Road-miles in town;

E_t is the PDL exposure in the town;

a and b are regression coefficients; and,

M_t is the model synthetic pure premium for the town.

[10] The relationship of traffic density to geographical variations in insurance experience has been observed in the literature (e.g., HLDI [9]; AIRAC [1]), as well as in some of the methodologies used in other states.

The regression parameters a and b were calibrated by a weighted linear least squares regression of I_t against D_t (weighted by compulsory coverage exposures in each town). A similar (but not identical) model has been used in all subsequent reviews.

The "traffic density" variable does not measure all components of traffic density. The numerator includes only a count of vehicles insured in a town. Unfortunately, reliable vehicle count data are not available from the Registry of Motor Vehicles, so the insured exposures were utilized, and any town-to-town variations in compliance with the state's compulsory insurance laws are assumed away. This is not perceived as a major modeling problem in Massachusetts. The vast majority of motorists (on the order of 95%) do purchase compulsory insurance. Furthermore, the insurance statistical plan does properly match insured exposures and insured losses, so that any systematic patterns of coverage should be captured by other elements of the analysis. The traffic density variable also omits the effect of one town's residents driving in another town. This omission was purposeful, as there was no intent to directly attribute to residents of one town the effects of congestion caused by non-resident drivers. Thus, while D_t is not traffic density in a wholly traditional sense, the MARB concluded that it was adequate and appropriate for the task at hand.

The calculated traffic densities vary significantly—by a factor of 50—from town to town, as illustrated in Exhibit 2 for a sample of towns, and the regression relationship explained a significant portion of the town-to-town variations[11] in I_t.

Other explanatory variables were explored. For the most part, these variables related to the size or socio-economic characteristics of a town. It did not prove possible at that time to identify a variable for which data were available and that contributed meaningfully to the explanatory power of the regression.

M_t, then, is an estimate of the town's claim frequency based on a modeling process; I_t reflects the actual claim frequency. The analysis utilized I_t, to the extent credible, and assigned the complement of the credibility to M_t.

[11] Boston data did not fit the regression relationship and thus were omitted from the calibration of regression parameters. The assignment of the Boston subdivisions to territories was judgmental, placing each section of Boston in an independent territory, as had been done for 1977.

EXHIBIT 1

1985 MASSACHUSETTS PRIVATE PASSENGER BASE RATES FOR EXPERIENCED OPERATORS

	Coverage				
Territory*	BI Liability (A-1)	No-Fault BI (A-2)	PDL	Collision	Comprehensive
1	$ 43	$10	$113	$147	$ 67
2	46	11	122	154	68
3	48	12	123	153	69
4	49	12	127	158	71
5	55	14	129	159	74
6	54	13	134	162	73
7	55	14	142	167	76
8	59	15	146	172	85
9	62	16	151	175	84
10	60	15	155	187	94
11	64	16	157	179	87
12	73	19	164	187	91
13	75	19	172	195	110
14	74	19	175	200	125
15	75	19	177	217	129
16	83	22	189	234	154
17	63	16	155	190	97
18	77	20	179	226	123
19	84	22	182	238	129
20	78	20	175	229	125
21	107	27	204	310	158
22	117	31	229	329	158
23	89	23	195	283	152
24	72	19	174	221	115
25	82	21	189	238	137
26	91	24	212	259	159

* Territories 17–26 are subdivisions of Boston.

EXHIBIT 2

EXAMPLE OF TRAFFIC DENSITY CALCULATIONS

Town Name	(1) 1983 PDL Exposures	(2) Road Miles	(3) Traffic Density (1) ÷ (2)
Hampden	2,968.9	57	52.1
Holland	914.0	37	24.7
Montgomery	374.4	31	12.1
Tolland	165.8	38	4.4
Wales	641.7	28	22.9
Amherst	10,022.2	121	82.8
Easthampton	8,556.3	83	103.1
Northampton	13,633.4	178	76.6
South Hadley	8,218.0	99	83.0
Ware	4,745.5	121	39.2
Belchertown	4,679.7	147	31.8
Hadley	3,034.7	80	37.9
Hatfield	2,023.0	59	34.3
Huntington	1,098.6	54	20.3
Williamsburg	1,492.7	48	31.1
Chesterfield	555.1	56	9.9
Cummington	439.5	62	7.1
Goshen	403.7	43	9.4
Granby	3,104.1	70	44.3
Middlefield	212.1	37	5.7
Pelham	658.7	43	15.3
Plainfield	294.4	49	6.0
Southampton	2.527.3	71	35.6
Westhampton	678.1	49	13.8
Worthington	622.1	64	9.7
Cambridge	30,201.7	142	212.7
Lowell	37,722.5	239	157.8
Everett	15,385.0	63	244.2
Malden	23,400.1	107	218.7
Medford	26,637.5	151	176.4
Newton	44,546.0	311	143.2
Somerville	26,169.1	105	249.2
Waltham	27,703.2	152	182.3
Watertown	17,042.1	77	221.3
Arlington	25,049.7	121	207.0
Belmont	14,010.8	82	170.9
Chelmsford	19,038.3	186	102.4
Concord	9,960.8	109	91.4
Dracut	12,546.6	112	112.0
Framingham	35,207.4	233	151.1
Hudson	9,345.9	79	118.3
Lexington	18,274.3	153	119.4
Marlborough	17,001.5	130	130.8
Melrose	15,376.9	82	187.5
Maynard	5,420.1	41	132.2

The credibility-weighted formula pure premium, F_t, for each town was calculated as

$$F_t = Z_t I_t + (1 - Z_t) M_t,$$

where Z_t is the credibility assigned to the data for town t.

Finally, the MARB review for 1981 territories introduced empirical Bayesian credibility procedures to assess the credibility to be assigned to the actual town data. Conceptually, the procedure treats the model pure premiums, M_t, as a "prior" estimate of the town experience, and the calculated synthetic pure premiums, I_t, as a subsequent observation. The credibility assigned to town data, I_t, was

$$Z_t = \frac{P_t}{P_t + K},$$

where P_t is an estimate of the town premium, and

K is the empirically determined credibility constant.

The credibility constant, K, is the ratio:

$$\frac{\text{an overall measure of year-to-year variations in town experience}}{\text{a measure of the extent to which actual town data, } I_t \text{, deviate consistently from the model, } M_t}.$$

This same *conceptual* formulation of K has been used in the subsequent territory reviews, although the actual procedures for estimating K have changed.[12] In each of these reviews, the derivation of K (or rather, the numerator and denominator of K) has relied on empirical methods that utilize the actual numerical values of the prior estimates and the observations.

The derivation of the credibility constant is beyond the scope of this paper (but see MARB [11]). The following interpretations, however, may be placed on the credibility formula and formula for K (see, for example, Hewitt [7]; and Hickman [8]):

(1) The magnitude of K is affected directly by the extent to which the density model, M_t, fits the actual data, I_t. If the model fits well, then the credibility algorithm concludes that little additional information is available from I_t. The denominator of K is small, K is large, and the credibilities assigned to I_t are relatively small.

[12] Only the current procedures for estimating K are detailed in this paper.

(2) Conversely, if the model, M_t, fits the data poorly, then the denominator of K is large, K is small, the credibilities assigned to I_t are relatively large, and the weights assigned to M_t are relatively small.

(3) If the town experience, I_t, varies significantly from year to year, the formulation concludes that I_t should not receive much weight. The numerator of K is large, K is large, and the credibilities assigned to I_t are small.

(4) The credibility formula structurally resembles the familiar $Z = P/(P+K)$ formula, which assigns more credibility to larger towns.

The factors described in (1), (2), and (3) are relative, not absolute. This highlights a major difference between the Bayesian credibility procedures used here and classical credibility: in the approaches used here, the credibility assigned to a set of data depends not only on the characteristics of that data, but also on the characteristics of the information that will be accorded the complement of the credibility.

The MARB proposal for 1981 continued the procedure of grouping together towns with similar values of the one-dimensional index (in this case, F_t) chosen to reflect town claims experience, although the details of the grouping procedure were somewhat different than in prior years.[13] Like the procedure used for 1977, the result was twenty-four rating territories: Territory 1 was the lowest rated territory, Territory 14 was the highest rated non-Boston territory, and Territories 15–24 were the ten subdivisions of Boston (not ranked in any particular order). Constraints on the movements of towns from their old territory assignments were reintroduced; however, restrictions applied only if an otherwise-indicated reassignment of a town would produce an unacceptably large rate change.[14]

In addition to identifying aspects of the territory analysis procedure in which methodological changes were needed, and proposing such changes, the data analyses undertaken in connection with the MARB proposal for 1981 territories indicated that the claims experience for towns shifted with sufficient rapidity that territory realignments should be evaluated regularly—preferably every other year.

[13] The details of the grouping procedure were virtually identical to those used by the Commissioner in subsequent years and in the MARB proposal for 1986, described below.

[14] Exhibit 1 displays the 1985 base rates by territory for experienced drivers, and provides a perspective on the rate implications of changing territories.

The State Rating Bureau recommendations for 1981 (as described in Massachusetts Division of Insurance [17]) concurred in the need for an updating of the 1977 territories, but did not embrace the methodological changes proposed by MARB. Rather, the SRB proposed either: (a) a simple updating of the 1977 territories based on later data, or (b) an updating of the town rankings based on later data and the introduction of a "territory within region" concept. Under this concept, each territory would be comprised of all towns having similar claims experience *and* located within a common geographic region of the state.

The Commissioner of Insurance, faced with this methodological dispute, chose the simple updating for 1981 and directed the parties to undertake a cooperative review and development of methodological changes (Massachusetts Division of Insurance [17]).

Review for 1982 Territories

For the development of 1982 territories, the parties did join in a cooperative effort, as well as continuing independent research efforts. Not the least of these research efforts was a master's thesis by one of the State Rating Bureau staff members (Siczewicz [27]). In this joint study for 1982, the work of Siczewicz provided most of the technical refinements to the treatment of credibility that had been developed in the MARB proposal for 1981. In general, the joint MARB-SRB-AG components of the proposal for the modification of rating territories for 1982 bore a strong resemblance to the MARB proposal for 1981. The major differences are summarized below.

In the review for 1982, the tabulation of the actual town data claims experience, the calibration of the density model, and the empirical determination of credibility parameters were conducted for each coverage separately, rather than for all coverages combined. This separate approach was intended to allow the credibility procedure to deal more fully with any differences between coverages in the stability of town claims experience. The town claim frequency (by coverage) was not converted into a pure premium at this stage, but rather was expressed as a claim frequency index.[15]

With the benefit of further study, the density model of claim frequency patterns was expanded to include two additional explanatory variables besides traffic density: a measure of the mix of driver classes in a town, derived from

[15] Claim frequency index = Town claim frequency/Statewide claim frequency.

the average classification relativity (*ACRF*) in the town; and a dummy variable that allowed the aberrant data in Boston to be included in the parameterization calculations without distorting the density regression coefficient.[16]

The *ACRF* variable is intended to reflect the fact that the claim frequency of the insureds in a town is affected by the mix of driver classifications in the town. For example, a town populated solely by senior citizens would be expected to have a lower claim frequency than an otherwise similar town comprised solely of operators with less than three years of experience. Actual towns fall somewhere between these extremes.

The ten subdivisions of Boston were observed to have claim frequencies significantly different from the claim frequencies of the 350 remaining towns in Massachusetts. These differences were not explained by the density and the class mix variables. In fact, differences *between* the ten subdivisions of Boston depart from the patterns which would be predicted by the traffic density model.

The form of the model proposed for 1982, and still utilized today, is:

$$\begin{aligned}\text{Model Frequency Index}_{c,t} = {} & A_{0,c} \\ & + A_{1,c} \times \text{Density}_t \\ & + A_{2,c} \times ACRF_{c,t} \\ & + A_{3,c} \times \text{Boston Dummy}_t;\end{aligned}$$

where A_0, A_1, A_2, A_3 are the regression parameters;
Boston Dummy = 1 in Boston,
0 elsewhere; and,
c refers to coverage, t refers to town.

The credibility procedure was refined so that the credibility parameters and the model regression parameters were determined simultaneously. As noted above in the discussion of the MARB proposal for 1981, the value of the credibility parameter depends on the characteristics of the claim frequency model, since the credibility parameter K depends on differences between the model and actual claim frequencies. In turn, in the review for 1982, the model regression parameters were determined by a weighted least squares regression, where the weights depended on the credibility assigned to the towns' data.

[16] The abnormalities of the Boston data were attributed to the high density of commercial vehicles in Boston (commercial vehicles are not captured in the traffic density variable or in any of the insurance data used in the territory analyses) and the small geographic size of the Boston subdivisions, which suggests that most driving is between subdivisions, not within a single subdivision.

In a broad sense, the use of regression weights dependent on the credibility assigned to a town is similar to the use of exposures, since exposures are a key factor in calculating credibility for a town.

The credibility for a particular town utilized a formula similar to 1981:

$$Z_{c,t} = H_{c,t}/[H_{c,t} + (\tau_c^2/\sigma_c^2)]$$

where H_t = exposures divided by claim frequency;

τ^2 is a measure of year-to-year variation in claim frequency; and,

σ^2 is a measure of the extent to which actual claim frequencies differ from model claim frequencies.

The use (for the 1981 review) of premiums to calculate the town credibility, Z_t, is replaced in this formula by H_t. Like premium, H_t produces larger credibility for towns with more exposures. With H_t, however, the higher the claim frequency, the less credibility is attributed to the actual data. This formulation of H_t assumes that the variability of claim frequency is proportional to claim frequency itself, and that the actual frequency in a town should be given less weight (credibility) as the variability of that claim frequency increases. This approach parallels the overall interpretation of the credibility constant, which is that less weight should be given to a body of data that exhibits instability. The specific methodology used to estimate σ^2 and τ^2 also was changed from the MARB review for 1981, based on Siczewicz [27]. That new methodology has been retained in subsequent reviews and is described below in connection with the MARB proposal for 1986 territories (see Appendix A).[17]

In the review for 1982, the formula frequency index in each town (for each coverage) was, as in the 1981 proposal, calculated as the credibility-weighted average of the actual claim frequency index and the model claim frequency index. In the next step, after this calculation, the effects of the class mix in each town were removed from the town's formula frequency index, since class effects are captured by classifications and classification relativities. The procedure for removing classification effects has been retained in subsequent reviews and is detailed in the Appendix A description of the latest methodology. A final formula claim frequency for each town and coverage is estimated by applying the town claim frequency index to the statewide claim frequency for the coverage.

[17] This credibility methodology also has been adapted for use in calculating Massachusetts private passenger class-territory rate relativities (MARB [15]).

The treatment of claim severity in the review for 1982 paralleled the implicit treatment in the previous year's review: for each coverage the statewide average claim severity, X_c, was assigned to each town, recognizing no variations in claim severity. This statewide average severity, applied to the town formula claim frequency, produced, for each coverage, a formula "pure premium" by town.

Finally, a one-dimensional index that combined all coverages was calculated for each town as

$$\frac{\sum_c E_{c,t} \times \text{Formula Pure Premium}_{c,t}}{\sum_c E_{c,t} \times \text{Statewide Pure Premium}},$$

where $E_{c,t}$ is the insured exposures for coverage c, town t.

This index calculates an all-coverages formula pure premium for the town and compares it to the statewide pure premium that would be observed if the town's coverage purchase patterns were observed statewide. The intent is to ascertain the extent to which a town is above or below average for the coverages purchased in that town.

An alternative formulation using actual statewide exposures in the denominator was rejected, since this alternative formulation would improperly differentiate between two towns identical in all respects except the extent to which physical damage coverages are purchased. Viewed another way, the residents of the town in which physical damage coverages are purchased heavily pay for those coverages directly and should not also pay indirectly by being placed in a higher rating territory.

The MARB, AG, and SRB joined in recommending this final index as the basis for establishing 1982 automobile rating territories (MARB [12]; AG [10]; SRB [21]), and the Commissioner adopted that recommendation (Massachusetts Division of Insurance [18]). With the exception of the treatment of claim severity, which has been refined in the subsequent two reviews, this methodology developed for 1982 has been retained in subsequent reviews and thus is set forth in greater detail in the Appendix A description of the most recent methodology.

The AG differed from the other parties in the method of using the final index to group towns. The AG proposed a clustering algorithm that would have placed *two* constraints on the towns in a territory: (1) the towns should have similar final index values, and (2) all the towns in a territory must be contiguous (AG [10]).

The addition of the contiguity constraint reflected, and imposed, the expectation that two adjacent towns would tend to have similar expected losses. This constraint was also intended to address concerns expressed by members of the driving public that sharp rate differentials between neighboring areas were unfair.

The resulting territories, while comprised of chains of contiguous towns, did not resemble tight clusters, as might have been hoped. In addition, the addition of the contiguity constraint cost a significant loss of homogeneity in the expected losses of towns in each territory. Various technical problems, beyond the scope of this paper, were also identified with the cluster algorithm.

Thus, the SRB and MARB recommended the continued use of town groupings based solely on similarity of town index values, and the Commissioner followed this recommendation. As in the prior revision, the reassignments of individual towns were constrained to avoid any unacceptably large rate changes from 1981 to 1982. The combination of the later data and the methodological changes resulted in territory reassignments for more than 250 towns.

Review for 1984 Territories

During the discussions that led to the joint recommendations for 1982, the parties agreed that a biennial review of territories would be appropriate. The agreement to follow a biennial schedule was based on several considerations:

(1) The claims experience of towns, relative to the statewide average, changes significantly over time. For example, one analysis performed by the MARB indicated that two years of later data (with no methodological changes) would produce indications that over 160 towns should be assigned to new territories, including 35 towns whose territory assignments should change by more than one territory. Thus, delaying a review beyond two years would allow miscategorization of many towns, and might necessitate unacceptably large rate effects when a territory revision did occur.

(2) A two-year interval provides adequate time for the parties to consider methodological improvements.

(3) Because of statistical coding procedures used in Massachusetts, insurance companies can accommodate territory realignments fairly easily, so that biennial revisions are not burdensome.

(4) Annual repetition of the entire territory review and decision process was viewed as impractical.

In accordance with the agreed biennial review schedule, representatives of the MARB, SRB, and AG met during 1983 to consider a possible revision of the territory definitions for 1984. Again the goals of the group were to review the methodologies previously used; to consider alterations and refinements to those methodologies; to review the data that had emerged since the prior review; and, to present to the Commissioner recommendations that had some common bases, even though it was not expected that complete unanimity would be achieved.

As in the previous review, the territory realignment process divided naturally into two major components: the determination of an index for ranking the towns, and the grouping of the ranked towns into territories. The work of the group led to a refinement in the index calculation and to complete unanimity as to the best index and rankings that could be devised for the 1984 review. The process of using the resulting index to group the towns into territories remained an area of some disagreement among the parties.

The index procedure agreed to by the parties, which was documented in the MARB filing [13], recommended only one methodological change to the approach used for the 1982 revision. Specifically, the treatment of severity was modified by assigning to each town the average claim severity for the town's *county*[18], rather than the *statewide* average claim severity. This refinement reflected the clear regional differences in average claim severity, but did so without introducing the instability observed in town claim severities. At that time, the parties had not been able to develop a credibility or modeling procedure that was satisfactory for incorporating by-town claim severities.

This methodological change had a significant impact on the final town index values, because the county average claim severities differ significantly from the statewide average claim costs, as shown in Exhibit 3. This exhibit, which displays the ratios of county average claim costs to statewide average claim costs, reveals for each coverage differences of at least 20% between the county with the lowest average claim severity and the county with the highest average claim severity.

[18] Or county group: in some cases small counties were combined.

EXHIBIT 3

ILLUSTRATIVE CLAIM COST VARIATIONS AMONG COUNTIES

	County Claim Cost Indices (1979–81)				
County Group	BI	PIP	PDL	Collision	Comprehensive
Barnstable, Dukes, Nantucket	.9983	1.0332	.9917	1.0761	.8250
Berkshire	1.0389	.9611	.9257	1.0608	.5895
Bristol	.8713	.9221	.9147	.9305	.9764
Essex	.9693	.9776	.9945	.9903	1.0315
Franklin, Hampshire	.9963	.9599	.9084	1.0672	.6030
Hampden	.9453	.9330	.9270	.9148	.8109
Middlesex	1.0070	1.0069	1.0261	.9902	1.0304
Norfolk	1.0628	1.0043	1.0468	1.0403	1.0309
Plymouth	1.0293	1.0329	1.0437	1.0978	1.0288
Suffolk	1.0718	1.1734	1.0983	.9480	1.3419
Worcester	1.0271	.9840	.9745	1.0355	.7701

Note: Indices calculated as follows:

A. For each year and each coverage, divide each county group average claim cost by statewide average claim cost.

B. For each county group and coverage, calculate an exposure weighted average of the resulting 1979, 1980 and 1981 indices.

The Commissioner (Division of Insurance [19]) adopted the parties' joint recommendations as to the calculation of the final town index values, and, as in the prior revision, selected town groupings based solely on similarity of town index values. This approach created sixteen non-Boston territories; the ten Boston territories were retained, as recommended by the SRB and MARB.[19]

Review for 1986 Territories

In preparing its recommendations for 1986 territories, the MARB retained the 1984 treatment of claim frequency, but again reviewed the handling of claim severity, in addition to incorporating updated data in the analysis.[20] This analysis introduced a newly-developed credibility procedure for claim severity which allowed, for the first time, the utilization of claim severity information by town. Of course, these data still were viewed as being less-than-fully credible, so that complementary data sources were also employed. The selected sources were the countywide[21] average claim severity and the statewide average claim severity.[22] Within each coverage, the claim severity relativity for a town was determined as a credibility-weighted average of the indications from the three sources. The credibility parameters were determined by a two layer hierarchical empirical Bayesian method, described more fully in Appendix A. The empirical Bayesian method compares the variation in relative severity within a town across years; the variation in relative severity across towns within a county; and, the variation in relative severity across counties within the state.

In this approach, the estimated severity for a town is the combination of the severity for the town, the severity for the county that contains the town, and the overall statewide severity. The town's own severity is used to the extent it is credible, with the complement of credibility being given to the estimated severity for the county. In turn, the estimated severity for the county is the credibility-weighted mean of the county to the extent it is credible, with the complement of credibility being given to the credibility-weighted severity overall.

[19] The AG recommended combining the ten subdivisions of Boston into three territories.

[20] These recommendations were developed by the MARB [14]. Recommendations of the AG and SRB were prepared and submitted separately.

[21] Actually, county groups in some cases. This component, taken alone, is equivalent to the standalone severity treatment used in the revision for 1984.

[22] This component, taken alone, is equivalent to the claim severity treatment used in the revision for 1982.

The introduction of this new procedure makes very little difference, of course, for small towns whose data is given little credibility and which therefore are assigned approximately the county average claim severity, as they were in the review for 1984. Similarly, the new procedure makes very little difference for a town with claim severities close to the county average. For larger towns that have average claim severities differing significantly from their county taken as a whole, the partial recognition of the town data can make a significant difference. In a few cases the credibility-weighted town severity is as much as 7% different from the county severity. Exhibit 4 illustrates the change in final town index values (for a selection of towns) due to this methodological change.

The other details of the MARB's methodology for 1986 are substantially the same as in the methodology used in the review for 1984. The entire procedure proposed by the MARB for calculating the town index values for use in establishing 1986 automobile rating territories is detailed in Appendix A.

The final town index values produced by the methodology are displayed in Exhibit 5 for a sample of towns. In this exhibit the towns are displayed in rank order, according to the final town index values, ranging from Buckland with a final index of .5034 (expected losses per car are about half the statewide average), to Chelsea with a final index of 1.9318 (expected losses per car are nearly twice the statewide average). The ten subdivisions of Boston are shown at the end of the exhibit and have final index values ranging from 1.2311 to 2.7791. These index values were used by the MARB in proposing 1986 territory definitions. As in prior years, the MARB recommended grouping towns having similar index values.

The AG's recommendations concurred with the MARB's new index calculations. The SRB did not offer a single specific index methodology, but rather expressed a concern that the revision of territories was occurring too frequently and that too many towns were being reassigned in each revision. The Commissioner's Decision (Massachusetts Division of Insurance [20]) employed the MARB index but, mirroring the SRB's concerns, imposed tight constraints on allowed reassignments of towns.

EXHIBIT 4

SAMPLE COMPARISON OF TOWN INDEX VALUES PRODUCED BY TWO METHODS (1980–1983 DATA)

Town Name	1984 Method	1986 Method	Difference
Hampden	.7659	.7708	.0049
Holland	.6475	.6567	.0092
Montgomery	.6424	.6503	.0079
Tolland	.6488	.6509	.0021
Wales	.8009	.8072	.0063
Amherst	.7211	.7298	.0087
Easthampton	.7546	.7325	−.0222
Northampton	.7512	.7299	−.0213
South Hadley	.7535	.7366	−.0169
Ware	.7604	.7694	.0090
Belchertown	.7820	.7879	.0060
Hadley	.6149	.6024	−.0125
Hatfield	.6583	.6548	−.0035
Huntington	.7498	.7923	.0425
Williamsburg	.7065	.7091	.0026
Chesterfield	.7251	.7339	.0087
Cummington	.6561	.6618	.0057
Goshen	.6323	.6565	.0243
Granby	.7600	.7763	.0163
Middlefield	.5793	.5786	−.0006
Pelham	.6884	.6680	−.0204
Plainfield	.6693	.6646	−.0047
Southampton	.6912	.7071	.0158
Westhampton	.7513	.7548	.0035
Worthington	.6231	.6292	.0061
Cambridge	1.3202	1.3130	−.0073
Lowell	1.1582	1.1488	−.0094
Everett	1.3963	1.4757	.0793
Malden	1.2804	1.3440	.0636
Medford	1.2249	1.2576	.0327
Newton	.9684	.9076	−.0609
Somerville	1.5165	1.5588	.0423
Waltham	1.0395	1.0412	.0017
Watertown	1.0894	1.0819	−.0075
Arlington	.9562	.9330	−.0233
Belmont	.9184	.8772	−.0412
Chelmsford	.7610	.7438	−.0171
Concord	.7150	.6968	−.0183
Dracut	.9456	1.0045	.0588
Framingham	.9564	.9359	−.0204
Hudson	.8916	.8935	.0019
Lexington	.7993	.7612	−.0381
Marlborough	.9501	.9526	.0024
Melrose	1.0178	1.0369	.0191
Maynard	.7719	.7571	−.0147

EXHIBIT 5
SHEET 1

MASSACHUSETTS
INDICATED 1986 RATING TERRITORIES
(For Selected Towns)

	Index						PDL	Observed Pure Premium (Latest Three Years)	
Town Name	BI	PIP	PDL	Coll.	Comp.	Combined	Exposures	Liability	Package
TERRITORY 1									
Buckland	0.5218	0.7366	0.4851	0.5470	0.3501	0.5034	952	$40.66	$109.89
Middlefield	0.6031	0.8149	0.5193	0.6633	0.4216	0.5786	212	32.76	100.02
Hadley	0.6028	0.7590	0.6461	0.6664	0.3850	0.6024	3,035	68.80	156.29
Worthington	0.4848	0.7371	0.5603	0.8248	0.5624	0.6292	622	45.35	164.78
Montgomery	0.6284	0.8956	0.5669	0.8384	0.4171	0.6503	374	61.44	213.45
Tolland	0.6041	0.8572	0.6676	0.7491	0.4391	0.6509	166	65.13	251.95
Hatfield	0.7726	0.7326	0.6637	0.7104	0.4177	0.6548	2,023	77.94	165.05
Goshen	0.5767	0.7435	0.6027	0.8800	0.4543	0.6565	404	29.52	199.34
Holland	0.5397	0.8712	0.6477	0.8328	0.4253	0.6567	914	69.30	190.59
Cummington	0.5592	0.7008	0.6663	0.8621	0.4390	0.6618	440	74.37	174.67
Plainfield	0.6870	0.7631	0.5609	0.8870	0.4990	0.6646	294	87.60	196.53
TOTAL (48 towns)						0.6235	81,045	$63.90	$161.92
TERRITORY 2									
Colrain	0.6147	0.7210	0.6560	0.8020	0.5095	0.6651	924	$81.64	$192.84
Pelham	0.7262	0.6530	0.6822	0.7369	0.4839	0.6680	659	75.40	163.79
Concord	0.6206	0.6773	0.7900	0.8217	0.4302	0.6968	9,961	73.91	198.98
Westminster	0.8263	0.7813	0.6824	0.8120	0.4102	0.7041	3,378	79.75	186.29
TOTAL (33 towns)						0.6900	94,683	$77.00	$188.88
TERRITORY 3									
Rockport	0.5922	0.7192	0.8276	0.7717	0.5292	0.7068	3,920	$69.03	$187.59
Southampton	0.8541	0.9494	0.6603	0.7553	0.4671	0.7071	2,527	69.55	179.61
Williamsburg	0.7943	0.7382	0.7546	0.7765	0.4272	0.7091	1,493	83.44	187.00
Amherst	0.6887	0.6539	0.7270	0.8517	0.6172	0.7298	10,022	78.67	204.44
Northampton	0.7930	0.9033	0.8373	0.7261	0.4467	0.7299	13,633	85.53	180.99
Easthampton	0.8141	0.8874	0.8341	0.7618	0.3950	0.7325	8,556	87.51	184.02
Chesterfield	0.7669	0.9490	0.5845	0.8806	0.6588	0.7339	555	75.75	217.71
South Hadley	0.8665	0.9271	0.8201	0.7660	0.4004	0.7366	8,218	85.80	183.24
Chelmsford	0.7609	0.6756	0.8297	0.7881	0.5589	0.7438	19,038	86.94	218.69
Brimfield	0.9462	0.9789	0.7243	0.7987	0.3833	0.7467	1,336	96.42	195.70
TOTAL (43 towns)						0.7313	192,994	$81.48	$200.11
TERRITORY 4									
Dalton	0.8673	0.7354	0.8972	0.7410	0.4462	0.7518	3,427	$86.45	$185.69
Westhampton	0.7902	0.9104	0.7261	0.9326	0.4372	0.7548	678	113.21	249.89
Maynard	0.7847	0.7828	0.8802	0.7766	0.5040	0.7571	5,420	89.74	203.76
Lexington	0.7151	0.6436	0.8919	0.8103	0.5826	0.7612	18,274	88.87	222.39
Ware	0.9327	0.9597	0.7733	0.8745	0.3557	0.7694	4,746	85.02	194.39
Hampden	0.8769	1.0627	0.7932	0.8385	0.4141	0.7708	2,969	100.69	219.45
Granby	0.8929	1.0815	0.7670	0.8488	0.4533	0.7763	3,104	93.16	205.79
Belchertown	0.9374	1.0049	0.7505	0.8813	0.4590	0.7879	4,680	95.83	212.63
TOTAL (52 towns)						0.7688	244,421	$88.71	$219.64

EXHIBIT 5
SHEET 2

MASSACHUSETTS
INDICATED 1986 RATING TERRITORIES
(FOR SELECTED TOWNS)

	INDEX							OBSERVED PURE PREMIUM (LATEST THREE YEARS)	
TOWN NAME	BI	PIP	PDL	COLL.	COMP.	COMBINED	PDL EXPOSURES	LIABILITY	PACKAGE
TERRITORY 5									
Freetown	0.7255	1.0134	0.7048	0.8121	0.8952	0.7923	4,429	$77.97	$214.40
Huntington	0.8181	1.0210	0.7228	0.9715	0.5001	0.7923	1,099	97.82	259.53
Wales	0.9791	1.1784	0.7330	0.8517	0.4766	0.8072	642	103.84	229.27
Cheshire	1.1968	0.8297	0.8353	0.7898	0.5492	0.8393	1,858	114.34	230.16
TOTAL (58 towns)						0.8133	377,184	$93.36	$225.84
TERRITORY 6									
Pittsfield	0.9237	1.0037	1.0512	0.7716	0.4955	0.8405	23,215	$103.34	$213.53
Belmont	0.8108	0.7705	0.9853	0.9192	0.7551	0.8772	14,011	94.29	242.68
Groveland	1.0068	0.9376	0.9187	0.9202	0.6729	0.8897	2,997	121.45	275.98
TOTAL (29 towns)						0.8623	246,318	$99.29	$237.42
TERRITORY 7									
Lynnfield	0.7559	0.7815	0.8793	0.9499	0.9536	0.8903	7,022	$89.51	$278.18
Hudson	1.0119	0.9559	0.9603	0.9667	0.5376	0.8935	9,347	113.63	262.89
Newton	0.8075	0.7563	1.0137	0.9686	0.7953	0.9076	44,546	97.82	265.62
Arlington	0.8520	0.8296	1.0094	0.9641	0.8836	0.9330	25,050	97.88	257.57
Framingham	0.8947	0.9556	1.0863	0.9804	0.6821	0.9359	35,207	110.40	270.40
Taunton	0.9870	1.1305	0.8896	0.8941	0.9970	0.9433	21,884	108.03	254.07
TOTAL (29 towns)						0.9155	316,422	$103.49	$259.77
TERRITORY 8									
Norwood	1.0023	0.8238	0.9998	0.9462	0.8767	0.9473	16,092	$107.88	$267.20
Marlborough	1.0850	1.0861	1.0098	0.9880	0.6390	0.9526	17,002	117.22	271.45
Wilmington	1.0063	0.9694	1.0451	0.9645	0.9674	0.9938	10,232	124.82	299.34
Tewksbury	1.0542	1.0173	1.0043	1.0518	0.8375	0.9973	13,205	118.43	301.59
TOTAL (11 towns)						0.9701	133,231	$110.64	$280.22
TERRITORY 9									
Marshfield	0.8233	1.0737	0.9463	1.0817	1.0827	1.0006	11,537	$104.63	$298.99
Dracut	1.0308	1.2132	1.0090	0.9948	0.9184	1.0045	12,547	128.22	295.68
Melrose	1.0408	0.9126	1.0519	0.9753	1.1626	1.0369	15,377	110.18	290.56
Waltham	1.0843	1.0354	1.1057	1.0299	0.9376	1.0412	27,703	118.94	288.15
Holyoke	1.3525	1.1436	1.1678	0.9313	0.8089	1.0597	17,069	132.77	261.19
TOTAL (21 towns)						1.0345	351,465	$116.81	$287.84
TERRITORY 10									
Haverhill	1.2649	1.1987	1.0834	0.9720	0.9866	1.0679	21,905	$132.68	$294.11
Watertown	1.0245	0.8988	1.1275	1.1080	1.0918	1.0819	17,042	111.42	293.96
Worcester	1.2873	1.0872	1.2669	1.0435	0.8536	1.1113	63,452	134.67	292.29
TOTAL (6 towns)						1.0914	149,630	$126.23	$297.59

EXHIBIT 5
SHEET 3

MASSACHUSETTS
INDICATED 1986 RATING TERRITORIES
(FOR SELECTED TOWNS)

	Index						PDL	Observed Pure Premium (Latest Three Years)	
Town Name	BI	PIP	PDL	Coll.	Comp.	Combined	Exposures	Liability	Package
TERRITORY 11									
Holbrook	1.1936	1.0925	1.1007	1.1166	1.1444	1.1276	6,136	$130.64	$325.24
Lowell	1.2058	1.2939	1.1749	1.1143	1.0629	1.1488	37,723	138.40	324.54
Quincy	1.1494	1.0244	1.1279	1.1782	1.4034	1.1907	39,832	121.99	330.82
TOTAL (7 towns)						1.1604	138,765	$124.57	$330.05
TERRITORY 12									
Springfield	1.6355	1.6528	1.2426	1.0985	0.9862	1.2463	62,300	$153.62	$309.41
Medford	1.0835	1.0678	1.1427	1.1778	1.7861	1.2576	26,638	125.71	365.73
Brockton	1.3352	1.2859	1.2262	1.2281	1.4207	1.2832	41,920	143.62	360.58
TOTAL (3 towns)						1.2604	130,857	$144.74	$337.27
TERRITORY 13									
Cambridge	1.1122	1.1053	1.1675	1.3506	1.7395	1.3130	30,202	$124.92	$362.73
Malden	1.2725	1.2228	1.2689	1.2197	1.7768	1.3440	23,400	140.38	378.93
Lynn	1.2561	1.1420	1.3693	1.1942	1.8444	1.3690	33,217	144.02	369.84
TOTAL (4 towns)						1.3415	91,504	$136.84	$370.15
TERRITORY 14									
Lawrence	1.4980	1.6148	1.3072	1.1900	1.9389	1.4421	23,935	$162.76	$394.42
Everett	1.3023	1.2718	1.3755	1.2769	2.2025	1.4757	15,385	147.74	424.12
TOTAL (3 towns)						1.4549	48,036	$154.29	$407.97
TERRITORY 15									
Somerville	1.3551	1.2398	1.3910	1.4373	2.3460	1.5588	26,169	$151.50	$436.19
TOTAL (1 town)						1.5588	26,169	$151.50	$436.19
TERRITORY 16									
Revere	1.5148	1.4559	1.4578	1.5443	3.1677	1.7956	17,396	$162.89	$547.65
Chelsea	1.6758	1.6544	1.5957	1.6970	3.3012	1.9318	7,307	183.39	573.08
TOTAL (2 towns)						1.8359	24,704	$168.95	$555.17
TERRITORY 17—West Roxbury (Boston)									
	1.2349	1.0351	1.1328	1.1949	1.4951	1.2311	12,867	$118.80	$335.09
TERRITORY 18—Roslindale (Boston)									
	1.4514	1.2973	1.4038	1.5675	2.4886	1.6536	10,769	$150.58	$457.66
TERRITORY 19—Jamaica Plain (Boston)									
	1.5896	1.6845	1.4404	1.7914	3.1083	1.8918	10,400	$155.81	$512.46
TERRITORY 20—Hyde Park (Boston)									
	1.3877	1.6163	1.3734	1.5910	2.4247	1.6534	11,481	$151.63	$463.63
TERRITORY 21—Dorchester (Boston)									
	2.1882	2.3175	1.6118	2.3901	4.2874	2.4664	33,479	$213.34	$727.76
TERRITORY 22—Roxbury (Boston)									
	2.4487	2.8154	1.8756	2.6857	4.8291	2.7791	6,538	$245.62	$818.45
TERRITORY 23—Boston Central (Boston)									
	1.6152	1.6337	1.4941	1.9746	3.6202	2.0513	16,904	$162.83	$613.69

EXHIBIT 5
SHEET 4

MASSACHUSETTS
INDICATED 1986 RATING TERRITORIES
(FOR SELECTED TOWNS)

Town Name	Index: BI	Index: PIP	Index: PDL	Index: Coll.	Index: Comp.	Index: Combined	PDL Exposures	Observed Pure Premium (Latest Three Years): Liability	Observed Pure Premium (Latest Three Years): Package
TERRITORY 24—Brighton (Boston)	1.2748	1.2898	1.3588	1.5606	2.0844	1.5361	15,873	$137.77	$422.43
TERRITORY 25—South Boston (Boston)	1.6416	1.6759	1.4753	1.9179	3.2148	1.9269	6,491	$168.47	$572.51
TERRITORY 26—East Boston (Boston)	1.7229	1.6995	1.8241	1.9951	3.9162	2.2038	10,780	$197.54	$669.83
TOTALS ALL TERRITORIES						1.0000	2,783,010	$109.48	$279.68

Perspective

The continuing evolution since 1976 of the improved methods described above, which are used for calculating a one-dimensional town index that reflects for each town the relative expected insured losses per car, has contributed to a trend towards satisfying the criteria set forth by the SRB in 1976 (and described in the introduction to this paper). Specifically, the criteria of (statistical) equity and reliability depend directly on the quality of the estimation of expected losses; the compatibility criterion has been satisfied by the decision to maintain a single set of territories; and the stability criterion has been addressed by scheduling regular territory reviews to minimize the number of dramatic territory changes, and by imposing constraints on any large changes that are indicated by the data. The criteria of homogeneity and discrimination depend on the accuracy of the estimation of expected losses, which serves as the basis for making territory assignments, but also depends on the selection of a grouping process, given the final town index. The next section discusses the grouping process.

4. GROUPING TOWNS INTO TERRITORIES

The presentation of territory recommendations in Massachusetts in the last nine years generally has involved two principal steps: first, developing a one-dimensional index that quantifies the relative claims experience in each town; and second, using the one-dimensional index to group towns into territories. The preceding section focused on the first step, from the use of composite physical damage claim frequencies in 1977 to the use for 1986 of a synthetic pure premium index computed from Bayesian estimates of town claim frequencies and claim severities by coverage.

This section discusses the methodology used to group towns into territories, given the one-dimensional final town index. Although various techniques have been discussed and proposed, the Commissioner has used basically the same approach in each of the territory revisions since 1977.

Principal Considerations

The principal considerations that have governed the proposals for groupings of towns into territories are:

(1) The homogeneity of competing territory configurations.

(2) The possible reintroduction of proximity constraints.

(3) The handling of the ten subdivisions of Boston.

(4) The magnitude of rate differentials between territories.

(5) The magnitude of individual town rate changes that would result from proposed realignments of territories.

(6) The number of territories.

(7) The size (number of exposures) of each territory.

The first of these considerations, homogeneity, has been defined in practice to refer to the extent to which individual town claims experience differs from the average claims experience for all towns in a territory. Several quantitative measures have been developed, as discussed below, to compare the overall homogeneity of competing territory configurations.

The second consideration, reintroduction of geographical constraints, has been suggested for several reasons, including improved public understanding of territories and social equity advantages of increasing the probability that apparently similar towns in the same area of the state would be placed in the same territory.

Suggested alternatives to the current ten independent territories for the ten subdivisions of Boston have involved combining some of the sections of Boston with one another and/or with non-Boston territories. Doing so would increase the exposure base used for pricing and resulting territories, would provide a degree of cross-subsidization between the combined Boston subsections, and would degrade the homogeneity of the territory configuration.

The fourth and fifth considerations, the magnitude of rate differentials between territories and the magnitude of individual town rate changes from year to year, have principally acted as constraints on otherwise-indicated territory changes.[23] In the grouping procedures actually adopted, these considerations generally have been incorporated by partially tempering the reassignments of a few towns for which the analysis indicated substantial changes, although a more restrictive constraint was employed by the Commissioner for 1986. These considerations have also contributed to the rejection of some proposals to reintroduce geographical constraints, since: (a) some of the geographic proposals could not be introduced without causing unacceptably large rate changes for certain towns (Massachusetts Division of Insurance [19]), and (b) with the large rate differentials between territories that would be implied by restrictions on the number of territories available to towns in a particular geographical area, small changes in data or methodology could cause a large rate change for a town. These implications of the geographic proposals follow from the fact that each geographic region of the state contains towns from a wide range of current territories.

The sixth consideration, the number of territories, is largely a practical one. Approximately two dozen territories have been viewed as enough to maintain a reasonable degree of homogeneity without the system becoming administratively cumbersome.

The final consideration, the number of exposures in each territory, has two aspects: each territory should provide a sufficient data base for the ratemaking process, and no territory should be dramatically larger than the remaining territories (since homogeneity might suffer). These factors have guided the development of proposals for grouping towns into territories.

[23] The rate changes discussed here affect individual towns only. All territory proposals are implemented so as to have no overall rate level effects.

Selected Grouping Methodology

The town grouping methodologies used in the 1977, 1982, 1984, and 1986 revisions all are generally similar (but with details differing). In essence, the towns are ranked in accordance with their final town index values, and index value breakpoints are selected. A territory is then defined as including all towns having index values between two consecutive breakpoints.

For the most part, the town index values form a continuum, with few obvious breakpoints, so that the breakpoints generally have been selected by a numerical algorithm. In the MARB proposal for 1986, for example, breakpoints initially were selected at an index value of unity (which is the statewide average index value) and at each integer power of 1.06; all towns with an index value below .665 (1.06^{-7}) are placed in Territory 1, and all towns with an index value above 1.504 (1.06^{7}) are combined in a single territory.[24]

The selection of the 1.06 factor was based on: (a) the number of territories it produced, (b) the sizes of the resulting territories, and (c) the homogeneity of the resulting territories (see below). Judgment, however, is superimposed on the territories at the high end of the index value range, where natural breakpoints are evident. Further, a judgment was made to continue the ten independent Boston territories.

Finally, a capping algorithm is applied to determine the rate impact on each town of the territory realignment. In the 1982 and 1984 revisions, any town seen as being subjected to an unacceptably large rate increase due to the realignment is reassigned to a territory closer (in territory number) to its current territory placement.

In the 1986 revision, the Commissioner imposed additional constraints: any town proposed by the MARB to move one territory up or down was not moved at all, while any town proposed by the MARB to move more than one territory up or down was constrained to move one territory (in the direction indicated). With these additional constraints, only 22 towns changed territories, and thus the 1986 territories are nearly identical to the 1984–85 territories.

Homogeneity Measures

Appendix B details several quantitative measures that have been designed to compare the relative homogeneity of alternative Massachusetts automobile

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[24] The algorithm used for 1982 and 1984 was similar. The revision for 1977 used a more complex algorithm to select breakpoints.

territory configurations. Each of the measures captures a slightly different dimension of homogeneity or heterogeneity, and no attempt has been made to calibrate the measures so that one measure can be compared to another; nor is there an absolute scale against which a territory configuration can be judged "homogeneous" or "not homogeneous." Rather, the appropriate comparison is among the results of a single homogeneity measure applied to various territory configurations. The territory configuration with a homogeneity value closer to zero is considered relatively more homogeneous by the standards of a particular measure.

These homogeneity measures have been used in three aspects of the territory review process in Massachusetts. First, they have been used to determine whether existing territory configurations are showing a significant[25] deterioration in homogeneity as they become outdated. Second, the measures have been used to compare different methods of constructing the final town index, to see which produces more homogeneous territories. Finally, and most obviously, the homogeneity measures have been used to evaluate alternative proposals for selecting territory groupings, given a set of final town index values. Exhibit 6 illustrates these uses of the homogeneity measures.

Outdated Territories

Exhibit 6 displays homogeneity measures for the 1982–83 territories, the 1984–85 territories, and the territories proposed by MARB for 1986.[26] The results indicate clearly that the 1982–83 and 1984–85 territories are significantly less homogeneous than are the territories proposed for 1986. It is not immediately evident from Exhibit 6, whether this difference is due to shifting claims experience or due to improving methodologies, but the inclusion on Exhibit 6, of the updated calculations based on the 1984 methodology makes it apparent that much of the difference is due to shifting claims experience.

Index Methodology

Exhibit 6 compares the homogeneity of territories produced by the 1984 town index methodology and by the 1986 town index methodology. Each is

[25] "Significant" in this context is a qualitative term, as statistical significance levels have not been determined for the homogeneity measures.

[26] Prior to the additional constraints that the Commissioner imposed on town movements.

EXHIBIT 6
SHEET 1
HOMOGENEITY MEASURES FOR TERRITORY GROUPINGS INDEX BASED ON 1984 INDEX METHOD UPDATED FOR NEW DATA VS. 1986 INDEX METHOD

	TERRITORY DIVISION INDEX INTERVALS; NO CAPPING								
	5%		5.5%		6%		6.5%		1984–85
HOMOGENEITY MEASURE*	1984 METHOD	1986 METHOD	1984 METHOD	1986 METHOD	1984 METHOD	1986 METHOD	1984 METHOD	1986 METHOD	TERRITORY GROUPING
1. P.P. Squared Diff. (Absolute)									
a) Liability	78.18	61.71	75.98	61.06	74.57	60.54	68.22	59.56	88.03
b) Package	370.20	345.74	403.21	301.84	373.66	277.07	320.06	288.38	427.04
2. P.P. Squared Diff. Cred. Weighted (%)									
a) Liability	.008722	.007414	.008370	.006662	.007982	.006506	.007391	.006314	.009365
b) Package	.005353	.004372	.005492	.004013	.005302	.003877	.004954	.003904	.007294
2A. P.P. Squared Diff. (%)									
a) Liability	.009681	.008415	.009276	.007451	.008769	.007234	.008047	.006994	.01035
b) Package	.005652	.004638	.005780	.004228	.005568	.004084	.005193	.004080	.007665
3. Index Squared Diff. (Absolute)	.000768	.001161	.000719	.000852	.000611	.000645	.000328	.000650	.001271
4. P.P. Maximum Diff. (Absolute)									
a) Liability	25.22	26.35	23.30	23.68	23.18	23.35	20.67	22.14	25.59
b) Package	52.41	46.59	52.72	45.48	51.99	45.91	48.79	43.07	51.43
5. P.P. Maximum Diff. Cred. Weighted (%)									
a) Liability	.2283	.2378	.2261	.2088	.2201	.2143	.2167	.2052	.3068
b) Package	.1613	.1493	.1567	.1444	.1533	.1337	.1450	.1417	.2198
5A. P.P. Maximum Diff. (%)									
a) Liability	.3170	.3700	.3028	.2831	.2691	.2761	.2489	.2522	.3859
b) Package	.1938	.1857	.1890	.1735	.1819	.1634	.1700	.1683	.2565
6. Index Maximum Diff. (Absolute)	.03572	.04294	.03416	.03335	.03104	.03114	.02983	.03096	.08221
7. Error Entropy	1.9366	1.9019	1.8101	1.9290	1.7993	1.9718	1.9038	1.9709	2.6141

* Refer to Appendix B for formulas.

EXHIBIT 6
SHEET 2

HOMOGENEITY MEASURES FOR TERRITORY GROUPINGS

	Using 1986 Index Methodology Territory Division Index Intervals; No Capping				MARB Recommendation			
Homogeneity Measure***	5%	5.5%	6%	6.5%	Uncapped Terr.*	Capped Terr.**	1984–85 Territory Grouping	1982–83 Territory Grouping
1. P.P. Squared Diff. (Absolute)								
a) Liability	61.71	61.06	60.54	59.56	59.15	60.05	88.03	127.07
b) Package	345.74	301.84	277.07	288.38	224.99	234.38	427.04	718.08
2. P.P. Squared Diff. Cred. Weighted (%)								
a) Liability	.007414	.006662	.006506	.006314	.006456	.006537	.009365	.01276
b) Package	.004372	.004013	.003877	.003904	.003738	.003885	.007294	.01176
2A. P.P. Squared Diff. (%)								
a) Liability	.008415	.007451	.007234	.006994	.007183	.007273	.01035	.01401
b) Package	.004638	.004228	.004084	.004080	.003941	.004101	.007665	.01226
3. Index Squared Diff. (Absolute)	.001161	.000852	.000645	.000650	.000315	.000346	.001271	.005063
4. P.P. Maximum Diff. (Absolute)								
a) Liability	26.35	23.68	23.35	22.14	23.20	24.02	25.59	27.27
b) Package	46.59	45.48	45.91	43.07	44.49	48.29	51.43	62.16
5. P.P. Maximum Diff. Cred. Weighted (%)								
a) Liability	.2378	.2088	.2143	.2052	.2151	.2183	.3068	.3635
b) Package	.1493	.1444	.1337	.1417	.1318	.1343	.2198	.2849
5A. P.P. Maximum Diff. (%)								
a) Liability	.3700	.2831	.2761	.2522	.2769	.2804	.3859	.6260
b) Package	.1857	.1735	.1634	.1683	.1614	.1700	.2565	.3349
6. Index Maximum Diff. (Absolute)	.04294	.03335	.03114	.03096	.02854	.04292	.08221	.1581
7. Error Entropy	1.9019	1.9290	1.9718	1.9709	1.9522	1.9844	2.6141	3.1465

* Reflecting judgmental adjustments to territories at high end of scale.
** Prior to imposition of Commissioner's additional constraints (see text).
*** Refer to Appendix B for formulas.

displayed with various alternatives to the 1.06 index value boundaries actually used for 1986. The results indicate that:

(a) For most of the homogeneity measures based on actual loss pure premiums, the 1986 method of treating claim severity substantially improves the homogeneity of the territories.

(b) For the homogeneity measures based on the constructed index values and for the error entropy measure, the 1986 and 1984 methodologies produce similar homogeneity values. However, since the dispersion of the index values has been increased by the recognition of claim cost variations by town, the index-based homogeneity measures and the error entropy measures probably have little useful value in comparing the homogeneity of the final territories produced by the two methods.

Territory Groupings

Exhibit 6, displays the homogeneity measures produced by territory groupings based on the selected 1.06 breakpoint factor as well as those based on alternative breakpoint factors of 1.05, 1.055, and 1.065. Generally, the homogeneity measures indicate that the breakpoint factors of 1.06 and 1.065 are to be preferred, with the 1.06 factor performing best on the measures that reflect a package of all major insurance coverages. The 1.06 factor actually was selected for the several reasons indicated above. Exhibit 6 also indicates that the MARB's judgmental adjustments to the territory breakpoints and the MARB's application of the "traditional" capping process produce only minor changes in the homogeneity measures.

By all measures, the proposed territories are far more homogeneous than the 1984–85 territories. However, the additional constraints imposed by the Commissioner nearly recreate, in 1986, the 1984–85 territory definitions and thus bear a nontrivial cost in terms of homogeneity.

Perspective

This loss of homogeneity usefully may be viewed as the cost of shifting the regulatory emphasis from the homogeneity criterion towards the stability criterion. This trade-off illustrates two general principles often encountered in classification issues (and other issues): that not all constraints can be satisfied simultaneously; and that the relative emphasis placed on the different constraints ultimately must be resolved by the application of judgment, even if complex methodologies are available to clarify the nature and implications of the necessary choices.

5. SUMMARY

The methodologies described in this paper may be useful specifically to practitioners in the automobile insurance field. In addition, particularly with regard to the empirical Bayesian credibility techniques, the formulas—or the concepts they implement—may be useful in other fields as well.

Two conclusions of the Massachusetts territory analysis are of particular interest in that they suggest a change to the conventional structure of automobile rating territories and a change to the frequency with which territories are reviewed. These two conclusions are:

(1) That claims experience varies significantly from town to town, even among neighboring towns with generally similar characteristics; and

(2) That claims experience of towns shifts materially over time and, therefore, that territory definitions should be reviewed regularly.

While the author expects that Massachusetts methodologies will continue to evolve in the future, the procedures and results of the current Massachusetts state of the art may prove useful elsewhere in the meantime.

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APPENDIX A
CALCULATION OF THE TOWN INDEX AND TOWN RANKING[27]

A1. SUMMARY

This appendix describes the calculation of the index that is intended to reflect a town's overall loss potential relative to the statewide average loss potential. The calculation methodology described here is that underlying the 1986 Massachusetts automobile rating territories, as described in the body of the paper. Exhibit 7 schematically displays the process of deriving the final town index used to rank the towns.

The starting point for the calculation of the town index is the actual experience (exposures, number of claims, and loss payments) of the vehicles insured in each town. This actual experience may be expressed in terms of claim frequency (average number of claims per insured exposure) and average claim cost (average cost per claim).

The analysis uses the actual claim frequency by coverage of each town, credibility weighted with model claim frequencies by town and coverage; the parameters of the model and the calibration of the credibility functions are based on an analysis of patterns and variations in claim frequency across towns and years. The claim frequency method of analysis is detailed in Section A2.

The analysis also utilizes average claim cost data by town, credibility weighted with average claim cost data by county and statewide. The procedure used to estimate the relative average claim cost by town is detailed in Section A3.

The resulting claim frequency and claim cost indications by town are combined to produce a pure premium index by town and coverage. These pure

[27] This Appendix was excerpted, with editing, from sections of the Massachusetts Automobile Rating and Accident Prevention Bureau's (MARB) *Filing for 1986 Private Passenger Territory and Classification Definitions*, July 1985, which was written by Dr. Richard Derrig, Howard Mahler, and the author of this paper. The Bayesian credibility procedures used in the claim frequency analysis were developed by the MARB and by Peter Siczewicz. The Two Layer Hierarchical Empirical Bayesian Method of analyzing claim severities (see below) was developed and prepared by Howard Mahler for the MARB *Filing for 1986 Private Passenger Territory and Classification Definitions*. Howard Mahler's work on the Two Layer Hierarchical Empirical Bayesian Method was based on P. Heckman, "Credibility and Solvency," *Pricing Property and Casualty Insurance Products*, CAS Discussion Paper Program, May 1980; and G. Venter, "Structured Credibility in Applications—Hierarchical, Multidimensional, and Multivariate Models," 1984 (unpublished).

EXHIBIT 7
SHEET 1

OUTLINE OF TOWN INDEX CALCULATIONS

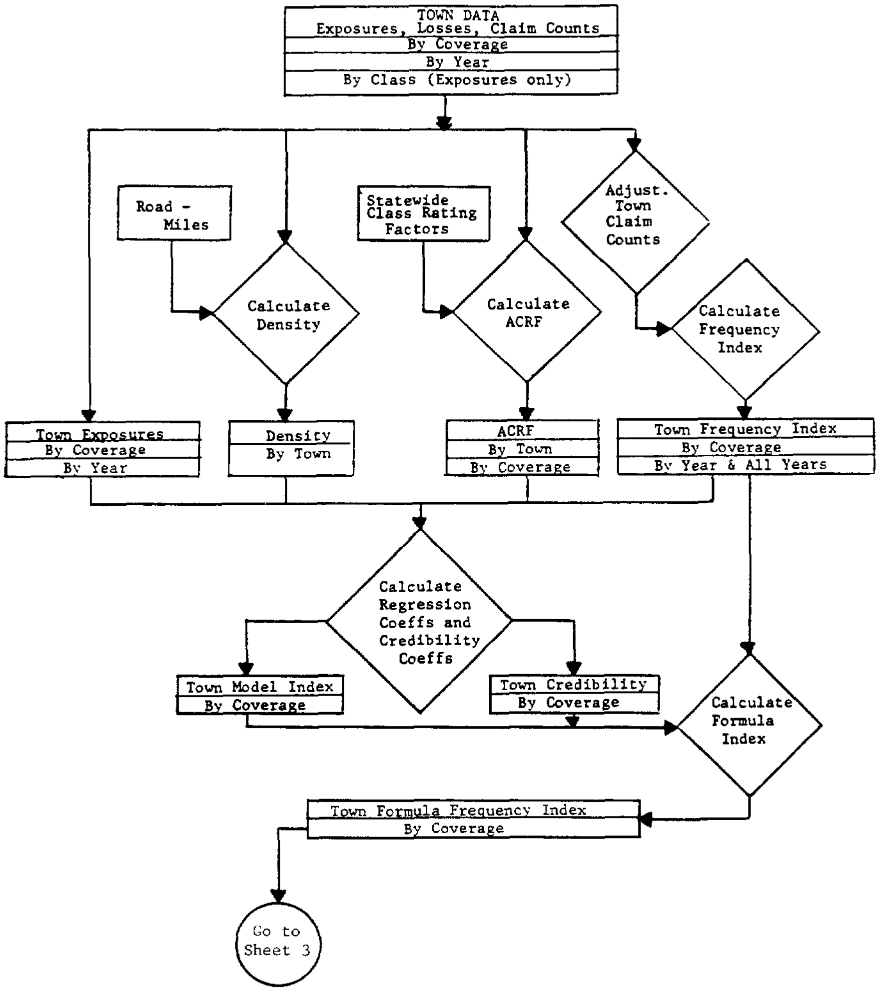


EXHIBIT 7
SHEET 2

OUTLINE OF TOWN INDEX CALCULATIONS

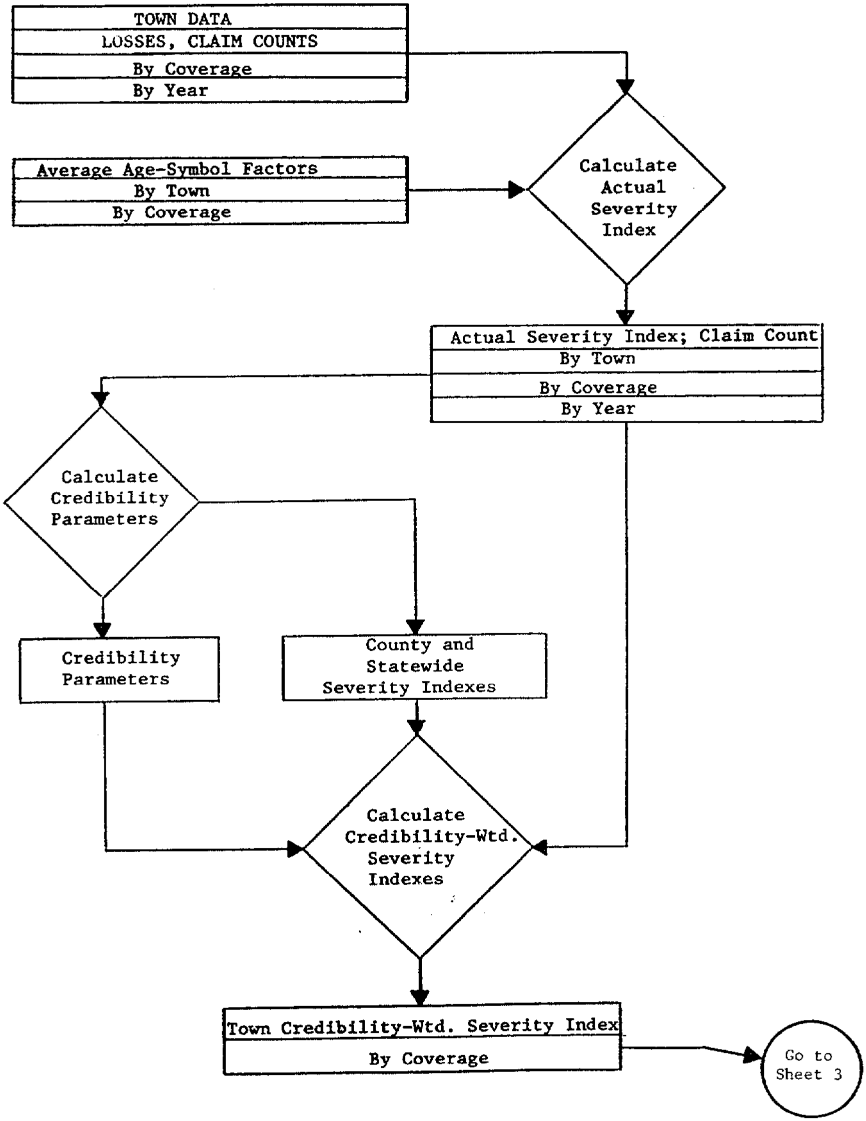
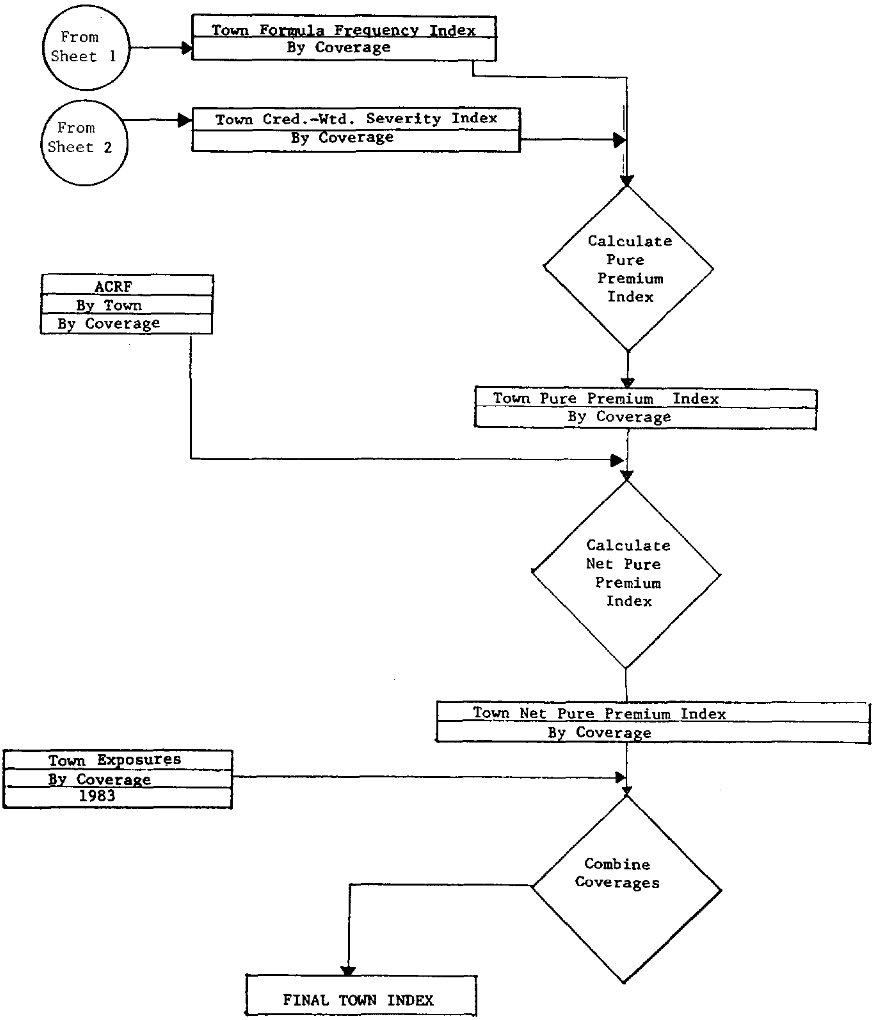


EXHIBIT 7
SHEET 3

OUTLINE OF TOWN INDEX CALCULATIONS



premium indices are then modified to the extent they reflect components of the town's driver classification mix already captured by other elements of the rating system.

As described in Section A4, the final town index is a weighted average of the pure premium indices for the five major coverages for which rates vary by territory.

A2. BUILDING THE CLAIM FREQUENCY INDEX

The details of the methodology used to determine the claim frequency index are described and illustrated in this section. Exhibit 8 details the formulas used.

a. Data

Exposures and claim counts by town and year (latest four years) for each of the coverages A-1, A-2, PDL, Collision, and Comprehensive are used. In order to ensure that the ultimate ranking of an individual town is not adversely affected by a single natural catastrophe, a listing of physical damage experience for each town by month is reviewed and compared with a list obtained from the Insurance Services Office of catastrophes assigned serial numbers during the experience period. The current review indicated that none of the serialized catastrophes produced unusual claim counts that might require adjustment or special treatment.

b. Actual Claim Frequency

The claim frequency in a town for a particular coverage and year is calculated as claims divided by exposures. The claim frequency *index* in a town for a particular coverage and year is the ratio of the town's claim frequency to the statewide claim frequency for the same coverage and year.

A claim frequency index for a town and coverage for all years combined is calculated as the average of the claim frequency index for each year, weighted by the town's exposure by year for the specified coverage. The resulting indices are re-balanced to produce an average index of unity across all towns.

c. Claim Frequency Model

Three explanatory variables affecting the claim frequency in a town are used in the claim frequency model: the traffic density in the town, the class mix in the town, and whether the town is part of Boston. The effect of each of these variables differs from coverage to coverage.

EXHIBIT 8
SHEET 1

CALCULATION OF CLAIM FREQUENCY INDEX MODEL PARAMETERS AND CREDIBILITY PARAMETERS FORMULAS[28]

The basic structure of the claim frequency model is:

$$\begin{aligned}\text{Frequency Index}_{c,t} = {} & A_{0,c} \\ & + A_{1,c} \times \text{Density}_t \\ & + A_{2,c} \times ACRF_{c,t} \\ & + A_{3,c} \times \text{Boston Dummy}_t\end{aligned}$$

where the subscripts c and t refer to coverage and territory;
Density$_t$ is the town density for non-Boston towns;
$ACRF_{c,t}$ is the average class rating factor for the coverage and town;
Boston Dummy$_t$ = 1 in towns which are part of Boston, 0 elsewhere; and
$A_{0,c}$; $A_{1,c}$; $A_{2,c}$; $A_{3,c}$ are regression coefficients.

The regression coefficients are determined separately for each coverage, so the c subscripts will be dropped in the remaining formulas.

With 360 towns in Massachusetts, it is convenient to perform the algebra in matrix notation, which parallels the structure of the APL program used in the analysis:

y is a 360×1 vector of actual town claim frequency indices.
$\hat{y}$ is a 360×1 vector of model claim frequency indices.
x is a 360×4 matrix of the independent variables in the claim frequency model, where
Column 1 is unity;
Column 2 is Density$_t$;
Column 3 is $ACRF_t$;
Column 4 is Boston Dummy$_t$.

[28] For a more detailed exposition of these formulas refer to Siczewicz [27]. Also see DuMouchel and Harris [4].

EXHIBIT 8
SHEET 2

x^T is the 4×360 transpose of x;
A is a 1×4 matrix of the regression coefficients;
W is a 360×360 diagonal matrix of the weights to be applied to each town in the weighted least squares regression; and,
W^{-1} is the inverse of W.

In practice, W is determined from W^{-1}; the entry for each town in W^{-1} is an estimate of the variance of the claim frequency in the town.

The first estimate of this variance in town t is

τ^2/H_t,

where τ^2 is a statewide measure of the year to year variations in claim frequency (see below), and H_t is

$$H_t = \frac{\text{Exposures}_t}{\text{Actual Claim Frequency Index}_t} \quad \text{(all years combined).}$$

That is, given the statewide claim frequency variation, a town with more exposures is estimated to have a lower variance, while a town with a high claim frequency is estimated to have a high variance in claim frequency.

The statewide value of τ^2 is calculated as:

$$\tau^2 = \frac{\sum_t \sum_y H_{t,y} \, (Y_{t,y} - Y_t)^2}{360 \times (\text{number of years of data} - 1)}$$

where
t = town;
y = year;
$H_{t,y}$ = exposures$_{t,y}/Y_t$;
Y_t = claim frequency index for town t, all years combined; and,
$Y_{t,y}$ = claim frequency index for town t, year y.

The first estimate of the regression coefficients A is calculated using a weighted least squares regression

$$A = (X^T WX)^{-1} \quad X^T WY.$$

EXHIBIT 8
SHEET 3

The second estimate of the variance in town t is:

$(\tau^2/H_t) + \sigma^2$,

where σ^2 is a measure of the variation of the model from the data. σ^2 is calculated as:

$$\sigma^2 = (\text{RSS} - (n-m))/\left(\left(\sum_t H_t/\tau^2\right) - \text{trace } (X^T W^2 X)\ ((X^T W X)^{-1})\right),$$

where RSS = Residual sum of squares

$$= \sum_t ((Y_t - \hat{Y}_t)^2/\ (\tau^2/H_t));$$

$\hat{Y}_t$ = Model claim frequency index for town t

$= (XA)_t$;

$n = 360$ = number of towns; and,

$m = 4$ = number of years of data.

With the revised values of W^{-1}, W is recalculated, and the final estimate of the regression parameters is

$$A^1 = (X^T WX)^{-1} \quad X^T WY.$$

The credibility assigned to the actual town frequency index is:

$$Z_t = \frac{\sigma^2}{\sigma^2 + \tau^2 \times (\hat{Y}_t \div \text{Exposures}_t)}$$

$$= \frac{(\text{Exposures}_t \div \hat{Y}_t)}{(\text{Exposures}_t \div \hat{Y}_t) + (\tau^2/\sigma^2)}.$$

The traffic density in a town is calculated as the ratio of insured exposures[29] in the town to road miles in the town.

The class mix in a town is quantified as the average of the rating class relativities underlying the current rates, weighted by the exposure distribution by class within the town; class mix factors (*ACRF*'s) are calculated by town separately for each coverage.

In order to reduce the possibility of Boston claim frequency patterns distorting the model for the remaining 350 towns, a "dummy" variable is introduced into the model; this variable has a value of unity in Boston, zero elsewhere. In addition, the traffic density variable is set equal to zero in Boston.

The structure of the claim frequency model is

$$\begin{aligned}\text{Model Frequency Index}_{c,t} = {} & A_{0,c} \\ & + A_{1,c} \times \text{Density}_t \\ & + A_{2,c} \times ACRF_{c,t} \\ & + A_{3,c} \times \text{Boston Dummy}_t,\end{aligned}$$

where the subscripts c and t refer to coverage and town, respectively.

d. Model and Credibility Parameters

The values of the model coefficients (the A values in the above equations) are determined empirically for each coverage using the latest four years of data. In addition, the credibility attributable to the actual claim frequency is determined by an analysis of the extent to which the actual claim frequency index contains meaningful information about town frequencies not captured by the model.

The values of the model coefficients are determined for each coverage separately by a weighted least squares regression of actual claim frequencies on density, *ACRF*, and the Boston Dummy variable. The weight applied in the regression analysis to the data for each town is essentially proportional to the credibility assigned to that data. The specific formulas used in this analysis, which determines both the regression parameters and the credibility parameters, are outlined in Exhibit 8.

[29] The latest year's PDL exposures are used.

The regression model parameters estimated in the latest review are:

	A-1	A-2	PDL	COMPRE-HENSIVE	COLLISION
Intercept ($A_{0,c}$)	−1.1233	−0.5227	−0.07902	−0.5680	−0.2486
Density coefficient ($A_{1,c}$)	.002142	.0007907	.002672	.002625	.002647
ACRF coefficient ($A_{2,c}$)	1.8124	1.3714	0.7270	1.1949	0.8816
Boston Dummy ($A_{3,c}$)	0.8052	0.6200	0.7320	1.7224	1.3393

For illustrative purposes, collision model claim frequency indices for Holland (rural), Wilmington (suburban), and Brighton (part of Boston) are calculated below:

	HOLLAND	WILMINGTON	BRIGHTON
(1) Town Density (×.002647)	24.7	97.5	0.0
(2) *ACRF* (×.8816)	.9682	1.0528	1.0014
(3) Boston Dummy (×1.3393)	0	0	1
(4) Intercept (−.2486)	−0.2486	−0.2486	−0.2486
(5) Model Claim Freq. Index	.6703	.9376	1.9735
(6) Balancing Factor to Produce Average Index of 1.000 (averaged over all towns, 4 years)	.98704	.98704	.98704
(7) Model Claim Frequency Index, Balanced	.6791	.9499	1.9994

The credibility to be assigned to the actual claim frequency index for a particular town and coverage is calculated as:

$$Z_{c,t} = \frac{H_{c,t}}{H_{c,t} + (\tau^2{}_c/\sigma^2{}_c)}$$

where $Z_{c,t}$ = credibility assigned to actual frequency index for coverage c, town t;

$H_{c,t} = E_{c,t} \div MFI_{c,t}$;

$E_{c,t}$ = exposures for coverage c, town t, all years combined;

$MFI_{c,t}$ = model claim frequency index for coverage c, town t;

τ_c^2 = a measure of the year to year variation in claim frequencies (see Exhibit 8); and,

σ_c^2 = a measure of the extent to which actual claim frequencies differ from model claim frequencies (see Exhibit 8).

The credibility parameters (τ_c^2, σ_c^2) determined in the latest review in accordance with the formulas outlined in Exhibit 8, are

	σ^2	τ^2
A-1	.03898	194.66
A-2	.03574	112.12
PDL	.01327	22.94
Comprehensive	.04078	25.01
Collision	.01816	13.59

Continuing the three town example, credibilities for collision are calculated as follows:

	HOLLAND	WILMINGTON	BRIGHTON
(1) Model Claim Frequency ($MFI_{c,t}$)	.6791	.9499	1.9994
(2) Exposures ($E_{c,t}$)	1,629.5	21,480.6	35,513.6
(3) Collision Credibility	.7623	.9680	.9596

$$\frac{(2) \div (1)}{((2) \div (1)) + (\tau^2/\sigma^2)}$$

e. Formula Frequency Index by Coverage and Town

The formula frequency index for each coverage is the weighted average of the actual frequency index and the model frequency index. The weight accorded the actual frequency index is the credibility, Z, determined in accordance with the above procedure; the model frequency index is calculated using the model parameters determined above.

Algebraically, the formula frequency is calculated as:

$$FF_{c,t} = (Z_{c,t} \times AFI_{c,t}) + ((1 - Z_{c,t}) \times MFI_{c,t})$$

where

$FF_{c,t}$ = Formula frequency index for coverage c, town t; and

$AFI_{c,t}$ = Actual claim frequency index for coverage c, town t.

Continuing the three town example, the formula frequency index values are:

	HOLLAND	WILMINGTON	BRIGHTON
(1) Actual Claim Frequency Index ($AFI_{c,t}$)	.7636	.9671	1.7234
(2) Model Frequency Index ($MFI_{c,t}$)	.6791	.9499	1.9994
(3) Credibility ($Z_{c,t}$)	.7623	.9680	.9596
(4) Formula Frequency Index ($FFI_{c,t}$) = (3)×(1) + (1.0−(3))×(2)	.7435	.9665	1.7346

A3. CALCULATING THE CLAIM COST INDEX

Separately for each coverage, claim severity relativities for each town are estimated. These relativities compare the estimated average claim severity for the town to the statewide average claim severity.

These claim severity relativities for each town are determined as a credibility-weighted average of the town, the county[30], and the statewide claim severity relativities indicated by historical data. The credibility parameters are determined by a Two Layer Hierarchical Empirical Bayes Method.

[30] Barnstable, Dukes, and Nantucket Counties are grouped together as Dukes and Nantucket are too small to remain ungrouped.

The estimated severity for a town is the combination of the severity for the town, the severity for the county that contains the town, and the overall statewide severity. The town's own severity is used to the extent it is credible, with the complement of credibility being given to the estimated severity for the county. In turn, the estimated severity for the county is the credibility-weighted mean of the county severity to the extent it is credible, with the complement of credibility being given to the credibility-weighted statewide severity.

The mechanics of the process are described in Exhibits 9 and 10. The calculated parameters are shown in Exhibit 11. Illustrative examples of the credibility-weighting process are included in Exhibit 12.

The input variables needed for this method of evaluating claim severities by town are:

1. Claims by coverage by year by town.
2. Relative average claim cost by coverage by year by town, modified by average age/symbol relativity by coverage by town[31]

$$= \frac{\text{average claim cost by coverage by year by town}}{\text{average claim cost by coverage by year, statewide}}$$

$$\div \frac{\text{average age/symbol relativity by coverage by town}}{\text{average age/symbol relativity by coverage statewide}}$$

3. County or county group assignments of the towns.

As shown in Exhibit 12, for the three town collision example the methodology yields:

	Holland	Wilmington	Brighton
Estimated Relative Severity	1.0846	1.0508	.9011

[31] The modification for average age/symbol is only needed for comprehensive and collision. This modification is intended to remove from the territory analysis variations between towns that are captured by another rating variable, age/symbol factors.

EXHIBIT 9
SHEET 1

CALCULATION OF THE CLAIM COST INDEX USING THE TWO LAYER HIERARCHICAL EMPIRICAL BAYESIAN CREDIBILITY MODEL SUMMARY OF FORMULAS[32]

Assume a *nested* series of groupings. In this specific implementation, the nested series of groupings is: towns, groups of these towns into counties (actually county groups), and the statewide group of all counties.

Assume an observed variable, X, for each town, for several time periods. (In the territory analysis, X is the relative claim severity.)[33] The intent is to estimate X in the future.

Let $X_{Cg}(t)$ represent X for time t, town g, in county C.

Similarly, let $P_{Cg}(t)$ represent the corresponding measure of exposure (in our case, number of claims).

The use of a dot, instead of a variable, denotes summation over that variable. For example:

$$P_{C\cdot}(\cdot) = \sum_{g,t} P_{Cg}(t), \text{ and}$$

$$W_{C\cdot} = \sum_{g} W_{Cg}.$$

The mean of X, weighted by P, is denoted by $\overline{X}$. For example:

$$\overline{X}_{Cg} = \frac{\sum_t X_{Cg}(t)\, P_{Cg}(t)}{\sum_t P_{Cg}(t)}, \text{ and}$$

[32] These formulas and their derivatives and implementation were developed and prepared by Howard Mahler and included in Massachusetts Automobile Rating and Accident Prevention Bureau, *Filing for 1986 Private Passenger Territory and Classification Definitions*, July, 1985.

[33] For the physical damage coverages, X is the relative claim severity divided by the relative average age/symbol relativity.

EXHIBIT 9
SHEET 2

$$\bar{X}_C = \frac{\sum_{g,t} X_{Cg}(t)\, P_{Cg}(t)}{\sum_{g,t} P_{Cg}(t)}.$$

Then, given certain assumptions, the least squares estimate for the variable X in the future, denoted by $t = 0$, is given by:

$$X_{Cg}(0) = W_{Cg}\bar{X}_{Cg} + (1 - W_{Cg})m_C$$

where $m_C = V_C M_C + (1 - V_C)\hat{m}$ = estimated relative severity for the county;

$$\hat{m} = \frac{\sum_C V_C M_C}{V_.} = \text{credibility weighted mean overall;}$$

$$W_{Cg} = \frac{P_{Cg}}{P_{Cg} + k_C} = \text{credibility for the town;}$$

$$V_C = \frac{W_{C.}}{W_{C.} + k/k_C} = \text{credibility for the county; and,}$$

$$M_C = \sum_g \frac{W_{Cg}\bar{X}_{Cg}}{W_C} = \text{credibility weighted mean for the county.}$$

The parameters k, k_C are to be estimated from the observed data.

It should be noted that the estimated severity for a town is the combination of the severity for the town, the severity for the county that contains the town, and the overall severity. The town's own severity is used to the extent it is credible, with the complement of credibility being given to the estimated severity for the county. In turn, the estimated severity for the county is the credibility weighted mean of the county to the extent it is credible, with the complement of credibility being given to the credibility weighted severity overall.

EXHIBIT 9
SHEET 3

Let $I(P) = 0$ if $P = 0$
$\quad 1$ if $P \neq 0$

and $$D_{1C} = \sum_{g,t} P_{Cg}(t)\,(X_{Cg}(t) - \overline{X}_{Cg})^2$$

$$D_{2C} = \sum_{g,t} P_{Cg}(t)\,(X_{Cg}(t) - \overline{X}_{C})^2$$

$$D_3 = \sum_{C,g,t} P_{Cg}(t)\,(X_{Cg}(t) - \overline{X})^2$$

Let $E(Y)$ represent the expected value of Y; $E(Y)$ will be *estimated* by the observed value of Y. For example, $E(D_{1C})$ will be estimated by the observed value for D_{1C}.

The estimates of the parameters are as follows:

$$s_C^2 = \frac{E(D_{1C})}{\left[\sum_{g,t} I(P_{Cg}(t))\right] - \left[\sum_{g} I(P_{Cg}(.))\right]}$$

$$k_C = \frac{P_{C.}(.) - \sum_{g} \dfrac{P_{Cg}^2(.)}{P_{C.}(.)}}{\dfrac{E\,(D_{2C})}{s_C^2} - \left[\sum_{g,t} I(P_{Cg}(t)) - I(P_{C.}(.))\right]}$$

$$k = \frac{\sum_{C} s_C^2 \left[P_{C.}(.) - \dfrac{P_{C.}^2(.)}{P_{..}(.)}\right]}{E\,(D_3) - \sum_{Cgt} s_C^2 \left[I\,(P_{Cg}(t)) - \dfrac{P_{Cg}(t)}{P_{..}(.)}\right] - \sum_{C} \left[s_C^2 \dfrac{P_{C.}(.) - \sum_{g} \dfrac{P_{Cg}^2(.)}{P_{..}(.)}}{k_C}\right]}$$

EXHIBIT 10
SHEET 1

CALCULATION OF THE CLAIM COST INDEX USING THE TWO LAYER HIERARCHICAL EMPIRICAL BAYESIAN CREDIBILITY MODEL IMPLEMENTATION

In the use of the Empirical Bayesian Credibility Model described in Exhibit 9 to calculate average claim costs by town, some fluctuations in the calculated values would be expected, since the parameters of the model are being calculated from only a limited quantity of data. For the practical implementation of the model, it is desirable to eliminate undue fluctuations.

Limitations on s_g^2

The parameters s_C^2 are estimated separately for each county (and each coverage). Since certain counties are relatively small, the computed value of s_C^2 can be subject to undue fluctuations.

$$s_C^2 = \frac{\sum_{g,t} P_{Cg}(t)\,(X_{Cg}(t) - \overline{X}_{Cg})^2}{\left[\sum_{g,t} I(P_{Cg}(t))\right] - \left[\sum_{g} I(P_{Cg}(.))\right]}$$

s_C^2 can be viewed as a weighted average of s_g^2 for each town g in the county C, where

$$s_g^2 = \frac{\sum_{t} P_{Cg}(t)\,(X_{Cg}(t) - \overline{X}_{Cg})^2}{\left[\sum_{t} I(P_{Cg}(t))\right] - I(P_{Cg}(.))};$$

the weights[34], w_g, are

$$w_g = \frac{\left[\sum_{t} I(P_{Cg}(t))\right] - I(P_{Cg}(.))}{\sum_{g,t} I(P_{Cg}(t)) - \sum I(P_{Cg}(.))}; \text{ and,}$$

[34] The weights for all towns are equal if every town has at least one claim in each year. Those towns in which no claims occurred in some years would receive less weight.

EXHIBIT 10
SHEET 2

s_C^2 is defined as

$$s_C^2 = \sum_g w_g s_g^2 .$$

Since s_C^2 is a weighted average of s_g^2 for individual towns, a reasonable way to limit variations in s_C^2 is to limit the contribution made by any individual town. This can be accomplished by restricting the value of s_g^2 that enters the computation of s_C^2 to lie between chosen minimum and maximum values. The minimum and maximum values can be chosen as a factor times the overall s^2 (which is a weighted average of s_g^2 over all towns in the state). Factors of 1/5 and 5 were chosen judgmentally.

Thus, in computing s_C^2 for each county, s_g^2 for each town was restricted to be within a range of 1/5 or 5 times the overall s^2 for all counties.

$$s^2 = \frac{\sum_{Cgt} P_{Cg}(t)\,(X_{Cg}(t) - \overline{X}_{Cg})^2}{\sum_{Cgt} I(P_{Cg}(t)) - \sum I(P_{Cg}(.))}$$

$$\tilde{s}_g^2 = \begin{cases} 1/5\ s^2 & \text{if } s_g^2 \leqq 1/5\ s^2 \\ s_g^2 & \text{if } 1/5 s^2 \leqq s_g^2 \leqq 5 s^2 \\ 5 s^2 & \text{if } s_g^2 \geqq 5 s^2 \end{cases}$$

$$\tilde{s}_C^2 = \sum w_g \tilde{s}_g^2$$

The resulting values of $\tilde{s}_C^2$ which were used in the review for 1986 are displayed in Exhibit 11.

Limitations on k values

Even with the application of these limitations, calculated k values may exhibit some fluctuations. Therefore, for each coverage, the credibility parameters k and k_C (k applies to the state, while there is a k_C for each county) are limited by the imposition of a maximum value and a minimum value. When

EXHIBIT 10
SHEET 3

the calculated value was less than the minimum, the value of the parameter was set equal to that minimum.[35] When the calculated value was more than the maximum value, the value of the parameter was set equal to the maximum value.

The choice of maximum and minimum values for k and k_C involves the use of some actuarial judgment, although tests indicated that the resulting combined indices for towns are relatively insensitive to these choices. A maximum value of 2500 claims and a minimum value of 100 claims were used for all coverages. The resulting values of k and k_C which were used in the latest territory review are displayed in Exhibit 11.

[35] In certain cases, the calculated value of the parameter k_c was a large negative number. This occurred when the calculated denominator was negative because the observed variations of the average claim costs between the towns within a county were small relative to the observed variation of the average claim costs within the individual towns from year to year. (For the overall k this would have occurred if the observed variations of the average claim costs between the different counties were small relative to the observed variation within a county from year to year.) This case was treated as an extension of the case where the calculated denominator was a very small positive quantity, and the calculated parameter was a very large positive quantity. Thus in those cases where the calculated parameter was negative, its value was set equal to the ,chosen maximum value. This choice has the appropriate effect on credibilities: it will assign less credibility to the towns within a county and more to the county.

EXHIBIT 11
SHEET 1

CALCULATION OF THE CLAIM COST INDEX USING THE TWO LAYER HIERARCHICAL EMPIRICAL BAYESIAN CREDIBILITY MODEL

	Credibility Parameter K, Severity				
County Group	BI	PIP	PDL	Comp.	Coll.
Overall	548	740	515	143	1026
Barnst., Dukes, Nant.	132	571	501	766	2500
Berkshire	100	268	631	1276	100
Bristol	309	339	165	929	153
Essex	1503	703	1319	100	387
Franklin	2500	2500	2500	317	211
Hampden	1812	356	2500	185	342
Hampshire	2500	2500	1731	898	202
Middlesex	2500	544	2500	125	199
Norfolk	2500	667	1272	245	269
Plymouth	421	409	978	380	646
Suffolk	1005	332	1157	696	2500
Worcester	2500	227	631	719	177

EXHIBIT 11
SHEET 2

CALCULATION OF THE CLAIM COST INDEX USING THE TWO LAYER HIERARCHICAL EMPIRICAL BAYESIAN CREDIBILITY MODEL

County Group	Credibility Parameter S^2, Severity				
	BI	PIP	PDL	Comp.	Coll.
Overall	1.7	2.0	1.2	4.1	1.6
Barnst., Dukes, Nant.	1.4	2.3	1.3	2.7	1.6
Berkshire	1.4	2.2	1.3	2.5	1.4
Bristol	1.8	2.6	1.1	7.2	1.7
Essex	1.5	1.7	1.1	3.4	1.1
Franklin	2.1	1.5	1.3	1.8	1.9
Hampden	1.7	1.7	1.0	4.2	2.1
Hampshire	1.6	2.2	1.3	2.0	1.8
Middlesex	1.8	2.0	1.3	4.4	1.3
Norfolk	1.7	1.7	1.4	3.9	1.6
Plymouth	1.6	1.7	1.2	5.4	2.0
Suffolk	1.4	1.8	1.7	13.6	2.0
Worcester	1.9	1.6	1.0	3.0	1.7

EXHIBIT 12
SHEET 1

PRICING EXAMPLES—SEVERITY METHODOLOGY

BRIGHTON
SUFFOLK TOWN'S PDL EXPOSURES 15,872.8

	BI	PIP	PDL	COMP.	COLL.
(1) Claims for Town	657.0000	1,197.0000	5,569.0000	6,770.0000	6,388.0000
(2) Cred. Weighted Mean for County Group	1.0944	1.1421	1.0718	1.3749	.9381
(3) Overall K (Claims)	548.0728	740.3648	515.3687	142.6349	1,026.1405
(4) Credibility for County Group	.9034	.8148	.9580	.9825	.9526
(5) Cred. Weighted Mean Overall	.9968	.9891	.9900	.8527	1.0684
(6) Est. Rel. Sev., Cnty. = (2) × (4) + (5) × 1 − (4)	1.0849	1.1138	1.0684	1.3657	.9443
(7) Actual Relative Sev. for Town	1.1094	1.0858	1.0178	1.1788	.8842
(8) K for County Group (Claims)	1,004.7278	331.8594	1,156.6019	696.4471	2,500.0000
(9) Cred. for Town = (1)/((1) + (8))	.3954	.7829	.8280	.9067	.7187
(10) Est. Rel. Sev. for County Group = (6)	1.0849	1.1138	1.0684	1.3657	.9443
(11) Est. Rel. Sev., Town = (7) × (9) + (10) × 1 − (9)	1.0946	1.0919	1.0265	1.1962	.9011

EXHIBIT 12
SHEET 2

PRICING EXAMPLES—SEVERITY METHODOLOGY

HOLLAND
HAMPDEN   TOWN'S PDL EXPOSURES 914.0

	BI	PIP	PDL	COMP.	COLL.
(1) Claims for Town	14.0000	56.0000	164.0000	105.0000	128.0000
(2) Cred. Weighted Mean for County Group	.9334	.9513	.9325	.7344	1.0438
(3) Overall K (Claims)	548.0728	740.3648	515.3687	142.6349	1,026.1405
(4) Credibility for County Group	.9167	.8349	.9743	.9514	.8162
(5) Cred. Weighted Mean Overall	.9968	.9891	.9900	.8527	1.0684
(6) Est. Rel. Sev., Cnty. = (2) × (4) + (5) × 1 − (4)	.9387	.9575	.9340	.7402	1.0483
(7) Actual Relative Sev. for Town	.9478	.9118	1.0217	.6832	1.1810
(8) K for County Group (Claims)	1,812.2624	355.5324	2,500.0000	184.7182	342.3312
(9) Cred. for Town = (1)/((1) + (8))	.0077	.1361	.0616	.3624	.2737
(10) Est. Rel. Sev. for County Group = (6)	.9387	.9575	.9340	.7402	1.0483
(11) Est. Rel. Sev., Town = (7) × (9) + (10) × 1 − (9)	.9388	.9513	.9394	.7195	1.0846

EXHIBIT 12
SHEET 3

PRICING EXAMPLES—SEVERITY METHODOLOGY

WILMINGTON
MIDDLESEX TOWN'S PDL EXPOSURES 10,232.4

	BI	PIP	PDL	COMP.	COLL.
(1) Claims for Town	419.0000	734.0000	3,010.0000	2,386.0000	2,170.0000
(2) Cred. Weighted Mean for County Group	1.0063	.9865	1.0308	.8848	1.0383
(3) Overall K (Claims)	548.0728	740.3648	515.3687	142.6349	1,026.1405
(4) Credibility for County Group	.9726	.9524	.9923	.9772	.9014
(5) Cred. Weighted Mean Overall	.9968	.9891	.9900	.8527	1.0684
(6) Est. Rel. Sev., Cnty. = (2) × (4) + (5) × 1 − (4)	1.0060	.9866	1.0305	.8841	1.0413
(7) Actual Relative Sev. for Town	1.0411	.9717	1.0661	.9980	1.0517
(8) K for County Group (Claims)	2,500.0000	543.7111	2,500.0000	124.9303	198.6265
(9) Cred. for Town = (1)/((1) + (8))	.1435	.5745	.5463	.9502	.9161
(10) Est. Rel. Sev. for County Group = (6)	1.0060	.9866	1.0305	.8841	1.0413
(11) Est. Rel. Sev., Town = (7) × (9) + (10) × 1 − (9)	1.0111	.9781	1.0499	.9923	1.0508

A4. FINAL TOWN INDEX

This section describes the combining of the claim frequency indices (from Section A2) and claim cost indices (from Section A3) and the determination of an overall index that incorporates all coverages.

The first step is to calculate a pure premium index by town and coverage. This index is simply the product of the claim frequency index and the claim cost index, and is interpreted as being a measure of the town's pure premium (average insurance loss dollars per vehicle) relative to the statewide average pure premium.

Any town-to-town variation in pure premiums that is captured by other rating variables, however, should not also influence a town's territory assignment. Therefore, each town's pure premium index is adjusted to remove the effects of the mix of insured drivers by driver classification[36] as measured by the *ACRF* described above. The resulting town net pure premium indices are re-balanced to unity within each coverage. In the three town example for collison:

	HOLLAND	WILMINGTON	BRIGHTON
(1) Claim frequency index	.7435	.9665	1.7346
(2) Claim cost index	1.0846	1.0508	.9011
(3) Pure premium index = (1) × (2)	.8064	1.0156	1.5630
(4) Average class rating factor	.9682	1.0528	1.0014
(5) Net pure premium index, re-balanced to unity = ((3) ÷ (4)) ÷ 1.00015	.8328	.9645	1.5606

Finally, an average index across all coverages (c) is calculated for each town (t) by weighting the coverage net pure premium indices. The weight assigned to each coverage depends on the number of exposures purchasing the coverage and on the statewide pure premium for the coverage:

$$\frac{\sum_c \text{Exposures}_{c,t} \times \text{Statewide Pure Prem}_c \times \text{Net Pure Prem Index}_{c,t}}{\sum_c \text{Exposures}_{c,t} \times \text{Statewide Pure Premium}_c}.$$

[36] As noted in Section A3, a corresponding adjustment to remove the effects of varying distributions by age and symbol is incorporated in the claim severity index calculation.

The resulting index is balanced to unity (on the latest year's PDL exposures) across all towns.

Applying the above formula to the three towns:

	HOLLAND	WILMINGTON	BRIGHTON
(1) Exposure (latest year)			
A-1, A-2, PDL	914.0	10,232.4	15,872.8
Comprehensive	533.7	7,176.6	11,806.0
Collision	422.2	5,794.7	9,679.2
(2) Net Pure Premium Index			
A-1	.5397	1.0063	1.2748
A-2	.8712	.9694	1.2898
PDL	.6477	1.0451	1.3588
Comprehensive	.4253	.9674	2.0844
Collision	.8328	.9645	1.5606
(3) Statewide Average Pure Premium			
A-1	38.61	38.61	38.61
A-2	14.92	14.92	14.92
PDL	62.01	62.01	62.01
Comprehensive	57.28	57.28	57.28
Collision	120.00	120.00	120.00
(4) Balancing Factor	1.0011	1.0011	1.0011
(5) Weighted average net pure premium index	.6567	.9938	1.5361

The resulting index is used to rank the 360 towns according to their loss potential. For the three town example the ranks are:

	RANK
Holland	30
Brighton	349
Wilmington	302

APPENDIX B
HOMOGENEITY AND HOMOGENEITY MEASURES[37]

B1. INTRODUCTION

As discussed in the body of this paper, one of the criteria by which alternative territory schemes are assessed is homogeneity; i.e., towns within the same territory grouping should possess similar inherent loss potential. If the territories are to be homogeneous then no town's loss potential measure should differ substantially from the average loss potential measure of all towns in that territory. This notion can be used formally to construct several quantitative indices which then can be used to guide the ratemaker in some of the grouping judgments which need to be made.

This appendix defines the indices that have been constructed for use in Massachusetts; all of them are referred to as *homogeneity measures* and are displayed in Exhibit 6.

B2. LOSS POTENTIAL

There are two readily available data sources which can be used to indicate a town's loss potential. One is the value of the combined index produced by the procedure described in Appendix A and displayed in Exhibit 5 for a sample of towns. Another is the actual latest three year experience pure premiums for the liability coverages and for the typical package of coverages.[38] Exhibit 5 also displays these pure premiums for a sample of towns. Each measure has relevance. The combined index is a true credibility weighted estimate of a synthetic pure premium relationship between towns, while the actual three year pure premiums are the data used to set territory relativities in the ratemaking process. Rather than choose between these two measures, both are used as homogeneity indicators.

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[37] This appendix was taken, with minor editing, from sections of the Massachusetts Automobile Rating and Accident Prevention Bureau's *Filing for 1986 Private Passenger Territory and Classification Definitions*, July, 1985. These sections of the MARB's filed analysis, including the specific homogeneity measures, were developed and prepared by Dr. Richard Derrig.

[38] The liability coverages consist of basic limits (10/20) A-1, PDL (5,000) and A-2. The package coverages consist of A-1 (10/20), PDL (5,000), A-2, Collision, and Comprehensive.

Homogeneity Measures

This section defines several measures of the homogeneity of a territory grouping procedure. In general, the measures test the difference between the town's loss potential and the average of the entire territory's loss potential. The measures utilize both the actual pure premium and the combined index values of loss potential. The first tests calculate both the average absolute squared difference (measure 1) and the percentage squared difference for the pure premium values. Since the latter will measure the percentage difference from the town's actual pure premium, which might be unstable for small towns, this measure is calculated with (measure 2) and without (measure 2A) a credibility weight for the reliability of the actual data. In order to test the average spread of the territory grouping, the next measures rely on the average maximum deviations of the town value from the territory average both using the absolute difference (measure 4), percentage difference with (measure 5) and without (measure 5A) a credibility weight, and the model combined index (measure 6). The precise definitions are listed in Exhibit 13. For all these measures, a homogeneity value closer to 0 indicates a more homogeneous set of territories.

B3. ERROR ENTROPY

One further measure of homogeneity can be defined based upon the information-theoretic concept of entropy. In general, entropy quantifies the degree of disorder or uncertainty in a system. An entropy-like measure is applied to determine the disorder or uncertainty in the difference between a town's combined index and the territory average index. In a sense, that difference is the "error" which results when the territory average index is assigned to the town. This is the assumption of perfect homogeneity. The entropy measure will then quantify the relative information about the concentration of these errors among territory grouping procedures. The notion of entropy has been used in a somewhat similar way by Garrison and Paulson [5] to compare concentrations in economic activity over time.

Consider a set of k categories $C_1, \ldots, C_k$ and a random sample of size n. Each observation of the sample falls into one of the categories C_i with some fixed probability $p_i > 0$; $i = 1, 2, \ldots, k$ with $\Sigma p_i = 1$, and in the sample a total of n_i observations fall into category C_i. Then the *entropy* or expected information of the system is defined by

$$H = \sum_{i=1}^{k} p_i \log p_i \, .$$

EXHIBIT 13
SHEET 1

HOMOGENEITY MEASURE DEFINITIONS

MEASURE	DEFINITION
1. Pure Premium Squared Diff.	$\sum_{\text{Town}_i} 83 \text{ EXP}_i (\text{Town PP}_i - \text{Terr PP}_i)^2 \div \sum_{\text{Town}_i} 83 \text{ EXP}_i$
2. Pure Premium Cred. Wgtd. Percentage Squared Diff.	$\sum_{\text{Town}_i} 83 \text{ EXP}_i \text{ Max Cred}_i \left(\frac{\text{Town PP}_i - \text{Terr PP}_i}{\text{Town PP}_i}\right)^2 \div \sum_{\text{Town}_i} 83 \text{ EXP}_i$
2a. Pure Premium Percentage Squared Diff.	$\sum_{\text{Town}_i} 83 \text{ EXP}_i \left(\frac{\text{Town PP}_i - \text{Terr PP}_i}{\text{Town PP}_i}\right)^2 \div \sum_{\text{Town}_i} \text{EXP } 83_i$
3. Index Squared Diff.	$\sum_{\text{Town}_i} 83 \text{ EXP}_i (\text{Town Ind}_i - \text{Terr Ind}_i)^2 \div \sum_{\text{Town}_i} 83 \text{ EXP}_i$
4. Pure Premium Maximum Diff.	$\sum_{\text{Terr}_i} 83 \text{ EXP}_i \max_i \lvert \text{Town PP}_i - \text{Terr PP}_i \rvert \div \sum_{\text{Town}_i} 83 \text{ EXP}_i$

EXHIBIT 13
SHEET 2

HOMOGENEITY MEASURE DEFINITIONS

MEASURE	DEFINITION
5. Pure Premium Cred. Wgtd. Percentage Max. Diff.	$\sum_{\text{Terr}_i} 83\ \text{EXP}_i \max_i \text{Max Cred}_i \left\lvert \frac{\text{Town PP}_i - \text{Terr PP}_i}{\text{Town PP}_i} \right\rvert \div \sum_{\text{Town}_i} 83\ \text{EXP}_i$
5a. Pure Premium Percentage Max. Diff.	$\sum_{\text{Terr}_i} 83\ \text{EXP}_i \max_i \left\lvert \frac{\text{Town PP}_i - \text{Terr PP}_i}{\text{Town PP}_i} \right\rvert \div \sum_{\text{Town}_i} 83\ \text{EXP}_i$
6. Index Max. Diff.	$\sum_{\text{Terr}_i} 83\ \text{EXP}_i \max_i \lvert \text{Town Ind}_i - \text{Terr Ind}_i \rvert \div \sum_{\text{Town}_i} 83\ \text{EXP}_i$
7. Error Entropy	$-\sum_{e_i} (\text{EXP}_{(e_i)}/\text{EXP}) \log (\text{EXP}_{(e_i)}/\text{EXP})$

EXHIBIT 13
SHEET 3

HOMOGENEITY MEASURE DEFINITIONS

NOTATIONAL CONVENTIONS

1. 83 EXP_i means the 1983 PDL exposure in earned car years for Town i.
2. Town PP_i means the pure premium of 1981–1983 losses divided by 1981–1983 earned car years for Town i.
3. Terr PP_i means the pure premium of 1981–1983 losses divided by 1981–1983 earned car years for all towns in the territory containing Town i.
4. Max $Cred_i$ means the maximum of the Empirical Bayes produced credibility values for all coverages (5 or 6) for Town i.
5. Town Ind_i means the model combined index for Town i.
6. $EXP_{(e_i)}$ means the total earned car years of exposure for all towns whose "error," Town Ind_i − Terr $Ind_i = e_i$, lies in the interval (e_i).
7. EXP means total exposure in earned car years.

The underlying probabilities p_i indicate the strength or concentration of the category C_i. On a sampling basis, for purposes of the current analysis, entropy is defined by the approximation[39]

$$h = - \sum_{i=1}^{k} (n_i/n) \log (n_i/n).$$

The greatest uncertainty occurs when H (or h) is the maximum value of log k, while the least uncertainty (most categorial information) occurs when H (or h) equals zero.

The construction of territories seeks the information content for the per exposure error in territory index assignment to towns. Assuming homogeneous towns, the sample size is the total exposure n. The categories are intervals of errors. (For this application, intervals of .01 were chosen to define categories.)

C_{-2}		C_{-1}		C_0		C_1		C_2
	$-.01$		0		.01		.02	

Thus, define:

$$n_i = \sum_t \text{Town Exposure}_t$$

when e_t = Town Index$_t$ − Territory Index$_t$ falls into C_i. Then, the entropy measure h will define the "concentration" of the errors e_t. The smaller the value of h, the more homogeneous the territory grouping will be. This is designated as homogeneity measure 7 and labelled the "Error Entropy" measure.

39 As usual, if $n_i = 0$ then $(n_i/n) \log (n_i/n) = 0$.