DISCUSSION BY ALFRED O. WELLER

"The 1979 NCCI Remarriage Table" is a concise account of the construction of the remarriage table itself and corresponding revisions of the Unit Statistical Plan. The article is rich in actuarial concepts and sophisticated techniques. It is straightforward and well organized in its presentation of the extensive work underlying the 1979 NCCI Remarriage Table. This discussion deals with two aspects of this work: (1) the use of parametric models to understand insurance processes better and to overcome deficiencies in available data, and (2) standards for construction of Unit Statistical Plan tables.

The construction of the 1979 NCCI Remarriage Table employs actuarial techniques suited to the dearth of available data. Compared to the 1958 CSO Tables with 59 million life-years of exposure and 400,000 deaths, the 150,000 years of exposure and 2,000 remarriages for the 1979 NCCI Remarriage Table are few—so few that direct use of observed remarriage rates would have been inappropriate and inaccurate due to large sampling variation. For example, the observed remarriage rate for a forty-seven year old widow is 250% of the rate for a widow age forty-six (both women being widowed for four years). However, a larger sample is not possible.

In order to balance smoothness and fit properly (distinguish signal from noise), the 1979 NCCI Remarriage Table is derived from a continuous force of remarriage defined by nine parameters. These nine parameters are determined by minimizing a quadratic loss function. The expression for the force of remarriage is:

$$\begin{aligned}\mu^{(r)}_{[x]+t} &= \text{prompt component} \qquad + \text{ delayed component} \\ &= p_1\, e^{-p_2 x}\, t^{p_8-1}\, e^{-p_3 t} \qquad + \theta\,(t-p_7)\, p_4\, e^{-p_5 x}\,(t-p_7)^{p_9-1} e^{-p_6(t-p_7)}\end{aligned}$$

where $\theta(t - p_7)$ is zero for $t$ less than $p_7$ and unity otherwise. The superscript $r$ denotes remarriage, the variable $x$ the widow's age (nearest birthday) on being widowed, and the variable $t$ the duration of widowhood.

Using the fitted values of parameters $p_1, p_2, \ldots, p_9$, the derived remarriage rates describe a general pattern of increasing rates during early widowhood followed by decreasing rates thereafter as the duration of widowhood increases. By analyzing derivatives (Exhibit I) it can be determined quickly that the force of remarriage increases during the first 1.6 years of widowhood, decreases after 5.3 years of widowhood, and attains a maximum (dependent upon age) between

these two durations. Such a conclusion would not be as apparent from alternative methods of graduation—inspection of individual rates of remarriage would be necessary to determine if the statement were true. In this respect, the parametric approach facilitates sharper insight than does graduation using difference equations, moving averages, or graphical methods. This advantage is demonstrated also by the analysis of trends in remarriage rates in Section VI of the article.

In a sense the work of casualty actuaries always has concerned parameters and their estimation. After all, rates are derived from estimates of such well-known parameters as pure premiums and expense ratios. If pure premiums are all that is directly observed, then intervening variables (i.e., entities somewhere between cause and observation) such as claim frequency and loss severity escape attention. An increase in rates might be attributed to an increase in pure premiums, but the reasons for the increase in pure premiums cannot be identified without further research. With respect to any individual phenomenon such as loss ratio or remarriage rate, the use of intervening variables can serve several purposes including (a) definition of current limits of knowledge (ability to explain causes and predict effects), (b) identification of hypotheses to test the accuracy of knowledge, (c) summary of observations and effects, and (d) improved communication, as when two psychoanalysts discuss a libido. Most importantly, by identifying intervening variables in today's models, we build a link to causal influences and gradually extend the realm of actuarial science. For instance, for the 1979 NCCI Remarriage Table, the nine intervening variables define prompt and delayed components, streamline the analysis of trends, and identify testable hypotheses with respect to updating the Table.

The article proceeds logically from the analysis of remarriage data to the construction of revised Unit Statistical Plan tables. These tables have been incorporated in the Unit Statistical Plan to apply to loss valuations made on or after November 1, 1980. The revised Unit Statistical Plan tables differ from their predecessors in two key respects.

1. Sources.
   A. The 1979 NCCI Remarriage Table replaces graduated remarriage rates for the U.S. Employees' Compensation System (Table 7 of Actuarial Study No. 55 of the Social Security Administration).
   B. The 1969–71 U.S. Decennial Life Tables for Total Females and Total Population replace the 1959–61 U.S. Decennial Life Tables for White Females and Total Population, respectively.

2. Format.
   A. A select and ultimate format replaces tabular values keyed to the middle of three years used in experience rating.
   B. Tables have been introduced for lump sum remarriage awards (dowries) and procedures for survivorship benefits have been refined.

The central role of the Unit Statistical Plan (USP) in workers' compensation insurance sets the structural standards for the revised tables. For certain legal obligations, the Unit Statistical Plan assigns a standard tabular value, representative of a cohort of claimants, to corresponding losses for purposes of manual ratemaking, experience rating and retrospective rating. USP tables are used to value further benefits until the process of claim settlement establishes actual values for individual claims. These values generally will differ from USP tables. For example, a lump sum settlement or a structured settlement can close claims with present values for future benefits different from USP tables. When a claim is settled, its actual liquidated value is reported. Thus, USP tables are designed to value consistently a legal obligation in light of current conditions without specific assumptions regarding claims settlement.

The "1979 NCCI Remarriage Table" is an apt solution to a difficult problem. Through the Unit Statistical Plan it contributes to the accurate and equitable pricing of workers' compensation insurance. Further, it is a thorough account of valuable actuarial work. We are grateful to Phil Heckman for presenting it to the Casualty Actuarial Society.

# EXHIBIT I

## ANALYSIS OF DERIVATIVES

Given: $\mu^{(r)}_{\{x\}+t}$ = force of remarriage

$$= p_1 e^{-p_2 x} t^{p_8-1} e^{-p_3 t} + \theta\{t - p_7\}\, p_4 e^{-p_5 x}(t - p_7)^{p_9-1} e^{-p_6(t-p_7)}$$

Assuming $\theta\{t - p_7\} = 1$ and taking derivatives:

$$\frac{d}{dt}\mu^{(r)}_{x+t} = p_1 e^{-p_2 x}\left[(p_8 - 1)\, t^{p_8-2}\, e^{-p_3 t} - p_3\, t^{p_8-1}\, e^{-p_3 t}\right]$$

$$+ p_4 e^{-p_5 x}\left[(p_9 - 1)\,(t - p_7)^{p_9-2}\, e^{-p_6(t-p_7)} - p_6(t - p_7)^{p_9-1}\, e^{-p_6(t-p_7)}\right]$$

$$= p_1 e^{-p_2 x} t^{p_8-1} e^{-p_3 t}\left(\frac{p_8 - 1}{t} - p_3\right)$$

$$+ p_4 e^{-p_5 x}(t - p_7)^{p_9-1} e^{-p_6(t-p_7)}\left(\frac{p_9 - 1}{t - p_7} - p_6\right)$$

Terms outside of large parentheses are positive for given parameter values. The derivative is set equal to zero and corresponding values of $t$ determined.

Prompt component: $t = (p_8 - 1)/p_3$
$= (2.313 - 1)/.7786$
$= 1.686$

Delayed component: $t = p_7 + (p_9 - 1)/p_6$
$= .583 + (1.809 - 1)/.171$
$= 5.314$

It follows that for $t < 1.6$ the force is increasing and for $t > 5.3$ the force is decreasing. A maximum occurs between these values.