THE OPTIMAL USE OF DEPOPULATION CREDITS IN THE PRIVATE PASSENGER AUTO RESIDUAL MARKET

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Abstract

This paper describes the depopulation credits that are available in the private passenger auto residual market plans of many states and develops two models that can be used by an insurer to optimize the use of those credits. Each model represents an extreme case, with the real world falling somewhere between the two extremes. An example of the use of the models also is included, as is some discussion of how to measure the benefit of optimally using depopulation credits.

INTRODUCTION

The private passenger automobile residual market plans of many states contain depopulation credit provisions. Under these provisions an insurer receives a fixed number of dollars credit against its residual market premium quota for each dollar of premium written voluntarily on specific categories of risks. In this paper, a risk that qualifies for such a credit is called an eligible risk. A company can use the depopulation credits on eligible risks to reduce its participation in the residual market. If the residual market in a particular state is consistently underpriced, this reduction in a company's quota can decrease its residual market underwriting losses. Thus, total underwriting income will be maximized by using the credits if the eligible risks written voluntarily by a company produce underwriting losses that are less than the reduction in underwriting losses attributable to the reduction in the residual market quota.

The problem, therefore, is determining which eligible risks to accept. The answer depends on the difference between total expected underwriting losses with the eligible risks written by the insurer in the voluntary market and total expected underwriting losses without them. This paper presents two models that give upper and lower bounds for the maximum loss and loss expense ratio that can be incurred on an eligible risk written voluntarily. The bounds are computed so as to maximize the total underwriting income of the company.

DEPOPULATION CREDITS AND RESIDUAL MARKET QUOTA

In most states an insurer can earn credits by voluntarily insuring the following types of risks:

Class 2: Youthful male principal operator/youthful male household resident operator.

Over 65: Operators aged 65 and over.

Keep-out: Any risk who is a previously uninsured resident of a compulsory insurance state.

Take-out: Any risk who is removed from the auto insurance plan (residual market) and written in the voluntary market.

Keep-out and Take-out credits are usually two-for-one credits. That is, two dollars of credit against the company's residual market premium quota is given for every dollar of premium written voluntarily on these risks. Class 2 and Over 65 credits are usually dollar-for-dollar credits. It is possible to "double-up" on credits. For example, a particular risk may be eligible for both Class 2 and Take-out credits. The company that voluntarily insures this risk gets both credits.

The formula used to determine each company's share of the residual market premium[1] is given below, based on the following notation:

$j = 1, 2, 3, 4$ denote the types of credits, i.e., $j = 1$ for Take-out credits, $j = 2$ for Keep-out credits, $j = 3$ for Class 2 credits and $j = 4$ for Over 65 credits;

$X_{j,i,y}$ denotes the premium eligible for type of credit j that is voluntarily written in year y by company i;

N_j denotes, for type of credit j, the number of credit dollars given per dollar of eligible premium written voluntarily;

$P_{i,y}$ denotes the voluntary market exposure penetration of company i in year y, i.e., the ratio of voluntary car-years insured by company i to the total number of car-years written in the voluntary market;

T_y denotes the total residual market premium to be assigned in the year y.

Take-out and Keep-out credits written in the current year reduce the current year's quota, while Class 2 and Over 65 credits take effect two years later. Thus the total credit in dollars for company i in year y is given by

$$K_{i,y} = \sum_{j=1}^{2} N_j X_{j,i,y} + \sum_{j=3}^{4} N_j X_{j,i,y-2} .$$

The credit in dollars for all companies combined is given by

$$K_y = \sum_i \sum_{j=1}^{2} N_j X_{j,i,y} + \sum_i \sum_{j=3}^{4} N_j X_{j,i,y-2}$$

The unadjusted quota for company i in year y is

$$q_{i,y} = \begin{cases} 0 \text{ if } P_{i,y-2}(T_y + K_y) - K_{i,y} \leq 0 \\ \dfrac{P_{i,y-2}(T_y + K_y) - K_{i,y}}{T_y} & \text{otherwise.} \end{cases}$$

$P_{i,y-2}$ appears in the calculation of the quota for the year y because the most current data available for calculating P_i is two years old.

As long as the proper time relationships are kept in mind (i.e., the penetration ratio $P_{i,y}$ is calculated using two year old data, and some credits apply in the

[1] "Quota Determination and Quota Fulfillment," Automobile Insurance Plans Services Office, New York. The formula given applies in every state except Florida, Hawaii, Missouri, Texas, Maryland, Massachusetts, New Hampshire, North Carolina, and South Carolina.

year in which they are written while others apply two years later) we can, for simplicity, drop the year subscript. Thus we have

$$q_i = \begin{cases} 0 \text{ if } P_i(T + K) - K_i \leq 0 \\ \dfrac{P_i(T + K) - K_i}{T} & \text{otherwise.} \end{cases}$$

If $P_i(T + K) - K_i \geq 0$ for all i (that is, no company writes a number of credits greater than the number necessary to eliminate its residual market assignment), then

$$q_i = \frac{P_i(T + K) - K_i}{T} \quad \text{for all } i \text{, and}$$

$$\begin{aligned} \sum_i q_i &= \sum_i \left(\frac{P_i(T + K) - K_i}{T}\right) \\ &= \left((T + K) \sum_i P_i - \sum_i K_i\right) \Big/ T \\ &= \frac{T + K - K}{T} \\ &= 1 \end{aligned}$$

If $P_{i'}(T + K) - K_{i'} < 0$ for some i', then $q_{i'} = 0$ since negative quotas are not allowed and $\Sigma_i q_i > 1$. In this case the positive quotas are divided by $\Sigma_i q_i$ so that the adjusted quotas sum to one. The quotas are further adjusted for over-assignments and under-assignments made in previous years.

Note that, since the current year's quota q_i is calculated using a penetration ratio P_i that is two years old, the current year's quota is unaffected by the volume of business written in the current year. However, the volume of business written in the current year will affect the future residual market quota.

Adjustments for over- and under-assignments will be ignored in this paper because they do not affect a company's overall participation; these adjustments only alter the allocation of that participation by year. Also, it is assumed that $P_i(T + K) - K_i \geq 0$ for all companies.

Under these assumptions the quota for company i is simply $(P_i(T + K) - K_i)/T$. Note that the quota is a function of the total credits written by all insurers, and therefore its value for company i depends on the actions of other insurers. This fact complicates the analysis because it is not always possible to anticipate these actions. This problem is most pronounced when credits are offered for the first time, such as Keep-out credits offered in conjunction with the passage of a compulsory insurance law. In these instances, no history is available upon which to anticipate the actions of the other companies.

Company behavior is the feature which distinguishes the two models presented here. Each model assumes an extreme case, with the real world falling somewhere between the two extremes.

In both models it is assumed that all eligible risks must be written either voluntarily or through the residual market. In neither model does the acceptance criterion consider investment income.

FIRST MODEL

Let g_i = break-even loss and loss expense ratio for company i;
g_t = break-even loss and loss expense ratio for residual market business;
r_j = expected loss and loss expense ratio at voluntary rates of risks eligible for credit j;
r_t = expected loss and loss expense ratio of residual market business.

The variables g_i, g_t, r_j, and r_t will be called loss ratios, although it is understood that they include loss adjustment expense.

The first model assumes that every eligible risk is written voluntarily by some company. It does not matter, however, which company or companies chooses to write this business. Also, we assume that all companies charge the same rates. Thus, K is constant, but the K_i may vary. Hence, an increase in $K_{i'}$ for some i' results in a decrease in $K_{i''}$ for some i''.

Consider a risk that is eligible for type of credit j. Let the premium for this risk, if written voluntarily, be denoted by X_j.

If this risk is not written voluntarily by company i, then the residual market quota for company i is given by

$$(P_i(T + K) - K_i)$$

If this risk is voluntarily written by company i, then the residual market quota is given by

$$(P_i(T + K) - K_i - N_jX_j).$$

Thus, the reduction in the quota due to writing this risk is N_jX_j. Since residual market business produces an underwriting loss of $r_t - g_t$, the reduction in the underwriting loss of residual market business due to writing this risk is

$$N_jX_j\ (r_t - g_t).$$

The expected voluntary underwriting loss incurred on this risk is equal to

$$X_j(r_j - g_i).$$

This risk should be written voluntarily whenever

$$(r_j - g_i) \leq N_j(r_t - g_t),$$

which is equivalent to

$$r_j \leq g_i + N_j(r_t - g_t).$$

Thus, for each type of credit we have expressed the maximum expected loss ratio that minimizes the net underwriting loss as a function of one variable—the expected loss ratio, r_t, of the residual market business.

The assumption that all companies charge the same premiums is not crucial; approximate equality is sufficient. The second model will not require this assumption.

SECOND MODEL

In contrast to the first model, which assumed that any eligible risk rejected by company i would be voluntarily insured by some other company, the second model assumes that any eligible risk rejected by company i must obtain insurance through the residual market. Formally, it is assumed that $\bar{K} = \Sigma_{m \neq i} K_m$ is constant.

In the real world some of the rejected individuals will be voluntarily insured by other companies, and some will not. Clearly then, the real world may be approximated by a linear combination of the two models.

Consider a risk eligible for type of credit j that, if voluntarily written by company i, would produce premium D_i and a loss and loss expense ratio r_j. If this risk were written through an assigned risk plan, the premium would equal FD_i, where F is the assigned risk rate level factor for this risk relative to company i's rates. The loss ratio on this risk would then equal r_j/F.

If company i voluntarily writes this business, then its assigned risk quota is reduced because of the credits that it earns.

Company i's quota without this credit is

$$P_i \left(T + \bar{K} + \sum_j N_j X_{j,i} \right) - \sum_j N_j X_{j,i} \; .$$

Company i's quota with this credit is

$$P_i \left(T + \bar{K} + \sum_j N_j X_{j,i} + N_j D_i \right) - \sum_j N_j X_{j,i} - N_j D_i \; .$$

The reduction in company i's quota due to writing this business voluntarily is

$$-P_i N_j D_i + N_j D_i \; .$$

The expected underwriting loss incurred on this risk if written voluntarily is

$$D_i(r_j - g_i).$$

Thus, the net loss attributable to the decision to voluntarily write this risk is given by

$$L_1 = D_i(r_j - g_i) + (P_i N_j D_i - N_j D_i)(r_t - g_t).$$

If the eligible risk were not voluntarily written, then the assigned risk quota for the company would not change, but the size of the assigned risk pool would increase. Company i's share of the additional loss is

$$L_2 = P_i F D_i[(r_j/F) - g_t].$$

Whenever $L_1 < L_2$, overall losses can be reduced by voluntarily writing this risk. The inequality will be satisfied when

$(r_j - g_i) + (N_j P_i - N_j)(r_t - g_t) < P_i F[(r_j/F) - g_t]$, which is equivalent to

$$r_j < \frac{g_i - P_i F g_t - (N_j P_i - N_j)(r_t - g_t)}{(1 - P_i)} \; .$$

As in the first model, we have expressed the maximum loss ratio r_j that optimizes use of the credits as a function of one variable—the expected loss ratio for residual market business.

It is interesting to compare $\bar{r}_j$, the maximum loss ratio at which the risk should be written voluntarily, as calculated using the two models. For the first model we have

$$\bar{r}_{j,1} = g_i + N_j(r_t - g_t).$$

For the second model we have

$$\bar{r}_{j,2} = \frac{g_i - P_iFg_t - (N_jP_i - N_j)(r_t - g_t)}{1 - P_i}\,.$$

The difference is:

$$\bar{r}_{j,2} - \bar{r}_{j,1} = \frac{P_i(g_i - Fg_t)}{1 - P_i}\,.$$

Thus, the two models are equivalent when $g_i = Fg_t$, and the first model gives an upper bound when $g_i - Fg_t$ is negative.

EXAMPLE

The following fictitious example illustrates the two models. Suppose a company wants to determine whether or not it should voluntarily write a particular risk that is eligible for a two-for-one credit ($N_j = 2$). The company's breakeven loss and loss expense ratio, g_i, is .70; its breakeven loss and loss expense ratio for residual market business, g_t, is .75; the assigned risk rate level factor for this risk is 1.2; and it insures 5% of the voluntary market ($P_i = .05$). The total residual market loss experience is given in Table 1.

TABLE 1

ASSIGNED RISK EXPERIENCE

	Earned Premium	Incurred Loss And Loss Expense Ratio
1975	$20,000,000	95.0%
1976	30,000,000	102.0
1977	35,000,000	98.0
1978	45,000,000	106.0
Total	130,000,000	101.2

Take-out and Keep-out credits written in the current year reduce the current year's quota, while Class 2 and Over 65 credits reduce the quota two years later. Thus, it is necessary to estimate the assigned risk loss ratio that is expected to prevail either in the current year or two years later, depending on what type of credit is being considered. It is not possible to estimate this loss ratio with a great deal of precision. Nevertheless, by looking at the total plan experience in recent years and considering trends and the promptness with which assigned risk rate changes have been approved and implemented in the past, one can formulate expectations of the likely range of the assigned risk loss ratios in the near future. Continuing with the example, suppose the assigned risk loss ratio is expected to fall in the range 95.0% to 105.0% for the next several years.

Using the low end of this range, 95.0%, in the first model, we get

$$\begin{aligned} \bar{r}_{j,1} &= g_i + N(r_t - g_t) \\ &= .70 + 2(.95 - .75) \\ &= 1.100. \end{aligned}$$

The second model gives

$$\begin{aligned} \bar{r}_{j,2} &= \frac{g_i - P_i F g_t - (NP_i - N)(r_t - g_t)}{1 - P_i} \\ &= \frac{.70 - (.05)(1.2)(.75) - (2(.05) - 2)(.95 - .75)}{.95} \\ &= 1.089. \end{aligned}$$

The optimal value of $\bar{r}_j$ in the real world is probably between the above two values, say $\bar{r}_j = 1.095$. Thus, the company should voluntarily write this risk if the risk's expected loss ratio at voluntary rates is less than 109.5 percent.

How much money will a company save by following this rule? The savings can be estimated roughly as follows. If the expected loss ratio of a risk is less than g_i, then that risk will be written voluntarily whether or not it is eligible for a credit. Thus, those risks written because of the credit will have expected loss ratios lying in the interval from g_i to $\bar{r}_j$. If we assume that the loss ratios are uniformly distributed over this interval, then the expected loss ratio for the group is $(\bar{r}_j + g_i)/2$. Suppose 2000 risks written because of the credit have an average premium of \$250 and an average expected loss ratio of 89.75 percent. Then the expected underwriting loss on this group is

$$2000 \times \$250\ (.8975 - .70) = \$98{,}750.$$

The reduction in the residual market expected underwriting loss is given by

$$2000 \times \$250 \times 2\ (.95 - .75) = \$200{,}000.$$

Thus, the use of the credits has reduced overall underwriting losses by an estimated $101,250.

The attractiveness of the credits increases as the expected assigned risk loss ratio increases and as the number of credits per dollar of premium increases.

SUMMARY

Both models have ignored the fact that a decision to write eligible risks voluntarily will increase the company's assigned risk quota because its voluntary business will increase.

The models developed here may not apply to the nine states that do not use the quota formula described above. Also, different types of credits may be offered in these states. In Massachusetts, for example, territory credits are available for voluntarily writing risks in Boston. However, it is possible to develop models for use in these nine states.

The optimal use of depopulation credits will not dramatically reduce underwriting losses in most states, but in those states with the largest and most underpriced residual markets, their use can be significant. If regulators expand the use of credits in an effort to depopulate the residual market, then the benefit of using those credits optimally will increase.