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SCALE ADJUSTMENTS TO EXCESS EXPECTED LOSSES

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Abstract

Loss distributions underlying increased limits factors are usually based on countrywide data, as state/class information is generally regarded as too sparse for this purpose. Yet the state/class average severities may be reliable, and the countrywide distributions can be adjusted for differences in this average, for example, by assuming that all losses move in the same proportion.

Such a scale change model can yield state/class increased limits factors; however, in many cases a single factor can be derived to adjust the countrywide increased limit factor incremental differences for the state or class average severity differential. This serves to appreciably simplify the application of the scale change model.

A complication arises in the case of Commercial Automobile Combined Single Limits: besides the BI and PD average severity differentials, the relative frequencies of BI and PD losses vary by state/class, leading to differences in the overall BI–PD mix of losses, and thus in the relationship between basic limits and excess losses. With reasonable additional assumptions, the required change in CSL increased limit factor incremental differences can again be represented by a constant factor which, in this case, reflects mix as well as scale differences.

Introduction

Science may perhaps be distinguished from technology by the use of models versus the use of techniques. A model in this context is a conceptual framework used to help order and comprehend events; a technique is a procedure used to produce a result, and may or may not have a conceptual foundation. "Actuarial science" in many instances amounts to a collection of techniques with either unstated models or no models at all at their foundation and, thus, would probably be better labeled "actuarial technology." This technology has been relatively successful in many cases; nonetheless, in order to develop a true actuarial science, the conceptual framework behind our techniques needs to be formulated more explicitly.

A fairly satisfactory approach to the scientific method, advocated by Popper [5], is to advance the simplest models not contradicted by the existing evidence, where one criterion of simplicity is the ease by which the model could be falsified by future adverse observation. Thus, seeing a red cardinal leads to the hypothesis "all cardinals are red," rather than "all but three cardinals are red," because the former could be more easily falsified, i.e., only one non-red cardinal would be needed.

In the domain of loss severity, a simple model would be that all risks have the same claim size distribution. However, this model has often been falsified by the observance of different average loss values for different lines, and for different states and classes within lines.

In many cases, the simplest non-falsified hypothesis is that the observed differences in average severity apply uniformly to all losses, that is, that the shapes of the loss size distributions under investigation are the same and only scale differs. This hypothesis might apply to states or classes for a given line of insurance or for different time periods for a given state/line combination.

This scale change model has been widely used in casualty insurance, for example, by Finger [2] and Miccolis [3]. Its use is not restricted to severity distributions; for instance, interim updates of Table M by the National Council on Compensation Insurance have often implicitly applied this model to aggregate claim distributions. There is also evidence (e.g., see [6]) that the scale change hypothesis does *not* hold for variation over time for long tailed casualty business, i.e., there is more to trend than simple monetary inflation.

In this paper the mathematics of the scale change model is developed explicitly, and the implications of this model for excess expected losses are explored.

Increased Limits Factors

When comparing one state or class to another, a scale difference would affect the base rates charged. But what is less obvious is that it should also affect the increased limits factors applied to those rates.

For example, consider an automobile claim cost distribution expressed in dollars versus the same distribution expressed in Swiss francs. The increased limits factor for 100,000 over 10,000 would differ depending on what currency were being referred to. In fact, if a dollar were worth two Swiss francs, the factor for \$100,000 over \$10,000 would be the same as the factor for SF 200,000 over SF 20,000.

The same concept can apply to simple inflation. If the U.S. experiences a 20% inflation between 1982 and 1984, the factor for \$120,000 over \$12,000 in 1984 dollars would be the factor for \$100,000 over \$10,000 in 1982 dollars. In a similar vein, if every loss in California were to cost exactly 1.5 times as much as the same loss in Louisiana, then the ratio of expected losses limited to \$150,000 to expected losses limited to \$15,000 in California would be the same as the ratio at limits of \$100,000 and \$10,000 in Louisiana, assuming everything else remains the same.

The excess loss costs then will be doubly affected by a scale change, first due to the increase in primary expected losses and second due to a change in the ratio of excess to basic losses. Quantifying this effect will be discussed below.

Mathematical Development

The simplest form of scale change, as illustrated by the Swiss franc example, is when one random variable is a scalar multiple of another; e.g., $Y = kX$. Then $E(Y) = kE(X)$, $\sigma_Y = k\sigma_X$, etc. Interestingly enough $CV(Y) \equiv \sigma_Y \div E(Y) = \sigma_X \div E(X) = CV(X)$, i.e., the scale change does not affect the coefficient of variation. Also the skewness will not be affected. This partially expresses the idea that the shape of the distribution is not affected by a scale change.

Now consider the cumulative distribution functions F and G for X and Y respectively. By definition $G(a) = \Pr[Y \leq a] = \Pr[kX \leq a] = \Pr[X \leq a \div k] = F[a \div k]$. Note that the transformation $Y = kX$ corresponds to the inverse transformation $G(a) = F(a \div k)$ on the distribution functions.

In some cases of interest, the random variables under study will be defined on different spaces, e.g., accidents in Louisiana versus accidents in California, so they cannot always be thought of as multiples of each other. In these cases, the relationship between distribution functions can be used to specify what is meant by a scale change.

Thus, if for random variables X and Y with distribution functions F and G there is a constant k such that $G(a) = F(a \div k)$, then Y will be called a scale change of X with constant k.

It is easy to show from the chain rule that the probability density functions f and g satisfy $g(a) = f(a \div k) \div k$, and the relationships between the moments discussed above for $Y = kX$ are readily derived from this definition.

Excess Losses

To calculate the effect on excess loss costs, it is very useful to refer to the concept of limited mean for a loss severity distribution. Intuitively this is the average loss size for losses limited to some specific amount a. For a distribution function F with density f the average severity limited to a will be denoted by $S_F(a)$ and is defined as

$$S_F(a) = \int_o^a t\, f(t)\, dt + \int_a^\infty a\, f(t)\, dt.$$

By change of variable in integration, it is easy to show that $S_G(a) = kS_F(a \div k)$ when the conditions in the above definition of scale change hold.

Now, when F is the severity distribution function, the expected loss increased limits factor for limit a over limit b (i.e., b is basic limits) is just $I_F(a;b) = S_F(a) \div S_F(b)$. This is because multiplying the numerator and denominator by the expected number of losses yields losses limited to a divided by basic limits losses.

Thus, for a scale change, we have $S_G(a) = kS_F(a \div k)$ and $S_G(b) = kS_F(b \div k)$, so $I_G(a;b) = S_G(a) \div S_G(b) = S_F(a \div k) \div S_F(b \div k) = I_F(a \div k;b \div k)$. This implies that the expected loss increased limits factor after an upward scale change is the factor for the scaled down limits before the change, and vice versa for a downward change. Dividing both sides of this last relation by $I_F(a;b)$ and simplifying yields $I_G(a;b) \div I_F(a;b) = I_F(a \div k;b \div k) \div I_F(a;b) = I_F(b;b \div k) \div I_F(a;a \div k)$. This gives a factor for adjusting the increased limits factor at a over b for X to that for Y which depends just on the distribution function F. With basic limits b this factor will in general depend on the increased limit a.

Simplifying the Application

The formula above shows, for example, how, under the scale change model, the increased limits factors for a state relate to the state to countrywide average severity differential and the countrywide increased limits factors. However, having separate tables of increased limits factors for each state could prove unwieldy. It turns out that, for many severity distributions, a single factor can be derived for each state, independent of limits, that will closely approximate the state to countrywide ratio of the difference between increased limits factors. The closeness of the approximation will usually depend on how wide a range of limits is chosen, as will be seen below. Such factors could be used to calculate state excess charges directly from countrywide, without having state tables of increased limits factors.

To facilitate discussion, define the excess layer factor $L_F(c,a;b)$ to be the difference in increased limits factors $I_F(c;b) - I_F(a;b)$ where again b is the basic limit. Then L_F represents the ratio of layer expected losses to basic limits losses. Now what is the adjustment factor for a scale change with constant k? This is the ratio $L_G(c,a;b) \div L_F(c,a;b)$ which can be expressed as

$$\frac{S_F(c \div k) - S_F(a \div k)}{S_F(c) - S_F(a)} I_F(b,\ b \div k).$$

This can be proved by expressing everything in terms of the S_F and S_G functions.

Now the ratio $[S_F(c \div k) - S_F(a \div k)]/[S_F(c) - S_F(a)]$ does not depend too strongly on a and c when both limits are in a reasonable range. Thus, approximating this ratio by a single factor d_F will allow the adjustment factor $L_G(c,a;b) \div L_F(c,a;b)$ to be expressed as $d_F I_F(b;b \div k)$ independently of limits a and c.

To explore the range of variability for the ratio $[S_F(c \div k) - S_F(a \div k)]/[S_F(c) - S_F(a)]$ it will be calculated for some specific loss severity distributions.

First consider the Pareto distribution $F(x) = 1 - (x \div r)^{-q}$; $r,q > 0$. If losses follow this distribution, even if only for the range of interest (for example, \$100,000 to \$1,000,000), we can calculate the ratio as follows:

$$f(x) = (q \div r)(x \div r)^{-(q-1)}$$

$$S_F(c) - S_F(a) = \int_a^c r^q q t^{-q}\, dt + c\left(\frac{c}{r}\right)^{-q} - a\left(\frac{a}{r}\right)^{-q}$$

$$= \frac{r^q}{q-1}\left(a^{-(q-1)} - c^{-(q-1)}\right);$$

and so

$$S_F(c \div k) - S_F(a \div k) = \frac{r^q}{q-1}\left((a \div k)^{-(q-1)} - (c \div k)^{-(q-1)}\right)$$

$$= \frac{r^q}{(q-1)k^{q-1}}\left(a^{-(q-1)} - c^{-(q-1)}\right).$$

Thus,

$$\frac{L_G(c,a;b)}{L_F(c,a;b)} = \frac{S_F(c \div k) - S_F(a \div k)}{S_F(c) - S_F(a)}\, I_F(b;b \div k)$$

$$= k^{1-q} I_F(b;b \div k).$$

Thus, in this case, the adjustment does not depend at all on the limits a and c.

The above argument requires that losses follow the Pareto at least in between $a \div k$ and c. This has been reported informally to be a reasonably close but not exact form for several lines of casualty losses in working excess layers. Distributions giving closer fits will generally not have this property (of allowing an exact single factor adjustment for scale independently of layer), but if losses are close to the Pareto it is reasonable to believe that a single factor can be found which is close to proper for each layer.

Consider next the lognormal distribution. A CV of 4 with an expected loss size of \$5,000 might represent a typical casualty line.

The ratios $[S_F(c \div k) - S_F(a \div k)]/[S_F(c) - S_F(a)]$ for a scale change (i.e., $k = 1.25$) calculated as above are shown for several excess layers in the top half of Appendix 1. (Recall that after the scale change a distribution with a mean of \$6,250 and a CV of 4 results.) The ratios are computed as follows. First, the lognormal parameters $\mu = 7.10059$ and $\sigma = 1.68322$ were derived from the equations $\sigma^2 = \ln(1 + CV^2)$ and $\mu = \ln E(x) - \sigma^2/2$. To calculate $S_F(a)$ the formulas $F(a) = \Phi((\ln a - \mu)/\sigma)$ and $\int_0^a t f(t)dt = E(x)\,\Phi[((\ln a - \mu)/\sigma) - \sigma]$ from [1] were used, where Φ is the standard normal cumulative distribution function.

As the table shows, most factors lie in the 120% to 135% range. This may or may not be a small enough range to consider constant, depending on the uses to which this analysis is to be put. A factor of 1.275 applied uniformly to all excess layer expected losses would seem to be a reasonable figure.

The shifted Pareto distribution discussed in [4] is also treated in Appendix 1. That distribution function is of the form

$$F(x) = 1 - \left(\frac{\beta}{x+\beta}\right)^{\delta};\ \beta,\delta > 0.$$

For the shifted Pareto:

$$E(x) = \frac{\beta}{\delta - 1}\ (\delta > 1),\ CV^2 = \frac{\delta}{\delta - 2},\ \delta > 2,$$

and

$$\int_o^a xf(x)dx = \frac{\beta}{\delta - 1}\left[(1 - \left(\frac{\beta}{a+\beta}\right)^{\delta}\left(\frac{\beta + a\delta}{\beta}\right)\right].$$

From this it can be shown that

$$S_F(a) = \frac{\beta}{\delta - 1}\left(1 - \left(\frac{\beta}{a+\beta}\right)^{\delta-1}\right).$$

The same mean and CV were assumed as in the lognormal case, and $\beta = 85{,}000/15$ and $\delta = 32/15$ were derived by matching moments. In this case, the ratios measured remained in the 126% to 128% area, as shown in Appendix 1. Thus, the constant ratio approximation is better for the shifted Pareto than for the lognormal. This distribution has generally been a more successful model for casualty loss severities than has the lognormal and is commonly used in increased limits ratemaking.

To obtain the final factor for adjusting excess layer factors for a scale change, the factor $I_F(b;b \div k)$ is needed for the basic limit b. Taking $b = 25{,}000$ gives 1.06 for the lognormal and 1.04 for the Pareto by applications of the above methods. The final adjustments to apply to excess layer factors are thus $1.06 \times 1.275 = 1.35$ for the lognormal and $1.04 \times 1.27 = 1.32$ for the shifted Pareto.

Calculating k

The factor k is the ratio of the average severity for Y to the average severity for X. One case of interest is where X is countrywide and Y is state loss size.

Then state excess factors can be calculated by applying a constant adjustment to the nationwide values by the method above. Estimating k is somewhat complicated by the fact that the average severities are available only for basic limits, i.e., $t = S_G(b) \div S_F(b)$ is known rather than $k = E(Y) \div E(X)$. Since $S_G(b) = kS_F(b \div k)$, it is possible to solve for k if the nationwide distribution F is known; e.g., $t = S_G(b) \div S_F(b) = kS_F(b \div k) \div S_F(b)$ or $tS_F(b) = kS_F(b \div k)$ gives an equation that can be solved for k if F and t are given.

For example in the Pareto case above ($\beta = 17{,}000/3$, $\delta = 32/15$),

$$S_F(b) = \frac{\beta}{\delta - 1}\left(1 - \frac{\beta}{b + \beta}\right)^{\delta - 1}$$

$$= 5000\,(1 - ((3b/17{,}000) + 1)^{-17/15}).$$

Suppose $t = 1.2$, i.e., the state in question has basic limit severity 20% above nationwide. Then with basic limits of \$25,000 the equation $tS_F(b) = kS_F(b \div k)$ becomes $1.2 \times 5{,}000\ (1 - ((75{,}000/17{,}000) + 1)^{-17/15}) = k \times 5{,}000\ (1 - ((75{,}000/k17{,}000) + 1)^{-17/15})$, i.e., $1.023 = k(1 - (1 + 4.412/k)^{-1.133})$. This can be solved iteratively to yield $k = 1.248$. Thus, with the given nationwide distribution and a state offset at basic limits of 1.2, a state scale factor of 1.248 results. Thus, from the above, an adjustment factor of approximately 1.32 should apply to excess layer factors.

A similar procedure could be used for other distributions. The calculation is somewhat easier for the Pareto because of the closed form for $S_F(a)$.

Scale Parameters

Often one of the parameters of a distribution can be used to effect a scale change. Such parameters are, therefore, called scale parameters. Beta for the Pareto and mu for the lognormal are examples. Thus, if F is the Pareto distribution function and $G(a) = F(a \div k)$, it is easy to see by direct substitution that G is a Pareto with just a different beta. A similar result can be derived for the lognormal and for many other distributions.

Combined Single Limits

For Commercial Automobile Combined Single Limits (CSL) an additional state offset to nationwide factors could be made to reflect the particular BI–PD mix in the state. For example, for the average state, \$5,000 limits PD losses might run about 80% of \$15,000 limits BI losses. Many companies have access to data of this type. One unpublished study available to the author indicated that for different states this percentage could fall anywhere in the range of 50% to 150%, or even outside this range, consistently by state. Possible explanations for this spread include differentials among the states in urban-rural mix and tort atmosphere. Property damage losses could reasonably be expected to predominate in urban areas where crowded conditions force lower speeds but lead to more encounters, while bodily injury may predominate on rural roads. So-called tort consciousness or propensity to sue could also lead to more bodily injury losses incurred in some areas.

Since BI and PD have quite different loss severity distributions, at least in average value, their mix could markedly affect the CSL severity distribution for the state.

Again the offset for state excess layer factors can be used as a single factor independent of layer for a suitable range of layers, but additional approximations are involved. Since there are numerous concepts to keep track of, some notation is necessary. Let B, P, and C refer to Bodily Injury, Property Damage, and Combined Single Limits, respectively, so $S_P(a)$ is Property Damage severity limited to a, $I_C(a;b)$ is the expected loss Combined Single Limits increased limits factor for a over b, and $L_B(c,a;b)$ is the Bodily Injury excess layer factor for the layer a to c with basic limits b. An asterisk will denote the concept for a state under consideration while non-asterisked variables will denote nationwide. A constant $t_C = L_{C*}(c,a;b) \div L_C(c,a;b)$ is sought where t_B and t_P, the similar constants for BI and PD, have already been determined.

The first approximation needed for this is $N_CS_C(a) = uN_BS_B(a) + vN_PS_P(a)$, where N is the expected number of losses for each category. This expression says the CSL limited losses can be approximated as a linear combination of the BI and PD limited losses.

At a given limit the CSL expected losses should be less than the sum of BI and PD expected losses at the same limit, because the CSL limit applies to the BI plus PD total rather than to each separately. The constants u and v are discount factors to reflect this. An example is provided by the so-called single limit rule, which for many limits and states is equivalent to $u = 1$, $v = .91$. A more compact form of the above expression arises if we introduce the notation $D(a)$ for the total expected loss dollars limited to a, i.e., $D(a) = NS(a)$. Then $D_C(a) = uD_B(a) + vD_P(a)$ is the approximation noted.

Now,

$$L_C(c,a;b) = (D_C(c) - D_C(a)) \div D_C(b)$$

$$= \frac{u\,(D_B(c) - D_B(a)) + v\,(D_P(c) - D_P(a))}{u\,D_B(b) + v\,D_P(b)}$$

$$= \frac{uL_B(c,a;b)\,D_B(b) + v\,L_P(c,a;b)\,D_P(b)}{u\,D_B(b) + v\,D_P(b)}$$

$$= w\,L_B(c,a;b) + (1 - w)\,L_P(c,a;b)$$

where $w = u\,D_B(b) \div (u\,D_B(b) + v\,D_P(b)) = 1 \div (1 + rv \div u)$ where r is the ratio of PD to BI losses at limit b.

Now u and v are reasonably believable as constants among states; that is, even though BI and PD constitute different percentages of the CSL losses from one state to the next, the same percentages of BI and PD losses at a given limit are eliminated by the CSL approach. Nonetheless, w will vary by state due to the varying BI–PD mix r^*.

Thus $L_{C^*} = w^*L_{B^*} + (1 - w^*)\,L_{P^*}$, suppressing the $(c,a;\mathrm{b})$, and t_C, the factor being sought, may be expressed as

$$t_C = L_{C^*} \div L_C = \frac{w^*L_{B^*} + (1 - w^*)\,L_{P^*}}{w\,L_B + (1 - w)\,L_P}$$

$$= \frac{w^*}{w}\,\frac{(L_{B^*} \div L_B) + ((1 - w^*) \div w^*)(L_{P^*} \div L_P)(L_P \div L_B)}{1 + ((1 - w) \div w)(L_P \div L_B)}$$

$$= \frac{1 + rv \div u}{1 + r^*v \div u}\,\frac{t_B + t_P(r^*v \div u)\,L_P \div L_B}{1 + (rv \div u)\,L_P \div L_B}$$

In the last formula, only the nationwide ratio $L_P(c,a;b) \div L_B(c,a;b)$ depends on c and a. The second approximation is to use a constant to represent this ratio. In a test intended to be representative (see Appendix 2) this ratio was found to vary from .142 for the layer from \$750,000 $= a$ to \$1,000,000 $= c$ to .190 for the layer from \$100,000 $= a$ to \$200,000 $= c$, where $b =$ \$25,000. The actual ratio t_C varies less than this because the term containing $L_P \div L_B$ is added to a larger term in both numerator and denominator.

Thus to recapitulate,

$$t_C = \frac{1 + rv \div u}{1 + r^*v \div u}\,\frac{t_B + t_P(r^*v \div u)\,q}{1 + (rv \div u)\,q}$$

where r and r^* are the nationwide and state ratios of PD to BI expected losses at basic limit b, v and u are constants used to linearly approximate CSL expected losses at any limit by BI and PD losses at the same limit, and q is a point approximation of nationwide PD excess layer factors over BI excess layer factors at the same limits. An example is discussed in Appendix 2.

Final Notes

It should be noted that the single factor approximations discussed above do not apply to increased limits factors. Rather they apply to the excess layer factors which are differences between two increased limits factors. If the approximation is good in a range that includes basic limits, then the adjustment factor could be applied to the part above 1.0 of a given increased limits factor, because that would be the excess layer factor for the layer from basic limits to the given limit. Even if this approach is not reasonable, an adjustment to the increased limits factor is still in order, but a constant factor adjustment will not be appropriate.

It should also be emphasized that the above formulas relate only to the expected loss portion of the premium. Loss expense and risk load are also important elements of excess charges that ought to be considered when applying the scale change model to excess pricing. Loss expense can probably be handled in a way consistent with the above constant adjustment factor approach.

It is questionable whether the appropriate risk load for a layer is the difference between ground up risk loads at the layer limits, and, thus, the loading approach should tie in closely with the specific application being considered.

One area for further study is the determination of the single limit discounts u and v. Respective values of 1.0 and .91 reflect current conventions, but as single limit occurrence distributions become available, better measurements should be possible.

Finally, the scale model, while a good working hypothesis in many cases, is not universally applicable. It is probably better than the identical distribution model in instances where consistent average value differences have been observed; but where there is reason to suspect that shape differences may exist, they should be investigated. In many lines, variation between classes (e.g., heavy trucks versus vans) is an area where shape differences in severity distributions may be found.

APPENDIX 1

EFFECT OF 25% SCALE CHANGE ON LAYER SEVERITIES

$$\frac{S_F(c \div k) - S_F(a \div k)}{S_F(c) - S_F(a)}$$

Lognormal Distribution

$E(x) = 5{,}000$ $\quad CV = 4$

$\mu = 7.10059$ $\quad \sigma = 1.68322$

a = Lower Layer Limit (000)	c = Upper Layer Limit (000)						
	200	250	300	400	500	750	1000
100	1.198	1.205	1.211	1.219	1.224	1.231	1.235
200		1.241	1.248	1.259	1.267	1.278	1.284
250			1.260	1.271	1.280	1.292	1.299
300				1.281	1.291	1.304	1.312
400					1.307	1.323	1.332
500						1.335	1.346
750							1.371

Shifted Pareto Distribution

$E(x) = 5{,}000$ $\quad CV = 4$

$$\beta = \frac{85{,}000}{15} \qquad \delta = \frac{32}{15}$$

a = Lower Layer Limit (000)	c = Upper Layer Limit (000)						
	200	250	300	400	500	750	1000
100	1.260	1.262	1.263	1.265	1.265	1.266	1.267
200		1.271	1.272	1.274	1.274	1.276	1.276
250			1.274	1.275	1.276	1.278	1.278
300				1.277	1.278	1.279	1.279
400					1.279	1.280	1.281
500						1.281	1.282
750							1.283

APPENDIX 2

COMBINED SINGLE LIMIT EXAMPLE

For nationwide PD severity the Pareto distribution $F_P(x) = 1 - (1 + x/\beta)^{-\delta}$ is used with $\beta = 335.023$ and $\delta = 1.35$. For BI a split Pareto severity distribution is used, i.e.,

$$F(x) = \begin{cases} 1.40935\ (1 - (1 + x/\beta)^{-\delta}) & x \leq 4{,}000 \\ 1 - .5913\ (1 + x/\beta)^{-\delta} & x \geq 4{,}000 \end{cases}$$

where $\beta = 5171.797$ and $\delta = 1.20848$. These parameters were chosen to be a realistic representation of the data once available. From the single limits rule $v \div u$ was taken at .91, and from a large unpublished sample a nationwide ratio of PD to BI \$25,000 losses of $r = .8$ was estimated. Suppose for a given state $t_B = 1.2$, $t_P = 1.0$, and $r^* = .6$ have been calculated. Then

$$t_C = \frac{1 + (.8)(.91)}{1 + (.6)(.91)} \times \frac{1.2 + (.6)(.91)q}{1 + (.8)(.91)q}\ .$$

By definition $q(c,a;b) = L_P(c,a;b) \div L_B(c,a;b)$ and $L(c,a;b) = (S(c) - S(a)) \div S(b)$. For PD,

$$S_P(a) = \frac{335.023}{.35}\left(1 - \left(\frac{a}{335.023} + 1\right)^{-.35}\right)$$

by the Pareto rule. A somewhat more complicated formula holds for S_B due to the split Pareto used. After some calculation, q is found to range from .142 to .190 for layers (a and c) in the \$100,000 to \$1,000,000 range. Selecting $q = .166$ yields $t_C = 1.287$. With q's of .142 and .190, t_C's of 1.294 and 1.280 arise respectively.

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