A NOTE ON LOSS DISTRIBUTIONS

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VOLUME LXIX

DISCUSSION BY CHARLES C. HEWITT, JR.

This is a remarkable piece of synthesis! It finds a thread running through many significant papers in our *Proceedings* and weaves an entirely new fabric which, when reviewed on an overall or on a modular basis, should make life much simpler for both the student and the practicing actuary. The author recognizes that mathematicians constantly strive for generalizations which provide solutions for *superficially* different problems.

He has achieved that goal in three basic elements found in this paper:

(1) Simplified (standardized?) notation,
(2) Recognition of a commonalty of approach, and
(3) Application of simplified notation to a gallimaufry of actuarial problems.

Standardized Notation

There is an opportunity here for the Casualty Actuarial Society to intervene on behalf of present and future generations of actuaries in the matter of actuarial notation. The textbook, currently in preparation under the auspices of the Actuarial Education and Research Fund (AERF), on distributions of a (single) loss will suggest a more mathematically oriented notation. Gary LaRose, with attribution to Robert Finger, suggests another notation.

Taking advantage of symbols readily available on computers and word processors, I have proposed still another:

(1) $F\#(x) = \int_0^x f(t)dt$

(2) $F\$(x) = \frac{1}{E(t)} \int_0^x t\, f(t)dt$ and

(3) $F\&(x) = F\$(x) + \frac{x}{E(t)} [1 - F\#(x)]$

where all three *cumulative* functions have a range from *zero* to *one*.

It is my feeling that this latter notation has a mnemonic value not possessed by the other forms. A comparison of the LaRose/Finger and Hewitt notations appears in the Appendix to this review.

Loss Distribution Table

A useful tool in valuing coverage limitation is a loss distribution table*.

LOSS DISTRIBUTION TABLE

Loss Amount (x)	Number $F\#(x)$	Amount $F\$(x)$	Loss Elimination Ratio (LER) $F\&(x)$
x_1	$F\#(x_1)$	$F\$(x_1)$	$F\&(x_1)$
x_2	$F\#(x_2)$	$F\$(x_2)$	$F\&(x_2)$
x_i	$F\#(x_i)$	$F\$(x_i)$	$F\&(x_i)$
∞	1	1	1

where $x_1 < x_2 < \ldots < x_i < \ldots < \infty$

The values in the "Number," "Amount," and "LER" columns are cumulative. Values of x are selected so as to make for easy calculation of frequently used deductibles (retentions) and limits. Formulas for commonly used actuarial expressions in both LaRose/Finger and Hewitt notations are contained in the Appendix.

Gary LaRose has earned the gratitude of many for the thought, research and clarity of expression which has gone into his effort.

* See Charles C. Hewitt, Jr. and Benjamin Lefkowitz, "Methods for Fitting Distributions to Insurance Loss Data," *PCAS* LXVI (1979), p. 147.

APPENDIX

Item	LaRose/Finger	Hewitt
Cumulative Distribution Function (c.d.f.)	$F(x)$	$F\#(x)$
Proportion of total losses on those losses whose amount is less than or equal to x	$X1(x)$	$F\$(x)$
Loss Elimination Ratios		
Deductible (Retention)	$X2(x)$	$F\&(x)$
Limit(s)	$X3(x)$	$1 - F\&(x)$
Interval Mean $\alpha = E(t)$	$\dfrac{X1(x_2) - X1(x_1)}{F(x_2) - F(x_1)} \alpha$	$\dfrac{F\$(x_2) - F\$(x_1)}{F\#(x_2) - F\#(x_1)} E(t)$
Value of a Layer ($x_2 > x_1$)		
Proportional	$X2(x_2) - X2(x_1)$	$F\&(x_2) - F\&(x_1)$
Absolute	$[X2(x_2) - X2(x_1)]\alpha$	$[F\&(x_2) - F\&(x_1)] \; E(t)$
Mean Value of Coverage Deductible (Retention) $= x_1$ Limit $= x_2$	$\dfrac{X2(x_2) - X2(x_1)}{1 - F(x_1)} \alpha$	$\dfrac{F\&(x_2) - F\&(x_1)}{1 - F\#(x_1)} E(t)$

Effect of changes in deductible (retention) from x_0 to x_1 and limit from x_2 to x_3	Modifiers LaRose/Finger	 Hewitt
On Frequency	$\dfrac{1 - F(x_1)}{1 - F(x_0)}$	$\dfrac{1 - F\#(x_1)}{1 - F\#(x_0)}$
On Severity	$\dfrac{[1 - F(x_0)][X2(x_3) - X2(x_1)]}{[1 - F(x_1)][X2(x_2) - X2(x_0)]}$	$\dfrac{[1 - F\#(x_0)][F\&(x_3) - F\&(x_1)]}{[1 - F\#(x_1)][F\&(x_2) - F\&(x_0)]}$
On Pure Premium	$\dfrac{X2(x_3) - X2(x_1)}{X2(x_2) - X2(x_0)}$	$\dfrac{F\&(x_3) - F\&(x_1)}{F\&(x_2) - F\&(x_0)}$