AN ANALYSIS OF EXPERIENCE RATING

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Abstract

Experience rating formulas that are currently in use have features that have no counterpart in the literature on Bayesian credibility. These features include the limiting of individual losses that go into the experience rating, separate treatment of primary and excess losses, and the gradual transition to self-rating. This paper analyzes the effect of these features using the collective risk model.

Most developments in Bayesian credibility assume that the variance of an individual insured's experience is inversely proportional to the size of the insured. This will not be the case if the parameters of the insured's loss distribution are changing over time. This paper analyzes the effect of this parameter uncertainty on the Bayesian credibility formulas.

Finally, Paul Dorweiler's method of testing experience rating formulas is updated using modern statistical methodology. The result is a very general method of evaluating the parameters of an experience rating formula.

1. INTRODUCTION

The passage of open competition laws for Workers' Compensation has indeed sparked a high degree of competition. Much of the competition is taking place on the individual insured level in the form of schedule and experience rating. In this new competitive environment the performance of these rating plans becomes crucial. The purpose of this paper is to examine the performance of some experience rating plans that are currently being used.

The predominant experience rating plan for Workers' Compensation is promulgated by the National Council On Compensation Insurance (NCCI). This plan is widely adhered to. In addition, the National Council performs the service of maintaining the experience and calculating the experience modification for each insured. These services relieve the insurance companies of considerable administrative expense.

For lines other than Workers' Compensation, an experience rating plan is promulgated by the Insurance Services Office (ISO). Variations from this plan by individual insurance companies are common. Also, ISO does not maintain experience for individual insureds. Getting reliable experience for new insureds is a real problem.

When designing experience rating plans, there are some administrative considerations that cannot be overlooked. The first is that experience ratings are done frequently and so simplicity is of paramount importance

A second consideration is that experience rating, as opposed to class rating, is very visible to the individual insured. A consequence of this is that the experience rating plans must give due consideration to what the insured perceives to be fair. Historically, see Snader [1], these considerations have included the following:

1. A single claim should change the experience modification by no more than a predetermined amount. This predetermined amount is known as the swing of the experience rating plan.
2. All insureds above a certain predetermined size are self-rated, that is they are rated entirely on the basis of their own experience.

In addition to the administrative considerations mentioned above, there are some mathematical considerations that should be made. The mathematical foundations of experience rating come from Bayesian estimation and credibility theory. As is the case with many other mathematical theories, a simplified mathematical model is proposed, and the optimal method of rating the insured is derived. The success of Bayesian estimation and credibility theory depend upon how closely the model represents reality.

The experience rating formulas derived from administrative considerations, hereafter referred to as "practical" formulas, may be different from those derived from the mathematical considerations, hereafter referred to as "theoretical" formulas. This paper investigates the compatibility of these two kinds of rating formulas. We would judge the formulas to be compatible if the accuracy of the "practical" formula is near that of the "theoretical" formula on the simplified models. While it is by no means certain that accuracy on simplified models implies accuracy in real life situations, inaccuracy on a simplified model should imply that something is wrong with the formula being tested.

Our first goal is to find "practical" formulas that perform well on simplified models. These formulas will depend upon unknown parameters which must be estimated from data. Our second goal is to show how these unknown parameters can be estimated. An example will be provided.

2. CURRENT EXPERIENCE RATING FORMULAS

We begin by briefly describing two experience rating plans that are currently in use. We will concentrate on the structure of the plans. The methods currently being used to derive the parameters of the plan are not really an issue at this time. In what follows, an experience modification will refer to the ratio of the premium after experience rating to the premium before experience rating.

2.1 The Workers' Compensation Experience Rating Plan

The Workers' Compensation Experience Rating Plan [2] has a long and rich history. Its development is described in detail by Perryman [3], Uhthoff [4] and Snader [1]. It is very much a "practical" experience rating plan and it has a strong appeal to common sense.

A feature of this plan is the partitioning of the actual losses into primary losses, denoted by A_p, and excess losses, denoted by A_e. In most states, the primary part, X_p, of a claim of amount X is given by the following formula:

$$X_p = X \qquad \text{if } X \leq 2000$$

$$X_p = \frac{10000 \times X}{X + 8000} \qquad \text{if } X > 2000.$$

The excess part of a claim, X_e, is equal to $X - X_p$. A_p is the total of the primary parts of all claims, and A_e is the total of the excess parts.

Let: E_p = expected primary loss;
E_e = expected excess loss; and
$E = E_p + E_e$.

Then the experience modification, *Mod*, is given by the following formula:

$$Mod = \frac{A_p + W \times A_e + (1 - W) \times E_e + (1 - W) \times K}{E + (1 - W) \times K}.$$

W is equal to zero for E less than some number Q, typically 25,000, and increases linearly to one as E increases to the self rating point S, which is usually around 500,000. K is generally set equal to 20,000.

E_p and E_e are products of expected loss rates and the amount of exposure for the insured. These expected loss rates are in the Workers' Compensation rating manual and are updated whenever there is a rate change.

This formula has some very appealing properties:

1. If $E \leq Q$, the formula simplifies to the following:

$$Mod = \frac{A_p + E_e + K}{E + K} . \qquad \text{(Equation 2.1)}$$

   This simplifies experience rating for small insureds.

2. Since X_p is always less than 10,000, the impact of a single large claim on the modification is limited.

3. The insured is self-rated for $E \geq S$. Also, the transition to self-rating is gradual.

4. It is generally believed that claim frequency rather than claim severity differentiates the good insured from the poor insured. The relatively greater impact of small claims is consistent with this belief.

2.2 The General Liability Experience Rating Plan

The General Liability Experience Rating Plan [5], like the Workers' Compensation plan, is very much a "practical" experience rating plan.

Let: ALR = adjusted actual loss ratio;
$AELR$ = adjusted expected loss ratio; and
Z = credibility factor.

Then the experience modification, *Mod*, is given by the following formula:

$$Mod = 1 + \frac{ALR - AELR}{AELR} \times Z.$$

The term "adjusted" refers to the fact that individual claim amounts are limited before entering the experience rating calculation. This limit increases with premium size. It is chosen so that a single large claim can change the experience modification by no more than .3.

Let: P = premium associated with the loss period; and
K = credibility constant (currently 100,000).

Then the credibility is given by the following formula:

$$Z = \frac{P}{P + K} .$$

If this formula were to apply for all values of P, no insured would ever be self-rated. Since self-rating is desired for very large insureds, the credibility formula changes to a linear function between a selected point, Q, and a selected self-rating point, S.

For $E > Q$:

$$Z = \frac{Q^2 + K \times E}{(Q + K)^2} . \qquad \text{(Equation 2.2)}$$

In the current General Liability experience rating plan, $K = 100{,}000$, $Q = 483{,}333$ and $S = 1{,}049{,}654$.

Currently, the premium used is collected basic limits premium. However, this is slated to be revised in 1985. The premium used in the adjusted expected loss ratio will be based on estimated prospective premium and adjusted for inflation and average exposure growth. Ideally, the premium should be based on the actual exposures of the experience period, but administrative considerations led to using estimated prospective premium. It should be noted that the plan contains optional provisions to use actual exposures if they are available.

When comparing the two experience rating plans, it should be noted that the Workers' Compensation plan is mandatory in most states. This includes many open competition states! The National Council can enforce the standards of their plan on all companies. They do this, of course, with the consent of the member companies.

3. MATHEMATICAL MODELS FOR EXPERIENCE RATING

Let X be a random variable which represents the total loss incurred by the insured. Let E be a measure of the size of the insured. E could be either the expected loss for the average insured or the premium of the insured which has been determined by a rating manual. Let $R = X/E$ and $\mu = \mathrm{E}[R]$, where E[ ] denotes expected value. R and μ are called the loss ratio and the expected loss ratio.

Experience rating is based on the premise that the expected loss ratio, μ, is different for each insured in a given classification. To model this, we assume that an insured has a loss ratio distribution, d, which is selected at random from a class of distributions, D. Each distribution d has its own mean, μ, and variance, V^2. Let $M = \mathrm{E}[\mu]$, $\tau^2 = \mathrm{Var}[\mu]$, and $\sigma^2 = \mathrm{E}[\nu^2]$, where these statistics are calculated over all distributions d in D.

This process is described by the following algorithm:

Algorithm 3.1

1. Select the distribution, d, along with μ and ν^2, at random from the class of distributions D.

2. Select the loss ratio, R, at random from the distribution d.

The goal of experience rating is to estimate the expected loss ratio, μ, given the loss ratio, R.

Two solutions to this problem are described by Bühlmann [6]. The first solution is the Bayesian solution:

$$B(R) = \mathrm{E}[\mu|R]$$

Bühlmann shows that the solution is optimal in the sense that

$$\mathrm{E}[(B(R) - \mu)^2]$$

is minimized.

A drawback to the Bayesian solution is that it requires knowledge of all the distributions d in D. The second solution, called the credibility solution, only requires knowledge of the quantities M, τ^2 and σ^2. It can be written in the form:

$$C(R) = Z \times R + (1 - Z) \times M.$$

Z is called the credibility factor. We want to choose Z so that

$$\mathrm{E}[(C(X) - \mu)^2]$$

is minimized. The solution, given by Bühlmann, is

$$Z = \frac{\tau^2}{\tau^2 + \sigma^2} \qquad \text{(Equation 3.1)}$$

Bühlmann goes on to show that the same choice of Z minimizes

$$\mathrm{E}[(C(R) - B(R))^2].$$

Thus the credibility solution can be characterized as the best linear approximation to the Bayesian solution. As Hewitt [7] and Mayerson [8] demonstrate, the Bayesian solution can be linear, and thus the credibility solution is identical to the Bayesian solution. However, Hewitt also gives an example where the Bayesian solution is different from the credibility solution. As we shall see below, the distinction can be important.

We shall use the collective risk model to describe the distribution of the losses. This model describes the total losses of an insured in terms of its claim count and claim severity distributions. This model has been described extensively by Meyers and Schenker [9], Heckman and Meyers [10], and Panjer [11].

Let N and S be random variables denoting the claim count and the claim severity for an insured, respectively. In its simplest form, the collective risk model can be described by the following algorithm:

Algorithm 3.2

1. Select the claim count, N, at random from a Poisson distribution.
2. Do the following N times:
   2.1 Select the claim severity, S, at random.
3. Set the total loss, X, equal to the sum of the claim amounts, S, selected in step 2.1.

Since credibility formulas are applied over a wide range of premium sizes, we need to be concerned with how the quantity σ^2 varies with premium. The usual assumption made is to let σ^2 vary inversely with premium. This is done mathematically by setting $\sigma^2 = \Sigma^2/E$, where Σ^2 is the constant of proportionality.

This assumption agrees with the intuition of many actuaries. One would certainly expect the variance of the loss ratio to decrease as E increases. This assumption can also be justified using collective risk theory. If we assume that the claim count distribution is Poisson for each insured and that the claim severity distribution is the same for all insureds, then it is demonstrated in Appendix A that σ^2 is inversely proportional to E.

Substituting Σ^2/E for σ^2 in Equation 3.1 yields the following expression for the credibility:

$$Z = \frac{E}{E + K} \qquad \text{(Equation 3.2)}$$

where $K = \Sigma^2/\tau^2$.

This formula for credibility is almost universally used in the actuarial literature on Bayesian credibility. An exception to this is in a paper by Robert A. Bailey and LeRoy J. Simon [12]. This exception is important and their demonstration is worth discussing in detail.

Using experience from the Canadian Merit Rating Plan, they were able to calculate empirical credibilities for the experience of a single private passenger car for one, two and three years of experience. These credibilities are given in the following table.

TABLE 3.1

EMPIRICAL CREDIBILITIES

Class	1 Year	2 Years	3 Years
1	.046	.068	.080
2	.045	.060	.068
3	.051	.068	.080
4	.071	.085	.099
5	.038	.050	.059

Let E denote the number of years in the merit rating period. Using the credibilities based on one year, the constant K in the credibility formula $Z = E/(E + K)$ is calculated. The credibilities for two and three years are then calculated using this value of K. The results are in the following table.

TABLE 3.2

DERIVED CREDIBILITIES $Z = E/(E + K)$

Class	K	2 Years	3 Years
1	20.7	.088	.126
2	21.2	.086	.124
3	18.6	.097	.139
4	13.1	.133	.187
5	25.3	.073	.106

We see, as Bailey and Simon observed, that the usual assumptions suggest that credibility should increase roughly in proportion to the number of years in the experience rating period. When comparing Tables 3.1 and 3.2 we see that the empirical credibilities are significantly less than what the usual assumptions would suggest!

Bailey and Simon attribute the failure of the usual assumptions to match the empirical credibilities, in part, to an "individual insured's chance for an accident changes from time to time within a year and from one year to the next." This phenomenon is very similar to that of parameter uncertainty, which is described by Meyers and Schenker [9]. In Appendix A it is demonstrated that the collective risk model with parameter uncertainty implies that σ^2 is of the form $\Sigma^2/E + \beta$, where $\beta > 0$. Substituting $\Sigma^2/E + \beta$ for σ^2 in Equation 3.1 yields the following expression for the credibility:

$$Z = \frac{E}{E \times J + K} \qquad \text{(Equation 3.3)}$$

where $J = 1 + \beta/\tau^2$ and $K = \Sigma^2/\tau^2$.

Using Equation 3.3, it is possible to take the credibilities for one and two years and solve for J and K. One can then calculate the credibility implied for three years. The results of these calculations are in the following table:

TABLE 3.3

DERIVED CREDIBILITIES $Z = E/(E \times J + K)$

Class	J	K	3 Years
1	7.7	14.1	.081
2	11.1	11.1	.068
3	9.8	9.8	.077
4	9.4	4.6	.091
5	13.7	12.6	.056

By comparing the above tables we see that the credibilities derived using Equation 3.3 come much closer to the empirical credibilities than those derived using Equation 3.2.

It should be noted that the maximum credibility obtainable in Equation 3.3 is $1/J$. Recall $J \geq 1$. Low maximum credibilities could be interpreted by saying that the insured is changing over time and that change is of a significant size when compared to differences between insureds.

Besides parameter uncertainty, there are other reasons why the usual assumptions may not be appropriate. The variable loss limit that is in the ISO experience rating plans is one such case. In Appendix A, it is demonstrated that the constant of proportionality, Σ^2, depends upon the second moment of the claim severity distribution. Since the effect of changing the loss limit is to change the claim severity distribution, one should not expect Σ^2 to be the same for all loss limits.

Since the loss limit increases with premium size, we would expect σ^2 to decrease slower than $1/E$ (See Appendix A.) Thus an attempt to impose a credibility formula of the form $Z = E/(E + K)$ will result in credibilities which are too small for the small insureds, and too large for the large insureds.

The formula $\sigma^2 = \Sigma^2/E + \beta$ also has the property that σ^2 decreases slower than $1/E$. Thus Equation 3.3 should provide a better estimate of the credibility. But the derivation of Equation 3.3 did not anticipate an increasing loss limit, and so one should not expect the estimated credibility to be perfect.

4. THE EFFICIENCY OF AN EXPERIENCE RATING PLAN

In the previous section we discussed optimal (for specific assumptions) experience rating plans. There are a number of reasons why an optimal plan might not be used. As discussed above, there may be several practical reasons for using some alternative plan. Another reason is that one must estimate the parameters M, τ^2 and σ^2. Estimation error will occur. The purpose of this section is to present a yardstick for comparing the performances of alternative experience rating plans.

The purpose of experience rating is to estimate the expected loss ratio, μ. If experience rating were not used, our estimate of μ would be M, which would be subject to error. A good measure, with historical precedent, would be to calculate the amount the expected error is reduced by a given experience rating formula.

Let F be an estimator of μ which results from an experience rating formula. F can be a function of any kind of loss experience of the insured such as total losses, claim count or limited losses. We then define the efficiency of F by the expression:

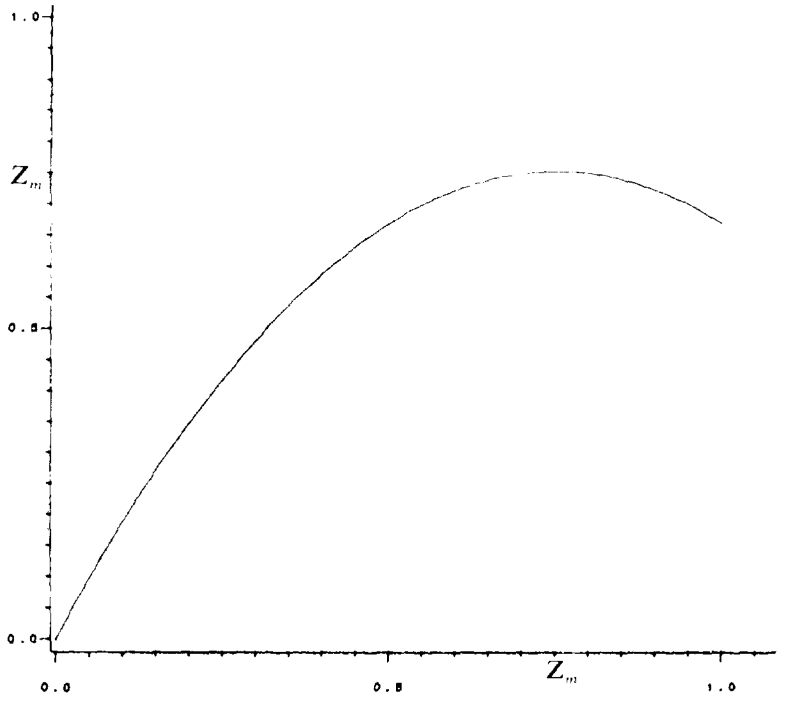
$$\frac{\mathrm{E}[(\mu - M)^2] - \mathrm{E}[(\mu - F)^2]}{\mathrm{E}[(\mu - M)^2]}$$

If F is a perfect estimator for μ, its efficiency will be equal to 1. If $F = M$, its efficiency will be equal to 0. It is possible, as we shall soon see, for the efficiency to be negative for a poorly chosen F.

One should note the similarity of this measure of efficiency with the statistic R^2 that is used in regression analysis. It is different from R^2 in that it does not automatically assume that F was chosen in some optimal manner.

If F is a credibility estimator of the form $Z \times R + (1 - Z) \times M$, it is shown in Appendix B that the efficiency of F is given by the expression $2 \times Z - Z^2/Z_m$, where Z_m is the optimal credibility given by Equation 3.1. A graph of the efficiency as a function of S is shown in Figure 1. This expression has the following properties:

FIGURE 1



1. The efficiency is maximized when $Z = Z_m$. This is Bühlmann's [6] result.

2. As a function of Z, the efficiency starts at 0 when $Z = 0$, raises to a maximum of Z_m when $Z = Z_m$ and falls to 0 when $Z = 2 \times Z_m$. The efficiency is negative for $Z > 2 \times Z_m$.

It is not difficult to see why credibility , even the non-scientific version, has been so popular. If $Z < 2 \times Z_m$ then a credibility estimate using Z will be more accurate than no experience rating. If $Z_m > 0.5$ then any choice of $Z \leq 1$ will guarantee an improvement in accuracy.

It should be noted that Z_m is not the maximum efficiency obtainable by any experience rating formula. As noted above, a Bayesian formula could be more accurate. As we shall soon see, it is also possible that an experience rating formula that uses detailed information such as claim count and claim severity can be even more accurate than the Bayesian formula.

5. THE GENERAL LIABILITY EXPERIENCE RATING PLAN

We now use the concepts developed above to analyze the General Liability experience rating plan. In particular, we will discuss the effect of self-rating and loss limits. Also, credibility and Bayesian estimation will be compared.

Let us suppose, for the sake of discussion, that the credibility formula $Z_m = E/(E + K)$, with $K = 100{,}000$ is the "correct" formula. Now suppose that instead of using Z_m for credibility we use $Z = (Q^2 + K \times E)/(Q + K)^2$, where $Q = 483{,}333$. Then the following table shows the efficiency of the formula for Z.

TABLE 5.1

E	Z	Efficiency of Z	Z_m
500,000	.8335	.8333	.8333
600,000	.8629	.8571	.8571
700,000	.8922	.8748	.8750
800,000	.9216	.8877	.8889
900,000	.9510	.8971	.9000
1,000,000	.9804	.9035	.9091

Examination of this table shows that there is minimal loss of efficiency when using Z instead of Z_m. If one accepts the credibility formula $Z = E/(E + K)$, the gradual shift to self-rating should also be acceptable.

We now turn to loss limits. The collective risk model will be used to describe the loss distributions. The mathematics will be less cumbersome if there is a finite number of loss amounts. For this reason, the claim count distribution will be binomial with N trials and the probability of a claim equal to p. The claim severity distribution will be a discrete version of the shifted Pareto, which is used to describe claim severity in many lines of casualty insurance. The probability, $F(x)$, that a claim will be less than or equal to x is given by:

$$F(x) = 1 - (b/(x + b))^q \quad x = 1, 2, \ldots, 49.$$

The remaining probability will be at the basic limit, 50.

The parameter q will be set equal to 1.25 for all prior distributions. The parameter b, in the claim severity distribution and p, in the claim count distribution may be different for each prior. N will reflect the size of the insured.

For a selected loss limit, L, the total losses can vary anywhere from 0 to $L \times N$. Using Panjer's algorithm [11] one can calculate the probability of each total loss for each prior distribution. One then calculates credibility and Bayes estimates of the experience modification for basic limits losses, as well as the efficiency of each estimate. Detailed calculations are given for one case in Exhibit 5.1. Efficiencies for several cases are given in Tables 5.2–5.4.

We first consider the case where only the claim count distributions vary. The efficiency is at a maximum for both the credibility estimator and the Bayes estimator when the loss limit is equal to 1, and decreases as the loss limit increases. This should come as no surprise. When the loss limit is 1, there is no random element due to claim severity. As we increase the loss limit, we increase the randomness in our measurements.

As expected, the Bayes estimator is more accurate than the credibility estimator. It is worth noting that the Bayes estimator is less affected by the increasing loss limit. The accuracy of the credibility approximation to the Bayes estimator gets worse as the loss limit increases.

We now turn to the case where only the claim severity distributions are varying. In this case, information about the distribution, as well as the random element, increases as the loss limit increases. The efficiency of both the credibility and the Bayes estimators is near maximum at a loss limit of 8. After that point the increase in efficiency is, at best, marginal. In fact it can decrease.

When both the claim severity and claim count distributions vary, the efficiency first increases and then decreases as the loss limit increases. The best loss limit is 4 for this example.

Attempting to draw conclusions about real life experience rating plans from models can be a risky undertaking. But accuracy is important, and not attempting to draw conclusions can also be risky. With this in mind, we proceed.

The first conclusion is that limiting the loss for an individual claim is a good idea. A well chosen loss limit will be large enough to capture differences in claim severity distributions. If the loss limit is too large, increased randomness will wipe out any extra information gained by the higher loss limit. This has been the traditional argument in favor of loss limits. It is gratifying to see it verified on a mathematical model.

While the Bayes estimator is more accurate, in practice we do not have enough information to use it. An alternative is to create conditions where the credibility estimator is a good approximation to the Bayes estimator. A loss limit serves this purpose.

The negative effects of high loss limits appear to be less pronounced for larger insureds. Perhaps this could be taken as justification for varying the loss limit. However one should not raise the loss limit indefinitely. Once the loss limit reaches a sufficient level to capture enough information on the claim severity, it should go no higher.

EXHIBIT 5.1

Credibility and Bayes Estimates

$N = 4$ Loss limit $= 4$

Prior#	Weight	p	b	q	Limited Sev. Mean	Limited Std. Dev.	Basic Limits Sev. Mean
1	0.25	0.20	0.25	1.25	1.24	0.69	1.48
2	0.25	0.30	0.50	1.25	1.47	0.93	2.03
3	0.25	0.40	0.75	1.25	1.68	1.07	2.57
4	0.25	0.50	1.00	1.25	1.85	1.16	3.09

Aggregate Probabilities

X	Prior#1	Prior#2	Prior#3	Prior#4
0	0.40960000	0.24010000	0.12960000	0.06250000
1	0.35481700	0.30735000	0.22576000	0.14488800
2	0.14376800	0.19673800	0.19920200	0.16774800
3	0.04484430	0.09761280	0.13228600	0.14045300
4	0.02853980	0.07571090	0.11665700	0.13708900
5	0.01326000	0.04745030	0.09041290	0.12598400
6	0.00364740	0.02021520	0.05102120	0.08841880
7	0.00090941	0.00771968	0.02505340	0.05248160
8	0.00044414	0.00442423	0.01612450	0.03707380
9	0.00013752	0.00186283	0.00844632	0.02315500
10	0.00002474	0.00053879	0.00323913	0.01092810
11	0.00000535	0.00016668	0.00122928	0.00481558
12	0.00000245	0.00008447	0.00068151	0.00287956
13	0.00000042	0.00002078	0.00021562	0.00111993
14	0.00000004	0.00000341	0.00004769	0.00029946
15	0.00000001	0.00000101	0.00001552	0.00010565
16	0.00000000	0.00000048	0.00000819	0.00006104

Limited Grand Mean = 2.28643 Credibility = .216312

EXHIBIT 5.1 (continued)

X	Prob(X)	Credibility *Mod*	Bayes *Mod*	Difference
0	0.21045000	0.7837	0.7325	0.0512
1	0.25820400	0.8783	0.8629	0.0154
2	0.17686400	0.9729	1.0177	−0.0448
3	0.10379900	1.0675	1.1504	−0.0829
4	0.08949920	1.1621	1.2006	−0.0385
5	0.06927680	1.2567	1.2722	−0.0155
6	0.04082570	1.3513	1.3487	0.0026
7	0.02154100	1.4459	1.4012	0.0447
8	0.01451670	1.5405	1.4218	0.1188
9	0.00840042	1.6352	1.4552	0.1799
10	0.00368269	1.7298	1.4880	0.2417
11	0.00155422	1.8244	1.5071	0.3173
12	0.00091200	1.9190	1.5152	0.4037
13	0.00033919	2.0136	1.5347	0.4789
14	0.00008765	2.1082	1.5498	0.5584
15	0.00003055	2.2028	1.5551	0.6477
16	0.00001743	2.2974	1.5605	0.7369

Expected Error: Bayes = .226192 Credibility = .230147
Efficiency: Bayes = .222546 Credibility = .208954

TABLE 5.2

COUNT DISTRIBUTIONS VARY

Prior	p	b	q
#1	0.2	0.75	1.25
#2	0.3	0.75	1.25
#3	0.4	0.75	1.25
#4	0.5	0.75	1.25

Loss Limit	Credibility			Bayes		
	$N = 4$	$N = 8$	$N = 16$	$N = 4$	$N = 8$	$N = 16$
1	.189	.317	.482	.190	.323	.496
4	.123	.218	.359	.152	.237	.377
8	.087	.160	.275	.147	.214	.309
12	.069	.130	.230	.146	.210	.288
16	.059	.112	.201	.145	.209	.281

TABLE 5.3

SEVERITY DISTRIBUTIONS VARY

Prior	p	b	q
#1	0.4	0.25	1.25
#2	0.4	0.50	1.25
#3	0.4	0.75	1.25
#4	0.4	1.00	1.25

Loss Limit	Credibility			Bayes		
	$N = 4$	$N = 8$	$N = 16$	$N = 4$	$N = 8$	$N = 16$
4	.026	.051	.096	.038	.059	.101
8	.035	.068	.127	.046	.077	.134
12	.035	.068	.127	.048	.079	.137
16	.034	.065	.122	.048	.080	.137

TABLE 5.4

COUNT AND SEVERITY DISTRIBUTIONS VARY

Prior	p	b	q
#1	0.2	0.25	1.25
#2	0.3	0.50	1.25
#3	0.4	0.75	1.25
#4	0.5	1.00	1.25

Loss Limit	Credibility			Bayes		
	$N = 4$	$N = 8$	$N = 16$	$N = 4$	$N = 8$	$N = 16$
1	.189	.317	.482	.190	.323	.496
4	.209	.344	.507	.223	.365	.536
8	.178	.301	.461	.223	.354	.510
12	.154	.267	.421	.223	.351	.495
16	.138	.242	.389	.223	.350	.488

6. THE WORKERS' COMPENSATION EXPERIENCE RATING PLAN

It was demonstrated in the last section that a loss limit can increase the accuracy of an experience rating plan. However, the Workers' Compensation Experience Rating Plan gradually introduces excess losses as the size of the insured increases. We now analyze this treatment of excess losses using the-collective risk model.

We shall use the Weibull distribution to model claim severity. The probability, $F(x)$, that a claim will be less than or equal to x is given by:

$$F(x) = 1 - e^{-(x/b)^c}.$$

The Poisson distribution will be used to model claim count. The probability of n claims, $P(n)$, is given by:

$$P(n) = e^{-\lambda} \times \lambda^n/n!$$

The parameter c will be set equal to .25 for all prior distributions. The parameter b for the claim severity distribution and the parameter λ for the claim count distribution will be independently chosen at random from the following table. Each parameter value is equally likely to be chosen.

TABLE 6.1

b	λ
30	40
40	70
50	100
60	130
70	160

It was necessary to resort to Monte Carlo methods in order to properly treat primary and excess losses. The following algorithm was repeated 10,000 times.

Algorithm 6.1

1. Select the Poisson parameter, λ, at random from Table 6.1.
2. Select the number of claims, n, at random from a Poisson distribution with parameter λ.

3. Select the Weibull parameter, b, at random from Table 6.1.

4. Do the following n times.
   4.1 Select a claim value, x, at random from a Weibull distribution with parameter b and $c(=.25)$.
   4.2 From x, calculate the primary loss, x_p, and the excess loss, x_e.

5. A_p is the sum of all the x_p's and A_e is the sum of all the x_e's.

It can be demonstrated by numerical integration of the severity distribution that $E_p = 48{,}000$ and $E_e = 72{,}000$.

In addition to the standard Workers' Compensation experience modification formula, we want to consider a modification formula in which the excess losses are ignored. This formula will take the following form:

$$M = \frac{A_p + K}{E_p + K} \qquad \text{(Equation 6.1)}$$

One should note the difference between this formula and formula 2.1. Using Hewitt's formulas [7], it can be demonstrated that the optimal value for K in this formula is 22,900.

For each trial in the simulation it is possible to calculate the modification for various formulas involving primary and excess losses. By comparing the calculated modification with the "true" modification one can estimate the efficiency of each formula. The results are in the following table.

TABLE 6.2

EFFICIENCY

	W	$K = 18{,}000$	$K = 23{,}000$	$K = 28{,}000$
Formula 6.1		0.68	0.68	0.66
Standard Formula	0.0	0.48	0.46	0.45
" "	0.1	0.51	0.50	0.49
" "	0.2	0.50	0.50	0.49
" "	0.3	0.44	0.44	0.44
" "	0.4	0.32	0.33	0.34
" "	0.5	0.13	0.15	0.17
" "	0.6	−0.12	−0.09	−0.07

Formula 6.1 is a clear winner in this case. There are two possible reasons for this. First, as demonstrated in the previous section, the primary losses seem to capture most of the information about the severity distribution. Second, the structure of the experience rating formula may very well be wrong! The Bayesian and credibility formulas described above are optimal under certain specified conditions. This author does not know of any conditions where the standard formula is optimal. At the very least, a proposal to retain the present formula should include a plausible model in which the present formula outperforms the competing formulas.

7. CHOOSING AN EXPERIENCE RATING FORMULA

So far, we have seen how modeling can give some good hints for the right form of an experience rating formula. Since we rarely, if ever, have the distributional information to do a pure Bayesian analysis, it appears that a good choice of an experience rating formula would be a credibility formula. The credibility could be given by either Equation 3.2 or Equation 3.3. A loss limit of some kind should definitely be used.

As of this writing, there is no nice clean way to pick an optimal loss limit. This author has had good luck with the Weibull distribution for severity in Workers' Compensation and the shifted Pareto distribution, see Patrik [13], for the severity in other lines of insurance. By trial and error on various models, as was done in the previous sections, one might come up with a reasonable loss limit, or loss limit formula. There is room for improvement here.

Once a loss limit has been selected one then gathers the limited losses and the expected limited losses for individual insureds over a period of years. This information is absolutely essential. Experience rating depends upon how well the experience of one year predicts that of another. With data such as this one can use the empirical Bayesian credibility procedure as described originally by Bühlmann and Straub [14], and later by the ISO Credibility Subcommittee [15] and Meyers [16].

A problem with these procedures is that they all assume that the σ^2 is inversely proportional to the expected losses, which results in using Equation 3.2 for the credibility. While these procedures might well be modified to handle more general assumptions about σ^2, the author would like to propose a different approach. This approach has the advantage that: (1) it is easy to modify the parameter estimation to accommodate alternative assumptions about σ^2; and (2) one can test the assumption made about σ^2. This approach has its origins in a study done by Paul Dorweiler [17].

In what follows we shall take the term "loss ratio" to mean current losses divided by the modified premium, where the experience modification is calculated from prior years' loss experience. We assume that the expected losses used in the experience rating formula are correct. If the loss ratio is positively correlated with the experience modification, then the credibility factors used are too low. Conversely, if the loss ratio is negatively correlated with the experience modification, then the credibility factors used are too high.

This can be justified by the following. Suppose an insured had a low experience modification and tends to have a lower than average loss ratio. Then to raise his loss ratio, one can give the insured a lower experience modification by giving more credibility to the experience. A similar argument applies when the insured tends to have a higher than average loss ratio.

Dorweiler tested the performance of an experience rating plan by partitioning insureds by manual premium size and modification size. For each premium size group he calculated the trend in loss ratio as the modification increased. The idea was to compare the number of times a positive trend occurred with the number of times a negative trend occurred. This method of testing credibility formulas is very general. No assumptions about the nature of the experience rating formula are required.

During the past fifty years, our understanding of statistics has vastly improved. Our computing capability today was unthinkable in Dorweiler's time. Today, Dorweiler's method might well be similar to the following.

Assume we have the correct form of the experience rating formula and we want to know if we have selected the right parameters. That is, we want to test the hypothesis

H_0: The parameters of the experience rating formula are correct

against the alternative hypothesis

H_1: At least one of the parameters of the experience rating formula is incorrect.

To test this hypothesis, we proceed as follows.

1. Partition the insureds into groups with similar modified premium size. Modified premium is used rather than manual premium because we want all insureds in the group to have the same loss ratio distribution. It is felt that expected losses rather than exposure is a better indicator of the loss ratio distribution.

2. Calculate the correlation coefficient between the loss ratio and the experience modification for each group.

   Kendall's τ, see Conover [18], is the preferred measure in this case. This correlation coefficient compares the number of pairwise increases with the number of pairwise decreases. Let τ_i denote Kendall's correlation coefficient for group i and let n_i be the number of insureds in group i. Under the null hypothesis, $\tau_i = 0$ for each group, the distribution of τ_i is approximately normal with mean 0 and variance $n_i(n_i - 1)(2n_i + 5)/18$. This is a nonparametric result.

3. Calculate the normalized correlation coefficient, for each group, and a combined normalized correlation coefficient. These terms are defined as follows.

   For each group i, set T_i equal to τ_i divided by its standard deviation. Under the null hypothesis T_i is approximately normal with mean 0 and variance 1. We call T_i the normalized correlation coefficient for group i. Let m be the number of groups. Set T equal to the sum of all the T_i's divided by the square root of m. T also has mean 0 and variance 1 under the null hypothesis. We will call T the combined normalized correlation coefficient.

4. Reject H_0 at significance level α if the percentile of T is outside the interval $(\alpha/2, 1 - \alpha/2)$. The percentile of T can be determined from the standard normal distribution.

   By noting that the confidence region of the parameters is the set of all parameters for which one fails to reject H_0, one can find a confidence region of the parameters by testing several sets of parameters. Acceptable parameters are those for which the percentile of T falls within the interval $(\alpha/2, 1 - \alpha/2)$. A best estimate of the parameters is one for which the percentile of T is equal to .5.

Let's see how this test works on live data. During the late seventies, the Individual Risk Rating Plan Committee at ISO issued a special call for individual insured data from actual experience ratings. ISO supplied the author with the following data elements from this call. For each of three years there was given the basic limits premium and the basic limits losses (adjusted for the loss limit). In addition, the adjusted expected loss ratio (*AELR*) was given. Ideally, one would like to have the losses that resulted from the policy that was actually experience rated, but we did the next best thing. The first two years of data were used to predict the third year.

Before doing the analysis, two adjustments to the ISO data were made. First, all insureds which did not have a full three years of experience were deleted. Second, the AELR was adjusted so that the total expected losses were equal to the total actual losses for the first two years. In all, there were 1,980 insureds which form 33 groups of 60 insureds.

Let us first assume that the credibility formula given by $Z = P/(P + K)$ is correct. Hypothesis tests were performed for a set of K values, with the following results.

TABLE 7.1

K	T	Percentile
16,000	−2.9442	.0016
18,000	−2.1870	.0144
20,000	−1.1476	.1256
22,000	−0.2060	.4184
24,000	0.2449	.5967
26,000	0.5558	.7108
28,000	1.0665	.8569
30,000	1.6194	.9473
32,000	2.1078	.9825
34,000	2.5697	.9949
36,000	2.9850	.9986

The best estimate for K will be between 22,000 and 24,000. The 95 percent confidence interval for K will range from slightly over 18,000 to slightly less than 32,000. Table 7.2 shows the T_i's for each group when $K = 22{,}000$.

Close examination of Table 7.2 reveals that the correlations are predominantly positive for the smaller insureds and very definitely negative for the larger insureds. This indicates that the credibility is too low for the smaller insureds and too high for the larger insureds. Thus the formula $Z = P/(P + K)$ is not the correct form of the credibility formula. This can be explained in terms of the changing loss limit and parameter uncertainty as described in Section 3 above. If we have the correct form of the credibility formula, the hypothesis test described above should apply equally well for any subset of groups.

Let us now examine the credibility formula $Z = P/(P \times J + K)$. In addition to calculating the combined normalized correlation coefficient for all insureds, we calculate the combined normalized correlation coefficient for the five different subsets of groups. The rationale for selecting the subsets will be discussed below.

Before discussing the above tables one should note that there are some small reversals in what might seem to be a clear pattern. These are random fluctuations caused by the insureds shifting groups with each set of parameters. Recall that the groups were based on modified premium.

Let us first examine the subsets consisting of Groups 1 to 5, Groups 6 to 19 and Groups 20 to 33. It can be observed that when $J = 1.0$, no value of K is in the 95 percent confidence region for each subset. The following pairs (J,K) are in the 95 percent confidence region for each subset.

		(4.0, 1000)
		(4.0, 2000)
		(4.0, 3000)
		(4.0, 4000)
	(3.0, 5000)	(4.0, 5000)
	(3.0, 6000)	(4.0, 6000)
	(3.0, 7000)	(4.0, 7000)
	(3.0, 8000)	(4.0, 8000)
	(3.0, 9000)	(4.0, 9000)
	(3.0, 10000)	
	(3.0, 12000)	
(2.0, 14000)	(3.0, 14000)	
(2.0, 16000)		

The details of the calculations for $J = 4.0$ and $K = 2{,}000$ are given in Table 7.3. As mentioned above, the derivation of the credibility formula $Z = P/(P \times J + K)$ does not anticipate a loss limit which increases as the size of the insured increases. Thus we should not expect this credibility formula to be exactly right

TABLE 7.2

<u>Headings</u>

MINMBLP —Minimum modified basic limits premium
MAXMBLP —Maximum modified basic limits premium
N —Number of insureds
TAU —Kendall's tau correlation coefficient between the loss ratio and the experience modification
MODPCT10 —10^{th} percentile of experience modifications
MODPCT50 —50^{th} percentile of experience modifications
MODPCT90 —90^{th} percentile of experience modifications
T —Normalized correlation coefficient

EXPERIENCE RATING ANALYSIS—GENERAL LIABILITY: K = 22,000 J = 1.00

OBS	MINMBLP	MAXMBLP	N	TAU	MODPCT10	MODPCT50	MODPCT90	T
1	10.0	250	60	0.10031	0.962409	0.98257	0.99638	1.1324
2	251.5	418	60	−0.01243	0.962809	0.97386	0.98161	−0.1403
3	422.4	604	60	0.10056	0.929858	0.95987	1.01518	1.1353
4	606.5	759	60	0.02147	0.908815	0.94638	0.97613	0.2424
5	762.8	884	60	0.01469	0.892809	0.93845	0.97723	0.1658
6	887.1	1032	60	−0.05537	0.879143	0.93260	1.01479	−0.6250
7	1036.8	1151	60	−0.04181	0.864093	0.92675	1.01257	−0.4720
8	1151.7	1279	60	0.10345	0.862320	0.92053	1.18874	1.1678
9	1286.5	1447	60	0.02318	0.824655	0.91998	1.10541	0.2616
10	1452.0	1603	60	−0.02373	0.839008	0.90577	1.06497	−0.2679
11	1609.1	1747	60	−0.05537	0.845583	0.90363	1.15102	−0.6250

TABLE 7.2 (continued)

OBS	MINMBLP	MAXMBLP	N	TAU	MODPCT10	MODPCT50	MODPCT90	T
12	1747.7	1910	60	0.20339	0.818599	0.89558	1.00920	2.2961
13	1913.8	2020	60	0.02938	0.803793	0.89252	1.17428	0.3317
14	2024.6	2169	60	0.14237	0.798667	0.90054	1.20914	1.6072
15	2169.8	2316	60	−0.02712	0.785614	0.86615	1.01141	−0.3061
16	2318.4	2495	60	0.14463	0.786776	0.86784	1.16782	1.6327
17	2498.1	2680	60	0.08475	0.743469	0.86263	1.23809	0.9567
18	2681.3	2885	60	0.08927	0.766698	0.85704	1.06101	1.0077
19	2885.0	3064	60	−0.00452	0.743238	0.84386	1.18246	−0.0510
20	3067.4	3346	60	0.07006	0.723105	0.84160	1.20551	0.7909
21	3352.6	3629	60	0.25085	0.717498	0.85007	1.24072	2.8318
22	3632.1	3880	60	0.02147	0.667270	0.85448	1.33079	0.2424
23	3883.3	4209	60	0.10734	0.713127	0.81707	1.26421	1.2118
24	4215.7	4580	60	0.01243	0.675236	0.80978	1.91797	0.1403
25	4581.6	5023	60	−0.04859	0.684537	0.82988	1.90822	−0.5485
26	5040.9	5529	60	0.05311	0.634504	0.77695	1.28748	0.5995
27	5533.5	6302	60	−0.08701	0.590834	0.83498	1.91602	−0.9822
28	6316.8	7390	60	−0.08023	0.641002	0.86242	1.95935	−0.9057
29	7405.5	8645	60	−0.35593	0.522315	0.80239	1.89673	−4.0181
30	8702.0	10808	60	−0.19955	0.468991	0.78968	1.97431	−2.2527
31	10847.9	15885	60	−0.20000	0.447977	1.06263	2.48548	−2.2578
32	16077.5	26102	60	−0.21695	0.622667	1.36827	2.34570	−2.4491
33	26118.2	297046	60	−0.26893	0.652677	1.52027	2.96620	−3.0359

TABLE 7.3

Headings

MINMBLP —Minimum modified basic limits premium
MAXMBLP —Maximum modified basic limits premium
N —Number of insureds
TAU —Kendall's tau correlation coefficient between the loss ratio and the experience modification
MODPCT10 —10^{th} percentile of experience modifications
MODPCT50 —50^{th} percentile of experience modifications
MODPCT90 —90^{th} percentile of experience modifications
T —Normalized correlation coefficient

EXPERIENCE RATING ANALYSIS—GENERAL LIABILITY: K = 2000 J = 4.00

OBS	MINMBLP	MAXMBLP	N	TAU	MODPCT10	MODPCT50	MODPCT90	T
1	9.9	230	60	0.07764	0.870207	0.90389	0.96442	0.8765
2	232.9	382	60	0.07910	0.862833	0.88592	0.90232	0.8929
3	389.4	572	60	0.01582	0.834214	0.86588	0.91637	0.1786
4	572.3	705	60	−0.07910	0.822935	0.85410	0.96478	−0.8929
5	709.2	821	60	−0.11073	0.812158	0.84499	0.89705	−1.2501
6	823.7	961	60	−0.26328	0.808248	0.83594	0.88607	−2.9721
7	971.3	1085	60	−0.02712	0.810787	0.83640	0.98953	−0.3061
8	1086.9	1232	60	−0.05483	0.810880	0.84006	1.03652	−0.6190
9	1243.2	1390	60	−0.11815	0.794511	0.83726	1.19594	−1.3337
10	1394.3	1553	60	−0.07910	0.799560	0.83418	1.06445	−0.8929
11	1555.2	1701	60	0.03277	0.803051	0.83198	1.09454	0.3699

TABLE 7.3 (continued)

OBS	MINMBLP	MAXMBLP	N	TAU	MODPCT10	MODPCT50	MODPCT90	T
12	1701.3	1830	60	−0.00339	0.799704	0.83866	1.35855	−0.0383
13	1833.6	1996	60	0.06893	0.795396	0.83288	1.33904	0.7781
14	1996.4	2142	60	0.11186	0.794848	0.82466	1.02667	1.2628
15	2146.2	2326	60	0.07797	0.788719	0.82349	1.01559	0.8802
16	2340.1	2518	60	0.10508	0.796236	0.82021	1.20622	1.1863
17	2518.6	2717	60	0.23616	0.792849	0.82674	1.27064	2.6660
18	2719.5	2924	60	−0.02486	0.787838	0.80931	1.04337	−0.2806
19	2927.5	3174	60	0.07232	0.785318	0.81290	1.21608	0.8164
20	3174.8	3446	60	0.13672	0.789134	0.82195	1.80638	1.5435
21	3450.3	3767	60	−0.06441	0.798985	0.85378	1.35756	−0.7271
22	3782.2	4143	60	0.15254	0.789817	0.90983	1.81165	1.7220
23	4147.5	4535	60	0.19096	0.782837	0.82072	1.32675	2.1557
24	4541.2	5042	60	−0.20791	0.790398	0.89006	2.10844	−2.3471
25	5043.4	5426	60	0.00452	0.783358	0.82616	1.84692	0.0510
26	5439.8	6073	60	0.05198	0.791312	0.84412	1.74888	0.5868
27	6084.6	6997	60	0.00339	0.788521	0.87782	2.34622	0.0383
28	7011.6	8064	60	−0.00339	0.785978	0.88043	2.35557	−0.0383
29	8115.5	9887	60	−0.11751	0.791988	0.98504	2.35900	−1.3266
30	9899.0	12103	60	−0.17467	0.783483	0.90444	2.02998	−1.9719
31	12344.9	17413	60	−0.04746	0.779974	0.90431	2.50209	−0.5357
32	17495.2	26177	60	−0.03164	0.810414	1.12274	1.73464	−0.3572
33	26221.3	206941	60	−0.00565	0.808315	1.05283	2.17655	−0.0638

TABLE 7.4

PERCENTILES OF T'S

$J = 1.0$

K	Groups 1:33	Groups 6:33	Groups 1:5	Groups 6:19	Groups 20:33
18000	.0144	.0051	.6795	.8646	.0000
20000	.1256	.0661	.7306	.9220	.0002
22000	.4184	.2411	.8716	.9677	.0022
24000	.5967	.3959	.8950	.9864	.0049
26000	.7108	.5197	.9050	.9875	.0149
28000	.8569	.7050	.9285	.9938	.0412
30000	.9473	.8665	.9374	.9976	.1063
32000	.9824	.9498	.9367	.9991	.2159
34000	.9949	.9795	.9609	.9994	.3594
36000	.9986	.9933	.9655	.9997	.5345

$J = 2.0$

K	Groups 1:33	Groups 6:33	Groups 1:5	Groups 6:19	Groups 20:33
8000	.0118	.0132	.2875	.6563	.0002
10000	.0843	.0863	.3785	.8020	.0027
12000	.2388	.2336	.4586	.8977	.0108
14000	.5767	.5186	.6505	.9662	.0391
16000	.7900	.7016	.7940	.9604	.1568
18000	.8964	.8358	.8235	.9809	.2449
20000	.9727	.9410	.8919	.9912	.4356
22000	.9893	.9754	.8951	.9954	.5696
24000	.9979	.9944	.9161	.9980	.7624

TABLE 7.4 (continued)

PERCENTILES OF T'S

$J = 3.0$

K	Groups 1:33	Groups 6:33	Groups 1:5	Groups 6:19	Groups 20:33
3000	.0136	.0152	.2913	.2652	.0075
4000	.0535	.0587	.3316	.4202	.0220
5000	.1626	.1677	.4024	.5796	.0590
6000	.3080	.2940	.4972	.7576	.0715
7000	.4104	.3837	.5471	.7854	.1134
8000	.5645	.5234	.6096	.9115	.1026
9000	.7601	.7388	.6183	.9435	.2481
10000	.8223	.8088	.6205	.9469	.3519
12000	.9430	.9229	.7546	.9597	.6054
14000	.9827	.9715	.8235	.9750	.7677
16000	.9963	.9935	.8421	.9931	.8535

$J = 4.0$

K	Groups 1:33	Groups 6:33	Groups 1:5	Groups 6:19	Groups 20:33
1000	.5953	.4443	.8293	.6717	.2602
2000	.5035	.5185	.4653	.6574	.3672
3000	.6111	.6415	.4471	.7501	.4356
4000	.7106	.7164	.5289	.8901	.3380
5000	.8044	.8108	.5472	.8704	.5467
6000	.8676	.8405	.6938	.7904	.7262
7000	.9215	.9317	.5450	.8689	.8373
8000	.9596	.9592	.6421	.9087	.8707
9000	.9878	.9886	.6526	.9732	.9014
10000	.9941	.9929	.7510	.9771	.9288

over the entire range of premium sizes. Examination of Table 7.3 reveals this to be the case. However, the results are superior to any that could be obtained with the credibility formula $Z = P/(P + K)$.

If loss limit increases with the size of the insured the credibility will increase more slowly than the formula $Z = P/(P \times J + K)$ would suggest. This is verified in Table 7.4 where the formula tends to assign too low a credibility to the medium size insureds in Groups 6 to 19 and too high a credibility to the large sized insureds in Groups 20 to 33. However, the formula tends to assign too high a credibility to the smaller sized insureds in Groups 1 to 5, and too low a credibility to the medium sized insureds in Groups 6 to 19. This is the opposite of what is expected.

We attempt to explain this reversal. We first note that there is a minimum premium size that qualifies an insured for experience rating. It is possible for an insured to have a sizeable decrease in exposure which will result in premiums which are below the minimum in the year being rated. But this happens rather infrequently. A far more common cause of insureds having a smaller size is for the insured to have low modification. This can be verified in Table 7.3 where over ninety percent of the insureds in Groups 1 to 5 have experience modifications which are less than 1.00.

Two possible explanations for the reversal can be given. First, since most insureds are those with good experience, the groups are more homogeneous (i.e. τ^2 is lower) and lower credibility is called for. Second, since the loss limit is assigned according to unmodified premium, σ^2 is not necessarily smaller for the smaller insureds. This would also have a tendency to lower credibility for the smaller insureds. If these explanations are correct, one should separate the very smallest insureds from the main part of the analysis. This is why the subsets were grouped in the above manner.

In summary, a very general way of analyzing data for experience rating has been proposed. Not only can it be used to determine parameters of an experience rating formula, but one can also test to see if the assumptions made in deriving the form of the credibility formula are valid.

8. SUMMARY AND CONCLUSIONS

This paper has attempted to use collective risk theory to analyze experience rating. Particular attention was paid to experience rating formulas used by the National Council on Compensation Insurance and Insurance Services Office. The first goal of this paper was to find an experience rating formula that worked well on mathematical models and would be easy to administer. An examination of the performance of experience rating plans on mathematical models led to the following conclusions.

1. A loss limit can be an effective tool for increasing the accuracy of an experience rating formula. Loss limits are particularly helpful when there are differences in claim frequency. Even if the only differences among the insureds are in claim severity, little accuracy will be lost with a loss limit.

2. The current formula in the Workers' Compensation Experience Rating Plan, which has a separate treatment of primary and excess losses, is less accurate than a formula which uses only primary losses.

3. There are some very plausible situations when the standard credibility formula $Z = E/(E + K)$ is not appropriate. These include parameter uncertainty over time and a loss limit which increases with the size of the insured. Failure to recognize this will result in overstating credibilities for larger insureds.

The author would recommend an experience rating formula based on the credibility formula $Z = E/(E \times J + K)$. A loss limit that does not vary by size of insured should be a part of the plan. Excess losses should not be a part of the plan. This formula is less complicated than current formulas and should be easier to administer.

It should be noted that the service performed by the NCCI in calculating the experience modification is probably more important than the choice of experience rating formulas. ISO would do well to perform a similar service, or at the very minimum, provide experience in a standard format so that individual insureds could calculate their own experience modifications.

A second goal was to show how the parameters of an experience rating plan could be estimated from data. This paper demonstrates, with live data, a very general procedure for testing the parameters of a proposed credibility formula. Systematic testing of various alternative parameters should enable one to derive a reasonably accurate formula. This method requires data with which one can compare actual losses with losses predicted by the proposed formula. The author considers this kind of data absolutely essential for accurate experience rating.

Experience rating has always been a combination of scientific and intuitive reasoning. While the intent of this paper is to put experience rating on a more scientific basis, it is hoped that the reader now has a better intuitive understanding of this very important subject.

9. ACKNOWLEDGMENTS

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APPENDIX A
WITHIN VARIANCE AND THE SIZE OF THE INSURED

In this appendix we discuss how the expected within variance, σ^2, depends upon the size of the insured.

Let: N be a random variable denoting the claim count;
λ be the expected number of claims;
χ be a random variable with $E[\chi] = 1$ and $Var[\chi] = c$;
S be a random variable denoting the claim severity; and
β be a random variable with $E[1/\beta] = 1$ and $Var[1/\beta] = b$.

The collective risk model with parameter uncertainty can be described by the following algorithm.

Algorithm A.1

1. Select χ at random.
2. Select the number of claims, N, at random from a Poisson distribution with parameter $\chi \times \lambda$.
3. Select β at random.
4. Do the following N times.
   4.1 Select the claim severity, S, at random.
5. Set the total loss, X, equal to the sum of the claim amounts, S, divided by β.

b and c measure uncertainty in the scale of the claim severity distribution and the mean of the claim count distribution, respectively.

Let $R_1 = X/E[X]$. Meyers and Schenker [9] show that

$$Var[R_1] = (1 + b) \times E[S^2]/(\lambda \times E^2[S]) + b + c + b \times c.$$

The size of the risk E is proportional to $E[X]$ and can be written as $C \times E[X]$. Thus we can then write:

$$Var[R] = \Sigma_1^2/E + \alpha_1 \qquad \text{(Equation A.1)}$$

where $\Sigma_1^2 = (1 + b) \times C \times E[S^2]/E[S]$ and
$\alpha_1 = b + c + b \times c$.

In keeping with the notation of Section 3, let d denote a distribution generated by the process described above. The linear relationship of Equation A.1

is preserved when taking expected values over all distributions, d. Thus we have

$$\begin{aligned} \sigma^2 &= \Sigma^2/E + \alpha & \text{(Equation A.2)} \\ \text{where } \Sigma^2 &= \mathrm{E}[\Sigma_1^2] \text{ and} \\ \alpha &= \mathrm{E}[\alpha_1]. \end{aligned}$$

Equation A.2 is used to derive the credibility formula 3.3. If b and c are equal to zero for each distribution d, then $\sigma^2 = \Sigma^2/E$. In this case the credibility formula 3.2 applies.

We see from Equation A.1 that Σ_1^2, and thus Σ^2, depend upon the severity distribution. An increase in the loss limit will increase Σ^2.

APPENDIX B

A FORMULA FOR THE EFFICIENCY

We prove that the efficiency is equal to $2 \times Z - Z^2/Z_m$. The proof is simply a rearrangement of concepts originated by Bühlmann [6] and discussed by ISO [15].

Lemma 1: $\mathrm{Cov}[X,\mu] = \tau^2$

Proof:
$$\begin{aligned}\mathrm{Cov}[X,\mu] &= \mathrm{E}[(X - M) \times (\mu - M)] \\ &= \mathrm{E}_\mu[\mathrm{E}[(X - M) \times (\mu - M)|\mu]] \\ &= \mathrm{E}_\mu[(\mu - M)^2] \\ &= \tau^2\end{aligned}$$

Lemma 2: $\mathrm{Var}[X] = \sigma^2 + \tau^2$

Proof:
$$\begin{aligned}\mathrm{Var}[X] &= \mathrm{E}_\mu[\mathrm{Var}[X|\mu]] + \mathrm{Var}_\mu[\mathrm{E}[X|\mu]] \\ &= \sigma^2 + \tau^2\end{aligned}$$

Theorem: Efficiency $= 2 \times Z - Z^2/Z_m$

Proof: Let F be an estimator for μ. By the definition of efficiency given in Section 4 we have:

Efficiency $= 1 - \mathrm{E}[(F - \mu)^2]/\tau^2$

In our case: $F = Z \times X + (1 - Z) \times M$. Thus we have:

Efficiency $= 1 - \mathrm{E}[(Z \times (X - M) - (\mu - M))^2]/\tau^2$

$= 1 - (Z^2 \times \mathrm{Var}(X) + \tau^2 - 2 \times Z \times \mathrm{Cov}(X,\mu))/\tau^2$

$= 1 - (Z^2 \times (\sigma^2 + \tau^2) + \tau^2 - 2 \times Z \times \tau^2)/\tau^2$

$= 2 \times Z - Z^2/Z_m$

Corollary: The efficiency is maximized when $Z = Z_m$.

Proof: d(Efficiency)/d$Z = 0$ when $Z = Z_m$.

This corollary is simply a restatement of Bühlmann's result.