A NOTE ON CALENDAR YEAR LOSS RATIOS

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Abstract

One important use of calendar year loss ratios is in the determination of rate changes. Two basic methods exist for calculating calendar year loss ratios. They are the standard calendar year loss ratio and the calendar year loss ratio by policy year contribution. This paper sets forth the mathematical definitions of these methods, examines the conditions under which the results equal those of a policy year or accident year approach, and examines the statistical variation of each method.

Introduction

Up until the early to mid 1970's, there was one basic method used to calculate calendar year loss ratios. This consisted of the paid losses plus change in loss reserves divided by the earned premium. At that point in time the National Council on Compensation Insurance (NCCI) introduced a new method of calculating calendar year loss ratios. This is referred to as calendar year loss ratios by policy year contributions. This calculation has been used by the NCCI in its rate filings since that time. The calendar year loss ratio is weighted 50%–50% with a policy year loss ratio in deriving the overall statewide rate change. However, no analysis has been presented as to why or if this procedure is superior. These are the questions examined herein.

Comparison of Average Results

The standard calendar year loss ratio on current benefit and rate level is

$$C_s = \sum_{i=0}^{-\infty} A_s \cdot (L_{i,1-i} - L_{i,-i})/P_s$$

where

C_s = standard calendar year loss ratio
A_s = factor to bring standard calendar year losses and premiums to current benefit and rate level
$L_{i,j}$ = reported incurred losses (includes a provision for IBNR) for policy year i evaluated at maturity j
P_s = calendar year earned premium.

It is well known that C_s will equal the ultimate accident year result if the *amount* of loss reserve adequacy has not changed over time.

The theoretical formula for the calendar year loss ratio by policy year contribution is:

$$C_p = \sum_{i=0}^{-\infty} A_i \cdot (L_{i,1-i} - L_{i,-i})/P_i$$

where

C_p = pure calendar year loss ratio by policy year contribution
A_i = factor to bring losses and premiums to a current benefit and rate level for policy year i
P_i = ultimate premium for policy year i

When put into this form it can be seen that C_p is really an estimate of the ultimate loss ratio for policy year (0) at the current benefit and rate level. The reserving method used in this formula relates developments in incurred losses between successive maturities to the earned premium for the particular policy year. By contrast, for policy years (-1) and (-2), in the NCCI rate filings, developments in incurred losses between successive maturities are related to the starting incurred loss value. Given this, one might question calling the result a calendar year loss ratio. However, the main purpose here is to examine under what conditions C_p gives an exact ultimate loss ratio. In Appendix I it is proven that C_p equals the ultimate policy year (0) on level loss ratio if the following two conditions hold:

(*i*) The ultimate on-level loss ratios for all policy years are equal.
(*ii*) The *percent* adequacy of the incurred losses for equal maturities is the same at successive policy years.

Hence, the standard calendar year approach is superior when the *amount* of incurred loss adequacy has not changed because it will then match the accident year loss ratio exactly. By contrast, the calendar year ratio by policy year contribution is more accurate when the *percent* of incurred loss adequacy has not changed since it will then match the policy year loss ratio exactly. In addition, for the policy year contribution method to be accurate, an additional condition must be imposed. We next examine the incurred loss adequacy conditions under which one method will be accurate and the other will not. These are set forth in Appendix III assuming an increasing premium volume. If premium volume is constant, then a constant amount adequacy will equal a constant percent adequacy. If premium volume is decreasing, then the low and high result would be interchanged.

The theoretical formula for the calendar year loss ratio by policy year contributions is not followed by the NCCI in its rate filings. The reason for this is that all loss developments past an 8th maturity are grouped together. The actual formula used by the NCCI is

$$C_n = \sum_{i=0}^{-7} A_i \cdot (L_{i,1-i} - L_{i,-i})/P_i$$

$$+ \sum_{i=-8}^{-\infty} A_{-8}/P_{-8} \cdot (L_{i,1-i} - L_{i,-i})$$

This formula is a hybrid of the standard calendar year loss ratio and the theoretical calendar year loss ratio by policy year contribution. In Appendix II it is shown that for this formula to provide the correct ultimate loss ratio, a constant percent incurred loss adequacy and on level loss ratio hold for maturities through 8. In addition, a constant amount incurred loss adequacy must hold after maturity 8. This would be expected in light of the conditions that underlie the components entering C_n.

Variance of Results

We next examine the statistical variance of the results under these two methods produced by random fluctuations in losses. It is shown in Appendix IV that the variance of C_p exceeds that of C_s when premiums are increasing, as has been the case for many years. This means that the pure calendar year loss ratio by policy year contributions will have larger swings from year to year than the standard calendar year loss ratio.

The reason for this is relatively simple. Theoretically, the same losses enter C_p and C_s. However, under C_p they are related to a smaller premium base and therefore have a larger variance. In practice, the actual losses entering may not be the same. This is because there are some companies that can report calendar year losses but are not able to split them into policy year components. Furthermore, it is relatively easy to show that Var (C_p) > Var (C_n) > Var (C_s).

Summary

The purpose of this paper is to compare the results of the calendar year loss ratio by policy year contribution and standard calendar year loss ratio calculations. In addition to the specific conclusions within, there is a universal one that can be drawn: No single ratemaking method can be best under all circumstances. The assumptions underlying each method have to be tested to see if they are met. If they are not, the extent of the deviation and the impact on the results need to be determined.

APPENDIX I

Derivation of Conditions Under Which Theoretical Calendar Year Loss Ratio by Policy Year Contributions Gives the Correct Result

We examine herein the conditions under which a pure calendar year loss ratio by policy year contribution will result in an unbiased result. An unbiased result is one in which all reserve adjustments for prior years cancel out. Hence, the loss ratio reflects only current underwriting conditions. The two necessary conditions are a constant on level ultimate loss ratio and a constant percent incurred loss adequacy for each year.

Let $L_{i,j}$ = incurred losses for policy year i evaluated at maturity j (These are the undeveloped incurred losses reported to the NCCI by individual companies. They include each company's own provision for case, reopened, and incurred but not reported loss reserves.)

P_i = ultimate premium for policy year i

$F_{i,j}$ = ratio of incurred losses evaluated at maturity j to ultimate incurred losses for policy year i

R_i = ultimate loss ratio for policy year i

A_i = factor to bring losses and premiums to a current benefit and rate level for policy year i

$L_{i,\infty}$ = ultimate incurred losses for policy year i

Maturity 1 is half a policy year, Maturity 2 is a just-completed policy year, etc. With the above definitions, we have:

$$L_{i,j} = F_{i,j} \cdot L_{i,\infty} \tag{1}$$

$$L_{i,\infty} = R_i \cdot P_i \tag{2}$$

$$L_{i,j} = F_{i,j} \cdot R_i \cdot P_i \tag{3}$$

The calendar year loss ratio by policy year contributions is:

$$\lim_{m \to -\infty} \sum_{i=0}^{m} A_i \cdot (L_{i,j+1} - L_{i,j})/P_i \tag{4}$$

where $i + j$ = constant, which because of the choice of indices above is 0.

$$\Rightarrow i + j = 0 \text{ or } j = -i \tag{5}$$

Substituting (3) and (5) into (4) we have:

$$\lim_{m \to -\infty} \sum_{i=0}^{m} A_i \cdot R_i(F_{i,1-i} - F_{i,-i})$$

if $A_i \cdot R_i = R'$ (constant on level loss ratio for all years)
(constant percent incurred loss adequacy) and $F_{i,j} = F'_j$

we have:

$$\lim_{m \to -\infty} R' \cdot \sum_{i=0}^{m} (F'_{1-i} - F'_i)$$

$$= R' \cdot \lim_{m \to -\infty} (F'_{1-m} - F'_0)$$

$$= R' \cdot (1 - 0) = R'.$$

APPENDIX II

Derivation of Conditions Under Which NCCI Calendar Year Loss Ratio by Policy Year Contributions Gives the Correct Result

In this Appendix we look at the conditions under which the NCCI calendar year loss ratio by policy year contribution will yield an unbiased result. We find that it is a combination of the conditions for the pure calendar year loss ratio by policy year contribution (Maturities 1 to 8) and the standard calendar year loss ratio (Maturities 8 and after).

$$C_n = \sum_{i=0}^{-7} A_i \cdot R_i \cdot (F_{i,1-i} - \mathrm{F}_{i,-i})$$

$$+ \lim_{m \to -\infty} \sum_{i=-8}^{m} \frac{A_{-8}}{P_{-8}} \cdot R_i \cdot P_i \cdot (F_{i,1-i} - F_{i,-i})$$

Let $F_{i,j} = F'_j$ for $i = 0$ to -8 (constant percent incurred loss adequacy)

$A_i \cdot R_i = R'$ for $i = 0$ to -8 (constant on level loss ratio)

$R_i \cdot P_i \cdot F_{i,j} = L_{i,\infty} + E_j$ for $i = -8$ to $-\infty$ (constant amount incurred loss adequacy)

where E_j = amount by which the maturity j incurred losses differ from the ultimate incurred losses.

Then $C_n = R' \cdot (F'_1 - F'_0 + F'_2 - F'_1 + \ldots + F'_8 - F'_7)$

$$+ A_{-8}/P_{-8} \cdot \lim_{m \to \infty} (E_9 - E_8 + E_{10} - E_9 + \ldots + E_{m+1} - E_m)$$

$$C_n = R' \cdot F_8' - A_{-8}/P_{-8} \cdot E_8$$

$$E_8 = -L_{-8,\infty} + R_{-8} \cdot P_{-8} \cdot F_{-8,8}$$

$$C_n = R' \cdot F_8' + A_{-8} \cdot L_{-8,\infty}/P_{-8} - A_{-8} \cdot R_{-8} \cdot F_{-8,8}$$
$$= A_{-8} \cdot R_{-8} = R'.$$

APPENDIX III

Comparison of Errors of Calendar Year Approaches Assuming Increasing Premium Volume

Theoretical Calendar Year Loss Ratio By Policy Year Contribution

Incurred Loss Adequacy	Constant Amount	Constant Percent
Excessive	Too Low	Exact
Inadequate	Too High	Exact

Standard Calendar Year Loss Ratio

Incurred Loss Adequacy	Constant Amount	Constant Percent
Excessive	Exact	Too High
Inadequate	Exact	Too Low

APPENDIX IV

Comparison of the Variances of Calendar Year Loss Ratio by Policy Year Contributions and the Standard Calendar Year Loss Ratios

Any type of loss ratio will include a certain amount of statistical variance due to random fluctuations in losses. The variances of the standard calendar year loss ratio and that of the pure calendar year loss ratio by policy year contributions are compared herein.

In addition to the definitions in Appendix I,

let C_p = calendar year loss ratio by policy year contributions
C_s = calendar year loss ratio calculated by standard methods
A_s = factor to bring standard calendar year losses and premiums to current benefit and rate level
P_s = standard calendar year premium
$D_{i,j}$ = difference in incurred losses for policy year i evaluated at maturities j and $j + 1$
$= L_{i,j+1} - L_{i,j}$

Then

$$C_p = \sum_{i=0}^{-\infty} A_i \cdot D_{i,-i}/P_i$$

$$\text{Var}\ (C_p) = \sum_{i=0}^{-\infty} \text{Var}\ (D_{i,-i})/(P_i/A_i)^2$$

assuming all the $D_{i,-i}$ are independent.

$$C_s = \sum_{i=0}^{-\infty} A_s \cdot D_{i,-i}/P_s$$

$$\text{Var}\ (C_s) = \sum_{i=0}^{-\infty} \text{Var}\ (D_{i,-i})/(P_s/A_s)^2$$

A number of items can be noted:

(*i*) Var $(D_{i,-i}) \geq 0$

(*ii*) For $i \leq -1$ it is almost certain that $P_i/A_i < P_s/A_s$ because of increasing premium volume.

(*iii*) Except in unusual cases, it is reasonable that:

$$P_s/A_s \approx (P_0/A_0 + P_{-1}/A_{-1})/2$$

(*iv*) It is reasonable to assume:

Var $(D_{0,0}) \leq$ Var $(D_{-1,1})$ since $D_{-1,1}$ includes reserve changes in addition to newly reported losses whereas $D_{0,0}$ includes only the latter.

(*v*) $1/(x + \epsilon) + 1/(x - \epsilon) > 1/(x/2)$ for $\epsilon \neq 0$

Given this it is easy to see that:

Var $(C_p) >$ Var (C_s)