# ACTUARIAL VALUATION OF PROPERTY/CASUALTY INSURANCE COMPANIES

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DISCUSSION BY ROBERT ROTHMAN AND ROBERT V. DEUTSCH

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## AUTHOR'S REPLY TO DISCUSSION

My paper points out that, in spite of voluminous readings in the Society of Actuaries literature, our *Proceedings* do not deal with this subject at all. The reviews by Mr. Lowe and by Messrs. Rothman and Deutsch have added significantly to the discussion, and subsequently to my deeper understanding of the underlying interrelationships affecting company valuations.

My paper is largely a synthesis of classical life literature. Thus, I was somewhat taken aback by the Rothman-Deutsch review suggesting that the present value of future cash flows was a better method than present value of future earnings. Nowhere in my review of the life insurance literature had this been suggested. Further, I had no intuitive understanding of what the real difference was between the two methods. So, I set about to reconcile the two approaches.

The reconciliation, presented below, is an algebraic representation of the cash and earnings process for a theoretical insurance enterprise.

First some definitions:

$V$ = Value of company
$S_t$ = Net Worth at time $t$
$CF_t$ = Cash flow at time $t$
$E_t$ = Statutory earnings at time $t$
$R_t$ = Reserves at time $t$ (not just loss reserves; all reserves)
$i$ = Assumed interest rate
$j$ = Risk-adjusted interest rate
$v$ = $1/(1 + i)$ = Interest discount rate
$u$ = $1/(1 + j)$ = Risk-adjusted discount rate

It immediately struck me that one major difference between the earnings and the cash flow methods is the interest versus the risk-adjusted discount rate. In the cash flow method, they are the same ($i = j$). In the classical earnings method, the use of a higher risk rate of return is stressed.[1]

The two methods can be formularized as follows for a theoretical enterprise.

CASH FLOW:

$$V = S_0 + R_0 + \sum_{t=1}^{\infty} u^t CF_t$$

or value is present cash (reserves and surplus), plus the present value of future cash.

EARNINGS:

$$V = S_0 + \sum_{t=1}^{\infty} u^t E_t$$

or value is present surplus (net worth) plus the present value of future earnings. (One thing that may not be clear from the text of my paper, but should be clear from the example, is that beginning surplus and future earned surplus contributions are not retained and compounded, but rather, present valued to the owner(s).)

Keeping in mind that earnings are equal to cash flow plus interest on reserves less reserve changes, the earnings formula can be restated as follows:

$$V = S_0 + \sum_{t=1}^{\infty} u^t [CF_t - (R_t - R_{t-1}) + iR_{t-1}].$$

The appended exhibit algebraically restates this formula as

$$V = [S_0 + R_0 + \sum_{t=1}^{\infty} u^t CF_t] - \sum_{t=1}^{\infty} (j - i) u^t R_{t-1}.$$

Thus, the earnings formula can be restated as equal to the cash flow formula less an interest penalty ($j - i$) on funds held in reserve. Therefore:

(1) If $j = i$, the two methods are equivalent, and

(2) If $j > i$, the earnings method introduces an interest penalty on reserves based on the assumption that such funds need to be invested conservatively. This penalty is analogous to the interest penalty on required

[1] James C. H. Anderson, "Gross Premium Calculations and Profit Measurement for Non-Participating Insurance," *Transactions Society of Actuaries*, Volume XI (1959), p. 378.

surplus, which is cited in the paper but not included in the above formulation.

In summary, the cash flow method can be seen as a special case of the classical earnings formula. When the discount rate equals the assumed interest earnings rate, the timing of the booking of earnings no longer affects present values. That is, the present values of statutory earnings, GAAP earnings, and cash flows are all the same.

## ALGEBRAIC REPRESENTATION OF EARNINGS FORMULA

$$(1)\ V = S_0 + \sum_{t=1}^{\infty} u^t[CF_t - (R_t - R_{t-1}) + iR_{t-1}]$$

This is algebraically equivalent to:

$$(2)\ V = S_0 + \sum_{t=1}^{\infty} u^t CF_t + \sum_{t=1}^{\infty} u^t(1+i)R_{t-1} - \sum_{t=1}^{\infty} u^t R_t$$

Removing the first term from the second summation:

$$(3)\ V = S_0 + \sum_{t=1}^{\infty} u^t CF_t + u(1+i)R_0 + \sum_{t=2}^{\infty} u^t(1+i)R_{t-1} - \sum_{t=1}^{\infty} u^t R_t$$

$$(4)\ V = S_0 + \sum_{t=1}^{\infty} u^t CF_t + \left[\frac{1+i}{1+j}\right] R_0 + \sum_{t=2}^{\infty} u^t(1+i)R_{t-1} - \sum_{t=1}^{\infty} u^t R_t$$

$$(5)\ V = S_0 + \sum_{t=1}^{\infty} u^t CF_t + R_0 - \left[\frac{j-i}{1+j}\right] R_0 + \sum_{t=2}^{\infty} u^t(1+i)R_{t-1} - \sum_{t=1}^{\infty} u^t R_t$$

Rearranging terms, and executing a change of variables on $t$ in the second summation:*

$$(6)\ V = [S_0 + R_0 + \sum_{t=1}^{\infty} u^t CF_t] - u(j-i)R_0 + \sum_{t=1}^{\infty} u^{t+1}(1+i)R_t - \sum_{t=1}^{\infty} u^t R_t$$

Then, combining the second and third summations:

$$(7)\ V = [S_0 + R_0 + \sum_{t=1}^{\infty} u^t CF_t] - u(j-i)R_0 - \sum_{t=1}^{\infty} u^t R_t \left[1 - \frac{1+i}{1+j}\right]$$

* This transformation is possible because the series converges to zero.

$$(8)\quad V = [S_0 + R_0 + \sum_{t=1}^{\infty} u^t CF_t] - u(j - i)R_0 - \sum_{t=1}^{\infty} u^t R_t \left[\frac{j - i}{1 + j}\right]$$

$$(9)\quad V = [S_0 + R_0 + \sum_{t=1}^{\infty} u^t CF_t] - u(j - i)R_0 - \sum_{t=1}^{\infty} u^{t+1} R_t (j - i)$$

Finally, bringing the $R_0$ term back inside the summation and executing a second transformation of the variable $t$:

$$(10)\quad V = [S_0 + R_0 + \sum_{t=1}^{\infty} u^t CF_t] - \sum_{t=1}^{\infty} (j - i) u^t R_{t-1}$$