it would be informative to investigate this sensitivity of accuracy with respect to the skewness and kurtosis of the various distributions applicable to the property/casualty lines of business.

AUTHOR'S REPLY TO DISCUSSIONS

Discussion by Albert J. Beer

My paper was originally written for the May, 1982, Casualty Actuarial Society Discussion Paper Program "Pricing, Underwriting and Managing the Large Risk." Mr. Beer reviewed my paper at that time. His discussion so enhanced my paper that I made my submission to the *Proceedings* conditional on his agreement to submit his discussion.

Mr. Beer's definitions of PML_α and $PMAL_\alpha$ easily clarify an area I treated by implication, but did not address directly. These definitions explain how the risk manager and the underwriter can have different PML's when they use the same loss distribution; the different PML's are due to different values for α.

Mr. Beer's generalizations of my results nicely complete the ideas I presented. It should be noted that his generalizations reduce the six methods presented to five, since "$X_{(r)}$" and "$X_{(r)}$ as an estimate of k_p" reduce to the same estimator.

Discussion by John S. McGuinness

Many of Mr. McGuinness's questions, particularly those concerning definitions, are answered if my paper and Mr. Beer's discussion are read as a unit, as intended. Mr. McGuinness has raised other issues, several of which I will address.

1) Sample Data

The sample data presented in Exhibit I of my paper are used to illustrate the calculation of the proposed estimators for probable maximum loss (PML), to ensure that the concepts are clearly understood by the readers. The source of the data is not relevant to the concepts presented in the paper.

The proposed estimators for PML were derived theoretically and tested on several sets of real and synthetic data. While of varying usefulness, I judged the estimators presented in the paper to be of sufficient interest to merit presentation to fellow actuaries.

I purposely did not choose data used by A. W. Whitney[1] or Ruth Salzmann[2]

[1] A. W. Whitney, "The Actuarial Theory of Fire Insurance Rates as Depending on the Ratio of Insurance to Sound Value, Hence the Determination of the Rates for Use with the Coinsurance Clause," *Transactions*, VI International Congress of Actuaries, Vol. 2, 1909, pp. 395–403.

[2] R. Salzmann, "Rating by Layer of Insurance," *PCAS* L, pp. 15–26.

to illustrate calculations because I wanted to get away from the idea that the PML concept applies only to property. Other actual data could not be presented in the paper because releases from clients could not be obtained in a timely fashion.

The data presented in my paper are computer-generated. They are the aggregate distribution of claims, the number of claims having a Poisson distribution, and the claim size having a lognormal distribution. Several papers in the *Proceedings* and other journals have stated the success of the Poisson and lognormal distributions in approximating number of claims and claim size, respectively[3]. Consequently, I feel the data presented in my paper are representative of a "real-life" situation where $PMAL_\alpha$ might be sought.

2) Order Statistics and Non-Normal Distributions

This paper is *not* based on the assumption of asymptotic normality, as Mr. McGuinness states. $E(X_{(r)})$ and $E(X_{(r)}) + 2(\text{Var}\ (X_{(r)}))^{1/2}$ as estimators for PML_α do require assumptions concerning the underlying distribution of the data. However, $X_{(r)}$, the upper bound of $E(X_{(r)})$, and the upper bound for κ_p are distribution-free.

3) Potential for Order Statistics

One of the reasons for writing this paper was to call attention to order statistics as a useful tool in actuarial work. As stated in the original paper:

> Order statistics are particularly useful for studying certain phenomena because quite a few of the results concerning the properties of $X_{(r)}$ and the properties of functions of some subset of the order statistics are distribution-free. If an inference is distribution-free, assumptions regarding the underlying population are not necessary.

If reliable information about the underlying population is available, it should, of course, be used to the greatest reasonable extent. However, it has been my experience and, I suspect, that of many other actuaries, that we usually know very little about the underlying population. Consequently, I find order statistics a frequently useful, and often underutilized, tool, particularly with reference to PML_α and $PMAL_\alpha$.

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[3] For excellent bibliographies, see:

A. L. Mayerson, "A Bayesian View of Credibility," *PCAS* LIII, pp. 85–104.

G. Patrik, "Estimating Casualty Insurance Loss Amount Distributions," *PCAS* LXVII, pp. 57–109.