AUTHOR'S REPLY TO DISCUSSION

One of my hopes in writing a paper on development factor analysis was that it would help to stimulate others in their research in this area. The subject is so important that if Charles Darwin were alive today, his contribution to link ratio analysis might be the discovery of the long-awaited missing link. In the absence of a re-vitalized Darwin, we are fortunate to have the review of Stephen P. Lowe and David F. Mohrman and the ideas and models they present.

Why is this subject so timely? Let us consider a commonly encountered situation. As we head out toward the more mature parts of our development triangles, and our data transition from the credible to the less credible, we are presented with several alternatives (presented in ascending order of preference):

1. Satisfy the actuarial craving to deploy a complex model which fits the given data points perfectly and wildly diverges as we attempt to use it to extrapolate beyond the historical experience.
2. Close your eyes, swallow hard, make an undocumentable selection, smile like a Cheshire cat, turn to the world at large and exclaim, "Trust me."
3. Rely on the indications of only two or three factors, each of which is often heavily impacted by large claims. The dictum, "When in doubt, throw it out," is often invoked here.
4. Use models which closely fit related data to extrapolate factors for later development factors based on earlier factors from more credible data. Some of the Lowe-Mohrman models could be very useful here.

It would have been helpful if the reviewers had provided some comparative tests of how well their models fit actual data, such as was provided for equation (5) and the salvage and subrogation data. I suspect that equation (5) often would represent a better fitting model than the basic inverse power function because each term behaves in the same manner as the inverse power curve and an extra parameter should increase accuracy. However, equations (6) through (8) present models which add complexity and may or may not increase accuracy. It is often true that more complex models improve accuracy within the range of historical data points. But, it is also true that they may tend to diverge from expected patterns when used for the purpose of extrapolation. The advantage of a simpler model is that its behavior for extrapolation purposes tends to be more reliable. This suggests that an important criterion in assessing different models is their ability to predict known factors for later periods of development based solely on earlier factors.

The reviewers' presentation of the hypothetical results obtained when a and b are set equal to unity in the two-parameter inverse power curve is quite interesting. It clearly illustrates the necessity of an eventual decline in the incremental amounts of development if convergence is to occur. This is generally not a problem as long as the historical data include later periods when the incremental amounts of development decline. If the incremental amounts are constant (as in Lowe's example) or increasing, the product series will diverge.

Lowe and Mohrman observe that the fitting method in the paper minimizes the errors in $\ln(f(t) - 1)$ and not the errors of $f(t)$. They further observe that differences between actual and fitted values are more significant when the development factors are close to 1.000 than when the development factors are significantly greater than 1.000. Thus, the fitting method puts more emphasis on factors for the more mature periods than for the earlier periods. This is usually a desirable result, since the estimated factors of consequence are those of the later development periods. For applications where greater accuracy is required for the earlier periods, the errors of $f(t)$ should be minimized instead of $\ln(f(t) - 1)$.

With regard to estimating a tail factor by multiplying together the extrapolated factors, the reviewers correctly note that this procedure results in a compounding of the errors. It should be noted, however, that a compounding of errors as the extrapolating proceeds further into the future is probably unavoidable as it would appear to be inherent in the process of foreseeing the distant future. The degree of uncertainty in our estimates will, in all likelihood, increase progressively as we forecast events occurring further away from the immediate present. Even if we are using the best possible model, the extrapolation is based on data of limited credibility and the results are very sensitive to statistical fluctuations in the historical experience.

In closing, it may be noted that the inverse power curve can easily be used for estimating the number of IBNR claims as an alternate method to that presented by Edward Weissner in his paper, "Estimation of the Distribution of Report Lags by the Method of Maximum Likelihood" (*PCAS* LXV, 1978). The procedure is much easier to apply and chi-square tests for goodness of fit indicate that a closer fit is obtained using the inverse power curve rather than maximum likelihood. A comparison of actual and fitted development factors for cumulative reported claims is included here as Exhibit 1.

EXHIBIT 1

Comparison of Actual and Fitted Reported Count Development Factors Using an Inverse Power Function

Year of Development	Medical Malpractice		Other Bodily Injury Liability		Auto Bodily Injury Liability	
	Actual	Fitted	Actual	Fitted	Actual	Fitted
2:1	2.094	2.162	1.274	1.295	1.160	1.163
3:2	1.179	1.199	1.062	1.060	1.013	1.014
4:3	1.099	1.071	1.027	1.024	1.004	1.003
5:4	1.032	1.034	1.014	1.012	1.001	1.001
6:5	1.021	1.019	1.006	1.007	1.000	1.001
7:6	1.010	1.012	1.005	1.005	1.000	1.000
8:7	1.008	1.008	1.003	1.004	1.000	1.000
9:8	1.007	1.006	1.003	1.003	1.000	1.000
10:9	1.004	1.004	1.001	1.002	1.000	1.000
R^2		.98759		.99038		.99483
a =		1.16241		0.29501		0.16288
b =		2.54727		2.29051		3.56889
c =		−1.00000		−1.00000		−1.00000